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SUBJECT: THE THEORY AND SOME APPLICATIONS OF THE HYDRAULIC
RAM AS APPLIED TO COUNTERCURRENT ION EXCHANGE

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INTRODUCTION

The principle of the hydraulic ram and its application to continuous countercurrent ion exchange was presented in ORNL-CF-53-1-213. Since that time the principle has appeared to be useful in other applications as well and this, among other reasons, has made it desirable to investigate the ram action on a theoretical basis.

SUMMARY

It has been shown on theoretical grounds that resin pressure in a hydraulic ram is distributed according to the equation

$$\ln \frac{\frac{4k}{D} P_{R1} + \frac{dp_L}{dz} - \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]}{\frac{4k}{D} P_{R2} + \frac{dp_L}{dz} - \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]} = \frac{4k}{D} (z_2 - z_1)$$

In a countercurrent ion exchange section of a column the critical fluidizing condition is

$$-\frac{dp_L}{dz} = \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]$$

which agrees very well with experimental evidence. The resin pressure distribution in a countercurrent section resembles, in many respects, that of the ram section.

In the ram section there is a maximum in P_{R2} as L is varied and D , Δp_L and P_{R1} are held constant. This fact has been made the basis of a graphical design procedure for a unit consisting of a ram and a countercurrent section.

A method and analytical procedure have been developed for determining the dynamic coefficient of sliding friction between resin-water and the walls of the column involved.

Illustrative applications of theory to experimental data are given.

THEORY

Consider an elemental section of a vertical tower of uniform cross section packed with ion exchange resin and water. Imagine that the resin moves smoothly as a dense bed down through the tower and that water moves downward through the resin bed. Then the force balance over the fluid phase in the elemental section dZ is

$$f \frac{\pi D^2}{4} dZ + \alpha \frac{\pi D^2}{4} (P_L + dP_L) = \alpha \frac{\pi D^2}{4} P_L + \alpha \frac{\pi D^2}{4} \frac{g}{g_c} \rho_L dZ \quad (1)$$

or

$$\frac{f}{\alpha} = - \frac{dP_L}{dZ} \quad (2)$$

where

$$P_L = P_L - \frac{g}{g_c} \rho_L Z \quad (3)$$

and

f = frictional drag, force/ft³ bed, due to water acting on resin and vice versa.

D = diameter of tower

Z = lineal distance measured down from top of tower

α = ft³ of water/ft³ of dense resin bed

P_L = total water pressure at any depth, Z , in bed

p_L = dynamic water pressure at any depth, Z , in bed

g = acceleration due to gravity

g_c = the proportionality constant in Newton's law stating the proportionality between force and rate of change of momentum.

ρ_L = density of water

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The force balance over the dense resin phase is

$$P_L (1 - \alpha) \frac{\pi D^2}{4} + P_R \frac{\pi D^2}{4} + \frac{\pi D^2}{4} f dz + \frac{g}{g_c} \rho_R \frac{\pi D^2}{4} dz =$$

$$\pi D k P_R dz + (P_L + dP_L)(1 - \alpha) \frac{\pi D^2}{4} + (P_R + dP_R) \frac{\pi D^2}{4} \quad (4)$$

or

$$\frac{dP_R}{dz} + \frac{4k}{D} P_R + (1 - \alpha) \frac{dP_L}{dz} - f - \frac{g}{g_c} \rho_R = 0 \quad (5)$$

where

$$p_R = P_R - \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] z \quad (6)$$

and

k = coefficient of sliding friction for wet dense resin bed moving over the wall of the tower

P_R = total resin pressure referred to cross area of bed and not to the cross area of the resin alone

p_R = dynamic resin pressure

ρ_R = bulk density of dense resin bed

Perhaps the best way to define the concept of resin pressure or pressure in the resin phase is to think in terms of a device consisting of a U-tube equipped with a porous piston placed in the bottom of the U and having one face connected through a long coil spring to the wall of the tube. Upon filling the tube with water the water passes through the pores of the piston and around the spring and there is no net force to cause compression or extension of the spring. If now one adds resin to the leg which does not contain the spring the resin will build up against the porous piston causing compression of the spring in accord with the head of resin. If one installs side arms protected by screens and pumps water down through the resin phase the spring will compress to an even greater extent. The degree to which the spring is compressed is a measure of the total force exerted on the piston

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and this, in conformity with prior stipulation, is related to the pressure through the area of the piston.

Since $\frac{dp_L}{dz}$ is constant for a given flow through a section of column of uniform cross section one can eliminate f from 5 using 2 and rearrange equation 5 to read

$$\int_{P_{R1}}^{P_{R2}} \frac{\frac{4k}{D} P_R + \frac{dp_L}{dz} - \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]}{dz} = - \frac{4k}{D} \int_{Z_1}^{Z_2} dz \quad (7)$$

Integration of 7 gives

$$\ln \frac{\frac{4k}{D} P_{R1} + \frac{dp_L}{dz} - \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]}{\frac{4k}{D} P_{R2} + \frac{dp_L}{dz} - \frac{g}{g_c} [\rho_R - (1 - \alpha) \rho_L]} = \frac{4k}{D} (Z_2 - Z_1) \quad (8)$$

which establishes the basic relation between resin pressures at any two points in a column of dense bed and uniform cross section with downflow.

If the resin flows down and the water flows up through the resin instead of down and if Z is measured from the bottom of the bed instead of the top, then in the previous treatment one need only change the sign in front of the g/g_c and $4k/D$ terms in equations 1, 4, 5, 7 and 8. In this case the critical fluidizing flow rate through the bed is that one for which the frictional lifting force equals the weight of the dense bed taking into account buoyancy of water, i.e.

$$f = \frac{g}{g_c} \rho_R + (1 - \alpha) \frac{dp_L}{dz} \quad (9)$$

In upflow

$$P_L = P_L - \frac{g}{g_c} \rho_L (1 - Z) \quad (10a)$$

$$p_R = P_R - \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] (1 - Z) \quad (10b)$$

where

l = total column height

Equations 2, 9 and 10a may be combined to give

$$- \frac{dp_L}{dz} = \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] \quad (11)$$

In this case equation 8 modified for upflow reduces to

$$P_{R2} = P_{R1} e^{+ \frac{4k}{D} (Z_2 - Z_1)} \quad (12)$$

Equation 11 is of interest primarily because it establishes a method for computing the dynamic quantity, $- dp_L/dz$ from the static values of ρ_R , ρ_L and α . Based on the static data for 20 mesh Dowex 50 presented in ORNL-CF-50-9-80, it is found from equation 11 that

$$- \frac{dp_L}{dz} = .0931 \text{ psi/ft}$$

This agrees very well with the experimental value, .093, read from Figure 1 to follow.

If one lets

$$a = 4k \frac{L}{D} \quad (13)$$

$$c = \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] \quad (14)$$

$$L = Z_2 - Z_1$$

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then for the ram section equation 8 may be written

$$P_{R2} = \frac{L}{a} \left(C - \frac{P_{L2} - P_{L1}}{L} \right) + \left[P_{R1} - \frac{L}{a} \left(C - \frac{P_{L2} - P_{L1}}{L} \right) \right] e^{-a} \quad (15)$$

If P_{R1} , D , and $(P_{L2} - P_{L1})$ are held constant while P_{R2} is differentiated with respect to L one finds for

$$\frac{dP_{R2}}{dL} = 0$$

that

$$\frac{P_{L2} - P_{L1}}{L} = \frac{a \left(\frac{a}{L} P_{R1} - C \right) e^{-a}}{\left[1 - (1 + a) e^{-a} \right]} = \frac{dp_L}{dz} \quad (16)$$

In other words, P_{R2} is a maximum for a given D , P_{R1} , and dp_L/dz when "a" satisfies equation 16. This is as it should be because P_{R2} increases at first with increases in L due to the increasing drag force of water on resin. With further increases in L , however, P_{R2} goes through a maximum and then decreases due to increasing effects of resin frictioning against the column wall.

The derivative of p_L may be eliminated between equations 15 and 16 to show that the maximum value of P_{R2} is

$$P_{R2} = P_{R1} e^{-a} + (1 - e^{-a}) \left[\frac{LC}{a} + \frac{L \left(C - P_{R1} \frac{a}{L} \right) e^{-a}}{1 - (1 + a) e^{-a}} \right] \quad (17)$$

which for P_{R1} equal zero becomes

$$\frac{P_{R2}}{L} = \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] (1 - e^{-a}) \left[\frac{1}{a} + \frac{e^{-a}}{1 - (1 + a) e^{-a}} \right] \quad (18)$$

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In the case of P_{R1} equal to zero equation 15 for the countercurrent section, i.e., for liquid upflow and resin downflow, becomes

$$\frac{P_{R2}}{L} = \frac{1}{a} \left\{ \frac{dp_L}{dz} + \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] \right\} (1 - e^{+a}) \quad (19)$$

Equations 16, 18 and 19 are plotted in Figures 2, 3 and 4.

Pressure drop data (see D. L. Foster, Secret Notebook #2767) for upflow through a 20 mesh Dowex 50 bed are plotted in Figure 1. The data were obtained by running water up through 28' of an annular bed contained between 1" and 4" I.D. pipes. The pressure drop over the resin bed was measured and the dynamic pressure drop was calculated for each flow rate. The bed length was also measured at each flow rate and divided into the dynamic pressure drop to obtain the value of dp_L/dz plotted.

The shape of the plot in Figure 1 is worth noting. In the text Richardson and Coulson, "Chemical Engineering", Vol. II, pp 24-25, a similar plot is given for granular solids in general. It shows that the linear plot eventually goes through a maximum above the fluidizing velocity and prior to the region of solids entrainment. It is evident in Figure 1 that this state of affairs was never obtained because data were not taken sufficiently far above the critical fluidizing velocity.

To make Figure 1 complete, one really should also measure the pressure drop in downflow particularly at the higher pressure drops where the pressure drop of a dense packed bed and a fluidized one should be distinctly different.

If one anticipates running a slurry through a bed, then Figure 1 should be determined using the slurry in question.

MEASUREMENT OF THE COEFFICIENT OF FRICTION, k

In order that the previous theory be useful it is necessary that the numerical value of k be known for the particular system consisting of solvent, resin and material of construction used in the column wall. One way of measuring k involves circulating a dense resin bed down through a vertical column of uniform cross section and suspending in this bed a thin long flat plate of the wall material in question. The plate is suspended by a wire attached to a scale for continuously monitoring the force exerted, which, of course, will be increased by the frictional drag of resin over

FIGURE 2

Plot of $\frac{P_{R2}}{L} = \frac{1}{a} \left\{ + \frac{g}{g_c} \left[\phi_R - (1-a) \phi_L \right] + \frac{dp_L}{dz} \right\} [1 - e^{-a}]$

for Countercurrent Section

$a = \frac{4k}{D} L; P_{R1} = 0$

FIGURE 4

Plot of $-\frac{dp_L}{dz} = \frac{\frac{E}{e_c} (\rho_R - (1 - \alpha)\rho_L) e^{-a}}{1 - (1 + a)e^{-a}}$

for the Hydraulic Ram

$P_{R1} = 0$

$a = 4\pi k \frac{L}{D}$

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the plate, the liquid drag being negligible. The plate need be long for purposes of increasing the accuracy of the measurement. The plate must be thin to make negligible the effects of resin and water phase pressures on the top and bottom ends of the plate.

For purposes of evaluating the results the previous theory must be extended to take into account that during the measurement the resin frictions against the plate as well as the column wall. In this case the counterpart of equation 8 is

$$\ln \frac{a' P_{R1} + b'}{a' P_{R2} + b'} = a' (Z_2 - Z_1) \quad (20)$$

where

$$a' = \frac{4k}{\pi D^2} [\pi D + 2w] \quad (21)$$

$$b' = \frac{dp_L}{dz} - \frac{g}{g_c} \left[\rho_R - (1 - \alpha) \rho_L \right] \quad (22)$$

w = width of suspended plate

1, 2 = subscripts now refer to upper and lower ends of the plate

If

F = force exerted on scale

M = mass of plate

then

$$F = \frac{g}{g_c} M + \int_{Z_1}^{Z_2} 2wkP_R dz \quad (23)$$

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If in equation 20 one uses P_R for P_{R2} and Z for Z_2 and substitutes the result in 23 and integrates one finds that

$$F = \frac{g}{g_c} M + \frac{2wk}{a'} \left[b' (Z_2 - Z_1) + (P_{R1} - \frac{b'}{a'}) (1 - e^{-a'(Z_2 - Z_1)}) \right] \quad (24)$$

If one selects Z_1 at the top of the resin bed where

$$P_{R1} = 0$$

then one need measure only w , F , M and $\frac{dp_L}{dZ}$ in order to calculate k from equation 24. Since $\frac{dp_L}{dZ}$ is constant along a column of uniform cross section, one may calculate $\frac{dp_L}{dZ}$ this slope from the formula

$$\frac{dp_L}{dZ} = \frac{P_{L2} - P_{L1}}{Z_2 - Z_1} \quad (25)$$

The foregoing theory has been applied to some rather crude measurements involving water, 20 mesh Dowex 50, and a cadmium plated plate. The value of k in this case is a sort of average value for the glass and cadmium surfaces and has been estimated to be

$$k = 0.06$$

DESIGN OF CONTINUOUS COUNTERCURRENT ION EXCHANGE COLUMN

A schematic of a continuous ion exchange column is shown in Figure 5. The two contracted sections are ram sections, each one acting to prevent fluidization in a countercurrent section below it. The two sections equipped with manometers are isolation sections, the manometers being employed to insure no liquid flow in these sections.

The length and diameter of the isolation sections are arbitrary since all that is involved here is the development of sufficient sensitivity in the manometers. From the standpoint of resin inventory the size of these sections would be kept to a minimum.

Fig. 5 Schematic of Continuous Column

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The remainder of the design of a column involves the design of two units each consisting of a hydraulic ram and its countercurrent section immediately below it. For each unit the values of P_{R2} for the ram and countercurrent section are equal. Since P_{R2} increases in both sections as P_{R1} increases the best choice for P_{R1} is

$$P_{R1} = 0$$

Thus, Figures 1-4 should be applicable to the design problem in the case of Dowex 50 ion exchange resin. To illustrate this point we will now undertake to design a typical unit consisting of a ram and a countercurrent section.

Let's assume that it is required to design a countercurrent section which is 2 ft long by 4" I.D. and is to operate at three times the fluidizing velocity. Based on Figure 1

$$3 \cdot 0.00233 = 0.00699 \text{ gpm/in}^2$$

and

$$-\frac{dp_L}{dz} = 0.38 \text{ psi/ft}$$

Assuming $k = .06$ one has that

$$a = \frac{4 \cdot .062}{1/3} = 1.44$$

From Figure 2

$$\frac{P_{R2}}{L} = .8 \text{ psi/ft}$$

Thus

$$P_{R2} = 1.6 \text{ psi}$$

The ram section design is based on Figure 3 and the fact that

$$P_{R2 \text{ ram}} = P_{R2 \text{ countercurrent section}}$$

The choices are summarized in the first four columns of Table 1. In the fifth column the ratio of liquid flow rates are given. This ratio is based on the formula

$$\text{Flow Ratio} = \frac{(QD^2)_{\text{countercurrent section}}}{(QD^2)_{\text{ram section}}}$$

Q and D are known for the countercurrent section. D for the ram is read from column 4 of Table 1. Q for the ram is determined by first reading the pressure gradient from Figure 4 and then Q from Figure 1.

TABLE 1
SPECIFICATIONS

a	$\frac{P_{R2}}{L}$ psi ft	L_{ram} , ft	D_{ram} , ft	$\frac{\text{Flow}}{\text{Ratio}}$, %
.02	9.3	.17	2.0	.82
.1	1.7	.91	2.1	
.2	.86	1.8	2.2	
.4	.40	3.9	2.3	
.6	.25	6.2	2.4	2.54
.8	.18	8.7	2.6	
1.0	.14	11.	2.7	
1.2	.11	14.	2.8	3.96

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One reason for using the foregoing example was that the 2' x 4" counter-current section was actually operated at three times fluidizing velocity with a ram about 2.5 ft x 4 in. Thus, it is obvious that the D in Table 1 are too large and the flow ratios are too small. This is due to the crude value used for k. Apparently k for Dowex 50 resin, water and glass approximates .009 more closely than it does .06. The computational procedure outlined remains unaffected, however. All that need be changed is the value of k. Figures 1-4 do not even involve k.

CALCULATION OF PRESSURE DROP INVOLVED IN THE PULSE CYCLE OF A SEMI-CONTINUOUS COLUMN

A schematic of a semi-continuous column is shown in Figure 6. The expanded sections are the column sections in which ion exchange normally takes place. The interconnecting pipes provide a path for resin flow during the pulse cycle. During the pulse cycle the valve in Figure 6 is positioned as shown and the pump circulates water down through the 8" column and around the loop to be withdrawn at a point above the valve and returned to the pump to repeat the circuit. As the water flows around the loop through the resin bed it tends by friction to drag the bed along with it.

There are two phenomena which occur in the pulse cycle that are unique. The first phenomenon is the disappearance of resin pressure at any point in the circuit where the bed fluidizes. Resin pressure can only exist when the resin particles are in contact to transmit pressure. Since there is no screen or equivalent to confine the resin at the uppermost resin level in the loop and since liquid and resin flow are cocurrent it is possible for fluidization to occur during the pulse cycle.

The second phenomenon is the "slip water" phenomenon. In spite of the fact that the resin moves during the pulse cycle the water always moves faster to maintain the drag force responsible for moving the resin in the first place. The difference between the water flow rate and the equivalent dense resin bed flow rate is the slip water flow rate. In an ideal system where buoyant and frictional effects are absent the slip water flow rate would be zero, i.e., the liquid and resin phases move as a homogeneous mass.

Returning now to Figure 6, one might suspect from the theory that since the uppermost level of the resin is unconfined the resin bed will move as a fluidized bed in the pulse cycle rather than as a dense bed. This is due to the fact that to lift a bed at all the fluidizing velocity must be exceeded. For a confined bed the flow rate would have to be higher to lift the bed because resin frictional drag against column wall must be overcome as well as the weight of the resin. Since fluidizing occurs at the lower flow rate for unconfined beds one would expect fluidization and loss of the

Fig. 6 Schematic Illustrating the Pulse Cycle of the Semi-Continuous Contactor

resin pressure phenomenon during the pulse cycle. As a consequence of this observation one should be able to calculate from Figure 1 alone the minimum overall pressure drop (across the pump) necessary to move the resin in the pulse cycle of Figure 6.

Tables 2 and 3 illustrate the calculation for Figure 6 based solely on Figure 1 and neglecting elbows in the lines.

A system modeled after Figure 6 was actually run in the laboratory. The calculated pressure drop is of the right order of magnitude.

The most impressive conclusion from the calculations is that 92.1% of the pressure drop occurs in the interconnecting piping and not in the columns proper.

TABLE 2
SPECIFIC PRESSURE DROPS DURING PULSE CYCLE

Pipe or Column Size, In.	$Q,$ gpm/in ²	$-\frac{dp_L}{dz}$ (Fig. 1) psi/ft
3"	$(\frac{12}{3})^2 \cdot .0023 = .037$	3.15
4"	$(\frac{12}{4})^2 \cdot .0023 = .020$	1.5
8"	$(\frac{12}{8})^2 \cdot .0023 = .0052$	.26
12"	.0023	.093

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TABLE 3
TOTAL PRESSURE DROP DURING PULSE CYCLE

Pipe or Column Size, In.	Total Length, Ft	Δp , psi
3"	4	12.6
4"	22	33.
8"	12	3.1
12	8.5	.8
Total		49.5 psi

In order to maintain a Δp of 49.5 psi, "slip water" will have to pass through the bed at the rate of

$$\pi \frac{(12)^2}{4} \cdot .0023 = .259 \text{ gpm}$$

To obtain the total pulse water flow rate (gpm) one must add to the slip water the dense resin bed flow rate.

A final point to be noted has to do with the possibility of entrainment occurring in the 3" and 4" lines connecting to the top of the 12" column in Figure 6. For this purpose the Stokes free settling velocities have been calculated for various resin particle sizes and tabulated in Table 4. It is obvious that Q for a 3" line in Table 2 is about 35 times too small for a 20 mesh particle to be entrained.

TABLE 4

STOKES FREE SETTLING VELOCITIES IN WATER AT ROOM TEMPERATURE

Resin Particle Size, Mesh	Q, $\frac{\text{gpm}}{\text{in.}^2}$
10	5.07
20	1.29
28	.657
60	.113
100	.0400