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September 17, 1968

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Prepared for U.S. Atomic Energy Commission under contract no. W-7405-Eng-48

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STATIC COMPRESSION OF RELATIVISTIC ELECTRON RINGS*

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I. Introduction

The principle of "collective ion acceleration"¹ discovered by the late V. Veksler is to accelerate ions trapped in a relativistic electron ring. By accelerating the electrons either by an external electric field or by expansion and axial acceleration of the ring in a static magnetic field it is in principle possible to impart to the ions an energy higher than the energy the electrons are accelerated by the ratio $(M/\gamma m)$ of the ion mass to the electron mass.

The process is practically realized in three steps.

An Astron type linear induction accelerator accelerates an electron beam of a few hundred amperes and an energy of a few MeV in short pulses of a few hundred nsec each. There are two such accelerators in existence with the following parameters:

LRL, Livermore	Dubna, USSR
Electron energy (MeV) 3.8	1.5
Beam current (A) 400	200
Pulse length (nsec) 300	200

In the second step the electrons are injected and trapped in a time varying magnetic field. Subsequently are compressed by a factor of 5 to 8. In the process the major and minor radius of the ring are compressed proportionally to $(B_0/B_f)^{1/2}$ where B_0, B_f are the initial and final value of the magnetic field during the compression phase. At the same time the electron energy, γ , expressed in rest mass units, is also increasing proportionally to $(B_f/B_0)^{1/2}$. The second step has been successfully accomplished in Dubna, USSR, where a 1.5 MeV ring has been compressed by a factor of 8 to a 12 MeV, 5 cm, 0.2 cm major

*Work performed under the auspices of the U.S. Atomic Energy Commission.

and minor radius respectively with a circulating current of 2000 A^2 . The ring is slightly ionized to approximately 1% neutralization. Further it has been observed that the ring maintains its integrity² during the 50 μsec the magnetic field remains constant following the compression phase. Experiments to trap and compress the ring have just started at LRL, Livermore using the Astron accelerator.

In the third step the ring should be extracted from the mirror field where it is trapped and subsequently accelerated electrically or magnetically to high energy. No extraction of the ring has been reported as yet.

In this note we shall deal with the second phase or step i.e., the formation and compression of the ring. A lot of theoretical work was done in this country on this subject.³

In my opinion in the second step it is attempted to accomplish two objectives at the same time:

1. To compress the ring in order to increase its electrostatic field which holds the ions, thus reducing the length required for subsequent acceleration to given energy.
2. To increase the electron energy in order to provide better stability conditions.

The second objective has two drawbacks. It reduces the mass ratio ($M/\gamma m$) by the compression ratio and it requires a time varying magnetic field which practically makes the compressor a betatron with variable orbit radius. If an operation at 60 pps is required the cost of the compressor can be formidable, while a betatron with constant radius should be much less expensive. Therefore it appears to me that a separation of the two functions is more practical from every aspect. Consequently a method is required to compress the electron ring without changing its energy in order to accomplish the first objective i.e., to increase substantially the electrostatic field of the ring. A method of compressing the ring in a static magnetic field at constant electron energy is described in the following section.

II. Compression of an Electron Ring in a Static Magnetic Field.

The two most known ways to compress or expand a charged particle orbit trapped in a magnetic field are:

1) Adiabatic expansion or compression. Charged particles trapped in a vacuum field and oscillating between two mirror regions conserve magnetic moment provided that their orbit radius is small in respect to the radius of curvature of the magnetic lines. Then

$$(p_{\perp}^2 / B) = \text{const} \quad (\text{II.1})$$

where $p_{\perp}$ is the component of the mechanical momentum transverse to the magnetic lines.

Since the radial momentum can be neglected i.e., $p_r \ll p_{\theta}$ and

$$p_{\theta} = \frac{e}{c} Br \quad (\text{II.2})$$

we find

$$Br^2 = \text{const} \quad (\text{II.3})$$

Since the canonical angular momentum is a constant of the motion it follows that the mechanical angular momentum is conserved, or

$$rp_{\theta} = \text{const} \quad (\text{II.4})$$

$$\frac{r_o}{r_f} = \sqrt{\frac{B_f}{B_o}} \quad (\text{II.4a})$$

$$\frac{p_{\theta f}}{p_{\theta o}} = \sqrt{\frac{B_f}{B_o}} \quad (\text{II.4b})$$

All the above equations are well known in the literature and are cited here for easy reference and comparison with what follows.

If a ring of electrons is moving in a magnetic field under conditions of adiabatic expansion or compression it preserves its integrity as long as axial focusing is provided by a gradient in the magnetic field. The field index n of the magnetic field

$$n = \frac{R \partial B_z}{B \partial r} \quad (II.5)$$

must be negative in the radial direction in order to be positive, thus focusing, in the axial direction and it should be larger than n_e which is the effective field index of the electron beam, namely

$$n_e = \frac{I}{8500} \left(\frac{R}{a} \right)^2 \cdot \gamma^{-3} \quad (II.6)$$

where R , a are the major and minor radius of the ring respectively, I is the ring current in A and γ is the electron energy in rmu (rest mass units). The index n_e is a measure, in the same units as the external field index, of the defocusing field of the ring. The defocusing field of the ring is equal to its self-field divided by γ^2 . We observe that the required field index is proportional to γ^{-3} , thus the axial focusing requirements are reduced drastically with increased electron energy.

2. Axial Acceleration or Deceleration at Constant Radius

In this case magnetic moment is violated and the only constants of the motion are the canonical angular momentum and the particle's energy. Magnetic field configurations providing orbits of constant radius for the electrons of Astron's E-layer have been reported ten years ago.⁴ The use of axial acceleration at constant radius in the injector phase of the electrons in the Astron was proposed in 1959.⁵ The required property of the magnetic field in order to maintain constant radius during axial acceleration or deceleration is that the betatron condition should be satisfied along the region the particles travel, at the equilibrium orbit, namely the change of the magnetic flux encompassed by the orbit should be twice the change of the axial magnetic field at the

orbit times the area enclosed by the orbit. When this condition is satisfied the radial derivative (dB_r/dr) of the radial component of the magnetic field must be zero at the orbit. This condition cannot be achieved with coils located externally to the particle's orbit. Therefore inside coils are required which are located in a cantilever enclosed by the particle's orbit. An electron ring can be accelerated at constant radius and maintain its integrity if the field index of the external field satisfies the condition described above. In contrast to the adiabatic expansion the mechanical angular momentum is not conserved and

$$\frac{p_{\theta f}}{p_{\theta 0}} = \frac{B_f}{B_0} \quad (II.7)$$

where the subscripts o, f indicate the initial and final values of the magnetic field and azimuthal mechanical momentum, respectively.

The combination of axial acceleration at constant radius followed by adiabatic compression makes possible the compression of an electron ring at constant energy in a static magnetic field. Let the initial values of the electron ring parameters be:

$$p_{\theta i} = \gamma mc, \quad p_z = 0$$

$$r_i^* = R \quad a^* = a_1$$

$$B = B_1$$

The ring undergoes at first acceleration at constant radius, during which the field is reduced to a value B_2 .

* r, a are the major and minor radius of the ring respectively

Then

$$p_{\theta 2} = (\gamma mc) \frac{B_2}{B_1} \quad (II.8)$$

$$p_{z2} = (\gamma mc) \left\langle 1 - \left(\frac{B_2}{B_1} \right)^2 \right\rangle^{1/2} \quad (II.9)$$

$$r_2 = R$$

$$a_2 = a_1 \sqrt{\frac{B_1}{B_2}}^* \quad (II.10)$$

Following the acceleration at constant radius the particles are allowed to undergo adiabatic compression in an increasing field until they stop. Then

$$p_{\theta 3} = \gamma mc \quad p_{z3} = 0$$

$$B_3 = B_2 \left(\frac{p_{\theta 3}}{p_{\theta 2}} \right)^2 \quad (II.11)$$

$$r_3 = r_2 \frac{p_{\theta 2}}{p_{\theta 3}} \quad (II.12)$$

*

The equations for the radial oscillations and radial momentum can be written in the form

$$r = R + a \sin (\omega t + \phi)$$

$$p_r = m\omega a \cos (\omega t + \phi)$$

If the expression is slow enough so that $(\dot{\omega}/\omega^2) \ll 1$ the adiabatic invariant is

$$J = \frac{m\omega a^2}{2}$$

Thus

$$a = \sqrt{\frac{2J}{m}}$$

or

$$a = \sqrt{\frac{2Jc}{e}} \cdot B^{-1/2}$$

The above equations yield

$$B_3 = \frac{B_1^2}{B_2}$$

$$r_3 = r_1 \frac{B_2}{B_3}$$

and

$$r_3 = r_1 \left(\frac{B_1}{B_3} \right) \quad (II.13)$$

$$a_3 = a_1 \left(\frac{r_3}{r_1} \right)^{1/2} \quad (II.14)$$

$$I_3 = I_1 \left(\frac{r_1}{r_3} \right) \quad (II.15)$$

$$E_3 = E_1 \left(\frac{r_1}{r_3} \right)^{3/2} \quad (II.16)$$

We observe that the self-field of the ring E (where $E = 60 I/a$, volts/cm) is increasing as the compression ratio in the $(3/2)$ power.

The static compression of the electron ring is possible because of the different properties of the magnetic field in acceleration at constant radius and adiabatic compression. During the acceleration at constant radius the axial component of the magnetic field at the equilibrium orbit varies linearly with the azimuthal mechanical momentum of the electrons. During the adiabatic compression, however the magnetic field at the ring radius is proportional to the square of the azimuthal mechanical momentum. Therefore if we allow the azimuthal momentum to oscillate from its maximum value γmc to $\epsilon \gamma mc$ and back to γmc in a succession of accelerations at constant radius and adiabatic compression respectively, at the end of each cycle the magnetic field increases by $1/\epsilon$ while the ring radius is compressed by ϵ . In a sequence of such cycles where the electrons "roll" and "coast" alternately it is possible to compress the ring to an arbitrarily small radius, thereby increasing considerably the self-field of the ring until a stability limit is reached, beyond which the ring may be unstable.

Following the compression phase the ring is maintained for a few microseconds at a small axial velocity (~ 0.01 c) in order to allow ions to be trapped within the electron ring.

Subsequently the ring can be accelerated electrically or in a decreasing magnetic field. Acceleration of the ring in a static magnetic field up to 1000 BeV is described in a separate report.⁶

III. Numerical examples

In this section the parameters of the compression and the electron beam are given for two extreme cases 4 MeV and 1000 MeV.

At 4 MeV the Astron facility can be used as a static compressor. An internal cantilever exists in the Astron tank and the existing coils and power supplies are adequate to compress the ring by a factor of 2.5:1 to a major radius of 16 cm. Further compression can be achieved by adding a new set of coils inside the Astron tank. I have assumed as the limit of the compression ratio the radius at which the self-field of the ring is equal and opposite to the external magnetic field.

	Low energy	High energy
Beam current (A)	400	1000
Injection energy (MeV)	4	1000
γ (rmu)	9	2000
Ring radius at injection (cm)	40	1000
Minor radius at injection (cm)	7	0.5
Beam quality (milliradian cm)	25π	0.25π
Magnetic field at injection (gauss)	380	3400
Self electrostatic field (E) of the ring (MeV/cm)	0.03	0.12
Effective field index of the electron beam (n_e)	0.1	$11.5 \cdot 10^{-5}$

	Low energy	High energy
First stage of compression		
R (cm)	16	200
a (cm)	0.63	.22
B (gauss)	950	17,000
E (MeV/cm)	0.10	1.34
Second stage of compression		
R (cm)	1.8*	34
a (cm)	0.21	.092
B (gauss)	8,550	100,000
E (MeV/cm)	2.56	19.2
ζ	1.0	.64
Ring current		
I(A)	9,000	29,400

The 4 MeV ring (first stage of compression) is expected to be achieved in the next few months following the completion of the improvements of the Astron accelerator on or about December 1, 1968. The new parameters of the Astron accelerator are:

$$E = 4 \text{ MeV}$$

$$I = 1000A$$

The 1000 MeV ring is expected to be achieved several years later, however, because it requires a new accelerator, probably a betatron using as injection the Astron accelerator, to accelerate 1000A to 1000 MeV. In concluding I would like to say that another advantage of the static field compressor is that it can produce a large number of rings per second depending only on the injection rate from the electron accelerator.

* At this radius the energy stored in the electromagnetic field of the ring is approximately 25% of the kinetic energy of the ring. A correction of the radius because of this effect is not included in the above calculation.

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