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KILOJOULE LASER SYSTEM SPATIAL FILTER ANALYSIS

By

H. Joseph Weaver

ABSTRACT

This report analyzes a spatial filter subsystem for the kilojoule laser system. It is assumed that the oscillator output beam has a Gaussian profile with cosinusoidal structure imposed on it. The report indicates how to choose the optical elements of the spatial filter system to remove this structure and how to analyze other forms of structure.

Introduction:

In this note a simple analysis is presented which illustrates the basic principles involved in the design of a spatial filter for the kilojoule laser system.

The purpose of this spatial filter is to remove the spatial structure in the oscillator pulse before it is injected into the beam shaping and preamplifier system.

Basic Principles of a Spatial Filter

A spatial filter consists of a lens (to separate structure from main beam profile) and an aperture (to remove the separated structure).

We first discuss the effect of a positive lens on a diverging beam. Such a situation is illustrated in Figure 1.

Figure 1

If we denote the optical disturbance function of the beam at plane 1 as

$$\Psi_1(\xi, \eta) = \Psi(\xi, \eta) \underbrace{e^{\frac{i\pi}{\lambda g} (\xi^2 + \eta^2)}}_{\text{divergence}}$$

Then we know from Fourier Optics that the optical disturbance function at plane 2 (Ψ_2) is given by:

$$\Psi_2(x, y) = e^{\frac{i\pi}{\lambda d} (x^2 + y^2)} \mathcal{F}_s \left[\Psi(\xi, \eta) e^{\frac{i\pi}{\lambda} \left(\frac{1}{d} + \frac{1}{g} - \frac{1}{f} \right) (\xi^2 + \eta^2)} \right] \quad (1)$$

where $\mathcal{F}_s$ denotes a scaled Fourier Transform with scale factor $s = 1/\lambda d$.

If now, the distance d and lens focal length f are chosen such that

$$d = \frac{gf}{(f-g)}$$

then equation (1) reduces to

$$\Psi_2(x, y) = e^{\frac{i\pi}{\lambda d} (x^2 + y^2)} \mathcal{F}_s [\Psi(\xi, \eta)] \quad (2)$$

Let us now consider the optical disturbance function of the oscillator output beam $[\Psi(\xi, \eta)]$. We will assume that the basic beam can be described by a circularly symmetric gaussian profile

$$\psi_3(\xi, \eta) = e^{-\frac{(\xi^2 + \eta^2)}{\sigma^2}}$$

where $\xi^2 + \eta^2 = r^2$, $0 \leq r \leq r_a$

Now we add a sinusoidal structure to this beam in one dimension.

This one dimensional structure simplifies the mathematics and more clearly illustrates the operation of a spatial filter. However, it is not at all an unreasonable choice of structure since two surface reflections (which are common) can be described by a sinusoidal function.

Thus the oscillator output beam will be described as

$$\psi(\xi, \eta) = e^{-\frac{(\xi^2 + \eta^2)}{\sigma^2}} (1 + A \cos w\xi) \quad (3)$$

NOTE: σ is the 1/e amplitude radius of the basic gaussian. It is related to the 1/e intensity radius σ_I by

$$\sigma = \sqrt{2} \sigma_I$$

thus σ is the $(1/e)^2$ intensity radius.

When $\psi(\xi, \eta)$ is described by equation (3) then equation (2) yields the following optical disturbance function at plane 2

$$\psi_2(x, y) = \pi \sigma^2 e^{\frac{i\pi(x^2 + y^2)}{\lambda d}} \left[e^{-\frac{(x^2 + y^2)}{\sigma^2}} + \frac{A}{2} \left(e^{-\frac{\left(x - \frac{\lambda d}{T}\right)^2 + y^2}{\sigma^2}} + e^{-\frac{\left(x + \frac{\lambda d}{T}\right)^2 + y^2}{\sigma^2}} \right) \right]$$

or if we set $y = 0$ and look only along x axis

$$\Psi_2(x) = \pi\sigma^2 e^{\frac{i\pi x^2}{\lambda d}} \left[e^{\frac{-x^2}{\hat{\sigma}^2}} + \frac{A}{2} \left(e^{\frac{-\left(x - \frac{\lambda d}{T}\right)^2}{\hat{\sigma}^2}} + e^{\frac{-\left(x + \frac{\lambda d}{T}\right)^2}{\hat{\sigma}^2}} \right) \right] \quad (4)$$

$$\hat{\sigma} = \frac{\lambda d}{\pi\sigma}$$

$$T = \frac{2\pi}{w}$$

A closer look at equation 4 indicates that at plane 2 there is a central gaussian beam and on either side an image of this gaussian. This is illustrated in Figures 2 and 3. These satellite profiles are the results of the beam structure. Obviously at plane 2 this structure has been separated from the main beam.

It is now obvious that to insure that the structure will not return to the beam as it propagates past plane 2 an aperture must be placed at plane 2, with radius r_0 , such that only the central gaussian beam passes.

We have implied here that the images are completely separated, however since they are gaussian profiles, they will always overlap to some degree. We therefore must now establish a criterion to determine an acceptable overlap. From previous work^[1] it is known that diffraction effects are negligible if a gaussian beam is apertured at an intensity level (relative to maximum) of 10^{-3} or less. Based on this fact we will choose the aperture radius r_0 such that

$$e^{-2(r_0/\hat{\sigma})^2} = 5 \times 10^{-4} \Rightarrow r_0 = 1.95 \hat{\sigma}$$

and require the image separation distance $\lambda d/T$ be such that at $r = r_0$ the intensity level of the satellite image is less than or equal to 5×10^{-4} .

i.e.

$$\frac{A^2}{4} e^{-\frac{2\left(r_0 - \frac{\lambda d}{T}\right)^2}{\hat{\sigma}^2}} \leq 5 \times 10^{-4}$$

or

$$r_0 - \frac{\lambda d}{T} \leq \sqrt{\frac{-\ln(2 \times 10^{-3}/A^2)}{2}} = a = .707 \sqrt{6.2146 + \ln A^2}$$

upon mathematical manipulation we eventually arrive at

$$\frac{T}{\sigma} \leq \frac{\pi}{a + 1.95} = R_c \quad (5)$$

We now have a maximum ratio (R_c) between the period of the beam structure and the $(1/e)^2$ intensity radius of the basic gaussian beam.

If the beam structure is such that

$$\frac{T}{\sigma} < R_c$$

Then the structure can be completely removed by placing an aperture of diameter

$$d_o = \frac{3.90 \lambda d}{\pi \sigma}$$

at plane 2.

Beam Radial Profile at Plane 1

Beam Radial Profile at Plane 2

Figure 2

Beam Radial Profile at Plane 1

Beam Radial Profile at Plane 3

Figure 3

If the structure is greater than this then there will be overlapping of the far field images and an aperture placed at plane 2 will cause unacceptable diffraction effects as beam propagates past plane 2. Figure 4 shows the case when $T/\sigma = R_c$. Figure 5 illustrates the case when $T/\sigma > R_c$.

It is of interest to consider the propagation of the beam (unapertured) past the far field at plane 2. A previous report^[2] for plane waves has shown that for every distance z past plane 1 there exists another (isomorphic) distance $\hat{z} = zf(f-z)$ such that the diffraction patterns at z and $\hat{z}$ are isomorphic. The region of interest is $z \geq f$ which implies that $\hat{z} < 0$, thus we see we can study the propagation past plane 2 by considering the negative (backward) propagation of the beam from plane 1. When $z = f$ (plane 2) the negative propagation distance $\hat{z}$ is equal to $-\infty$ and the beam is in its far field pattern. As z moves past plane 2 and approaches infinity, $\hat{z}$ returns from infinity and goes to $-f$. Therefore the lens focal length f determines the propagation characteristics of the beam past plane 2. If f is chosen such that the beam propagating backward from plane 1 a distance f develops into its far field pattern then as the beam propagates past plane 2, it will remain in its far field.

The previous remarks for a plane wave can be applied to a diverging beam if we note that a diverging beam described by

$$\psi(\xi\eta)e^{\frac{i\pi}{\lambda g}(\xi^2 + \eta^2)}$$

incident upon a positive lens of focal length f can be considered the same as a plane wave incident upon a lens of focal length

$$fg/(g-f).$$

Beam Profile at Plane 1

t_1

Beam Profile at Plane 2

Beam Profile 5 Meters past Plane 2

Figure 4

Beam Profile at Plane 1

I_1

Beam Profile at Plane 2

Beam Profile 5 Meters Past Plane 2

Figure 5

Computer studies indicate that a beam described by equation 3 will develop into its (negative) far field pattern in a distance $\hat{d}$ given by

$$\hat{d} = 1.5 \times 10^4 \sigma^2 \quad (6)$$

(Note: σ is $(1/e)^2$ intensity radius)

The previous remarks indicate a way of eliminating the aperture in the spatial filter system. Recall that the only purpose of the aperture was to remove the satellite images so that as the beam propagated past plane 2 they would not recombine with the central gaussian and generate the beam structure which was just removed.

However, if we choose

$$fg/(g-f) \geq \hat{d} = 1.5 \times 10^4 \sigma^2 \quad (7)$$

Then as the beam propagates past plane 2 it will remain in the far field and thus propagate essentially unchanged. The aperture, therefore, is not required.

When an aperture was used at plane 2, we recall that the image separation had to be great enough to eliminate truncation diffraction effects. However, since the beam will eventually be shaped by an apodized aperture which will use only a small on axis portion of the beam, the image separation distance can be reduced. This will be discussed in more detail later.

From plane 2 (spatial filter plane) the beam must be propagated to the beam shaping plane (plane 3). At plane 3 an apodized (wt) aperture is placed. The intensity transmission profile of this aperture is described by an equation of the form

$$e^{-(r/ra)^N}$$

We now consider the incident (to aperture) beam requirements. (See Figure 6)

Figure 6

The operation of the wt aperture has been described in detail [1] and here we briefly review the basic parameters

r_p is the clear (hard) radius of the wt aperture

α_p is the half-angle divergence to r_p .

Both of the above parameters are dictated by the preamplifier and a disk system. Their values are

$$r_p = .878 \text{ cm}, \quad \alpha_p = 1.7 \text{ mR}$$

r_3 is the $(1/e)^2$ intensity radius of the incident gaussian beam at plane 3 and α_3 is the half angle divergence to r_3 .

Although the value of r_3 is not critical, it should be approximately described by

$$r_3 = \frac{.82 r_p}{\sqrt{-\frac{\ln .85}{2}}} \quad (8)$$

α_3 is then given as

$$\alpha_3 = \frac{r_3 \alpha_p}{r_p} \quad (9)$$

using our values for r_p and α_p , we find

$$r_3 = 2.53 \text{ cm}$$

$$\alpha_3 = 4.9 \text{ mR}$$

returning to plane 2, we see

$$\delta = \frac{\lambda d}{\pi \sigma} = (1/e)^2 \text{ intensity radius} \quad (10)$$

$$\hat{\alpha} = \frac{\delta}{d} = \frac{\lambda}{\pi \sigma} = \text{half angle divergent to } \delta \quad (11)$$

Therefore, as the beam propagates from plane 2 to plane 3 both the half-angle divergence and $(1/e)^2$ intensity radius must be changed as indicated above.

We now present two simple optical systems to accomplish this

A) Single negative lens

A single negative lens is placed at plane 2 as shown in figure 7

Figure 7

Simple geometrical optics yields

Focal length of lens (to alter divergence) is given as

$$f = \frac{\delta}{\alpha_3 - \hat{\alpha}} \quad (12a)$$

Distance between planes (to increase beam radius) is given as

$$d = \frac{r_3 - \delta}{\alpha_3} \quad (12b)$$

B) Double Lens (Galilean) System

A Galilean telescope, as shown in figure 8, will also accomplish the required transformations

Figure 8

Again, a geometrical optics analysis yields

- 1) Choose distances d_2 and d_3 to be reasonable and convenient.
- 2) Positive lens focal length is given as

$$f_2 = \frac{d_2(r_3 - \alpha_3 d_3)}{(r_3 - \delta) - \alpha_3(d_2 + d_3)}$$

- 3) Negative lens focal length is given as (13)

$$f_1 = \frac{\delta d_2}{(r_2 - \delta) - (\alpha_3 d_3 + \hat{\alpha} d_2)}$$

We note that changing d_2 and d_3 offers some degree of flexibility in fine tuning the divergence α_3 .

Because of its simplicity, the negative lens system (A) is preferred. However, the distance between the planes is usually so large that this system is not practical.

APODIZED APERTURE AS SPATIAL FILTER

At this point, we will assume that the lens at plane 1 is selected according to equation (7) (far field is frozen) and that the beam is unapertured at plane 2. (Once the far field is established at plane 2, neither lens system A or B will disturb it.) We will now show that the aperture at plane 3 can be used to remove any structure on the beam.

Shown in figure 9 is the typical far field pattern (at any distance past plane 2).

Figure 9

As can be seen from the figure (or derived mathematically), the minimum value of the sum of the images is located at $\lambda \hat{d}/2\tau$. Thus, the intensity profile is monotonic from the axis ($r = 0$) to $r = \lambda d/2\tau$.

The aperture passes (and shapes) only a small on axis portion of the beam and in addition, the clear aperture radius r_p is

$$r_p = 1.22 \tilde{\sigma} \sqrt{\frac{-\ln .85}{2}} = .348 \tilde{\sigma} = .348 \frac{\lambda \hat{d}}{\pi \sigma}$$

to ensure that the beam remains monotonic within the wt clear aperture, we require the location of the minimum intensity to be outside r_p or

$$\frac{\lambda \hat{d}}{\tau} > r_p = .348 \frac{\lambda \hat{d}}{\pi \sigma}$$

or

$$\frac{\tau}{\sigma} \leq \frac{\pi}{.348} = 9.03 \quad (14)$$

If we examine equation 14, we can see period of the structure is so large compared with the $(1/e)^2$ radius of the basic gaussian that the beam can be considered free from structure. Thus we can see for all practical purposes that when the wt aperture is used as a spatial filter, all beam structure can be removed.

The previous remarks on the use of the wt aperture as the spatial filter indicate that this approach is to be preferred since it eliminates all the beam structure. However, in the kilojoule system an amplifier must be placed between plane 2 and plane 3 and the aperture of this amplifier will limit the size of the beam. There are two cases of interest.

CASE 1 Structure can be separated

In this case, equation 5 is satisfied and the structure can be separated from the basic gaussian beam. The total radius of the beams far field pattern can, in fact, become quite large depending upon $\lambda d/\tau$. Thus, to fit this beam through any reasonable sized amplifier aperture, the satellite images should first be removed by an appropriate aperture at plane 2.

CASE 2 Structure can not be separated

In this case, equation 5 is not satisfied and the satellite images cannot be separated and, therefore, cannot be removed by an aperture at plane 2 without causing serious diffraction effects. Therefore, the entire beam (central gaussian and overlapping satellite images) must be propagated through the amplifier. We now perform a simple worst case analysis to determine the amplifier aperture required to pass this beam.

We assume that equation 5 just fails
i.e.,

$$\frac{T}{\sigma} = \frac{\pi}{a + 1.95}$$

If we now define (see Figure 9) $\tilde{r}$ as the radial location where the satellite image falls to 10^{-3} of the central (on axis) maximum, we can show

$$\tilde{r} = \sigma \left(\sqrt{\frac{-\ln(2 \times 10^{-3}/A^2)}{2}} + \sqrt{\frac{-\ln(4 \times 10^{-3}/A^2)}{2}} + 1.95 \right) \quad (15)$$

Let us now apply our analysis to the physical dimensions and limitations of the kilojoule system.

Energy Density Limitation

Self-phase modulation and self-focusing considerations require that the energy density of the beam be kept below a certain critical value ($e_{\max}$.) We here set this value, $e_{\max}$ for optical elements at .2 joules/cm² for a 100 picosecond pulse. We note that in air this value is much higher.

Let us assume that the beam structure relative amplitude A is .5 and fill in the other physical dimensions of the system to determine what a typical spatial filter would look like.

OSCILLATOR OUTPUT PARAMETERS

$$\sigma = .07 \text{ cm} = (1/E)^2 \text{ intensity radius}$$

$$\alpha = .924 \text{ mRAD} = 1/2 \text{ angle divergence to } \sigma$$

thus, $g = 75.8 \text{ cm}$ (see Equation 1)

$$e_1 = \text{peak (on axis) energy density} = .19 \text{ J/cm}^2$$

BEAM PARAMETERS AT PLANE 2

Regardless of which expansion system (negative lens or Galilean telescope) an optical element will be located at Plane 2. Therefore the energy density must be kept below $.2 \text{ J/cm}^2$ at plane 2.

Some of the energy will be distributed to the satellite images and thus the on axis energy density will be reduced. If we assume the image separation is large enough that the satellite images do not contribute to the on axis energy density then we can write

$$c_1 = \text{reduction factor} = 1 - A^2/2 = .875$$

thus for the peak energy density e_{max} to remain less than $.2 \text{ J/cm}^2$ we require

$$\hat{\sigma} = \sqrt{\frac{c_1 e_1}{e_{\text{max}}}} \quad \sigma = .064 \text{ cm}$$

using equation 10 we find the distance between plane 1 and plane 2 to be

$$d = \frac{\pi \hat{\sigma}^2}{\lambda} = 132.8 \text{ cm}$$

the 1/2 angle divergence to $\hat{\sigma}$ is found via equation (11)

$$\hat{\alpha} = \frac{\hat{\sigma}}{d} = .48 \text{ mrad}$$

To "freeze" the far field pattern as the beam propagates past plane 2, equation (6) requires

$$d \geq 1.5 \times 10^4 \sigma^2 = 73.5 \text{ cm}$$

therefore, we see equation (6) is satisfied and the beam will propagate essentially unchanged (in shape) past plane 2.

We now use a rearrangement of equation (7) to determine the focal length of the lens at plane 1.

$$f = \frac{dg}{d+g} = 48.3 \text{ cm}$$

EXPANSION SYSTEM

We now must expand the beam as it propagates from plane 2 to plane 3.

1) Single Lens System

Using equations 12a and 12b, we arrive at,

$$f = \frac{\hat{\sigma}}{\alpha_3 - \hat{\alpha}} = 14.5 \text{ cm}$$

$$d = \frac{r_3 - \hat{\sigma}}{\alpha_3} = 503.3 \text{ cm}$$

2) Galilean System

Using equations 13 and choosing $d_2 = 80 \text{ cm}$ and $d_3 = 20 \text{ cm}$, we obtain

$$f_2 = \frac{d_2 (r_3 - \alpha_3 d_3)}{(r_3 - \hat{\sigma}) - \alpha_3 (d_2 + d_3)} = 98.5 \text{ cm}$$

$$f_1 = \frac{\hat{\sigma} d_2}{(r_3 - \hat{\sigma}) - (\alpha_3 d_3 + \hat{\sigma} d_2)} = 2.2 \text{ cm}$$

The Galilean expansion system is to be preferred in this case since it reduces the total distance required by about 4 meters.

The critical structure ratio given by equation 5 is

$$\frac{T}{\sigma} = \frac{\pi}{a+1.95} = .897$$

or

$$T = .063 \text{ cm}$$

thus, if $T < .063$ then the satellite images can be apertured out and just the structure free central gaussian propagated to plane 3.

If on the other hand $T \geq .063$ then we cannot aperture out satellite images without the possibility of introducing severe diffraction structure into the central gaussian. In this case the entire beam must be propagated to plane 3.

The maximum radius ($\tilde{r}$) of this beam is described by equation 15 and for the worst case $T = .063$ cm given as

$$\tilde{r} = .316 \text{ cm}$$

However, the beam must pass through a preamplifier of gain approximately equal to 8, thus to keep energy density below $e_{\max}$ the area of the beam must be increased by 8 or the radius must be increased by $\sqrt{8} = 2.83$. Thus the preamplifier must be able to accommodate a beam of 1.79 cm in diameter, if this worst case is ever reached.

CONCLUSIONS

In this report a spatial filter system was analyzed for a particular (realistic) beam structure. Other beam structures can be studied in exactly the same manner. These other various structures will possibly lead to other forms of far field patterns, but the basic principles for removal of the structure are identical to that presented in this report.

REFERENCES

- [1] Beam Shaping and Propagation Study for Kilojoule Laser System
Report Number SSL 73-236.
- [2] Effect of Optical Systems on Diffraction
Report Number SSL 73-333.

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