Westinghouse

No: WANL-TMI-458

Date January 19, 1963

ASTRONUCLEAR LABORATORY

TO: F. D. Retallick
J. G. Gallagher

From A. B. Rothman
Reactor Analysis
Subject Measurement of the Shutdown
Reactivity in Subcritical
Configurations

cc: W. H. Esselman W. Freidhof
W. H. Arnold J. D. Cleary
E. J. Ney J. Ravets
J. L. Beene D. W. Drawbaugh
C. Barksdale H. Krug
P. Blake R. E. Uhrig, University of Florida

SUMMARY

An experimental method of measuring the shutdown reactivity of a subcritical core is outlined. The method consists of two measurements:

1. A pulsed neutron measurement of Rossi- α
2. Decay of the neutron population following source withdrawal from a subcritical configuration.

Since such measurements, if feasible, make initial approaches to critical less hazardous, a pulsed neutron source would be a good addition to our facility.

INTRODUCTION

Reactivity worths of assemblies close to critical usually present no problem. Perturbations from critical are usually calibrated with control elements by reactivity measurements of two types:

1. Positive or negative reactivity determinations with an assumed

Inhour equation

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

REFERENCE

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

2. Rossi- α measurements using pulsed neutron sources

The use of a calibrated drum in the presence of highly poisoned cores, as for example those cores which are poisoned with shipping wires, would be more complicated in that the drum worth is affected by the presence of poison.

For this reason a method is proposed for measuring shutdown reactivity without the use of calibrated drums or for that matter for subcritical cores which are far removed from critical. In this analysis certain refinements such as the effect of contributions from photoneutrons and higher flux modes have been deferred to a later date.

THEORY

The usual one group space independent reactor kinetics equations for a core with an extraneous source are given by

$$\frac{dn}{dt} = \frac{\Delta k_{eff} n(t)}{l^*} - \frac{\beta \gamma k_{eff} n(t)}{l^*} + \sum_i \lambda_i C_i(t) + S_o$$

$$\text{and} \quad \frac{dC_i}{dt} = -\lambda_i C_i(t) + \frac{\beta_i \gamma_i k_{eff} n(t)}{l^*} .$$

Here the quantities have their usual definitions with

$$\begin{aligned} \gamma_i &= \text{effectiveness of the } i\text{th delayed neutron group} \\ \text{and } \beta \gamma &= \bar{\beta} = \sum_i \beta_i \gamma_i . \end{aligned}$$

Initially for the steady state condition with a source present we have

$$\frac{dn}{dt} = \frac{dC_i}{dt} = 0$$

$$\frac{\Delta k_{\text{eff}} n(0)}{l^*} - \frac{\beta \gamma k_{\text{eff}} n(0)}{l^*} + \sum_i \lambda_i C_i(0) + S_o = 0$$

$$\lambda_i C_i(0) = \frac{\beta_i \gamma_i k_{\text{eff}} n(0)}{l^*}$$

which gives the well known relation

$$n(0) = - \frac{S_o l^*}{\Delta k_{\text{eff}}}$$

If the core is left at the same k_{eff} or Δk_{eff} and the source removed the kinetics equations become

$$\frac{dn(t)}{dt} = \frac{\Delta k_{\text{eff}}}{l^*} n(t) - \frac{\beta \gamma k_{\text{eff}}}{l^*} n(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = -\lambda_i C_i(t) + \frac{\beta_i \gamma_i}{l^*} k_{\text{eff}} n(t)$$

Taking the Laplace transforms of these equations yields

$$s \tilde{n}(s) - n(0) = \frac{\Delta k_{\text{eff}}}{l^*} \tilde{n}(s) - \frac{\beta \gamma k_{\text{eff}}}{l^*} \tilde{n}(s) + \sum_i \lambda_i \tilde{C}_i(s)$$

$$s \tilde{C}_i(s) - C_i(0) = -\lambda_i \tilde{C}_i(s) + \frac{\beta_i \gamma_i}{l^*} k_{\text{eff}} \tilde{n}(s)$$

Solution of these simultaneous equations for the transform of the neutron density

yields

$$\frac{n(s)}{n(0)} = \frac{1 + \sum_i \frac{\beta_i \gamma_i k_{\text{eff}}}{l^* (s + \lambda_i)}}{s \left(1 + \sum_i \frac{\beta_i \gamma_i k_{\text{eff}}}{l^* (s + \lambda_i)} \right) - \frac{\Delta k_{\text{eff}}}{l^*}} \quad (1)$$

Recalling that the Rossi- α for the core is given by

$$\alpha = \frac{(1 - k_{\text{eff}}) + \bar{\beta} k_{\text{eff}}}{l^*} = \frac{-\Delta k_{\text{eff}} + \bar{\beta} k_{\text{eff}}}{l^*}$$

we have

$$\frac{k_{\text{eff}}}{l^*} = \frac{\alpha}{\beta \gamma} + \frac{\Delta k_{\text{eff}}}{l^*}$$

and letting

$$A_i = \frac{\beta_i \gamma_i}{\beta \gamma}$$

Equation 1 becomes

$$\frac{\tilde{n}(s)}{n(0)} = \frac{1 + \left(\alpha + \frac{\Delta k_{\text{eff}}}{l^*}\right) \sum_i \frac{A_i}{s + \lambda_i}}{s \left[1 + \left(\alpha + \frac{\Delta k_{\text{eff}}}{l^*}\right) \sum_i \frac{A_i}{s + \lambda_i} \right] - \frac{\Delta k_{\text{eff}}}{l^*}}$$

Taking the inverse transform gives

$$\frac{n(t)}{n(0)} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{st} \frac{\left[1 + \left(\alpha + \frac{\Delta k_{\text{eff}}}{l^*}\right) \sum_i \frac{A_i}{s + \lambda_i} \right]}{s \left[1 + \left(\alpha + \frac{\Delta k_{\text{eff}}}{l^*}\right) \sum_i \frac{A_i}{s + \lambda_i} \right] - \frac{\Delta k_{\text{eff}}}{l^*}} ds \quad (2)$$

Integration of Equation 2 is accomplished by finding the residues of the Laurent series equivalent expression of Equation 2, namely

$$\frac{n(t)}{n(0)} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \sum_i \frac{R(s_i) e^{s_i t}}{s - s_i} ds \quad (2a)$$

so that

$$\frac{n(t)}{n(0)} = \sum_i R(s_i) e^{s_i t} \quad (2b)$$

It is understood that the integral is taken over enough of the complex plane so as to include all the poles, one pole for every delayed neutron group plus the pole s_o , whose residue, $R(s_o) e^{s_o t}$, describes the asymptotic decay of neutron population after source withdrawal so that

$$\left(\frac{n(t)}{n(0)} \right)_{\text{assym}} = R(s_o) e^{s_o t}$$

where

$$s_o - \frac{\Delta k_{\text{eff}} / l^*}{1 + (\alpha + \frac{\Delta k_{\text{eff}}}{l^*}) \sum_i \frac{A_i}{s_o + \lambda_i}} = 0 \quad (3)$$

Recognizing Equation 2 to be in the form

$$\frac{n(t)}{n(0)} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{q(s) e^{st}}{p(s)} ds$$

where $q(s)$ and $p(s)$ are two analytic functions, a theorem in complex variables together with Equation 3 gives

$$R(s_o) e^{s_o t} = \frac{q(s_o)}{p'(s_o)} = \frac{(\Delta k_{\text{eff}} / l^* s_o) e^{s_o t}}{\frac{\Delta k_{\text{eff}}}{l^* s_o} - s_o (\alpha + \frac{\Delta k_{\text{eff}}}{l^*}) \sum_i \frac{A_i}{(s_o + \lambda_i)^2}}$$

This then gives

$$\left(\frac{n(t)}{n(0)} \right)_{\text{assym}} = \frac{(\Delta k_{\text{eff}} / l^* s_o) e^{s_o t}}{\frac{\Delta k_{\text{eff}}}{l^* s_o} - s_o (\alpha + \frac{\Delta k_{\text{eff}}}{l^*}) \sum_i \frac{A_i}{(s_o + \lambda_i)^2}} \quad (4)$$

Since s_0 may be obtained from the relation

$$s_0 = \frac{d \ln \left(\frac{n(t)}{n(0)} \right)_{\text{assym}}}{dt}$$

since α may be obtained from a pulsed neutron measurement, since the λ_i are known, and since

$$A_i = \frac{\beta_i \gamma_i}{\beta \gamma} \approx \frac{\beta_i}{\beta}$$

where the β_i are known, we may use Equation 4 to determine $\Delta k_{\text{eff}}/l^*$. Also, we have

$$\alpha + \frac{\Delta k_{\text{eff}}}{l^*} = \frac{\beta \gamma}{l^*} k_{\text{eff}}$$

and

$$\frac{\Delta k_{\text{eff}}/l^*}{\alpha + \frac{\Delta k_{\text{eff}}}{l^*}} = \frac{\Delta k_{\text{eff}}/k_{\text{eff}}}{\beta \gamma} \quad (5)$$

The final equation, Equation 5, is then an expression for the shutdown reactivity in dollars divided by k_{eff} . This then suggests considering a measurement on the decay of neutron population after source withdrawal, together with a pulsed neutron source measurement of α as a means of determining the shutdown reactivity of a core poisoned with shipping wires or of subcritical cores on the initial approach to critical.

CONCLUSIONS

This report suggests two experimental measurements which may be carried out on one subcritical core, to determine the shutdown reactivity of that

core. The approach involves simplified assumptions about the transient behavior of the core. Another approach, which will be considered is a step reactivity perturbation to a subcritical core with a source.

It should be noted that the reactivity could be determined in a shutdown core, if this method is feasible, without using a combination of two common crutches, namely:

1. Reciprocal counts versus loading
2. Drums calibrations

While these methods should be used as an added set of measurements for other reasons, they have certain weaknesses in determining shutdown reactivities. The method in this report outlines a determination of shutdown reactivity at the time of the measurements provided adequate data processing is available.

SIGNED A. B. Rothman
A. B. Rothman