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REPORT NO. RN-S-0332
TO
AEC - NASA SPACE NUCLEAR PROPULSION OFFICE

YEAR-END REPORT ON
DIGITAL-PROGRAM DEVELOPMENT
SEPTEMBER 1966

NERVA PROGRAM

CONTRACT SNP-1

MASTER

AEROJET-GENERAL CORPORATION

SACRAMENTO, CALIFORNIA

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YEAR-END REPORT ON
DIGITAL-PROGRAM DEVELOPMENT
SEPTEMBER 1966

ROCKET ENGINE OPERATIONS - NUCLEAR

NERVA PROGRAM

CONTRACT SNP-1

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ABSTRACT

A family of digital-computer programs has been developed and is being maintained for NERVA analysis. These programs include a transient engine program, a steady-state engine program, supporting component programs, and miscellaneous specialty programs.

The programs have been upgraded by using the latest NERVA test data and applying the newest analytical techniques. A system has been developed for controlling changes to ensure continuous operation on all engine models.

C. M. Rice
Program Manager
Rocket Engine Operations - Nuclear

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1.0 INTRODUCTION

This report summarizes the development status of the digital-computer programs being used for NERVA systems analysis. Fifteen such programs are discussed, each of which has a specific purpose and is actively used for NERVA analysis.

The principal program is a transient type that can predict engine startups, power ramps, and power holds. Supporting this program is a steady-state program for quickly calculating power holds, as well as a family of component programs for detailed studies. The component programs are essentially subroutines of the larger programs, with the capability of separate input. These programs are used to study special component tests and to improve analytical techniques. New equations or routines developed from these studies can be quickly used as input to the larger programs.

In addition to the principal programs, various special programs have been developed to resolve specific NERVA problems. The system programs, for reasons of economy and computer capacity, must make simplifying assumptions in many of the analytical approaches. The special programs can perform more comprehensive analyses on specific problems but require a greater expenditure of time and money.

2.0 SUMMARY

Thirteen major additions have been made to the NERVA transient digital model to improve its operating speed, the number of problems it can analyze, and the accuracy of the calculation techniques. Five improvements have been incorporated in the steady-state program for similar reasons. In addition, many minor improvements have been made in each program.

Data from the CFDTS, NRX/EST, and in-house component tests have been studied to improve the analytical techniques used in the digital programs. A significant upgrading has been achieved by the development of good low-speed pump correlations for startup analysis. These were developed by using the CFDTS data and NRX/EST data. The remainder of the program, which had been previously matched to BF_4 -D, BF_4 -E, NRX-A1, NRX-A2, and NRX-A3 data, was reviewed and found to be in good shape.

Routines have been developed for computing helium and nitrogen properties, and the hydrogen decks have been extended to 7000°R and 5000 psia. In addition, the TabT and TabH hydrogen decks have been combined so that pressure and temperature or pressure and enthalpy can be used to obtain properties without an iterative loop. This has greatly improved operating time, as the decks are entered 80,000 to 100,000 times during each complete engine balance.

New steady-state and transient programs have been developed for NERVA studies based on the technology-engine models. The programs are operational and are currently being used for parametric design studies. New special programs have been developed for statistical data analysis, checkout of the common analog model, turbopump analysis, chilldown analysis, nozzle flow mapping, and gas injection for thrust-vector control.

Work has been initiated on a second-generation transient system program using all differential equations, which will be more accurate and will perform more comprehensive NERVA system studies. The program is available for limited studies at present and is being expanded to perform complete engine studies.

3.0 DISCUSSION

3.1 GENERAL DISCUSSION - ALL PROGRAMS

The prime digital program used by Aerojet for NERVA engine system studies is the transient engine simulation program (Reference 1). This program uses a finite-difference technique to solve the flow, pressure, and temperature distribution within a variety of NERVA Technology Engine configurations. The program can predict the operation of tests of the reactor only (NRX) or of engine systems (NRX/EST and XE), which include the turbopump, hot-bleed port, engine tanks, and all lines.

The program is highly versatile and can handle a variety of engine system calculations. Engine operation through startup and into the operating range can be predicted by using tank conditions and stored engine energy (material heat capacitances). Engine power transients can be calculated from turbine-power-control-valve position and nuclear power, or from the desired chamber pressures and temperatures. Power ramps can be calculated over the entire range of engine operation. The engine model has been mated with a facility model, and interaction studies can be conducted. The response of facility and engine valves can be supplied as input to the program, and engine operation under emergency or blowdown conditions can be calculated. Steady-state operation of the engine can also be predicted without having to go through a power ramp. The major limitation of the model at present is its inability to predict the operation of the engine control system.

The principal backup digital program for the transient engine simulation program is the NERVA engine balance steady-state program (Reference 2). This program computes the steady-state operation for the reactor only or for engine system tests. Fluid conditions in the various engine plenums are computed by the program, but material and fluid temperature distributions in the reactor system are not computed.

The program has proved to be a useful backup by providing quick, economical, steady-state data and serving as a double check for system predictions. The complexity of the input to the larger transient program makes it susceptible to human errors in preparing input or in operating the program. Before making major systems predictions, the transient and steady-state programs are checked against each other at the design point. A marked discrepancy between the two requires a complete re-examination of the engine system model and any recent programing changes. No official predictions are made until the discrepancy has been resolved.

To ensure a valid analysis of system operation, good component models must be maintained. The mathematical models used in the system programs for three major components have been separated into individual programs for analysis of component test data, as follows:

- (1) Hot-bleed port - Reference 3
- (2) Nozzle tube - Reference 4 and Section 3.3.4
- (3) Turbopump - Section 3.5.4

The component programs use identical configuration input and engineering coefficients. As the analytical techniques are improved from analysis of component test data, the upgraded equations or engineering coefficients can be used as direct input to the systems program.

In addition to the component programs directly supporting the transient and steady-state programs, a series of special digital programs have been developed to analyze specific problems for the NERVA program, as follows:

Statistical Data Analysis - These programs discussed, in Section 3.5.2, take test data directly from NTO tapes and make best estimates of actual values from multiple readings by means of statistical techniques.

Digital Check of Analog Results - This program, discussed in Section 3.5.3, makes a digital check of analog equations and compares the answer with the analog output. The two answers are compared against each other and against an allowable error band.

Chilldown Analysis - This program, discussed in Section 3.5.5, includes a comprehensive fluid-flow and heat-transfer model containing terms disregarded in normal analysis. The program was intended to analyze the flow oscillations that occur when cryogenic hydrogen flows into ambient lines that add energy to the fluid as a result of the temperature difference between fluid and lines.

Nozzle Flow Mapping - This program, discussed in Section 3.5.6, relates the flow through a nozzle to the chamber pressure, chamber temperature, and nuclear power. The program plots the results on maps that are used for quick estimates of nuclear power during actual engine tests.

Gas Injection (see Section 3.5.7) - This program was developed to study gas injection for NERVA thrust-vector control. The program includes the latest equations available in the literature for this relatively new process.

Differential-Equation Program (see Section 3.5.8) - This program, which is under development, is a second-generation NERVA transient program that solves engine balances by using a direct series solution of the differential equations describing one-dimensional fluid flow rather than using finite-difference techniques. The program will greatly increase the dynamic-response capability of the digital analysis.

3.2 CONTROL OF PROGRAMS

3.2.1 General

Each digital system program is designed to handle a number of different engine designs. The various designs are described by a large number

of inputs that detail the various components. The programs also have a large number of options by which engine balances can be made. Both the engine configurations and the types of analysis being performed by the programs are undergoing constant change. A tightly controlled change system has been found necessary to prevent errors from occurring in the operational programs and associated inputs. The control system must prevent errors but still provide the freedom necessary for continual program improvement.

3.2.2 System

A control system has been initiated that is operated by three engineers:

Program Development Engineer - Responsible for developing and maintaining records of basic program equations and logic.

System Operation Engineer - Responsible for conducting analysis on a particular engine configuration, such as XE or NRX.

Configuration Control Engineer - Initiates and maintains the data book for each engine configuration. Certifies that changes have not adversely affected program output.

3.2.3 New Program Control

Systems-analysis management will authorize a new program by issuing a charge number, stating the objectives of the program, setting a due date, and assigning a program development engineer to develop the required equations and to work with a programmer. This engineer will obtain input values from the configuration-control group to match his program-input "L" locations. These values are entered into the proper locations by the engineer and a member of the input group to check out the program.

When the programmer and the program development engineer are satisfied that the program is functioning smoothly, a copy of the program deck will be given to the system-operation engineer. The program development engineer will keep a book of all changes made to the program, which will give the equations, input units, description, and location. All changes that are made to any program, such as closure methods and equations, must be recorded in this book and approved by the engineer responsible for that particular program.

The system-operation engineer will make the following runs to check out the program:

- (1) Steady-State Program
 - (a) 100% power
 - (b) 50% power
 - (c) 30% power
- (2) Transient Program
 - (a) 100% power
 - (b) 50% power
 - (c) 30% power
 - (d) Bootstrap and transient ramp to full power

When the data output meets the required steady-state conditions, the system-operation engineer will supply the configuration control group with a summary of all calculations and the input data.

After verifying that the data output meets the required steady-state values, the configuration-control group will issue a document showing all the input values and "L" locations and will release the input and program decks for production runs. A copy of the input deck and the program absolute deck will be kept by the configuration-control group.

The system-operation engineer is responsible for using only these verified, published, input values in his input deck.

3.2.4 Changes in Production Program

Should the program be altered because of desired changes or errors, the program development engineer must make the correction and obtain new input values from the configuration-control group, check out the program for verification of steady-state values, and follow the same procedure as set forth in Section 3.2.3 for release of input and production decks.

After verifying that the data output meets the required steady-state values, the configuration-control group will issue an amended input document to release the input and program decks for production runs.

3.2.5 Modification to Input

If the input is modified by the configuration-control group, this group will submit the changes in writing to the system-operation engineer, who will make runs and proceed as outlined in Section 3.2.3. After verifying that the data are valid and the program operates satisfactorily, the configuration control group will issue an amended input document to release the input decks for production runs.

3.3 UPGRADING FLEXIBILITY OF EXISTING PROGRAMS

This section presents the results of modifications that have been made to the existing digital programs to increase their flexibility. Included in this category are items that improve the program speed, increase the number of conditions that can be studied, increase the program accuracy, and make it easier to use the programs.

3.3.1 Properties

(a) Hydrogen

The first hydrogen-properties deck used in the systems programs was obtained from Otis Farmer from the Los Alamos Scientific Laboratory and is now about three years old. This original program was written in FORTRAN II and FAP and used an iterative process to determine property values when using pressure and temperature as inputs. The program was limited in range (38 to 3000°R and 0 to 1500 psia), would not always converge (even within its boundaries), and was slow.

Farmer's Tab(T) and Tab(H) decks, which were received next, covered a wider range (38 to 5000°R and 0 to 1500 psia) and were four times faster than the previous program because of the inherent speed advantages of tabular data. The Tab(T) and Tab(H) decks were in FORTRAN IV and MAP. The core storage required is 6000 locations, and the program interpolates 35,000 property values per minute.

These decks have been modified at Aerojet by combining the two decks and extending the range (38 to 7000°R and 0 to 5000 psia). The range was increased by curve fitting and by extrapolating data obtained from Reference 5. This was done so that system operating characteristics at higher pressures and temperatures could be computed and evaluated.

The core storage now required is 6200 cells, and the program now interpolates 28,000 property values per minute. The program can be entered with pressure and enthalpy or pressure and temperature, and the following information can be obtained:

0 - Pressure	P	6 - Density	ρ
1 - Specific vol	V	7 - Specific heat	C_p
2 - Enthalpy	H	8 - Conductivity	K
3 - Entropy	S*	9 - Viscosity	μ
4 - Quality	X	10 - Specific-heat ratio	γ
5 - Temperature	T	11 - Sonic velocity	C*

When entering with pressure and temperature, either saturated-liquid or saturated-vapor properties are obtained.

A later deck has been received from Otis Farmer that is written in FORTRAN IV and has a range of 0 to 5000 psia and 36 to 5000°R. This program has been checked out and is planned for use in the systems-analysis programs when the IBM 360 computer is in operation.

Farmer's hydrogen-properties decks have proved themselves in the systems-analysis programs, are not range-limited, and are faster than any other known system for finding hydrogen properties.

(b) Nitrogen

A digital routine has been developed that will give the thermodynamic properties of nitrogen when a pressure from 0 to 3000 psia and a temperature from 113.7 to 9000°R are given. The program will not give a quality result in the two-phase region but is designed to give either saturated-gas or -liquid properties when in the two-phase region.

The program uses two techniques to calculate the specific heat, enthalpy, entropy, density, and the velocity of sound. Below 468°R, a modified form of the equations recommended in Reference 6 is used; and above 540°R, a modified form of the equation from Reference 7 is used. In the intermediate range, both sets of equations are used, and the results are averaged by a linear weighting technique.

Two transport properties, viscosity and thermal conductivity, are calculated from empirical expressions. These values are less accurate than the thermodynamic properties.

Description of Equations for Specific Heats, Enthalpy, Entropy, Density, and Velocity of Sound

The method of determining the thermodynamic properties of nitrogen follows the traditional method of combining equations that describe the ideal-gas properties and equations that include empirical corrections to obtain the actual pressure-temperature-density relationship. From these two basic sets, values of density, enthalpy, entropy, heat capacity at constant pressure and volume, and velocity of sound are directly calculated in closed form. The program, as developed, requires temperature and pressure as independent input variables.

Below 540°R, the program calculates properties by using a modified form of the technique recommended in NBS Circular 129 (Reference 6). The principal modification is that the critical temperature is assumed to be 127.122 rather than 126.2. This assumption (plus small corrections to the equations) smooths the program output and prevents discontinuities that give problems in any digital program.

The basic technique used is to calculate the thermodynamic properties at the desired temperature and at zero pressure by using curve fits of ideal-gas constant-pressure heat capacities. The properties are then adjusted to the actual pressure by using standard thermodynamic equations based on pressure, temperature, and density. Density is obtained from the input pressure and temperature by using a modified Benedict-Rubin-Webb equation of state obtained from Reference 6.

The basic program routine is as follows:

- a. The program is entered with a pressure and temperature.
- b. By using the equations outlined in Figure 1, the ideal-gas thermodynamic properties of nitrogen at the entering temperature and zero pressure are calculated. The vapor pressure is then computed by using the empirical equations shown in Figure 2, if the temperature is below 127.122°K.

c. The density at the actual pressure and temperature is then determined by using the equation of state shown in Figure 3. Determination of the density is an iterative loop. A density is assumed and the pressure is calculated by using the equation in Figure 3.

d. Below the critical temperature, three densities are obtained: (1) the saturated-liquid density, (2) the saturated-gas density, and (3) the density at the actual pressure and temperature.

e. These densities are determined by using the equation of state (Figure 3) and the vapor pressure (Figure 2). The following sketch illustrates how the equation of state will relate pressure and density in the two-phase region.

f. To determine the densities at the three points, a table has been supplied as input to the computer routine, which gives the first guesses for density shown as A, B, and C in the above sketch.

g. From points A and B, the program will close on the calculated vapor pressure. The closure routines are such that, starting from point A, the saturated-vapor density will be obtained. Starting from point B, the saturated-liquid density will be obtained. When starting from point C, the program will iterate on the entering pressure.

h. Once the program has solved for the densities, the thermodynamic properties are calculated by using the equations shown in Figure 4.

i. The terms involving the equation of state (Figure 3) are invalid in the two-phase region, as shown in the sketch of paragraph e; so the program uses the equation of state from p_0 to the saturated-vapor density and from the saturated-vapor density to the density at actual pressure and temperature. Between the saturated-vapor and saturated-liquid densities, a modified Clapeyron relation (Figure 5) is used.

j. The total value is then the sum of the values between the three limits.

k. Above 468°R, a viral equation of state is used, one involving pressure and temperature as the direct variables (equations in Figure 6). The viral equation is that given in NBS Circular 564 (Reference 7). The tabulated coefficients (as functions of temperature) were fitted to three polynomial sets, with $1/T$ as the variable, to cover the range up to 9000°R. The polynomials are not truncated (as are the NBS data), since they normally converge to zero as temperature increases; the smoothness of the data is thus improved. Above 1540 psia (105 atmospheres), the calculated compressibility-factor isotherm is extrapolated by means of a straight-line tangent to the isothermal curve at 1540 psia. This permits the higher-temperature P-V-T surface to extend to the 3000-psia limit of the low-temperature P-V-T surface. Both low- and high-temperature P-V-T surfaces in the crossover region are quite close together at 3000 psia.

l. Between 468 and 540°R, both equations of state are used, and the calculated properties are averaged by using a linear-weighting method to provide an absolutely smooth surface over the entire region for all thermodynamic properties.

m. Program limits are from solid nitrogen (63.15°K) to 9000°R and from 0 to 3000 psia. All values in and out are handled in Rankine, pounds, psia, Btu's, feet, and seconds.

Description of Equations for Transport Properties

Viscosity is calculated from the equations shown in Figure 7. Viscosity in the low-temperature region is calculated from the NBS Circular 564 (Reference 7) expression involving temperature and density. Viscosity for liquid and dense nitrogen is calculated by using an empirical Thodos correlation method (Reference 19). Available experimental data were plotted, and log functions of viscosity deficiency were empirically fitted. The overall fit ($\pm 5\%$) was not very good, but a reasonably smooth viscosity-pressure-temperature surface was obtained.

Thermal conductivity is calculated by using the empirical equations shown in Figure 8. An identical technique was used for thermal conductivity. Here the fit to experimental data was better, except for some points in the dense-gas region. High-temperature values are obtained by logarithmic extrapolations of the one-atmosphere curves, with the same density correction functions used. Both low- and high-temperature calculated viscosity and thermal-conductivity values agree well with the plotted curves in Reference 8. The smoothness of the surfaces has not been checked in detail. Thermal-conductivity units are $\text{Btu/hr-ft-}^{\circ}\text{R}$, and viscosity units are lb/ft-sec . The equations give values over the same temperature-pressure region as that defined for the thermodynamic properties.

(c) Helium

The thermodynamic properties of helium are calculated directly in closed form by using a viral form of the Beattie-Bridgeman equation of state and the single-valued ideal-gas heat capacity. The constants used are those tabulated by Aikin in Reference 9. This equation is the same as used by Wilson and others in their tables of thermodynamic properties.

The equation is used in the low-temperature region as an approximation of the real gas behavior. This was done as a storage measure until more accurate equations in the low-temperature region were available. Mann's equation in Reference 10 did not appear to offer any advantages at the time over the accepted Beattie-Bridgeman equation. The equations are summarized in Figure 9.

Arbitrary pressure-temperature limits of 0 to 3000 psia and 36 to 5000°R are written into the program. The accuracy of calculated values for densities greater than 5 lb/cu ft in this region appears to be poor (up to +20%). Since the program was to be used specifically as a heat-transfer subroutine, as is the case of the hydrogen Tab(r) and Tab(H) programs, accuracy was not critical.

Viscosity and thermal conductivity are calculated as functions of temperature, and viscosity is independent of pressure. The viscosity equation given by Aikin (Reference 9) is used. Thermal conductivity is calculated from viscosity, pressure, and heat capacity at constant volume by using equations given by Aikin.

All thermodynamic values are in units of degrees Rankine, pounds, Btu's, seconds, and feet. Viscosity is in lb/hr-ft, and thermal conductivity is in Btu/°R-hr-ft.

3.3.2 Program 12013 - Transient Program

The 12013 program is the operational transient program being used for NRX, NRX/EST, and XE transient studies. New requirements are always being made of the program, and the following pages summarize some of the more important features added to the program during the past year.

(a) Blowdown Option

One of the methods used to cool down the engine system is to flow gas at ambient temperature through the system. The gas is supplied from a large tank that is initially at a high pressure and is allowed to simply blow down through the engine. The blowdown option consists of a model of this system contained as a subroutine in the transient engine simulation program.

The initial mass, temperature, and pressure of the gas in the tank are entered into the program. The program calculates new pressures and temperatures as mass is removed from the tank. The gas flows out through a pressure-control valve that can limit its pressure to any desired value; if the tank pressure falls below the desired outlet pressure, the tank pressure is then used. A variable loss coefficient is used to simulate the piping and valve.

The gas enters the engine system either at the pump inlet or in the pump discharge line, depending on input. The pump flow can be routed out through a vent valve whose position can vary with time. The discharge-line loss coefficient in the area between the vent valve and the point where the blowdown flow is introduced can be varied to simulate a pump shutoff valve or a check valve.

With the above arrangement, it is possible to simulate a changeover to the cooldown system at the end of a power run. At the desired time, the blowdown-line valves are ramped open and the pump discharge is routed out through the vent valve until the turbine power-control valve shuts and the pump stops turning.

(b) Engine Test Stand 1 (ETS-1) Digital Model

To determine the effects of an XE emergency engine cooldown in terms of facility-to-engine interaction and to optimize the facility capabilities in terms of a desirable engine-cooldown operating profile, a digital model of the NTO ETS-1 facilities has been combined with the current XE digital engine model for simulation of the total XE engine emergency cooldown system. This 12013 digital engine simulation model has been modified to include the characteristics of the facility tankage, lines, valves, and associated plumbing through which the working fluids flow to reach the engine system during cooldown.

Specifically, the normal five-part XE engine digital simulation program has been expanded to nine parts. The additional four parts are used to describe the facility fluid systems. Four facility fluid systems, which include their respective transport properties, have been incorporated into the XE model. These fluids are LH_2 , LN_2 , GHe , and GH_2 .

The facilities model data described in detail in Reference 5 include a description of the specific fluid system considered, the specific component boundaries, the method of data calculation, specific input values, and their respective program locations.

The detailed information required for the facility description was obtained from the preliminary issue of the ETS-1 Facilities Data Book (Reference 11) and from discussions with cognizant facilities personnel when specific facility information was not available in the data book.

The ETS-1 line configurations were separated into four separate systems with various parts common to each other. The four systems considered were liquid nitrogen, helium, gaseous hydrogen, and high-pressure liquid hydrogen. The facilities are handled in the 12013 program by the addition of parts (see Reference 1) in front of the engine system.

With the facilities included in the analysis, the available pressures and fluid quantities will be included in the analysis; and by using the pipe temperatures calculated in the program, the engine will receive the propellant at a temperature that is a function of heat transfer in the facilities. The model is currently being used for XE engine cooldown studies.

(c) Calculation of TPA Efficiency from Test Data

This calculation is an option of the transient-analysis program used to aid in the reduction of data obtained from startup runs and to calculate turbopump operating information.

The turbopump-speed input is entered as a function of time from the test data. Also in the input is other system control information such as TPCV gate position and times for various valve operations. A curve of the relationship between pump speed, head rise, and flow rate is known and is an input in the program.

Since speed is known at every calculation time, the pump head rise and total work delivered are functions of only flow rate and pump efficiency. The transient program solves for the flow through the system as a function of inlet pressure, and through an iterative process, the flow and pressure delivered by the pump are balanced by the flow and pressure required by the engine.

On the basis of engine pressures and temperatures, the amount and state of the turbine inlet flow can be calculated. With this information, the total energy being supplied to the turbine at various times during the test is calculated. Because the total energy to the turbine and the total work from the pump are known at this point, the overall turbopump efficiency can be computed. The shaft horsepower is not known, and the individual pump and turbine efficiencies still cannot be determined. The input turbine efficiency correlation relating U/C_o to efficiency is assumed to be valid, and the shaft horsepower is computed by using this correlation. The program then computes the pump efficiency and prints out the results for each time point.

(d) Addition of Heat Energy in Lines

The transient program was originally designed to calculate transient system operation during power runs. After the engine has been run for a short period of time, the major lines in the system have chilled. The heat-transfer coefficients on the outside of the lines due to natural convection are low enough so that any heat pickup in the lines can be ignored.

However, when an attempt was made to accurately predict system operation during a bootstrap type of startup, the heat gained by the fluid in chilling the lines was found to be significant. A calculation was added to include this effect. An average line heat-transfer coefficient is calculated by using an experimentally determined correlation for turbulent flow through a tube:

$$h_c = 0.0217 \frac{K}{D} \left[\frac{VD\rho}{\mu} \right]^{0.8} \left[\frac{\mu_p}{K} \right]^{0.4} \left[\frac{2T}{T+\theta} \right]^{0.34},$$

where: h_c = film coefficient, Btu/in.²-sec-°R,
 K = fluid thermal conductivity, Btu/sec-hr-°R,
 ρ = fluid density, lb/in.³,
 C_p = fluid specific heat, Btu/lb-°R,
 μ = fluid viscosity, lb/in.-sec
 D = tube diameter, in.,
 T = fluid temperature, °R,
 V = velocity, in./sec,
 θ = wall temperature, °R.

The mass initial temperature and heat-transfer area of the line are entered. The convection relationship is used to calculate the temperature change of the pipe over the program computing interval and to calculate the resulting increase in enthalpy of the fluid flowing through the line.

$$\theta_f = \theta_i - \frac{h_c \times A_p (\theta_i - T_i)}{m C_r},$$

where

$$\begin{aligned}\theta_c &= \text{initial wall temp, } ^\circ\text{R,} \\ \theta_f &= \text{final wall temp, } ^\circ\text{R,} \\ T_i &= \text{fluid temperature at inlet, } ^\circ\text{R,} \\ M &= \text{pipe mass, lb,} \\ C_R &= \text{pipe specific heat, Btu/lb-}^\circ\text{R,} \\ A_p &= \text{pipe heat-transfer area, in.}^2;\end{aligned}$$

and

$$T_o = T_i + \frac{h_c A_p (\theta_i - T_i)}{c_p \dot{W}},$$

where

$$\begin{aligned}T_o &= \text{fluid outlet temperature, } ^\circ\text{R,} \\ \dot{W} &= \text{fluid flow rate, lb/sec.}\end{aligned}$$

(e) Pump Tailoff Option

It is presently planned to shut down the X-engine by scrambling the reactor and controlling the nozzle chamber temperature by using feed hydrogen. A chamber temperature will be demanded, and the control system will regulate the TPCV position to pump liquid hydrogen into the engine as required to maintain the desired temperature.

An option has been added to the 12013 program to control the engine to a fixed chamber temperature as the nuclear power decays. The TPCV position is the outer loop in the program closure routines, and it is possible to add a valve response curve to better simulate cooldown. This option is being studied for use as an aid in XE test planning.

(f) Implicit Heat-Transfer Solution

The derivation of the finite-difference solution for the transient-conduction equation used to compute material temperatures in the transient engine system program is covered in Reference 1. This solution is expressed in the explicit form; that is, the temperature at some future time and point can be solved for directly in terms of temperatures and material properties calculated at the previous time. Based on the material properties and node arrangement, there is some maximum time interval over which a valid new temperature can be obtained. If a time step greater than this maximum interval is attempted, the solution becomes unstable and diverges from the correct solution of the differential equation.

If the heat-balance equations are rewritten in terms of the temperatures for which a solution is desired, a new set of finite-difference equations is obtained. An iterative type of solution is required to determine the temperatures, but the equations are stable for any time step.

When certain parts of the engine are very cold, their properties are such that, in some cases, the maximum time step with the explicit equations is very small. Several hundred passes through the temperature calculations may be required to advance 1 sec in time. The implicit equations might require 20 iterations for the same 1-sec time advancement. Thus, running time is considerably improved by using the implicit equations. In other cases, the explicit equations might require only one pass for a 1-sec interval and would represent a time saving.

Both sets of equations are now available in the 12013 program and are an input option.

(g) Chamber Pressure and Temperature Option

The transient engine simulation program was previously set up so that control over engine operation was obtained by changing two variables: TPCV position and the rate of internal heat generation. This system duplicates the actual

means by which the engine is controlled. However, most test plans are made by specifying planned temperature and pressure ramps for the nozzle chamber. To provide for easy pretest prediction of these ramps, an option has been added to the transient program to operate at demanded chamber pressures and temperatures.

To compute the engine system operation at a particular chamber pressure (P_c) and chamber temperature (T_c), initial guesses are first made for TPCV position and heat generation. An engine operating point is calculated, and the P_c is compared with the desired P_c . If they are different, a new TPCV position is tried, and a new operating point is calculated. This procedure is continued until a correct P_c is found. The chamber temperature is then checked, and, if necessary, a new heat-generation guess is made. At this point, the program starts over, again varies TPCV position to close the P_c to the desired value, and rechecks T_c . When both P_c and T_c are at the proper values, the time point calculation is considered to be complete.

Either the P_c or the T_c closure can be used alone. For example, a P_c demand and a heat-generation rate could be used as input; or, an open-loop system (no turbopump) could be used as input with flow rate and T_c used as the control variables.

(h) Simplified Hot-Bleed Port

The hot-bleed-port subroutine used in the transient engine simulation program (Reference 1) is a detailed model of the actual hot-bleed port. Because of the complexity of the calculations, a significant portion of the overall calculation time is spent on this subroutine. A study of the results of the routine showed that answers of acceptable accuracy could be obtained over a limited range of engine operation by using a greatly simplified hot-bleed-port model. In this simplified routine, the flows are calculated by using a pipe-flow loss coefficient in the equation:

$$\Delta P = \frac{K \dot{W}^2}{2g\rho A^2},$$

where

K = loss coefficient,

$\dot{W}$ = flow rate, lb/sec,

g = gravitational constant, ft/sec²,

ρ = fluid density, lb/ft³,

A = flow area, ft².

The mix-point pressure is assumed, and the hot, diluent, and turbine flows are calculated. The mix flow is then varied until all flows and pressures balance. An enthalpy balance is performed at the mix point to determine turbine inlet temperature.

The loss coefficients are obtained from runs made by using the detailed hot-bleed-port program for engine operating conditions near those where the simplified program is to be used. During any transient ramp, an occasional time point is checked with the detailed hot-bleed-port to ensure that accuracy is being maintained.

(i) Plotting Routine

Hand plotting of finished transient calculations has been found very time consuming. The 12013 program has been modified to prepare a tape for a program that plots key parameters against time.

As the program calculates engine-system operating data over the time range specified, all computed values are stored on a magnetic tape for each time point. This tape is read back by the computer at the finish of the transient run, and another program sorts the data, extracts specific values, changes them to a particular format, and writes them on another magnetic tape. This second tape is read by a CalComp plotter, and graphs of the data are prepared automatically.

The plot program is presently programed to give a series of nine plots for each run. These plots contain information on system flow rates,

fluid pressures, fluid temperatures, and turbopump operation. If desired, the output tapes from a series of transient runs can be processed together to make a composite plot.

(j) Steady-State Option

A single program was desired that would calculate both transient engine operation and steady-state points. A number of modifications were therefore incorporated in the transient engine simulation program to give it steady-state capability.

The transient-heat-conduction equations were rewritten with the heat-storage term set equal to zero. The resulting equation form requires successive iterations until the solution relaxes to the steady-state values. The fluid-flow solution is basically a steady-state analysis and so is left unchanged from the transient option.

The fluid-flow and material-conduction answers depend on each other. To make certain that a steady state has been reached, a number of passes are made through the system. The fluid flow and heat transfer are alternately upgraded on the basis of data from the preceding pass. When the temperatures and pressures stop changing, a steady-state point has been reached.

One advantage of combining the steady-state program with the transient program is that a transient run can be started from a calculated steady-state point. After a steady state has been reached, the program will automatically switch to transient operation and continue with the run.

(k) Restart Tape for Program Input

To make it easier to set up the input for a computer run, data from previous runs are stored on a magnetic tape that can be read by the computer. The tape contains system pressures, temperatures, flow rates, TPA operating parameters, and other necessary data as functions of time. A time point can be

selected from the tape that will supply initial values that are reasonable for the iteration routines in the program to use as first guesses. The tape is read as input, the proper values at the desired time are selected by the program, and the calculation begins.

By using the restart tape, an excessively long run can be broken up into short runs. The tape is used to supply information needed to start each section of the run.

(1) Iteration Routine Improvement

The closure logic in the transient engine simulation program is very complex. Originally a number of iteration routines were used in the program to generate new guesses for variables on the basis of their previous values. These routines were combined into a single master subroutine called by various portions of the program. This routine takes two previous values for a variable and uses a linear projection to a point of zero error. In most of the iterations, minimum and maximum limits are set and moved in to restrict the range of iteration and to speed up the closure.

To facilitate checkout of various parts of the main program, a special print option is included in the iteration routine. With this option, the results of any selected iteration or group of iterations within the program can be brought out. This capability is used to check such items as convergence rates and effects of changes on other closure routines that might be effected.

(m) Nozzle-Tube Choking

During normal engine operation, the hydrogen flow chokes at the main nozzle, but during unusual operating conditions when the inlet gas is much warmer than normal, choking may also occur in the nozzle tubes, as is the case during the first part of a bootstrap startup and during some of the cooldown operations.

To account for this effect, a test is made to determine the ratio of flow rate and choked flow rate both in the nozzle tubes and in the main nozzle. When the nozzle tubes choke, they become the point that determines the flow through the system.

At present, only one point in the system can be choked. If the nozzle tubes choke, their flow rate is used through the system. No tests for other choke points or pressure discontinuities are made. A change is being prepared that will allow multichoking by using the first choke point to calculate flow and finding the minimum step pressure drop after each choke point that is necessary to initiate choking at the next throat.

3.3.3 Program 1108 - Steady-State Program

(a) Improved Closure Routines

Difficulty was being encountered with the steady-state program in closing upon a solution for many low-power operating points. The cause of this problem was determined not to be in the system model but in the closure routines of the various iteration loops.

The following iteration closures were revised:

- (1) Bolt coolant flow
- (2) Hot-bleed-port stagnation exit pressure
- (3) Turbine flow
- (4) Hot-bleed-port Mach number
- (5) Hot-bleed flow
- (6) Diluent flow
- (7) Heat fluxes in the bleed port and nozzle tubes

With these changes, the program is now able to run low-power operating points.

(b) Addition of Bolt Coolant

Approximately 5 to 7% of the flow entering the nozzle-manifold bypasses around the nozzle tubes is used to cool the nozzle closure bolts and then enters the reflector inlet plenum. This flow is controlled primarily by orifices in the three bolt-coolant lines.

A one-channel bypass flow was added to the 1108 program to simulate the bolt-coolant bypass. Pressure losses are all taken at one point by using one standard loss coefficient.

No heat is added in the bypass line, and the nozzle outlet flow is mixed in the reflector inlet plenum with no losses. The overall nozzle pressure drop is determined with an iterative loop by guessing the flow distribution between the bolt coolant and main nozzle until the pressure drops match within closure tolerances.

(c) Unchoking of Turbine Exit

In several NRX/EST tests conducted at low power, the flow through the turbine was small and the turbine exit nozzle unchoked. The unchoked condition of the nozzle has a major effect on the turbine pressure ratio and, consequently, on work output of the turbine.

To properly predict low-power operation, turbine unchoking had to be accounted for at the exit of the turbine blades. The program normally computes the turbine exit pressure by using a fixed pressure ratio. A test was added, after normal closure, to back-calculate the pressure at the turbine blades by starting from the turbine exhaust. By using the calculated turbine flow and temperature, the pressure losses that occur upon leaving the turbine exhaust plenum and passing through the various lines are added to the ambient pressure. If this calculated pressure is greater than the exhaust pressure obtained from the fixed pressure ratio, the turbine is assumed to be unchoked, and the turbine work must be recomputed by using the higher back pressure. If the computed pressure backing up

through the lines is lower than the pressure from the fixed pressure ratio, the turbine exit is assumed to be choked, and expansion waves are assumed to occur shortly aft of the rotating blades. If there are restrictions in the turbine exhaust system, the first choking point is picked up to back-calculate the turbine exhaust pressure.

(d) Direct Printing of Steady-State Summaries

The 1108 program is used to prepare the steady-state maps for pretest predictions. During preparation of the maps, many steady-state runs are made, and hand tabulation of the data has been found to be very time-consuming.

A routine has been added to the 1108 program to print out each completed steady-state point in a form acceptable for publication. A sample output is shown in Figure 10.

(e) Closure on Turbine Inlet Temperature

A steady-state point can be calculated from a known tank pressure and temperature, a known diluent control valve setting and a desired nozzle chamber pressure and temperature, or a TPCV position and nuclear power. Design data points are generally specified by the tank conditions, the nozzle chamber pressure and temperature, and the turbine inlet temperature.

To obtain the desired turbine inlet temperature, it has been necessary to add a routine that adjusts the diluent-control-valve setting if the turbine inlet temperature is not correct after closing on nozzle chamber pressure and temperature.

3.3.4 Nozzle-Tube Analysis

A family of programs is available for nozzle-tube analysis. The programs being used and the special features that have been added to each are as follows:

(a) Program 18110, Steady-State Single-Channel Analysis

This program contains the same equations as used in the 1108 steady-state program. Component test data are studied with this program, and upgraded heat-transfer and fluid-flow coefficients are determined that can be added directly into the 1108 program. Special features that have been added are as follows:

(1) The program can analyze hydrogen, nitrogen, and water tests and hydrogen-oxygen combustion tests.

(2) The program can perform all calculations by using stagnation or static pressures. Analysis of NRX/EST data (see Section 3.4.3) has shown that the use of stagnation pressures for property lookups in the low-power range results in unacceptable errors.

(3) The program checks for choking. When warm gas is flowing, choking occurs at the throat, giving an increased flow resistance. During chill-down or warmup, it is necessary to check for the time at which choking occurs.

(4) The program input can include flow, inlet temperature, and either inlet or outlet pressure (or both) on the cold side. This option was found desirable in analyzing engine tests, where the outlet pressure may be more reliable than the inlet, or where both pressures may be known and an expansion wave is occurring within the nozzle. The program will compute the expansion-wave position and magnitude when operating with inlet and outlet pressures.

(5) The program will compute either cocurrent or counter-current flow. This option was added for dome cooling studies and for some NERVA applications where cocurrent flow was considered.

(6) A variety of cold-side heat-transfer relations can be used. Several means are available for calculating the coefficients and it has been found desirable to study the data by means of various relationships to ensure that the best analytical results are obtained. Options have been added to use any of the following:

1 - 2 Thompson A and B
3 Lewis
4 A modified Hess-Kuntz

(b) Program 18136, Parallel-Passage Steady-State Nozzle-Tube Program

This is called a separate program but is actually the same as the 18110 program except for a second passage on the cold side to simulate the tubes bisected by the hot-bleed port. To simplify programing, many of the special options available in the 18131 program have not been included in this program. The second simulated flow passage has a special subroutine for the hot-bleed port to more accurately determine temperatures and pressure losses.

(c) Program B-12013, Single-Passage Transient Nozzle-Tube Program

This program uses equations similar to those in the 12013 transient program. The program has just been completed and will be used to upgrade heat-transfer and pressure-loss coefficients for the transient program.

(d) Program 16183, Extension Skirt

This is a new nozzle-tube routine for the NERVA skirt analysis. Special features are discussed in Section 3.5.1.

3.4 UPGRADING OF EXISTING PROGRAMS BY USE OF TEST DATA

3.4.1 Introduction

The NERVA digital analytical models include the latest engineering techniques available. However, some simplifying assumptions must be made because of time limitations in developing the model computer-core storage capacity, and the cost of conducting the analyses. Coefficients and simplifying assumptions exist in the programs that are based on prior test data and must be reviewed as new test data become available. A continual effort is maintained to improve the reliability and confidence in analysis techniques for the NERVA models. The following sections summarize recent efforts in this area.

3.4.2 CFDTS Tests

Results of model improvements obtained from the CFDTS tests are discussed in detail in Reference 11. The outstanding accomplishment in these studies was development of low-speed turbopump correlations. Low-speed turbopump data had previously been obtained by extrapolating high speed data, and the extrapolations were inadequate. With the improved correlations, the transient program can now properly calculate the startup or bootstrap of the experimental NERVA engines. The program is currently being used in XE startup studies as an aid in test planning and for pretest predictions.

3.4.3 NRX/EST Tests

Studies are now being made to upgrade the digital programs by using the NRX/EST test data. The programs accurately predicted overall system performance (see Reference 12), but there were areas that required improvements. Improvements that have already been incorporated are as follows:

(1) The nozzle model has been modified to include a loss coefficient for the tube exit orifices.

(2) The technique for calculating temperature rise in the tubes has been changed. The fluid heat capacity (C_p) was used to calculate fluid temperature changes. This procedure caused no problem in the higher pressure regions, where there are neither rapid changes in C_p nor latent-heat considerations.

This assumption caused no problem in analyzing such tests as NRX/A3, where all holds were at relatively high power levels. However, in NRX/EST, a number of data points were obtained at low power; and obvious errors became apparent in the nozzle predictions. The specific-heat method had been originally used in the program because a property deck was not available with pressure and enthalpy lookup capabilities. With the merged tab properties deck now available (see Section 3.3.1.a), it is possible to readily change the fluid temperature routine. An enthalpy rise is now calculated that is based on the heat transfer, and temperature is determined from the pressure and enthalpy. The equations are as follows:

was:

$$T_{N+1} = T_N + \frac{q}{\dot{W} C_p};$$

changed to:

$$H_{N+1} = H_N + \frac{q}{\dot{W}};$$

find T_{N+1} from H_{N+1} and P_{N+1} ,

where

T = temperature, °R,
 q = heat flux, Btu/sec,
 $\dot{W}$ = flow rate, lb/sec,
 C_p = heat capacity, Btu/lb-°R,
 H = enthalpy, Btu/lb,
 P = pressure, psia.

In the two-phase region, quality is also determined from the enthalpy. From this quality, two-phase densities are calculated.

(3) The technique for looking up properties in the nozzle routine has been changed. Other deviations at low power were believed to be caused by the lookup of coolant properties at stagnation temperatures and pressures. In the aluminum jacket and the large U-tube configuration, velocity-head changes were not large enough to cause errors. Even with the small U-tube configuration, the only error induced at high pressure was in the density lookup. This error could be readily compensated for in the friction factor by a 5% adjustment. In the lower pressure regions tested in NRX/EST, all properties were more pressure sensitive. The programs are being modified to compute static pressures for property lookups.

(4) The curve for TPCV loss coefficient vs position for NRX/EST was found to be in error. The NRX/EST engine gave a different relationship than that obtained during in-house tests. Studies of the two systems indicated the in-house tests were conducted without the turbine blocking valve in the line, and its absence apparently affected the operation of the TPCV. The X-engine will be operated with the blocking valve downstream of the TPCV; so the NRX/EST data cannot be used directly in the programs for the X-engine predictions.

The X-engine valve will be tested with the blocking valve in its appropriate position for the final X-engine predictions. Pending receipt of these data, the original in-house data are being used for X-engine predictions.

Studies of the NRX/EST data are continuing, and additional minor changes are expected to be made in the digital system models on the basis of these data.

3.4.4 SN 21 Nozzle Analysis

Nozzle SN 21 was subjected to a comprehensive series of in-house tests with water and liquid oxygen and hydrogen. The nozzle was instrumented with pressure and temperature probes at various positions in the nozzle and afforded a good means

of checking the nozzle profiles predicted by the NERVA component programs (Section 3.3.4). A comprehensive comparison of test data and analytical predictions is presented in Reference 13. On the basis of these analyses, it was concluded that the nozzle model was accurately predicting nozzle performance, and no model changes were made.

3.4.5 Special Data Studies

The heat-transfer analysis presented in Reference 14 was studied to ensure that the best heat-transfer relations were being used in the nozzle-tube program. The equations of Hess and Kuntz appeared to give the best relation, but these equations still tended to predict a low heat-transfer coefficient below about 85°R.

The data were reviewed, and a reduction in the viscosity correction was found to greatly improve the prediction of low-temperature heat-transfer coefficients (see Figures 11 and 12).

The following equation gives the improved prediction:

$$h = 0.0164 \frac{K_f}{D} \left(\frac{D \rho_f v_B}{\mu_f} \right)^{0.8} \left(\frac{C_p \mu}{K} \right)_f^{0.4} \left(1 + \frac{.01457 \mu_w}{\mu_b} \right) \left(\frac{\mu_B}{\mu_{CR}} \right)^{0.67},$$

where

- C_p = specific heat, Btu/lb,
- D = hydraulic diameter, in.,
- K = thermal conductivity, Btu-in./in.²-°R-sec,
- h_c = heat-transfer coefficient, Btu/sec-°R-in.²,
- v = velocity, in./sec,
- ρ = density, lb/in.³,
- μ = viscosity, lb/in.-sec,

and subscript notation is as follows:

- B - evaluated at bulk fluid temperature,

- f - evaluated at film temperature ($T_f = \frac{T_{Bulk} + T_{Wall}}{2}$),
- w - evaluated at wall temperature,
- CR - evaluated at critical point.

The new equation will be added to all systems programs in the near future.

3.5 NEW PROGRAMS

3.5.1 NERVA

The NERVA engine is different enough from the technology engine to make it difficult to perform NERVA engine calculations with the same program deck used for the technology engines. Because of the large number of changes required, separate program decks have been prepared for both the transient and steady-state programs. The transient program has been changed to the 16178 program, and the NERVA steady-state program is called the 16180 program.

Both programs include the basic logic and equations of the technology-engine programs. The following paragraphs summarize the principal changes that have been incorporated or are planned:

(1) The input descriptions of flow area and configuration for all components have changed. The inputs are summarized in a data book that will be published when the engine design is better established. A record system has been initiated to keep track of the various design modifications made as development progresses.

(2) The basic engine model has been altered to include the removal of structural coolant from the pump discharge. This removal can be controlled by a control valve, can be varied to obtain a desired flow, or can be maintained as a fixed percentage of the pump discharge flow.

(3) The engine model provides for two hot-bleed ports and two turbine power-control valves. Each of the TPCVs can operate in parallel with a fixed orifice to give a limited control range.

(4) The nozzle model provides for cooling of various segments with different streams. Cocurrent or countercurrent cold- and hot-side flow can be handled. Extensive effort has been expended to develop the subroutine for handling skirt calculations with the turbine exhaust flowing through the skirt.

The routine developed is entered with a turbine exhaust pressure and temperature. The routine then computes the heat-transfer and pressure losses through the skirt until Mach one is reached. Mach one must be reached at the skirt exit, and the turbine exhaust pressure is adjusted until this condition is met.

Pressure calculations based on static pressure generally become unstable near Mach one, as very small increments of heat transfer or pressure loss (or both) will cause large changes in Mach number. Therefore, smaller nodes are required as Mach one is approached. A technique has been developed in which variable node sizes are used to prevent program failures as Mach one is approached.

In each node, the increasing area tends to lower the Mach number, but the friction losses and the heat addition tend to drive the Mach number to one. Each node is analyzed with friction, heat addition, and area change taken into account. The assumption is then made that the summation of effects of heat addition and area change would be small with respect to frictional changes. Thus, a Fanno line check is made of node size before the start of the node. If the value of $4 fL/D$ for the fluid is not twice that of the node, the input node length is halved until this requirement is met. By this procedure, the program is allowed to determine the necessary node size.

All initial pressure guesses are stored. If the fluid approaches Mach one before reaching the end of the tube, this pressure guess is noted as being too low. If the fluid reaches the end of the tube with a Mach number of less than one, this pressure guess is noted as being too high. The program iterates until pressure is within the input tolerance. (Closures to 0.01 psid have been achieved in less than 1 min with a 50% error in the initial guess.)

Two options are available with this routine. The first, as described above, calls for choking at the tube exit; the second calls for choking at the throat of a nozzle at the tube exit. In the second case, the Mach number at the end of the tube is determined by using the area ratio between the tube and the throat. The program iterates upon this Mach number.

The model can handle any type of turbopump and has been used to study six combinations of turbopumps, as follows:

- (1) High-efficiency centrifugal pump with a three-stage turbine
- (2) High-efficiency centrifugal pump with a seven-stage turbine
- (3) Low-efficiency centrifugal pump with a three-stage turbine
- (4) Low-efficiency centrifugal pump with a seven-stage turbine
- (5) Axial pump with a three-stage turbine
- (6) Axial pump with a seven-stage turbine

Both the steady-state and the transient programs are operational. Parametric studies are being conducted on the various turbopumps, and these studies include line sizes. Nozzle studies are in process, and a complete optimization program can be conducted with the model.

3.5.2 Statistical Data Analysis

Engine-system tests at NRDS develop multiple measurements on many variables. Three digital programs have been developed to statistically analyze the data and give best-estimate analyses. A technical memorandum is planned on these programs in which a more detailed description will be presented.

(a) Program 14081, NRX/EST Power Determination

This program determines best estimates of power, flow rate, chamber pressure, and chamber temperature by determining four estimates of each of these parameters. Three of the estimates are obtained by using three pairs of the remaining three variables, and the fourth estimate is the actual test measurement. For example, power, Q , is estimated from the following relations:

$$\begin{aligned} Q &= \text{test} \\ Q &= f(P_c, T_c) \\ Q &= f(P_c, \dot{W}) \\ Q &= f(T_c, \dot{W}) \end{aligned}$$

The uncertainty of each estimate is also computed by including in the input the uncertainties of each of the variables that influence the estimate. Thus, the uncertainty of Q depends on the uncertainties of chamber temperature, chamber pressure, the lumped constant of the nozzle choked-flow equation, turbine flow, diluent flow, enthalpy at the pump, and diluent enthalpy.

(b) Program 16155, Normal-Population Data Test Program

This program processes test data from magnetic tape or punched cards and calculates and plots the mean value and the confidence interval for the true mean. The data can be modified by applying a constant, if necessary, to each measurement array to account for bias correction. An "outlier" test is provided that tests the assumption that redundant measurements come from the same normal population. Data found to be outside the normal are not considered in the calculations. If a measurement continually fails the outlier test, it may be rejected from further calculations.

(c) Program 16160, Steady-State Mean and Standard-Deviation Calculations

Means and standard deviations are computed for specified time intervals for given parameters from NRX data tapes. The NTO data tapes are converted to usable form by the 16155 program, and provision is made for deleting extreme values. Measurements on the data tape are made at four samples per second, and all data for each desired channel are listed, as are the deleted measurements.

3.5.3 Analog Checkout

(a) Introduction

A digital program (coded for the IBM 1130) was developed to check the solutions obtained from the CAM II common analog model (Reference 15). The program is operative for the NRX/EST analog model and is currently being upgraded for the X-engine.

(b) Checking Method

The basic approach to checking the system for consistency is to take the analog results and run the values back through the basic equations simulated by the analog computer. If the analog is operating properly, the percent error between the analog values and the computed check value is small ($< 1\%$).

The steady-state equations are given in Figure 13, and several points should be noted:

(1) Equations marked with an asterisk (*) are not for a true steady-state value. Each of these variables is an integral that could not be evaluated directly at steady-state conditions. Therefore, the value of the integrand was evaluated. In all cases, these values should be small.

(2) Some parameters were not checked. However, space has been provided in both the equation set and in the computer so that these checks may be added later.

(3) Equation 21 has not been used. It may also be added later.

(4) Two parameters require functional data; namely P_5 and P_{15} . In the case of P_5 , two functions are required, $F_1(p_5)$ and $F_2(p_5)$. Curve fits were made to each of these as follows:

1. $F_1(p_5) = -0.06666 + 4.94999 p_5 + 1.91667 p_5^2$ for $0.2 < p_5 < 2$.
2. $F_2(p_5) = -18.016 p_5 + 268.85 p_5^2 - 18.85 p_5^3$ for $0 < p_5 < 1.4$.
 $F_2(p_5) = -499.667 + 678.33$ for $1.4 < p_5 < 2$.

The function $F_1(\theta_v)$ for P_{15} is determined by doing a polynomial interpolation on tabular data.

(c) Program

The program is written in card FORTRAN and is made up from five separate programs:

1. The Main Program

This program contains the input-output sections of the program. Data are read into the computer and checked for certain errors. If these errors are found, the following statement is printed out: "Input Error Type X." A list of the error types is given in Figure 14.

2. Subroutine CVONE

This subprogram solves Equations 1 through 40.

3. Subroutine CVTWO

This subprogram solves Equations 41 through 86.

4. Function THETA

This subprogram interpolates the tabulated values of $F_1(\theta_v)$ required for P_{15} .

5. Subroutine TEST

This subprogram checks values prior to taking the square root. If the value is negative, the following statement is printed out: "Argument of Square Root in Equation for CV (X) is negative and equal to Y." X corresponds to the equation number used in Figure 2, and Y is the actual value. The value of the argument is then set to positive, and the calculations are continued.

For convenience, variables are numbered the same as the equations.

(d) Program Input

The program input is divided into four groups:

1. The Theta Tables

These tables are values of $F_1(\theta_v)$ as functions of θ_v . The required format is given in Figure 14, Section a. The arrangement of data is given in Figure 14, Section d.

2. The Variable Name Table

These tables provide the variable names that are printed on the output sheets. Card format is given in Figure 14, Section c; and the order of the data is given in Figure 14, Section c.

3. The Scale-Factor Table

This table has the scale factor to be applied to the input data. The card format required is given in Figure 14, section b; and the data arrangement in Figure 14, Section f.

4. Problem Data Cards

This set of data contains the variables generally used in a given problem and those that will change from problem to problem. Two different formats are used. The comment card (first card) uses the format given in Figure 14, Section c; and the remaining cards use the format of Figure 14, Section b. All cards use the data arrangement in Figure 14, Section g.

As many sets of data as required may be run, provided no changes are made to groups A, B, and C of the data.

The order of the input is groups A, B, C, and D, in order.

A sample of the input data is given in Figure 15.

(e) Program Output

The input data are printed out for each variable in the output. In addition, the check value for each variable and the error as a percentage of the input value are also printed out. For convenience, a check of the error is made. The percentage errors are printed out in three separate columns. The groups 0 to 1%, 1 to 2%, and over 2% are printed in column 1, 2, or 3 of the error section of the output. A sample of the output is given in Figure 16.

3.5.4 Turbopump Analysis

A digital program was developed to aid in the analysis of turbine and pump performance. Experience in analyzing CFDTS data (see Section 3.4.2) pointed out the need for this special program. Data analysis programs previously existed for computing pump parameters, but in the case where the data were inadequate, such as for CFDTS, the existing programs could not be used to iterate and solve the pump efficiencies.

The basic problem in CFDTS data analysis was the lack of adequate means to determine the shaft horsepower because of poor outlet temperature data on both the pump and turbine, which made it difficult to determine the pump and turbine efficiencies. The subject program greatly assisted in this analysis because of the following features:

(a) The shaft horsepower can be calculated from input turbine correlations that compute efficiency as a function of U/C_o , where:

$$U = \text{blade tip speed,} \\ C_o = \text{spouting velocity} = 223.797 \sqrt{C_p \times \text{inlet temp}} \left\{ 1 - \left[\frac{1}{\text{press. ratio}} \right]^{\frac{\gamma - 1}{\gamma}} \right\}.$$

This procedure eliminates the need for the turbine outlet temperature.

(b) The shaft horsepower can be computed from pump temperatures and pressures.

$$\text{SHP} = \frac{1.4149 \times \text{pump flow rate}}{\text{actual enthalpy, pump inlet} - \text{isentropic enthalpy, pump outlet}}$$

(c) The shaft horsepower can be computed from a pump efficiency that is based on an input correlation (see sketch on next page) that relates the efficiency to speed and flow rate:

η_p = pump efficiency,
 N = speed, rpm,
 $\dot{W}$ = flow rate, lb/sec.

This correlation eliminates the need for a pump outlet temperature.

The above options were used by alternately computing pump and turbine efficiencies from either data or best-estimate correlations. This process was iterated until correlations were developed that appeared reasonable from an engineering standpoint, gave outlet temperatures that were logical, and the resultant correlations would reasonably predict engine startup in a transient program.

The program was written in FORTRAN IV for the IBM 1130 computer, which permitted a rapid turn-around time for the iterative studies.

3.5.5 Chilldown Analysis

A digital program to predict flow oscillations and pressure surges that occur during chilldown of cryogenic-fluid transfer lines has been developed that is based on system configuration and operating conditions and uses a one-dimensional flow model and conventional empirical equations for determining heat and momentum transfer. The detailed development of the analytical methods is given in Aerojet Report RN-TM-0246.

Comparison of the results of chilldown analysis with the available experimental data shows that the exact frequency and amplitude of flow oscillations cannot be predicted because of the lack of understanding of the nature of two-phase flow, especially for cryogenic fluids. However, the analytical results show that the gross hydrodynamic behavior of systems during chilldown may be studied with such a model.

The analytical results obtained thus far show that pressure surges twice as high as the supply-tank pressure may be expected, whereas the experimental results indicate that the pressure surges are only about 40% higher than the tank pressure. Similarly, the frequency of flow oscillations obtained from chilldown analysis is about 2 cps as compared with the experimental values of about 1 cps. A typical plot of pressure vs time during chilldown of the XE-engine inlet line is shown in Figure 17.

One of the serious limitations of the program for transient analysis of the entire NERVA engine system is the requirement for an extremely small computing time interval (0.01 sec), which would require a prohibitively long computer time for the transient systems analysis programs. Also, transfer lines consisting of grossly dissimilar flow passages cannot be treated. However, a study of the initial transients by using the chilldown analysis program in conjunction with one of the transient digital programs is contemplated. The details of carrying out such an analysis have not been worked out yet.

The byproducts of the chilldown analysis program are the system chilldown time and the propellant requirements to achieve a given degree of chilldown. Studies are being continued to determine the effects of such items as valve opening time and external heat fluxes on the frequency and amplitude of flow oscillations.

3.5.6 Choke-Flow Solving

(a) Introduction

Rapid and accurate estimation of power is always a problem during a nuclear-reactor test. One means of quickly checking the power is to use nozzle chamber conditions to compute power. For a given nozzle of known dimensions and for a known enthalpy entering the engine, power, flow, nozzle chamber pressure, and nozzle chamber temperature have a fixed relationship. From any two that are known, the remaining two can be calculated. A program has been developed that computes these relationships and prepares plots over the full range of conditions expected in NERVA Engines.

The choke-flow solver program is an IBM 7094 digital-computer program written in FORTRAN IV. The program is given a range for any two of the four parameters and, in turn, will compute the remaining pair. The program also computes thrust and specific impulse for each set of computed parameters. The program may also be used to generate a plot control tape for the Calcomp plotter that will give a visual display of the output. All combinations of power, flow, temperature, and pressure plots are available to any desired scale, as are thrust and specific-impulse plots. There is also an option available to plot voltage in place of temperature to correspond to raw test data.

(b) Discussion

General

The program has six options for input. The following table summarizes the various options:

Option	Variable			
	Input		Output	
	A	B	C	D
1	Pressure	Temperature	Flow	Power
2	Pressure	Flow	Temperature	Power
3	Pressure	Power	Temperature	Flow
4	Temperature	Flow	Pressure	Power
5	Temperature	Power	Pressure	Flow
6	Flow	Power	Pressure	Temperature

On the plots obtained from the program, the X and Y axes will always be the same as the calculated variables. The plots will include constant lines of the input variables. Specific impulse and thrust will be on the X and Y axes, with the input variables showing as constant lines on the plot.

The scales can be set at any magnitude desired. The CalComp plotter has an 11-in. limit on one axis, but the program will prepare partial plots that can be laid side by side to form a large figure.

The program will also convert any of the variables to a voltage reading if a curve or table of voltage vs the subject variable is provided as input. This feature was used on the plots prepared for the NRX/EST tests where only a nonlinear voltage was available to the X-Y plotter used with the test diagnostic console.

The program uses the Los Alamos hydrogen-properties subroutine discussed in section 3.3.1(a) to provide the thermodynamic and transport properties of hydrogen. At this time, the hydrogen-properties tables are limited to a temperature range from 36 to 5000°R and a pressure range from 0 to 1500 psi; however, additional properties can be added by changing only one control subroutine.

The program starts by holding input parameter A constant and moving by increments through the range of parameter B, solving each point by using the equation shown in Figure 18. If the point is within the valid region, thrust

and specific impulse are calculated, and the point is stored in a plot array table. After range B is exhausted, A is covered by increments and range B is restarted.

Printed output is produced as the points are being computed, and after all points are finished, as many plots as desired may be made by using the computed table and additional input-plot scaling cards.

The entire program has been written for ease of modification and broad general use by having many input options. The plot scaling can be computed or may be used as input for external control.

All calculations are performed in single, precision, floating-point mode, which will generally maintain greater accuracy than that of the input parameters. The convergence tolerances for the Newton iterations used can be externally controlled through input to save extra running time when extreme accuracy is not needed.

In using Option 6 and calculating a maximum-size map of 2500 points, the program ran to completion in 3.17 min with 6 output plots being made.

The program has numerous checks on input and data validity. A complete listing of the error messages can be found in Figure 19; a condensation of the name-list input is shown in Figure 20.

Main Program

The main program contains all the controls that affect the total flow through the main logic. Permanent BCD input is read and stored. The first input case is read, initialized, and checked for validity. Output headings are called on the first input case entry, and selected optional equations are computed for the number of points desired. Thrust and specific impulse are computed when wanted, and point computation is stored in the plot tables as well as being written on the output print if it is not suppressed. After all points for a given case are computed, control is transferred to the plot-control subroutine, where any

desired plots are made by using this new data array. When control is returned after plotting, the data and some controls are reinitialized and the next input case is read. When all input cases are completed, the program is closed and the job is finished, with the computer control being returned to the system monitor.

Subroutine Plot Control

The plot-control subroutine contains the logic to make all decisions necessary to draw plot frames and curves from the computed array.

This subroutine must call either the standard or nonstandard plot frame routines and read in the input scaling when the nonstandard frame is wanted. Valid and invalid points are separated so that points where computations did not converge are not plotted. The array is separated for the nonstandard plots so that only the points that are within the scale are used. Curves are plotted from temporary push-down stacks, with increment reversing to minimize the plot-pen dead-head transport time. The title-card read flag and plot length are reset for each new plot so that several very long plots cannot be made accidentally. Titles are changed on the axis labels to match the option used to compute the array.

The entire process is repeated for the thrust and specific-impulse plots when they are called. The main array is one-dimensional to increase speed. Therefore, stepping intervals must be calculated with the skipping interval such that isotherms and isobars, or the other steady parameters, can be plotted.

Subroutine Datum

This subroutine is used to preset initial conditions and standard input parameters prior to execution of the main program.

Subroutine Print

This subroutine is used to keep the line-count and page-count logic. It also prints the standard Aerojet heading the first time through and the

new page headings when they are needed. This subroutine in turn calls for printing of either input or output as desired and contains print suppression and maximum-page-flush logic.

Subroutine Print Input

This subroutine is used to print the input parameters at the start of each case. Multiple entry points are entered from print, and the number of lines printed is returned for the line-count logic.

Subroutine Print Output

This subroutine differs from print input only in the context of the material printed.

Subroutine Photo

(map)

This subroutine is used to communicate between a FORTRAN source and a map subroutine that is needed to call the Star plot. The Star-plot subroutine was not in FORTRAN source language at the time the photo subroutine was written.

Subroutine *Frame

(map)

This subroutine is the standard *Frame routine used at Aerojet to scale and plot curves without using extensive external controls. The standard version has been modified so that variable x and y storage increments can be given. Many of the control parameters have been placed in block common so that they are accessible in the FORTRAN plot-control subroutines.

Subroutine Aerojet Header

This subroutine is used in conjunction with the subroutine program number to write a standard Aerojet letter heading at the start of each run.

Subroutine Program Number

(map)

This subroutine picks up time of day, program number, and program user from the system nucleus and sends this information to be used in the heading.

Subroutine Antipa

This subroutine is used to check the range of input parameters to the parahydrogen tables. Values above or below are set to the table limits, warning messages are written if called for, and the proper tab deck is entered. Any tab deck can be used by changing this one subroutine. When the tab-T deck is entered with a temperature above 5000°R, enthalpy only is extrapolated so that some of the iteration will close near the table limit. The called-for properties, temperature, or enthalpy are returned in the calling sequence; and all properties are available in a block common section until the next deck entry is made. This common is cleared internally in the properties deck upon each new entry.

Subroutine Tab-T and Tab-H

(Prest map)

The current version of the merged properties deck has two entry points, one for an entry with temperature and pressure, the other for entry with pressure and enthalpy. The number of properties returned can be limited by setting an input flag. This procedure is used to save time by not looking in the tables for properties that are not needed. All properties that are returned are placed in a block common region. The current merged properties deck is based on work by O. Farmer at Los Alamos.

Input for Program 16161

Cards 1, 2, 3, 4, and 5

These five cards are permanent BCD input data used for the standard plot tables. The data are furnished with the deck and need not be changed unless it is wished to rename the standard plot tables.

The input format is 12A6, and values on these five cards are read into an array called BCD with a dimension of 60; if details are desired, please see "program listing."

Start of Normal Input

For additional information on using the name list input, see Figure 20. A description of the variables and their use follows this section, which describes the input format only.

Card Type A

These cards are the first of each new input case and have the following format:

Col 1	=	b (left blank)
Col 2	=	\$
Col 3	=	N
Col 4	=	U
Col 5	=	C
Col 6	=	A
Col 7	=	S
Col 8	=	E
Col 9	=	b (left blank)
Col 10 to col 80.		May have values for any variables in the Dump (i) array (normally not used). This card type is ended by placing a \$ sign in a col greater than two.

Card Type B (one card only)

This card is the title that will be printed at the start of this case output. This card can have any alpha-numeric data in col 1 to col 72 and must be included even if it is all blank.

Card Type C

This card (one or more) is used to provide the input for all of the program control parameters that direct the logic to be used for the following case:

Col 1	=	b (left blank)
Col 2	=	\$
Col 3	=	C
Col 4	=	Ø
Col 5	=	N
Col 6	=	T
Col 7	=	R
Col 8	=	L
Col 9	=	b (left blank)
Col 10 to 80		May have values for any variables in the K or V (or both) arrays.

This card type is ended by placing a \$ sign in a col greater than two.

Card Type D

This card (one or more) is used to input the actual parameters for this case that are different from either the present input or the last case, whichever applies.

Col 1	=	b (left blank)
Col 2	=	\$
Col 3	=	I
Col 4	=	N
Col 5	=	P
Col 6	=	U
Col 7	=	T
Col 8	=	b (left blank)

Col 9 to Col 80 May contain values for any of the variables listed below

QMAX ,	A
Hi ,	NØA
AT ,	DELA
CN ,	B
EPSIL,	NØB
PA ,	DELB
AE ,	

This card type is ended by placing a \$ sign in a col greater than two.

Optional-Plot Card Sets

These optional-plot card sets are required for each plot requested in K(6) and K(7) of the preceding input. The total number of sets required will be:

$$\{ K(6) + K(5) \} * K(11),$$

where K(6) and K(5) are either 1 or zero.

If no plots are made, these cards are not needed.

Card Type E

This card (one or more) is used to provide input for the plot control logic. Most of these are not used when it is elected to use the standard scaling option, and most variables are needed when it is elected to externally set the scaling.

Col 1	=	b (left blank)
Col 2	=	\$
Col 3	=	P
Col 4	=	L
Col 5	=	Ø

Col 6 = t
Col 7 = b (left blank)
Col 8 - 80 May contain values for any of the variables
 listed below

IASKIP

IBSKIP

XNOT

YNØT

DELX

DELY

INCHES*

NOTITL*

This card type is ended by placing a \$ sign in a col greater
than two.

Card Type F

Col 1 to Col 36 Any alpha-numeric data that are to be used for
the title on this plot.

* See description of the variables inches and notitl, as they are exceptions to the
normal scatter loading.

3.5.7 Gas Injection

A study was conducted to determine the side force that could be exerted on the NERVA engine by injecting hot gas into the nozzle expansion cone. The techniques currently available for analyzing hot-gas injection are limited, and it was necessary to find a mathematical model and develop a program for calculating side forces and thrust changes.

The mathematical model developed in Reference 16 and extended in Reference 17 is used in this program. Conical shock-wave relationships are entered into the program as second-degree polynomial equations derived from graphical data by the least-squares curve-fit method. The Newton-Raphson iteration routine, presented in Reference 18, is used to solve the simultaneous equations.

The basic assumptions of this model are illustrated in Figure 21. The gas is assumed to flow into the nozzle perpendicular to the free stream, thereby producing a separated-flow area and a shock wave. Both the separated-flow area and the shock wave are initiated from a point upstream and form a cone from that point (apex) back to the injection port. The portion of the shock wave and flow-separated area aft of the injection port are ignored. The model assumes that the effective area producing side force under each of these cones can be simulated by projecting the area that is bounded by the intersection of the cones and the surface of the nozzle wall. The projected areas and the average pressure differences between the free stream and the areas under the two cones are then computed by using semiempirical equations. The total side force is the sum of (1) the force (area times pressure differential) for the region between the shock wave and the separated-flow boundary, (2) the force for the separated-flow region, and (3) the thrust due to the injected gas. A listing of the equations is shown in Figure 22.

The program is operational and was used during studies on NERVA to determine the feasibility of using turbine exhaust or an engine bleed stream for thrust-vector control.

3.5.8 Advanced Transient Program

Work was initiated on a more complex transient program for NERVA engine analysis. The program is built basically around the comprehensive solution of the eight simultaneous differential equations reported in Section 3 of Reference 7. These equations have no omissions for one-dimensional fluid-flow analysis with area change, energy transfer, and friction.

The program will improve the accuracy of the overall system and will allow the analysis of wave propagation with fluids at extremely high frequencies. With this type of program, the oscillations noted at low power on NRX/EST can be predicted in advance.

Basic coding and logic networks have been completed, and the program is being checked. Problems still remain to be solved, but some operating data have been obtained that indicate there will be no additional computing costs required with the new program.

The program is expected to be available for limited analysis in the next increment. There is no present plan to phase out the current transient digital program, as it has many options built into it (Section 3.3.2) that require considerable effort to add and check out.

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IDEAL-GAS (NO PRESSURE EFFECT) CORRELATIONS FOR NITROGEN THERMODYNAMIC PROPERTIES*

A. BASIC EQUATIONS

1. $C_p = A + BT + CT^2 + DT^3 \dots + HT^7;$
2. $H = AT + \frac{BT^2}{2} + \frac{CT^3}{3} + \frac{DT^4}{4} \dots + \frac{HT^8}{8} + H_o;$
3. $S = A \ln T + BT + \frac{CT^2}{2} + \frac{DT^3}{3} \dots + \frac{HT^7}{7} + S_o;$
4. $U = H - RT = H - 0.29678419T;$
5. $C_v = C_p - 0.29678419;$

where C_p = specific heat at constant pressure,
 H = enthalpy,
 S = entropy,
 U = internal energy,
 C_v = specific heat at constant volume.

Fitted-range:

lower T, °K	60	250	800	1300
upper T, °K	270	850	1600	5000
Program range, °K	60-250	250-800	800-1500	1500-5000
A	+1.0381732 E+00	1.0277390 E+00	1.3446104 E+00	+7.036734 E-01
B	-3.3542924 E-06	3.1823095 E-05	-1.9405777 E-03	+8.442185 E-04
C	+5.5150190 E-08	8.4276021 E-07	+4.5430063 E-06	-5.570147 E-07
D	-2.9295079 E-10	-6.9609199 E-09	-4.8092449 E-09	+2.228971 E-10
E	+5.6600875 E-13	2.1441671 E-11	+2.7603232 E-12	-5.612205 E-14
F	-0-	-3.0397145 E-14	-8.3752678 E-16	+8.732061 E-18
G	-0-	+2.0758716 E-17	+1.0574123 E-19	-7.688294 E-22
H	-0-	-5.5840862 E-21	-0-	+2.934251 E-26
H _o	1.5087518 E-02	+1.5178322 E+02	+1.1207774 E+02	+2.400940 E+02
S _o	-1.4997738 E+00	-1.4560852 E+00	-2.8368271 E+00	+2.310950 E-01

*Taken from NBS Circular 564.

Figure 1

Ideal-Gas (No Pressure Effect) Correlations for
Nitrogen Thermodynamic Properties

EMPIRICAL EQUATION FOR CALCULATING THE VAPOR PRESSURE OF NITROGEN AT LOW TEMPERATURES

Vapor pressure below 126°K

$$1. \quad \text{Log}_{10} P_{vp} = A + B/T + CT + DT^2 + ET^3 + FT^4 + GT^5,$$

where

P_{vp} = vapor pressure, atmospheres;

T = temperature, °R.

A	+0.527805	E+00
B	-.30507339	E+03
C	+.16441101	E+00
D	-.31389205	E+02
E	+.29857103	E-04
F	-.14238458	E-06
G	+.27375282	E-09

Vapor pressure from 126 to 127.122°F

$$2. \quad P_{vp} = P_{vp}^1 - (TN)^2 \times (T.1786028 - 0.12046123 \times TN),$$

where

P_{vp} = vapor pressure,

P_{vp}^1 = vapor pressure from Equation 1,

TN = $T - 126.0$,

T = temperature, °R.

Figure 2

Empirical Equations for Calculating the Vapor Pressure of
Nitrogen at Low Temperatures

BENEDICT-RUBIN-WEBB EQUATION OF STATE USED TO CALCULATE
NITROGEN DENSITY BELOW 540°R

$$\begin{aligned}
 P = & + \text{SCT}(1) * \rho \\
 & + \text{SCT}(2) * \rho^2 \\
 & + \text{SCT}(3) * \rho^3 \\
 & + \text{SCT}(4) * \rho^4 \\
 & + \rho^{-0.0056} \rho^2 \times [\text{SCT}(5) \times \rho^3 + \text{SCT}(6) * \rho^5] \\
 & + \text{SCT}(7) * \rho^6;
 \end{aligned}$$

$$\text{SCT}(1) = R * n_1 * T,$$

$$\text{SCT}(2) = R * n_1 * T + n_2 + n_3/T + n_4/T^2 + n_5/T^4,$$

$$\text{SCT}(3) = R * n_6 * T,$$

$$\text{SCT}(4) = n_8 * T,$$

$$\text{SCT}(5) = n_9/T^2 + n_{10}/T^3 + n_{11}/T^4,$$

$$\text{SCT}(6) = n_{12}/T^2 + n_{13}/T^3 + n_{14}/T^4,$$

$$\text{SCT}(7) = n_{15},$$

where

P = pressure, atmospheres;

ρ = density, gm-moles/liter;

T = temperature, °R.

R	+0.820574	E-01
n_1	+0.3371608442	E-01
n_2	-0.5771942866	E+00
n_3	-0.1142108127	E+03
n_4	+0.8522634899	E+03

Figure 3 (Page 1 of 2)

Benedict-Rubin-Webb Equation of State Used to Calculate
Nitrogen Density Below 540°R

BENEDICT-RUBIN-WEBB EQUATION OF STATE USED TO CALCULATE
NITROGEN DENSITY BELOW 540°R (cont.)

n ₅	+0.3440176200	E+07
n ₆	+0.1650365874	E-02
n ₇	-0.1578905910	E-01
n ₈	+0.4168356912	E-05
n ₉	+0.3211549057	E+03
n ₁₀	+0.1080120452	E+06
n ₁₁	-0.1066657899	E+08
n ₁₂	-0.3304489192	E+01
n ₁₃	+0.1223693626	E+04
n ₁₄	-0.5693539048	E+05
n ₁₅	+0.1675167178	E-05

Figure 3 (Page 2 of 2)

BASIC THERMODYNAMIC EQUATIONS
USED TO CALCULATE NITROGEN PROPERTIES

$$(1) \quad H = U(\text{ideal}) + \sum U \text{ corrections} + 3.6167832 * \frac{P}{\rho},$$

$$(2) \quad S = S(\text{ideal}) + \sum S \text{ corrections},$$

$$(3) \quad C_p - C_v = \frac{3.6167832 \times T \times \left(\frac{\partial P}{\partial T} \right)_\rho}{\rho^2 \times \left(\frac{\partial P}{\partial \rho} \right)_T},$$

$$(4) \quad C_p = C_v \text{ ideal} + \sum C_v \text{ Corr} + (C_p - C_v),$$

$$(5) \quad U (\text{Corr}) = 3.6167832 \times \int_{\rho_1}^{\rho_2} \frac{P_{eq} d\rho}{\rho^2} - T \int_{\rho_1}^{\rho_2} \frac{\left(\frac{\partial P}{\partial T} \right)_\rho}{\rho^2} d\rho,$$

$$(6) \quad S(\text{Corr}) = 3.6167832 \times \int_{\rho_1}^{\rho_2} \frac{\left(\frac{\partial P}{\partial T} \right)_\rho}{\rho^2} d\rho - 0.29678419 \times \ln (RT \rho),$$

$$(7) \quad C_v(\text{Corr}) = -3.6167832 \times \int_{\rho_1}^{\rho_2} \frac{\left[\frac{\partial^2 P}{\partial T^2} \right]_\rho}{\rho^2} d\rho \times T,$$

where H = enthalpy at desired pressure and temperature,

$U (\text{ideal})$ = internal energy at desired temperature and zero pressure,

obtained from Equation 4, Figure 1,

Figure 4 (Page 1 of 3)

Basic Thermodynamic Equations Used to Calculate Nitrogen Properties

BASIC THERMODYNAMIC EQUATIONS
USED TO CALCULATE NITROGEN PROPERTIES (cont.)

correction = sum of the internal energy corrections calculated in going from zero pressure to actual pressure (above the critical temperature, Equation 5 is used for the correction; below the critical temperature, Equation 5 is used from zero pressure to the vapor pressure and again from the vapor pressure to the actual pressure. The equations in Figure 5 are used to go from saturated gas to saturated liquid at the vapor pressure.

$\sum U \text{ correction } V = \text{sum of the three corrections.}),$

P = pressure,

ρ = density,

S = entropy,

S ideal = entropy at actual temperature and zero pressure obtained from Equation 3, Figure 1,

$\sum S \text{ corrections} = \text{sum of the entropy corrections in going from zero pressure to actual pressure. (Above the critical temperature, this is obtained from Equation 6; below the critical temperature, a three-step correction is used: from zero pressure to the vapor pressure, Equation 6 is used; from saturated vapor to saturated gas, Equation 2 of Figure 5 is used; from saturated liquid to compressed liquid at actual pressure, Equation 6 is used.}$

$\sum S \text{ corrections} = \text{sum of the three corrections.}),$

C_p = specific heat at constant pressure,

C_v = specific heat at constant volume,

T = temperature, °R,

Figure 4 (Page 2 of 3)

BASIC THERMODYNAMIC EQUATIONS
USED TO CALCULATE NITROGEN PROPERTIES (cont.)

$\left(\frac{\partial P}{\partial T}\right)_\rho$ = partial derivative of the equation of state (Figure 2) with respect to temperature at constant density,

$\left(\frac{\partial P}{\partial \rho}\right)_T$ = partial derivative of the equation of state (Figure 2) with respect to density at constant temperature,

$\Sigma C_v \text{ corr}$ = sum of the constant-volume specific-heat corrections from zero pressure to the actual pressure (Above the critical temperature, one correction is obtained by using Equation 7. Below the critical temperature, Equation 7 is used from zero pressure to vapor pressure; Equation 3, Figure 5, is used from saturated vapor to saturated liquid; and Equation 7 is used again from saturated vapor to the actual pressure. ΣC_v = sum of the three corrections.),

$U (\text{corr})$ = internal-energy correction from zero pressure to any desired pressure,

P_{eq} = equation of state (Figure 2),

$d\rho$ = differential of density,

$S(\text{corr})$ = entropy correction from zero pressure to any desired pressure.

R = gas constant,

$C_v(\text{corr})$ = constant-volume specific-heat correction from zero pressure to any desired pressure,

$\left(\frac{\partial^2 P}{\partial T^2}\right)_\rho$ = second partial derivative of the equation of state with respect to temperature at constant density.

Figure 4 (Page 3 of 3)

EQUATIONS FOR DETERMINING NITROGEN THERMODYNAMIC-PROPERTY CORRECTIONS
GOING FROM SATURATED VAPOR TO SATURATED LIQUID

$$(1) \quad U \text{ corr} = \Delta V \times (P_{\text{v.p.}} - T \times \frac{dP_{\text{vp}}}{dT}),$$

$$(2) \quad S \text{ corr} = \Delta V \times \frac{dP_{\text{vp}}}{dT},$$

$$(3) \quad C_v \text{ corr} = -T \times \Delta V \times \frac{d^2 P_{\text{vp}}}{dT^2} + CVX,$$

$$(4) \quad \Delta V = 3.6167832 \times \left(\frac{1}{\rho_{\text{sat vapor}}} - \frac{1}{\rho_{\text{sat liq}}} \right),$$

$$(5) \quad CVX = 3.6167832 \times \left[CVY(1) \times CVZ(1) - CVY(2) \times CVZ(2) \right. \\ \left. + (T \times \frac{dP_{\text{vp}}}{dT} - P_{\text{vp}}) \times (CVZ(2) - CVZ(1)) \right],$$

$$(6) \quad CVY = T \times \left(\frac{\partial P}{\partial T} \right)_\rho - P_{\text{vp}},$$

$$(7) \quad CVZ = \frac{\left(\frac{\partial P}{\partial T} \right)_\rho - \frac{dP_{\text{vp}}}{dT}}{\rho^2 \times \left(\frac{\partial P}{\partial \rho} \right)_T},$$

where U corr = internal-energy correction for nitrogen going from saturated gas to saturated liquid (state 1 to state 2),

ΔV = change in specific volume (see Equation 4),

P_{vp} = vapor pressure,

T = temperature,

$\frac{dP_{\text{vp}}}{dT}$ = derivative of the vapor-pressure equation (Figure 2) with respect to temperature,

$S \text{ corr}$ = entropy correction going from saturated gas to saturated liquid,

$CV \text{ corr}$ = constant-volume specific-heat correction going from saturated gas to saturated liquid,

Figure 5 (Page 1 of 2)

Equations for Determining Nitrogen Thermodynamic-Property
Corrections Going from Saturated Vapor to Saturated Liquid

EQUATIONS FOR DETERMINING NITROGEN THERMODYNAMIC-PROPERTY CORRECTIONS
GOING FROM SATURATED VAPOR TO SATURATED LIQUID (cont.)

$$\frac{d^2 P_{vp}}{dT^2} = \text{second derivative of vapor-pressure equation (Figure 2) with respect to temperature,}$$

$$CVX = \text{correction factor (see Equation 5),}$$

$$\rho_{\text{sat vapor}} = \text{saturated-vapor density,}$$

$$\rho_{\text{sat liq}} = \text{saturated-liquid density,}$$

$$CVY = \text{correction factor,}$$

$$CVZ = \text{correction factor,}$$

$$\text{subscript (1)} = \text{sat vapor,}$$

$$\text{subscript (2)} = \text{sat liquid,}$$

$$\left(\frac{\partial P}{\partial T} \right)_{\rho} = \text{partial derivative of the equation of state (Figure 2) at constant density,}$$

$$\left(\frac{\partial P}{\partial \rho} \right)_T = \text{partial derivative of the equation of state (Figure 2) at constant temperature.}$$

EMPIRICAL EQUATIONS FOR CALCULATING HIGH-TEMPERATURE
THERMODYNAMIC PROPERTIES OF NITROGEN (ABOVE 468°R)

$$(1) \quad PV = Z RT,$$

$$(2) \quad Z = 1 + F_1(T)P + F_2(T)P^2 + F_3(T)P^3$$

Below 105 Atm

$$(3) \quad F_1(T) = \frac{A1}{T} + \frac{B1}{T^2} + \frac{C1}{T^3} + \frac{D1}{T^3} + \frac{E1}{T^5} + \frac{F1}{T^6} + \frac{G1}{T^7} + \frac{H1}{T^8}$$

$$(4) \quad F_2(T) = \frac{A2}{T^2} + \frac{B2}{T^3} + \frac{C2}{T^4} + \frac{D2}{T^5} + \frac{E2}{T^6} + \frac{F2}{T^7} + \frac{G2}{T^8},$$

$$(5) \quad F_3(T) = \frac{A3}{T^3} + \frac{B3}{T^4} + \frac{C3}{T^5} + \frac{D3}{T^6},$$

Above 105 Atm

$$(6) \quad F_1(T)_{\text{new}} = F_1(T)_{\text{old}} - 0.11025E + 05 \times F_3(T)_{\text{old}},$$

$$(7) \quad F_2(T)_{\text{new}} = F_2(T)_{\text{old}} + 210. \times F_3(T)_{\text{old}},$$

$$(8) \quad F_3(T)_{\text{new}} = 0,$$

$$(9) \quad H = H_{\text{ideal gas}} - 0.29678419 \int_p \left(\frac{\partial Z}{\partial T} \right) dP,$$

$$(10) \quad S = S_{\text{ideal gas}} - 0.29678419 \left[\int_p Z dP + T \int_p \left(\frac{\partial Z}{\partial T} \right) dP + \log P \right],$$

$$(11) \quad C_p = C_{p \text{ ideal gas}} - 0.29678419 T \left[2 \int_p \left(\frac{\partial Z}{\partial T} \right) dP + T \int_p \left(\frac{\partial^2 Z}{\partial T^2} \right) dP \right]$$

$$(12) \quad C_v = C_p - (C_p - C_v),$$

$$(13) \quad C_p - C_v = 0.1013249 T \left(\frac{\partial V}{\partial T} \right)_p^2 - \left(\frac{\partial V}{\partial P} \right)_T,$$

$$(14) \quad \text{Vel sound} = \sqrt{2.96.78393 \cdot \gamma \cdot \left(\frac{\partial P}{\partial \rho} \right)_T},$$

Figure 6 (Page 1 of 3)

Empirical Equations for Calculating High-Temperature
Thermodynamic Properties of Nitrogen (above 468°R)

EMPIRICAL EQUATIONS FOR CALCULATING HIGH-TEMPERATURE
THERMODYNAMIC PROPERTIES OF NITROGEN (ABOVE 468°R) (cont.)

$$(15) \quad \left(\frac{\partial V}{\partial T} \right)_P = \frac{2.9290297 \left[Z + T \left(\frac{\partial Z}{\partial T} \right)_P \right]}{P},$$

$$(16) \quad \left(\frac{\partial V}{\partial P} \right)_T = \frac{2.9290297 \times T \left[\left(\frac{\partial Z}{\partial P} \right)_T - Z/P \right]}{P},$$

$$(17) \quad \left(\frac{\partial P}{\partial \rho} \right)_T = \left(\frac{P}{2.9290297 \times Z \times T} \right)^2 \left(\frac{\partial V}{\partial P} \right)_T,$$

where

P = pressure, atmospheres,

V = specific volume, cc/gm,

Z = correction factor (function of pressure and temperature),

R = gas constant,

T = temperature, °K,

$F_1, F_2, \text{ \& } F_3$ = functions of temperature,

$\left. \begin{array}{l} A_1 \longrightarrow H_1 \\ A_2 \longrightarrow G_2 \\ A_3 \longrightarrow D_3 \end{array} \right\} = \text{constants, as follows:}$

$A_1 - 3.581956 \text{ E} - 01$	$A_2 - 1.7944294 \text{ E} - 02$	$A_3 + 1.045052 \text{ E} - 01$
$B_1 + 2.711694 \text{ E} + 02$	$B_2 - 3.082885 \text{ E} + 01$	$B_3 - 3.139792 \text{ E} + 02$
$C_1 - 5.016151 \text{ E} + 05$	$C_2 + 2.213507 \text{ E} + 05$	$C_3 + 1.196195 \text{ E} + 05$
$D_1 + 3.528861 \text{ E} + 08$	$D_2 - 1.590002 \text{ E} + 08$	$D_3 - 8.281343 \text{ E} + 06$

Figure 6 (Page 2 of 3)

EMPIRICAL EQUATIONS FOR CALCULATING HIGH-TEMPERATURE
THERMODYNAMIC PROPERTIES OF NITROGEN (ABOVE 468°R) (cont.)

$$E_1 - 1.563224 \text{ E} + 11 \quad E_2 + 5.732878 \text{ E} + 10$$

$$F_1 + 4.136719 \text{ E} + 13 \quad F_2 - 1.089192 \text{ E} + 13$$

$$G_1 - 5.964381 \text{ E} + 15 \quad G_2 + 8.318625 \text{ E} + 14$$

$$H_1 + 3.595164 \text{ E} + 17$$

H = enthalpy at actual temperature and pressure,

H ideal gas = enthalpy at actual temperature and zero pressure (see Figure 1),

$$\left(\frac{\partial Z}{\partial T}\right)_p, \left(\frac{\partial^2 Z}{\partial T^2}\right)_p = \text{first and second partial derivatives of equation 2 with respect to temperature at constant pressure,}$$

S = entropy at actual temperature and pressure,

S ideal gas = entropy of the ideal gas at actual temperature and one-atmosphere pressure (see Figure 1),

Cp = specific heat at constant pressure,

Cp ideal gas = constant-pressure specific heat at actual temperature and zero pressure,

Cv = constant-volume specific heat,

$$\left(\frac{\partial V}{\partial T}\right)_p = \text{partial derivative of specific volume with respect to temperature at constant pressure,}$$

$$\left(\frac{\partial V}{\partial P}\right)_T = \text{partial derivative of specific volume with respect to pressure at constant temperature,}$$

$$\left(\frac{\partial P}{\partial \rho}\right)_T = \text{partial derivative of pressure with respect to density at constant temperature,}$$

$$\gamma = \text{ratio of specific heats, CP/CV.}$$

Figure 6 (Page 3 of 3)

EMPIRICAL EQUATIONS
FOR CALCULATING THE VISCOSITY OF NITROGEN

Above 1500°K

$$(1) \text{ Visc} = 0.8137811 \text{ E} - 04 \times T^{(0.564581048)} + 1.5 \text{ E} - 04 \times \rho$$

Below 1500°K

$$(2) \text{ Visc (1 atm)} = A + BT + CT^2 + DT^3 + ET^4 + FT^5,$$

$$(3) \text{ Visc} = \text{Visc (1 atm)} + \text{corrections.}$$

Density Corrections

If density is below 0.1132 gm/cc,

$$(4) \text{ Corr 1} = (1 + VB \times \rho) \times 0.175 + (VB \times \rho)^2 \times 0.8651,$$

$$(5) \text{ VB} = \left(\frac{\sqrt{T}}{\text{visc (1 atm)}} \right)^{1.5} \times 0.775 \text{ E} - 07$$

If density is between 0.1132 and 0.376 gm/cc,

$$(6) \text{ corr 2} = + 3.6505 \text{ E} - 04 \times (\rho)^{1.3622926}.$$

If density is greater than 0.376 gm/cc,

$$(7) \text{ corr 3} = + 2.5528 \text{ E} - 03 \times (\rho)^{3.3502366},$$

Where visc = viscosity, poises,

T = temperature, °K,

ρ = density, gm/cc,

visc (1 atm) = viscosity of the ideal gas at 1 atmosphere,

VB = correction factor,

corr 1 - corr 3 = density correction factors,

A - F = constants, as follows:

Figure 7 (Page 1 of 2)

Empirical Equations for Calculating the Viscosity of Nitrogen

A = - 4.1269284 E -06,
B = + 8.1599805 E -07,
C = - 8.9865456 E -10,
D = + 8.0346912 E -13,
E = - 3.9516078 E -16,
F = + 7.8557164 E -20.

EMPIRICAL EQUATIONS
FOR CALCULATING THE THERMAL CONDUCTIVITY OF NITROGEN

Below 300°K

$$K = \frac{T}{1 + \frac{2.17.6 \times 10^{-7.75}}{T}} \times 2.45320 \text{ E } -05 + \text{density correction}$$

Above 300°K (t = T - 273.2; t is in degrees centigrade)

$$K = 2.4147 \text{ E } -04 + 7.558 \text{ E } -07 \times t - 3.2116 \text{ E } -10 \times t^2 + 6.3507 \text{ E } -14 \times t^3 + \text{density correction.}$$

Density correction

Below a density of 0.5 gm/cc,

$$\text{density correction} = 0.13131 \text{ E } -02 (\rho)^{1.448817};$$

Above a density of 0.5 gm/cc,

$$\text{density correction} = 0.20625 \text{ E } -02 (\rho)^{2.1002351};$$

where

K = thermal conductivity, Joules/°K-sec-cm,

T = temperature, °K,

ρ = density, gm/cc

Figure 8

Empirical Equations for Calculating the Thermal Conductivity of Nitrogen

EMPIRICAL EQUATIONS FOR CALCULATING THERMODYNAMIC PROPERTIES OF HELIUM

A. Equation of State

$$(1) \quad P = F_1(T) \rho + F_2(T) \rho^2 + F_3(T) \rho^3,$$

$$(2) \quad F_1(T) = RT,$$

$$(3) \quad F_2(T) = -\frac{CR}{T^2} + B_0 RT - A,$$

$$(4) \quad F_3(T) = \frac{CB \cdot R}{T^2} + a_0 A.$$

B. Ideal-Gas Equations (No pressure Corrections)

$$(5) \quad C_p \text{ ideal} = 1.2412262,$$

$$(6) \quad C_v \text{ ideal} = 0.7447357,$$

$$(7) \quad U_{\text{ideal}} = 0.7447357T + 5.8811,$$

$$(8) \quad S_0 \text{ ideal} = 1.2412262 \log(T) + 1.02870$$

C. Thermodynamic parameters

The thermodynamic parameters are calculated with equations 1 to 7 in Figure 4, except Equation 1 is substituted for the equation of state in Figure 2 and Equations 5 through 8 are used for the base ideal-gas characteristics.

D. Viscosity

$$(9) \quad \text{Visc} = 6.7 \text{ E} - 04(T)^{0.68},$$

$$(10) \quad \text{Thermal conductivity}$$

$$K = 2.44 C_v \times \text{Visc},$$

where

ρ = density,

$F_1(T), F_2(T), F_3(T)$ = functions of temperature,

T = temperature, $^{\circ}\text{R}$,

A = 5.0906,

a_o = 0.23963,

B_o = 0.056063,

C = 934.17,

R = 2.6829,

C_p ideal = specific heat at constant pressure of an ideal gas
(no pressure correction),

C_v ideal = specific heat at constant volume of an ideal gas

V ideal = internal energy of an ideal gas with no pressure correction,

S_o ideal = entropy of an ideal gas at 1 psia,

Visc = viscosity,

K = thermal conductivity.

NERVA ENGINE SYSTEM

OVERALL ENGINE THRUST, LB	51918
MAIN THRUST CHAMBER THRUST, LB	50158
TURBINE EXHAUST NOZZLE THRUST, LB	1760
OVERALL ENGINE SPECIFIC IMPULSE, SEC	583
CHAMBER SPECIFIC IMPULSE, SEC	622
TURBINE NOZZLE SPECIFIC IMPULSE, SEC	212
TURBINE POWER CONTROL VALVE POSITION, DEG	72.5
AMBIENT PRESSURE, PSIA	12.76
REACTOR POWER LEVEL, M.W.	1069
TURBOPUMP POWER, M.W.	5.8
PUMP EFFICIENCY, PCT.	67.2
TURBINE EFFICIENCY, PCT. (TOTAL/STATIC)	39.5
NPSP, PSI	12.9
TURBOPUMP SHAFT SPEED, RPM	22620
NOZZLE EXPANSION RATIO	10
TURBINE EXHAUST NOZZLE AREA RATIO	1.0

NO	STATION	FLOW LB/SEC	(PS)PRESSURE(P0)		TEMP DEG R	HEAT FLUX M.W.
			STATIC PSIA	TOTAL PSIA		
1	TANK OUTLET	89.0	32.8	32.8	37.0	
2	PUMP INLET	89.0	31.6	32.8	38.4	
3	PUMP OUTLET	89.0	1003	1016	51.5	
4	NOZZLE MANIFOLD INLET	89.0	948	956	51.5	
						24.1
5	NOZZLE OUTLET	89.0	771	771	115	
						25.1
6	REFLECTOR OUTLET	89.0	727	727	193	
						1.09
7	SHIELD OUTLET	83.1	711	711	196	
						1043
8	CORE OUTLET	83.1	588	588	3425	
9	NOZZLE CHAMBER	80.7	588	588	3425	
10	DILUENT BLEED INLET	5.89	718	718	194	
11	DILUENT BLEED OUTLET	5.89	679	680	242	
12	HOT BLEED	2.40	588	588	3425	
13	MIXING RING OUTLET	8.30	564	576	1188	
15	TPCV INLET	8.30	510	524	1151	
16	TURBINE INLET	8.30	262	496	1151	
17	TURBINE OUTLET	8.30	73	139	962	
18	TURBINE EXHAUST NOZZLE	8.30	19.41	36.76	962	

HESS AND KUNZ CORRELATION

$$h = \text{EXPERIMENTALLY-DETERMINED HEAT-TRANSFER COEFFICIENT} = \frac{Q/A}{T_b - T_f}$$

$$h_c = 0.0208 \frac{k_f}{D} \left(\frac{D \rho_f v_b}{\mu_f} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)_f^{0.4} (1 + 0.01457 \frac{v_w}{v_b} \frac{\Delta T_f}{T_b})$$

RECOMMENDED EQUATION TO NORMALIZE BULK TEMPERATURE EFFECTS USING
 $C_L = K(T_b)$

AS TABULATED

$$h_c = 0.0203 C_L \frac{k_f}{D} \left(\frac{D \rho_f v_b}{\mu_f} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)_f^{0.4} (1 + 0.01457 \frac{v_w}{v_b})$$

COOLANT TEMPERATURE, °R	C_L
50	2.0
55	1.73
60	1.48
65	1.26
70	1.07
75	.93
80	.87
85	.85

Ratio of Experimentally Determined Heat-Transfer Coefficient to Calculated Coefficient, Hess and Kunz Correlation, Cryogenic Hydrogen

REDUCED-VISCOSITY CORRELATION

$$h = \text{Experimentally-determined heat transfer} = \frac{Q/A}{T_f}$$

$$h_c = 0.0164 \frac{k_f}{D} \left(\frac{\rho_f v_b}{\mu_f} \right)^8 \left(\frac{c_p \mu}{k} \right)_f^{.4} \left(1 + .01457 \frac{\mu_w}{\mu_b} \right) \left(\frac{\mu_b}{\mu_{cr}} \right)^{.67}$$

Ratio of Experimentally Determined Heat-Transfer Coefficient to Calculated Coefficient, Viscosity Modified, Hess and Kunz Correlation, Cryogenic Hydrogen

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL

$$P_2 = P_T - 5.36 \times 10^{-3} \dot{W}_p^2. \quad (1)$$

$$P_3 = P_2 + 1.41 \times 10^{-6} N^2 + 3.01 \times 10^{-4} \dot{W}_p N - 4.17 \times 10^{-2} \dot{W}_p^2. \quad (2)$$

$$P_4 = P_3 - (\dot{W}_p / 11.72)^2. \quad (3)$$

$$P_5 = F_1 (P_5) T_5 - T_2 (P_5). \quad (4)$$

$$P_8 = 5.33 P_8 T_8. \quad (5)$$

$$P_{14} = 5.32 P_{14} T_{15}. \quad (6)$$

$$P_{15} = (P_{14} - P_b) f_1 (\theta_v). \quad (7)$$

$$*P_5 = \dot{W}_{nc} - \dot{W}_6. \quad (8)$$

$$*P_8 = 30,600 \dot{W}_{c.3e} + \dot{W}_{trT} - \dot{W}_n - \dot{W}_{12}. \quad (9)$$

$$*P_{14} = \dot{W}_v - \dot{W}_t. \quad (10)$$

$$\dot{W}_p = 11.72 (P_3 - P_4)^{\frac{1}{2}}. \quad (11)$$

$$\dot{W}_{nc} = 35.8 P_5 (P_4 - P_5)^{\frac{1}{2}}. \quad (12)$$

$$\dot{W}_t = 0.567 P_{15} / (T_{15})^{\frac{1}{2}} \quad (13)$$

$$\dot{W}_{db} = 26.5 [P_6 (P_6 - P_8)]^{\frac{1}{2}} \quad (14)$$

$$\dot{W}_v = 3.9 [P_{14} (P_8 - P_{14})]^{\frac{1}{2}} \quad (15)$$

Figure 13 (Page 1 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL (cont.)

$$\dot{W}_n = 7.98 P_8 / T_8^{\frac{1}{2}}. \quad (16)$$

$$\dot{W}_{12} = \dot{W}_{13} - \dot{W}_{db} \quad (17)$$

$$T_2 = T_T. \quad (18)$$

$$T_3 = T_2 + 2.92 \times 10^{-8} * N^2. \quad (19)$$

$$T_5 = (\phi_n + 6 \dot{W}_{nc}^{0.2} T_3) / (1 + 6 \dot{W}_{nc}^{0.2}). \quad (20)$$

$$\text{Not used} \quad (21)$$

$$T_{11} = (T_6 + 0.0845 \phi_h) / 1.0845. \quad (22)$$

$$T_{13} = 0.78 T_{11} (\dot{W}_{db} / \dot{W}_{13}) + 1.05 T_8 (\dot{W}_{12} / \dot{W}_{13}). \quad (23)$$

$$T_{15} = (T_{13} + 0.074 \phi_h) / 1.074. \quad (24)$$

$$T_{tr} = (\phi_{tro} + 1.730 \phi_{ssj} + 1.531 T_{ci}) / 4.755. \quad (25)$$

$$T_{tre} = 2 T_{tr} - T_{ci}. \quad (26)$$

$$T_h = (\phi_{pgj} + 0.9849 \phi_{ss}) / 1.9849. \quad (27)$$

$$T_h^I = (\phi_{sgo} + \phi_{0.2}) / 2. \quad (28)$$

$$\phi_h = (0.226 \dot{W}_{db} T_{11} + 0.254 \dot{W}_t T_{15}) / (0.226 \dot{W}_{db} + 0.254 \dot{W}_t). \quad (29)$$

$$\phi_n = (0.322 \dot{W}_n^{0.8} T_8 + 0.657 \dot{W}_{nc}^{0.8} T_5) / (0.322 \dot{W}_n^{0.8} + 0.657 \dot{W}_{nc}^{0.8}). \quad (30)$$

$$\phi_{pg} = 0.5464 \phi_{sg} + 0.4536 \phi_{pgj} + 1.5107 (n/2). \quad (31)$$

Figure 13 (Page 2 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL (cont.)

$$\phi_{pgj} = 0.1685 \phi_{pg} + 0.8315 T_h. \quad (32)$$

$$\phi_{sg} = 0.9716 (n/2) + \phi_{sgo}. \quad (33)$$

$$\phi_{sgo} = 0.3827 \phi_{sg} + 0.6173 T_h'. \quad (34)$$

$$\phi_{ss} = (\phi_{ssj} + 0.0151 T_n) / 1.0151. \quad (35)$$

$$\phi_{ssj} = (\phi_{ss} + 2.357 \dot{W}_{tr} T_{tr}) / (1 + 2.357 \dot{W}_{tr}). \quad (36)$$

$$\phi_{tr} = 1.7886 (n/2) + \phi_{tro}. \quad (37)$$

$$\phi_{tro} = (\phi_{tr} + 38.53 \dot{W}_{tr} T_{tr}) / (1 + 38.53 \dot{W}_{tr}). \quad (38)$$

$$\phi_{sso} = 0.9849 \phi_{ss} + 0.0151 T_h. \quad (39)$$

$$H_{sg} \quad \text{Not checked} \quad (40)$$

$$M_p = 1.71 \times 10^{-6} N^2 + 1.0 \times 10^{-4} \dot{W}_p N + 9.04 \times 10^{-2} \dot{W}_p^2. \quad (41)$$

$$M_t = 3.58 P_{15}. \quad (42)$$

$$*_N = M_t - M_p. \quad (43)$$

$$n/2 \quad \text{Not checked} \quad (44)$$

$$\theta_v \quad \text{Not checked} \quad (45)$$

$$T_8 = 0.937 T_{c.3e} + 0.0617 T_{tre} \quad (46)$$

$$P_6 = 9196.8 P_6 T_6 / (1.0 - 321.0 P_6) - 6.39 \times 10^8 P_6^2. \quad (47)$$

Figure 13, (Page 3 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL (cont.)

$$P_{c.1e} = 9196.8 P_{c.1e} T_{c.1e}. \quad (48)$$

$$P_{c.2e} = 9196.8 P_{c.2e} T_{c.2e}. \quad (49)$$

$$P_{ci} = 9196.8 P_{ci} T_{ci}. \quad (50)$$

$$*P_6 = \dot{W}_6 - \dot{W}_{cit} - \dot{W}_{db} - \dot{W}_{trT}. \quad (51)$$

$$P_c = 1/3 (P_{ci} + 3 P_{c.1e} + 3 P_{c.2e} + P_{c.3e}). \quad (52)$$

$$*P_{c.1e} = \dot{W}_{c.1e} - \dot{W}_{c.2e}. \quad (53)$$

$$*P_{c.2e} = \dot{W}_{c.2e} - \dot{W}_{c.3e}. \quad (54)$$

$$P_{c.3e} = P_8 9196.8 T_{c.3e}. \quad (55)$$

$$*P_{ci} = \dot{W}_{ci} - \dot{W}_{c.1e}. \quad (56)$$

$$P_{tre} = P_8 / 9196.8 T_{tre}. \quad (57)$$

$$\dot{W}_6 = [3.689 \times 10^5 (P_5 - P_6)]^{\frac{1}{2}}. \quad (58)$$

$$\dot{W}_{c.1} = (\dot{W}_{c.1e} + \dot{W}_{ci}) / 2. \quad (59)$$

$$\dot{W}_{c.2} = (\dot{W}_{c.2e} + \dot{W}_{c.1e}) / 2. \quad (60)$$

$$\dot{W}_{c.3} = (\dot{W}_{c.3e} + \dot{W}_{c.2e}) / 2. \quad (61)$$

$$\dot{W}_{c.1e} = [0.01084 (P_{ci} - P_{c.1e}) P_{c.1e}]^{\frac{1}{2}}. \quad (62)$$

$$\dot{W}_{c.2e} = [0.008643 (P_{c.1e} - P_{c.2e}) P_{c.2e}]^{\frac{1}{2}}. \quad (63)$$

$$\dot{W}_{c.3e} = [0.01357 (P_{c.2e} - P_8) P_{c.3e}]^{\frac{1}{2}}. \quad (64)$$

$$\dot{W}_{ci} = \dot{W}_{cit} / 30,600. \quad (65)$$

Figure 13 (Page 4 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL (cont.)

$$\dot{W}_{cit} = [3.0662 \times 10^5 (P_6 - P_{ci}) P_{ci}]^{\frac{1}{2}}. \quad (66)$$

$$\dot{W}_{tr} = [0.2594 (P_6 - P_8) / (\frac{25.06}{P_6} + \frac{5.297}{P_{tre}})]^{\frac{1}{2}}. \quad (67)$$

$$\dot{W}_{trT} = 289.0 \dot{W}_{tr}. \quad (68)$$

$$T_6 = 2 T_r - T_5. \quad (69)$$

$$T_7 = T_6 + 0.2 (\phi_s + T_s). \quad (70)$$

$$T_{c.1} = 0.4814 T_{ci} + 0.5184 \phi_{c.1} + 38.45 \left(\frac{\dot{W}_{c.1} - \dot{W}_{c.1e}}{\dot{W}_{c.1}} \right). \quad (71)$$

$$T_{c.2} = 0.6812 \left(T_{c.1e} - \frac{T_{c.1}}{4} \right) + 0.4891 \phi_{c.2} + 72.54 \left(\frac{\dot{W}_{c.1e} - \dot{W}_{c.2e}}{\dot{W}_{c.2}} \right). \quad (72)$$

$$T_{c.3} = 0.6608 \left(T_{c.2e} - \frac{T_{c.2}}{4} \right) + 0.5043 \phi_{c.3} + 1110.0 \left(\frac{\dot{W}_{c.2e} - \dot{W}_{c.3e}}{\dot{W}_{c.3}} \right). \quad (73)$$

$$T_{c.1e} = 2 T_{c.1} - T_{ci}. \quad (74)$$

$$T_{c.2e} = 2 T_{c.2} - T_{c.1e}. \quad (75)$$

$$T_{c.3e} = 3 T_{c.3} - 3 T_{c.2e} + T_{c.2}. \quad (76)$$

$$T_{ci} = T_7. \quad (77)$$

$$T_r = T_5 + 0.8367 \phi_r / 1.8367. \quad (78)$$

$$T_s = \phi_s - (T_7 - T_6) / 0.2. \quad (79)$$

$$\phi_c = (3 \phi_{c.1} + 2 \phi_{c.2} + 3 \phi_{c.3}) / 8. \quad (80)$$

$$\phi_{c.1} = T_{c.1} + 0.01600 [(n/2) / (\dot{W}_{c.1})]. \quad (81)$$

$$\phi_{c.2} = T_{c.2} + 0.01256 [(n/2) / (\dot{W}_{c.2})]. \quad (82)$$

Figure 13 (Page 5 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

EQUATIONS USED IN THE DIGITAL-COMPUTER CODE
TO CHECK THE COMMON ANALOG MODEL (cont.)

$$\phi_{c.3} = T_{c.3} + 0.00627 \left[(n/2) / (\dot{W}_{c.3}) \right]. \quad (83)$$

$$\phi_s = 26.97 \left[(n/2) / (\dot{W}_{cit}) \right] + T_s. \quad (84)$$

$$\phi_r = T_r + 38.03 \left[(n/2) / (\dot{W}_6) \right]. \quad (85)$$

$$\phi_d \quad \text{Not checked} \quad (86)$$

Figure 13 (Page 6 of 6)

Equations Used in the Digital-Computer Code to Check the
Common Analog Model

INPUT INFORMATION, DIGITAL CHECK OF
COMMON ANALOG MODEL

Section a

<u>Type</u>	<u>Meaning</u>
1	Scale-factor cards out of order or missing
2	Comment card out of order or missing
3	Input data card out of order or missing
4	Theta table values missing
5	Number of values in theta table exceeds 50
6	Theta value cards out of order or missing
7	Table cards out of order or missing
8	Base data card out of order or missing

Section b

<u>Input Data Format</u>		
<u>Columns</u>	<u>Name</u>	<u>Format</u>
1 - 3	Card ID	I3
11 - 20	WORD - 1	E10.4
21 - 30	WORD - 2	E10.4
31 - 40	WORD - 3	E10.4
41 - 50	WORD - 4	E10.4
51 - 60	WORD - 5	E10.4
61 - 70	WORD - 6	E10.4

Section c

<u>Comment Card Format</u>		
<u>Column</u>	<u>Name</u>	<u>Format</u>
1 - 3	200	I3
4 - 72	WORD	A3, 11A6

Figure 14 (Page 1 of 4)

Input Information, Digital Check of Common Analog Model

INPUT INFORMATION, DIGITAL CHECK OF
COMMON ANALOG MODEL (cont.)

<u>Variable Name and Format</u>			
<u>Column</u>	<u>Name</u>	<u>Format</u>	
1 - 3	ID	13	
9 - 16	NAME 1	A6, A2	
17 - 24	NAME 2	A6, A2	
25 - 32	NAME 3	A6, A2	
33 - 40	NAME 4	A6, A2	
41 - 48	NAME 5	A6, A2	
49 - 56	NAME 6	A6, A2	
57 - 64	NAME 7	A6, A2	
65 - 72	NAME 8	A6, A2	Section d

<u>Theta Cards</u>		
First card	ID	050
	WORD 1	Number of data sets
Second card	ID	051
	WORD 1	θ_v first point
	WORD 2	$f(\theta_v)$ first point
	WORD 3	θ_v second point
	WORD 4	$f(\theta_v)$ second point
	WORD 5	θ_v third point
	WORD 6	$f(\theta_v)$ third point
Third card	ID	052
	etc....	

Figure 14 (Page 2 of 4)
Input Information, Digital Check of Common Analog Model

INPUT INFORMATION, DIGITAL CHECK OF
COMMON ANALOG MODEL (cont.)

Section e

	<u>Variable Name Card</u>
First card	ID 080
	WORD 1 P2
	WORD 2 P3
	.
	.
	.
	WORD 8 RHO 5
Second card	ID 081
	WORD 1 RHO 8
	WORD 2 RHO 14
	.
	.
	.
	WORD 8 WN
	etc...

Section f

	<u>Scale-Factor Cards</u>
First card	ID 100
	WORD 1 Scale factor for P2
	WORD 2 Scale factor for P_5
	.
	.
	.
	WORD 12 Scale factor for W_{nc}
	etc.

Section g

	<u>Input Data Cards</u>
First card	ID 200
	WORD Problem description

Figure 14 (Page 3 of 4)
Input Information, Digital Check of Common Analog Model

INPUT INFORMATION, DIGITAL CHECK OF
COMMON ANALOG MODEL (cont.)

	<u>Input Data Cards</u>
Second card	ID 201 WORD 1 PT WORD 2 PB WORD 3 TT
Fourth card	ID 202 WORD 1 Value of P2 WORD 2 Value of P3 . . . WORD 6 Value of P14
Fifth card	ID WORD 1 Value of P15 WORD 2 Value of P_5 . . . WORD 12 Value of W_{nc} etc...

Figure 14 (Page 4 of 4)
Input Information, Digital Check of Common Analog Model

COMPUTING SERVICES DIVISION

DATA INPUT SHEET

NAME _____ DATE _____

IDENTIFICATION _____

1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
2.00. PROGRAM DESCRIPTION																
2.01.	P _T				P _B				T _T							
2.02.	P ₂				P ₃				P ₄				P ₅			
2.03.	P ₁₅				P ₅				P ₈				P ₁₄			
2.04.	W _T				W _{DB}				W _K				W _N			
2.05.	T ₃				T ₅				T ₁₁				T ₁₃			
2.06.	T _{cr}				T _{cre}				T _L				T _{L'}			
2.07.	φ _{pg}				φ _{pgi}				φ _{sg}				φ _{sgo}			
2.08.	φ _{tr}				φ _{tro}				φ _{ss0}				H _{sg}			
2.09.	N				n ₂				Q				T ₅			
2.10.	P _{c.2e}				P _{ci}				P _e				P _e			
2.11.	P _{c.3e}				P _{ci}				P _{cre}				W _e			
2.12.	W _{c.3}				W _{c.1e}				W _{c.2e}				W _{c.3e}			
2.13.	W _{tr}				W _{trT}				T _e				T _T			
2.14.	T _{c.3}				T _{c.1e}				T _{c.2e}				T _{c.3e}			
2.15.	T _s				φ _c				φ _{c.1}				φ _{c.2}			
2.16.	φ _r				θ _d											

2-23-66 5-45 PM

PT = 70.0 TT = 38.0 PR = 12.7

NO.	VARIABLE	VOLTAGE	SCALE FACTOR	VALUE	CHECK VALUE	PER CENT ERROR
1	P-2	0.3889	1.000E+00	3.889E+01	3.878E+01	0.2799
2	P-3	-0.6285	1.500E+01	9.427E+02	9.367E+02	0.6455
3	P-4	0.6000	1.500E+01	9.000E+02	9.003E+02	-0.0386
4	P-5	0.7350	1.000E+01	7.350E+02	7.252E+02	1.3370
5	P-8	-0.5623	1.000E+01	5.623E+02	5.569E+02	0.9664
6	P-14	0.2625	2.000E+01	5.250E+02	5.245E+02	0.0963
7	P-15	0.4133	1.000E+01	4.133E+02	4.143E+02	-0.2422
8	RHO-5	0.5125	2.000E-02	1.025E+00	1.100E-01	
9	RHO-8	0.4438	5.770E-04	2.561E-02	-4.662E-01	
10	RHO-14	0.4426	2.000E-03	8.852E-02	8.000E-03	
11	W-P	-0.7519	1.015E+00	7.632E+01	7.663E+01	-0.4083
12	W-NC	-0.7656	1.000E+00	7.656E+01	7.781E+01	-1.6350
13	W-T	-0.7086	1.000E-01	7.086E+00	7.022E+00	0.9046
14	W-DB	-0.5459	1.000E-01	5.459E+00	5.425E+00	0.6272
15	W-V	-0.7094	1.000E-01	7.094E+00	7.087E+00	0.1039
16	W-N	0.6975	1.000E+00	6.975E+01	7.007E+01	-0.4632
17	W-12	-0.1625	1.000E-01	1.625E+00	1.635E+00	-0.6154
18	T-2	-0.3807	1.000E+00	3.807E+01	3.800E+01	0.1839
19	T-3	-0.5155	1.000E+00	5.155E+01	5.149E+01	0.1116
20	T-5	0.1380	1.000E+01	1.380E+02	1.376E+02	0.3006
21		0.0000	0.000E-01	0.000E-01	0.000E-01	0.0000
22	T-11	0.2637	1.000E+01	2.637E+02	2.637E+02	0.0081
23	T-13	-0.4549	2.500E+01	1.137E+03	1.140E+03	-0.2069
24	T-15	0.4455	2.500E+01	1.114E+03	1.112E+03	0.1697
25	T-TR	0.4692	1.000E+01	4.692E+02	4.666E+02	0.5599
26	T-TRE	-0.3508	2.000E+01	7.016E+02	7.090E+02	-1.0547
27	T-H	0.1871	5.000E+01	9.355E+02	9.329E+02	0.2783
28	T-H PRI	0.6867	5.000E+01	3.434E+03	3.435E+03	-0.0364
40	H-SG	-0.3538	2.000E+01	7.076E+02	0.000E-01	0.0000
41	M-P	0.7405	2.000E+01	1.481E+03	1.476E+03	0.3245
42	M-T	-0.2714	5.500E+01	1.493E+03	1.480E+03	0.8767
43	N	0.4288	5.000E+02	2.144E+04	1.170E+01	
44	N/2	-0.5034	2.000E+00	1.007E+02	0.000E-01	0.0000
45	TH-TPCV	0.4565	1.000E+00	4.565E+01	0.000E-01	0.0000

Data Output for CAM II Check

Figure 16

Page 93

Predicted Oscillations, Chlldown of the X-Engine Inlet Line

Figure 17

Page 94

LIST OF EQUATIONS, NOZZLE FLOW PROGRAM

1. $\gamma = 0.209 + \sqrt{1.462 - 7.2728 \times 10^{-5} T_c}$
2. $C^* = \frac{1}{\gamma} \frac{\gamma + 1}{2} \left(\frac{\gamma}{\gamma - 1} \right) \sqrt{\frac{2 g R \gamma T_c}{\gamma + 1}}$
3. $\dot{W} = \frac{P_c A_t g C_n}{C^*}$
4. $h_c = h(P_c, T_c)$. (from hydrogen-properties table)
5. $Q = \dot{W} (h_c - h_i)$
6. $T_c (\text{guess}) = \left(\frac{A_t P_c .1396}{\dot{W}} \right)^2$
7. $T_c (\text{guess}) = \left(\frac{Q}{P_c A_t (0.1396)(3.5)} \right)^2$
8. $\dot{W} = \frac{Q}{(h_c - h_i)}$
9. $P_c = \frac{\dot{W} C^*}{A_t g C_n}$
10. $P_c (\text{guess}) = \frac{Q}{\sqrt{t_c} A_t (0.1396)(3.5)}$
11. $T_c (\text{guess}) = \frac{Q}{\dot{W} (3.5)}$
12. $Q_{mw} = \frac{Q}{947.321}$
13. $Q\% = \frac{Q_{mw} (100)}{Q_{max}}$

Figure 18 (Page 1 of 2)

List of Equations, Nozzle Flow Program

LIST OF EQUATIONS, NOZZLE FLOW PROGRAM (cont.)

$$\text{I.} \quad \left(\frac{A_t}{A_e} \right)^2 = \frac{2}{\gamma - 1} \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma + 1}{\gamma - 1}} \left[\left(\frac{P_e}{P_c} \right)^{\frac{2}{\gamma}} - \left(\frac{P_e}{P_c} \right)^{\frac{1 + \gamma}{\gamma}} \right] .$$

$$\text{II.} \quad T = \left\{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right) \left(\frac{\gamma + 1}{\gamma - 1} \right) \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma - 1}{\gamma}} \right] + \frac{P_e - P_a}{P_c} \left(\frac{A_e}{A_t} \right) \right\} A_t P_c .$$

Symbols

T_c	=	throat temperature, °R
R	=	gas constant, ft-lb/lb-°R
g	=	gravitational constant, ft/sec ²
P_c	=	chamber pressure, psia
A_t	=	area, throat, in. ²
C_n	=	efficiency coefficient
h_c	=	chamber enthalpy, Btu/lb
h_i	=	inlet enthalpy, Btu/lb
Q	=	power, Btu/sec
$\dot{W}$	=	flow rate, lb/sec
Q_{mw}	=	power, megawatts
A_e	=	area at the exit, in. ²
P_e	=	pressure at the exit, psia
T	=	thrust
P_a	=	ambient pressure, psia

Figure 18 (Page 2 of 2)

LIST OF ERROR MESSAGES, NOZZLE FLOW PROGRAM

1. K(9) became zero; all further printing has been suppressed.
2. All further printing suppressed by page count exceeding K(13) input.
3. Program flushed when number of print statements exceeded K(14) input.
4. xx was the option no. from input *****; this is less than 1 or greater than 6. Skip to next case.
5. Temperature of xxxxxx was below the table limits. Temperature was set to table limit and program returned to execution.
6. Pressure of xxxxxx was beyond the table limits. Pressure was set to table limits and program returned to execution.
7. Enthalpy of xxxxxx was beyond the table limits. Enthalpy was set to table limit and program returned to execution.
8. xx iterations have failed. Check input and epsilon for errors. Program skipped to next input case.
9. xx values have been sent to paragraph-H that are outside table limits -----; program is skipping to next case.
10. The iteration failed to close for the next line of output. *** I2 or epsilon was set too small or input data was bad.
11. The thrust iteration failed to close. Either epsilon or I2 was too small. The last two values of PE agreed within xxxxxx.
12. Temperature closed at or beyond the table limit.
13. The computed values are invalid since temperature is above table limit *****; above 5000 deg the introduced error increases with temperature.
14. NOA * NOB cannot be less than 1 *; skipping to next case.
15. Variable gamma iteration failed to close. *** last values differed by xxxxxx.
16. Storage table exceeded *** NOA and NOB must be less than 501, and NOA * NOB must be less than 2501; skip to next input case.

Figure 19

List of Error Messages, Nozzle Flow Program

CONDENSATION OF NAME LIST INPUT, NOZZLE FLOW PROGRAM

The name-list input used in this program is the Version 13 FORTRAN IV scatter-loading scheme. For a complete description of its use refer to the IBM Version-13 FORTRAN IV manual for the 7094 computer, form 28-6390-0.

Cards a, c, and d use the name-list input, and some comments can be made about this scheme in general.

After each read statement, the card reader searches through consecutive input cards looking for a \$ in Col 2 followed by the name-list name called for in the read statement and followed in turn by at least one blank col.

Example:

After a match of this type is found, any data included between the last blank col and the next encountered \$ sign in a col greater than 2 are read, converted to the type of the receiving field, and stored over the last value in this particular field. If no new values are read for a given field, the last value remains in the program ready to start the next case.

Example:

Figure 20 (Page 1 of 3)

Condensation of Name List Input, Nozzle Flow Program

CONDENSATION OF NAME LIST INPUT, NOZZLE FLOW PROGRAM (cont.)

In the above example:

A will be loaded with floating point 4.2
K(4) will be loaded with fixed point 5
V(2) will be loaded with floating point 0.0
V(3) will be loaded with floating point 0.0
V(4) will be loaded with floating point 0.0
V(5) will be loaded with floating point 0.0
V(6) will be loaded with floating point 0.0
I will be loaded with fixed point 4 (truncated)
X will be loaded with floating point 5.0
\$ will end this read statement.

If the array name V has been used without the subscript, the full number of points must be loaded to fill the entire array.

Example:

Dimension Z(100)

\$LIST Z, 52 * 0.0, 4.7, 3.5, 45 * 10.0 \$

full 100 values to fill the Z array.

Many of the variables used in the input are initialized to a standard value before the program starts. Only those values that are different from the standard case or the last case need be included in the input. Once a value is entered, it will remain unless it is changed in the course of completing the case, as are some values in the K array and some plot parameters.

Figure 20 (Page 2 of 3)

CONDENSATION OF NAME LIST INPUT, NOZZLE FLOW PROGRAM (cont.)

The following list of precautions will help in avoiding some of the common pitfalls in using the name-list read.

1. Avoid using col 1 or col 80, as they sometimes cause IBM system errors.
2. Do not run a variable off the end of one card and onto the next card. Separate card where you might normally have a blank.
3. Do not embed blanks in a data field.
4. If the input key is punched, make sure the operators do not punch a sequence number in columns 73 to 80, as the card reader scans all 80 columns and will not recognize these characters.

Area Producing Side Force, Obtained from Basic Assumptions of Gas-Injection Model

Figure 21

EQUATIONS FOR SECONDARY-GAS-INJECTION
THRUST-VECTOR-CONTROL PROGRAM

Mathematical Model for Calculating Side Force
Induced by Injection of a Gas Into a Supersonic Nozzle Stream

A. From semiempirical relations for the behavior of turbulent boundary layers,

$$(1) \quad SS^2 = K(S\phi)^2. \quad (\text{Reference 20})$$

B. For a given Mach-No. station within the nozzle, calculate the pressure ratio required for boundary-layer separation from the relation

$$(2) \quad \frac{PSS}{P\phi} = 1 + \frac{(S\phi)^2}{2} \left\{ \frac{(1 - K)}{1 + \left[(G\phi - 1)/2 \right] (S\phi)^2} \right\}. \quad (\text{Reference 20})$$

C. The pressure ratio at the outer boundary of a conical three-dimensional separated region required for separation is assumed to be the same as that for the two-dimensional case presented in Reference 20.

$$(3) \quad \frac{PF}{P\phi} = \frac{PSS}{P\phi} \left\{ 1 + 0.328 \frac{K (S\phi)^2 (\text{DELTA})}{1 + \left[(G\phi - 1)/2 \right] K (S\phi)^2} \right\}.$$

The separation angle delta is unknown, so an approximation of $PF/P\phi$ is obtained from the relation

$$(4) \quad \frac{PF}{P\phi} = \frac{PSS}{P\phi} \left[\frac{1 + G}{1 + (PSS/P\phi) G} \right], \quad \text{where } G = -0.328 \frac{K \sqrt{(S\phi)^2 - 1}}{1 + \left(\frac{G\phi - 1}{2} \right) K (S\phi)^2}.$$

A value of delta is obtained for conical shock relations for the value of $PF/P\phi$ found in Equation 4. This value of delta is then substituted in Equation 3 to obtain a better approximation to $PF/P\phi$. The values of delta and theta, the wave angle, are then found from conical shock relations for the adjusted value of $PF/P\phi$.

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Equations for Secondary-Gas-Injection Thrust-Vector-Control Program

The results for P_F/P_ϕ , δ , θ , and P_2/P_ϕ as functions of S_ϕ obtained from the solution of Equations 3 and 4 are presented graphically for $G_\phi = 1.4$ in Reference 16 and are shown in Figures 23, 24, 25, and 26. The following curve fits based on these graphical data were used in the computer-program solutions:

Pressure at the boundary of the separated flow region (Figure 21) is

$$(5) \quad P_F = P_0 \left[-0.0467 + 1.293 S_0 - 0.1287 S_0^2 \right].$$

Average pressure between the shock cone and the separated-flow region (Figure 21) is

$$(6) \quad P_2 = P_0 \left[-0.07535 + 1.2498 S_0 - 0.12437 S_0^2 \right].$$

Shock-cone-deflection and cone angles (Figure 21) are

$$(7) \quad \begin{aligned} \delta &= 24.596 - 1.8742(S_0) - 0.04693(S_0)^2; \\ \theta &= 66.178 - 16.578(S_0) + 1.3997(S_0)^2. \end{aligned}$$

D. The accommodation height (h) for the injected fluid must be calculated to establish the location of the oblique shock wave. The injected gas is known to expand rapidly, so the pressure is assumed to vary parabolically from the injected pressure to the separation pressure. The average pressure in the separated-flow region is then

$$(8) \quad \begin{aligned} P_S &= P_G - 2/3 (P_G - P_F) \\ P_S &= 0.333 \left[P_G + 2(P_F) \right]. \end{aligned}$$

After injection, the gas is assumed to make a sharp turn and to flow parallel to the nozzle wall without mixing with the main stream. Furthermore, the secondary flow is assumed to take a semicylindrical shape after turning. The accommodation height is then obtained from:

$$(9) \quad H = (2A/\pi)^{1/2}, \text{ where } A = A^4_{\text{final}}, \text{ as defined below.}$$

EQUATIONS FOR SECONDARY-GAS-INJECTION
THRUST-VECTOR-CONTROL PROGRAM (cont.)

E. A_4 is the cross-sectional area of the semicylindrical shape at the injection plane, and P_4 is the pressure acting on this area. Two equations in these two unknowns can be written on the basis of conservation of mass, conservation of energy, stream momentum balance for the primary and injector streams, and geometrical and boundary conditions. These relationships are presented in Reference 16 and are extended in Reference 17; the final-form simultaneous equations are shown below as the solutions for Z and V in the notation adapted for the computer solution:

(10) Solution for P_4 and A_4 :

a. $P_4 = P_0$ (first guess).

b. A_4 (first guess) obtained from solution of Equation 10d for $P_4 = P_0$.

$$c. \quad Z = \frac{P_4 (A_0 - A_4)}{G_0} \left[Y - P_4 (A_0 - A_4) - P_S (A_4) + W \right] \times$$

$$\left\{ 1 + \frac{U}{G_0 (P_4) (A_0 - A_4)} \left[Y - P_4 (A_0 - A_4) - P_S (A_4) + W \right] \right\}.$$

$$d. \quad V = \frac{(P_4)^2 (A_4)^2}{G_G} \left[\frac{P_S}{P_4} - \frac{W}{P_4 (A_4)} - 1 \right] \times$$

$$\left[1 + \frac{T}{G_G} \left(\frac{P_S}{P_4} - \frac{W}{P_4 (A_4)} - 1 \right) \right].$$

where:

$$Z = (P_0)^2 (A_0)^2 (S_0)^2 \left[1 + U (S_0)^2 \right],$$

$$Y = (P_0) (A_0) \left[1 + G_0 (S_0)^2 \right],$$

$$W = (F_G) (V_G) \left[\sin (ALF + EPS) \right],$$

$$U = (G_0 - 1)/2,$$

$$V = (P_G)^2 (A_G)^2 (S_G)^2 \left[1 + T (S_G)^2 \right],$$

$$T = (G_G - 1)/2.$$

Equations 10c and 10d are solved by iteration, the closure tolerance for successive values of P_4 and A_4 being 0.001.

EQUATIONS FOR SECONDARY-GAS-INJECTION
THRUST-VECTOR-CONTROL PROGRAM (cont.)

F. The side force produced by gas injection is assumed to be the sum of three components. The first component results from the pressure increase in the separated-flow region (Equation 8). The second is due to a similar increase occurring between the shock and the separated region (Equation 6). The third component is due to the momentum of the injected gas. It is assumed that, downstream of the injection part, there is no contribution to the side force, as any forces present in this region balance each other.

The areas on which the pressure differences act to cause the side force are bounded by the intersection of the shock cone with the nozzle wall. For simplicity, and for consistency with earlier assumptions used in this model, the projected area bounded by the intersection of a shock cone and a cylinder is used in the analysis. The derivations of the projected areas over which the different pressures act are shown in Reference 16; included in the derivations are expressions for the distance between the shock vertex and the injection-port center line, the angle of injection, the accommodation height, the shock angle, and geometrical relationships between the shock and the cylinder.

Shock standoff distance (Figure 21):

$$(11) \quad X = H \left[\frac{1}{\tan \delta_{\text{ela}}} + \tan (\alpha_{\text{LF}} + \epsilon) \right].$$

Projected area (Figure 21):

$$(12) \quad \text{Dela} = H (X)^2 + \tan \epsilon.$$

EQUATIONS FOR SECONDARY-GAS-INJECTION
THRUST-VECTOR-CONTROL PROGRAM (cont.)

Injector side force:

$$(13) \quad \text{Force} = \left\{ \left[\left(\frac{P_2}{P_0} \right) - 1 \right] \left[\text{de}1a - X(H) \right] + \left[\left(\frac{P_F}{P_0} \right) - 1 \right] \left[\left(\frac{X}{H} \right) - \frac{AG}{2} \right] \right\} \\ + \left\{ (P_0) \cos (ALF) \right\} + \left\{ (AG) (P_0) \left[\left(\frac{P_G}{P_0} \right) - 1 \right] \right\} \cos (ALF) \\ + \left[FG (VG) / GFP \right] \cos (ELF) ,$$

where:

$$AG = f [FG, TTG, PTG, GG, \& R] ,$$

$$PG = f [PTG] ,$$

$$\text{and } VG = f [GG, R, TTG] .$$

One-dimensional, isentropic flow relationships for sonic throat conditions.

G. The thrust-vector deflection angle produced by the side force and the thrust augmentation due to secondary injection are found by using the following relations:

Thrust-vector deflection angle:

$$(14) \quad \tan GL = \arctan (TRATO), \text{ where } TRATO = \text{FORCE} / \text{THRUST}.$$

Augmentation of engine thrust by secondary injection (Reference 21), with injector-nozzle discharge coefficient and vacuum thrust efficiency both assumed to be unity:

$$(15) \quad \text{TAUG} = \text{PTG}(AG)(CF) \left\{ - \cos (ALF + ELF2) \right. \\ \left. + \frac{\pi}{1 + GG(SG)}^2 \left[\frac{1}{2} - \frac{ELF2}{180} \right] \left[\sin (ALF + ELF2) \right] \right\} \\ - \text{PTG}(AG)(P_0/PTG)(E) \\ \left\{ - \cos (ALF + ELF2) + \left[\cot (ELF2) \right] \left[\sin (ALF + ELF2) \right] \right\} \\ + \text{PTG}(AG) \left\{ 0.4311(W3) \left(\frac{S}{DP} \right)^{\frac{1}{2}} \left[\frac{PTP(AP)}{PTS(AG)} \right]^{\frac{1}{4}} \sin (ALF) \right\}$$

where

$$W1 = \left[SO / (1 + 0.2 SO^2) \right]^{1.25},$$

$$W2 = \frac{1}{2} + 1 / \left[0.56 (SO)^2 \right] + \left[1 + 0.2 (SO)^2 \right] / 0.4 (SO)^2$$

$$+ (SG/SO) \left\{ \left[1 + 0.2 (SO)^2 \right] / \left[1 + 0.2 (SG)^2 \right] \right\}^{\frac{1}{2}} \cos (ELF2),$$

$$\text{and } W3 = W1 (W2)^{0.75}.$$

Also included in the program are equations for calculating the effects of secondary injection on the overall specific impulse of the engine system. Since these equations will vary, depending on the engine design, only the general form is shown here.

$$(16) \quad I_{sp} = I_{sp} (\text{Engine} + \text{SITVC system}) - I_{sp} (\text{Engine}).$$

DESCRIPTION OF TERMS USED IN EQUATIONS

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
AG	Injector throat area	in.^2
AISP	Engine-system specific impulse without secondary injection	$\frac{\text{lbf-sec}}{\text{lbm}}$
AISPF	Total specific impulse of engine with secondary injection	$\frac{\text{lbf-sec}}{\text{lbm}}$
ALF	Nozzle half-angle	degrees
AMAS	Ratio of injectant flow to primary nozzle flow	---
AO ($A \phi$)	Cross-sectional area of nozzle at injection port location	in.^2
AP	Throat area of primary nozzle	in.^2
AXIS	Axial distance between throat and exit plane of primary nozzle	in.
CF	Ideal, unaugmented, vacuum thrust coefficient of injector nozzle	---
Delta(δ)	Separation angle	degrees
DISP	Loss in specific impulse due to secondary injection	$\frac{\text{lbf-sec}}{\text{lbm}}$
DP	Throat diameter of primary nozzle	in.
E	Expansion ratio	---
EPS	Angle of injection, measured ccw from perpendicular to nozzle (primary) center line	degrees
ELF	$\text{EPS} + (90^\circ - \text{ALF})$	degrees
FG	Mass-flow rate of injectant	lbm/sec
FINDC	Injection side force due to pressure disturbance in primary nozzle	lbf
FO	Mass-flow rate in primary nozzle	lbm/sec
FORCE	Total side force produced by secondary injection	lbf
FREAC	Injection side force due to momentum of secondary stream	lbf
GFP	Proportionality factor, 32.17	lbm-ft/lbf-sec^2
GG	Specific-heat ratio of injectant gas	---
GO ($G \phi$)	Specific-heat ratio of primary engine exhaust gases	---
K	Semiempirical constant (0.55)	---
L	Code number - input data cards	---
M	Iteration count	---

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<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
P2	Average static pressure between shock cone and separated-flow region	psia
PF	Static pressure at outer boundary of separated-flow region	psia
PG	Static pressure of injectant stream at entry point into nozzle	psia
PO (P_ϕ)	Static pressure of nozzle exhaust stream at injection plane	psia
PS	Average static pressure at injector in separated-flow region	psia
PSS	Static pressure at nozzle wall required for boundary-layer separation	psia
PTG	Total pressure in injector plenum	psia
PTP	Engine total pressure	psia
R	Gas constant (766.66 for H_2)	ft-lbf/lbm-°R
RS	Throat radius of circular, sonic injector	in.
S	Available nozzle wall downstream of injector-port leading edge	in.
SS	Mach number at which separation will occur	---
SG	Mach number of injectant stream at entry point into nozzle	---
SO (S_ϕ)	Mach number of nozzle exhaust stream at injection plane	---
TANGL	Deflection angle of engine thrust vector due to secondary injection	degrees
TAUG	Augmentation of engine thrust due to secondary injection	lbf
Theta(θ)	Conical shock angle	degrees
TRATO	Ratio of injector side force to undisturbed engine thrust	---
TRUST	Undisturbed axial thrust of main nozzle	lbf
VG	Velocity of injectant stream at entry point into nozzle	ft/sec
W1,W2,W3	Blast-wave-theory correlation parameters	---
XXLL	X/L location of injectors	---

Figure 22 (Page 8 of 8)

$S\phi$	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3
$PF/P\phi$	1.51	1.71	1.905	2.085	2.24	2.38	2.51	2.645	2.75	2.85	3.94	3.0	3.03	3.045	3.055

$S\phi$	4.5	4.7	5.1	5.3	5.5	5.7	5.9	6.0
$PF/P\phi$	3.075	3.08	3.1	3.103	3.115	3.12	3.175	3.13

Static-Pressure Ratio of the Outer Boundary of a Conical Separated Region vs Mach No.

$S\phi$	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9
P_2/P_ϕ	1.5	1.66	1.81	1.98	2.11	2.25	2.39	2.51	2.62	2.76	2.80	2.86	2.90

$S\phi$	4.0	4.5	5.0	5.5	6.0
P_2/P_ϕ	2.91	2.95	2.96	2.99	3.01

Average Static-Pressure Ratio between Shock and Separated Region vs Mach No.

$S\phi$	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9
δ	19.8	20.5	21.0	21.2	21.0	20.6	20.1	19.5	19.0	18.4	17.7	17.0	16.5

$S\phi$	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.5	5.7	5.9	6.0
δ	15.9	15.3	14.9	14.4	14.0	13.5	13.2	12.9	12.6	12.4	12.25

Separation Angle vs Mach No.

Figure 25

$S\phi$	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9
θ	49.2	43.9	40.4	37.5	35.2	33.3	31.5	29.9	28.4	27.0	25.8	24.6	23.5

$S\phi$	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.5	5.7	5.9	6.0
θ	22.4	21.5	20.8	20.0	19.4	18.6	18.0	17.6	17.4	17.1	17.05

Shock Angle vs Mach No.

Figure 26