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PART ONE: THE REACTION $K^-d \rightarrow \bar{K}^0 nn$, FROM
THRESHOLD TO 300 Mev/c.

PART TWO: K^+p ELASTIC SCATTERING: 80-280 Mev/c.

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B.S., University of Illinois, 1960

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THESIS

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PART ONE: THE REACTION $K^-d \rightarrow \bar{K}^0nn$, FROM
THRESHOLD TO 300 MeV/c.

PART TWO: K^+p ELASTIC SCATTERING: 80-280 MeV/c.

Robert Maurice Lansford, Ph.D.
Department of Physics
University of Illinois, 1967

PART ONE

The cross section for the reaction $K^-D \rightarrow \bar{K}^0nn$ has been measured for the laboratory momentum interval from threshold (96.7) to 300 MeV/c.

The cross section is found to rise quickly above threshold to around 6 mb at 150 MeV/c, then decrease slowly from 150 to 300 MeV/c.

The experimental values are found to be consistently lower (about 20%) than the recent values calculated by Hetherington and Schick using Faddeev techniques.

PART TWO

An experiment has been performed using the Berkeley 15 inch Hydrogen bubble chamber to measure the low energy K^+p elastic scattering cross section.

Although the differential cross section is generally lower than those obtained by Stubbs,^{1/} the experiments are consistent to within the quoted experimental errors.

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PART ONE: THE REACTION $K^-d \rightarrow \bar{K}^0nn$, FROM
THRESHOLD TO 300 MeV./c.

I. INTRODUCTION

For some time, the deuterium charge exchange cross section has been of theoretical interest.

Early interest was stimulated by a suggestion due to Pais^{1/} that the $(K^-, \bar{K}^0)$ parity might be odd. Subsequent calculations indicated that the reaction

$$K^-d \rightarrow \bar{K}^0 nn \quad (I.1)$$

should be sensitive to the $(K^-, \bar{K}^0)$ parity.^{2,3/} These results were confined to the expected threshold behavior, which experimentally is a difficult region to investigate.

Nevertheless, other indirect evidence has accumulated which lends weight to the general belief that the $(K^-, \bar{K}^0)$ parity is even.

Recently, rather sophisticated three-body calculations have been made for reaction (I.1) over an extended momentum range.^{4/} Thus, there is now possible a comparison with theory in a more readily accessible experimental region.

This is a report of a bubble chamber study of the cross section for the reaction

$$K^-d \rightarrow \bar{K}^0 nn ,$$

for the momentum interval from threshold (96.7) to 300 MeV/c.

II. EXPERIMENTAL DETAIL

A. Physical Facilities

The experiment was performed using the BNL-Columbia 30 inch bubble chamber. Approximately 80,000 pictures were obtained, of which 18,000 have been used in this sample. The average K^- flux was about 3.5 K^- 's per frame. The ratio of background π 's to K^- 's was about 10:1.

1. Beam

The physical layout of the beam is seen in Fig. 1. The beam was tuned to select 760 MeV/c K^- mesons. A 19.5 inch block of Carbon absorber was inserted just before the beam entered the chamber (Fig. 1), so as to degrade the beam to about 300 MeV/c at the entrance fiducial plane. The resulting beam momentum spread extended from nearly 125 MeV/c to above 400 MeV/c. The actual beam profile obtained is displayed in Fig. 2.

2. Chamber

The useful volume of the 30 inch chamber is defined roughly by a cylinder of radius 25 cm whose axis is perpendicular to the viewing window, and a depth of about 30 cm. See Appendix E.

Three separate cameras are used, utilizing bright field illumination.

The normal magnetic field of the chamber is about 12 kilogauss. See Appendix A.

3a

Figure 1. The beam transport system leading to the
BNL-Columbia 30 inch bubble chamber.

Figure 2. The beam momentum spectrum at the entrance
to the fiducial volume.

See Section III.A.

B. Scanning

All scanning was carried out by two physicists (the author and R. B. Muir) using Prevost scanning tables. The film was scanned two views at a time. A second scan on a portion of the film was carried out using the third view in combination with one of the two previous views. On the basis of this second scan we believe the scanning efficiency to be very nearly 100%. (That is, both scans produced identical events.)

1. Scanning Criteria

a. Potential difficulties and their resolution. Actual scanning consists of detecting the chain of events;

$$K^- d \rightarrow K_1^0 nn$$

$$\quad \quad \quad \downarrow \pi^+ \pi^-$$

(See Appendix D.)

Notice that detection of K_2^0 decays is neither feasible (much longer lifetime) nor desirable. ($K_2^0 \rightarrow 2$ charged + neutral, so we cannot fit them as K_2^0 .)

Entering K^- 's are rather easily separated from the background π 's since the K^- 's, as opposed to the π 's, are several times minimum ionizing.

Each K^- thus detected is followed until it either;

1. leaves the chamber,
2. decays,
3. scatters, or
4. interacts.

Classes 1 through 3 cause no difficulty and can be quickly discarded on the scanning table.

However, the desired sequence could be confused with;

We may eliminate these events on the scanning table by requiring a visible gap between the K^- and the V.

3. The rare reactions producing Λ 's,

The problem is now reduced to separating protons from positive pions.

To assist in doing this, the following properties of the above reactions are utilized (in order of usefulness).

Opening angle. $\bar{K}^0$ production does not occur at rest, so the $\bar{K}^0$ emerges with low momentum in the laboratory ($< 300 \text{ MeV/c}$). Thus the two- π decay of the K_1^0 will have a very wide opening angle ($> 150^\circ$).

On the other hand, Λ production can occur at rest, so Λ 's thus produced will usually have considerable momentum. The Λ decay opening angle will most likely be $< 150^\circ$.

Ionization. K_1^0 decay is a high Q reaction, each π receiving about 200 MeV/c in the K_1^0 CM system. The π 's are thus minimum ionizing.

The Q of a Λ decay is low, producing a proton of 100 MeV/c, so the proton is several times minimum ionizing.

Range. Positive tracks which do not leave the chamber and do not have a secondary prong are protons, (stopping π^+ 's would decay).

By judicious application of the above criteria, it was possible to avoid measuring most of the Λ events. However, if an event was ambiguous on the scanning table, it was measured.

The effectiveness of these criteria was checked by measuring all V's in a sample of film. After fitting, the results were compared with the results of the scan. In this sample, the application of these criteria correctly eliminated Λ decays only.

b. Scanning summary.

1. A visible gap is required.
2. An event with a large opening angle is most likely a K_1^0 .
3. The positive prong of a K_1^0 decay will be minimum ionizing.
4. Positive prongs must leave the chamber.

2. Scanning Bias Check

By comparing the scanning lists with the results of the reconstruction, we confirmed our ability to distinguish $\bar{K}^0$'s from Λ 's at the scanning stage.

The possibility still remained that certain configurations might be missed.

To check for any such bias, use was made of the fact that the distribution of $\bar{K}^0$'s is expected to be azimuthally symmetric about the beam. Thus, the production plane angle was calculated for each event. This angle is the angular orientation of the normal to the production plane, $\hat{k}_- \times \hat{k}_0$, ($\hat{k}_-$, $\hat{k}_0$ are unit vectors in the directions of the K^- , and the K^0_1 , respectively) with respect to a suitably chosen fiducial line. This line was taken to be $\hat{k}_- \times \hat{z}$, where $\hat{z}$ is defined along the camera axes and toward the cameras.

The K^0_1 distribution of events in the angle ρ , where

$$\cos \rho = (\hat{k}_- \times \hat{z}) \cdot (\hat{k}_- \times \hat{k}_0) ,$$

is thus expected to be uniform if the events are unbiased. The angle ρ may be conveniently viewed as the azimuthal angle of a coordinate system obtained from the standard "bubble chamber system" by two rotations. (Fig. 3.)

The experimental distribution, folded into one quadrant, is displayed in Fig. 4. The events have been corrected for the imposed 1 mm projected gap length cutoff. (pp. 21-25).

- Figure 3. The coordinate systems for the production plane calculation. $\hat{k}_-$ and $\hat{k}_0$ are, respectively, unit vectors in the directions of the K^- and K_1^0 .
- Typical orientations in the bubble chamber system. The beam is nominally along $\varphi_- = 90^\circ$.
 - Rotation about the z axis by angle φ_- to the new system (x', y', z') . The beam is now in the x', z' plane. ($\hat{k}_0$ has been omitted for clarity.)
 - Rotation about y' by the angle $\pi/2 - \lambda$ to the new system (x'', y'', z'') . z'' is now along $\hat{k}_-$; y'' is in the old x, y plane. The angle ρ is measured from y'' , in the x'', y'' plane.

a.

b.

c.

Figure 4. The number of corrected K_1^0 events per angular interval plotted against the angle ρ of Fig. 3. The angle is formed by y'' and $\hat{k}_- \times \hat{k}_0$, the normal to the production plane. The data have been folded about 90° . The solid line is the straight line fit to the data.

The distribution appears satisfactorily flat except for the drop in the region $75 - 90^\circ$. This dip may not be attributed to the cutoff since any bias due to the cutoff should appear near 0° . $75 - 90^\circ$ is, in fact, the region where one would expect to detect gaps most easily. That is, the production plane is nearly perpendicular to the camera axes. See Fig. 3.

Statistically, the distribution is consistent with azimuthal symmetry. The horizontal line is the straight line fit to the data.

C. Measuring

Events selected by the scanning process were then measured by physicists.

The measuring machine used is of local design, consisting basically of the stage of a Gaertner toolmaker's microscope. The film is viewed using a Bausch and Lomb zoom-lens binocular viewer. The stage is electrically driven by a "joy" stick arrangement. Coordinates are read by Datex encoding digitizers of least count 1 micron. All output from the measuring stage is on punched cards.

The number of remeasurements was small and confined for the most part to events we felt were genuine $\bar{K}^0$'s. Aside from measurement blunders, the rejects were due to the difficulty in locating the decay vertex. This was caused by the wide opening angle and the minimum ionizing character of the pions. Because of the relatively high π momentum, the π tracks are frequently very "gappy" near the vertex. As explained in section B-1, this problem is much less frequent in the case of Λ decay.

For these failures, an extrapolation technique using a template was utilized in order to improve on the vertex location.^{19/} If necessary, an event was remeasured several times to obtain acceptable reconstruction.

All candidates were ultimately reconstructed satisfactorily.

D. Data Reduction

All the data manipulating programs used are descendants of the Berkeley programs: Panal, Package, EPC, and Sumx. See Appendix B for a more complete description.

All events passing the scan criterion were measured and then fit to;

1. $K^0_1 \rightarrow \pi^+ \pi^-$ (3C)
2. MM $\rightarrow \pi^+ \pi^-$ (0C)
3. $\Lambda \rightarrow p \pi^-$ (3C)
4. MM $\rightarrow p \pi^-$ (0C)

(At the second vertex.) Here "X"C gives the number of constraints imposed. For example, for number 1, only the magnitude of the K^0_1 momentum is unmeasured.

MM means a missing mass calculation; the invariant mass is calculated and no constraints are imposed.

In addition, in order to insure a two-body decay, the coplanarity of each event was calculated. Thus, given unit vectors, $\hat{k}_0, \hat{\pi}_+, \hat{\pi}_-$ in the direction of the respective particles, the coplanarity is;

$$\text{COPLAN} = \hat{k}_0 \cdot (\hat{\pi}_+ \times \hat{\pi}_-) .$$

Also the error, $\delta \text{ COPLAN}$ is calculated. An event was rejected if COPLAN differed from 0 by more than three standard deviations; i.e., if

$$|\text{COPLAN}| / \delta \text{ COPLAN} > 3 .$$

In the sample considered, this proved to be a vacuous condition as no events were more than two standard deviations from coplanarity.

This condition was checked by using various combinations of the two π 's from the decay

$$K^- \rightarrow \pi^+ \pi^- \pi^- .$$

All of these events failed the coplanarity test, (as they should, except for badly measured events and some pathological cases).

This insures that the only remaining ambiguity is between the K^0_1 and Λ decays.

The separation was made using the chi-square from the fit and the calculated missing mass.

It was usually the case that the chi-squares for the two processes differed by at least a factor of 10. Likewise, because of the large Q difference in the K^0_1 decay, the missing mass was also sensitive to the mass interpretation of the neutral particle.

We thus required that the two fits have chi-squares differing by a factor of 10 and that one chi-square lie within the 1% confidence level region for 3C fits, (11.3). If the event was still ambiguous, the missing mass calculations were considered. This was necessary for 2 of 75 events.

In this way, 75 events were determined to be K_1^0 decays.
from a zero-prong vertex.

III. EXPERIMENTAL RESULTS

The experimental expressions for the cross sections to be investigated are:

$$\sigma_i = \frac{n_i f}{\rho t_i}$$

$$\frac{d\sigma_{ij}}{d\Omega} = \frac{n_{ij} f}{2\pi \rho t_i \delta(\cos\theta_j)}$$

$n_i(j)$ = the corrected number of events in momentum interval i
(angular interval j).

$\delta(\cos\theta_j)$ = the angular interval, j .

t_i = the accumulated path length (in cm) in momentum interval i .

ρ = the deuterium density in nuclei/cc, $= 1/24.8 \times 10^{-24}$, (Appendix C).

$f = r_1 * r_2 = 2.89$.

r_1 = Branching ratio $\left(\frac{K_1^0 \rightarrow \text{all modes}}{K_1^0 \rightarrow \pi^+ \pi^-} \right) = 1/.693 \quad 2/$

r_2 = Ratio $\left(\frac{\bar{K}^0}{K_1^0} \right) = 2$.

A. Path-Length Determination

The path-length (t_i) was obtained by measuring all K_1^0 tracks in every twentieth frame. Each track was measured until,

1. it "ended" by
 - a. scattering,
 - b. decaying,
 - c. interacting, or
2. it left the fiducial volume. (Appendix E)

It is then easy to accumulate the path-length per momentum interval in the fiducial volume by use of the measured momentum and the range-energy relation.

Aside from systematic errors, considered later, the uncertainty in the path-length can be ascribed to two sources:

1. Measurement error. Several measurements of the same track do not give the same momenta.
2. Even assuming no measuring error, upon repeating the experiment, the number of tracks per momentum bin will fluctuate statistically.

Source 2 was estimated by assuming a multinomial distribution in the momentum at the entrance to the fiducial volume. That is, N tracks are distributed in n momentum bins. By the usual manipulations, the errors shown in Table 1 were obtained.

The contribution to the error due to measurement errors is complicated by the fact that many tracks are near stopping. This leads to a truncated Gaussian type of error distribution.

Rather than try to sort this out analytically, we decided to use an empirical method.

This was accomplished by generating a random number which appeared to have been drawn from a normal population of mean zero and variance one. This was then scaled by the momentum error (Pang, Appendix B).

The central momentum used was then,

$$P^0 = P + \delta P * (\text{Random number}).$$

This momentum, P' , was then used in the subsequent path-length calculation.

The results of this investigation are shown in Fig. 5. Several runs of this nature produced the numbers of Table 1. The final column contains those errors from the random generation, combined with those from the multinomial distribution.

B. The Corrected Number of Events, $n_{i(j)}$

The observed number of events cannot be taken as the number produced because we have imposed a projected gap length cutoff. In addition, there is the possibility of escape from the fiducial volume before decay.

The survival probability from the production point to the position x is,

$$P(x) = \exp(-m*x/(t_o*c*p)),$$

where,

x = the distance from the production point,

m = mass of the decaying particle, 497.9 MeV,

p = particle momentum in MeV,

t_o = restframe lifetime; $(.843 \pm .013)*10^{-10}$ sec, ^{6/}

c = the velocity of light.

For a projected minimum length x_o , a dip λ , and distance x_{fp} to the nearest fiducial plane, the corresponding detection probability is now,

$$P(p, \lambda, x_o, x_{fp}) = \exp(-m*x_o/(t_o*c*p*\cos(\lambda))) \\ - \exp(-m*x_{fp}/(t_o*c*p)) .$$

Figure 5. The path length per 25 MeV/c plotted against laboratory momentum. The errors shown include both measurement and statistical errors.

Table 1. Path length summary.

Interval (MeV/c)	Path (m)	Errors (m)		
		Multinomial	Random	Combined
96.7 to 125	268.07	3.11	18.00	18.25
125 to 150	527.81	5.84	9.62	11.25
150 to 175	946.90	9.04	6.07	10.90
175 to 200	1533.11	12.80	12.59	17.98
200 to 225	2292.52	17.52	3.44	17.80
225 to 250	3107.76	26.08	12.75	28.90
250 to 275	3341.95	39.17	36.19	50.70
275 to 300	2486.88	46.11	18.67	49.80

Each accepted event (accepted meaning a projected length $\geq x_0$ and within the fiducial volume) is given a weight,

$$w = 1/P(p, \lambda, x_0, x_{fp}) .$$

The corrected number of events, $n_{i(j)}$, is then the accumulated weight, Σw in the $i(j)$ interval.

1. Choice of Cutoff Length, x_0

x_0 is chosen by observing the change in $N = \Sigma w(x_0)$, where the sum is over all accepted events, as x_0 is varied.

The general behavior expected is that for values of x_0 near the optimum, N will fluctuate little, the change in the w 's compensating correctly for the change in the accepted number of events. For larger values of x_0 , large fluctuations may take place as the accepted number of events is decreasing rapidly. For small values of x_0 , N might be expected to decrease, for in this region, scanning and measuring biases become important.

The desire is to choose x_0 as small as possible, to increase the statistics, consistent with high scanning and measuring efficiency.

Fig. 6 shows the variation of N as a function of x_0 . From this information, x_0 was chosen to be .1 cm.

2. Event Distribution

Fig. 7 displays the resulting event distribution in laboratory momentum, before and after weighting.

Figure 6. Corrected number of events as a function of the projected length cutoff. The errors indicated are statistical only.

Figure 7. The number of events per 25 MeV/c as a function of laboratory momentum. The crosshatched region corresponds to the uncorrected data, the open area to the same data corrected for the projected length cutoff. The interval, $96.7 \leq P \leq 125$ MeV/c includes an event whose measured momentum is 87.4 ± 5 MeV/c, as explained in the text.

As was noted before, no fitting can be done at the first vertex, so the errors in the interaction momentum are fairly large. As a consequence, an event whose momentum was 87.4 ± 5 MeV/c and thus well below the charge exchange threshold of 96.7 MeV/c, but which appeared to be a K^0_1 on the basis of the fit, the missing mass, and the ionization, was included in the 96.7 to 125 MeV/c interval. Thus, the numbers in this region must be viewed as upper limits in the subsequent calculations.*

C. Cross Section

Fig. 8 is a plot of the cross section as a function of laboratory momentum, which for statistical reasons has been averaged over 50 MeV/c intervals.

The upper curve in Fig. 8 is the fit for the K^-p charge exchange data.^{8,9/} It is apparent that the influence of the neutron is profound. Even above 200 MeV/c the proton and deuterium data differ by a factor 2. The modifications near threshold are even more striking.

The results of the cross section calculations are included in Table 2.

*Two possible explanations of the origin of this event arise.

1. Straggling has not been included in the 5 MeV/c error quoted for this event. From Ref. 7, the contribution to the error from straggling is estimated to be ± 15 MeV/c. The event is then consistent with deuterium production and should be included.

2. The deuterium might have a contamination due to H_2 , so the event might be $K^-p \rightarrow \bar{K}^0 n$. Of course, if this is the case, it should not be included.

Figure 8. The cross section (mb) for the reaction $K^-d \rightarrow \bar{K}^0 nn$ as a function of laboratory momentum. Threshold is 96.7 MeV/c. Both measurement and statistical errors are included.

□ This experiment

○ Hetherington and Schick^{4/} (3-body calculation)

◇ Chand^{10/} (with effective range term)

Δ Kim I^{8/} (K^-p data)

▽ Cook^{3/} (for odd K^- , $\bar{K}^0$ parity)

Table 2. Summary of Cross Sectional Data

Interval	(Events)		σ	σ	$\delta\sigma$
(MeV/c)	Raw	Corrected	(millibarns)		
96.7 to 125	1	1.28	3.00		
				2.42	1.56
125 to 150	1	1.36	1.84		
150 to 175	3	4.00	3.03		
				5.94	1.40
175 to 200	14	18.99	8.85		
200 to 225	12	15.74	4.90		
				4.61	0.88
225 to 250	15	18.70	4.32		
250 to 275	16	19.87	4.25		
				3.10	0.64
275 to 300	6	6.83	1.96		

Several authors have calculated the deuterium charge exchange cross section.^{2-4,10/} The earlier calculations were motivated by a suggestion due to Pais^{1/} that some experimental data might best be interpreted as requiring odd (K^- , $\bar{K}^0$) parity. Subsequently, calculations were performed to estimate the effects of parity on the cross section.^{2,3/}

Because of the requirements of the Pauli principle on the (n,n) final state, the reaction $K^-d \rightarrow \bar{K}^0 nn$ may be expected to be sensitive to the relative (K^- , $\bar{K}^0$) parity, particularly near threshold.^{2,3/} The results of these calculations indicated that the momentum dependence near threshold was strongest for odd relative parity.

Experimentally, this is a very difficult region in which to work and no decisive conclusions as to the relative parity can be drawn from the data presented, (our data do fall significantly below Cook's odd parity calculations) other than to remark that the magnitude observed appears most consistent with even parity.

In any event, the success of the symmetry theories (which assume even K^- , $\bar{K}^0$ parity) in organizing the accumulated high energy data strongly implies even parity.

Therefore, we shall not be concerned further with this question.

More relevant to the present discussion are the calculations of Chand,^{10/} and more recently, that of Hetherington and Schick.^{4/}

The Chand calculation is in the spirit of the Dalitz and Tuan analysis of the low energy K^-p data.^{11/} The calculation is claimed to be reliable in the range, $150 \leq P_{lab} \leq 300$ MeV/c.

A set of parameters for an effective range expansion is obtained from K^-p data and then used to calculate differential and integrated deuterium charge exchange cross sections.

Chand's deuterium cross sections seem to suffer in much the same way as do those which he calculates for the proton data (Fig. 8). That is, the magnitude is correct but the slope is too steep.

From Ref. 10, it appears that Chand's calculations are quite sensitive to the range parameter used. Because this parameter was obtained by interpolating between two sets of data, it might be worthwhile to repeat this calculation using the more recent K^-p data of Kim.^{8/}

In the calculation of Hetherington and Schick,^{4/} the Faddeev equations are used to solve exactly the three-body break-up problem.

To do so, the two particle potentials were taken to be of the S-wave, non-local-separable type,^{12/} i.e., Yukawa, parameterized by a strength, λ , in each isospin state, and a range parameter, β . The parameters for the n,p triplet potential were taken from Ref. 12. The scattering lengths of Kim^{8/} were used to fix the K-nucleon parameters. The K^- , $\bar{K}^0$ mass difference has been neglected.

The cross sections thus calculated reproduce the shape of the data reasonably well but the magnitude is about 20% high. Because the K^- , $\bar{K}^0$ mass difference was not included, the threshold behavior cannot be correct. There is some reason to believe that the magnitude is sensitive to this mass difference and that the effect is in the right direction and of the correct size so as to explain the present

discrepancy.^{4/} It is hoped that such a calculation might become available in the future.

D. Differential Cross Section

The experimental angular distributions are displayed in Figs. 9 and 10 for three laboratory momentum intervals. The errors shown are purely statistical.

The solid curves shown in the case of the K^-d cm data are the calculations of Chand,^{10/} scaled to give the integrated experimental cross section. The theoretical behavior at the forward angles is due to the restrictions of the Pauli principle on the triplet (n,n) final state.

At present, no three-body differential cross section calculations have been reported for the deuterium charge exchange reaction.

- Figure 9. The experimental differential cross section.
 $d\sigma/d\Omega$ (mb/str) vs. $\cos\theta^*$. θ^* is the
scattering angle in the K^-d cm system. The
solid curves have been derived from those of
Chand^{10/} by a scale factor and serve as a guide
only.
- a. $150 \leq P_{\text{lab}} \leq 200$ MeV/c
 - b. $200 \leq P_{\text{lab}} \leq 250$ MeV/c
 - c. $250 \leq P_{\text{lab}} \leq 300$ MeV/c

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Table 3. Summary of K^-d cm Differential

Cross Sectional Data

Angle Interval	150-200 MeV/c			200-250 MeV/c			250-300 MeV/c		
	Num.	$\frac{d\sigma}{d\Omega}$	$\delta(\frac{d\sigma}{d\Omega})$	Num.	$(\frac{d\sigma}{d\Omega})$	$\delta(\frac{d\sigma}{d\Omega})$	Num.	$\frac{d\sigma}{d\Omega}$	$\delta(\frac{d\sigma}{d\Omega})$
-1. to -.6	1	.14	.14	2	.17	.12	1	.09	.09
-.6 to -.2	3	.50	.29	4	.40	.20	3	.17	.10
-.2 to +.2	3	.61	.35	6	.47	.19	5	.34	.15
.2 to .6	5	.69	.27	8	.58	.21	9	.54	.18
.6 to 1.	5	.82	.37	7	.54	.21	5	.34	.15

IV. SUMMARY AND CONCLUSIONS

Data have been presented on the reaction $K^-d \rightarrow \bar{K}^0 nn$. The cross section is consistent with even (K^- , $\bar{K}^0$) parity, and agrees reasonably well (20%) with recent three-body calculations.

Angular distributions have been presented which indicate that low partial waves are taking part (predominately S-wave).

Work is now in progress on the remaining film with the hope that the statistics will be trebled.

At the same time, it would be helpful if the present three-body calculations were extended to include the K^- , $\bar{K}^0$ mass difference, as well as the differential cross sections at several momenta.

PART TWO: K_p^+ ELASTIC SCATTERING: 80-280 MeV/c.

I. INTRODUCTION

Ever since Kaon beams became available for experimental use, it has been of fundamental interest to determine the sign and strength of the K-nucleon interaction.

To determine the strength, one measures the cross section for elastic scattering. Then by examining the interference with the Coulomb field at low momenta, it is possible to deduce the sign of the interaction.

For these purposes, the reaction

$$K^+p \rightarrow K^+p$$

offers several advantages. Unlike the situation in K^-p , absorption effects do not take place at low momenta. Furthermore, inelastic effects do not occur, for the π production threshold is not reached until 510 MeV/c K^+ laboratory momentum.

Initially, the first data were obtained using emulsion techniques.^{2/} Since then, both counter and bubble chamber experiments have been reported.^{3-5/}

For the momentum interval necessary for this experiment, a bubble chamber has several practical advantages over emulsion or counter techniques. In the case of nuclear emulsions, one is faced with the difficulty of separating the hydrogen scatters from the more numerous scatters from the heavier nuclei.

On the other hand, below 200 MeV/c, counter experiments suffer because of the limited range of the final state particles.

In the bubble chamber, however, such particles will stop and can be detected over a near 4π steradian range.

This experiment was undertaken with two purposes. First, to verify the results of the previous experiments in the momentum range, 80 to 280 MeV/c. Secondly, to improve the accuracy of the results by use of larger statistics and a more refined experimental method.

As will become clear, the first purpose was achieved, but the method chosen was inadequate to accomplish the second.

II. EXPERIMENTAL DETAILS

A. Physical Layout

The experiment was performed using the "stopping" K^+ beam of the Bevatron at the Lawrence Radiation Laboratory.^{6/} The beam transport system was tuned for 650 MeV/c K^+ mesons. To obtain a "stopping" beam near the entrance of the 15 inch chamber, a tungsten moderator was inserted in the beam path immediately before the chamber. The resulting momentum spread at the entrance to the fiducial volume (Appendix B) extended from 75 to above 300 MeV/c.

Approximately 33,000 pictures were obtained at an average flux of 2 K^+ 's per frame. The background was believed to be less than 2%.^{1/}

The 15 inch chamber is viewed with 4 cameras, arranged at the corners of a rectangle. The cameras were constructed so that two views appear on a single film. The magnetic field of the chamber was approximately 10 kilogauss.

B. Scanning

We wish to identify the elastic scattering reaction,

$$K^+p \rightarrow K^+p .$$

In particular, we are interested in small forward angles, so as to observe the Coulomb-nuclear interference. In addition, the momentum region, $80 \leq P_{lab} \leq 280$ MeV/c is of interest. In these regions the scattered proton will frequently not receive sufficient momentum to make one bubble, so that the event may appear as a "one-prong".

Because many incoming beam tracks are nearly stopping and at rest reactions are not possible, there will be a large background of decaying particles. Much of this background may be eliminated by use of the ionization of the second track. However, a significant fraction of events will remain ambiguous, particularly in the forward direction. In most cases, kinematic fitting will be of little help.

In order to eliminate these background decays, it was decided to require all scatter candidates to be followed by a visible decay of the scattered particle (Fig. 1).

In the determination of the differential track length of K^+ mesons, beam tracks were also required to end with a decay. It was hoped that in this way, we might take advantage of the fact that a large fraction of the beam tracks stop in the chamber. For a track known to stop, the entrance momentum may be obtained from the range of the track. This momentum determination is more accurate than the momentum obtained from the measured curvature of the track.

The identification of entering K^+ 's was particularly simple. In particular, the beam was known to be contaminated by less than 2% π 's and μ 's. In addition, K^+ 's at these momenta are quite heavily ionizing, while the background is near minimum ionizing.

Once an entering K^+ was identified, it was followed until it "ended" by,

1. leaving the chamber,
2. scattering; the scattered particle then leaving the chamber,
3. scattering; followed by a decay of the scattered particle, or
4. decaying.

Figure 1. Event type 63. Shown are the principal variants of this event. The top figure illustrates the case when the proton is seen. In the lower figure, insufficient momentum is transferred to the proton to make a visible stub. The dotted tracks indicate the decay products.

a.

b.

Each track was followed in all 4 views because of the possible view dependency of detecting small angle scatters and because of the presence of "flares", or poor optics. Only tracks with ending 3. were included as scatter candidates, while those of 4. comprised the beam track data.

All scatter candidates were entered on a master scan list. In addition, a measuring sketch of the event was made. This sketch, headed by the event identification, included a table of views to be used for each track. This was necessary because the reconstruction program, Pang,^{7/} will not accept more than two views for a given track.

If the scan indicated that measuring difficulties might be encountered, a note was made of the anticipated difficulty, as well as possible corrective measures.

C. Measuring

All measuring was carried out on a digitized microscope of local design. Basically, the machine consists of the stage of a Gaertner toolmakers microscope. The stage has a useful travel of 80 mm. The lead screws are driven electrically by a "joy" stick and are directly connected to Datex encoding digitizers. All output from the measuring machine is on punched cards.

Although the setting error, determined by measurement of a very accurate test line, was less than 3 microns, the accuracy obtained in practice when measuring fiducials and actual bubble chamber tracks was nearer to 7.5 microns.

The possible causes of event failure necessitating remeasurement are many, but in practice over 80% of the failures fell into two definite categories.

First, those caused by operator blunders. Usually these errors were obvious and easily corrected.

Second, errors induced by use of an incorrect view pair. The reconstruction program requires that the line joining the axes of the camera pair used be as nearly perpendicular to the track as possible to insure correct depth reconstruction. For these errors, it was necessary to remeasure the event after selecting the correct view pair(s) for the offending track(s).

D. Data Processing

The data processing programs used are direct descendants of the Berkeley programs: Pang,^{7/} Kick,^{8/} EPC,^{9/} and Sumx.^{10/}

Each event was fitted by Kick to the elastic scattering hypothesis

$$K^+ p \rightarrow K^+ p ,$$

provided that the proton was seen. Since both proton and K^+ will not interact and thus will likely come to rest if they do not escape the chamber, 4 fits were performed. Each outgoing track was considered to first stop, and then to "leave" the chamber. If one or the other of the outgoing particles did stop, the quality of the corresponding fit was much improved. The fit used in subsequent calculations was chosen on the basis of confidence level.^{12/}

If the proton is so short as to be unmeasured, the fit becomes 1C, (one constraint). In this case Kick would fit for the unknown momentum and direction of the proton. A constraint may be regained, however, by using the fact that if the proton is not seen, the projected momentum is restricted to be less than that required to form one bubble (~ 1 mm or ~ 80 MeV/c). Otherwise a stub would be seen. This constraint is imposed by Kick.

All corrections and the imposition of the fiducial volume (Appendix B) were performed in Sumx.^{10/}

III. EXPERIMENTAL DETAILS

A. Introduction

The following expression is to be used to calculate the differential cross section from the experimental data.

$$\frac{d\sigma_{ij}}{d\Omega} = \frac{n_{ij}}{2\pi\rho t_i \delta\cos\theta_j},$$

where,

$n_{i,j}$ = the number of corrected events in momentum interval i , angular interval j ,

ρ = the density in nuclei per cc, = 3.55×10^{22}

$\delta\cos\theta_j$ = the width in $\cos\theta$ of the angular interval, j .

t_i = the accumulated path length in momentum interval i .

B. Path Length

In order to determine the path length per momentum interval, 6% of the tracks determined to be beam tracks from the scan were measured. Because of certain biases inherent in the scan, to be discussed now, it is not a straightforward procedure to generate the path length from these tracks.

1. The sample of measured beam tracks is strongly biased towards momenta below 220 MeV/c at the entrance plane. As was discussed in Section II-B, K^+ tracks were only considered to be beam tracks if they ended with a decay.

The residual range of a K^+ entering with a momentum ~ 220 MeV/c is ~ 27 cm. Since the fiducial volume (Appendix B)

is only slightly larger than this, tracks above 220 MeV/c will only rarely be detected. The reason for this may be seen in Fig. 2, where the probability of decay in the fiducial volume is plotted against incident momentum. It is clear that such a track is unlikely to decay within the fiducial volume.

One way out of this difficulty is to only consider scatters with entrance momenta ≤ 220 MeV/c. This is not useful because in the sample of scatters detected, only ~ 50 or $< 5\%$ were in this category. This is because the peak in entrance momentum appears to be slightly above 220 MeV/c with the result that most of the path length will be contributed by tracks in this category.

A second possibility is to consider the subset of all scatters which are then followed by a decay. One might imagine that these scatters and the corresponding subset of beam tracks are related to each other in the same way as the parent samples. Unfortunately, this is not the case and the reasons why it is not will be discussed further in the next section.

Thus, because most of the beam track decays are in-flight decays, it is not possible to use the momentum from range and then treat the corrections as small, thus forming the path length by an iterative process. Instead, the momentum from curvature is used to calculate the probability of decay within the chamber. Each track is then weighted by the inverse of this probability. (See Appendix A).

The entrance plane momentum distribution after this correction is seen in Fig. 3.

Figure 2. The detection probability plotted against the momentum at the entrance plane. For momenta above 226 MeV/c, the residual range is sufficient to allow escape from the fiducial volume (~ 27.5 cm).

Figure 3. At the top is the corrected momentum spectrum at the entrance plane; below, the uncorrected spectrum. The shift in the corrected peak illustrates the presence of the scan bias towards events with momenta less than 220 MeV/c.

The resulting path length distribution is shown in Fig. 4, and the data are summarized in Table 1.

The statistical fluctuation of numbers of entering tracks was estimated by assuming a multinomial distribution at the entrance plane.

To estimate the effect of measurement errors on the momentum from curvature, each beam track was changed by a random (Gaussian) error and the path length was then recomputed.

C. Corrected Number of Events, $n_{i,j}$

The observed number of events must be corrected for several effects. The first of these is due to the imposed projected angle cutoff.

As the laboratory scattering angle (on the scanning table) decreases, the scanning efficiency decreases correspondingly. To avoid this region of poor scanning efficiency, only those events were accepted for processing whose projected scattering angle exceeded 3° . Then using the azimuthal symmetry of the distribution of events about the beam, a correction factor was calculated for each event (Appendix A).

In order to insure a reliable fit, it was necessary to require that the length of the scattered K^+ exceed some minimum, x_0 . However if this is done, for a given incident momentum, all angular intervals may not be kinematically accessible. Fig. 5 illustrates this point. The parameter is the minimum length, x_0 . For a given x_0 , each curve is the locus of scattering angle ($K^+ p$ cm) as a

Figure 4. The accumulated path length per 20 MeV/c momentum interval. The errors shown are those from the multinomial distribution combined with those from the random error generation.

Table 1. Path Length Summary

Interval (MeV/c)	Path length (10^4 cm)	No. Entering	Errors (10^4 cm)
80-100	1.63	15	.02
100-120	3.05	23	.21
120-140	5.35	56	.61
140-160	9.01	86	.98
160-180	14.16	138	.76
180-200	21.07	221	1.20
200-220	28.23	312	1.15
220-240	30.10	233	.86
240-260	23.02	136	1.31
260-280	11.26	91	2.36

Figure 5. The kinematic limits for

$$x_0 = 1., 2.5, \text{ and } 5 \text{ mm.}$$

For example, for the interval,

$$-.6 \leq \cos \leq -.2, \text{ and}$$

$$100 \leq P_{\text{lab}} \leq 120 \text{ MeV/c,}$$

the cross-hatched region is not accessible.

function of laboratory momentum. Each number, $n_{i,j}$ is then corrected for the kinematically allowable phase space (that to the right of the curve for a given $\delta p \delta \cos \theta$ interval).

A correction must also be made for those scattered K^+ 's with residual range $\gg x_0$, but which decay with a range $< x_0$ and thus are lost. (Appendix A).

A final correction was made to compensate for those scatters whose scattered K^+ leaves the chamber before decaying, and hence, is not included in the scan. (Appendix A.) This correction is quite dependent on the scattering angle. It is for this reason that the subsample of events with scatters and subsequent decays (uncorrected) will not yield the same cross section as obtained by using all scatters, whether or not a decay is seen.

As an example, consider events whose momentum at the entrance plane is above 220 MeV/c, so that it is probable that the track will leave the chamber without a decay, provided no scatter takes place. The situation is changed little for small forward scatters; the residual range of the K^+ after the scatter is very nearly the same as before. As the scattering angle increases, the residual range of the scattered K^+ decreases due to the increased momentum transfer to the proton. In this case a decay is much more likely to be seen.

IV. THE DIFFERENTIAL CROSS SECTION

The experimental differential cross sections are shown in Fig. 6 for five laboratory momentum intervals. To improve the statistics, the data have been averaged over 40 MeV/c intervals. The second set of data is from references 1 and 5.

In general, the two experimental results agree within errors, although the present results appear to be consistently lower than the earlier data.

In an attempt to determine the sign of the Coulomb nuclear interference and obtain the corresponding phase shifts, the data were fit by the method of least squares to the expression:

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2} \left| e^{i\delta} \sin\delta - \frac{\alpha}{2 \sin^2 \frac{\theta}{2}} \exp(-i\alpha \ln \sin^2 \frac{\theta}{2}) \right|^2$$

where, $k = K^+ p$ cm momentum

θ = the $K^+ p$ cm scattering angle

$$\alpha = \frac{e^2}{h v_{\text{rel}}}$$

v_{rel} = the relative $K^+ - p$ velocity

δ = the s-wave phase shift.

Because of the marginal statistics, the values of δ obtained were very sensitive to the input data. The result of this calculation is thus inconclusive and is not included.

Figure 6. The differential cross section (mb/ster)
for five laboratory momentum intervals
as a function of the K^+p cm scattering angle.

The angular intervals are:

$$-1.0 \leq \cos\theta \leq -.6$$

$$-.6 \leq \cos\theta \leq -.2$$

$$-.2 \leq \cos\theta \leq .2$$

$$.2 \leq \cos\theta \leq .6$$

$$.6 \leq \cos\theta \leq .8$$

$$.8 \leq \cos\theta \leq .95$$

The data from this experiment are indicated
by $\bullet$. Superimposed on this are the data ($\blacksquare$)
of Goldhaber, et al.^{5/} The angular intervals
are the same as those given above.

V. SUMMARY AND CONCLUSIONS

Data have been presented for the elastic scattering of K^+ mesons on protons. The differential cross section is found to be flat and to vary little with increasing momentum. These results are consistent with the previously reported bubble chamber experiments.^{1,5/}

Because of the limited statistics available, it was not possible to obtain reliable values for the phase shifts.

PART ONE: APPENDICES

APPENDIX A. MAGNETIC FIELD DETERMINATION

Because it is not possible to perform a fit at Vertex I, it is vital that the momentum from curvature be as accurate as possible. This was complicated by the fact that due to technical difficulties, the magnetic chamber of the bubble chamber was not operated at its normal value during this exposure.

For the BNL 30-inc. chamber, the field is quite uniform, so that,

$$B(x,y,z) = B_0(1 + B_1 * f(x,y,z) + \dots)$$

where $B_1 * f < 4\%$ at maximum $f(x,y,z)$.

Thus it is adequate to accurately fix B_0 . To do this, two methods were used.

1. Tau decays. Several hundred well measured tau decays ($K^- \rightarrow \pi^- \pi^- \pi^+$) were fit (4 constraints). The field was adjusted to give the "pulls" mean 0 and variance 1.^{15/} This was sufficient to allow us to separate $\bar{K}^0$'s from Λ 's. (A constrained fit is less sensitive to the field than is an unconstrained fit.) After the events were separated, method 2 was used to sharpen the field value.

2. Missing mass from $\bar{K}^0$ decay. For two particle decay, the missing mass is sensitive to the opening angle between the decay products and to the momenta of the products. This is particularly true of $\bar{K}^0$ decay because of the small mass and large Q .

Consider the following diagram.

We have $P = P_1 + P_2$, (4-momenta)

$$\begin{aligned} \text{so that } MM^2 &= P^2 = (P_1 + P_2)^2 \\ &= 2M^2 + 2E_1E_2 - 2P_1P_2x \end{aligned}$$

where $x = \cos\theta$,

M = the masses of 1,2; assumed equal.

Since $P \propto B_0$, upon calculating δMM , we find,

$$\delta MM/MM = \left(\frac{P_1P_2}{MM^2} \right) \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} - 2x \right) \frac{\delta B_0}{B_0}$$

Here, P_1, P_2 are the magnitudes of the 3-momenta and $\beta_i = P_i/E_i$.

Using the $\bar{K}^0$ mass, $\delta B_0/B_0$ is calculated for each event.

After forming $\langle \delta B_0 \rangle$, a new B_0' is derived using this correction and the process is iterated. The missing mass spectrum resulting from the parameters decided upon is shown in Fig. 11.

Figure 11. The experimental K_1^0 mass from 68 events using the final value of the magnetic field. The solid line corresponds to the accepted value of 497.8 MeV; the dotted line to the weighted average of the experimental values, 499.7 ± 1.8 MeV.

APPENDIX B. PROGRAM DESCRIPTIONS

The following are brief, informal descriptions of the principal data processing programs used in this experiment.

PANAL^{13/} the Panal program is basically a 3 part formatting and bookkeeping program.

1. Format. In this section, a multiscard event is reformatted, if necessary, and written out as one record. Checks are made on the consistency of the data, and errors may cause the event to be rejected.
2. Sort. The section of Panal consists of a sorting pass to order the data on the basis of keys provided by the programmer. Following the sort, an ordered list of condensed data is written to aid in the bookkeeping process.
3. Merge. Pass 3 allows sorted data to be merged, so as to build up a library file for the experiment.

PACKAGE; Package is a combination of two programs; PANG^{14/} and KICK^{15/}

1. PANG^{14/} In the Pang program, individual tracks are reconstructed using the measured film coordinates from the various views. A least squares fit is performed, taking into account the energy loss in the liquid. Corrections for film distortions, optical distortions, and magnetic field variations are incorporated.

Included in the output from Pang are the momentum and angles of the track, as well as the errors propagated from the measurement and those due to Coulomb scattering.

2. KICK.^{15/} In Kick, a least squares fit of tracks at a vertex is performed, using the information derived in Pang. When possible, the fit is constrained by imposing conservation of momentum, which is included by the use of Lagrangian multipliers.

The output quantities consist of the fitted momentum and angles of the various tracks at the vertex as well as the corresponding error matrices.

EPC.^{16/} EPC is a program which allows the physicist to easily use the standardized output from Package to compute more meaningful quantities, i.e., to perform Lorentz transformations.

SUMX.^{17/} This program is a rudimentary Information and Retrieval system. It allows the experimentalist to select subsets of data, form new lists, make histograms and ideograms, and so forth.

APPENDIX C. DEUTERIUM RANGE AND DENSITY

The range-energy relation was calibrated for deuterium by using ranges. The μ 's used in the sample were produced in the sequence;

$$K^- \rightarrow \pi^- \pi^- \pi^+,$$

$$\pi^+ \rightarrow \mu^+ \nu,$$

$$\mu^+ \rightarrow e^+ \nu \bar{\nu}.$$

Events were considered provided that;

1. the tau was well measured, (4C)
2. the π^+ was nearly stopping (1 std. dev.)
3. the μ^+ dip was $\geq 30^\circ$.

These criteria were imposed to improve the probability that the decay took place near rest and to minimize reconstruction difficulties. On the basis of 97 events,^{19/} the range was found to be consistent with that of Yoon;^{18/} $.959 \pm .001$ cm.

The corresponding deuterium density is $.135 \pm .002$ g/cc.

In terms of nuclei per cc,

$$\rho = 1/24.8 \times 10^{-24}.$$

APPENDIX D. EVENT TYPE 30

Figure 12. Schematic drawing of a typical event type 30.

The track interpretations are:

Track	Mass
1	K^-
2	K^0_1 or Λ
3	π^-
4	π^+ or p

EVENT TYPE 30

APPENDIX E. FIDUCIAL VOLUME

The peculiar fiducial volume shown in Fig. 13 was chosen because of technical difficulties with the chamber window. Vacuum pump oil was condensing on the window so that in the areas outside the volume, scanning was difficult.

The beam direction was such that the beam principally entered through plane 1, although the spatial spread was large enough to allow entry through plane 2 also.

The following table gives the coefficients of the chosen planes in the form; $Ax + By + Cz + D = 0$.

Plane	A	B	C	D
1	-.25	1.	0.	29.5
2	.60	1.	0.	32.0
3	1.00	0.	0.	25.5
4	.60	1.	0.	32.0
5	-.60	1.	0.	32.0
6	1.00	0.	0.	-20.0
7(front)	0.	0.	1.	-13.75
8(back)	0.	0.	1.	15.0

#7 = front plane $z = 13.75$

#8 = back plane $z = -15.0$

PART TWO: APPENDICES

APPENDIX A. CORRECTIONS TO THE DATA

1. Projected Angle Cutoff

Because only those events were processed which had a projected angle $\geq \psi$, the sample must be corrected for the missed events. To derive this probability, we use the expected azimuthal symmetry of the scattered particle about the beam direction.

The locus of scatters of angle θ about the beam direction is a cone, with the apex at the point of the scatter and the axis along the beam direction. Certain angular regions (measured around the base of the cone) will not be accessible because of the angular cutoff. These angular regions are formed by the intersection of the cone and two planes perpendicular to the scanning plane, and making an angle $\pm \psi$ with the projected beam. If angles are measured from the $\hat{z}$ axis of Fig. 7, we may express the probability of observing an event in terms of two angles, ϕ_1, ϕ_2 , by using the symmetry about the plane formed by the beam and the projected beam. Then,

$$P = \left| \frac{\phi_1 - \phi_2}{\pi} \right|.$$

The problem is thus to determine the extreme angles, ϕ_1 and ϕ_2 .

The "bubble chamber" system is shown in Fig. 7a. The z-axis is along the optic axis. The calculation may be simplified by performing the two rotations of the coordinate system as indicated in Fig. 7. Any scanning bias present will not depend on ϕ_1, ϕ_2 but only on $\phi_1 - \phi_2$, so we may start from the system of Fig. 7b.

Figure 7. a) The "bubble chamber" coordinate system.

$\hat{k}_i, (\hat{k}_o)$ refer to the incoming (outgoing) K^+ .

The z-axis is along the direction of the magnetic field. The beam enters along the y-axis.

b) The system generated by a rotation of ϕ_i about the z-axis. $\hat{k}_i$ now lies in the x', z' plane.

c) The system obtained from 7b by a rotation about y' by $\pi/2 - \lambda_i$ to the ρ, ξ, η system. $\hat{k}_i$ is now along η .

a.

b.

c.

Now in the primed system, the event will be detected provided that,

$$\tan \psi \leq \tan \varphi_0$$

$$= \frac{\hat{y}' \cdot \hat{k}_0}{\hat{x}' \cdot \hat{k}_0}$$

Following the second rotation, we have

$$\hat{\rho} = \hat{x}' \sin \lambda_1 - \hat{z}' \cos \lambda_1$$

$$\hat{\xi} = \hat{y}'$$

$$\hat{\eta} = \hat{x}' \cos \lambda_1 + \hat{z}' \sin \lambda_1$$

and $\hat{k}_1 = \hat{\eta}$

$$\hat{k}_0 = \hat{\rho} \cos \theta \cos \varphi + \hat{\xi} \cos \theta \sin \varphi + \hat{\eta} \sin \theta$$

θ is the polar angle in the $\hat{\rho}, \hat{\xi}, \hat{\eta}$ system and hence, the laboratory scattering angle.

Making the obvious substitutions and then collecting terms, we have:

$$\begin{aligned} \hat{k}_0 = & \hat{x}' (\sin \lambda_1 \cos \theta \cos \varphi + \cos \lambda_1 \sin \theta) \\ & + \hat{y}' \sin \theta \cos \varphi \\ & + \hat{z}' (\sin \lambda_1 \sin \theta - \cos \lambda_1 \cos \theta \cos \varphi). \end{aligned}$$

Thus, we must have,

$$\tan \psi = \frac{\sin \varphi}{(\sin \lambda_1 \cos \varphi + \cos \lambda_1 \tan \theta)}$$

for the critical values, φ_1 and φ_2 .

Solving for φ , we find,

$$x^2(1-B^2) - 2xA - (B^2-A^2) = 0,$$

where $x = \tan\varphi$,

$$A = \tan\psi \sin\lambda_i,$$

$$B = \cos\lambda_i \tan\theta \tan\psi.$$

Using a well known trigonometric identity, we define X form,

$$X = \tan(\varphi_+ - \varphi_-) = \frac{\tan\varphi_+ - \tan\varphi_-}{1 + \tan\varphi_+ \tan\varphi_-}$$

$$= \frac{x_+ - x_-}{1 + x_+ x_-}$$

$$= \frac{-2 \sqrt{A^2 + (1-B^2)(B^2-A^2)}}{(1-B^2) - (B^2-A^2)}$$

$$\text{Thus, since } P = \frac{|\varphi_1 - \varphi_2|}{\pi}$$

$$P = \begin{cases} 1/\pi \operatorname{Arctan}(X), & X \geq 0, \\ 1-1/\pi \operatorname{Arctan}(|X|), & X < 0. \end{cases}$$

2. The Survival Probability, g.

In this and the following sections, it is useful to have an analytic expression for the momentum, P as a function of the residual range, L. Fig. 8 shows the fit obtained to the data from the range-energy table of Pang, by the form,

$$\frac{P}{P_0} = \left(\frac{L}{L_0}\right)^\alpha, \quad (1)$$

Figure 8. The fit obtained to the data from the Pang
momentum to range table by the form,

$$\frac{R}{R_0} = \left(\frac{P}{P_0}\right)^\beta ,$$

with

$$R_0 = 1.47 \text{ cm},$$

$$P_0 = 100 \text{ MeV}/c,$$

$$\beta = 3.55 .$$

with

$$P_o = 100 \text{ MeV/c},$$

$$L_o = 1.47 \text{ cm},$$

$$\alpha = 0.282 \pm .003 .$$

We now wish to compute the probability, g , that a particle of mass m , momentum p (hence residual range $L(p)$) at $x = 0$, will survive a distance $x < L$.

For mean free path $\lambda(p)$, the probability for a decay in the interval dx is $dx/\lambda(p)$. Using relation (1), and performing the integration, we obtain:

$$g(L, x) = \exp\left(-\beta\left(\frac{L}{L_o}\right)^\gamma\left(1-\left(\frac{L-x}{L}\right)^\gamma\right)\right) .$$

Here,

$$\beta = \frac{m L_o}{\gamma P_o t_o} = .0276,$$

t_o = the mean lifetime in the restframe (1.23×10^{-8} sec),

$$\gamma = 1 - \alpha = .718.$$

If x_{fp} is the arc length along the track from the entrance plane to the exit fiducial plane, the probability of a decay (and hence of detecting the track during the scan) is

$$P_{\text{detection}} = \begin{cases} 1 & , \quad L \leq x_{fp} \\ 1-g(L, x_{fp}), & L > x_{fp} \end{cases} .$$

$1/P_{\text{detection}}$ is the weight given to each beam track. (Because beam tracks followed by scatters comprise less than 2% of the beam tracks, this effect has been ignored.)

3. Corrections to Scatters

In the case of scatters, the above is modified by the fact that a minimum length x_0 was required. In addition, the scattered K^+ may leave the chamber. Thus the detection probability is

$$P_{\text{detsct}}(L) = \begin{cases} g(L, x_0); & L \leq x_{fp} \\ g(L, x_0) - g(L, x_{fp}); & L > x_{fp} \end{cases}$$

This is a position dependent function. To approximately take this fact into account, the azimuthal symmetry about the beam is used; the average probability is formed.

$$P_{\text{detsct}}(X) = \int_0^{2\pi} \frac{P(X, L, \phi) d\phi}{2\pi}$$

Each scatter is then weighted by the inverse of this expression.

APPENDIX B. FIDUCIAL VOLUME

The fiducial volume was chosen to be as large as consistent with good scanning efficiency. The fiducial volume was applied after the reconstruction and fitting so as to be independent of scanning.

The following table gives the fiducial volume as a series of planes of the form $Ax + By + Cz + D = 0$.

Plane	A	B	C	D
1. Beam In	0	-1.	0.	13.5
2. Side	-1.	0.	0.	12.0
3. Beam Out	0.	1.0	0.	14.0
4. Side	1.0	0.	0.	12.5
5. Top	0.	0.	-1.0	2.0
6. Bottom	0.	0.	-1.0	24.0

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VITA

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