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MATHEMATICAL ANALYSIS OF A TWO-BEAM GAMMA DENSITOMETER

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by

G. D. Lassahn

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ABSTRACT

A mathematical analysis was done for a two-beam gamma densitometer used with simple two-phase flow geometries. The ambiguity and errors are discussed for the particular system considered, and data interpretation charts are given. Generally applicable mathematical techniques are outlined, and a pertinent computer program is listed.

SUMMARY

For the densitometer geometry used in this work, a simple graphical scheme can be used to determine the flow regime and the average density of the two-phase fluid in the pipe, if certain assumptions are made about the nature of the fluid flow. Essentially, unambiguous interpretations of the densitometer readings can be done with the restrictive assumptions used here. Interpretation with less restrictive assumptions would require a rather complicated analysis and considerable knowledge of how the fluid flow regimes depend on the temperature, pressure, and velocity of the fluid.

CONTENTS

ACKNOWLEDGEMENT	ii
ABSTRACT	iii
SUMMARY	iv
1. INTRODUCTION	1
2. SYSTEM ANALYSIS	1
2.1 Densitometer Description	1
2.2 Fluid Flow Model	3
2.3 Calculated Densitometer Response	3
2.4 Ambiguity	12
2.5 Noise and Errors	15
3. CONCLUSIONS	20
4. FUTURE WORK	20
5. REFERENCES	20
APPENDIX A – OUTLINE OF MATHEMATICAL TREATMENT OF GAMMA DENSITOMETER	21

FIGURES

1. Geometry of the two-beam gamma densitometer which was analyzed	2
2. Beam 1 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at standard temperature and pressure	4
3. Beam 2 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at standard temperature and pressure	5

4.	Beam 1 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at standard temperature and pressure	6
5.	Beam 2 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at standard temperature and pressure.	7
6.	Beam 1 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at blowdown temperature and pressure	8
7.	Beam 2 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at blowdown temperature and pressure.	9
8.	Beam 1 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at blowdown temperature and pressure	10
9.	Beam 2 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at blowdown temperature and pressure	11
10.	Interpretation chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at standard temperature and pressure	12
11.	Interpretation chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at blowdown temperature and pressure	13
12.	Average density chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at standard temperature and pressure	14
13.	Supplemental chart for ambiguous region in which the beam 2 sensor reading is at its minimum. For an inner annulus radius greater than 1.15 inches, there is no ambiguity; for a radius less than 0.32 inch, there is no information at all	15
14.	Supplemental chart for ambiguous region in which the beam 1 sensor reading is at its minimum. For liquid depths less than 2.66 inches, there is no ambiguity	16

15.	Beam 1 sensor reading versus liquid depth for several values of curvature of the liquid-gas interface at standard temperature and pressure	17
16.	Beam 2 sensor reading versus liquid depth for several values of curvature of the liquid-gas interface at standard temperature and pressure	18
17.	Interpretation chart for two-beam densitometer, assuming a variable radius of curvature of the liquid-gas interface, at standard temperature and pressure	19
18.	Average density chart for a two-beam densitometer, assuming a variable radius of curvature of the liquid-gas interface, at standard temperature and pressure	19
A-1		24
A-2		25
A-3		29

TABLES

A-I	Computer Input Variables	32
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1. INTRODUCTION

This report describes work supported by the Semiscale Project done to increase understanding of gamma densitometers in general and to analyze a particular semiscale densitometer.

A gamma densitometer, as discussed in this report, consists of sources, detectors, and shielding which define one or several beams of gamma radiation passing through a pipe containing a fluid. The attenuation of any of the beams as it traverses the fluid gives an indication of the average density of the fluid along the path of the radiation beam, assuming the mass attenuation coefficient of the fluid is known. Several aspects of gamma densitometer theory have been discussed by T. C. Piper^[1].

At Aerojet Nuclear Company, the primary application of gamma densitometers is to pipes carrying an inhomogeneous mixture of liquid and gaseous water. Usually the distribution of the two phases in the pipe is not known beforehand. Without knowing this distribution, it is usually not possible to interpret unambiguously the reading from a single-beam gamma densitometer. Therefore, several gamma beams are used simultaneously to try to determine the liquid-gas distribution, or flow regime, and the density distribution inside the pipe. It is not very realistic to expect an unambiguous determination of an arbitrary density distribution with a finite number of gamma beams. However, a few beams can be expected to distinguish between the several flow regimes which actually occur in real two-phase systems and to give a reasonably accurate indication of the fluid density average across the pipe.

This analysis was done for a particular two-beam densitometer designed for use with the semiscale system, with the object of determining how well the densitometer performed with certain flow regimes. However, the mathematical treatment, the computer program, and the general techniques of the analysis are of general interest.

2. SYSTEM ANALYSIS

2.1 Densitometer Description

The system analyzed is sketched in Figure 1. The pipe is three-inch, Schedule 80, 316 stainless steel pipe. The radiation source contains 30 curies of cesium-137, which yield about 9.5×10^{11} gamma photons per second. The detectors are scintillation crystal-photomultiplier assemblies, which detect about 25% of the incident gamma photons. The collimators are made of tungsten, lead, and steel. The construction details of the real system are given elsewhere^[2], but they are not relevant here since in this analysis the collimator materials are assumed to be perfect absorbers.

Fig. 1 Geometry of the two-beam gamma densitometer which was analyzed.

2.2 Fluid Flow Model

For the first part of the study, two types of fluid flow were considered: simple stratified flow, with liquid in the bottom of the pipe and foam in the top, with a flat interface between the two fluids; and simple annular flow, with liquid covering the inside surface of the pipe and a circular cross section region of foam centered in the pipe. The foam in each case is assumed to have a void fraction variable from zero to one.

2.3 Calculated Densitometer Response

For these flow regimes, the response of each detector was calculated separately, for various stratified liquid depths; annulus inner radii, and foam void fractions. The sensor output was normalized so that the reading was unity for an empty pipe. The results of these calculations are indicated in Figures 2 through 5 for water at room temperature and atmospheric pressure, and in Figures 6 through 9 for water at 1,560 psi and 600°F (this represents the initial flow conditions in a blowdown from 600°F, which is actually higher than the usual 540°F in a semiscale blowdown).

2.31 Interpretation Chart. From the data presented in Figures 2 through 5, one can construct the diagram in Figure 10. This interpretation chart allows one to determine the flow regime, the void fraction of the foam, and the dimensions of the two fluid regions for a given pair of sensor output values. On this chart there will never be sensor output values that fall in the extreme upper left or lower right corners. Points along the diagonal dashed line represent the case where there is no liquid, but only foam in the pipe. Points above this dashed line indicate stratified flow, and points below the dashed line indicate annular flow. For example if the readings are 0.84 for beam 1 and 0.86 for beam 2, the flow is annular; the foam region has a radius of about 1.35 inches, and the foam void fraction is about 0.75. Readings of 0.7 for beam 1 and 0.985 for beam 2 indicate stratified flow with a liquid depth of about 1.60 inches and a foam void fraction of about 0.95.

Figure 11 is a similar interpretation chart for 1,560 psi and 600°F. Note that the interpretation charts in Figures 10 and 11 are valid and meaningful only if the fluid flow conforms to the simple annular or simple stratified liquid and foam model previously specified.

2.32 Average Density Chart. Knowing the flow geometry and foam void fraction values obtained from the interpretation chart, the average density of the fluid in the pipe can be calculated for a given pair of sensor readings. By this method, one can construct a chart which gives directly the average density of the fluid in the pipe for a given pair of sensor readings. Figure 12 shows such a chart for standard temperature and pressure. Of course, the use of this average density chart still depends on the previously mentioned assumptions about the flow regime.

Fig. 2 Beam 1 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at standard temperature and pressure.

Fig. 3 Beam 2 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at standard temperature and pressure.

Fig. 4 Beam 1 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at standard temperature and pressure.

Fig. 5 Beam 2 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at standard temperature and pressure.

Fig. 6 Beam 1 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at blowdown temperature and pressure.

Fig. 7 Beam 2 sensor reading versus liquid depth for simple stratified flow for several foam void fraction values at blowdown temperature and pressure.

Fig. 8 Beam 1 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at blowdown temperature and pressure.

Fig. 9 Beam 2 sensor reading versus annulus inner radius for simple annular flow for several foam void fraction values at blowdown temperature and pressure.

Fig. 10 Interpretation chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at standard temperature and pressure.

2.4 Ambiguity

There is some limited ambiguity in Figures 10, 11, and 12 involving flow regimes which are not expected to occur in real systems. The curves which intersect the bottom or left-hand edges of these charts do then bend and follow along the edge to the lower left corner. Therefore, points which fall on the bottom edge or on the left-hand edge of the charts can represent different combinations of fluid geometry and void fraction and different average densities. For example, if the readings were 0.7 for beam 1 and 0.745 (the minimum) for beam 2, a void fraction of 0.48 and an annulus inner radius of 1.15 are possible, or a void fraction of 1.00 and an annulus inner radius of 0.70, or an infinite number of combinations of values between these are possible. This is slightly clarified by Figure 13. For points in the right-hand portion of this supplemental chart, there is no ambiguity in the interpretation of the densitometer readings; these points all lie away from the edges of the charts in Figures 10 and 12. Points in the left-hand portion of Figure 13 represent complete ambiguity; all of these points lie on the single point at the lower left corner of Figures 10 and 12. These points correspond to an annulus core too small to interact with either radiation beam. The points in the middle section of Figure 13 represent the limited ambiguity caused by annulus cores large enough to interact with beam 1 but too small to interact with beam 2. These points lie on the bottom edge of Figures 10 and 12.

Fig. 11 Interpretation chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at blowdown temperature and pressure.

Figure 13 can be used to obtain some information about the average density in the cases of limited ambiguity. For readings of 0.7 for beam 1 and 0.745 for beam 2, as in the previous example, it is known that the values of the void fraction and annulus inner radius must be one of the pairs that lie on the solid line labeled 0.7 in Figure 13. The average density for points on this line varies between 0.70 and 0.77 g/cm³, as indicated by the dashed lines. Thus, while the average density is not known exactly, the ambiguity is greatly reduced by the use of Figure 13.

Figure 14 is a similar chart for the ambiguity that results in stratified flow when the liquid-foam interface is high enough so the foam cannot interact with beam 1. These points lie on the left-hand edge of Figures 10 and 12. There is no ambiguity for the points in the left portion of Figure 14, and there are no cases of complete ambiguity as there are for annular flow.

The cases of ambiguity in annular flow are not expected to occur in reality; only thin water annuluses are expected. Then the only ambiguity in the average density is for stratified flow, and this can be reduced to about 0.01 g/cm³ with the aid of Figure 14.

2.41 Other Fluid Flow Models. All of the foregoing has been for the limitation of simple annular or simple stratified flow with liquid and foam. Other realistic cases include

Fig. 12 Average density chart for two-beam densitometer, assuming simple annular or simple stratified flow with a variable foam void fraction, at standard temperature and pressure.

stratified liquid-gas flow with a curved interface, and annular liquid-gas flow with the gas core centered above the pipe center. Both of these geometries can be described by specifying the depth of the liquid above the center of the bottom of the pipe and the radius of curvature of the liquid-gas interface.

The response of the two-beam system was calculated for this type of flow, neglecting the unrealistic cases in which the gas core is centered below the pipe center in annular flow. The calculated sensor readings are indicated in Figures 15 and 16. As for the previous regime model, an interpretation chart (Figure 17) and an average density chart (Figure 18) can be constructed for this flow regime model.

There is ambiguity along the top edge of Figure 18 and also either ambiguity or very poor resolution at points near the left edge of Figure 18. However, the more important point to note is this: The average density curves in Figure 18 are quite different from those in Figure 12. That is, the interpretation of the two-beam densitometer output is strongly model-dependent. And of course there are reasonable flow models other than those considered here, which add more uncertainty to the interpretation. The possibilities are too numerous to consider in a formal analysis. Perhaps the most reasonable approach is to use other information, such as knowledge of common flow regimes at the temperature, pressure, and velocity being considered, to select a flow regime model for the interpretation of two-beam densitometer output.

Fig. 13 Supplemental chart for ambiguous region in which the beam 2 sensor reading is at its minimum. For an inner annulus radius greater than 1.15 inches, there is no ambiguity; for a radius less than 0.32 inch, there is no information at all.

2.5 Noise and Errors

Apart from the uncertainty in the interpretation of the two-beam densitometer output, some consideration of the expected errors is in order. The two major sources of error are expected to be shot noise, or statistical fluctuations in the radiation beam intensities, and radiation scattering.

2.51 Radiation Scattering. For this radiation (0.662 MeV gamma) in water, essentially all of the primary beam attenuation is due to scattering. A detailed and accurate analysis of the effects of the scattered radiation would be very difficult. However, some estimates can be made of the magnitudes of the effects.

The primary effect of the scattered radiation is to decrease the apparent linear attenuation coefficient of the water. Then all of the radiation sensor readings will be closer to 1 than is indicated by calculations that neglect scattered radiation. This effect is small, probably about 1% of the full scale reading or less. However, it will probably not be noticed in most cases, even if it is quite large, because it amounts to an approximately linear change of scale for the sensor output, and such changes of scale are eliminated by the initial calibration procedures for the densitometers.

Fig. 14 Supplemental chart for ambiguous region in which the beam 1 sensor reading is at its minimum. For liquid depths less than 2.66 inches, there is no ambiguity.

Another effect of scattered radiation is to introduce some crosstalk between the two beams. An order of magnitude calculation indicates that if one beam changes from minimum to maximum reading, the other should change by much less than 1%. This crosstalk could easily be as small as 0.01%.

In short, it appears that the effects of scattered radiation should be negligible. It should be noted that good beam collimation is generally very important in minimizing the effects.

2.52 Shot Noise. The calculations indicate that there are at least 1.7×10^6 counts per second in each detector from the primary beam, or 1,700 counts per millisecond (msec). This implies that statistical fluctuations should give a noise level of about 2.4% of the signal level on a 1-msec time scale; that is, one should be able to obtain roughly a 2.4% noise level with a 1-msec time constant or with 1-msec time resolution. The noise level should be about 0.77% for a 10-msec time constant or about 7.7% for a 1/10-msec time constant.

Fig. 15 Beam 1 sensor reading versus liquid depth for several values of curvature of the liquid-gas interface at standard temperature and pressure.

Fig. 16 Beam 2 sensor reading versus liquid depth for several values of curvature of the liquid-gas interface at standard temperature and pressure.

2.53 Error Propagation. The propagation of this error due to noise into the determination of flow geometry or average density is easy to estimate. For given readings, one locates the point on the interpretation chart or the average density chart. A small ellipse is then drawn around this point. The semiaxes of this ellipse are as large as the probable error in the readings. This is done on Figure 10 for the two example points previously mentioned, assuming an error of $\pm 0.3\%$ in each beam at the point (0.84, 0.86) and an error of $\pm 1\%$ in each beam at the point (0.70, 0.985). The range of probable values for the flow geometry, void fraction, or average density is just that region inside the ellipse.

Fig. 17 Interpretation chart for two-beam densitometer, assuming a variable radius of curvature of the liquid-gas interface, at standard temperature and pressure.

Fig. 18 Average density chart for a two-beam densitometer, assuming a variable radius of curvature of the liquid-gas interface, at standard temperature and pressure.

2.54 Mathematical Approximations. As is discussed in Appendix A, there are some mathematical approximation and numerical integration errors included in this analysis. The total effect of these errors is estimated to be not more than 1% of full scale in all the figures except perhaps Figures 13 and 14; these were drawn with rather limited data, and they may contain more errors than the others.

3. CONCLUSIONS

The major source of error in using gamma densitometers is due to lack of knowledge of the distribution of the fluid mass in the pipe. If the general nature of the fluid distribution (the flow regime) is known, then a simple graphical scheme can be used to determine the average fluid density and some details of the fluid distribution from the densitometer readings. If the flow regime is not known beforehand, then the interpretation of the densitometer readings is subject to considerable ambiguity, which can result in errors of about 0.15 g/cc in the determination of the average density of the fluid in the pipe for the particular system studied here.

4. FUTURE WORK

A prerequisite for profitable continuation of this work is a better understanding of what flow regimes are expected with various temperature, pressure, and fluid velocity combinations. Then the densitometer geometry can be optimized for those flow regimes which are likely to occur, and the response of the densitometer can be calculated for those several flow regimes. Interpretation charts and average density charts can then be constructed. The goal would be to have interpretation charts with no ambiguity and average density charts which are the same for all of the probable flow regimes. Failing this, it would be possible to estimate such things as the most probable value and the standard deviation of the average density in the pipe. Of course there could be other information, such as temperature and pressure, which would aid in the interpretation of the densitometer readings. However, such a project would be time consuming and expensive, and it might not give better results than we have now.

5. REFERENCES

1. T. C. Piper, *Dynamic Gamma Attenuation Density Measurements*, ANCR-1160 (May 1974).
2. The two-beam gamma densitometer is described in more detail in a drawing by H. W. Heiselmann, June 5, 1974, and in a notegram from H. W. Heiselmann to C. B. Parsons, February 19, 1974. Other relevant drawings are BSK-1810-1, DSK-1811-02, SK-TOMs-1143, SK-TOMs-1212, and SK-TOMs-191.

APPENDIX A
OUTLINE OF MATHEMATICAL TREATMENT
OF GAMMA DENSITOMETER

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APPENDIX A

OUTLINE OF MATHEMATICAL TREATMENT OF GAMMA DENSITOMETER

The system to be analyzed consists of a radiation source, a radiation detector, a collimator which limits the spatial extent of the radiation beam between the source and the detector, and a pipe with some fluid which attenuates the radiation beam. Note Figures A-1 and A-2. The radiation source and collimator tubes are cylindrical. The radiation beam axis and the pipe axis are perpendicular, but they do not necessarily intersect.

The source axis is similarly perpendicular to the pipe axis. It happens that in the cases of immediate interest, there is a line which perpendicularly intersects both the source axis and the pipe axis, as in Figure 11, but this condition is not required in the mathematics.

All scattered radiation is neglected, and the collimator materials are assumed to be perfect absorbers. Then the output of the radiation detector is:

$$E = K \iiint d^3 r_1 \iint d^2 r_2 \frac{1}{4\pi |\vec{r}_2 - \vec{r}_1|^2} \exp \left(- \int_{\vec{r}_1}^{\vec{r}_2} \alpha \, d\ell \right) .$$

Here K is the product of the radiation source intensity (photons per time per volume of source) and the detector efficiency [volts per (photons per time)]; α is the linear attenuation coefficient which appears in the classic equation $I(x) = I(0) e^{-\alpha x}$; the integral of α is to be taken along a straight line between $\vec{r}_1$ and $\vec{r}_2$; the integral over $\vec{r}_1$ is to be taken over that part of the source which would be included in the collimating tube if the collimating tube were extended; and the integral over $\vec{r}_2$ is to be taken over that part of the detector surface which is in front of the collimating tube. The source intensity and the detector efficiency are assumed to be independent of $\vec{r}_1$ and $\vec{r}_2$.

A reasonable evaluation of this expression requires some manipulation and approximation. The worst of these approximations has already been made in the range of the $\vec{r}_1$ integral. Some regions of the source outside of the region specified can serve as sources of rays which can travel within the collimator tube and reach the detector. The pertinent source volume that has been ignored is roughly 2.5% of the correct source volume, but the error in the total radiation intensity is only about 1%. This approximation has two effects: first, it tends to give less weight to the effects of the fluids near the axis of the radiation beam, relative to the effects of the fluids near the edge of the beam; second, it decreases all the calculated values of E by about 1%. This second error effect is generally inconsequential, since the detector output is usually normalized to remove the effects of variations in source intensity and detector sensitivity anyway.

Let the (u, v, w) coordinate system origin be at the center of the detector face, with the u -axis pointing along the beam axis toward the source, as indicated in Figure A-2. Then (u_1, v_1, w_1) are the rectangular coordinates of the point $\vec{r}_1$ in the source, and $(0, v_2, w_2)$ are the coordinates of the point $\vec{r}_2$ on the detector face.

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Fig. A-1

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Fig. A-2

The collimator tube diameter and the source diameter are small compared with r_o , the u-coordinate of the center of the source. Then,

$$|\vec{r}_2 - \vec{r}_1|^2 \approx r_o^2 \left(1 + 2 \frac{u_1 - r_o}{r_o} \right).$$

This approximation introduces an error of 0.3% in the worst case and much less on the average.

It will be shown later that $\int a \, d\ell$ does not depend significantly on u_1, w_1, w_2 ; then it can be written as $f(v_1, v_2)$, and

$$E = \frac{K}{4\pi} \int_{-R_c}^{R_c} dv_1 \int_{-R_c}^{R_c} dv_2 e^{-f(v_1, v_2)} \int_{-\sqrt{R_c^2 - v_2^2}}^{\sqrt{R_c^2 - v_2^2}} dw_2 \int_{r_o - \frac{\sqrt{R_s^2 - w_1^2}}{\cos \phi}}^{r_o + \frac{\sqrt{R_s^2 - w_1^2}}{\cos \phi}} dw_1 \frac{1}{r_o^2 \left(1 + 2 \frac{u_1 - r_o}{r_o} \right)} du_1.$$

Here R_c is the radius of the collimator tube, R_s is the radius of the source, and ϕ is the angle between the source axis and the perpendicular to the beam axis, as in Figure A-2. The limits on the w_1 integral are $\pm R_s$ if

$$|v_1| < \sqrt{R_c^2 - R_s^2}, \text{ or } \pm \sqrt{R_c^2 - v_1^2} \text{ if } \sqrt{R_c^2 - R_s^2} < |v_1| < R_c.$$

The u_1 integral is equal to

$$\frac{1}{2r_o} \left[\ln \left(1 + \frac{2 \sqrt{R_s^2 - w_1^2}}{r_o \cos \phi} \right) - \ln \left(1 - \frac{2 \sqrt{R_s^2 - w_1^2}}{r_o \cos \phi} \right) \right]$$

which can be approximated by

$$\frac{2 \sqrt{R_s^2 - w_1^2}}{r_o^2 \cos \phi}$$

using $\ln(1+x) \cong x$ for $x \ll 1$. For the worst conditions considered here, the error in this approximation is about 0.03%.

Now the integrals over w_1 and w_2 can be done to give

$$E = \frac{K}{\pi r_o^2 \cos \phi} \int_{-R_c}^{R_c} dv_1 \int_{-R_c}^{R_c} dv_2 \sqrt{R_c^2 - v_2^2} G(v_1^2) e^{-f(v_1, v_2)}$$

where

$$G(v_1^2) = \sqrt{R_c^2 - v_1^2} \sqrt{R_s^2 - R_c^2 + v_1^2} + R_s^2 \arcsin\left(\frac{\sqrt{R_c^2 - v_1^2}}{R_s}\right) \text{ if } R_c^2 - R_s^2 < v_1^2 < R_c^2,$$

$$\text{or } G(v_1^2) = \pi R_s^2/2 \text{ if } v_1^2 < R_c^2 - R_s^2.$$

The function $f(v_1, v_2)$ can be considered as the sum of three terms of the form $a_i \ell_i$ where a_i is the linear attenuation coefficient of the i^{th} medium (metal pipe, liquid in the pipe, or gas in the pipe) and ℓ_i is the path length through that medium. Of course the a_i are constants. To find ℓ_i , it is convenient to first define the parameter S to be a perpendicular distance between the pipe axis and the ray from $\vec{r}_1$ to $\vec{r}_2$. For $w_1 = w_2$, an exercise in geometry gives

$$S = (v_2 - v_c) \cos \psi + u_c \sin \psi.$$

Here, (u_c, v_c) is the position of the pipe axis (a constant) and ψ is the angle between the beam axis and the ray in question, as indicated in Figure A-2. This can be approximated as

$$S = -v_c + \frac{u_c}{r_o} v_1 + \left(1 - \frac{u_c}{r_o}\right) v_2.$$

In the worst cases, the error in this approximation is $\pm 1.3\%$; however, this error tends to average to zero for the different values of u_1 (u_1 is the variable that was eliminated in this approximation), and the total error is estimated to be less than 0.1% .

For $w_1 \neq w_2$, this expression for S should be multiplied by

$$\left[\frac{(w_2 - w_1)^2 + (v_2 - v_1)^2 + (u_2 - u_1)^2}{(v_2 - v_1)^2 + (u_2 - u_1)^2} \right]^{1/2}.$$

Neglecting this factor contributes an error of only 0.05% in the worst case.

The path length of the ray through the metal pipe wall is

$$\ell_m = 2 \left(\sqrt{R_1^2 - S^2} - \sqrt{R_2^2 - S^2} \right)$$

with the understanding that if any of the quantities in the square root signs are negative, the square root is to be interpreted as zero. R_1 and R_2 are the outside and inside radii of the pipe.

For the liquid and gas path lengths, we need to know the angle θ between the ray and the vertical. This is

$$\theta = \theta_0 - \psi \approx \theta_0 - \frac{v_1 - v_2}{r_0}$$

where θ_0 is the angle between the beam axis and the vertical. The approximation for θ introduces an error within $\pm 0.16\%$, which averages to a much smaller total error.

For simple stratified flow, the path length in the liquid is

$$\ell_L = \sqrt{R_2^2 - S^2} - S \tan \theta + (d - R_2)/\cos \theta$$

and the path length in the gas is

$$\ell_G = 2\sqrt{R_2^2 - S^2} - \ell_L$$

Here, d is the depth of the liquid at the vertical centerline of the pipe (see Figure 2).

If the interface between the liquid and the gas is curved with radius of curvature R_3 , as in Figure A-3, the calculation of path lengths is a little more complicated. First, the parameter S_1 is defined to be

$$S_1 = S - (R_3 + d - R_2) \sin \theta$$

which is the distance from the center of curvature of the gas-liquid interface to the ray. Then the path length of the ray through the liquid is $\ell_L = Q_4 + Q_5$, where

$$Q_4 = \sqrt{R_2^2 - S^2} - \sqrt{R_3^2 - S_1^2} - (R_3 + d - R_2) \cos \theta$$

and

$$Q_5 = \sqrt{R_2^2 - S^2} - \sqrt{R_3^2 - S_1^2} + (R_3 + d - R_2) \cos \theta$$

with the understanding that Q_4 and Q_5 are to be called zero if the values calculated from these formulas turn out to be negative. The path length through the gas is then

$$\ell_G = 2\sqrt{R_2^2 - S^2} - \ell_L$$

The knowledge of these various path length equations and of the values of the various α_i allow the calculation of $f(v_1, v_2)$ for given (v_1, v_2) ; this allows the evaluation of E in principle, although in practice it is more reasonable to evaluate the integral numerically.

ANC-A-4485

Fig. A-3

It is estimated that the total error in the normalized E due to the several approximations made in this analysis is about 0.3% or less. The total error in the absolute value of E (not normalized) is probably about 1.5%. As mentioned before, this last error is usually removed in choosing a scale factor to make the reading equal to full scale on the meter for an empty pipe.

In addition to the radiation detector output, it is desirable to know the average density of the fluid in the pipe; it is

$$\rho_{av} = F\rho_G + (1 - F)\rho_L$$

where F is the fraction of the pipe area that is occupied by gas (or foam), ρ_G is the density of the gas or foam, and ρ_L is the density of the liquid. F must be calculated for three distinct cases:

Case 1: Annular Flow; there is a complete, but not necessarily uniformly thick annulus of liquid, as in (a) and (b) of Figure A-3. In this case,

$$F = R_3^2/R_2^2$$

Case 2: Stratified Flow; there is a flat, level interface between the liquid and the gas. In this case,

$$F = \frac{1}{\pi} \left[\arccos\left(\frac{y}{R_2}\right) - \frac{y}{R_2} \sqrt{1 - \left(\frac{y}{R_2}\right)^2} \right]$$

where $y = d - R_2$.

Case 3: Partial Annulus, or Curved Interface; the gas-water interface is curved, as in (c), (d), and (e) in Figure A-3. In this case,

$$F = \frac{1}{\pi} \left[\arccos\left(\frac{y_2}{R_2}\right) - \frac{y_2}{R_2} \sqrt{1 - \left(\frac{y_2}{R_2}\right)^2} \right] + \frac{R_3^2}{\pi R_2^2} \left[\arccos\left(\frac{y_3}{R_3}\right) - \frac{y_3}{R_3} \sqrt{1 - \left(\frac{y_3}{R_3}\right)^2} \right]$$

where

$$y_2 = \frac{R_2^2 - R_3^2 - d_a}{2 d_a}$$

$$y_3 = d_a - y_2$$

$$d_a = d + R_3 - R_2$$

With these equations, the calculation of the average fluid density in the pipe is easily incorporated into the computer program that does the integration over v_1 and v_2 .

A program suitable for these calculations is listed at the end of this appendix. It is written in GE Timesharing Fortran IV. Table A-I lists the input parameters for this program. The meanings of most of these are evident from the table and the figures, but some need explanation.

It is more efficient to evaluate the integral simultaneously for several combinations of a_L and a_G . Therefore, the program requests the number (NV) of pairs of values of a_L and a_G (AL and AG) and the values for each pair, for which the integral will be simultaneously evaluated.

The program is arranged to evaluate the integral for a rectangular array, ND x NR in size, of pairs of values of the liquid depth and the radius of curvature. D30 and R31 are the values of the depth and the radius for the first element in this array. The values of d and R_3 in successive elements of this array are increased by increments of DELD and DELR, respectively. The program automatically discards any combination of values which correspond to annular flow with the gas core centered below the pipe axis. R_3 must always be greater than zero. If the value of R_3 is greater than 10^4 times the value of R_2 , then R_3 is interpreted as being infinite, corresponding to simple stratified flow.

All of the variables should be expressed in inches, degrees, or inverse inches. If a different unit of length is used consistently, the calculated response of the radiation detector will be correct, but the average density will not.

For each pair of d, R_3 values, the computer will list on one line: d, R_3 , and E for the empty pipe; on each successive line: a_L , a_G , E, normalized E, and the average fluid density.

After the integration is done for one set of d, R_3 values, the program will request new data specifying a new set of d, R_3 values. If a negative value is put in for the first variable in any input list, the program branches back to the previous input statement.

TABLE A-I

COMPUTER INPUT VARIABLES

Quantity	Variable Name	Computer Variable Name	Value		
			Beam 1	Common	Beam 2
Pipe Outer Radius	R_1	R1		1.75	
Pipe Inner Radius	R_2	R2		1.45	
Collimator Tube Radius		01, 02		0.1875	
Source Radius		0S		0.125	
Source Intensity x Detector Efficiency	K	AI		0.64×10^{13}	
Linear Attenuation Coefficient for Gamma Rays in Pipe Metal	α_m	AM		1.55	
Source to Detector Distance	r_o	UO		9.9	
Coordinates of Pipe Axis	u_c	UC	7.2		7.46
	v_c	VC	0.49		-1.26
Angle Between Beam Axis and Vertical	θ_o	TO	19.74		57.33
Angle Between Source Axis and Normal to Beam Axis	ϕ	FEE	10.26		27.33
Number of Subintervals in v_1, v_2 Integral Range		N1, N2		30,30	
Number of Pairs of Values of Liquid and Gas Attenuation Coefficients		NV			
K^{th} Liquid and Gas Attenuation Coefficients	α_L, α_G	AL(K), AG(K)		0.204 ^[a] , 0.0 for STP 0.148, 0.01335 for blowdown	
Initial Liquid Depth	d	D30			
Liquid Depth Increment		DELD			

TABLE A-I (contd.)

Quantity	Variable Name	Computer Variable Name	Value		
			Beam 1	Common	Beam 2
Number of Liquid Depth Values		ND			
Initial Radius of Curvature	R_3	R31			
Radius of Curvature Increment		DELR			
Number of Radius Values		NR			

[a] This value of $\alpha_L = 0.204/\text{inch}$ is an average of three values obtained from the literature. It has since been learned that the value of $0.218/\text{inch}$ is in better agreement with experiment.

```

1000 IMPLICIT REAL (L)
1010 REAL AL(12),AG(12),SA(12)
1020 149 READ, R1,R2,01,02,0S,A1,AM
1030 IF (R1.LE.0.) STOP
1040 PRINT 131, R1,R2,01,02,0S,A1,AM
1050 PRINT 131
1060 141 READ, U0,UC,VC,T0,FEE
1070 IF (U0.LE.0.) GO TO 149
1080 PRINT 131, U0,UC,VC,T0,FEE
1090 PRINT 131
1100 A3=A1/(COS(FEE/57.29578)*U0*U0)/3.14159
1110 RR1=R1*R1
1120 RR2=R2*R2
1130 001=01*01
1140 002=02*02
1150 00S=0S*0S
1160 C1=UC/U0
1170 C2=1.-C1
1180 131 FORMAT (/1X,E13.6)
1190 T0=T0/57.29578
1200 142 READ, N1,N2,NV,(AL(K),AG(K),K=1,NV)
1210 IF (N1.LE.0) GO TO 141
1220 D1=2.*01/FLOAT(N1)
1230 D2=2.*02/FLOAT(N2)
1240 A2=A3*D1*D2
1250 143 READ, D30,DELD,ND,R31,DELR,NR
1260 IF (D30.LT.0.) GO TO 142
1270 DO 197 ID=1,ND
1280 DDS=D30+DELD*FLOAT(ID-1)
1290 R30=R31
1300 DO 196 IR=1,NR
1310 R3=R30+DELR*FLOAT(IR-1)
1320 IF (R3.LE.0.) GO TO 196
1330 IF (R3.LT.(R2-DDS)) GO TO 196
1340 IF (R3.LE.0.) GO TO 196
1350 RR3=R3*R3
1360 SM=0.
1370 DO 190 K=1,NV
1380 190 SA(K)=0.
1390 DO 199 I=1,N1
1400 V1=D1*(FLOAT(I)-.5)-01
1410 VV1=V1*V1
1420 IF (VV1.GT.(001-00S)) GO TO 101
1430 F1=1.570796327*00S
1440 GO TO 110
1450 101 IF (V1.GT.01) GO TO 199
1460 Q1=SQRT(001-VV1)
1470 QS=SQRT(00S-001+VV1)
1480 F1=Q1*QS+00S*ATAN2(Q1,QS)
1490 110 CONTINUE

```

```

1500 DO 198 J=1,N2
1510 V2=D2*(FLOAT(J)-.5)-02
1520 F=F1*SQRT(002-V2*V2)
1530 S=C1*V1+C2*V2-VC
1540 T=T0+(V2-V1)/U0
1550 Q1=RR1-S*S
1560 111 Q1=SQRT(Q1)
1570 Q2=RR2-S*S
1580 IF (Q2.GT.0.) GO TO 113.
1590 LM=2.*Q1
1600 GO TO 120
1610 113 Q2=SQRT(Q2)
1620 LM=2.*(Q1-Q2)
1630 IF (R3.LT.(R2*10000.)) GO TO 114
1640 LL=Q2+(DDS-R2)/COS(T)-S*TAN(T)
1650 IF (LL.LT.0.) LL=0.
1660 IF (LL.GT.(2.*Q2)) LL=2.*Q2
1670 GO TO 115
1680 114 S1=S-(R3+DDS-R2)*SIN(T)
1690 Q3=RR3-S1*S1
1700 IF (Q3.GT.0.) GO TO 301
1710 LL=2.*Q2
1720 GO TO 115
1730 301 Q3=SQRT(Q3)
1740 Q4=Q2-Q3-(R3+DDS-R2)*COS(T)
1750 IF (Q4.LT.0.) Q4=0.
1760 Q5=Q2-Q3+(R3+DDS-R2)*COS(T)
1770 IF (Q5.LT.0.) Q5=0.
1780 IF (Q5.GT.(2.*Q2)) Q5=2.*Q2
1790 LL=Q4+Q5
1800 115 LG=2.*Q2-LL
1810 DO 116 K=1,NV
1820 116 SA(K)=SA(K)+F*EXP(-AL(K)*LL-AG(K)*LG-AM*LM)
1830 120 SM=SM+F*EXP(-AM*LM)
1840 198 CONTINUE
1850 199 CONTINUE
1860 SM=SM*A2
1870 IF (R3.LT.10000.*R2) GO TO 182
1880 Y1=DDS-R2
1890 IF (Y1.GT.R2) Y1=R2
1900 ARG=(ATAN2(SQRT(RR2-Y1*Y1),Y1)-Y1*SQRT(RR2-Y1*Y1)/RR2)/3.14159
1910 GO TO 152
1920 182 IF ((2.*R3+DDS).GT.(2.*R2)) GO TO 151
1930 ARG=RR3/RR2
1940 GO TO 152
1950 151 IF (DDS.LT.(2.*R2)) GO TO 153
1960 ARG=0.
1970 GO TO 152
1980 153 DA=R3+DDS-R2
1990 Y1=(DA+(RR2-RR3)/DA)/2.

```

```

2000 Y2=DA-Y1
2010 ARG=(RR3*ATAN2(SQRT(RR3-Y2*Y2),Y2)-Y2*SQRT(RR3-Y2*Y2) +
2020& RR2*ATAN2(SQRT(RR2-Y1*Y1),Y1)-Y1*SQRT(RR2-Y1*Y1))/RR2/3.14159
2030 152 ARL=1.-ARG
2040 PRINT 118
2050 PRINT 118, DDS,R3,SM
2060 DO 117 K=1,NV
2070 RHO=4.9*(ARG*AG(K)+ARL*AL(K))
2080 SAK=SA(K)*A2
2090 UAK=SAK/SM
2100 117 PRINT 118, AL(K),AG(K),SAK,UAK,RHO
2110 118 FORMAT (/5(1X,E13.6))
2120 PRINT 118
2130 PRINT 118
2140 196 CONTINUE
2150 197 CONTINUE
2160 GO TO 143
2170 END

```

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