BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

CRISP 73-23

ACCELERATOR DEPARTMENT
Informal Report

BUNCHED BEAM STABILITY IN PROTON-PROTON STORAGE RINGS

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November 8, 1973

ABSTRACT

Bunched beams can be unstable against longitudinal collective oscillations. In particular, the interaction of a beam with perfectly conducting chamber walls can force the beam into a condition of latent instability. In such a state, growth of oscillation modes can ensue as a result of wall resistivity or as a consequence of the interaction of the beam with resonant structures in the ring. The dependence of the stability criteria on energy, rf voltage, and rf frequency is given. It is shown how a high frequency rf system tends to stabilize the bunches. A specific set of parameters for an essentially nonstacked bunched beam p-p storage ring, parameters that are consistent with using the AGS as an injector, is considered.

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1. Introduction

Recently, a set of criteria for the stability of bunched beams against longitudinal collective oscillations was developed.¹ It was shown that the interaction of a beam with perfectly conducting chamber walls could force the beam into a condition of latent instability. What this means is that such a force will produce a collective beam oscillation, and if the force is sufficiently strong, will leave the beam at the edge of stability. Any force with a resistive component will then cause exponential growth of the oscillations. Vacuum chamber resistivity and resonant structures situated in the rings are sources for such a resistive component.

It has been suggested that the AGS be used as an injector into a pair of intersecting storage rings for p-p collisions.² It is a simple matter to show that the AGS bunches in stationary buckets at 28 GeV are unstable (for reasonable bunch areas). Now this is not of great concern in the AGS because, first, at 28 GeV, the AGS bunches are in moving buckets, which has a stabilizing influence; and, second, the unstable state exists for only a short time, on the order of 100 msec, and growth times are generally considerably longer than this. However, for a storage ring, transfer is made into stationary buckets and, of course, the beam is stored in stationary buckets. This means stabilization with moving buckets³ is not possible. Furthermore, beams are to be stored for, perhaps, hours. Thus, although for accelerators, one might conceivably allow the beams to be unstable as long as growth times were long enough (this is the case for the ISR bunches being accelerated into the stack¹); for stored bunches, a more stringent condition is required. In this case, the beams must be stable against small resistive forces.

The AGS beam can be stabilized by using a larger stable phase angle (moving bucket stabilization). We will show that by choosing a sufficiently high frequency in the storage ring, transfer into stationary buckets can result in a stable beam against small resistive forces. We will also discuss the constraint on shunt impedances arising from resonant devices in the rings.

In section 2, we modify the stability criteria in Ref. (1) so that they are simplified and suited to our purposes; and in section 3, we develop a set of storage ring parameters which provide for a stable bunched beam.

2. Stability Criteria for Bunched Beams

Perfectly Conducting Walls

If the walls are perfectly conducting, then the image force induced on the beam is nonresistive and no growth results. However, as we have previously indicated, nonresistive forces can bring the beam into a condition of latent instability. The beam would then be unstable against small resistive force components such as arise from the chamber wall resistivity. To insure against such a state prevailing, we impose a limit on the average current in the beam. The condition can be derived from the Landau damping criterion which states essentially that the frequency shift resulting from the force must be less than the spread in synchrotron frequency in the bunches, or¹

$$\frac{\Delta\omega_{sc}}{\Omega_s} < \frac{S}{4\Omega_s} \quad , \quad (2.1)$$

where $\Delta\omega_{sc}$ is the space charge frequency shift (i.e. arising from perfectly conducting walls and real),

Ω_s is the central synchrotron frequency (in units of radians/sec),
and

S is the spread in synchrotron frequency in the bunch.

If we assume that the bunches are well within the bucket, then the spread in synchrotron frequency comes primarily from the octupole component of the synchrotron force. In these circumstances we can write for the spread,

$$\frac{S}{\Omega_s} = \frac{\pi^2 h^2 B^2}{16 M^2} \quad , \quad (2.2)$$

where h is the harmonic number of the rf system,

M is the number of equally spaced bunches, and

B is the bunching factor [bunch length/bunch separation].

The real space charge frequency shift can be written,¹

$$\frac{\Delta\omega_{sc}}{\Omega_s} = (0.152) \left[\frac{\pi^3 E_0 Z_0}{16 f_0^2} \right] \frac{g_c IB}{\beta \gamma |\eta| A_M^2} , \quad (2.3)$$

where I is the average beam current,

A is the invariant bunch area in eV-sec,

$$\eta = 1/\gamma_{tr}^2 - 1/\gamma^2,$$

γ is the energy in proton mass units,

β is the velocity in units of the speed of light,

f_0 is the proton frequency, in revolutions per sec,

E_0 is the proton mass (938 MeV),

Z_0 is impedance of free space (377 ohms), and

g_c is a geometrical factor, given by

$$g_c = 1 + 2\ln(r/b) , \quad (2.4)$$

with r and b the vacuum chamber radius and beam radius respectively.

Taking $r/b = 2.5$, $\beta = 1$, and $f_0 = 123.9$ kHz (corresponding to $\beta = 1$ and a circumference of $3 \times C_{AGS} = 2422$ m), we obtain from (2.1),

$$I < 8.0 \times 10^{-3} \gamma |\eta| A^2 h^2 B . \quad (2.5)$$

In deriving this expression, we have assumed elliptical bunches; therefore, the peak voltage, V , and bunching factor are related by

$$Vh \cos \varphi_s = \frac{8f_0^2}{\pi^3 E_0} \frac{|\eta| A^2 h^4}{\gamma B^4} , \quad (2.6)$$

with φ_s the stable rf phase angle.

Resistive Walls

We restrict ourselves to dipole oscillation modes; this is a reasonable assumption for small bunching factors.¹ For a stainless steel chamber, a frequency $f_0 = 123.9$ kHz, and a chamber radius of 2.5 cm, the frequency shift due to resistive walls is given by

$$\frac{\Delta\omega_{RW}}{\Omega_s} = 0.04 \frac{\gamma IB^4}{|\eta| A_M^{5/2}} G \left(\frac{\nu_s + n}{M} \right) , \quad (2.7)$$

where ν_s is the synchrotron number ($\Omega_s = 2\pi f_0 \nu_s$),
 n is the particular bunch to bunch coupling mode ($1 \leq n < M$),
 G is a bunch function, which is roughly

$$G(q) \approx e^{i2\pi q}, \quad (2.8)$$

and the dependence on f_0 is $f_0^{-5/2}$. For small B , the magnitude of the frequency shift due to the wall resistivity is generally much smaller than the basic space charge shift, Eq. (2.3). Thus, in determining the stability criterion, i.e. current limit for stability, we can neglect this term. It would be a more significant effect if the beam were in a state of latent instability, as we have previously discussed. Then the resistivity of the walls introduces a resistive component, i.e. $\text{Im}(\Delta\omega_{RW}) \neq 0$. Thus, the beam would be unstable against dipole oscillations, with a growth rate, for a beam which is far from stability, given by

$$\frac{1}{\tau_{RW}} = \text{Im}(\Delta\omega_{RW}) = 0.84 \frac{IB^2}{AM^{\frac{1}{2}}}, \quad (2.9)$$

where we have taken the worst mode, corresponding to $\text{Im}(G) \approx 1$; and the synchrotron number is

$$\nu_s = \frac{\Omega_s}{2\pi f_0} = 2.7 \times 10^{-5} \frac{A|\eta|M^2}{\gamma B^2}, \quad (2.10)$$

(ν_s has a linear dependence on f_0 and so τ_{RW} goes like $f_0^{3/2}$).

Resonant Interactions

If we assume a small bunching factor, B , and a low enough resonance frequency, then only the dipole mode is significant.¹ A simple quantitative criterion for this is

$$f_{RES} < \frac{M}{2B} f_0, \quad (2.11)$$

or if r is the resonant harmonic,

$$f_{RES} = r f_0, \quad (2.12)$$

and (2.11) is

$$r < \frac{M}{2B}. \quad (2.13)$$

The frequency shift (which is a measure of the image force induced by the resonant structure) can be written¹

$$\frac{\Delta\omega_{RES}}{\Omega_s} = 0.095 \frac{\gamma_B^4 r_R I}{|\eta| A_M^2} D, \quad (2.14)$$

where R_s is the shunt impedance for the resonant structure; D is a complex quantity which is characteristic of the type of resonant device; and we have assumed $\beta = 1$, and a frequency, $f_o = 123.9$ kHz. (Note that $\Delta\omega_{RES}/\Omega_s$ has a f_o^{-2} dependence.)

From (2.1) (interpreting $\Delta\omega_{SC}$ as $\Delta\omega_{RES}$) and (2.2) we obtain the stability criterion,

$$rR_s < \frac{1.6|\eta|A_M^2h^2}{I|D|\gamma_B^2}. \quad (2.15)$$

In reality, of course, we must add the frequency shifts induced by all image forces before applying (2.1) (i.e. with $\Delta\omega_{TOTAL}$). However, it is consistent with the spirit of our approximations to treat each force separately.

Without information about the particular resonant structure, we must be vague about our choice for D and r . But we do know that¹

$$|D| < 1. \quad (2.16)$$

Thus, it is most difficult to satisfy the stability criterion for narrow band resonators, i.e. $|D| \sim 1$. In this case, we have

$$rR_s < \frac{1.6|\eta|A_M^2h^2}{I\gamma_B^2}. \quad (2.17)$$

As a means of evaluating the need to design for stability, we provide an estimate of the growth rate for the case of no Landau damping:

$$\frac{1}{\tau_{RES}} = \text{Im}(\Delta\omega_{RES}) = \frac{2B^2 r_R I}{A_M^2}, \quad (2.18)$$

where we have taken $\text{Im}(D)$ to be its maximum value of unity; and the dependence on f_o of τ_{RES} is linear.

3. Bunched Beam p-p Storage Rings

We consider the basic storage ring design suggested in Ref. (2). Thus, 3 AGS pulses are transferred in sequence into a ring which has a

circumference 3 times the AGS circumference. So, $f_0 = 123.9$ kHz at high energy. With matched transfer, γ , A and B are fixed. With $\gamma = 30$, $\eta = 0.002$, $A = 1.0$ eV-sec/bunch, $\varphi_s = 0$, and $M = 36$, the AGS beam is matched with

$$V_h = 5238 \text{ kV} \quad . \quad (3.1)$$

This means a bunching factor, $B^{-1} = 10.26$. The stability criterion for the basic space charge force, Eq. (2.1), thus becomes

$$I < 4.68 \times 10^{-5} h^2 \text{ (Amperes)} \quad . \quad (3.2)$$

The minimum value of h is 36 since 36 AGS bunches are to be transferred to each storage ring. However, with $h = 36$, the current limit is $I < 0.06$ A, which is more than an order of magnitude less than the design current of 0.72 A.

By writing the stability criterion in the form (3.2), we can see how to stabilize the transferred beam. If we take

$$h = 6M = 216, \quad (3.3)$$

then the current limit becomes

$$I < 2.2 \text{ A} \quad . \quad (3.4)$$

The matching voltage, from (3.1) is, $V = 24.25$ kV. Now, this process of increasing h and decreasing V is clearly an effective stabilizing technique. The limit is, of course, the fact that the buckets enclosing the bunches are shrinking. The bucket area is given by

$$A_b = 0.25 \sqrt{\frac{V\gamma}{|\eta|h^3}} \quad . \quad (3.5)$$

For the above choice of h and V , $A_b = 1.5$ eV-sec, and results in a bucket filling of

$$\frac{A}{A_b} = 66\% \quad .$$

The filling factor can be reduced by increasing the voltage (adiabatically, after beam transfer). With $V = 100$ kV per turn, $A_b = 3.1$ eV-sec and the filling factor is 32.7%. However, this is done at the expense of limiting current. Since the bunching factor becomes $B^{-1} = 14.62$, the space charge stability limit then becomes $I < 1.5$ A, a factor of 2 higher than the design current.

During acceleration and for storage at high energy, the current limit increases, the former through moving bucket stabilization, the latter due to the higher energy. Thus, at 200 GeV, the bunching factor becomes, $B^{-1} = 21.2$, the bucket filling factor becomes 15.5%, and the space charge current limit becomes $I < 10.6$ A.

The resistivity of the chamber walls introduces no significant current limitation, as can be seen by comparing the magnitude of the frequency shift, Eq. (2.7), with that arising from the basic space charge, Eq. (2.3).

Given the design current, $I = 0.72$ A, (2.17) gives an upper limit to the shunt impedance resulting from a resonant device interacting with the beam. Since the maximum allowable shunt impedance decreases with energy (like $\gamma^{-\frac{1}{2}}$), we give only the requirement at high energy, for the stored beam. Thus, for $\gamma = 200$, we obtain

$$rR_s < 0.9 \text{ M}\Omega .$$

For $r = 350$, i.e. a resonant frequency, $f_{\text{RES}} = 43.4$ MHz, we have for the shunt impedance stability limit,

$$R_s < 2.6 \text{ k}\Omega .$$

Since the critical frequency¹ for the dipole mode is 47 MHz, that is, for higher frequency the shunt impedance stability limit decreases, the most dangerous interacting structure would be one resonating at 47 MHz, and the limiting shunt impedance is $R_s < 2.4 \text{ k}\Omega$.

In the event that the current is above the stability limit, or if the shunt impedance for some resonant structure is above the stability limit, then growth of the unstable oscillation modes will follow. Considering first the resistive wall effect, (2.9) gives for the growth time,

$$\tau_{\text{RW}} = 107.3 \text{ minutes},$$

where we have interpreted I in (2.9) as the quantity of current above the stability limit and taken $I = 0.5$ A. For resonant structures, we can find growth time estimates from (2.18). If we interpret R_s as the quantity of shunt impedance above the stability limit, we find for a $100 \text{ }\Omega$ impedance, resonating at 47 MHz, a growth time,

$$\tau_{\text{RES}} = 10.6 \text{ seconds},$$

for $\gamma = 200$. Such a short growth time reflects the importance of achieving the required stability limit for each resonant structure.

4. Conclusions

RF design parameters for a pair of bunched beam p-p storage rings have been suggested. Stability from the basic space charge interaction can be achieved by using a high rf frequency, i.e. a harmonic number 6 times the number of bunches. The effect of resistivity in the chamber walls provides no difficulties. On the other hand, resonant structures in the ring must be carefully assessed. The critical resonant frequency is 47 MHz and the shunt impedance limit at this frequency is 2.4 k Ω .

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