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MASS TRANSFER IN LOW VELOCITY GAS
STREAMS

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NOMENCLATURE

A	Constant in series solution
C	Concentration, volume per cent carbon dioxide in air
C_A	Measured average effluent concentration from test section
d	Differential operator
D	Molecular diffusion coefficient, sq ft/sec
D_p	Particle diameter, ft
E	Eddy diffusivity, sq ft/sec; thermal conductivity cell impulse
$\bar{E}$	Total diffusivity = $D + E$, sq ft/sec
G	Mass velocity, lbs/hr/sq ft
h	Interval size in semi-numerical solution
J_0	Zero order Bessel function of first kind
J_1	First order Bessel function of first kind
k	Number of position of radial point
n	Index of summation; slope of the log C versus log E thermal conductivity cell calibration curve
N	Total number of intervals in semi-numerical solution, reciprocal of h
N_A	Rate of diffusion, lbs/sec/sq ft
P	Weight function = $\frac{u}{V} \theta$
Pe	Peclet number = $\frac{D_p V}{E}$
r	Radial distance from axis of pipe, with r_0 the pipe radius, ft

- v
- R Function representing the radial variation of concentration
- Re Reynolds number = $\frac{2r_o G}{\mu}$
- t Radius of injection tube, ft; time of diffusion, sec
- u Point velocity, ft/sec; point velocity in axial direction, ft/sec
- V Average velocity in axial direction, ft/sec
- x,y,z Directional coordinates in rectangular coordinate system
- z Axial distance from injection tube, ft
- Z Function representing the axial variation of concentration
- ∂ Partial differential operator
- ∇ Gradient operator
- λ_n Constant in analytical solution; function of axial variable in semi-numerical solution
- ρ Density, lbs/cu ft
- θ Angular coordinate in cylindrical coordinate system
- θ Dimensionless radial position, r/r_o , with θ_t the injection tube inside wall radial position
- μ Absolute viscosity, lbs/ft sec

MASS TRANSFER IN LOW VELOCITY GAS STREAMS*

John E. Frandolig and R. W. Fahien

ABSTRACT

Experimental studies of the mixing of coaxial streams of carbon dioxide and air at ambient temperature and atmospheric pressure were made at Reynold numbers of 1695, 4510, and 7605.

The test section consisted of a vertical, 4-inch galvanized iron pipe. Carbon dioxide was introduced into the center of an air stream free of moisture and carbon dioxide by means of an injection tube which extended the full length of a calming section 14 feet 4 inches long. Radial concentration profiles of carbon dioxide were measured at three positions downstream. Gas samples were withdrawn by a pitot type probe and analyzed by a thermal conductivity cell. Velocity distributions were determined by means of a five-loop, circular hot-wire anemometer.

Total diffusivities of carbon dioxide in air for the entire radius of the pipe were computed by a semi-numerical method. The total diffusivities were found to vary significantly with position in the pipe and to be rather complicated functions of local flow conditions.

An approximate analytical solution of the general mass transfer equations describing the system was employed to compute total diffusivity values representative of the overall mass transfer process. This solution was derived assuming the total diffusivity and velocity to be independent of radial position.

*This report is based on an M. S. thesis by John E. Frandolig submitted June, 1957, to Iowa State College, Ames, Iowa. This work was done under contract with the Atomic Energy Commission.

INTRODUCTION

With the advent of highly complex industrial processing schemes, the need for more accurate design procedures has become evident. In the past design engineers have depended heavily on past experience and pilot plant experiments, looking upon theory as primarily of academic importance. As a result almost all theoretical advances have been made by academic institutions. However, in recent years industrial interest in theoretical studies has boomed, with the idea that better design procedures can be worked out by employing highly theoretical concepts. As a result almost all of the basic chemical and physical mechanisms have recently come under varying degrees of scrutiny by industrial and research organizations. In the field of chemical engineering a great need for studies of the mechanisms of heat, mass, and momentum transfer has been realized. The presence of one or a combination of these transfer mechanisms is found in all processing operations.

Present mass transfer correlations are based on overall effects in various physical systems. This method has been employed primarily because of a lack of the knowledge of the exact mechanisms. Such correlations have been found both inaccurate and inadequate for the design of modern processing units. Thus, the need for data on internal effects in various systems is evident. It is for this reason that the present study was undertaken.

This investigation is limited to the study of the transfer of mass in low velocity coaxial gas streams. Its purpose is twofold: first, to determine the relative magnitude of the mass transfer due to turbulence in low velocity gas streams as compared with that due to the molecular process; and second, to determine the variation of the total mass transfer diffusivity with position in the containing duct.

Carbon dioxide was introduced at the axis of a stream of air flowing in a cylindrical duct. Radial concentration profiles were established at various distances downstream from the point of injection of the carbon dioxide. Gas samples were removed by means of a pitot type probe and analyzed by passing them through a thermal conductivity cell. A hot-wire anemometer was used to establish the radial velocity distribution for each flow rate. These data were then treated in accordance with two unique solutions of the general mass transfer differential equations described herein, yielding values of the total mass transfer diffusivity.

LITERATURE REVIEW

The physics of the exchange of energy, mass, and momentum between fluids in motion has recently come under varying degrees of investigation. The most significant aspect of a study of the transfer of any one of these quantities lies in the fact that certain analogous relationships, even though not complete, have been developed for these mechanisms. Thus, a study of one of these transfer processes is not only important in itself, but also because of its relation to the other two. The information gained from studies in this field is fundamental in many applications to reactors, absorption towers, condensers, and numerous other unit operations.

The process of mass transfer, or diffusion, although itself quite involved, has become the primary subject of investigation because it offers such a clear description of turbulence. However, virtually all previous studies of the diffusion mechanism have been concerned with the consideration of fully developed turbulent flow.

The theories of both molecular and turbulent diffusion have become applicable in chemical engineering problems to the extent that general introductory treatments are now found in such chemical engineering texts as Sherwood and Pigford (20) and Treybal (24). Perry's Chemical Engineers' Handbook (14) presents a good comprehensive survey of both theories.

Bakhmeteff (1) presents a comprehensive discussion of the theoretical concepts of turbulence suggested by Prandtl, von Karman, and Taylor. With these introductory concepts, rapid advances have been made in the field of turbulent, or eddy, diffusion. The Reynolds analogy first disclosed the basic similarity between energy transfer, mass transfer, and momentum transfer (fluid friction). Sherwood (19) presents theory and experimental data relating these three transfer processes. Dryden (5) makes use of the Reynolds analogy and turbulence theory to apply the defining characteristics of turbulence, scale and intensity, to a definition of turbulent diffusivity.

Experimental studies of the diffusion process have been made primarily in three systems: (1) coaxial gas streams in circular ducts, (2) wetted-wall columns, and (3) beds of particles, both packed and fluidized. These systems are representative of the majority of the unit operations used throughout the process industries. However, the study of mass transfer in coaxial gas streams may be thought of as being the most basic since, in effect, all processes deal essentially with the flow of fluids in interstices of one kind or another.

The first measurements of mass transfer by the turbulent process in a gaseous system were made by Towle and Sherwood (23). Carbon dioxide and hydrogen were introduced at the axis of a cylindrical duct, and concentration traverses over the central third of the duct diameter were obtained at vari-

ous distances downstream. The results were interpreted in terms of values of eddy diffusivity, obtained by comparing the data with an adaption of the Wilson equation for heat dissipation from a continuous-point heat source in a moving stream of fluid. Eddy diffusivity values obtained were of the order of 100 times the molecular diffusivity, and were the same for both hydrogen and carbon dioxide. The diffusivity was found to approach an asymptote with increase in axial distance from the tracer-gas injection tube. The asymptotic values of diffusivity increased approximately proportionally with Reynolds number.

Sherwood and Woertz (21) studied the interphase transfer of material in a rectangular duct. Water was transferred through a flowing air stream to a falling film of calcium chloride solution. The eddy diffusivity was reported to be essentially constant over the main central portion of the gas stream. The product of eddy diffusivity and gas density was found to be approximately 1.6 times the eddy viscosity, the latter being obtained from a velocity traverse. Under the experimental conditions, 28-57 per cent of the total resistance to diffusion was found to be in the main turbulent "core" of the fluid stream, with the remainder divided about equally between the two films at the duct walls.

Linton and Sherwood (11) studied mass transfer from cast tubes, cylinders, plates, and spheres to water in streamline

and turbulent flow. Only overall mass transfer coefficients were computed. The effect of the length of column to column diameter ratio on the mass transfer coefficient was shown to some extent for turbulent flow in round tubes.

Bernard (2) was the first to investigate turbulent diffusion in packed beds of solids. A tracer-gas injection technique was employed for both liquid and gaseous systems. An analytical expression, yielding values of $V/\underline{E}$, was derived from considerations of the partial differential equation representing the diffusion process and boundary conditions of a confining wall and a point injection tube. Pressure-drop data (friction factor) and diffusivity (Peclet number = $\frac{D_p V}{\underline{E}}$) were correlated satisfactorily with Reynolds number by the use of particle diameter as the length parameter, indicating interstitial turbulence is correctly interpreted as a function of packing size.

Fahien (6) and Dorweiler (4) present quite a complete review on mass transfer in packed beds.

Fahien (6) solved the general mass transfer differential equations for steady state mass transfer, allowing both total diffusivity and velocity to vary with radial position. The solution, considering boundary conditions of a confining wall and a finite tracer-gas injection tube, was effected by replacing the differential equation with a set of homogeneous linear difference equations, yielding eigen functions and

eigen values which express concentration as a function of position in the packed bed. The Peclet number was found to increase with radial position, the increase depending on the ratio of the particle diameter to the duct diameter. An explanation of the variation of Peclet number with radial position was presented on the basis of an increase in void fraction with radial position. Average values of Peclet number were found to increase with the particle diameter to duct diameter ratio and to be substantially independent of mass velocity for mass velocities above 500 lbs/hr/sq ft.

Dorweiler (4) extended the mass transfer data for packed beds to very low flow rates. The variation of Peclet number with radial position was shown for these low flow rates. Average values of Peclet number were correlated with Reynolds number. The interaction of the molecular and eddy mechanisms was illustrated by defining molecular and eddy Peclet numbers, and correlating them with Reynolds number.

Schwarz and Hoelscher (18) constructed a wetted-wall column which produced a falling film essentially free of rippling. Radial concentration profiles of water vapor in the column with fully developed turbulent flow of air were measured for several positions downstream from the inlet. The air Reynolds number was 25,000. The velocity distribution was established by a total head pitot tube. These data were treated in accordance with a graphical technique, yielding

point values of eddy diffusivity and eddy viscosity as functions of radial position. The effect of column height on local mass transfer was shown. The data indicated that the mass transfer rate did not become constant until a certain downstream distance was reached.

Hanratty et al. (7) presented a theoretical analysis of homogeneous isotropic systems involving turbulent diffusion. The diffusion characteristics of a fluid contained in a differential volume were assumed to be dependent on its previous history. The analysis concluded that the eddy diffusivity becomes constant only for relatively large times of diffusion. Although this analysis was used to describe turbulent diffusion in particulates fluidized beds of solids, no characteristic parameter for a fluidized bed was involved. Thus, a similar analysis may be applied to any field of turbulent diffusion. Experimental work consisted of measuring the spreading of a tracer dye from a very small injection tube in glass-sphere beds fluidized in water. From these data, mixing parameters - eddy diffusivity, scale, and intensity of turbulence - were established.

Lynn et al. (12) studied the mixing of coaxial streams of natural gas and air at atmospheric pressure and ambient temperature at Reynolds numbers of 44,000 and 79,000. Air was passed through a 6-inch cylindrical duct concentric with a 1/8-inch annulus through which natural gas was introduced.

Measurements of concentration and velocity were made as functions of the radius of the channel and the distance downstream from the point of initial mixing. From the considerations of concentration and velocity as functions of θ^2 , values of the eddy diffusivity and eddy viscosity were established as functions of position in the flow channel for the central portion of the flowing stream. The use of θ^2 rather than θ as the independent variable permitted the direct calculation of center-line values of the eddy properties. These eddy properties were found to be rather complicated functions of the particular flow conditions and channel geometry.

EQUIPMENT AND PROCEDURE

Figure 1 shows a schematic diagram of the apparatus used in this investigation. The equipment can be divided functionally into an air cleaning and purifying section, an air metering section, a carbon dioxide metering section, and a gas analyzing section.

Air from a high pressure source was used as the bulk stream gas. Since it was necessary to have air quantitatively free of moisture and carbon dioxide, a series of purification steps preceded the air metering section. A double oil filter consisting of the following two parts was used to remove serious oil contamination: (1) a one foot length of 1 1/4-inch pipe packed with steel wool acting as a course filter and (2) a Fischer Governor, Type 361, porous stone air filter acting as a final filter. The air was next passed through two 6-foot towers of 4-inch flanged pipe, the first containing 12-mesh silica gel for the removal of water vapor and the second containing 8-mesh soda lime for the removal of carbon dioxide. These small particles had the advantage of greater contact area. However, they presented the problem of dealing with fluidized beds. Satisfactory operation was realized by filling the towers to a height of only 3 feet and placing 6 inches of steel wool at the top of each.

The main air stream was metered by a series of three Brooks rotameters covering a range of flow rates from 1.5 to

Figure 1. General flow and sampling diagram.

35.0 SCFM. The ends of each rotameter were fixed to short lengths of 3/4-inch pipe terminated by a union fitting. Thus, the rotameters could be easily changed by connecting the union fittings on the ends of the short pipes to matching stationary union fittings on the flow piping arrangement. One Bourdon-tube pressure gage preceded the permanent rotameter connections and another followed the oil filter. The rotameters were operated between pressures of 18.0 and 22.0 psig, using calibration charts supplied by the manufacturer and perfect gas law pressure corrections. Two Reynolds spring-loaded diaphragm pressure regulators, NR8250EV18 and R8200EV18, were used for pressure reduction and regulation. The air rate was controlled by a 3/4-inch globe valve.

Carbon dioxide was obtained from 50 pound commercial cylinders, with vapor pressures of approximately 1000 psig. The gas was metered by means of a series of three small Brooks rotameters operated at 18.0 psig and covering a flow range from 500 to 12,000 SCCM. The rotameters were calibrated by means of a standard wet test meter. The gas rate was regulated by both a pressure reducing valve on the cylinder and a 3/8-inch needle valve following the rotameters. Frequent adjustments were necessary to maintain a constant flow rate. Since the liquid carbon dioxide cooled considerably on vaporization, the reducing valve on the cylinder as well as the first 3 feet of the carbon dioxide line were wrapped with ni-

chrome wire heating tape. The temperature of the heating tape was regulated by means of a Variac powerstat until the gas temperature at the entrance of the rotameter bank became constant at approximately room temperature.

The test section, shown in Figure 2, consisted of a vertical 4-inch galvanized pipe with a base height of 14 feet 4 inches. The carbon dioxide injection tube, a $1\frac{13}{32}$ -inch i.d.-1/2-inch o.d. hard copper tube, entered the base of the test section and extended to a point 2.6 inches above the top. It was held in the center of the pipe by means of a spoked disc near the base of the test section and two sets of equally spaced 1/16-inch steel guide wires. The end of the injection tube was beveled at an angle of 75° to one-half the wall thickness.

The test section was designed so that the sampling height above the end of the injection tube could be varied. Additional sections of pipe could be attached by placing them on top of the base section and tightening threaded rods extending through lugs on both sections. Machined overlapping grooves on all pipe sections helped to minimize connection roughness.

A discharge section was placed on top of the column to eliminate the effects of convection currents and to build up a slight positive pressure for forcing out gas samples.

Figure 2. Details of test section

Gas samples were removed by means of a sharp-edged pitot probe (3/16-inch i.d. and 3/8-inch i.d., depending on the bulk stream flow rate) housed in a 6-inch section of 4-inch pipe. This assembly was made with machined overlapping grooves so that it could be inserted in the test section. Samples could be removed from various radial positions in the pipe by manipulation of the graduated external section of the pitot probe. Different angular positions could be obtained by loosening the threaded tie rods holding the sections together and rotating the housing assembly to the desired position.

Gas samples removed by the pitot probe were passed through a Gow-Mac M/T-T-8, improved standard thermal conductivity cell for analysis. The cell is an eight filament unit containing two filaments in each of four arms of a Wheatstone bridge. An operating bridge current of 138 milliamperes was supplied by an arrangement of two 12-volt direct current sources connected in parallel. The current was adjusted by means of a microdial heliopot and decade box resistance until the desired value was obtained as measured by the drop across a standard 10.0 ohm resistor. The drop across the standard resistor was measured by means of a Rubicon, high precision, type B potentiometer and a portable lamp and scale galvanometer. Variations of -0.2 per cent were observed over 4-8 hour operating periods. The cell response, that is, the unbalanced emf of the Wheatstone bridge for carbon dioxide-air samples passing

through one side of the cell and purified reference air taken from a point just after the purification section passing through the other, was read on a Brown Electronic strip chart recording potentiometer. Linear strip chart paper was used on the recording potentiometer. Therefore, the unbalanced emf of the Wheatstone bridge was read in arbitrary units instead of in millivolts. Interpretation of this response in terms of concentration was made by a calibration of the cell.

The extreme sensitivity of the thermal conductivity cell to various operating conditions required a rigorous operating procedure so as to obtain satisfactory cell calibration and performance. The three major variables were found to be ambient temperature, gas flow rate through the cell, and the main air stream rate. Control of the approximately 200 SCCM sample flow rates through the cell was accomplished by using 1/4-inch needle valves preceding the small Brooks rotameters in the sample flow lines, resulting in a maximum deviation of flow rates of approximately 5 per cent. A satisfactory cell calibration method finally alleviated the effects of the ambient temperature and the main air stream rate.

Calibration of the cell was accomplished by determining the carbon dioxide concentration from carbon dioxide and air stream flow rates and by relating to cell impulses obtained by analyzing a thoroughly mixed sample of the gas. A log-log plot of concentration versus cell impulse showed a straight

line relationship for a given main air stream rate and a given ambient temperature. A series of parallel lines were determined for various main air stream rates and ambient temperatures. The main air stream rate was thought to affect the displacement of the calibration lines because of the varying efficiency of the silica gel tower with the air rate through it. The final working calibration of the cell showed the relation between concentration and cell impulse to be of the form

$$C/C_A = (E/E_A)^n,$$

where n is the slope of a $\log C$ versus $\log E$ plot and may be best determined by the method of least squares. Although the value of n varied slightly during initial experimentation, it was found to remain constant at 0.957 during the final experimental runs.

A five-loop, circular hot-wire anemometer, shown in Figure 3, was used to determine the velocity distribution across the 4-inch test section. The anemometer was of the constant current type and was constructed in accordance with the procedures outlined by Ower (13) and King (9) and modeled after the ones used by Schwartz (16). It consisted of five concentric loops of 0.006-inch platinum wire supported by means of thin mica strips cemented to a machined Plexiglass mount. The entire assembly was housed in a 6-inch section of 4-inch brass pipe. All copper-platinum wire joints within the housing were spot-welded for permanence of resistance. The power

Figure 3. Hot-wire anemometer.

supply for the anemometer circuit consisted of four 6-volt automotive-type batteries connected in series. Voltages across the anemometer wires and the standard resistance were measured by a Rubicon, high precision, type B potentiometer in conjunction with a lamp and scale galvanometer.

Calibration of the anemometer is based primarily upon the principles of fluid flow through a pipe. For low velocities the anemometer was calibrated by placing it in a vertical, galvanized 4-inch pipe which had an undisturbed length of 14 feet 4 inches. It was assumed that the flow was parabolic for Reynolds numbers less than 2100. Thus, the point velocities corresponding to the anemometer loop diameters are given by

$$u = 2V (1 - (r/r_o)^2)$$

where V is the average velocity as calculated from the flow rate measured by the main air stream rotameters. At higher velocities, a flat profile was introduced across nearly the entire pipe diameter by placing a 100-mesh screen one inch below and one inch immediately above the anemometer. The velocity for the four inner loops was then given by the average velocity divided by 0.942 (22) and that of the outer loop was given by the average velocity multiplied by 0.800.

After each anemometer loop was annealed for a period of six hours at a dull red heat in still air, the anemometer was placed on top of the vertical pipe and a 36-inch section of pipe was placed on top of it to eliminate discharge effects

inside the anemometer housing. After the flow was adjusted to the desired level, the anemometer was brought into the circuit. The current was adjusted by first manipulating a decade resistance box until the approximate value was read on an ammeter and then making a final adjustment by varying a fine resistance until the voltage drop across a standard resistance was equal to 0.555 volts as measured by the potentiometer. The voltage drop across the anemometer loop was then measured. Each anemometer loop was run through the low velocity region before going to the next. When the calibration for the low velocity region was completed, the screens were placed in the pipe and the higher velocity calibrations were performed in the same manner as described above.

In general the experimental procedure consisted of establishing flow rates and the thermal conductivity bridge current and in reading cell responses to changes in concentration. The thermal conductivity cell was operated continuously throughout the experimental runs. Because there was some change in the cell response with the main air stream rate, the air rate was set approximately 8 hours before a run was begun. The carbon dioxide rate was set to correspond to the approximate linear velocity of the main air stream at the center of the tube. Approximately 30 minutes were allowed for the carbon dioxide to reach an equilibrium temperature for the desired flow rate.

After equilibrium flow rates were established and the cell response for zero concentration was recorded, the carbon dioxide was admitted to the test column. Approximately ten minutes were allowed for the sampling flow system to reach equilibrium before the cell response corresponding to the average effluent concentration in the column was recorded. Samples were then withdrawn through the pitot probe at specified radial positions. Traverses were made in regular order from wall to center to wall. After a complete diameter had been traversed, the cell responses to the average effluent concentration and zero concentration were again recorded.

The thermal conductivity cell used in this investigation was a "fast acting cell". However, it was observed on the strip chart continuously recording the cell impulse that a period of approximately 3 minutes was required to obtain a full cell response for a given sample. This was because the sampling lines had to be completely flushed by each new sample before the cell received a representative sample from a given sampling point. The major factor in interpreting cell response was found to be that the concentration at any one point on the sampling traverse did not remain constant but fluctuated slightly in a random manner as small changes in the flow pattern occurred. Thus, the cell impulse representing the average concentration of each gas sample was determined by a graphical integration of the strip chart; that

is, the impulse was arithmetically time-weighted over a 6-9 minute sampling period.

Point velocities across the 4-inch test column were obtained at a single height above the injection tube for the three mass velocities studied by inserting the anemometer assembly into the test section in place of the pitot probe assembly. After the flow rate was adjusted to the desired value, the standard current was established in the circuit. The voltage drop across each anemometer loop was then measured by the potentiometer. Velocities corresponding to these measured voltages were then obtained from the calibration curves.

The data obtained by the above described procedure were treated by two mathematical techniques described in this paper.

METHODS OF CALCULATION

General Equations of Mass Transfer

In a moving fluid, mass may be transferred by either molecular diffusion, turbulent diffusion, or by a combination of both mechanisms. Molecular diffusion may be defined as the process by which concentration gradients disappear in a stagnant fluid. Turbulent (or eddy) diffusion is effected by the random motion of discrete "lumps" of fluid which mix with the surrounding fluid until their properties become identical with the average properties of the fluid in the vicinity.

Except in very special cases, both transfer processes are considered to take place simultaneously in a moving fluid. The total rate of material transfer by the two combined diffusional processes may then be described mathematically by the equation

$$N_A = - (D + E) \nabla C = - \underline{E} \nabla C , \quad (1)$$

where N_A = rate of diffusion per unit area

C = concentration of diffusing material

E = eddy diffusivity

D = molecular diffusion coefficient

$\underline{E}$ = total diffusivity

The development of the basic differential mass transfer equations considers the rate of transfer of material into a differential cube by means of diffusion and convection across

its boundaries and the net rate of change of material within the cube (as encountered in a reacting system or at unsteady-state).

Consider the non-reacting, steady-state system in which the diffusion and convection processes are effective in all directions. The net rate of transfer in each direction is then

$$\text{x-direction: } \left[\frac{\partial}{\partial x}(u_x C) - \frac{\partial}{\partial x} (E_x \frac{\partial C}{\partial x}) \right] dx dy dz ,$$

$$\text{y-direction: } \left[\frac{\partial}{\partial y}(u_y C) - \frac{\partial}{\partial y} (E_y \frac{\partial C}{\partial y}) \right] dx dy dz ,$$

$$\text{z-direction: } \left[\frac{\partial}{\partial z}(u_z C) - \frac{\partial}{\partial z} (E_z \frac{\partial C}{\partial z}) \right] dx dy dz ,$$

while the accumulation in the differential element is zero.

The general equation for mass transfer in a non-reacting, steady-state system is then

$$\begin{aligned} \frac{\partial}{\partial x} (u_x C) + \frac{\partial}{\partial y} (u_y C) + \frac{\partial}{\partial z} (u_z C) &= \frac{\partial}{\partial x} (E_x \frac{\partial C}{\partial x}) + \frac{\partial}{\partial y} (E_y \frac{\partial C}{\partial y}) \\ &+ \frac{\partial}{\partial z} (E_z \frac{\partial C}{\partial z}) , \end{aligned}$$

or expressed in cylindrical coordinates,

$$\begin{aligned} \frac{\partial}{\partial r} (C u_r r) + \frac{\partial}{\partial \theta} (C u_\theta) + r \frac{\partial}{\partial z} (C u_z) &= \frac{\partial}{\partial r} (r E_r \frac{\partial C}{\partial r}) + \frac{\partial}{\partial \theta} (E_\theta \frac{\partial C}{\partial \theta}) \\ &+ r \frac{\partial}{\partial z} (E_z \frac{\partial C}{\partial z}) . \quad (2) \end{aligned}$$

Equation 2 together with the continuity equation,

$$\frac{\partial}{\partial r} (r \rho u_r) + \frac{\partial}{\partial \theta} (u_\theta \rho) + r \frac{\partial}{\partial z} (u_z \rho) = 0, \quad (3)$$

represents the entire process of material transfer in a non-reacting, steady-state, moving fluid system. Thus, the simultaneous solutions of these two equations are essentially the solutions to all problems of material transfer in the above described system. However, to date, because of the complexity of Equations 2 and 3, it has been necessary for investigators to make various simplifying assumptions before solutions can be effected.

Analytical Solution of Equations 2 and 3

From experimental data the total diffusivity can be determined from a solution of Equations 2 and 3. However, analytical solutions to date have involved the following assumptions:

$$u_r = 0$$

$$\partial C / \partial \theta = 0$$

$$\frac{\partial u_z}{\partial z} = 0$$

$$\frac{\partial u_\theta}{\partial \theta} = 0$$

$$\underline{E}_z = 0$$

$$u_z = \text{constant} = V$$

$$\frac{\partial \underline{E}_r}{\partial r} = 0.$$

It is implicit in the above that the specific weight must be constant in order that the radial velocity component be zero. Equations 2 and 3 then reduce to

$$\frac{\partial^2 C}{\partial r^2} + \frac{\partial C}{r \partial r} = \frac{v}{E} \frac{\partial C}{\partial z}, \quad (4)$$

where E represents the total diffusivity.

The following boundary conditions, describing experimental conditions are imposed:

I. At $z = 0$, the plane of the tip of the injection tube, the concentration over the face of the tube is constant at C_f , and outside of the tube is zero:

$$C(r, 0) = C_f \quad (0 < r < t)$$

$$C(r, 0) = 0 \quad (t < r < r_0) .$$

II. As the height above the injection tube becomes great, the concentration at any radial position approaches the average concentration:

$$C(r, \infty) = C_A .$$

III. At the wall, no diffusion takes place:

$$\frac{\partial C}{\partial r} = 0 \quad \text{at } r = r_0 .$$

IV. The concentration distribution is symmetrical such that:

$$\frac{\partial C}{\partial r} = 0 \quad \text{at } r = 0 .$$

A solution of Equation 4 in the form of an infinite Fourier-Bessel series,

$$\frac{C}{C_A} \frac{C_A}{C_M} = 1 + \frac{2}{t} \sum_{n=1}^{\infty} \frac{J_1(\lambda_n t)}{\lambda_n J_0^2(\lambda_n r_o)} J_0(\lambda_n r) e^{-\lambda_n^2 z E / V}, \quad (5)$$

was presented by Bernard (2) and modified by Kurihara (10). C_A is the measured average concentration, C_M is the mean integral concentration (defined analytically by Equation 15), t is the injection tube radius, r_o is the column radius, z is height above the plane of the injection tube, and $\lambda_n r_o$ is the root of $J_1(\lambda_n r_o) = 0$. Bernard (2), Kurihara (10), and Dorweiler (4) used a form of Equation 5 in treating concentration data from a packed bed. Schlinger and Sage (15) applied it to mass transfer in turbulent gas streams.

The problem of dealing with an infinite series is circumvented by the rapid convergence of the series for the experimental conditions where the ratio of $C_{\max}/C_{\min}$ is sufficiently small, that is, z is sufficiently large. The reduced form of Equation 5 is

$$\frac{C}{C_A} \frac{C_A}{C_M} = 1 + \frac{2 J_1(\lambda_1 t)}{t \lambda_1 J_0^2(\lambda_1 r_o)} J_0(\lambda_1 r) e^{-\lambda_1^2 E z / V}. \quad (6)$$

By introducing a new radial variable, $\theta = r/r_o$, Equation 6 may be written as

$$\frac{C}{C_A} \frac{C_A}{C_M} = 1 + \frac{2 J_1(\lambda_1 \theta_t)}{\theta_t \lambda_1 J_0^2(\lambda_1)} J_0(\lambda_1 \theta) e^{-\lambda_1^2 \bar{E} z/r_o^2} V. \quad (7)$$

Considering the concentration at the column center, C_o , where $\theta = 0$,

$$\frac{C_o}{C_A} \frac{C_A}{C_M} = 1 + \frac{2 J_1(\lambda_1 \theta_t)}{\theta_t \lambda_1 J_0^2(\lambda_1)} e^{-\lambda_1^2 \bar{E} z/r_o^2} V. \quad (8)$$

By plotting C/C_A versus $J_0(\lambda_1 \theta)$, the value of C_o/C_A can be obtained analytically. The intercept of the linear plot is the term C_M/C_A . Since all the other terms are known, $\bar{E}$ can be calculated. Equation 8 was applied in this investigation by fitting the experimental concentration data to a linear relationship with the zero order Bessel function by the method of least squares. A sample calculation is shown in Appendix A.

Semi-numerical Solution of Equations 2 and 3

Under the rigid assumptions that the mass transfer process is a function of local flow conditions, Fahien (6) solved equations 2 and 3 allowing both the total diffusivity, $\bar{E}$, and the velocity, u_z (or u), to vary with radial position. The

solution is in terms of finite difference equations and may be a more realistic description of the actual diffusion mechanism. Although Fahien applied the solution to concentration and velocity data from a packed bed, the mathematics involved no parameters characterizing the packing in the bed and are also applicable to this problem.

Under the preceding assumptions, the reduced differential mass transfer equation may be written

$$\frac{\partial}{\partial \theta} \left(E \theta \frac{\partial C}{\partial \theta} \right) = r_o^2 u \theta \frac{\partial C}{\partial z} . \quad (9)$$

Equation 9 was replaced by a system of difference equations written about points over the radius. This system of difference equations is linear in the eigen functions R at each interval about the point, and the eigen value λ_n . The solution of Equation 9 is of the form

$$C = \sum_{n=0}^N A_n R_n e^{-\lambda_n z / r_o^2} , \quad (10)$$

where A_n are constants, R_n are the eigen functions, λ_n are the eigen values, and N is the number of increments into which the radius has been divided.

A simplification of the solution is presented by Dorweiler (4). Since the terms in Equation 10 involve a negative exponent term in z and λ_n , and since λ_n increases with n ,

succeeding terms in the series become smaller. Thus, for z sufficiently large, only two terms in the series need be considered:

$$C = A_1 R_1 Z_1 + A_2 R_2 Z_2 \quad (11)$$

By applying boundary conditions I. through IV., and imposing the orthogonality of the eigen functions, a direct solution for $\underline{E}$ is of the form

$$\underline{E}_{(k + \frac{1}{2})h} = \frac{\lambda_1 h^2 V \sum_{n=0}^k P_k (C_k - C_M)}{\theta_{(k + \frac{1}{2})h} C_k - C_{k+1}}, \quad (12)$$

where k designates the interval position on the radius ($k=0$ at the center and $k = N$ at the wall), h is the interval size ($h = 1/N$), θ is the radial position, and P is the weight function ($P = \frac{u}{V} \theta$). The value of λ_1 is given by

$$\lambda_1 = - \frac{r_o^2}{z} \ln Z_1, \quad (13)$$

where Z_1 is given by

$$Z_1 = \frac{\sum_{k=0}^N P_k (C_k - C_M)^2 \sum_{k=0}^{\theta_t} P_k}{\sum_{k=0}^{\theta_t} P_k (C_k - C_M) \sum_{k=0}^N C_k P_k} \quad (14)$$

C_M is defined as the mean concentration, obtained by a volumetric flow rate-weighted average of point concentrations:

$$C_M = \frac{\sum_{n=0}^N C_k P_k}{\sum_{n=0}^N P_k} \quad (15)$$

In order to apply the preceding semi-numerical solution, concentration and velocity data need to be known at radial positions such that the radius is divided into an integral number of equal divisions. Such point values were determined by plotting the experimental data and reading off the values at the desired radial positions. The "smoothed" values of C/C_A and u/V used for the calculations in this work are tabulated in Tables 4 and 5 respectively.

The application of the semi-numerical solution is illustrated in Appendix B.

RESULTS AND DISCUSSION

Experimental Data

The experimental work in this investigation was performed at ambient temperature and atmospheric pressure. During the period of experimentation, the temperature varied from a minimum of 73.4°F to a maximum of 82.4°F. The temperature was considered only in the calculation of mass velocities. Although it is well known that temperature has some effect on the rate of mass transfer, this effect was not considered in this study since the temperature range was only 9.0°F.

Experiments were conducted for mass velocities of 228, 612, and 1022 lbs/hr/sq ft corresponding to respective Reynolds numbers of 1695, 4510, and 7605.

Three significant points should be considered with respect to the concentration data: (1) the time variation of cell impulse for a given radial position, (2) the accuracy of the concentration data, and (3) the angular symmetry of the concentration profiles.

Figure 4 shows the relative variation of thermal conductivity cell impulse (concentration) with time for given radial positions over a 9 minute sampling period. This variation is much more pronounced near the wall of the pipe as shown by the graphs. Two physical factors to which this variation may be attributed are: (1) the considerable roughness of the pipe

$\theta = 0.000$ $\theta = 0.438$ $\theta = 0.938$

Figure 4. Effect of radial position on variation of cell impulse with time

wall, and (2) pressure variations (flow rate changes) of ± 0.5 psig on the incoming air stream during an experiment. A time-weighted average of the cell impulse for each sampling point was taken as the value most representative of the average concentration from the given radial position. Such an averaging process was accomplished by a graphical integration of the cell impulse versus time curves.

Since the concentration data were determined from thermal conductivity cell response, they are subject to the accuracy of the cell calibration. The cell calibration was effected by determining cell response for thoroughly mixed gas samples whose concentrations were calculated from carbon dioxide and air flow rates. An absolute relationship established by such a calibration procedure is consequently dependent upon the accuracy of the flow meters. However, by using the relative relationship

$$C/C_A = (E/E_A)^n ,$$

some elimination of the absolute error is provided. The application of the method of least squares to the calibration data was thought to yield the best representative value of the exponent n . Also, Equations 8 and 12 indicate that C/C_A values can be used directly in the calculation of values of $\underline{E}$. Experimental values of C/C_A are tabulated in Table 7.

Typical radial concentration profiles are shown in Figure 5. Asymmetry is quite evident. This warping of the concen-

Figure 5. Typical concentration profiles

tration profiles can be attributed primarily to the eccentricity in both the injection tube and the containing pipe. Because of this asymmetry, the assumption of negligible angular transfer in the solutions of the general mass transfer equations is lacking to some extent. However, a good representation of the mass transfer throughout the entire cross section of the pipe was thought to be acquired by averaging the concentrations for the two radial positions sampled.

Since the change in velocity with axial distance was neglected in both solutions of the general mass transfer equations, the velocity distribution was obtained at only one height ($z = 46.4$ inches) above the plane of the injection tube for each mass velocity studied. These data are tabulated in Table 8.

Schlenger and Sage (15) investigated the variation of velocity with radial position in an empty pipe at various axial distances from an injection tube. The flow was completely turbulent. Their data indicated a very high peak velocity in the center portion of the pipe (over an area approximately equal to the cross section of the injection tube) at small distances from the injection tube. As the axial distance was increased, the effect of the injection tube decreased. A similar effect was thought to be present in this work. However, since the smallest of the five anemometer loops was located at $\theta = 0.316$ and the radius of the injection tube extended to only $\theta = 0.1009$, no such effect could be detected.

Experimental Results

The molecular diffusion coefficient for carbon dioxide in air at 25°C is 0.164 sq cm/sec (1.765×10^{-4} sq ft/sec) (14). By comparing this value with the values of the total diffusivity reported herein, a large contribution of turbulent exchange, even in low velocity gas streams, may be observed.

Before the results of the calculations are discussed, the difference between $\bar{E}$ as determined by Equation 8 and $\bar{E}$ as determined by Equation 12 should be emphasized. In the derivation of Equation 12 several assumptions were made: (1) the radial component of velocity is zero; (2) angular symmetry is present; (3) the velocity is independent of axial position; and (4) diffusion in the direction of gross motion is negligible.

The first assumption is true unless unique disturbances arise. Angular symmetry is only affected by the eccentricity of the circular ducts. Except for the effects of the injection tube, the velocity is quite independent of axial distance. Diffusion in the direction of gross motion is only approximately negligible, but becomes less important at large axial distances where $\partial C / \partial z$ and $\partial^2 C / \partial z^2$ are very small. Hence, the semi-numerical equation gives values of $\bar{E}$ very closely approximating what may be considered the true physical values.

However, in the derivation of Equation 8, it is assumed, in addition to the above assumptions, that velocity and total

diffusivity are independent of radial position in the containing duct. As has been shown, this is not the case. Therefore, the value of the total diffusivity as determined by the analytical equation may be visualized as a hypothetical diffusivity. Its specific value lies in that it helps to visualize the overall results of the diffusion process. Although Equation 8 does not represent the physical situation, it does fit the experimental concentration data reasonably well. This is shown in Figure 6 in which the concentration is plotted against its functional variable $J_0(\lambda_1 \theta)$.

Until a more exact mathematical analysis is derived, the semi-numerical solution of the general mass transfer equations may be considered to give the best representation of the diffusion process. However, by no means should it be accepted as a complete description of the mechanism.

An inspection of Figure 7 reveals, at least qualitatively, the importance of entrance effects on the mass transfer process in the flow range studied. The most significant aspect is that the greatest relative effect is on the flow rate corresponding to $Re = 1695$. Of course, the axial distances for the three flow rates are not the same. Perhaps, if the data for $Re = 1695$ were extended to a greater axial distance, $\bar{E}$ might approach a limiting value less than that for either $Re = 4510$ or $Re = 7605$. This would be in line with the existing belief that $\bar{E}$ increases with Reynolds number for undis-

Figure 6. Variation of concentration with
Bessel function of zero order

Figure 7. Variation of total diffusivity with axial distance

turbed flow in a cylindrical channel. However, when an attempt to establish a concentration profile at a greater axial distance was made, it was found that no significant slope, $\partial C / \partial z$, could be detected. Thus, a value for E could not be calculated.

Figure 8 shows the same data of Figure 7 plotted as E versus time of diffusion in the field (diffusion time = z/V). The time of diffusion may be considered a better correlating parameter for all fluid velocities. Figure 8 may be considered indicative of the accepted theory that the lower the flow rate the longer the time required in an undisturbed channel before steady-state flow conditions are achieved. This may explain the large slope of the E versus z and E versus diffusion time curves in such a short diffusion time for $Re = 1695$.

The data of Lynn *et al.* (12) indicated an increase in the total diffusivity with axial distance for much higher flow rates. In general, the trend would be expected to be the same as in this work. However, their work involved the introduction of a tracer gas through a small annulus at the wall of the containing pipe instead of from a small injection tube located at the center of the pipe. Evidently, the entrance effects for the two systems were not the same.

It is probable that many of the discrepancies in the literature concerning mass transfer data result from failure

Figure 8. Variation of total diffusivity with diffusion time

to consider entrance effects. Previous data have been correlated by the evaluation of coefficients and exponents of a dimensionless equation, in which an overall mass transfer coefficient is expressed. No consideration is given to the length of the column in question as a parameter of the problem. Thus, it may be of much importance to study new correlation methods involving the length parameter or perhaps, even better, the length to diameter dimensionless ratio.

Figure 9 presents a good representation of the variation of total diffusivity with radial position for undisturbed flow in a cylindrical channel. There seems to be no significant entrance effects for any of the three levels. As was expected, the total diffusivity tends toward the value of the molecular diffusion coefficient as the wall of the pipe is approached. In the center portion of the pipe the total diffusivity is reasonably constant as might be dictated by the local flow conditions.

Figures 10 and 11 also show significant variation of the total diffusivity with radial position. These results indicate quite clearly the expected diminishing entrance effects on the mass transfer process as the axial distance from the injection tube is increased. They also indicate that this decreasing characteristic of $\bar{E}$ as depicted in Figures 7 and 8 is primarily due to the decrease in point values of $\bar{E}$ near

Figure 9. Variation of total diffusivity with radial position

Figure 10. Variation of total diffusivity with radial position

Figure 11. Variation of total diffusivity with radial position

the center of the pipe (where the effect of the injection tube should be the greatest) as the axial distance is increased.

No point value of the total diffusivity at the center-line of the pipe could be calculated by Equation 12. However, the point value of $\underline{E}$ at the center-line of the pipe is believed to be of the same magnitude as $\underline{E}$ for adjacent radial positions. Schwarz and Hoelscher (18) state that the total diffusivity at the center-line of the pipe must tend toward infinity. Furthermore, they say that the total diffusivity has no meaning at the center-line because of the manner in which it is defined. Therefore, a close study of the defining equation,

$$N_A = - \underline{E} (\partial C / \partial r),$$

is in order. At the center-line of the pipe, $\partial C / \partial r = 0$, and thus $N_A = 0$ if $\underline{E}$ is finite. However, at a very small radial distance from the center-line, $\partial C / \partial r \neq 0$, $N_A \neq 0$, and both are finite, which makes it implicit that $\underline{E} \neq \infty$. Therefore, a good definition of $\underline{E}$ at the center-line of the pipe would be

$$\underline{E} = \lim_{r \rightarrow 0} \frac{N_A}{-\partial C / \partial r}.$$

Such reasoning gives $\underline{E}$ a finite value everywhere and makes it a continuous function in r . Thus, if a function representing $\underline{E}$ as a function of position in the pipe were known, the mass

transfer equation above could be integrated exactly over the entire cross section of the pipe.

Lynn et al. (12) graphically integrated Equation 12, with the integrations being made with respect to θ^2 instead of θ . By such a transformation, it was possible to compute values of the total diffusivity at the center-line of the pipe. Their results indicated $\underline{E}$ to be reasonably constant for the center portion of the pipe for Reynolds numbers of 44,000 and 79,000.

Although there have been numerous experimental and theoretical studies of the turbulent process, many questions remain unanswered. The greatest hindrance in describing the turbulent mechanism seems to lie in the lack of a complete working mathematical analysis of the transfer of energy, mass, and momentum in any given system. Therefore, with the further development of the mathematics of fluid dynamics will come a better understanding of the diffusional mass transfer mechanism.

CONCLUSIONS

The major conclusions drawn from the results of this investigation may be summarized as follows:

1. A considerable contribution of turbulent exchange in the flow range where the molecular process of mass transfer might be expected to be controlling was observed.
2. The total diffusivity was found to be a rather complex function of system geometry and local flow conditions. Specifically, the total diffusivity was found to vary with both radial and axial position.
3. The total diffusivity was found to vary from values several times the molecular diffusion coefficient in the central portion of the gas stream to values approaching the molecular diffusion coefficient near the wall of the pipe.
4. Entrance effects were found to be of considerable importance in the mass transfer process under the particular experimental conditions of this investigation. Thus, many of the discrepancies in the literature concerning mass transfer data may be the result of failure to consider such entrance effects.

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APPENDIX A

Sample Calculations

Analytical Solution

Equation 8 was used to determine values of the total diffusivity. By plotting C/C_A against $J_0(\lambda_1\theta)$, and determining the best linear relationship for the data by the method of least squares, the intercept will give C_M/C_A . Then, since for $C = C_0$, $J_0(\lambda_1 0) = 1.0$, the value of $\underline{E}$ can be determined.

From boundary condition III, when $r = r_0$, $\theta = 1.0$, and

$$J'_0(\lambda_1) = J_1(\lambda_1) = 0.$$

The first root of $J_1(x) = 0$ is 3.8317. Hence, $x = \lambda_1 = 3.8317$.

The method of least squares form for Equation 8, representing the C/C_A ordinate by y and the $J_0(\lambda_1\theta)$ abscissa by x is

$$\begin{vmatrix} \sum_{i=1}^n x_i & \sum_{i=1}^n y_i & \sum_{i=1}^n 1 \\ \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i y_i & \sum_{i=1}^n x_i \end{vmatrix} = 0.$$

The final form of the preceding determinant is

$$y = mx + b,$$

where m and b are the slope and intercept needed. Table 1 shows the application of the method of least squares to the experimental data of a given run.

Experimental conditions: $z = 78.3$ inches,
 $\theta_t = 0.40625/4.026 = 0.1009$, $G = 1022$ lbs of air/hr/sq ft
 ($V = 3.823$ fps).

The best line thus established is

$$C/C_A = 0.7877 J_0(\lambda_1 \theta) + 0.9265 .$$

The value of C_o/C_M is then

$$\frac{(0.7877 + 0.9265)}{(0.9265)} = 1.8502 .$$

The form of Equation 8 for the center concentration is

$$C_o/C_M = 1 + \frac{2}{\theta_t \lambda_1} \frac{J_1(\lambda_1 \theta_t)}{J_0^2(\lambda_1)} e^{-\lambda_1^2 z \frac{E}{V} r_o^2} .$$

Substituting the functional values,

$$1.8502 = 1 + 6.0497 e^{-890.50 \frac{E}{V}} ,$$

from which $\frac{E}{V} = 0.002204$ sq ft/sec.

The results computed by the preceding method are given in Table 2.

Table 1. Application of method of least squares to experimental concentration data

θ	C/C_A	$\lambda_1 \theta$	$J_0(\lambda_1 \theta)$	$J_0^2(\lambda_1 \theta)$	$\frac{C}{C_A} J_0(\lambda_1 \theta)$
0.000	1.745	0.000	1.0000	1.0000	1.7450
0.063	1.731	0.241	0.9855	0.9712	1.7059
0.188	1.616	0.720	0.8744	0.7646	1.4130
0.313	1.426	1.199	0.6716	0.4510	0.9577
0.438	1.208	1.678	0.4106	0.1686	0.4960
0.563	0.995	2.157	0.1344	0.0181	0.1337
0.688	0.833	2.636	-0.1137	0.0129	-0.0947
0.813	0.724	3.115	-0.2966	0.0880	-0.2147
0.938	0.640	3.594	-0.3912	0.1530	-0.2504
	<u>10.918</u>		<u>3.2750</u>	<u>3.6274</u>	<u>5.8915</u>

Table 2. Analytical solution results

G	Re	V	z(in.)	t	$\underline{E} \times 10^3$
1022	7605	3.823	106.8	2.328	2.370
1022	7605	3.823	90.4	1.971	2.514
1022	7605	3.823	78.3	1.707	2.204
612	4510	2.313	90.4	3.257	1.620
612	4510	2.313	70.4	2.536	1.750
612	4510	2.313	53.8	1.938	1.851
228	1695	0.849	25.2	2.473	2.690
228	1695	0.849	17.4	1.708	3.185
228	1695	0.849	12.4	1.217	3.738

APPENDIX B

Sample Calculations
Semi-numerical Solution

The details involved in the use of the semi-numerical solution are shown in Table 3. The order of the entire table for a given radial increment is from left to right, with the order of increase in radial position downward.

Experimental conditions: $z = 90.4$ inches,
 $\theta_t = 0.40625/4.026 = 0.1009$, $h = 1/8$, $N = 8$, $G = 1022$ lbs of
 air/hr/sq ft ($V = 3.823$ fps).

The mean integral average concentration is given by

$$C_M = \frac{\sum_{k=0}^N C_k P_k}{\sum_{k=0}^N P_k} = \frac{4.5218}{4.1210} = 1.097 .$$

Referring to Equation 14, the terms thus far calculated in Tables 3a and 3b can be applied in calculating Z_1 and λ_1 .

$$\sum_{k=0}^{\theta_t} P_k = 0.0210 + \frac{0.1009}{0.1250} (0.1676) = 0.1563 .$$

$$\sum_{k=0}^{\theta_t} P_k (C_k - C_M) = 0.0095 + \frac{0.1009}{0.1250} (0.0706) = 0.0665$$

Table 3a. Calculation of mean integral concentration

k	θ	C_k/C_A	u_k/V	P_k	$C_k P_k$
0	0.0000	1.547	1.341	0.0210 ^a	0.0325
	0.0625		1.341		
1	0.1250	1.518	1.341	0.1676	0.2544
	0.1875		1.335		
2	0.2500	1.428	1.330	0.3325	0.4748
	0.3125		1.322		
3	0.3750	1.294	1.311	0.4916	0.6361
	0.4375		1.295		
4	0.5000	1.168	1.278	0.6390	0.7464
	0.5625		1.251		
5	0.6250	1.054	1.211	0.7569	0.7978
	0.6875		1.157		
6	0.7500	0.963	1.081	0.8108	0.7808
	0.8125		0.954		
7	0.8750	0.895	0.770	0.6738	0.6031
	0.9375		0.486		
8	1.0000	0.860	0.000	0.2278 ^b	0.1959
				$\sum_{k=0}^N P_k =$	$\sum_{k=0}^N C_k P_k =$
				4.1210	4.5218

^a P_0 does not follow the definition of velocity times radial position since the product thus obtained would be zero. Physically, this cannot be true since the P function is a weighted average over an increment. Hence, by graphical definition,

$$P_0 = \frac{1}{2} (P(h/4)) = \frac{1}{2} (1.341) \left(\frac{1}{4} \cdot \frac{1}{8} \right) = 0.0210 .$$

^bSimilarly, $P_N = P_8$ is given by graphical definition as

$$P_8 = \frac{1}{2} P((N - \frac{1}{2})h) = \frac{1}{2} P(7\frac{1}{2}h) = \frac{1}{2} (0.486)(0.9375) = 0.2278 .$$

Table 3b. Determination of eigen value, λ_1

k	θ	$C_k - C_M$	$P_k(C_k - C_M)$	$P_k(C_k - C_M)^2$
0	0.0000	0.450	0.0095	0.0043
	0.0625			
1	0.1250	0.421	0.0706	0.0297
	0.1875			
2	0.2500	0.331	0.1101	0.0364
	0.3125			
3	0.3750	0.197	0.0968	0.0191
	0.4375			
4	0.5000	0.071	0.0454	0.0032
	0.5625			
5	0.6250	-0.043	-0.0325	0.0014
	0.6875			
6	0.7500	-0.134	-0.1086	0.0146
	0.8125			
7	0.8750	-0.202	-0.1361	0.0275
	0.9375			
8	1.0000	-0.237	-0.0540	0.0128

$$\sum_{k=0}^N P_k(C_k - C_M)^2 =$$

0.1490

$$1/Z_1 = \frac{\sum_{k=0}^N C_k P_k}{\sum_{k=0}^N P_k (C_k - C_M)^2} \times \frac{\sum_{k=0}^{\theta_t} P_k (C_k - C_M)}{\sum_{k=0}^{\theta_t} P_k} = \frac{(4.5218)(0.0665)}{(0.1490)(0.1563)} = 12.912$$

$$\ln(1/Z_1) = 2.55817$$

$$\lambda_1 = \frac{r_o^2}{z} \ln(1/Z_1) = \frac{(0.16775)^2}{(7.5333)} (2.55817) = 0.0095558 \text{ ft}$$

$$\lambda_1 h^2 v = (0.0095558)(0.015625)(3.823) = 0.00057081 \text{ sq ft/sec}$$

Referring to Equation 12, with the constant coefficient $\lambda_1 h^2 v$ evaluated, the point diffusivity can be calculated as shown in Table 3c.

$$\frac{E(k + \frac{1}{2})h}{\theta(k + \frac{1}{2})h} = \frac{\lambda_1 h^2 v \sum_{k=0}^k P_k (C_k - C_M)}{C_k - C_{k+1}}$$

The smoothed values of C/C_A used in the semi-numerical solution calculations are given in Table 4. The smoothed values of u/V are given in Table 5. Point values of the total diffusivity are tabulated in Table 6.

Table 3c. Calculation of point diffusivity

k	θ	$\sum_{k=0}^k P_k (C_k - C_M)$	$C_k - C_{k+1}$	$\underline{E}$
0	0.0000	0.0095	0.029	
	0.0625			0.002992
1	0.1250	0.0801	0.090	
	0.1875			0.002709
2	0.2500	0.1902	0.134	
	0.3125			0.002593 ^a
3	0.3750	0.2870	0.126	
	0.4375			0.002972
4	0.5000	0.3324	0.114	
	0.5625			0.002959
5	0.6250	0.2999	0.091	
	0.6875			0.002736
6	0.7500	0.1913	0.068	
	0.8125			0.001976
7	0.8750	0.0552	0.035	
	0.9375			0.000960
8	1.0000	0.0012		

$$a_{\underline{E}} = \frac{(0.0005708)(0.1902)}{(0.3125)(0.134)} = 0.002593$$

Table 4. Smoothed values of C/C_A used in
semi-numerical calculations

$G = 1022$			
θ	$z = 106.8 \text{ in.}$	$z = 90.4 \text{ in.}$	$z = 78.3 \text{ in.}$
0.000	1.341	1.547	1.745
0.125	1.315	1.518	1.685
0.250	1.244	1.428	1.524
0.375	1.153	1.294	1.321
0.500	1.062	1.168	1.093
0.625	0.988	1.054	0.909
0.750	0.931	0.963	0.776
0.875	0.882	0.895	0.678
1.000	0.852	0.860	0.624
C_M/C_A	1.019	1.097	0.992

$G = 612$			
θ	$z = 90.4 \text{ in.}$	$z = 70.4 \text{ in.}$	$z = 53.8 \text{ in.}$
0.000	1.317	1.463	1.734
0.125	1.303	1.452	1.716
0.250	1.257	1.395	1.634
0.375	1.184	1.287	1.475
0.500	1.096	1.151	1.267
0.625	0.998	1.000	1.032
0.750	0.893	0.840	0.777
0.875	0.803	0.687	0.534
1.000	0.757	0.615	0.426
C_M/C_A	1.016	1.025	1.065

Table 4. (Continued)

G = 228			
θ	z = 25.2 in.	z = 17.4 in.	z = 12.4 in.
0.000	1.162	1.306	1.461
0.125	1.157	1.294	1.447
0.250	1.142	1.262	1.408
0.375	1.108	1.203	1.325
0.500	1.060	1.117	1.204
0.625	1.004	1.006	1.042
0.750	0.951	0.896	0.848
0.875	0.909	0.813	0.684
1.000	0.888	0.781	0.617
C_M/C_A	1.036	1.063	1.107

Table 5. Smoothed values of u/V used in
semi-numerical calculations

θ	G = 1022 V = 3.823	G = 612 V = 2.313	G = 228 V = 0.849
0.0000	1.341	1.450	2.000
0.0625	1.341	1.450	1.992
0.1250	1.341	1.450	1.969
0.1875	1.335	1.442	1.930
0.2500	1.330	1.440	1.875
0.3125	1.322	1.438	1.805
0.3750	1.311	1.430	1.719
0.4375	1.295	1.401	1.617
0.5000	1.278	1.360	1.500
0.5625	1.251	1.304	1.367
0.6250	1.211	1.220	1.219
0.6875	1.157	1.117	1.055
0.7500	1.081	1.000	0.875
0.8125	0.954	0.870	0.680
0.8750	0.770	0.690	0.469
0.9375	0.486	0.442	0.242
1.0000	0.000	0.000	0.000

Table 6. Point values of diffusivity, $\underline{E} \times 10^3$ (sq ft/sec),
calculated by semi-numerical solution

$G = 1022$			
θ	$z = 106.8$ in.	$z = 90.4$ in.	$z = 78.3$ in.
0.0625	2.288	2.992	2.292
0.1875	2.317	2.709	2.380
0.3125	2.523	2.593	2.651
0.4375	2.707	2.972	2.569
0.5625	2.951	2.959	2.816
0.6875	2.806	2.736	2.813
0.8125	1.781	1.976	2.032
0.9375	0.727	0.960	0.921

$G = 612$			
θ	$z = 90.4$ in.	$z = 70.4$ in.	$z = 53.8$ in.
0.0625	2.508	4.791	4.588
0.1875	2.200	2.718	2.942
0.3125	2.060	2.174	2.310
0.4375	1.976	2.019	2.082
0.5625	1.699	1.750	1.786
0.6875	1.236	1.293	1.298
0.8125	0.813	0.773	0.781
0.9375	0.415	0.418	0.448

$G = 228$			
θ	$z = 25.2$ in.	$z = 17.4$ in.	$z = 12.4$ in.
0.0625	6.484	5.936	8.088
0.1875	6.225	6.288	8.331
0.3125	4.078	5.011	5.848
0.4375	3.211	3.860	4.573
0.5625	2.438	2.706	3.171
0.6875	1.760	1.898	1.927
0.8125	1.029	1.176	1.109
0.9375	0.409	0.640	0.574

APPENDIX C

Table 7. Experimental concentration data, C/C_A

$G = 1022$			
θ	$z = 106.8$ in.	$z = 90.4$ in.	$z = 78.3$ in.
0.938	0.948	0.971	0.623
	0.775	0.770	0.669
0.813	0.983	1.026	0.700
	0.827	0.828	0.749
0.688	1.039	1.103	0.815
	0.878	0.913	0.851
0.563	1.086	1.201	0.971
	0.957	1.014	1.018
0.438	1.162	1.304	1.176
	1.046	1.154	1.240
0.313	1.242	1.424	1.412
	1.159	1.299	1.440
0.188	1.314	1.523	1.602
	1.253	1.441	1.630
0.063	1.345	1.563	1.733
	1.337	1.498	1.728
0.000	1.338	1.563	1.745

$G = 612$			
θ	$z = 90.4$ in.	$z = 70.4$ in.	$z = 53.8$ in.
0.938	0.831	0.549	0.326
	0.707	0.717	0.569
0.813	0.905	0.689	0.512
	0.796	0.831	0.788
0.688	1.002	0.854	0.762
	0.886	0.991	1.046
0.563	1.096	1.011	1.031
	1.001	1.132	1.276
0.438	1.175	1.173	1.271
	1.094	1.272	1.498
0.313	1.251	1.324	1.476
	1.196	1.368	1.638
0.188	1.301	1.409	1.623
	1.266	1.444	1.710
0.063	1.316	1.451	1.715
	1.303	1.457	1.737
0.000	1.320	1.463	1.745

Table 7. (Continued)

G = 228				
θ	z = 25.2 in.	z = 17.4 in. ^a	z = 17.4 in. ^a	z = 12.4 in.
0.891	0.895	0.840	0.794	0.647
	0.914	0.776	0.812	0.720
0.783	0.932	0.902	0.877	0.772
	0.938	0.840	0.869	0.831
0.674	0.975	0.965	0.965	0.952
	0.996	0.954	0.963	0.999
0.565	1.032	1.080	1.082	1.115
	1.032	1.047	1.090	1.154
0.457	1.066	1.145	1.157	1.245
	1.089	1.136	1.163	1.297
0.348	1.120	1.214	1.230	1.327
	1.124	1.199	1.241	1.358
0.239	1.141	1.254	1.272	1.404
	1.145	1.251	1.276	1.425
0.131	1.156	1.285	1.304	1.437
	1.160	1.284	1.306	1.452
0.000	1.162	1.292	1.320	1.465

^aDiameters perpendicular to each other

Table 8. Experimental velocity data^a

θ^2	G = 1022 V = 3.823	G = 612 V = 2.313	G = 228 V = 0.849
0.10	5.06	3.32	1.53
0.30	4.81	3.06	1.18
0.50	4.50	2.51	0.85
0.70	3.43	1.87	0.51
0.90	1.50	0.89	0.17

^aThe data for each velocity distribution were corrected such that an exact material balance on the flow rate was obtained. This involved multiplying each velocity for a given distribution by the ratio

$$\frac{\text{Actual average velocity}}{\int_0^1 u \, d(\theta^2)}$$

This operation did not change the u/V distribution.