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SUBJECT: Integral Equations for the Flux Density Near
a Thin Foil and for Scattering in Air in the
Presence of the Ground

AUTHOR: F. H. Murray

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Integral Equations for the Flux Density Near a Thin Foil
And for Scattering in Air in the Presence of the Ground

F. H. Murray

The foils to be considered are composed of dense materials, and may be absorbing and scattering according to some anisotropic law. In the approximate analysis employed, a foil is assumed to be so thin that the flux changes are inappreciable over a small part of the foil surface ΔS , which is also so large that an average chord of the boundary is very much larger than the thickness h of the foil.

The flux density per steradian will be defined in terms of surface source densities on the foil; for the region outside the foil material the surface source density is $(n^+ \cdot v)g(xyz, v)$ where the normal n^+ is taken outward from the foil, and $g(xyz, v)$ is the limiting flux at the surface. (Cf. Case, De Hoffmann, and Placzek, Introduction to the Theory of Neutron Diffusion, 129-130.) Because of the thinness of the foil, one may collapse the surface exterior to surface element ΔS under consideration, and assume that the foil exterior to ΔS contributes a surface source density $(n^+ \cdot v)g(xyz, v)$ which is the sum, defined for a common normal direction n^+ , of the source densities for the upper and lower foil surfaces. A set of integral equations will be found to express the dependence of the source density to be contributed by the part of the foil within ΔS , on the remaining sources and source densities; the solution of this set of integral equations, by numerical calculation or otherwise, defines the flux everywhere outside the foil.

Let the normal n^+ be the direction of the z axis, on ΔS ; on the upper surface $z = h$, on the lower $z = 0$. Assume that the known sources and the surface source densities outside ΔS contribute the flux densities on the lower and upper surfaces within ΔS

$$G_1 = \sum_{rs} A_{rs} Y_{rs}(v) \quad z \leq 0$$

$$G_3 = \sum_{rs} B_{rs} Y_{rs}(v) \quad z \geq h$$

The actual flux above or below the foil within ΔS may be calculated approximately by the methods used for a stratum composed of the foil material and without boundaries in the x or y directions, for $0 < z < h$. (Cf. ORNL 55-5-2.) The total flux densities per steradian expressed in terms of eigenfunctions become, for any subscript s ,

$$H_{1s} = G_{1s} + \sum_{t,r} a_{tr} p_t \exp(p_t z) X_{rs}^t Y_{rs} \quad z \leq 0, p_t > 0$$

$$\sum_{w,u} b_{wu} k_w \exp(k_w z) Z_{us}^w Y_{us} \quad 0 < z < h \quad (1)$$

$$H_{3s} = G_{3s} + \sum_{q,m} c_{qm} p_q \exp(p_q z) X_{ms}^q Y_{ms} \quad z > h, p_q < 0$$

From the boundary condition at $z = 0$ expressing the continuity of $(n.v)$ flux one obtains

$$b_w = c_w^0 + \sum_t H_{wt}^0 a_t$$

$$H_{wt}^0 = \sum_u h_u^1 z_{us}^w x_{us}^t$$

$$c_w^0 = k_w^{-1} \sum_u z_{us}^w h_u^2 a_{us}(0)$$

where h_u^1 and h_u^2 are defined by the absorption and coefficients of the differential cross sections for scattering in the exterior region (1) and in the foil (2). The equation can be written in vector notation

$$[b] = [c^0] + H^0 [a] \quad (2)$$

The similar boundary condition at $z = h$ leads to the equation

$$[c] = [c'] + H' [b] \quad (3)$$

$$c_m' = -p_m^{-1} \exp(-p_m h) \sum_u h_u^1 x_{us}^m B_{us}$$

$$H_{mw}' = \exp((k_w - p_m)h) \sum_u h_u^2 x_{us}^m z_{us}^w$$

Hence from (2) and (3)

$$[c] = [c'] + H' [c^0] + H' H^0 [a] \quad (4)$$

In this set of equations the components c_q corresponding to positive eigenvalues p_q are zero, and the corresponding equations define the set of non-vanishing components a_t ; the non-vanishing components c_q are then defined by the remaining equations of (4).

It is clear that the method of computation is the same as if a set of sources exterior to the foil within ΔS defined a flux density everywhere in space, including the foil interior to ΔS , and then additional flux densities were added with formulas (2), (3), to make the flux crossing the upper and lower surfaces of ΔS continuous. Hence for thin dense foils approximately $G_1 = G_3$, $A_{rs} = B_{rs}$.

If $I^0(P)$ is the flux defined in the exterior region in the absence of the foil, and $\sum_{nw} f_{nm}(Q)Y_{nm}(v)$ is the source density at a point Q of S per unit surface area, let $\sum_{rs} g_{rs}(Q,P)Y_{rs}$ be the resulting flux density at P . The particular source density for which $f_{nm}(Q) = 1$, with all other components of the source at Q equal to zero, then contributes $\sum_{rs} g_{rs,nm}(Q,P)Y_{rs}$ to the flux at P . Clearly

$$\sum_{rs} g_{rs}(Q,P)Y_{rs} = \sum_{rsnw} g_{rs,nm} f_{nm}(Q)Y_{rs},$$

$$g_{rs}(Q,P) = \sum_{nm} g_{rs,nm} f_{nm}(Q).$$

$$A_{rs}(P) = \sum_{nw} \iint_{S'} g_{rs,nm}(Q,P) f_{nm}(Q) dS_Q + I_{rs}^0(P) \quad (5)$$

Now the direct unscattered flux from the nearby surface contributes nothing to the flux on S , S' denoting the surface exterior to ΔS , since the factor $(n.v)$ in the definition of surface sources makes the direct tangential flux zero at P ; hence the integrals converge when $S' = S$ above.

With the approximation $A_{rs} = B_{rs}$ on the interior of ΔS , and the coefficients a_t, c_q given by (4), one can write

$$H_{1s}(P) = \sum_m A_{ms}(P) Y_{ms} + \sum_{r,m} K_{mr}^s A_{rs}(P) Y_{ms}$$

$$H_{3s}(P) = \sum_m A_{ms}(P) Y_{ms} + \sum_{r,m} L_{mr}^s A_{rs}(P) Y_{ms}$$

Now the source densities for the uncollapsed upper and lower foil surfaces of ΔS are respectively

$$(n^-.v)H_{1s}, \quad (n^+.v)H_{3s}$$

Taking n^+ on the upper surface in the direction of the positive z axis, the sum of these source densities which represents the source density for the exterior region contributed by the part of the foil within ΔS becomes

$$\begin{aligned} \sum_{nw} f_{nm}(P) Y_{nm} &= (n^+.v)(H_3 - H_1) = (n.v) \sum_{mrs} (L_{mr}^s - K_{mr}^s) A_{rs}(P) Y_{ms}(v) \\ &\equiv \sum_{nsr} M_{nk}^s (L_{kr}^s - K_{kr}^s) A_{rs}(P) Y_{ns}(v) \\ &\quad k \end{aligned}$$

Hence

$$f_{ns}(P) = \sum_{k,r} M_{nk}^s (L_{kr}^s - K_{kr}^s) A_{rs}(P) = \sum_r T_{nr}^s A_{rs}(P) \quad (6)$$

where T_{nr}^s is a constant matrix.

From (5) there results

$$f_{ns}(P) = \sum_r T_{nr}^s \left[\sum_{uw} \iint_S \varepsilon_{rs,uw}(Q,P) f_{uw}(Q) dS_Q - I_{rs}^0(P) \right] \quad (7)$$

Air-ground Scattering

If an isotropic source is located in air above the ground at such a height that the variation in flux over the ground surface is inappreciable for a separation between two points P and Q which is considerably greater than the mean free path in the ground, - or the flux in the ground is reduced to a very small fraction of the flux at the surface, at depths small compared to the distance PQ, for which the flux remains appreciably constant, then the analysis is similar to the above. In place of equation (4) used above, equation (3) defines the flux density $H_{3s}(P)$. The effective source density in space is then $(n^+ \cdot v)H_{3s}$, and H_1 is replaced by zero in the analysis from this point on. The set of integral equations (7) is formally the same, if

$$T_{nr}^s = \sum_k M_{nk}^s (L_{kr}^s + \delta_{kr})$$

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