ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

March 15, 1958

Technical Report No. 15
(Research and Development Report)

ELECTROSTATIC EFFECTS IN THE DEPOSITION
OF AEROSOLS ON CYLINDRICAL SHAPES

By

George S. Dawkins

Contract No. AT(11-1)-276
U. S. Atomic Energy Commission

H. F. Johnstone, Technical Director

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This report is a part of the thesis submitted by George S. Dawkins to the Graduate College of the University of Illinois for the Degree of Doctor of Philosophy in Chemical Engineering, June, 1957. Mr. Dawkins was a research assistant on the Contract from September, 1956 to June, 1957.

H. F. Johnstone, Technical Director

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I. INTRODUCTION

Electrostatic effects are often important in the collection of small particles from streams of aerosols. While the maximum collection efficiency by inertial impaction on a single collector is unity, when coulombic forces exist between the particles and a collector, the efficiency may be much larger. For collection of very small particles, when a high efficiency of removal is needed some kind of electrostatic precipitation is often necessary.

Kraemer studied the properties of electrically charged aerosols and experimentally measured the deposition of aerosols on spheres caused by electrostatic effects (1). This work is an extension of Kraemer's work to electrostatic collection of small particles on cylinders. It is applicable to the precipitation of particles by charged filter mats, the collection of charged aerosols, and the electrostatic precipitation of charged particles on a rod maintained at a high electrical potential. Characteristic parameters for the various electrostatic forces were developed. These parameters show the effect of the size of the particles, the diameter of the collecting cylinder, the charge of the particles, the relative velocity between the aerosol stream and the collector, and the electric field of a cylindrical collector. The collection parameters are defined as dimensionless ratios of the electrostatic forces between the particle and the collector and the fluid resistance acting on the particle.

The equations of motion for a small particle were solved by a digital computer for the case of potential flow past a cylinder for various electrostatic forces. Graphs and equations giving efficiencies of collection as a function of the different parameters are reported.

Efficiencies of collection of dioctylphthalate aerosols on cylinders were measured. The amount collected agreed well with that predicted by theory.

The results of some brief experiments with fiber mats, mats of charged wires, and mats of tangled wire dipoles were reported.

II. HISTORICAL

Ranz discussed the forces that cause collection of small particles (2). He points out that the efficiency of collection may be expressed as a function of dimensionless characteristic parameters which are ratios of the force causing collection to the fluid resistance acting on the particle. The mechanisms of collection considered were inertial impaction, interception, settling, Brownian diffusion, thermal forces, and electrostatic forces.

Kraemer studied the collection of small particles on spheres by electrostatic attraction (3); Wilson studied electrostatic collection on the walls of tubes (4); while Lipscomb and Daniel studied electrostatic deposition on plane surfaces (5), (6).

Chen studied the efficiency of collection by a single fiber and the mutual effect of neighboring fibers in a filter mat (7).

Rossano worked on the collection of aerosols by charged mats using charged and uncharged aerosols (8). He found increased efficiencies of collection when either the aerosol or the mat was charged.

LaMer showed that the penetration of liquid aerosol particles through gas mask filters decreases as the average charge on the particle increases (9).

Lowe and Lucas discuss the reentrainment of solid particles from the standpoint of molecular adhesion of solid surfaces (10).

III. THEORETICAL

The efficiency of deposition of particles on a cylinder from a moving air stream is of interest. The trajectory of a particle can be calculated if the forces acting on it are known. If the trajectory intercepts the cylinder the particle is considered to be collected. The equations of motion for the particle were derived by equating the electrostatic forces acting on it to the forces of fluid resistance. Expressions were developed for the different electrostatic forces. The differential equations describing the motion were solved with a digital computer. Approximate solutions to the equation were developed for a few limiting cases.

A. Equations of Motion

The drag resistance of a sphere for Reynolds numbers in the range $10^{-3} < N_{Re} < .5$ is

$$\vec{F} = - \frac{3\pi\mu D_p(\vec{u} - \vec{v})}{C} \quad (1)$$

where $\vec{u}$ and $\vec{v}$ are the velocities of the particle and the air stream. C is the Cunningham correction which compensates for the molecular slip. For particles in the micron range, C can be approximated by

$$C = (1 + 1.6 \times 10^{-1} / D_p) \quad (2)$$

In an electric field and in the absence of inertial, gravitational, magnetic, and thermal forces, the equations of motion for a small particle are

$$\frac{dr}{dt} = V_r + \frac{C F_r}{3\pi\mu D_p}$$

(3)

$$r \frac{d\theta}{dt} = V_\theta + \frac{C F_\theta}{3\pi\mu D_p}$$

F_r and F_θ are the radial and angular components of the electrostatic forces acting between the particle and the collector. Expressions for v_r and v_θ can be derived from equations for potential flow around a cylinder (11), (12). The velocity components in polar coordinates are

$$V_r = \frac{dr}{dt} = V_0 \left(\frac{\tilde{r}^2 - 1}{\tilde{r}^2} \right) \cos \theta$$

$$V_\theta = \tilde{r} \frac{d\theta}{dt} = -V_0 \left(\frac{\tilde{r}^2 + 1}{\tilde{r}^2} \right) \sin \theta \quad (4)$$

In rectangular coordinates they are

$$V_x = V_0 \left\{ 1 - \frac{(\tilde{x}^2 - \tilde{y}^2)}{(\tilde{x}^2 + \tilde{y}^2)^2} \right\} \quad (5)$$

$$V_y = V_0 \frac{2\tilde{x}\tilde{y}}{(\tilde{x}^2 + \tilde{y}^2)^2}$$

Combining Equations 3 and 4 and dividing by v_0 to make the equations dimensionless gives

$$\frac{d\tilde{r}}{d\tilde{t}} = \frac{(\tilde{r}^2 - 1)}{\tilde{r}^2} \cos \theta + \frac{C F_r}{3\pi\mu D_p V_0} \quad (6)$$

$$\frac{d\theta}{d\tilde{t}} = -\left(\frac{\tilde{r}^2 + 1}{\tilde{r}^3} \right) \sin \theta + \frac{C F_\theta}{3\pi\mu D_p V_0}$$

For viscous flow around a cylinder the theoretical collection efficiencies are slightly lower than those derived from the equations for potential flow. This is

because of increased upstream separation by the collector. Detailed studies of flow around a cylinder have been published by Davies and Peetz (13).

B. Definition of Collection Efficiency

Equations 6 are first order and single valued and the trajectories do not cross. There is a limiting trajectory for a particle starting at $x = \infty$. If the particle starts at a distance greater than y^* it will not be stopped. See Figure 1.

The efficiency of collection is defined as the ratio of the area of the stream from which all particles are removed to the projected cross-sectional area of the collector. For a cylinder, the efficiency is

$$\eta = \frac{2y^*}{D_c} \quad (7)$$

It is assumed that there is no turbulence or other stochastic diffusional process present, and that the forces are described by Equation 6.

This representation is valid when the magnitude of random motion is small compared to the collector diameter.

C. Fluid Streamlines in a Duct

Cornish gives the velocity profile for fluid flowing in a duct (14).

$$V = V_0 \left[\frac{1 - \frac{y^2}{a^2} - \frac{32}{\pi^3} \left\{ \frac{\cosh(\pi x/2a)}{\cosh(\pi b/2a)} \cos\left(\frac{\pi y}{2a}\right) - \frac{1}{3^3} \frac{\cosh(3\pi x/2a)}{\cosh(3\pi b/2a)} \cos\left(\frac{3\pi y}{2a}\right) + \dots \right\}}{1 - \frac{32}{\pi^3} \left\{ \frac{1}{\cosh(\pi b/2a)} - \frac{1}{3^3 \cosh(3\pi b/2a)} + \dots \right\}} \right] \quad (8)$$

2a and 2b are the height and width of the duct.

FIG.1. PARTICLE TRAJECTORIES NEAR A CYLINDRICAL OBSTACLE WITH ELECTROSTATIC FORCES

Fig. 2. Diagram of Duct.

For positions near the center of the duct the flow is nearly parabolic and can be described by the usual parabolic equation

$$V = V_0 \left(1 - y^2/a^2 \right) \quad (9)$$

D. Electrostatic Forces

The aerosol particles are considered to be spheres. For small particles the charge is spread uniformly over the surface. A sphere with a charge uniformly distributed on the surface is equivalent to a point charge of equal magnitude located at the center.

The presence of other particles has little effect on the attraction between one particle and a cylinder. Debye (15) has shown that a pseudo-dielectric constant may be used for a cloud of conducting particles in a gas. This is defined by

$$\frac{(\epsilon'' - 1)}{(\epsilon'' + 2)} = \frac{\pi N_p D_p^3}{6} \quad (10)$$

For aerosol particles, a concentration of 10^5 particles per cc. and a diameter of 10^{-4} cm. is reasonable. This gives a value of 10^{-7} , for $\frac{\pi}{6} N_p D_p^3$ and ϵ'' is close to 1. For ϵ'' equal to 1 there is no influence of the surrounding particles.

The overall electrostatic force between the particle and the collector is the sum of the following electrostatic forces.

1. When the collector and the particle are charged there is a coulombic force. The charge on a cylinder may be considered as a line charge. The equation for the coulombic force between a line charge and a point charge is shown in Appendix A3.

2. If the aerosol particles are charged and the cylinder is at a constant voltage the particles induce a line charge of opposite sign in the collecting cylinder. The magnitude of the line charge may be approximated by considering the collector to be surrounded by an atmosphere of charged particles forming a cylinder of radius R_0 . An expression for this force is derived in Appendix A9, Equation 22A.

3. The line charge induces a dipole in dielectric particles. The attraction between the induced dipole and the line charge is shown in Appendix A6.

4. The charged particles also form an image in the collecting cylinder. The maximum attraction between the image and a point charge is shown in Appendix A7.

5. There is a small force on the particles acting towards the collector resulting from the space occupied by the collector in the atmosphere of charged particles. An estimate of this force is shown in Appendix A8.

6. There is also the force of the space charge caused by the surrounding aerosol particles acting from the center towards the outer boundary. This force, which may be significant, depends on the presence of nearby grounded surfaces. The equation for the space charge repulsion for concentric cylinders is derived in Appendix A9. The equation for the effect of space charge repulsion on the efficiency of collection by a cylinder in a duct is derived in Appendix A10.

E. Equation of Motion for a Particle Near a Cylinder Under the Influence of Electrostatic Forces

Combining all the terms for the electrostatic forces except that of the space charge, which will be considered as a special case, the total force acting between the individual particles and the collector becomes

$$\begin{aligned} \vec{F}_r = & - \frac{Q_p \rho (R_0^2 - r_c^2)}{4 \epsilon_0 \ln(R_0/r_c)} \frac{1}{r} + \frac{Q_p V}{\ln(R_0/r_c)} \frac{1}{r} - \frac{\rho D_c^2 Q_p}{8 \epsilon_0 r} \\ & - \frac{(1 - \epsilon') Q_p^2}{(1 + \epsilon') 16 \pi \epsilon_0 (r - D_c/2)^2} - \frac{(\epsilon - 1) D_p^3 Q_p^2}{(\epsilon + 2) 8 \pi \epsilon_0 r^3} \end{aligned} \quad (11)$$

The electrostatic forces can be written in terms of dimensionless collection parameters and distances. Combining Equations 6 and 11 gives

$$\begin{aligned} \frac{d\tilde{r}}{d\tilde{t}} &= \frac{(\tilde{r}^2 - 1)}{\tilde{r}^2} \cos \theta + \frac{K_E}{\tilde{r}} - \frac{K_I}{\tilde{r}^3} - \frac{K_M}{(\tilde{r} - 1)^2} - \frac{K_G}{\tilde{r}} - \frac{K_S}{\tilde{r}} \\ \frac{d\theta}{d\tilde{t}} &= - \left(\frac{\tilde{r}^2 + 1}{\tilde{r}^3} \right) \sin \theta \end{aligned} \quad (12)$$

The collection parameters are defined as follows:

$$\begin{aligned} K_E &= \frac{(Q_p/D_p)_{ma} Q_d 2C}{2\pi \epsilon_0 D_c 3\pi \mu V_0} \\ K_I &= \frac{(\epsilon - 1) D_{KI}^2 Q_p^2 8C}{(\epsilon + 2) 8\pi \epsilon_0 D_c^3 3\pi \mu V_0} \\ K_M &= \frac{(1 - \epsilon') Q_p (Q_p/D_p)_{ma} 4C}{(1 + \epsilon') 16\pi \epsilon_0 D_c^2 3\pi \mu V_0} \\ K_G &= \frac{\rho (Q_p/D_p)_{ma} (R_0^2 - r_c^2) 2C}{4\epsilon_0 \ln(R_0/r_c) D_c 3\pi \mu V_0} \\ K_S &= \frac{\rho (Q_p/D_p)_{ma} D_c 2C}{8\epsilon_0 3\pi \mu V_0} \end{aligned} \quad (13)$$

The parameter K_E can be evaluated if the charge per unit length, Q_1 is known. An equation relating Q_1 to the voltage and geometry of the system is shown in Appendix A2 for the case of concentric cylinders. Q_1 must also be known in order to evaluate K_I . If the aerosol does not surround the cylinder uniformly, K_G can be approximated by using a suitable radius, R_0 , that represents a reasonable outer boundary for the aerosol cloud.

The correct averages of D_p and Q_p for the collection parameters are shown in Appendix C.

The efficiency of collection by a cylinder can be expressed as a function of the characteristic parameters. Approximate solutions of Equation 12 for K_E and K_I are given in Appendix B.

Equation 12 was solved by a digital computer for selected values of the collection parameters. The method used is discussed in Appendix D. The solutions for the different cases are shown in Appendix E and the predicted efficiencies as shown in Figures 3, 4, 5, and 6.

F. Effect of Neighboring Collectors

For the collection of aerosol particles by a group of cylinders, the mutual effect of the cylinders on each other is important. If a cylinder is at the center of a group of cylinders at the same charge there is no resultant field and therefore there can be no coulombic forces. The central cylinder is similar to a cylinder at constant voltage surrounded by a concentric cylinder at the same voltage. Equation 17A shows that the coulombic force is zero for this case.

Kraemer and Johnstone suggest that a mat of a tangled wire dipole should have a high collection efficiency (16). However, if the dipole wires are arranged

randomly, the resultant field of the mat is zero and the only field affecting the collection is that between the two wires forming the dipole.

FIG. 3. PREDICTED EFFICIENCIES OF COLLECTION OF CHARGED AEROSOL PARTICLES BY A CHARGED CYLINDRICAL COLLECTOR

FIG. 4, PREDICTED EFFICIENCY OF COLLECTION OF UNCHARGED AEROSOL PARTICLES BY A CHARGED CYLINDRICAL COLLECTOR

FIG. 5. PREDICTED EFFICIENCY OF COLLECTION OF CHARGED AEROSOL PARTICLES BY AN UNCHARGED CYLINDRICAL COLLECTOR

FIG.6. PREDICTED EFFICIENCY OF COLLECTION OF AEROSOL PARTICLES ON CYLINDER BY INTERCEPTION, $\eta = (1+R) - 1/(1+R)$

IV. EXPERIMENTAL

A. Equipment and Procedure

A flow diagram of the experimental apparatus is shown in Figure 7. A LaMer-Sinclair-type generator was used to produce a homogeneous dioctylphthalate aerosol (17). The size distribution of the particles is given in Table 3A.

The aerosol particles were charged by the corona device shown in Figure 8. With care, no more than 10 percent of the aerosol was precipitated during the charging. The central wire was positive.

The charge on the particles was measured with the charge spectrometer developed by Kraemer (18), which is shown in Figure 9, and also with a Millikan cell. The results obtained by the two methods agreed closely.

The ratio of charge-to-diameter for the particles was found by measuring the deflection of the aerosol stream in an electrostatic field. A parabolic velocity profile was assumed and calculations of Q_p/D_p were made according to the theory developed by Kraemer. The aerosol velocity was adjusted to that of the air stream at the center of the duct.

The geometric standard deviation of the charge distribution was obtained from the Millikan cell measurements. The naturally occurring charge on the aerosol particles was also measured by the Millikan cell. From 2 to 30 percent of the particles had a natural charge of one or more electrons, the larger particles having the higher charge. About 50 percent of the particles had a positive charge.

FIG. 7. FLOW DIAGRAM OF EXPERIMENTAL EQUIPMENT

NICHROME
WIRE .0035" DIA.

BRASS CYLINDER
0.435" I.D.

2-56 M.S.

$\frac{28}{15}$ BALL
JOINT

WIRE

GASKET

SUPPORT FOR
COAXIAL CABLE

GASKET

SPRING

GASKET

AEROSOL
OUT

10-32 M.S.

0.3"

FIG. 8. DEVICE FOR CHARGING AEROSOLS

338 025

FIG. 9. CHARGE SPECTROMETER

A high-velocity cascade impactor and a Millikan cell were used to determine the particle size distribution. An average value for D_p was used in evaluating the collection parameters. A light scattering device known as the "owl", which is described by Sinclair (19), was used to check the particle size during each run.

The average particle size was obtained with the four-stage cascade impactor described by Ranz and Wong (20). The dimensions of the jets used were as follows:

Jet Number	Length cm.	Area sq. cm.	V_0 cm./sec.	θ microns
1	.0711	.0711	2340	1.66
2	.0526	.0526	3170	1.22
3	.0273	.0341	4890	0.71
4	.0205	.0256	6510	0.53

The average diameters obtained by the impactor agreed with those obtained by measuring the velocity of fall for 50 particles in the Millikan cell. The results from the Millikan cell were used to obtain the geometric standard deviation of the particle size distribution.

The concentration of particles in the aerosol stream was found by passing a measured volume through the electrostatic precipitator shown in Figure 10. The collection was quantitative when the corona current of the precipitator was 90 microamperes.

The collecting cylinder was a stainless steel rod, 6 inches long, mounted in a Lucite frame between two brass plates which were grounded as shown in Figure 11.

The potential on the cylinder was maintained by a 0-to 20-kv. power supply. The amount of aerosol deposited per unit length of the cylinder was found by removing the deposit from the center with alcohol and analyzing the solution by means of an ultraviolet electrospectrometer.

FIG.10. ELECTROSTATIC PRECIPITATOR FOR SAMPLING AEROSOLS

FIG. II. COLLECTING CYLINDER AND SUPPORT

The glass fiber mats were air-formed unbonded mats obtained from Glass Fibers, Inc., of Toledo, Ohio. The size distribution of the glass fibers is reported by Chen (21). The glass fiber used had a true density of 2.5 g./cc., a geometric mean diameter of 0.95 microns, and a geometric standard deviation of 2.1. The holder for the filter mat was the one described by Chen.

The tangled-wire mats were made from 10-mil enameled copper wire. The mats were grounded, or kept at a positive or negative potential. The tangled-wire dipole mats were made by twisting together equal lengths of 10-mil enameled copper wire and forming the dipole into a mat.

The mats were 1 1/2 inches deep and were placed in a 24-mm. glass tube and held in place by friction only. The collection efficiency was determined by measuring the concentration of the aerosol stream before and after the mats.

B. Sample Calculations

The following is a complete set of calculations for Run 75.

1. Cunningham Correction Factor, C

$$C = 1 + 1.6 \times 10^{-1} / D_p = 1 + .16 / .56 = 1.29$$

2. Centerline Velocity, v_o

The velocity at the center of the duct was calculated by assuming a parabolic velocity profile for which the velocity at the center is 1.5 times the average velocity. The area of the duct was 14.1 sq. cm.

$$v_o = \frac{1.5 \text{ g} \times 1000}{60 \times 14.1} = 1.77 \text{ g} = 4.5 \text{ cm/sec.}$$

3. Efficiency of Collection, η

The rate of aerosol deposition on the collector, W_c , is

$$W_c = \int_0^{y^*} N_p v_y \times dy = N_p \times \int_0^{y^*} v_o (1 - y^2/b^2) dy \quad (14)$$

The total flow of aerosol in the center section of the duct is

$$W_T = \int_0^b N_p \times v_y dy = N_p \times \int_0^b v_o (1 - y^2/b^2) dy \quad (15)$$

Combining Equations 14 and 15 and substituting $y^* = \frac{\eta D_c}{2}$ gives

$$\frac{W_c}{W_T} = \frac{N_p v_o \times [\eta D_c/2 - \eta^3 D_c^3/24 b^2]}{N_p v_o \times [2b/3]} = \frac{3}{4} \frac{\eta D_c}{b} - \frac{\eta^3 D_c^3}{16 b^3} \quad (16)$$

The efficiency of collection is calculated from

$$\frac{\eta^3}{2} \left(\frac{D_c}{2b} \right)^3 - \frac{3\eta}{2} \left(\frac{D_c}{2b} \right) + \frac{W_c}{W_T} = 0 \quad (17)$$

The assumption of a parabolic velocity profile is necessary when the efficiency is high. For low efficiencies, Equation 17 reduces to

$$\eta = \frac{2 W_c}{3 W_T} \left(\frac{26}{D_c} \right) \quad (18)$$

For Run 75, solution of Equation 17 gives $\eta = 0.553$.

4. Average Diameters

The correct average diameters to be used for the collection parameters are discussed in Appendix C.

$$D_g = (D_{g \text{ cascade impactor}} + D_{g \text{ Millikan cell}}) / 2 = (0.56 + 0.54) / 2 = 0.55 \text{ microns}$$

$$D_n = D_g \exp(0.5 \ln^2 \sigma_g) = 0.55 \exp(0.5 \times 0.033) = 0.56 \text{ microns}$$

$$D_w = D_g \exp(3.5 \ln^2 \sigma_g) = 0.55 \exp(3.5 \times 0.033) = 0.62 \text{ microns}$$

$$D_{KI} = D_g \exp(4.0 \ln^2 \sigma_g) = 0.55 \exp(4.0 \times 0.033) = 0.63 \text{ microns}$$

$$\begin{aligned} (Q_P/D_P)_{ma} &= (Q_P/D_P)_n \exp\left(\frac{1}{2} \ln^2 \sigma_g + 3 \rho_r \ln \sigma_g' \ln \sigma_g\right) \\ &= (56.7/0.56) \exp\left(\frac{1}{2} (.122)^2 + 3 \times .8 \times .122 \times .182\right) = 108 \frac{\text{electrons}}{\text{micron}} \end{aligned}$$

5. Collection Parameters

The collection parameters were calculated from Equation 13. The dielectric constant of dioctylphthalate is 5.4, which is the value reported by Kraemer (22) from measurements made by L. A. Knecht at the University of Illinois. The collecting cylinder was considered to be surrounded by a concentric grounded cylinder of radius $R_0 = b\sqrt{2} = 1.57 \text{ cm}$. The conversion factors used were 300 volts per esu/cm, 10^4 microns per centimeter, 3×10^9 esu per coulomb, and 4.8×10^{-10} esu per electron. The density of dioctylphthalate is 0.983 grams per cc.

$$K_E = \frac{2C(QP/OP)_{ma} V}{3\pi \mu V_0 D_c \ln(2R_0/D_c)} = \frac{2 \times 1.29 \times 108 \times (-300) \times 4.8 \times 10^{-10} \times 10^4}{3\pi \times 1.8 \times 10^{-4} \times 4.5 \times .318 \times \ln(7\sqrt{2}) \times 300} = -0.241$$

$$K_I = \frac{4C\epsilon_0 D_{II}^2 (\epsilon-1) V^2}{3\mu V_0 D_c^2 (\epsilon+2) \{\ln(2R_0/D_c)\}^2} = \frac{4 \times 1.29 \times 8.85 \times 10^{-21} \times (.63)^2 \times 4.4 \times (-300)^2 \times 10^{-8} \times 9 \times 10^{18}}{3 \times 1.8 \times 10^{-4} \times 4.5 \times (.318)^3 \times 7.4 \times \{\ln 7\sqrt{2}\}^2 \times 9 \times 10^4} = 2.33 \times 10^{-6}$$

$$\rho = \frac{W_T Q_P \times 6 \times 10^{12} \times 4.8 \times 10^{-10} \times 10^{-6}}{8 \times 10^3 \times \pi D_w^3 \rho_P \times 3 \times 10^9} = \frac{90 \times 56.7 \times 6 \times 10^{12} \times 4.8 \times 10^{-10} \times 10^{-6}}{2.54 \times 10^3 \times \pi \times (.62)^3 \times .986 \times 3 \times 10^9} = 2.61 \times 10^{-12} \frac{\text{conl.}}{\text{cc.}}$$

$$K_G = \frac{C\rho(QP/OP)_{ma} [R_0^2 - r_c^2]}{6\pi \mu V_0 D_c \epsilon_0 \ln(2R_0/D_0)} = \frac{1.29 \times 2.61 \times 10^{-12} \times 108 [1.57^2 - (.159)^2] \times 10^4 \times 4.8 \times 10^{-10}}{6\pi \times 1.8 \times 10^{-4} \times 4.5 \times .318 \times \ln 7\sqrt{2} \times 3 \times 10^9 \times 8.85 \times 10^{-21}} = 1.49 \times 10^{-2}$$

$$K_P = \frac{C D_c \rho (QP/OP)_{ma}}{2 \epsilon_0 3\pi \mu V_0} = \frac{1.29 \times .318 \times 2.61 \times 10^{-12} \times 108 \times 10^4 \times 4.8 \times 10^{-10}}{4.5 \times 3 \times \pi \times 1.8 \times 10^{-4} \times 2 \times 8.85 \times 10^{-21} \times 3 \times 10^9} = 1.37 \times 10^{-3}$$

$$\exp(\Delta \bar{x} K_P) = \exp\left(\frac{9 \times 2.54 \times 2}{.318} \times 1.37 \times 10^{-3}\right) = \exp(.198) = 1.22$$

$$R = D_w/D_c = (.62/.318) \times 10^{-4} = 1.94 \times 10^{-4}$$

$$\Sigma K_i / \exp(K_P \Delta \bar{x}) = \frac{(-K_E + 1.01 K_I + K_G + K_S + \pi R)}{\exp(K_P \Delta \bar{x})} = \frac{.241 + .015}{1.22} = 0.211$$

C. Results

1. Collection of Aerosol Particles by Single Cylinders

The results of the experiments on aerosol collection on single cylinders are shown in Figure 12. The numerical solution of Equation 12 with the digital computer showed that, for values of K_I and R less than 0.1, if K_I was multiplied by 1.01 and R was multiplied by $\pi/2$, the efficiencies of collection would be the same. For this reason, the parameters were multiplied by these factors and the data were plotted against the combined collection parameters. The parameter K_M was too small to influence the efficiency of collection.

2. Collection of Aerosol Particles by Fiber Mats

The efficiency of collection by the fiber mats as a function of the time the mat had been used is shown in Figures 13, 14, 15, and 16.

3. Collection of Aerosol Particles by Wire Mats

The efficiency of tangled wire mats shown in Table 5 and Figure 17 as a function of the potential on the mat.

The efficiency of tangled wire dipole mat is shown in Figure 18 as a function of the dipole voltage.

FIG.12. COLLECTION OF AEROSOL PARTICLES ON CYLINDRICAL COLLECTORS

FIG. 13. COLLECTION OF AEROSOLS ON GLASS FIBER MATS

FIG. 14. COLLECTION OF AEROSOLS ON GLASS FIBER MATS

FIG. 15. COLLECTION OF AEROSOLS ON GLASS FIBER MATS

FIG. 16. COLLECTION OF AEROSOLS ON GLASS FIBER MATS

FIG. 17. COLLECTION OF AEROSOLS ON TANGLED WIRE MATS

FIG. 18. COLLECTION OF AEROSOLS ON TANGLED WIRE DIPOLE MATS

V. DISCUSSION AND CONCLUSIONS

A. Theoretical Treatment

Exact solutions of Equation 12 were obtained by the computer for the cases of potential flow around a cylinder for the coulombic, induced dipole, and image effects.

For the experimental conditions used, when K_E was less than 1.0, the coulombic attraction for the particles by a cylinder in a duct was found to be the same as the coulombic attraction for a cylinder in free space.

For the parameter K_E , the approximate solution of Equation 12 shown in Appendix C agrees with the solution given by the computer; for the parameter K_I , Figure 4 shows that the approximate equation and the computer solution are the same only for values of K_I larger than 30.

The validity of combining the collection parameters to calculate the efficiency of collection was verified. The efficiency calculated by combining K_E and K_I of approximately equal values is about six percent higher than the theory predicts. The calculated efficiencies for selected combinations of the collection parameters are given in Appendix E1.

B. Collection of Aerosol Particles by Single Cylinders

The experimental results agree closely with the theory for the deposition of aerosols on a single cylinder. Figure 12 shows that the data fall close to the predicted line.

The accuracy of measurements was poorest for the uncharged aerosols because of the error in determining the natural charge and the deviation of the data was the greatest in this case.

Several runs were made with the collecting cylinder not grounded. For these, it was not possible to evaluate the collection parameters but the efficiencies were generally much lower than those obtained for runs made under the same conditions with the cylinder grounded. This shows the importance of the parameter K_G .

There was no collection on the cylinder when it was charged with the same polarity as the aerosol. The results for Runs 69-111 are the most consistent, as the values of the parameters were determined most accurately for these runs and the conditions of collection were such that the analytical errors were smallest.

C. Collection of Aerosol Particles by Fiber Mats

1. Uncharged Aerosols

Figures 13, 14, and 15 show that the collection efficiency of uncharged aerosols by filter mats decreased at first and eventually increased as the mats became clogged. This is contrary to the results of Blasewitz and Judson who found that the amount of methylene blue aerosols removed by glass fiber mats increases with time (23).

2. Charged Aerosols

In this case the collection efficiency of the filter mats increased rapidly (Figures 13, 15, and 16). This is probably caused by the space charge built up on the mat by the charged particles which tends to repel the particles from the center of the space charge towards the wall of the mat holder. Some precipitation on the walls was noticed in these runs.

In the filtration of charged aerosol particles, there are two opposing effects. The fibers acquire the same charge as the aerosol and thus tend to repel the

particles, but the space charge repels the aerosol towards nearby grounded surfaces and enhances the collection.

For the experimental conditions used, the effect of the space charge seems to be the most important. Figure 16, however, shows the results of one run in which the efficiency of collection of the charged aerosol decreases slightly after an initial increase.

The importance of electrostatic charges on the efficiency of filtration by bag filters has been observed in industrial installations. It has been reported that filter bags made with conducting fibers give higher efficiencies for the filtration of charged particles than those in which all of the fibers are made of dielectric material.

3. Tangled Wire Mats

For this case the highest efficiencies were obtained when the filter mat was grounded (Figure 17). The electrostatic field of the mat was weak and thus the coulombic forces were small.

4. Tangled Wire Dipole Mats

For this case the efficiency of the mats improved slowly as the voltage on the dipole was increased (Figure 18). The random positioning of the tangled wire dipole resulted in a weak electrostatic field.

D. Considerations for Industrial Aerosol Collectors

A strong electrostatic field is of prime importance in promoting electrostatic deposition of aerosols. There are several different ways of developing a strong electrostatic field. The high efficiency of collection reported for resin-wool filters is caused by the electrostatic field formed by the charged resin particles and the oppositely charged wool fibers (24).

Screens of alternately positive and negative wires should produce a strong field. A three dimensional array of cylinders, each surrounded by four cylinders of opposite polarity should give high efficiencies of collection for charged aerosol. Grounded or charged wires placed in filter mats may increase the efficiency of the mats.

In all equipment depending on electrostatic mechanisms for aerosol removal, the time that the particles are in the electrostatic field is important, as the collection of the particles is essentially by diffusion and the particles should remain in the electric field for finite intervals.

VI. SUMMARY

A basic study of the mechanism of deposition of small particles on cylindrical collectors in the presence of electrostatic forces has been made.

Theoretical equations predicting the efficiency of collection have been derived and solved with a digital computer. The solution of these equations are shown in Figures 3, 4, 5, and 6.

The deposition of liquid aerosol particles on cylindrical collectors was measured experimentally for a range of electrostatic conditions. The experimental results agree with the theory.

An experimental study was made of the effect of electrical charges on aerosol particles on the filtration efficiency of glass fiber mats.

Experiments were also made on the effect of placing a potential on a mat of fine wire and on filtration by a mat of a tangled wire dipole.

APPENDIX A. THE SOLUTIONS OF SOME ELECTROSTATIC PROBLEMS

1. Field of a Point Charge

The electrostatic field created by a point charge is

$$\vec{E}_r = Q_p / 4 \pi \epsilon_0 r \quad (1A)$$

2. Field of a Line Charge

A charged cylinder of infinite length acts as if all of the charge is located at the axis. The cylinder is described by a line charge of strength Q_l per unit length. The radial field strength caused by the line charge is

$$\vec{E}_r = Q_l / 2 \pi \epsilon_0 r \quad (2A)$$

Q_l depends on the voltage and the geometry of the system. For concentric cylinders, Q_l is given by

$$Q_l = 2 \pi \epsilon_0 V / \ln(2 R_o / D_c) \quad (3A)$$

3. Coulombic Attraction Between a Point Charge and a Line Charge

For a point charge Q_p at a distance r from the axis of a cylinder of infinite length with charge Q_l per unit length the coulombic force is

$$\vec{F}_r = \frac{Q_p Q_l}{2 \pi \epsilon_0 r} \quad (4A)$$

$$\vec{F}_\theta = 0$$

4. Image Force Caused by a Point Charge Near a Conducting Plane

The point charge induces an image at an equal distance from the plane. The image is equal to the point charge in magnitude and is of opposite sign. Thus the force of attraction is the coulombic force between the two point charges

$$\vec{F}_r = -\frac{Q_p^2}{4\pi\epsilon_0 r^2} \quad (5A)$$

5. Image Force Due to a Point Charge Near a Dielectric Plane

A point charge near a dielectric plane with a dielectric constant ϵ' induces a charge of magnitude $Q_p \frac{(1-\epsilon')}{(1+\epsilon')}$ at a distance equal to the point charge from the plane. The coulombic force of attraction between the point charge and its image is

$$\vec{F}_r = -\frac{(1-\epsilon') Q_p^2}{(1+\epsilon') 4\pi\epsilon_0 r^2} \quad (6A)$$

6. Force Due to the Image of a Line Charge in a Dielectric Sphere

A line charge near a dielectric sphere induces a dipole in the sphere. The dipole is oriented in the radial direction and the force in the θ direction is zero. Harnwell gives an expression for the moment of the dipole induced in a dielectric sphere by a line charge (25),

$$\vec{p} = \frac{\pi\epsilon_0(\epsilon-1) D_p^3 \vec{E}_r}{2(\epsilon+2)} \quad (7A)$$

The force due to a dipole in an electric field is given by

$$\vec{F} = \vec{p} \text{ grad } \vec{E} \quad (8A)$$

$\frac{\partial E}{\partial \theta}$ and $\frac{\partial E}{\partial z}$ are zero, and

$$\frac{\partial}{\partial r} \left(\frac{Q_l}{2\pi\epsilon_0 r} \right) = -\frac{Q_l}{2\pi\epsilon_0 r^2} \quad (9A)$$

Combining Equations 7A, 8A, and 9A gives

$$\vec{F}_r = -\frac{(\epsilon-1) D_p^3 Q_l^2}{(\epsilon+2) 8\pi\epsilon_0 r^3} \quad (10A)$$

7. Attraction Due to the Image of a Point Charge in a Cylinder

This is a non-symmetrical case and there is no simple analytic solution. Weber (26) and Smythe (27) give solutions in terms of harmonic Fourier series and Bessel functions. The theory of images cannot be used here with accuracy as in the case of a point charge and a sphere because of the asymmetry. The force of attraction will always be less than that of the image of a point charge in a plane. Thus, as an approximation, the collecting cylinder can be considered a plane. This is accurate only when the aerosol particle is near the surface of the collector and when the collector is much larger than the particle. Since the attractive force is proportional to $1/r^2$ it is usually small in comparison with other forces unless the particle is close to the cylinder. When the ratio D_p/D_c is small this is a good approximation. If the force is small in comparison to the other forces there is no effect of the image force on the collection of charged aerosol particles.

The image charge is located at a distance of $2(r - D_c/2)$ from the point charge. From Equation 6A the attractive force therefore will be

$$\vec{F}_r = - \frac{(1 - \epsilon') Q_p^2}{(1 + \epsilon') 16 \pi \epsilon_0 (r - D_c/2)^2} \quad (11A)$$

8. Attraction by Region Occupied by the Collector

The attraction of the particle towards the region of zero charge density caused by the presence of the collector is a force of minor importance. The magnitude of the force can be estimated from the coulombic attraction between the

particle and a line charge generated by filling the collector with oppositely charged particles of the same charge density as the aerosol. Thus, the force is

$$\vec{F}_r = - \frac{\rho D_c^2 Q_p}{8 \epsilon_0 r} \quad (12A)$$

9. Concentric Cylinders

A useful case for which the electrostatic field can be found is that of concentric cylinders. Many actual cases can be approximated by this model. Consider a cylinder of diameter D_c surrounded uniformly by a large number of small spherical particles. These in turn are surrounded by a large grounded cylinder of radius R_o .

To obtain the force that acts on an individual particle the electrostatic field at the position of the particle must be known. In the model, the space between the cylinders can be considered at a constant charge density since it is assumed that the particles are uniformly distributed and homogeneously charged.

The voltage of the system must obey Poisson's law.

$$\nabla^2 V = - \frac{\rho}{\epsilon_0} \quad (13A)$$

Writing the equation in cylindrical coordinates and noting that all terms in θ are zero because of the symmetry, we have

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\rho}{\epsilon_0} = 0 \quad (14A)$$

This is a nonhomogeneous linear differential equation with variable coefficients.

Since it is linear, particular solutions can be superimposed to give a general solution.

Using the substitution, $P = \frac{dv}{dr}$, gives

$$r \frac{dP}{dr} + P = - \frac{r\rho}{\epsilon_0} \quad (15A)$$

which is a first order linear equation. For the homogeneous equation

$r \frac{dP}{dr} + P = 0$ we get the solution, $P = \frac{B}{r}$, where B is the integration constant.

A particular solution which satisfies the inhomogeneous case is, $P = - \frac{\rho r}{2\epsilon_0}$.

These two solutions can be combined to give a general solution since Equation 15A is linear.

$$P = \frac{dv}{dr} = \frac{B}{r} - \frac{\rho r}{2\epsilon_0} \quad (16A)$$

which gives on integration

$$V = - \frac{\rho r^2}{4\epsilon_0} + B \ln r + G \quad (17A)$$

To find the value of B, boundary conditions must be imposed. When the outer cylinder is at zero potential and the collector is charged to a voltage V_1

Equation 17A becomes

$$V_1 = - \frac{\rho r_c^2}{4\epsilon_0} + B \ln r_c + G \quad (18A)$$

$$0 = - \frac{\rho R_o^2}{4\epsilon_0} + B \ln R_o + G$$

and B is given by

$$B = \left[- \frac{\rho (R_o^2 - r_c^2)}{4\epsilon_0} + V_1 \right] / \ln(r_c/R_o) \quad (19A)$$

The electric field at any point is

$$\vec{E} = - \vec{\nabla} V = - \frac{dV}{dr} \quad (20A)$$

and the force on a point charge is

$$\vec{F} = \vec{E} \cdot Q_p \quad (21A)$$

Solving for F gives

$$\vec{F}_r = Q_p \left[\frac{r\rho}{2\epsilon_0} - \frac{\rho (R_o^2 - r_c^2)}{4\epsilon_0 \ln(R_o/r_c)} \frac{1}{r} + \frac{V_1}{\ln(R_o/r_c)} \frac{1}{r} \right] \quad (22A)$$

The first term of the equation is due to the repulsion of the space charge of the aerosol. The force is directed radially towards the grounded shell. The second term is the attraction between the particle and the line charge which the charged particles induce in the cylinder. The third term is the force of coulombic attraction between the cylinder and the charged particles. When the collector is charged, this term usually predominates.

9. Effect of Space Charge Under the Conditions Used in the Experimental Work

When the aerosol is charged, the particles are repelled towards the grounded surfaces. The force of repulsion is zero at the center of the space charge and is a maximum near the walls of the container.

Consider a duct to be filled uniformly with charged aerosol particles of charge density ρ . There will be a force of repulsion towards the walls of the duct. The repulsive force will be zero at the center of the duct because of the symmetry. Consider an aerosol particle at the distance y from the center of the duct. The electrostatic field at the position of the particle is caused by the charged aerosol in the region between the center of the duct and the particle.

The total flux Ψ is

$$\Psi = 4\pi Q_T = 4\pi \rho y A \quad (23A)$$

and the field at any point is

$$\vec{E} = \Psi / 4\pi \epsilon_0 A \quad (24A)$$

Combining Equations 23A and 24A gives

$$\vec{E} = \frac{\rho y}{\epsilon_0} \quad (25A)$$

$$\vec{F}_y = \frac{Q_p \rho y}{\epsilon_0} \quad (26A)$$

A particle near the axis moves through the duct at a uniform velocity. The deflection caused by the space charge in the distance, $\Delta \tilde{x}$, may be found by writing the equations of motion in dimensionless terms. The stream is considered to be flowing toward the collecting cylinder from the negative $\tilde{x}$ direction.

$$\begin{aligned} \frac{d\tilde{x}}{d\tilde{t}} &= 1 \\ \frac{d\tilde{y}}{d\tilde{t}} &= \frac{dy}{v_0 dt} = \frac{c}{3\pi\mu D_p v_0} \frac{\tilde{y} D_c}{2} \frac{\rho Q_p}{\epsilon_0} = K_p \tilde{y} \end{aligned} \quad (27A)$$

Combining the two equations gives

$$\frac{d\tilde{y}}{d\tilde{x}} = K_p \tilde{y} \quad (28A)$$

and solving for the deflection gives

$$\ln \frac{\tilde{y}_2}{\tilde{y}_1} = K_p (\tilde{x}_2 - \tilde{x}_1) = K_p \Delta \tilde{x} \quad (29A)$$

or

$$\tilde{y}_2 = \tilde{y}_1 \exp(K_p \Delta \tilde{x}) \quad (30A)$$

In addition to the other forces, the space charge acts as a force and causes a deflection. A particle originally at $\tilde{y}_1$ would be at $\tilde{y}_2$ when it approaches the collector, if $\Delta \tilde{x}$ is the length of the duct. Since η is defined as $\tilde{y}_1$ for a cylinder

$$\eta_{\text{theoretical}} = \eta_{\text{measured}} \exp(K_p \Delta \tilde{x}) \quad (31A)$$

$\tilde{y}_1$ is the actual area from which the aerosol is collected and $\tilde{y}_2$ is the predicted area from which it is collected. Since the theoretical collection efficiency is directly proportional to the collection parameters for the range of experimental conditions the parameters should be divided by $\exp(K_p \Delta \tilde{x})$ so that they become the same as the measured efficiencies of collection.

10. The Solution of Equations for the Electrostatic Field of a Cylinder

Smythe has developed an equation for the voltage distribution of the line charge between two grounded parallel plates using a Schwarz transformation (28). When the line charge is centered between the grounded plates the equation for the voltage reduces to

$$V = \frac{Q\ell}{2\pi\epsilon_0} \tanh^{-1} \left[\frac{\sin\left(\frac{\pi y}{2a}\right)}{\cosh\left(\frac{\pi x}{2a}\right)} \right] \quad (32A)$$

The electric field at any point can be determined from Equation 32A since

$$\vec{E} = -\nabla V \quad (33A)$$

Solving for $\vec{E}_x$ and $\vec{E}_y$ gives

$$\begin{aligned} \vec{E}_x &= -\frac{\partial V}{\partial x} = +\frac{Q\ell}{2\pi\epsilon_0} \left\{ \frac{1}{1 - \left[\frac{\sin(\pi y/2a)}{\cosh(\pi x/2a)} \right]^2} \right\} \frac{\frac{\pi}{2a} \sin\left(\frac{\pi y}{2a}\right) \sinh\left(\frac{\pi x}{2a}\right)}{\cosh^2\left(\frac{\pi x}{2a}\right)} \\ \vec{E}_y &= -\frac{\partial V}{\partial y} = -\frac{Q\ell}{2\pi\epsilon_0} \left\{ \frac{1}{1 - \left[\frac{\sin(\pi y/2a)}{\cosh(\pi x/2a)} \right]^2} \right\} \frac{\frac{\pi}{2a} \cos\left(\frac{\pi y}{2a}\right)}{\cosh\left(\frac{\pi x}{2a}\right)} \end{aligned} \quad (34)$$

This field reduces to that of a line charge in space near the cylinder. The force on a point charge in the field is given by

$$\vec{F} = \vec{E} Q_p \quad (35A)$$

11. Equation of Motion for a Charged Particle in a Duct Containing a Charged Collecting Cylinder Centered Between Grounded Plates

Combining the equations for potential flow around a cylinder in a duct (Equations 5 and 9) with those for the force caused by the coulombic attraction of a point charge in the electrostatic field (Equations 34A and 35A) gives

$$\begin{aligned} \frac{d\tilde{x}}{d\tilde{t}} = & + \left[1 - \frac{(\tilde{x}^2 - \tilde{y}^2)}{(\tilde{x}^2 + \tilde{y}^2)^2} \right] \left[1 - \frac{\tilde{y}^2 D_c^2}{4a^2} \right] \\ & - K_E \left\{ \frac{1}{1 - \left[\frac{\sin(\frac{\pi}{2} + \frac{\pi \tilde{y} D_c}{4a})}{\cosh(\frac{\pi \tilde{x} D_c}{4a})} \right]^2} \right\} \frac{\sin(\frac{\pi}{2} + \frac{\pi \tilde{y} D_c}{4a}) \frac{\pi D_c}{4a} \sinh(\frac{\pi \tilde{x} D_c}{4a})}{\cosh^2(\frac{\pi \tilde{x} D_c}{4a})} \\ \frac{d\tilde{y}}{d\tilde{t}} = & - \left[\frac{2\tilde{x}\tilde{y}}{(\tilde{x}^2 + \tilde{y}^2)^2} \right] \left[1 - \frac{\tilde{y}^2 D_c^2}{4a^2} \right] \\ & + K_E \left\{ \frac{1}{1 - \left[\frac{\sin(\frac{\pi}{2} + \frac{\pi \tilde{y} D_c}{4a})}{\cosh(\frac{\pi \tilde{x} D_c}{4a})} \right]^2} \right\} \frac{\frac{\pi D_c}{4a} \cos(\frac{\pi}{2} + \frac{\pi \tilde{y} D_c}{4a})}{\cosh(\frac{\pi \tilde{x} D_c}{4a})} \end{aligned} \quad (36A)$$

These equations describe the trajectory of a particle for potential flow around a cylinder in a duct when coulombic forces are present.

APPENDIX B. APPROXIMATE COLLECTION EFFICIENCIES

Approximate equations for the collection efficiencies can be derived for several cases. These are valid for high collection efficiencies when the collecting cylinder has little influence on the limiting trajectory. The following assumptions were made: the velocity field is uniform, the interception parameter R is zero, and the collector has no effect on the stream.

1. Coulombic Attraction of Cylinder

The equations of motion are

$$\frac{dr}{dt} = V_0 \cos \theta + \frac{CF_r}{3\pi\mu D_p} \quad (37A)$$

$$r \frac{d\theta}{dt} = -V_0 \sin \theta + \frac{CF_\theta}{3\pi\mu D_p}$$

the boundary conditions are $y = y^*$ at $x = -\infty$, and the electrostatic forces are zero at $x = -\infty$ so that Equation 37A reduces to

$$\frac{dr}{dt} = V_0 \cos \theta \quad (38A)$$

$$r \frac{d\theta}{dt} = -V_0 \sin \theta$$

For coulombic attraction F_θ is zero and F_r is $Q_p Q_{1/2} \pi \epsilon_0 r$. Combining Equations 37A gives

$$\frac{d\tilde{r}}{\tilde{r} d\theta} = -\cot \theta - \frac{K_E}{\tilde{r} \sin \theta} \quad (39A)$$

Using $\tilde{r} \sin \theta$ as an integrating factor and integrating from $\theta = \pi$ to 0 gives

$$\frac{d(\tilde{r} \sin \theta)}{d\theta} = \sin \theta \frac{d\tilde{r}}{d\theta} + \tilde{r} \cos \theta = -K_E \quad (40A)$$

$$\eta = \frac{zy^*}{D_c} = \tilde{r} \sin \theta = -K_E \pi + B \quad (41A)$$

The outermost trajectory will end at $\tilde{r} = 1$, $\theta = 0$. Therefore $B = 0$ and

$$\eta = -\pi K_E \quad (42A)$$

2. Induced Dipole Attraction Caused by a Charged Cylindrical Collector

Combining the equation of motion and the attraction term for the induced dipole gives

$$\frac{d\tilde{r}}{\tilde{r} d\theta} = -\cot \theta + \frac{K_I}{\tilde{r}^3 \sin \theta} \quad (43A)$$

Using $\tilde{r}^3 \sin^3 \theta$ as an integrating factor gives

$$\frac{\partial (\tilde{r}^3 \sin^3 \theta)}{\partial \theta} = \frac{\partial \eta^3}{\partial \theta} = 3 K_I \sin^2 \theta \quad (44A)$$

$$\eta^3 = \frac{3\pi K_I}{2} - 3 K_I \sin(2\pi) + B \quad (45A)$$

Since $\theta = 0$ at $\tilde{r} = 1$, $B = 0$ and

$$\eta = \left(\frac{3\pi K_I}{2} \right)^{1/3} \quad (46A)$$

3. Coulombic Attraction in a Duct

Using the parabolic velocity profile for flow in a duct gives

$$\frac{d\tilde{r}}{\tilde{r} d\theta} = -\cot \theta - \frac{K_E \tilde{R}_0^2}{\tilde{r} \sin \theta \left(\frac{4a^2}{D_c^2} - \tilde{r}^2 \sin^2 \theta \right)} \quad (47A)$$

Using $\tilde{r} \sin \theta$ as an integrating factor gives

$$\frac{\partial (\tilde{r} \sin \theta)}{\partial \theta} = \frac{-K_E \tilde{R}_0^2}{\left(\frac{4a^2}{D_c^2} - \tilde{r}^2 \sin^2 \theta \right)} \quad (48A)$$

which gives, on integration,

$$\eta^3 - \frac{12a^2}{D_c^2} \eta - \frac{12\pi K_E a^2}{D_c^2} = 0 \quad (49A)$$

When K_E is less than one, the equation simplifies to $\eta = -\pi K_E$.

APPENDIX C. AVERAGE DIAMETERS TO BE USED IN ELECTROSTATIC EQUATIONS

The experimental efficiencies of collection are based on the amount of aerosol deposited on the cylinder. This corresponds to a mass average efficiency.

The aerosol particles were not homogeneous either in size or charge. Both the particle size and particle charge distributions were approximated by the log normal distribution function.

By definition, the efficiency of collection for all particles is

$$\eta = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n_i D_p^3 f(Q_p, D_p) dQ_p dD_p}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p^3 f(Q_p, D_p) dQ_p dD_p} = \frac{E(n_i D_p^3)}{E(D_p^3)} \quad (50A)$$

where $f(Q_p, D_p)$ is a joint probability distribution by number and $E(x)$, the expectation of x , is an abbreviation for the double integral.

The correct averages may be obtained from the integrated and average forms of the collection equations.

For the parameter K_E , the efficiency of collection is $\eta = -\pi K_E$. Equating the average and integrated forms gives

$$\eta = B\left(\frac{\bar{Q}_p}{D_p}\right) = \frac{E(B(Q_p, D_p^2))}{E(D_p^3)} \quad (51A)$$

$$\left(\frac{\bar{Q}_p}{D_p}\right) = \frac{E(Q_p, D_p^2)}{E(D_p)^3} = \left(\frac{Q_p}{D_p}\right)_{ma} \quad (52A)$$

By assuming that $\ln(Q_p/D_p)$ and $\ln D_p$ both obey a normal bivariate probability distribution with a correlation coefficient of ρ_r , Kraemer derives an approximation of the mass average (Q_p/D_p) in terms of the number median (29).

$$\left(\frac{Q_p}{D_p}\right)_{ma} = \left(\frac{Q_p}{D_p}\right)_n \exp \left[\frac{1}{2} (\ln^2 \sigma_g' + 6 \rho_r \ln \sigma_g' \ln \sigma_g) \right] \quad (53)$$

The correlation coefficient was shown experimentally to be approximately +0.8.

For the parameter K_G

$$\eta = B \rho \left(\frac{\bar{Q}_p}{D_p} \right) = \frac{E \rho (Q_p, D_p^2)}{E (D_p^3)} \quad (54A)$$

and

$$\left(\frac{\bar{Q}_p}{D_p} \right) = \frac{E (Q_p, D_p^2)}{E (D_p^3)} = \left(\frac{Q_p}{D_p} \right)_{ma} \quad (55A)$$

The charge per unit volume, ρ , is independent of the charge and the size distribution of the aerosol and is calculated by using the number average charge and the mass mean particle diameter.

If it is assumed that $\eta = BK_T$, as shown by the numerical solutions for η in the experimental range, then the efficiency of collection is given by

$$\eta = B \bar{D}_p^2 = \frac{E (B D_p^5)}{E (D_p^3)} \quad (56A)$$

$$D_{KI} = \left\{ \frac{E (D_p^5)}{E (D_p^3)} \right\}^{1/2} = \left\{ \frac{\sum D_{pi}^5 n_i}{\sum D_{pi}^3 n_i} \right\}^{1/2} \quad (57A)$$

To get D_{KI} in terms of the number mean diameter and the geometric standard deviation, Equation 57A must be integrated over all diameters, using for the distribution function the log normal probability distribution (30). Upon integration, Equation 57A gives

$$D_{KI} = \left\{ \frac{\exp(5 \ln D_g + 12.5 \ln^2 \sigma_g)}{\exp(3 \ln D_g + 4.5 \ln^2 \sigma_g)} \right\}^{1/2} = D_g \exp(4 \ln^2 \sigma_g) \quad (58)$$

Where D_g is the geometric mean particle diameter. For log normal distributions the following identities are valid

$$D_n = D_g \exp(0.5 \ln^2 \sigma_g) \quad (59A)$$

$$D_w = D_g \exp(3.5 \ln^2 \sigma_g) \quad (60A)$$

The correct average diameter to be used in the K_I parameter therefore is

$$D_{KI} = D_n \exp(3.5 \ln^2 \sigma_g) = D_w \exp(0.5 \ln^2 \sigma_g) \quad (61)$$

The efficiency of collection caused by interception is given by Ranz and Wong (31)

$$\eta = (1 + R) - 1/(1 + R) \cong 2R \quad (62A)$$

To find the correct average diameter

$$\eta = 2 \frac{\bar{D}_p}{D_c} = \frac{E\left(\frac{2}{D_c} D_p^4\right)}{E(D_p^3)} \quad (63A)$$

and

$$D_R = \frac{E(D_p^4)}{E(D_p^3)} = \frac{\sum n_i D_{pi}^4}{\sum n_i D_{pi}^3} = D_w \quad (64A)$$

Table 1A is a summary of the different average diameters and their relationships for a log normal distribution.

Table 1A. Diameter Relationships for Log Normal Distribution of Sizes

$$D_g = D_{nmd} = \text{geometric mean diameter by number}$$

$$D_n = D_g \exp(0.5 \ln^2 \sigma_g) = \text{number mean diameter}$$

$$D_w = D_R = D_g \exp(3.5 \ln^2 \sigma_g) = \text{mass average diameter}$$

$$D_{KI} = D_g \exp(4.0 \ln^2 \sigma_g)$$

$$D_g' = D_{mmd} = D_g \exp(3.0 \ln^2 \sigma_g) = \text{geometric mean diameter by mass}$$

$$(Q_p/D_p)_{ma} = (Q_p/D_p)_n \exp(0.5 \ln^2 \sigma_g' + 3 \rho_r \ln \sigma_g' \ln \sigma_g)$$

APPENDIX D. SOLUTION OF UNABRIDGED EQUATIONS OF MOTION BY COMPUTER

Two programs were used to calculate the trajectories of the particles. The first program was similar to that described in detail by Kraemer (32). It was used to calculate the trajectories for Equation 12.

The aerosol particle was started at $x = -700$ and successive trajectories were generated until the difference in efficiency from two trajectories, one just touching and one missing the collector, was less than 0.003 percent. About 20 trajectories were needed for each solution with a given set of parameters. Each computation required about 10 minutes with the ILLIAC, the University of Illinois digital computer.

The second program was used for the solution of the equations for coulombic attraction between a cylinder centered in a duct and a charged particle (Equation 36A). In this program the floating decimal subroutine of the ILLIAC was used. The program gave trajectories by the Runge-Kutta method of integration described by Scarborough (33). Trajectories were calculated until the difference in efficiency between two successive trajectories was less than one percent.

APPENDIX E. COLLECTION EFFICIENCIES OF CYLINDERS CALCULATED BY DIGITAL COMPUTER FOR SELECTED VALUES OF PARAMETERS¹

1. Solutions to Equation 12

$(-K_E + K_G + K_S)$	K_I	K_M	R	η (1)
+1+02	0	0	0	+3029+02
+1+00	0	0	0	+3141+00
+1-02	0	0	0	+3168-02
+1+01	0	0	0	+3137+01
+3+01	0	0	0	+9386+01
+3-03	0	0	0	+959 -03
+1-03	0	0	0	+329 -03
+3-02	0	0	0	+944 -02
+1-01	0	0	0	+3143-01
+1+01	0	0	+2+00	+3137+01
+1+00	0	0	+2+00	+5374+00
+1-01	0	0	+2+00	+3825+00
+1+01	0	0	+1+00	+3137+01
+1+00	0	0	+1+00	+3748+00
+1-01	0	0	+1+00	+2069+00
+1+00	+1+00	+1-02	0	+6022+00
+1+01	+1+01	0	0	+3641+01
+1-01	+1-01	0	0	+6042-01
+1+01	+1-01	0	0	+3144+01
+1-01	+1+01	0	0	+1207+01
+1+01	+1-01	+1-02	0	+3150+01
+1+00	+1+00	0	0	+5171+00
0	+5+00	0	0	+7904+00
0	+5-02	0	0	+1532-01
0	+5-03	0	0	+1589-02
0	+1+02	0	0	+334 +01
0	+1-02	0	0	+313 -02
0	+1-01	0	0	+300 -01
0	+1+00	0	0	+243 +00
0	+15+01	0	0	+1466+01
0	+15-03	0	0	+486 -03
0	+5-04	0	0	+18 -03
0	+5-01	0	+5+00	+8553+00
0	+5+00	0	+5+00	+1051+01
0	+5-01	0	+2+00	+4068+00
0	+5-01	0	+1+00	+2447+00
0	+5-03	0	+1-02	+286 -02
0	0	+1+01	0	+2936+01
0	0	+1+00	0	+127 +01
0	0	+1-01	0	+577 +00
0	0	+1-02	0	+267 +00
0	0	+1-04	0	+1445-01
0	0	+1-03	0	+1104+00
0	0	+3-01	0	+8384+00
0	0	+3-02	0	+3851+00
0	0	+3-03	0	+1783+00
0	0	+1-06	0	+163 -03

¹ See footnote, p. 58

$(-K_E + K_G + K_S)$	K_I	K_M	R	η^1
0	0	0	+1+00	+1909+00
0	0	0	+2+00	+3666+00
0	0	0	+5+00	+8333+00
0	0	0	+1-01	+1999-01
0	0	0	+1-02	+2000-02

2. Solutions to Equation 36A, $2a/D_0 = 7$

$(-K_E + K_G + K_S)$	η
+1+01	+342+01
+14+01	+511+01
+1+00	+318+00
+1-01	+314-01

- (1) These numbers are in floating decimal. The number +A+OP is equal to $A \times 10^P$ where A is a fraction between 1/10 and 1. The number +3029+02 is equal to 30.29.

¹See footnote page 58

APPENDIX F. DATA ON AEROSOL COLLECTION¹

Table 2A. Collection of Aerosols on Single Cylinders

Run	D _c cm.	V volts	Time, min.	Q _p electrons	q l./min.	W _c Rate of collection g./min.	W _t Rate of flow g./min.
3	+318+00	-100+04	+500+01	+161+03	+211+01	+646+02	+925+02
4	+238+00	-100+04	+800+01	+161+03	+212+01	+669+02	+925+02
7	+318+00	-100+04	+100+02	+108+03	+212+01	+583+02	+120+03
8	+238+00	-100+04	+100+02	+108+03	+212+01	+545+02	+120+03
10	+318+00	-100+04	+500+01	+295+03	+206+01	+773+02	+108+03
11	+238+00	-100+04	+600+01	+295+03	+210+01	+829+02	+108+03
14	+318+00	-100+04	+300+01	+232+03	+203+01	+549+02	+820+02
15	+238+00	-100+04	+300+01	+232+03	+203+01	+505+02	+820+02
17	+318+00	-100+04	+300+02	+206+00	+210+01	+665+00	+122+03
18	+238+00	-100+04	+380+02	+206+00	+210+01	+458+00	+122+03
19	+318+00	-465+04	+330+02	+206+00	+210+01	+158+01	+122+03
20	+238+00	-465+04	+400+02	+206+00	+210+01	+160+01	+122+03
21	+238+00	-100+04	+400+02	+117+00	+501+01	+618+00	+291+03
22	+238+00	-465+04	+300+02	+117+00	+501+01	+216+01	+291+03
23	+318+00	-465+04	+250+02	+117+00	+501+01	+260+01	+291+03
24	+318+00	-100+04	+410+02	+117+00	+501+01	+703+00	+291+03
26	+318+00	-100+04	+450+01	+234+03	+498+01	+129+03	+256+03
27	+238+00	-100+04	+400+01	+234+03	+499+01	+114+03	+256+03
30	+238+00	-100+04	+300+01	+174+03	+498+01	+828+02	+246+03
31	+318+00	-100+04	+300+01	+174+03	+499+01	+105+03	+246+03
33	+318+00	0	+210+02	+150+03	+486+01	+965+00	+273+03
34	+318+00	-100+04	+300+01	+150+03	+486+01	+102+03	+273+03
35	+238+00	-100+04	+300+01	+150+03	+486+01	+986+02	+273+03
36	+238+00	0	+170+02	+150+03	+486+01	+259+01	+273+03
37	+238+00	-100+04	+140+03	+978-01	+486+01	+493+00	+400+03
38	+238+00	-465+04	+300+02	+978-01	+486+01	+182+01	+400+03
39	+318+00	-465+04	+510+02	+978-01	+486+01	+206+01	+400+03
40	+318+00	-100+04	+350+02	+978-01	+486+01	+670+00	+400+03
41	+318+00	0	+400+02	+171+03	+486+01	+171+01	+283+03
42	+318+00	-100+04	+400+01	+171+03	+486+01	+125+03	+283+03
43	+238+00	-100+04	+400+01	+171+03	+486+01	+115+03	+283+03
44	+238+00	0	+180+02	+171+03	+486+01	+148+01	+283+03
45	+238+00	-100+04	+400+02	+291-01	+204+01	+190+00	+188+03
46	+238+00	-465+04	+250+02	+291-01	+204+01	+494+00	+188+03
47	+238+00	0	+350+02	+980+02	+204+01	+176+01	+112+03
48	+238+00	-100+04	+400+01	+980+02	+204+01	+559+02	+112+03
49	+318+00	-100+04	+450+01	+980+02	+205+01	+563+02	+112+03
50	+318+00	0	+360+02	+980+02	+205+01	+201+01	+112+03
51	+318+00	0	+300+02	+980+02	+448+01	+739+00	+112+03
52	+318+00	-100+04	+400+01	+980+02	+448+01	+534+02	+112+03
53	+318+00	-100+04	+580+02	+291-01	+447+01	+120+00	+189+03
54	+318+00	-465+04	+250+02	+291-01	+447+01	+443+00	+189+03
55	+318+00	-465+04	+310+02	+291-01	+209+01	+460+00	+185+03
56	+318+00	-100+04	+400+02	+291-01	+209+01	+135+00	+185+03
57	+318+00	0	+320+02	+108+03	+209+01	+172+01	+117+03
58	+318+00	-100+04	+300+01	+108+03	+209+01	+678+02	+117+03
59	+238+00	-100+04	+300+01	+108+03	+209+01	+576+02	+117+03
60	+238+00	0	+300+02	+108+03	+209+01	+177+01	+117+03

¹ See footnote, page 58

Table 2A. (Continued)

Run	D _c cm.	V volts	Time, min.	Q _p electrons	q l./min.	W _c Rate of collection g./min.	W _t Rate of flow g./min.
61	+318+00	0	+300+02	+108+03	+428+01	+887+00	+117+03
62	+318+00	-100+04	+400+01	+108+03	+428+01	+527+02	+117+03
63	+318+00	0	+500+02	+303+03	+463+01	+222+00	+672+02
64	+318+00	-100+04	+400+01	+303+03	+463+01	+499+02	+672+02
65	+318+00	-100+04	+420+02	+140+00	+464+01	+317+00	+107+03
66	+318+00	-465+04	+260+02	+140+00	+474+01	+828+00	+107+03
67	+318+00	0	+300+02	+232+03	+474+01	+148+00	+960+02
68	+318+00	-100+04	+400+01	+232+03	+474+01	+475+02	+960+02
69	+318+00	-500+04	+280+02	+195-01	+254+01	+294+00	+157+03
70	+318+00	-200+04	+600+02	+195-01	+254+01	+950-01	+157+03
71	+238+00	-200+04	+680+02	+195-01	+254+01	+102+00	+157+03
72	+238+00	-500+04	+300+02	+195-01	+254+01	+129+01	+157+03
73	+318+00	-500+04	+300+02	+195-01	+552+01	+148+00	+158+03
74	+318+00	-200+04	+450+02	+195-01	+552+01	+563-01	+158+03
75	+318+00	-300+03	+400+01	+567+02	+254+01	+103+02	+900+02
76	+318+00	-100+03	+600+01	+567+02	+254+01	+391+01	+900+02
77	+318+00	-300+02	+100+02	+567+02	+254+01	+190+01	+900+02
78	+318+00	-100+02	+150+02	+567+02	+254+01	+971+00	+900+02
79	+318+00	0	+300+02	+567+02	+254+01	+654+00	+900+02
80	+238+00	0	+370+02	+567+02	+254+01	+582+00	+900+02
81	+238+00	-100+02	+150+02	+567+02	+254+01	+718+00	+900+02
82	+238+00	-300+02	+100+02	+567+02	+254+01	+133+01	+900+02
83	+238+00	-100+03	+600+01	+567+02	+254+01	+306+01	+900+02
84	+238+00	-300+03	+400+01	+567+02	+254+01	+823+01	+900+02
85	+318+00	-300+03	+400+01	+567+02	+552+01	+760+01	+108+03
86	+318+00	-100+03	+600+01	+567+02	+552+01	+253+01	+108+03
87	+318+00	-300+02	+800+01	+567+02	+552+01	+158+01	+108+03
88	+318+00	-100+02	+150+02	+567+02	+552+01	+691+00	+108+03
89	+318+00	0	+300+02	+567+02	+552+01	+422+00	+108+03
90	+318+00	0	+210+02	+400+03	+259+01	+160+01	+275+03
91	+318+00	-100+02	+210+02	+400+03	+259+01	+407+01	+275+03
92	+318+00	-300+02	+100+02	+400+03	+259+01	+671+01	+275+03
93	+318+00	-100+03	+700+01	+400+03	+259+01	+189+02	+275+03
95	+318+00	-300+03	+700+01	+400+03	+259+01	+457+02	+275+03
96	+318+00	-300+03	+700+01	+400+03	+259+01	+367+02	+275+03
97	+318+00	-100+03	+900+01	+400+03	+259+01	+146+02	+275+03
98	+318+00	-200+04	+800+02	+208+00	+259+01	+141+01	+315+03
99	+318+00	-100+04	+160+03	+208+00	+259+01	+740+00	+315+03
100	+318+00	-400+04	+130+03	+208+00	+259+01	+375+01	+315+03
101	+318+00	-100+04	+608+03	+208+00	+259+01	+103+01	+315+03
102	+318+00	-400+04	+130+03	+208+00	+896+01	+112+01	+473+03
103	+318+00	-100+02	+140+02	+409+03	+896+01	+262+01	+473+03
104	+318+00	-300+02	+100+02	+409+03	+896+01	+405+01	+473+03
105	+318+00	-100+03	+800+01	+409+03	+896+01	+205+02	+473+03
106	+318+00	-300+03	+600+01	+409+03	+896+01	+649+02	+473+03
107	+318+00	-300+03	+600+01	+409+03	+896+01	+709+02	+473+03
108	+318+00	-100+03	+800+01	+409+03	+896+01	+261+02	+473+03
109	+318+00	-300+02	+100+02	+409+03	+896+01	+798+01	+473+03
111	+318+00	0	+200+02	+409+03	+896+01	+146+01	+473+03

Table 2A. (Continued)

Run	D _n microns	exp (K _p Δx̄)	K _I	K _G	-K _E	$\frac{\Sigma K's}{\exp(K_p \Delta \bar{x})}$	h
3	+789+00	+177+01	+575-04	+420-01	+180+01	+105+01	+358+01
4	+789+00	+176+01	+107-03	+495-01	+212+01	+124+01	+500+01
7	+789+00	+140+01	+572-04	+245-01	+121+01	+888+00	+237+01
8	+789+00	+140+01	+107-03	+292-01	+143+01	+105+01	+293+01
10	+128+01	+144+01	+129-03	+267-01	+192+01	+135+01	+370+01
11	+128+01	+142+01	+236-03	+306-01	+223+01	+159+01	+534+01
14	+112+01	+134+01	+107-03	+213-01	+180+01	+137+01	+340+01
15	+112+01	+134+01	+201-03	+253-01	+214+01	+162+01	+413+01
17	+128+01	+100+01	+126-03	+142-07	+131-02	+170-02	+254-01
18	+128+01	+100+01	+236-03	+168-07	+155-02	+215-02	+233-01
19	+128+01	+100+01	+273-02	+142-07	+609-02	+911-02	+604-01
20	+128+01	+100+01	+511-02	+168-07	+721-02	+127-01	+813-01
21	+107+01	+100+01	+675-04	+507-08	+451-03	+811-03	+132-01
22	+107+01	+100+01	+146-02	+507-08	+210-02	+386-02	+463-01
23	+107+01	+100+01	+780-03	+426-08	+177-02	+278-02	+416-01
24	+107+01	+100+01	+361-04	+426-08	+381-03	+636-03	+113-01
26	+107+01	+123+01	+363-04	+151-01	+767+00	+638+00	+246+01
27	+107+01	+123+01	+678-04	+180-01	+906+00	+755+00	+286+01
30	+107+01	+112+01	+679-04	+954-02	+673+00	+614+00	+213+01
31	+107+01	+112+01	+362-04	+799-02	+567+00	+517+00	+205+01
33	+102+01	+109+01	0	+611-02	0	+667-02	+165-01
34	+102+01	+109+01	+449-04	+611-02	+548+00	+511+00	+179+01
35	+102+01	+109+01	+840-04	+727-02	+649+00	+606+00	+230+01
36	+102+01	+109+01	0	+727-02	0	+781-02	+590-01
37	+102+01	+100+01	+840-04	+455-08	+425-03	+823-03	+767-02
38	+102+01	+100+01	+182-02	+455-08	+197-02	+412-02	+282-01
39	+102+01	+100+01	+971-03	+382-08	+167-02	+288-02	+240-01
40	+102+01	+100+01	+449-04	+382-08	+358-03	+639-03	+781-02
41	+102+01	+112+01	0	+828-02	0	+871-02	+282-01
42	+102+01	+112+01	+449-04	+828-02	+627+00	+569+00	+212+01
43	+102+01	+112+01	+840-04	+986-02	+743+00	+674+00	+259+01
44	+102+01	+112+01	0	+986-02	0	+102-01	+325-01
45	+636+00	+100+01	+675-04	+999-08	+509-03	+755-03	+629-02
46	+636+00	+100+01	+146-02	+999-08	+237-02	+402-02	+163-01
47	+636+00	+216+01	0	+674-01	0	+352-01	+980-01
48	+636+00	+216+01	+675-04	+674-01	+171+01	+829+00	+324+01
49	+636+00	+215+01	+359-04	+561-01	+144+01	+702+00	+246+01
50	+636+00	+215+01	0	+561-01	0	+301-01	+835-01
51	+636+00	+117+01	0	+118-01	0	+116-01	+308-01
52	+636+00	+117+01	+164-04	+118-01	+659+00	+573+00	+232+01
53	+636+00	+100+01	+165-04	+176-08	+196-03	+346-03	+296-02
54	+636+00	+100+01	+356-03	+176-08	+912-03	+141-02	+109-01
55	+636+00	+100+01	+761-03	+787-08	+195-02	+285-02	+116-01
56	+636+00	+100+01	+352-04	+787-08	+420-03	+588-03	+339-02
57	+636+00	+253+01	0	+681-01	0	+311-01	+687-01
58	+636+00	+253+01	+352-04	+681-01	+155+01	+646+00	+288+01
59	+636+00	+253+01	+658-04	+811-01	+184+01	+765+00	+319+01
60	+636+00	+253+01	0	+811-01	0	+362-01	+943-01
61	+636+00	+125+01	0	+162-01	0	+151-01	+354-01
62	+636+00	+125+01	+172-04	+162-01	+758+00	+623+00	+216+01

Table 2A. (Continued)

Run	D _n microns	exp (K _p Δx)	K _I	K _G	-K _E	K' exp(K _p Δx)	h
63	+112+01	+108+01	0	+572-02	0	+631-02	+154-01
64	+112+01	+108+01	+471-04	+572-02	+103+01	+962+00	+384+01
65	+112+01	+100+01	+470-04	+195-08	+477-03	+764-03	+138-01
66	+112+01	+100+01	+994-03	+186-08	+217-02	+342-02	+361-01
67	+112+01	+106+01	0	+457-02	0	+516-02	+718-02
68	+112+01	+106+01	+460-04	+457-02	+773+00	+731+00	+241+01
69	+556+00	+100+01	+648-03	+308-08	+139-02	+216-02	+874-02
70	+556+00	+100+01	+104-03	+308-08	+554-03	+782-03	+282-02
71	+556+00	+100+01	+194-03	+367-08	+657-03	+102-02	+406-02
72	+556+00	+100+01	+121-02	+367-08	+164-02	+303-02	+510-01
73	+556+00	+100+01	+298-03	+657-09	+638-03	+106-02	+436-02
74	+556+00	+100+01	+477-04	+657-09	+255-03	+427-03	+166-02
75	+556+00	+122+01	+233-05	+149-01	+241+00	+211+00	+533+00
76	+556+00	+122+01	+259-06	+149-01	+804-01	+797-01	+202+00
77	+556+00	+122+01	+233-07	+149-01	+241-01	+338-01	+985-01
78	+555+00	+122+01	+259-08	+149-01	+804-02	+206-01	+503-01
79	+556+00	+122+01	0	+149-01	0	+141-01	+339-01
80	+556+00	+122+01	0	+177-01	0	+164-01	+402-01
81	+556+00	+122+01	+485-08	+177-01	+952-02	+242-01	+496-01
82	+556+00	+122+01	+436-07	+177-01	+286-01	+397-01	+919-01
83	+556+00	+122+01	+485-06	+177-01	+952-01	+942-01	+212+00
84	+556+00	+122+01	+436-05	+177-01	+286+00	+250+00	+569+00
85	+556+00	+105+01	+107-05	+378-02	+111+00	+110+00	+328+00
86	+556+00	+105+01	+119-06	+378-02	+370-01	+394-01	+109+00
87	+556+00	+105+01	+107-07	+378-02	+111-01	+148-01	+684-01
88	+556+00	+105+01	+119-08	+378-02	+370-02	+775-02	+255-01
89	+556+00	+105+01	0	+378-02	0	+424-02	+182-01
90	+142+01	+159+01	0	+341-01	0	+249-01	+271-01
91	+142+01	+159+01	+184-07	+341-01	+200-01	+374-01	+691-01
92	+142+01	+159+01	+165-06	+341-01	+598-01	+625-01	+114+00
93	+142+01	+159+01	+184-05	+341-01	+200+00	+150+00	+321+00
95	+142+01	+159+01	+165-04	+341-01	+598+00	+401+00	+775+00
96	+142+01	+159+01	+165-04	+341-01	+598+00	+401+00	+623+00
97	+142+01	+159+01	+184-05	+341-01	+200+00	+150+00	+247+00
98	+142+01	+100+01	+735-03	+105-07	+207-02	+316-02	+209-01
99	+142+01	+100+01	+184-03	+105-07	+104-02	+157-02	+110-01
100	+142+01	+100+01	+294-02	+105-07	+415-02	+746-02	+555-01
101	+142+01	+100+01	+184-03	+105-07	+104-02	+157-02	+152-01
102	+142+01	+100+01	+849-03	+132-08	+120-02	+241-02	+111-01
103	+145+01	+105+01	+802-08	+323-02	+640-02	+101-01	+259-01
104	+145+01	+105+01	+722-07	+323-02	+192-01	+223-01	+400-01
105	+145+01	+105+01	+802-06	+323-02	+640-01	+652-01	+202+00
106	+145+01	+105+01	+722-05	+323-02	+192+00	+188+00	+640+00
107	+145+01	+105+01	+722-05	+323-02	+192+00	+188+00	+700+00
108	+145+01	+105+01	+802-06	+323-02	+640-01	+652-01	+257+00
109	+145+01	+105+01	+722-07	+323-02	+192-01	+223-01	+787-01
111	+145+01	+105+01	0	+323-02	0	+396-02	+144-01

¹ All values are expressed in floating decimals. The expression

+A+OP signifies $A \times 10^P$, where A is between .1 and 1.0. Thus,

+925+02 represents the number +92.5 and -925-02 represents the number -0.00925.

Table 3A. Particle Size Distribution of Aerosols

Runs	D _s from Millikan cell, microns	D _g from Cascade impactor, microns	D _n microns	D _w microns	D _{KI} microns	σ _g	σ _{g'}
1-8	0.76	0.79	0.79	0.86	0.88	1.19	1.13
9-20	1.24	1.31	1.28	1.33	1.34	1.12	1.13
21-32	1.09	1.04	1.07	1.09	1.10	1.09	1.13
33-44	1.02	0.98	1.02	1.17	1.20	1.24	1.13
45-62	0.65	0.61	0.64	0.67	0.67	1.13	1.13
63-68	1.18	1.03	1.12	1.19	1.21	1.16	1.13
69-89	0.56	0.53	0.56	0.62	0.63	1.20	1.13
90-102	1.32	1.42	1.42	1.75	1.81	1.30	1.18
103-111	1.46	1.26	1.45	2.09	2.22	1.42	1.26

Table 4A. Experimental Conditions in Tests on Filter Mats

Run	Type of mat	Porosity, % void space	V _{av} <u>cm.</u> sec.	D _n microns	σ _g	W _t <u>g.</u> min.	Q _p elec- trons	Thick- ness cm.
II	glass fiber	0.965	6	0.56	1.2	90	58	0.1
III	glass fiber	0.965	6	0.56	1.2	90	58	0.1
Va	glass fiber	0.965	6	0.56	1.2	90	0	0.1
Vb	glass fiber	0.965	18	0.56	1.2	108	58	0.1
VI	glass fiber	0.965	19	1.32	1.3	290	380	0.1
VII	tangled wire	0.936	10	1.32	1.3	275	380	3.75
VIII	tangled wire dipole	0.948	10	1.32	1.3	275	380	3.75

Table 5A. Collection of Aerosols by Mats

1. Run II. Collection efficiency, %

Time min.	Aerosol uncharged	Aerosol charged	ΔP cm. oil
25	0.893		1.05
95	0.872		1.30
165	0.843		1.50
305	0.852		1.50
360	0.851		1.60
405		0.942	1.70
450		0.923	1.70
495		0.912	1.75
580		0.923	2.00
665		0.930	2.00
755		0.940	2.00
805		0.935	2.50
840	0.780		2.50
1010	0.805		2.50
1100	0.764		2.75
1760	0.923		3.10
1795	0.923		3.10
1815	0.930		
1835		1.000	
1905		0.980	
1945		1.000	
2050		0.992	
2120		0.994	
2195	0.917		

2. Run III. Aerosol uncharged

20	0.892
65	0.846
105	0.842
185	0.858
230	0.804
265	0.846
315	0.830
385	0.796
445	0.786
555	0.788
575	0.782
755	0.778
785	0.742
825	0.694
865	0.638
955	0.606
985	0.682

Table 5A. (Continued)

Collection efficiency, %

Time min.	Aerosol uncharged	ΔP cm. oil
1090	0.638	3.70
1120	0.662	4.10
1325	0.636	4.45
1355	0.696	4.75
1650	0.624	6.10
1685	0.702	6.45
2225	0.710	11.65
2260	0.698	13.00
2450	0.752	15.30
2485	0.748	18.50

3. Run Va. Aerosol uncharged

Time min.		
0	0.923	0.8
0	0.891	0.8
1015	0.802	3.3

4. Run Vb. Aerosol charged

Time min.		
35	0.878	4.2
46	0.903	4.2
57	0.903	4.4
175	0.980	4.4
186	0.983	4.4
197	0.985	4.4
345	0.993	5.0
356	0.995	5.0

5. Run VI. Aerosol charged

Time min.		
7	.9924	
18	.9950	
29	.9969	
54	.9975	
70	.9969	
81	.9963	
104	.9944	
126	.9930	
137	.9941	
		338 066

Table 5A. (Continued)

6. Run VII. Aerosol charged

Mat potential, volts	Collection efficiency, %
0	26.0
0	16.0
+ 30	12.0
+ 30	14.0
+150	9.0
+150	10.0
+150	3.5
+ 30	14.5
+ 30	2.5
- 30	15.0
- 30	6.0
-150	15.0
-150	10.5
0	17.0
0	21.0

7. Run VIII. Aerosol charged

Dipole potential,
volts

0	40
0	44
30	61
30	58
30	58
30	43
30	44
0	33
0	23
150	69
150	69

VII. NOMENCLATURE

A	= area, sq. cm.	
B	= constant	
C	= Cunningham factor, $(1+1.6 \times 10^{-1}/D_p)$	
$\mathcal{D}$	= characteristic diameter for cascade impactor jet, microns	
D_c	= diameter of collector, cm.	
D_g	= geometric number mean diameter of particles, microns	
$D_{g'}$	= geometric mass mean diameter of particles, microns	
D_{KI}	= mean diameter of particles for parameter K_I , microns	
D_n	= number mean diameter of particles, microns	
D_p	= diameter of particles, microns	
D_r	= mean diameter of particles for parameter R, microns	
D_w	= mass mean diameter of particles, microns	
$\vec{E}$	= electrostatic field, esu/cm. ²	
$\vec{F}$	= force on particles, dynes	
G	= constant	
K_E	$= \frac{(Q_p/D_p)_{ma} \ 2 \ C}{2 \ \pi \ \epsilon_0 \ D_c \ 3 \ \pi \ \mu \ V_0}$	= parameter for coulombic force, dimensionless
K_G	$= \frac{\rho(Q_p/D_p)_{ma} (R_0^2 - r_c^2) \ 2 \ C}{4 \ \epsilon_0 \ \ln(R_0/r_c) \ D_c \ 3 \ \pi \ \mu \ V_0}$	= parameter for induced electrification, dimensionless
K_I	$= \frac{(\epsilon - 1) \ D_{KI}^2 \ Q_p^2 \ 8 \ C}{(\epsilon + 2) \ 8 \ \pi \ \epsilon_0 \ D_c^3 \ 3 \ \pi \ \mu \ V_0}$	= parameter for induced dipole, dimensionless
K_M	$= \frac{(1 - \epsilon') Q_p (Q_p/D_p)_{ma} \ 4 \ C}{(1 + \epsilon') \ 16 \ \pi \ \epsilon_0 \ D_c^2 \ 3 \ \pi \ \mu \ V_0}$	= parameter for image force, dimensionless
K_P	$= \frac{\rho(Q_p/D_p)_{ma} \ C \ D_c}{2 \ \epsilon_0 \ 3 \ \pi \ \mu \ V_0}$	= parameter for space charge, dimensionless
K_S	$= \frac{\rho(Q_p/D_p)_{ma} \ D_c \ 2 \ C}{8 \ \epsilon_0 \ 3 \ \pi \ \mu \ V_0}$	= parameter for space charge, dimensionless
N_p	= concentration of aerosol particles, particles/cc.	
N_{Re}	= Reynolds number	

- ΔP = pressure drop across fiber mats, cm. Merriam oil
- Q_1 = charge on collector, coulombs
- Q_p = charge on particles, coulombs, or electron units
- $(Q_p/D_p)_{ma}$ = mass average ratio of charge to diameter, electrons/micron
- $(Q_p/D_p)_n$ = number average ratio of charge to diameter, electrons/micron
- R = parameter for interception, dimensionless
- R_o = radius of outer grounded cylinder, cm.
- $\tilde{R}_o$ = $2 R_o/D_c$
- V = charge on collecting cylinder, volts
- W_c = rate of aerosol collection, micrograms/min.
- W_t = total rate of aerosol passing collector, micrograms/min.
- a = $1/2$ height of duct, cm.
- b = $1/2$ width of duct, cm.
- $\vec{p}$ = dipole moment induced in sphere
- q = total flow of aerosol, $\ell./min.$
- r = radial distance from collector, cm.
- $\tilde{r}$ = $2r/D_c$ = radial distance from collector, dimensionless
- r_c = radius of collector, cm.
- t = time, sec.
- $\vec{u}$ = velocity of aerosol particle, cm./sec.
- $\vec{v}$ = velocity of air stream, cm./sec.
- v_{av} = average velocity through filter mat, cm./sec.
- v_o = velocity at center of duct, cm./sec.
- x = distance in direction of flow, cm.
- Δx = $\Delta x/r_c$ = length of duct, dimensionless
- y = distance normal to direction of flow, cm.

- y^* = boundary value at $x = -\infty$, cm.
 ϵ = dielectric constant of aerosol particle
 ϵ^i = dielectric constant of a cylinder
 $\epsilon^{i'}$ = pseudo-dielectric constant of entire aerosol
 ϵ_0 = permittivity of free space, 8.85×10^{-21} coulombs²/dyne-cm.²
 η = collection efficiency, ratio of the area of the stream from which all particles are removed to the projected cross-sectional area of the collector, or the percentage of aerosol removed by a filter mat.
 θ = angular position from positive x-axis
 μ = viscosity of air, or abbreviation of micron
 ρ = charge density of aerosol, electrons/cc.
 ρ_p = density of particle, g./cc.
 ρ_r = correlation coefficient between distribution of $\ln D_p$ and $(\ln Q_p/D_p)$
 σ_g = geometric standard deviation in diameter of aerosol particles
 $\sigma_{g'}$ = geometric standard deviation in the ratio (Q_p/D_p) of aerosol particles
 ψ = total electric flux through a surface, coulombs
 ∇ = vector differential operator
 $\ln$ = natural logarithm
 $\exp(x) = e^x$

VIII. BIBLIOGRAPHY

1. Kraemer, H. F., Ph.D. thesis, University of Illinois, 1954.
2. Ranz, W. E., and Wong, J. B., Ind. Eng. Chem., 44, 1371 (1952).
3. Kraemer, H. F. and Johnstone, H. F., Ind. Eng. Chem., 47, 2426 (1955).
4. Wilson, I. B., J. Colloid Sci., 2, 271 (1947).
5. Lipscomb, W., et al., J. Appl. Phys., 18, 72 (1947).
6. Daniel, J., and Brackett, F., J. Appl. Phys., 22, 542 (1951).
7. Chen, C. Y., Chem. Revs., 55, 595 (1955).
8. Rossano, A. T., and Silverman, L., Air Cleaning Laboratory, School of Public Health, Harvard University, Rept. NYO-1594, May 11, 1955.
9. LaMer, V. K., Columbia University, Rept. NYO-514, June 30, 1952.
10. Lowe, H. J., and Lucas, D. H., Brit. J. Appl. Phys., Sup. No. 2, S40, (1953).
11. Streeter, V. L., "Fluid Dynamics," p. 67, McGraw-Hill, New York, 1948.
12. Ranz, W. E., University of Illinois Engineering Experiment Station, Rept. No. 3, Contract AT(30-3)-28, p. 13, March 31, 1951.
13. Davies, C. N., and Peetz, C. V., Proc. Roy. Soc. (London), A234, 269 (1956).
14. Cornish, R. J., Proc. Roy. Soc. (London), A120, 691 (1928).
15. Debye, P., "Polar Molecules," Dover Publications, New York, 1945.
16. Kraemer, H. F., and Johnstone, H. F., Ind. Eng. Chem., 47, 2426 (1955).
17. Sinclair, D., and LaMer, V. K., Chem. Revs., 44, 245 (1949).
18. Kraemer, H. F., Ph.D. thesis, p. 56, University of Illinois, 1954.
19. Sinclair, D., "Handbook on Aerosols", p. 97, U. S. Atomic Energy Commission, Washington, D. C., 1950.
20. Ranz, W., and Wong, J. B., Arch. Ind. Hyg. and Occupational Med., 5, 464 (1952).
21. Chen, C. Y., "Filtration of Aerosols by Fibrous Media," p. 36, Annual Report on Contract, DA18-108-CML-4789, Engineering Experiment Station, University of Illinois (1954).

22. Kraemer, H. F., Ph.D. thesis, p. 74, University of Illinois, 1954.
23. Blasewitz, A. G., and Judson, B. F., Chem. Eng. Prog. 51, 6 (1955).
24. Walton, W. H., Chemical Defense Experimental Station, Porton, England, Rept. 2465, (1942).
25. Harnwell, G. P., "Principles of Electricity and Electromagnetism," p. 68, McGraw-Hill, New York, (1938).
26. Weber, E., "Electromagnetic Fields," p. 523, John Wiley and Sons, New York, 1938.
27. Smythe, W. R., "Static and Dynamic Electricity," p. 174, McGraw-Hill, New York, 1950.
28. *ibid.*, p. 83.
29. Kraemer, H. F., Ph.D. thesis, p. 72, University of Illinois, 1954.
30. Herdan, G., "Small Particle Statistics," p. 116, Elsevier, New York, 1953.
31. Ranz, W. E., and Wong, J. B., Ind. Eng. Chem., 44, 1371 (1952).
32. Kraemer, H. F., Ph.D. thesis, p. 91, University of Illinois, 1954.
33. Scarborough, J. B., "Numerical Mathematical Analysis," 2nd Ed., p. 299, Johns Hopkins University Press, 1950.