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COMMENTS ON OBTAINING A BOUNCE AVERAGE FOKKER-PLACK EQUATION

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April 30, 1974

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COMMENTS ON OBTAINING A BOUNCE AVERAGE FOKKER-PLANCK EQUATION

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April 19, 1974

ABSTRACT

A procedure is suggested for obtaining directly a Fokker-Planck equation in the space of constants of the bounce motion which thereby avoids the relatively laborious task of bounce averaging the velocity space Fokker-Planck equation.

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In applying the Fokker-Planck equation to calculations of the classical loss from a mirror machine, any effects due to variation of magnetic field and ambipolar potential along a field line are usually neglected. An exception to this was the effort of Marx to perform the required bounce average numerically¹. However, this method is wasteful of computer time, and a method which looks directly at diffusion in the space of the constants of the bounce motion is strongly to be desired.

Any F-P equation is derived on the basis that there are a set of variables (the three velocity components in the usual example) whose time of change due to a weak fluctuating field is long compared to the auto-correlation time of that field, τ_{AC} . Thus, for the usual velocity space diffusion, the time derivative of $f(\underline{v})$ is expressed in terms of the averages

$$\left\langle \frac{\Delta \underline{v}(\tau)}{\tau} \right\rangle \quad \text{and} \quad \left\langle \frac{\Delta \underline{v}(\tau) \Delta \underline{v}(\tau)}{\tau} \right\rangle$$

where the time τ over which the average is taken is short compared with characteristic time of $f(\underline{v})$, but long compared with τ_{AC} . These averages are obtained by directly integrating the single particle equation of motion using simple perturbation theory.

For ion scattering by shielded Coulomb collisions

$$\tau_{AC} \approx \lambda_{De} / V_{Ti} \ll \omega_{ci}^{-1}$$

at most densities of interest. Thus, strictly speaking, the F-P equation may be used to describe the diffusion of an ion in the process of experiencing a cyclotron orbit. However, because the collision time is so much longer than ω_{ci}^{-1} , one normally cyclotron averages the equation for $f(\underline{v})$,

taking f to be independent of orbit angle, and thus obtains an equation for the evolution of $f(v_{\perp}, v_{\parallel})$ where $v_{\perp}, v_{\parallel}$ are constants of the cyclotron motion. This process of averaging is so simple as to usually not merit comment. However, it might be useful to build upon the idea that the same result could have been obtained directly by initially considering a distribution $f(v_{\perp}, v_{\parallel})$ whose time derivative is expressed in terms of

$$\left\langle \frac{\Delta v_{\perp}(\tau)}{\tau} \right\rangle, \left\langle \frac{\Delta v_{\parallel}(\tau)}{\tau} \right\rangle, \left\langle \frac{\Delta v_{\perp}(\tau) \Delta v_{\perp}(\tau)}{\tau} \right\rangle, \text{ and } \left\langle \frac{\Delta v_{\parallel}(\tau) \Delta v_{\parallel}(\tau)}{\tau} \right\rangle.$$

In this case τ is chosen not only to be long compared with τ_{AC} but also compared with ω_{ci}^{-1} . From the point of view of the notion we hope to generalize this latter approach directly obtains a F-P equation in the space of constants of the cyclotron motion.

The extension to directly obtaining an F-P equation in the constants of the bounce motion is now apparent. One chooses as variables a set of constants of this latter motion, such as energy and magnetic moment or velocity and pitch angle at mid plane, and solves the single particle equation on a time scale long compared with the bounce time. Hopefully this procedure is easier than the alternative of expressing all the derivatives of the usual velocity space F-P equation in terms of constants of the bounce motion and then performing the bounce average directly. One reason for this optimism is that in following the suggested procedure one is calculating directly the coefficients of the derivatives with respect to the constants of the motion. Procedures for solving single particles' equations of motion asymptotically for large times are well established,^{2,3}

and extension of these treatments to include weak rapid fluctuating electric fields is straightforward. We have neglected the drift time scale in the above, taking it to be infinite compared to the bounce time. However, slow drifts are trivially included following the suggested procedure.

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