**THE FRACTURE TOUGHNESS
OF THIN BERYLLIUM SHEET**

George E. Tardiff, Jr.

February 28, 1974

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George E. Tardiff, Jr.

(Ph.D. Thesis)

(Michigan Technological University)

MS. date: February 28, 1974

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PREFACE

It is not possible to properly or completely document the contributions of the many individuals at the Lawrence Livermore Laboratory who made the present work possible. I wish, however, to attempt an expression of appreciation to those individuals who made particularly important contributions.

I wish to thank my wife Ruby for her encouragement and stamina over the course of the work. Ron, my son, I hope will understand and forgive the many instances when I was present but unavailable due to preoccupation. I wish to thank Lee Roberts for making the work possible. It would be difficult to overstate the technical contributions made by Bernhard Kuhn who did the fracture toughness testing and Sam DiGiallonardo who did the electron microscopy. Only Richard Kelly and I realize the extent of his many contributions.

I want to express special appreciation to three other individuals. I owe a debt of gratitude to my advisor, Professor Lloyd Heldt of the Michigan Technological University, for his support and advice. I wish to thank Bob Juntz not only for his assistance but also for a valued friendship over the course of the work. Finally, I wish to thank Doris Edwards for her consistent enthusiasm and fortitude in the editing and typing of the manuscript.

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THE FRACTURE TOUGHNESS OF THIN BERYLLIUM SHEET

ABSTRACT

The ambient-temperature fracture toughness of thin beryllium sheet (0.80 mm thick) was measured for a wide variety of metallurgical conditions. Various experiments were also carried out to develop an understanding of the factors that account for and limit the magnitude of the fracture toughness.

In order to measure the fracture toughness over the widest possible range in yield-strength and fracture-toughness levels, it was necessary to develop fracture test methods for the large-scale plastic yielding regime. Three independent methods, one developed previously by others (J integral) and two new methods (based on a compliance technique and a plastic work approach), were found to give equivalent results.

The fracture toughness of the ingot source sheet under investigation varied from 15 to 40 MPa·m^{1/2} depending on the effect of heat treatment on the microstructure. The low toughnesses were associated with the presence of small subgrains in the microstructure. It was concluded that these small subgrains enabled the bend-plane splitting mechanism to reduce the toughness. A study of the effect of crack root diameter on the fracture toughness supports the conclusion that, in material containing subgrains, the conditions that govern yielding in the grains immediately adjacent to the crack tip control the magnitude of the fracture toughness.

The high toughness conditions of the ingot sheet were associated with complete recrystallization and the absence of subgrains in the microstructure. The magnitude of the toughness in the latter case was accounted for in terms of a plane-stress fracture model based upon the Dugdale analysis, and the flow strength and capacity for through-thickness contraction measured in the tensile test. The strains measured ahead of the crack tip were found to be consistent with the plane-stress model and the assumption that the ability to contract through the thickness determines the magnitude of the fracture toughness.

The magnitude of the fracture toughness determined for powder-origin beryllium sheet was also accounted for in terms of the plane-stress model and the flow strength and through-thickness contraction measured in the tensile test. Lower toughnesses were observed for the powder sheet (~30 MPa·m^{1/2}) than for the tough condition of the ingot sheet. This difference was associated with a smaller capacity for through-thickness contraction in the case of the powder sheet because of its more pronounced preferred orientation.

INTRODUCTION

The extension to metals of the Griffith approach to characterization of the fracture resistance of glass began in earnest in the 1950's and has evolved into a widely accepted and burgeoning field of study. The bulk of the effort to date has been to develop valid methods of experimentally assessing fracture resistance (through critical strain energy release rate or, equivalently, critical stress intensity measurements). Experimental measurements of fracture resistance are based on linear elasticity and help to form a rational basis for structural design with brittle materials. Mere characterization of the fracture resistance, however, does not provide an understanding of the basic sources of fracture resistance. Such understanding is required if improvements in fracture resistance by microstructural control are to be made. In addition, significant variation in fracture resistance is generally observed in a given batch or heat of material and wider variations would normally be expected from heat to heat. While it is conceptually possible to statistically sample the fracture resistance to attain any required degree of reliability, such a procedure can be inordinately expensive and can result in overly stringent material specifications. A basic understanding of the sources of variation, on the other hand, would lead to the most efficient material specifications required to assure a given level of fracture resistance.

Understanding of the basic sources of fracture resistance in beryllium is especially lacking. The very large reported variation in measured values of fracture resistance might, in part, be attributed to variations in grain and subgrain size, dislocation substructure, preferred orientation, and the disposition of impurity elements. No convincing association of such variations with measurements of fracture resistance have been made for beryllium. Preliminary testing has indicated that some of the observed variability may be due to improper analysis of fracture resistance test results. These preliminary tests have shown that beryllium sheet can exhibit large nonlinear deflections in the fracture test. The effect of nonlinearity on the validity of a data analysis by linear elastic fracture methods is unknown, preventing an unequivocal comparison of the fracture resistance of specimens in different microstructural states.

It is the purpose of this thesis research to investigate and elucidate some of the basic origins of fracture resistance in beryllium sheet in order to develop an improved understanding of the sources of variation. It is hoped that this will provide some basis for control and improvement of fracture resistance by microstructural control. A prerequisite to this task is the evaluation of linear and nonlinear fracture resistance test methods. This problem, therefore, must receive major attention in the thesis research. Having established a valid method of measuring the fracture resistance, an attempt is made to account for the magnitude of the measured toughness in terms of the ordinary uniaxial tensile properties, microstructure, and fracture mode.

1. REVIEW OF FRACTURE MECHANICS

Theory of Compliance Methods for Measuring Fracture Resistance

A valid interpretation of fracture resistance measurements is required if effects of metallurgical variations are to be unequivocally established. It is necessary, therefore, to begin this study by reviewing the theoretical basis for measuring fracture resistance. Irwin and Kies,¹ Irwin,² and Irwin and Wells,³ have discussed the basis for experimental determination of fracture resistance and the following review is taken from their developments.

If a cracked body such as that illustrated in Fig. 1 is loaded in the range where the body is predominantly elastic then a linear relation is found between the applied force, F , and deflection, Δ , such as that illustrated in Fig. 2. In the following discussion the amount of energy absorbed in plastic deformation at the crack tip is assumed to be negligible in comparison with the elastic energy stored in the crack tip stress field. Real metals exhibit various departures from this idealization and the effect of this is discussed in a subsequent section on experimental methods.

If the crack is extended by a small increment da then the specimen becomes more compliant and the load-deflection behavior changes as noted in Fig. 2. It is possible, using the information given in Fig. 2, to compute the energy made available to do work as a consequence of the crack extending from a to $a + da$ under constant force F . This is given by the work done by the applied load in moving through the deflection $d\Delta$ less the increase in elastic strain energy V stored in the specimen as a consequence of crack extension:

$$\begin{array}{l} \text{energy made available to do} \\ \text{work due to crack extension} \\ \text{from } a \text{ to } a + da \end{array} = Fd\Delta - dV. \quad (1)$$

The increase in elastic strain energy is given by the difference between the areas under the load-deflection curves corresponding to crack lengths $a + da$ and a , or:

$$dV = \frac{1}{2} (\Delta + d\Delta) \cdot F - \frac{1}{2} \Delta \cdot F = \frac{1}{2} Fd\Delta. \quad (2)$$

The strain energy release rate, $\mathcal{G}$, is now defined as the energy made available to do work per unit area of crack extension and so is given by (1) divided through by da . Using this and substituting (2) into (1) and also taking the thickness of the specimen as unity the following is obtained:

$$\begin{array}{l} \text{energy made available to do} \\ \text{work per unit area of crack} \\ \text{extension} \end{array} = \mathcal{G} = \frac{F}{2} \frac{d\Delta}{da}. \quad (3)$$

Now the compliance, λ , is defined by $\Delta = \lambda F$ so that (3) becomes:

Fig. 1. Specimen with a through-thickness edge crack.

Fig. 2. Force vs deflection behavior for a specimen cracked under constant force.

Fig. 3. Force vs deflection behavior for a specimen cracked under constant deflection.

$$\mathcal{G} = \frac{F^2}{2} \frac{d\lambda}{da}. \quad (4)$$

since F was held constant. Equation (4) provides the basic experimental basis for measuring fracture resistance. If an elastic specimen containing a crack of length a is dead weight loaded to a point where crack extension becomes unstable then the fracture resistance may be found using the load required for onset of unstable fracture and the value of $d\lambda/da$ determined from a prior compliance calibration as follows:

$$\begin{aligned} \text{fracture resistance} = R &= \mathcal{G}_{\text{crit}} \\ &= \frac{(F \text{ at fracture})^2}{2} \left(\frac{d\lambda}{da} \right) \text{ evaluated at } a_0. \end{aligned} \quad (5)$$

There has been argument in the literature regarding the proper use of symbols for fracture resistance. The fracture resistance as given by Eq. (5) above ($\mathcal{G}_{\text{crit}}$ or R), however, is completely unequivocal in meaning and simply gives the strain energy release rate at which unstable fracture commences. It is noted, however, that the value of $\mathcal{G}_{\text{crit}}$ measured is in general very dependent on the detailed condition of the crack tip, that is, the crack front shape and sharpness. This point will be discussed in a following section. The compliance calibration referred to above can be carried out by measuring the compliance $\lambda = \Delta/F$ as a function of crack length. The value of $d\lambda/da$ corresponding to the crack length in the specimen being fracture tested can then be determined by measuring the slope of the curve defined by λ vs a at that crack length. An experimental compliance calibration developed for the single edge notched (SEN) specimen is given in Ref. 4. An important finding in that experimental study was that $d\lambda/da$ became independent of the gage length when compliance measurements were made over a gage lengths greater than twice the width of the specimen ($>2W$). The elastic stress field does not become uniform over gage lengths less than $2W$ with cracks of reasonable size present and this results in a dependence of $d\lambda/da$ on gage length for short gage lengths.

The above development was carried out for a specimen which undergoes crack extension under constant force. Most testing machines are stiff, however, and this results in a reduction in load as the specimen deflects because of cracking. This results because the testing machine normally has a high effective elastic modulus and also because the crossheads of the testing machine require a finite period of time to respond to the specimen deflection. The extreme of this behavior is cracking under fixed displacement conditions and it is necessary to consider this problem in order to assess the consequence of using a stiff machine for fracture testing. Figure 3 illustrates the drop in load which occurs when crack extension takes place under constant displacement. Under this condition the applied load does no work as the specimen cracks so that the strain energy release rate is given by the loss in the strain energy stored in the specimen per unit area cracked as follows:

$$\mathcal{G} = \frac{1}{2} \Delta \cdot \frac{(F_1 - F)}{da} = \frac{1}{2} \Delta \frac{dF}{da}. \quad (6)$$

Differentiating the relation $\Delta = \lambda F$ with respect to a at constant Δ , the following is obtained:

$$\lambda \frac{dF}{da} + F \frac{d\lambda}{da} = 0, \quad (7)$$

so that

$$\frac{dF}{da} = -\frac{F}{\lambda} \frac{d\lambda}{da}. \quad (8)$$

Substituting (8) into (6) and again noting that $\Delta = \lambda F$, the same expression found for fixed force conditions results:

$$\mathcal{G} = \frac{1}{2} F^2 \frac{d\lambda}{da}. \quad (4)$$

Since the same expression for $\mathcal{G}$ is obtained assuming both fixed load and fixed displacement it is not expected that machine stiffness will affect the critical conditions for the onset of unstable fracture.

Equivalence Between Compliance Methods and Stress Analysis

The compliance method of the determination of fracture resistance does not require any special knowledge of stress analysis. It is conceptually clear, however, that the same results ought to be derivable from elastic stress analysis since strain energy density and hence strain energy release rates depend only on the elastic stress field. Irwin was the first to show the fundamental connection between compliance methods and stress analysis methods and the following development follows his work.^{2,5}

Irwin⁵ developed the stress field equations giving the stresses at the tip of a cracked body under a tensile stress starting with a method due to Westergaard.⁶ The details of this development are based on classical linear elasticity theory which involves the discovery of an Airy stress function which will satisfy the equations of compatibility and equilibrium.⁷ Westergaard gave solutions which allowed the stress fields associated with various plane strain and plane stress crack problems to be constructed. Irwin generalized the result and found that for cracks under tensile loading (no applied shear stresses resolvable in the crack plane) the stresses and displacements can be given by:

$$\sigma_x = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right).$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right),$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2},$$

$$\sigma_z = \nu (\sigma_x + \sigma_y), \quad x_3 = y_3 = 0. \quad (9)$$

and

$$u = \frac{K_I}{G} \left(\frac{r}{2\pi} \right)^{1/2} \cos \frac{\theta}{2} \left(1 - 2\nu - \sin^2 \frac{\theta}{2} \right),$$

$$v = \frac{K_I}{G} \left(\frac{r}{2\pi} \right)^{1/2} \sin \frac{\theta}{2} \left(2 - 2\nu - \cos^2 \frac{\theta}{2} \right),$$

$$w = 0. \quad (10)$$

where the equations have been written for the plane strain case ($w = 0$) and the symbols have the following definition:

$\sigma_x, \sigma_y, \sigma_z$ - normal stress components

$\tau_{xy}, \tau_{xz}, \tau_{yz}$ - shear stress components

r, θ - polar coordinates

u, v, w - displacement components

G - shear modulus

and

K_I is the opening mode stress intensity parameter which depends on the details of the crack geometry and loading and can be found, in principle, by stress analysis.

The coordinate system appropriate to Eqs. (9) is given in Fig. 4. Equations (9) are given in the form used by Paris and Sih⁸ to conform with current practice. The K_I

Fig. 4. Coordinate system for crack-tip stresses.

parameter is referred to as a stress intensity factor because its magnitude governs the intensity of the stress field described by Eq. (9). The subscript I indicates that the applied stress intensity opens the crack normal to its plane. Stress field equations corresponding to applied shears in the plane of the crack are given in Ref. (8) and this type of loading leads to definitions of stress intensity factors in shear, K_{II} and K_{III} . The latter modes of stress application will not be considered in the present work.

Irwin's approach to showing the equivalence between compliance methods and the stress analysis approach involve the calculational model depicted in Fig. 5. In this model the work required to close a small increment of crack of length α is computed from the known stresses and displacements. At closure the stresses applied to the virtual crack surfaces, S_y , must be infinitesimally greater than the y-direction stresses given by Eqs. (9) on $\theta = 0$. Using this and taking into account the shift in coordinate system in Fig. 5 the closure stress is found:

$$S_y = \left[\frac{p K_I}{2\pi(\alpha - r)} \right]^{1/2}, \quad (11)$$

where p is a proportional loading factor which varies from 0 to 1 as the crack is closed.

The displacement, v , over which crack closure occurs is found from (10) for v with $\theta = 180^\circ$:

$$v = (1 - p)(1 - \nu) \frac{K_I}{G} \left(\frac{2r}{\pi} \right)^{1/2}, \quad (12)$$

where $(1 - p)$ varies from 1 to 0 as the crack is closed.

The work per unit thickness required to close an element of length, dr , along α is:

$$\text{work to close along } dr = -2 \int_{v=v_{\max}}^{v=0} (S_y dr) dv, \quad (13)$$

where the factor of 2 takes into account both crack surfaces.

From (11) and (12) Eq. (13) may be written:

$$\begin{aligned} \text{work to close along } dr &= -2 (S_y, \max dr) v_{\max} \int_0^1 -p dp \\ &= S_{y_{\max}} v_{\max} dr. \end{aligned} \quad (14)$$

Fig. 5. Model for computing relation between energy rate and stress intensity factor.

Inserting the maximum values of S_y and ν into (14) and using $G = E/2(1 + \nu)$ where E is Young's modulus, the total work done in closing α can be found:

$$\begin{aligned} \text{total work to close } \alpha &= \frac{2(1 - \nu^2)}{\pi} \left(\frac{K_I^2}{E} \right) \int_0^\alpha \left(\frac{r}{\alpha - r} \right)^{1/2} dr \\ &= (1 - \nu^2) \frac{K_I^2}{E} \cdot \alpha. \end{aligned} \quad (15)$$

Since $\mathcal{G}_I$ is defined as the energy released in an increment of crack extension per unit area of crack extension, it follows that $\mathcal{G}_I$ is given by (15) divided by α , or:

$$\begin{aligned} \mathcal{G}_I &= \frac{(1 - \nu^2) K_I^2}{E} \text{ for plane strain:} \\ \mathcal{G}_I &= K_I^2/E \text{ for plane stress.} \end{aligned} \quad (16)$$

Equation (16) gives the relationship between the compliance which is experimentally determinable and the stress intensity factor which is derivable from methods of stress analysis. It is noted that for plane stress the factor $(1 - \nu^2)$ does not appear.⁸

An example of an experimental verification of the equivalence between an experimental determination of $\mathcal{G}_I$ by the compliance method and that computed from stress analysis is given in Ref. 4.

Critical Stress Intensity Factor

Together with solutions for the stress intensity factor, the relations (4), (9), and (16) have formed the foundation for the subsequent development of the field of fracture mechanics. Solutions for the stress intensity factor developed by stress analysis and in some cases supported by compliance measurements are available for a wide variety of specimen configurations.⁹ An experimental measurement of fracture resistance involves the loading of a specimen such as the SEN specimen shown in Fig. 1 to the point of fracture. The fracture load, crack length, and specimen dimensions are then used to calculate a critical value for the stress intensity, K_{crit} ,^{*} using the stress intensity solution applicable to the specimen configuration under test. The particular solution for stress intensity for the finite width SEN specimen as given in Ref. 9 is reproduced below:

^{*}In this study critical stress intensity and critical strain energy release rate values are given by K_{crit} and $\mathcal{G}_{crit}$, respectively. The subscript I is dropped because all of the present work involves the opening mode of crack extension.

$$K_I = \frac{Y \cdot F a^{1/2}}{bW},$$

$$\text{where } Y = 1.99 - 0.41 \frac{a}{W} + 18.70 \left(\frac{a}{W}\right)^2 - 38.48 \left(\frac{a}{W}\right)^3 + 53.85 \left(\frac{a}{W}\right)^4$$

a = crack length

F = force

h = thickness

W = width.

(17)

This particular solution resulted from stress analysis⁹ and it has been shown¹⁰ to give K values which are within a few percent of those deduced from compliance measurements over the range of a/W from 0.2 to 0.4. The critical stress intensity factor is also referred to as fracture toughness in the literature and this usage will be used in the present work. The validity of the above procedure depends on the size of the plastic zone being very small with respect to the dimensions of the elastic stress fields described by Eqs. (9). At present there is no strictly theoretical basis for taking into account the effects of crack tip plasticity on K_{crit} measurement. The approach to this problem has been primarily empirical and is described in following sections.

Effect of Notch Sharpness and Thickness

An increase in crack tip sharpness or thickness would be expected, on an intuitive basis, to make relaxation of stress by plastic flow at the crack tip more difficult. On this basis one would expect the fracture toughness to be sensitive to crack tip sharpness and thickness. These expectations are borne out by experiment. Figure 6, taken from Ref. 11, shows the substantial decrease in toughness which occurs as the crack tip root radius is decreased for H-11 steel. These data indicate that there exists a lower limiting value of root radius below which the toughness does not decrease. A similar relationship has been reported for 7075-T6 aluminum alloy.³

The effect of thickness on fracture toughness is typified by the data reported by Irwin¹² given in Fig. 7. The increase in toughness with thickness at small thickness, the maximum, and the decrease to a limiting value at large thickness as shown in Fig. 7 is found to be typical behavior for a wide variety of metallic materials.¹³ The rise in the curve can be associated with the increase in volume of deforming metal at the crack tip with increasing thickness.¹⁴ The peak value marks the beginning of the effect of transverse constraint in limiting of volume of plastically deforming metal at the crack tip.¹² Fracture in cases of all thicknesses less than the thickness corresponding to peak toughness will be defined as plane stress. Plane stress implies the absence of through-thickness constraint. The limiting value at large thickness is considered to be a basic material property and has been termed the plane strain fracture toughness, K_{Ic} .⁹

Fig. 6. Dependence of fracture toughness on crack-tip root radius found for H-11 modified steel. Based on data from Ref. 11.

Fig. 7. Thickness dependence of fracture toughness for 7076-T6 aluminum alloy. Based on data from G. R. Irwin.¹²

Plane Strain Fracture Toughness - K_{Ic}

The proposition that K_{Ic} represents a lower limiting value of fracture toughness (in reasonable thicknesses) has led to substantial efforts aimed at defining the conditions necessary for valid K_{Ic} testing. This effort has resulted in an ASTM standard practice for plane strain fracture testing (designation ASTM E399). Much of the work pertinent to this standard is reviewed in Refs. 9 and 15. The principle requirements of this method include fatigue precracking as a means of notch preparation and the use of specimens of minimum specified dimensions. Fatigue precracking is required to insure the sharpest possible crack tip condition and therefore that the lowest possible value of fracture toughness, K_{Ic} , will be measured. The specimen size requirement includes the condition that both the thickness, h , and crack length, a , will exceed a value given by:

$$h \text{ and } a > 2.5 \left(\frac{K_{Ic}}{\sigma_{ys}} \right)^2, \quad (18)$$

where σ_{ys} is the yield strength measured in uniaxial tension.

The latter condition is found by experiment to be necessary to insure that the fracture toughness value will be applicable to specimens (or hardware) with dimensions beyond those used for the specimens.^{9,15} There has been some discussion which suggests that (18) may not be sufficient to guarantee this independence of size.

The basis for (18) lies in the requirement that the plastic zone size must be very small compared with the spatial extent of the elastic stress field ahead of the crack tip in order that the linear elastic fracture approach reviewed above be applicable. An order of magnitude estimate of plastic zone size can be found from Eqs. (9) for σ_y on $\theta = 0$ with σ_y set equal to σ_{ys} .³ This gives an approximation to the plastic zone size, r_y , for plane stress conditions as follows:

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2. \quad (19)$$

Equation (19) leads to the expectation that the plastic zone size will increase in proportion to the parameter $(K_I/\sigma_{ys})^2$. If the zone of plasticity at fracture is to be kept small with respect to relevant specimen dimensions, then it is clear that these dimensions must exceed some multiple of $(K_{Ic}/\sigma_{ys})^2$. This idea together with a large amount of experimentation⁹ led to the semiempirical rule given by (18). The ASTM standard practice also includes limits on the extent of deviation from linearity between load and deflection measured in a fracture test.

Plane Stress Fracture Toughness

No standard method exists for fracture testing under circumstances where (18) is not satisfied, e.g., in the case of all thicknesses less than ~19 mm for the 7075-T6 data given in Fig. 7. The applicability of linear elastic fracture methods for such cases must in general be experimentally evaluated. Since an assessment of the linear elastic approach to measuring fracture toughness must begin with a phenomenological understanding of the fracture test, the different types of observed behavior will be briefly reviewed. Figure 8 shows various types of load-deflection behavior which have been observed in tests of precracked metallic specimens.^{9,13} Other intermediate types of behavior have also been observed.¹⁵ The deflections may be measured either at the crack mouth (ASTM E399) or over a gage length spanning the crack plane as indicated in Fig. 9 (Ref. 4). For a specimen behaving in an elastic manner the displacement is expected to be directly proportional to load as shown at a in Fig. 8. The deviations from linearity indicated in Fig. 8 can occur due to crack extension or plastic deformation or a combination of both.⁹ It is expected that evaluation of fracture toughness from the force at which first observable cracking occurs would be least ambiguous in the cases of records a and b in Fig. 8 because no significant deviation from linearity occurs prior to crack extensions.⁹ For records such as c, d, and e, however, the possibility that plastic deformation preceded or accompanied crack growth prevents this simple interpretation of the records in terms of linear elastic fracture mechanics. If crack extension measurements are made during the test, however, they can be used together with the corresponding forces to compute a K_{crit} or $\mathcal{G}_{crit}$ value corresponding to that particular crack length. To assess the relative contribution of plastic deformation to nonlinearity the compliance V/F (where V is crack mouth opening and F is the associated force) can be determined and compared to the change in compliance expected on the basis of the amount of crack extension from correlations between compliance and crack length made previously. An example of such a correlation for the edge cracked specimen geometry is reproduced in Fig. 10. If the contribution of plasticity to nonlinearity is small, then the fracture resistance in terms of either $\mathcal{G}_{crit}$ or K_{crit} can be determined at each crack length. This fracture resistance can then be plotted vs crack extension to obtain what is known as an R curve for the material.¹⁶ Examples of such R curves are given in Fig. 11. The important conclusions from Fig. 11 are that the fracture toughness can depend on crack extension and that the critical value of fracture toughness can, therefore, depend on specimen configuration. The R curve labeled a is shown together with the $\mathcal{G}$ curves (lines) corresponding to three stress levels. As the stress increases the slope of the $\mathcal{G}$ vs a line increases. Unstable fracture occurs when the following conditions are met¹⁶:

$$\text{and } \left. \begin{array}{l} \mathcal{G} = R \\ \frac{\partial \mathcal{G}}{\partial a} > \frac{2R}{\partial a} \end{array} \right\} \begin{array}{l} \text{condition for} \\ \text{fracture instability.} \end{array} \quad (20)$$

Fig. 8. Typical observed force-deflection behaviors.^{9,13} The peaks indicate unstable fracture.

Fig. 9. Displacement measuring positions.

Fig. 10. Correlation between compliance measured at the crack mouth and crack length for the edge cracked specimen.⁹

Fig. 11. Examples of R curves; a, b, and c.

Since the relation between $\mathcal{G}$ and a can have curvature depending on specimen size and configuration it follows that the point of tangency between the $\mathcal{G}$ and R curves can vary and therefore that the measured value of $\mathcal{G}_{crit}$ will depend on specimen design.¹⁶ The R curve b shows pop-in behavior which occurs as a consequence of a small amount of unstable cracking. The crack arrests because the R curve shape is such that: $\partial R / \partial a > \partial \mathcal{G} / \partial a$ after a small amount of crack extension. The R curve c shows behavior expected from a brittle material. It is not expected that the $\mathcal{G}_{crit}$ value would be dependent on specimen geometry in such cases.

Understanding of the mechanism of the growth of fracture resistance with crack extension is imperfect but this behavior is generally attributed to the growth and work hardening of the plastic zone as the crack extends.¹⁶

For force-deflection behavior such as that indicated by curve e, in Fig. 8, the observed deviation from linearity is due entirely or substantially to plastic deformation at the crack tip. For such records there is no available method for interpreting the results using linear elastic methods.

Nonlinear J-Integral Method

A method of interpretation of nonlinear response due to plasticity such as that illustrated by curve e in Fig. 8 has been proposed and tested by Begley and Landes¹⁷ (B&L). The basis of this approach lies in Rice's¹⁸ development of the mathematics of nonlinear elastic fracture mechanics (two dimensional) which also applies to elastic-plastic metals when treated by the deformation theory of plasticity. The deformation theory reduces to the nonlinear elastic case except in cases of unloading.¹⁹ Rice proved the existence of a line integral, J , the value of which is independent of the path of integration taken around the crack tip. J is given by:

$$J = \int_{\Gamma} W dy - T \left(\frac{\partial u}{\partial x} \right) ds, \quad (21)$$

where: Γ is the path of integration surrounding the crack tip,

W is the elastic strain energy density,

T is the normal stress acting on Γ ,

u is the y direction displacement,

and ds is an increment of path length.

Rice also showed that the value of J gives the potential energy, U , difference between identically loaded specimens containing cracks of different sizes, or:

$$J = \frac{dU}{da}. \quad (22)$$

For the case of nonlinear elastic bodies J as given by (22) is equivalent to $\mathcal{G}$, the strain energy release rate for the linear elastic case. The variables contained in (21) are all

descriptive of the nonlinear elastic stress field, J can then be viewed as a parameter that characterizes the nonlinear stress field in much the same way that $\mathcal{G}$ characterizes the linear elastic field, viz: $\mathcal{G} = K^2/E$. B&L point out that the analytic expressions developed by Hutchinson²⁰ and Rice and Rosengren²¹ to describe the nonlinear stress and strain fields support the idea that J is a parameter which describes the nonlinear field. The latter led B&L to postulate that measurement of a critical value of J would provide a method of characterizing the fracture state which is equivalent to characterization of fracture in the linear elastic case by $\mathcal{G}_{crit}$.

The experimental approach used by B&L was to evaluate U for various crack lengths by measuring the work done in loading cracked specimens to given deflections. This work is the area under the force-deflection curve at the given deflection. From (22) the J value corresponding to that deflection was then found from the slope of the plot of the work done to a given deflection vs crack length. The critical value of J (J_{IC} as used by B&L) was then determined from this by noting the value of J corresponding to the measured deflection at fracture. The experimental work done by B&L resulted in J_{IC} values for small nonlinear (25 and 50 mm thick) fracture specimens of a Ni, Cr, Mo, V steel that were the same, within material variation, as $\mathcal{G}_{IC}$ values determined from much larger, linear elastic specimens (200 mm thick). This was taken as evidence by B&L that the critical J concept can be used to characterize fracture in circumstances of nonlinear load deflection behavior and that the critical value of J thus determined will be equal to the linear elastic fracture resistance, $\mathcal{G}_{crit}$.

It is noted that the procedure used by B&L to evaluate J_{crit} is, in the case of linear behavior, identical to the basis for the measurement of $\mathcal{G}_{crit}$ by the linear elastic compliance method described previously.

II. ORIGIN OF FRACTURE RESISTANCE IN METALS

Crack Tip Plasticity

Most metals do not suffer a dramatic loss in strength due to the presence of small cracks or defects as do glass-like solids. It has been long recognized that the relative insensitivity of the strength of metals to the presence of defects results because of their ability to deform plastically at the crack tip. Crack tip plasticity acts to reduce the high stresses predicted for purely elastic behavior [Eqs. (9)] to the order of the flow stress in uniaxial tension. Orowan²² and Irwin²³ suggested that in metals the value of $\mathcal{G}_{crit}$ represented the plastic work done per unit area of crack advance. Orowan pointed out in Ref. 22 (in a note added in proof on p. 214) that the measured extent of plastic strain adjacent to the crack plane is consistent, on this basis, with the known effect of crack size on brittle fracture in mild steel. It is now commonly accepted that the extension of the Griffith²⁴ theory for glass, which predicts fracture when the strain energy release rate just exceeds the surface energy, γ , for both crack surfaces ($\mathcal{G}_{crit} = 2\gamma$), to metals but with $\mathcal{G}_{crit}$ representing the plastic work done per unit area of crack advance is valid and provides a method of characterizing the brittleness of metals. It is suggested that the long time between the development of the Griffith theory and the acceptance of this method for metals (~40 years) is partly due to the fact that $\mathcal{G}_{crit}$ for a metal is a measure of the ability of the metal to flow plastically at a crack tip without cracking and this does not have any obvious significance as a fundamental material property. Surface energy, on the other hand, is such a property and its value can be determined by a variety of independent methods. The value of $\mathcal{G}_{crit}$ for metals, however, has important engineering significance because it provides the basis for rational engineering design.²⁵

Dugdale Elastic-Plastic Model

It is apparent that a basic understanding of the origin of fracture resistance in metals must begin with methods which are able to predict the form and extent of crack tip plasticity as a function of $\mathcal{G}$ or K . This together with a critical strain or critical stress failure criterion might, in principle, allow the critical value of $\mathcal{G}$ to be computed. The recognition of this possibility has led to substantial efforts to characterize the elastic-plastic, crack tip stress and strain fields by continuum mechanics methods¹⁸ as well as methods based upon the mathematical theory of dislocation arrays.²⁶ These methods can predict the spatial extent of the plastic zone for the idealized cases of plane stress and plane strain deformation as a function of $\mathcal{G}$. The distribution and intensity of strain within the plastic zone near to the crack tip where the critical fracture processes occur are as yet beyond the reach of these analytical methods.

A widely used elastic-plastic model due to Dugdale,²⁷ subsequently extended by Goodier and Field,²⁸ and used to account for the magnitude of plane stress fracture toughness by Hahn and Rosenfield,¹⁴ has proven very useful in the interpretation of the

origin of fracture resistance. The basis for and principle results of this model are reviewed here.

The model considered by Dugdale is shown in Fig. 12. In the calculational model the crack is imagined to have extended to the tip of a narrow plastic zone which formed in response to the crack tip stress concentration. A stress equal to the flow stress of the metal at the crack tip is considered to act on the virtual crack surfaces. The plastic material is assumed to be rigid-plastic with flow strength equal to Y . The stress analysis for this case then reduces to the purely elastic problem of a sheet with an edge crack of length $a + \rho$ with a stress equal to Y acting to close the crack surfaces from $x = a$ to $x = a + \rho$. The remote applied stress is equal to σ . In order for the crack not to extend, the displacement of the virtual crack surfaces must be accommodated over a finite effective gage length g as indicated in Fig. 12b.

It is apparent from the model (Fig. 12a) that the y direction stress acting at $x = a + \rho$ must not exceed Y , otherwise the plastic zone would extend. Dugdale found, using Muskhelishvili's solution²⁹ for the y -direction stress field, that this condition defined the plastic zone length in equilibrium with the applied stress:

$$\frac{\rho}{a + \rho} = 2 \sin^2 \frac{\pi \sigma}{4Y} \quad (23)$$

(after Dugdale²⁷),

or

$$\frac{\rho}{a} = \sec \frac{\pi \sigma}{2Y} - 1 \quad (24)$$

(after Hahn and Rosenfield¹⁴).

Dugdale²⁷ found that (23) described very closely the length of plastic zone as a function of applied stress and crack length for a particular mild steel.

Goodier and Field²⁸ using Muskhelishvili's method found the crack wall displacement for the Dugdale model:

$$v = \frac{(a + \rho)}{\pi E} Y \left[\cos \theta \ln \frac{\sin^2 \left(\frac{\pi \sigma}{2Y} - \theta \right)}{\sin^2 \left(\frac{\pi \sigma}{2Y} + \theta \right)} + \cos \frac{\pi \sigma}{2Y} \ln \frac{\left(\sin \frac{\pi \sigma}{2Y} + \sin \theta \right)^2}{\left(\sin \frac{\pi \sigma}{2Y} - \sin \theta \right)^2} \right] \quad (25)$$

where: v is the y -direction displacement

$$\text{and } \theta = \arccos \frac{x}{a + \rho}.$$

At the crack tip:

$$\delta_t = 2v_t = \frac{8Ya}{\pi E} \ln \sec \frac{\pi \sigma}{2Y} \quad (26)$$

[Eqs. (25) and (26) after Hahn and Rosenfield¹⁴].

Fig. 12. Crack propagation models; (a) Dugdale-Muskhelishvili model, (b) idealized plastic zone model.

At low stresses ($\sigma/Y \lesssim 0.6$) Eq. (26) reduces to

$$\delta_t = \frac{\pi a \sigma^2}{EY} \quad (27)$$

after Hahn and Rosenfield¹⁴.

Substituting into (27) the stress intensity solution for the single edge notch specimen,⁸ $K = \sigma\sqrt{\pi a}$, and recalling that $\mathcal{G} = K^2/E$, the following is obtained:

$$\mathcal{G} = Y \cdot \delta_t. \quad (28)$$

Equation (28) relates the macroscopic applied strain energy release rate to the flow stress and crack opening displacement. The equation has a simple physical significance. The magnitude of the displacements in the elastic medium surrounding the plastic zone at the crack tip determines the applied strain energy release rate. These displacements are dependent on the plastic properties of the crack tip plastic zone. The larger Y is for a given δ , the greater is the force opposing crack opening and consequently the displacements (or $\mathcal{G}$) in the elastic medium must be greater. Since $\mathcal{G}$ (or K) increases with the elastic displacements these parameters must increase with Y at constant δ . The same qualitative argument can be made regarding the influence of δ at constant Y .

Hahn and Rosenfield¹⁴ point out that Eq. (28) was first recognized by Wells.³⁰ Various forms of (28) were subsequently used by Knott and Cottrell,³¹ Tetelman,³² and Hahn and Rosenfield.¹⁴

A more general form of (28) for the Dugdale Model was proved by Rice using the path independent integral (Eq. 21). His result is written:

$$\mathcal{G} = \int_0^{\delta_t} \sigma(\delta) d\delta. \quad (29)$$

In (29) Y is written as $\sigma(\delta)$ since the closure stress will in general be a function of δ due to strain hardening and reduction in area.

The fracture resistance or critical value of strain energy release rate can be associated via Eq. (28), with a critical value of crack tip displacement, δ_t^* , required to cause fracture:

$$\mathcal{G}_{crit} = Y \cdot \delta_t^* \quad (30)$$

and from Eqs. (16)

$$K_{crit} = \sqrt{E'Y \cdot \delta_t^*}$$

where: $E' = E$ for plane stress,

$$E' = \left(\frac{1}{1 - \nu^2} \right) E \text{ for plane strain.}$$

Equation (30) provides a link between the macroscopic parameter useful in engineering design with a microscopic parameter, δ_t^* , which is a measure of the ability of a metal to stretch locally at the tip of a sharp crack. The principle problem in understanding the effect of microstructure on fracture resistance reduces to understanding the effect of microstructure on δ_t^* . Two important factors which influence the magnitude of δ_t^* are: 1) the dimension over which the displacement is accommodated (effective gage length) and 2) the capacity of this effective gage length to stretch (effective elongation). Most of the understanding of the latter has been developed empirically and is discussed in subsequent sections.

The Nature of Plane Stress Fracture

Hahn and Rosenfield¹⁴ (H&R) carried out an extensive study of the nature of the plastic deformation which developed from the roots of edge slots (~0.08 mm root radius) in 3% silicon steel. The etching response of silicon steel is sensitive to plastic strain and this behavior allowed H&R to establish the pattern, extent, and intensity of strain developed at the slot tip as a function of applied stress. The deformation patterns found by H&R are schematically illustrated in Fig. 13. Deformation was observed to begin at low stress levels in the hinge character illustrated in 13a and then to develop into the 45° shear mode (13b) as the stress level was increased. As a consequence of the 45° shear, the width of the plastic zone on the surface (g in Fig. 12) was about equal to the sheet thickness. H&R found that the 45° shear became well developed with a shape approximating the Dugdale strip when the length of the zone became about equal to four times the sheet thickness. Since unrestrained through thickness shear should result in relaxation of through thickness stress, the condition for plane stress suggested by H&R for the silicon steel was:

$$\rho \geq 4h, \quad (31)$$

where ρ is the length of the plastic zone and
 h is the sheet thickness.

H&R tested the predictions of the Dugdale model for ρ (Eq. 24) and crack tip displacement δ (Eq. 26) with values deduced from their measurements of zone length, width, and strain intensity and found good agreement. H&R also noted that K_{crit} calculated from (30) with Y equal to the uniaxial yield stress and:

(a) Hinge-type

(b) 45° Shear-type

Fig. 13. Schematic drawing of plastic zone deformation types; (a) hinge type, (b) 45° shear type. Based on the work of Hahn and Rosenfield.¹⁴

$$\delta_t^* = \left(\frac{RA \cdot h}{2} \right) \quad (32)$$

where RA, the reduction in area at fracture in uniaxial tension,

is in reasonable agreement with experimental measurement of plane stress fracture toughness for the case of an aluminum alloy and a steel.¹⁴

In a subsequent paper Rosenfield, Dai, and Hahn³³ modified the Dugdale model to take into account the variation in closure stress (Fig. 12a) along the plastic zone which results from strain hardening and area reduction. Using this improved model and the criterion for instability that the crack tip strain equal the strain to failure measured in a tensile test the authors accounted quite accurately for experimental measurements of plane stress fracture toughness in terms of uniaxial properties for 4330 M steel and 2219-T87 aluminum. The value of zone width or the effective gage length was taken equal to h for the aluminum and to h and h/2 for the steel. The comparison between the measured and computed toughnesses was best with zone width h/2 in the case of the steel but the computed values were still within about 25% of the measured values with width taken equal to h. A relatively weak dependence of K_{crit} on the value taken for h is apparent from Eqs. (32) and (30) which show that $K_{crit} \propto \sqrt{h}$.

Plane Stress Fracture of Metals

It is of interest to examine the general applicability of the H&R interpretation of the origin of plane stress fracture resistance. For this the value of Y in Eq. (30) is taken as the average between the uniaxial yield and ultimate strength: $Y = \sigma_{ys} + \sigma_u/2$. Also, the effective gage length is taken equal to thickness h, and the effective capacity for strain is taken as the fractional elongation to failure in a tensile test e_f , so that $\delta_t^* = h e_f$. Equation (30) then gives a relation between the commonly measured uniaxial tensile properties and plane stress fracture toughness, as follows:

$$K_{cT} = \sqrt{Eh \left(\frac{\sigma_{ys} + \sigma_u}{2} \right) e_f} \quad (33)$$

where K_{cT} is the estimate for plane stress fracture toughness from tensile properties.

The relation (33) contains the assumptions that the effect of strain hardening can be accounted for by using an average strength and that the elongation in the tensile test is homogeneous over the gage length. The applicability of (33) was checked by comparing the predicted values of fracture toughness with those reported in the literature for the materials listed in Table 1. The thickness dependence predicted by Eq. (33) was verified by plotting the measured plane stress fracture toughness vs $\sqrt{h}$ in Fig. 14. In Fig. 15 the measured fracture toughnesses and computed fracture toughnesses, both normalized to $\sqrt{h}$, are compared for the materials listed in Table 1. The comparison

Table 1. Summary of plane stress fracture toughness and uniaxial tensile data.

Material	Sheet thickness, mm	Measured K_{IC} MPa·m ^{1/2}	Thickness, mm		Uniaxial tensile properties			Normalized plane stress fracture toughness, MPa	
			At measured K_{IC}	At onset of flat fracture	Yield strength, MPa	Ultimate strength, MPa	Elongation, %	Measured $K_{IC}/\sqrt{b}$	Computed $K_{IC}/\sqrt{b}$
B120VCA 50% cold reduced ^a	0.58	91	1.78	1.78	1200	1311	6.9	3280 ^b	3110
	1.09	88							
	1.09	115							
	1.09	126							
	1.78	132							
	1.78	136							
2024-T62 ^a	0.63	40	2.54	6.35	338	428	13.0	1830 ^b	1950
	1.14	56							
	2.54	66							
	2.54	64							
	2.54	69							
HP9-4-45 Martensitic ^a	0.51	158	1.47	3.56	1587	1704	10.5	6210 ^b	5982
	0.63	183							
	1.40	201							
	1.40	178							
	1.47	273							
	2.21	209							
	2.21	229							
4340 318°C temper ^a	0.76	147	1.27	3.56	1442	1828	6.5	5620 ^c	4690
	0.76	168							
	1.27	180							
	1.27	209							
	1.27	212							
B120VCA aged, 404°C ^a	0.61	76	^d	1.02	1118	1242	8.8	3380	3380
	0.61	77							
	0.61	99							
Maraging 1900 MPa yield ^a	0.71	124	^d	2.79	1897	1960	7.0	4670	5290
7075-T6	0.41	74	0.81	4.75	504 ^f	573 ^f	10 to 20 ^f	3800 ^c	2035 to 2860
	0.81	111							
2024-T3	3.17	191	6.35	>6.35	345 ^f	485 ^f	18 ^f	3210	2380
	6.35	238							

^aReference 34.^bFrom slope of data in Fig. 14a.^cFrom slope of data in Fig. 14b.^dNot well defined.^eReference 35.^fNot reported in Ref. 35. Data taken from Ref. 36.

Fig. 14. Measured plane stress fracture toughness vs square root of thickness for various metals.

Fig. 15. Comparison between measured plane stress, fracture toughness, K_c , and fracture toughness estimated from tensile properties, K_{cT} . Toughnesses are normalized to root thickness.

between the two is good considering the approximations inherent in (33). It is likely that if the modifications used by H&K and an experimentally derived value for the width of the accommodation zone were included in the calculated values that the agreement would be even better. The correlation suggests that the development of a plastic zone by 45° shear in the plane stress fracture mode is a good approximation to reality for many materials. It is noted, however, that the shape of the plastic zone varies from the simple strip shape and depends on the particular material.³⁷

The above indicates that plane stress fracture can, to a reasonable degree of approximation, be accounted for in terms of uniaxial tensile properties. The effect of microstructure on plane stress failure is, therefore, indirect and is exerted through its effect on uniaxial tensile properties.

Plane Strain Fracture

The phenomenological condition for plane strain fracture of metals, as described by Fig. 7, is reached when the fracture toughness becomes independent of thickness. For all known cases of plane strain fracture the fracture surface is flat and square with the specimen surfaces and the through thickness strain at failure is minute. This limit implies that the plastic work done per unit area of cracking will be constant and independent of thickness beyond the plane strain thickness. There is, as yet, no known method of predicting plane strain sharp crack fracture resistance in terms of uniaxial tensile properties or microstructure nor is it yet possible to predict the thickness at which the transition from plane stress to plane strain fracture will occur. The intuitive expectation that increasing yield stress at the expense of ductility will generally reduce fracture resistance is, however, often observed.¹⁵ It has also been found that correlations of plane strain fracture resistance of steels which fail by cleavage with the ratio or difference between σ_F^* and σ_{ys} result in simple empirical relationships.³⁸ σ_F^* is the so-called micro-cleavage fracture stress which is sometimes³⁹ taken as the stress for cleavage fracture measured in uniaxial tension. As would be expected the plane strain fracture toughness decreases as the ratio σ_F^*/σ_{ys} or difference $\sigma_F^* - \sigma_{ys}$ decreases.

One popular interpretation³⁹⁻⁴¹ of plane strain failure depends on the hypothesis that fracture will occur when stress intensification due to plastic constraint, and/or strain hardening and strain rate at the notch root raises the peak stress to the level of σ_F^* . The idea that plastic constraint can significantly intensify stresses at the root of a notch is also the basis for the traditional explanation for the increase in ductile-brittle transition temperature which attends the introduction of a notch.²² One fairly strong suggestion that the above hypothesis has some validity is provided by the experiments of Knott.^{40,42} Knott used an analytical result which showed that the stress intensification (σ_y/σ_{ys}) at general yield for a notched bend specimen depends on the included angle of the notch:

$$\frac{\sigma_y}{\sigma_{ys}} = \left(1 + \frac{\pi}{2} - \frac{\theta}{2}\right)$$

where θ is the included angle of the notch

σ_y is the y direction stress (Fig. 4)

σ_{ys} is the uniaxial yield stress.

Equation (34) was derived by Green and Hundy⁴³ using plane strain slip line field theory for an ideal-rigid plastic material. Knott fractured smoothly notched specimens, containing different included angles, at various temperatures to determine the included angle at which fracture was just coincident with general yield as a function of temperature. Knowing σ_{ys} and θ for general yield fracture, Knott was able to calculate σ_F^* via (34). He found that σ_F^* was sensibly constant although σ_y varied from 147 to 207 MPa and θ for fracture at general yield varied from 45° to 120°. The constancy of σ_F^* (for cleavage) with temperature is in line with the suggestion that the cleavage fracture stress is relatively independent of temperature.²² Also, the magnitude of stress intensification found was in fair accord with theoretical expectations.

An indication that attainment of a critical stress level in the plastic zone at the tip of a sharp crack controls the fracture resistance is provided by the work of Hahn and Rosenfield.³⁹ In this work H&R discovered an empirical correlation:

$$\frac{\sigma_F^*}{\sigma_{ys}} = 1 + 3.19 \frac{K_{Ic}}{\sigma_{ys}}, \quad (35)$$

where σ_F^* was taken as the cleavage fracture stress in uniaxial tension.

Equation (35) fit K_{Ic} measurements obtained for cleavage fracture of steel over a wide range of temperature and strain rate. Equation (35) is in qualitative accord with the previous ideas expressed in this section. Zero K_{Ic} in (35) would imply fracture as soon as the yield stress was achieved at the crack root and that a stress intensification factor $\sigma_F^*/\sigma_{ys} = 1$ would be sufficient to initiate fracture. While this limit may not be reached for real metals which always undergo some finite plastic strain to develop the conditions required for fracture,³¹ it is reasonable conceptual extrapolation. More significantly (35) shows that for higher toughness conditions the stress intensification factor required for fracture increases.

The ideas discussed above are physically appealing and promote some understanding of the basis for the successful empirical correlations which have been discovered. A difficulty arises, however, when fracture occurs prior to general yield. In this case it is found that the peak stress (at the elastic-plastic boundary) is well above the micro-cleavage fracture stress found for notched bars which fail at the point of general yielding.⁴¹ It has been suggested in explanation that the fracture stress is dependent on plastic strain and that fracture occurs within the plastic zone where the

strain is higher (and the stress lower) in cases of fracture prior to general yield. This idea is consistent with observations which indicate that the resolved normal stress for cleavage failure depends on the amount of prior shear strain. As an example, the critical resolved normal stress for basal plane fracture of zinc single crystals is dramatically reduced as the amount of prior shear strain is increased.⁴⁴ For fracture at general yield the region of peak stress extends over a greater proportion of the plastic zone thus fracture can occur at the position of greatest strain at the peak stress level.⁴¹ This explanation is largely speculative, however, and other explanations include the idea that the smaller plastic zone associated with fracture prior to general yield is less likely to contain a weak fracture nucleation site.

It is evident that a convincing and useful continuum mechanics approach to the problem of understanding plane strain fracture toughness does not yet exist. In most real cases fracture occurs between the maximum plane stress value and the plane strain limit. For these cases the plastic strain varies through the thickness and no analytic solutions are available to deal with this situation. It may be more productive, therefore, to seek understanding of the sources of fracture toughness through attempts at correlation with uniaxial properties and an evaluation of the effect of microstructure, composition, and processing with the help of the simple Dugdale model and insights gained by the plane strain fracture studies.

III. REVIEW OF PROPERTIES AND METALLURGY OF BERYLLIUM

Variability in Fracture Toughness

Figure 16, taken from a review by Conrad, *et al.*,⁴⁵ illustrates the very large range in fracture toughness values reported for beryllium. As pointed out in this review, some of the reported variation reflects differences due to precracking procedure. Also, some of the variation is undoubtedly due to metallurgical differences associated with processing practice and impurity content. No convincing association of fracture toughness with metallurgical differences was, however, made in the review.

It is possible that some of the data included in the review by Conrad, *et al.* are erroneous (as the authors themselves pointed out) due to the failure to adhere to the basic tenets of linear elastic fracture mechanics. Data from two of the sources cited (Albertin and Harris and Dumegan), for example, indicate that the nominal net section stress at fracture was near or exceeded the material yield stress in many cases. This would indicate that nonlinear deflections should have been evident in the fracture test. No indication was given in these sources, however, of the extent of nonlinearity observed. In Fig. 17 the effect of using the equations of linear elastic fracture mechanics in situations where failure is preceded by large scale plastic yielding (limit load failure) is illustrated. For this illustration the single edge notched specimens for which the stress intensity solution is given by Eq. (17) is used. The dependence of nominal stress on load and specimen dimensions is given by the equation in Fig. 17. A value for the apparent fracture toughness in terms of the nominal stress at failure and specimen dimensions is obtained by combining the latter two equations. The resulting relationship and the dependence of the apparent fracture toughness on specimen width for the case of a limit load failure which occurs at a nominal stress of 172 MPa is shown in Fig. 17. The figure shows that the apparent fracture toughness is very dependent on specimen size in the case of a limit load failure and that smaller specimens lead to lower apparent toughnesses. It is important to note that the value of K apparent in the case of a limit load failure bears no obvious relation to the actual value of the fracture toughness except that it may be a lower limit. It is possible that this effect is responsible for some of the low fracture toughnesses reported by Albertin for ingot sheet beryllium. In this regard, it is noted that Albertin used narrow specimens for the tests (38 and 17.8 mm width) and that fracture of the lower strength ingot sheet specimens occurred at nominal stresses which approximated the material yield stress. In the case of the higher strength ingot sheet specimens (~276 MPa yield) fracture did occur at relatively low nominal stress levels ($\sigma_{\text{nom}}/\sigma_{\text{yield}} < 0.6$) and it is probable that in these cases the tests were linear elastic. The metallurgical factors responsible for this relatively high yield strength may have also been responsible for the low toughness exhibited by this higher strength ingot sheet.

Fig. 16. Beryllium fracture toughness data from many sources collected by H. Conrad et al. in Ref. 45. Note range of scatter.

Fig. 17. Apparent fracture toughness derived from limit load failure of ingot sheet at 172 MPa nominal stress.

It is also noted that Albertin used surface flawed specimens to evaluate the toughness of beryllium sheet. It is possible that a different fracture toughness value will result from tests of surface flawed specimens and specimens containing through-the-thickness cracks due to differences in constraint which may accompany development of a plastic zone in each case. Surface flawed specimens, for example, may give rise to more nearly plane strain conditions and to lower fracture toughness. Stress relaxation due to plastic flow may, on the other hand, more readily lead to plane stress conditions and a higher fracture toughness in cases of through-the-thickness cracks. In this connection it is noted that the fracture toughnesses of powder origin beryllium sheet reported by Albertin are much lower ($20.6 \text{ MPa}\cdot\text{m}^{1/2} \text{ max}$) than those found for powder sheet of similar thickness by Brush Wellman, Inc. and NASA-Lewis (see Fig. 16) using through-the-thickness cracks. It is also possible that metallurgical effects can account for the differences in measured toughness but there was too little characterization of the sheet to examine this possibility.

In view of the above discussion, the statement by Conrad, et al.,⁴⁵ that the fracture toughness values for powder sheet "are 2 to 3 times larger than that (sic) reported for ingot sheet" cannot be taken as conclusive evidence that ingot sheet is more brittle than powder sheet. Also contained in the review by Conrad, et al., is a plot of fracture toughness vs elongation measured in a uniaxial tensile test for beryllium sheet which purports to show that fracture toughness increases with uniaxial tensile elongation. This plot carries with it the strong inference that ingot source sheet will be less tough than powder source sheet in general since ingot sheet generally exhibits much lower uniaxial elongation than powder sheet.

While it is possible that the actual toughness of ingot sheet could be less than powder sheet due to metallurgical effects, it is also possible to conclude that ingot sheet could be the tougher material. For example, ingot sheet can have a much less developed preferred orientation and therefore can have a greater capacity for through-the-thickness contraction than powder sheet (to be discussed in following sections). As discussed in Ch. II, greater fracture toughness might be expected from a material which has a greater ability to contract through the thickness at least in cases of plane stress fracture.

An important goal of the present thesis research was to assess the fracture resistance of ingot sheet to establish a basis for comparison of the toughness of ingot sheet and powder sheet. This required the development of methods of nonlinear fracture analysis since preliminary experiments showed that ingot sheet can exhibit large nonlinear deflections in a fracture test. It was also necessary to consider the effects of metallurgical factors on the fracture toughness since it is possible that the fracture toughness of either class of materials can vary due to microstructural influences. The metallurgical factors which might influence the fracture toughness include: preferred crystallographic orientation, grain and subgrain size and shape, dislocation substructure, and the kind, amount and distribution of impurity elements. To establish a basis

for the experimental program, a review of the microstructural aspects of the flow and fracture of beryllium is given in the following sections.

Mechanical Behavior of Crystals

The preferred orientation which develops in fabricated beryllium sheet and the phenomenological characteristics of fracture in beryllium can be best understood in terms of the mechanical behavior of single crystals. For this reason a review of single crystal behavior is given here.

Predominant Slip Modes - The early work on the phenomenological aspects of the deformation and fracture of beryllium single crystals reported by Lee and Brick^{46,47} and Tuer and Kaufmann⁴⁸ (hereafter referred to as the LBT&K work) still provides a good starting point for a review of the mechanical behavior of beryllium. LBT&K grew beryllium single crystals from the melt and studied their deformation and fracture characteristics as a function of orientation and temperature. The planes of easy slip at room temperature were found to be the basal planes $\{0001\}$ and the first order prism planes $\{1\bar{1}00\}$ of the hexagonal close packed lattice. These planes are illustrated in Fig. 18. The slip direction was established as $\langle 2\bar{1}\bar{1}0 \rangle$ for both slip planes. LBT&K found that basal slip produced relatively straight slip lines and that the stress resolved in the slip plane and along the slip direction necessary to initiate basal slip was around 14 MPa for their crystals. For crystals oriented with the basal plane parallel to the tensile axis LBT&K found that first order prism plane slip initiated at a resolved shear stress of about 69 MPa and that wavy or jagged slip traces developed on the crystal surfaces. The capacity for prism slip was found to be substantial and a drawing illustrating this is given in Fig. 19. The particular crystal shown in Fig. 19 was oriented with the basal planes and the $[10\bar{1}0]$ direction parallel to the tension axis as shown in Fig. 20. In this situation prism slip will begin on one of the systems (a or b). To accommodate extension, the prism planes on which active slip begins will rotate toward the tensile axis moving the active planes away from the 45° position and thus reducing the resolved shear stress on these planes. The conjugate planes move toward the 45° position and so increase the resolved stress on the conjugate family of planes. A simple construction shows that the resolved shear on a plane inclined at an angle θ to the tensile axis is given by:

$$\tau = F \cos \theta \sin \theta,$$

where τ is the $\bar{A}_0$ resolved shear stress,

F is the axial force, and

A_0 is the cross sectional area.

(36)

Fig. 18. Important crystallographic planes and directions in beryllium.

Fig. 19. Top and side views of a beryllium single crystal specimen with $10\bar{1}0$ parallel to the tensile axis. From a photograph by Tuer and Kaufmann.⁴⁸

Fig. 20. Orientation that causes conjugate slip on two families of first order prism planes.

This function maximizes at $\theta = 45^\circ$ showing that for a given applied force the maximum resolved shear stress lies on a plane inclined at 45° to the tensile axis. This has the effect of causing an alternating rotation of the two families of prism planes such that the overall orientation of the crystal with respect to the tension axis is maintained as extension occurs. This type of behavior led to the nearly knife edge failure pictured in Fig. 19. The reduction in area corresponding to the neck shown in Fig. 19 is about 68%, but the deformation is highly anisotropic as no contraction occurred along the c-axis (see Fig. 19b). In the case of a crystal oriented with $[2\bar{1}\bar{1}0]$ along the tensile axis as shown in Fig. 21, the first prism system to begin operating rotates toward the 45° inclination which increases the resolved shear. This favors continued slip on the system which first begins to slip. The rotation causes bending stresses to develop and this leads to failure near the grips after 8 to 12% elongation. Tuer and Kaufman also observed that slip traces corresponding to only one family of first order prism planes occurred in crystals tested in this orientation.

Reorientation Due to Slip — LBT&K note that a crystal with an arbitrary orientation with respect to the tensile axis is always observed to reorient during extension so that the basal planes become nearly parallel to the direction of extension. This process can be visualized with the aid of Fig. 22. The tensile axis, T in Fig. 22 will rotate toward $(11\bar{2}0)$ by slip on the basal plane in the $(11\bar{2}0)$ direction as extension proceeds. As the basal plane becomes more parallel to the tensile axis the resolved shear stress it experiences falls off while the resolved shear stress on the first order prism plane increases. At some point, marked P in Fig. 22, prism slip becomes favored; the direction of slip changes accordingly and the tensile axis begins to rotate toward the $(2\bar{1}\bar{1}0)$ axis. Just beyond the $(10\bar{1}0)$ axis slip becomes favored on the conjugate family and the tensile axis rotates back toward $(11\bar{2}0)$. This process repeats until finally the tensile axis lies along $[10\bar{1}0]$ as in Fig. 22. Continued extension beyond this point would cause no further change in orientation as discussed in the previous section. This review of reorientation was given because it provides a basis for understanding preferred orientations which develop during sheet rolling. This topic is discussed in a subsequent section.

Twinning — It has been established by LBT&K that the twinning direction is $\langle 10\bar{1}1 \rangle$ in $\{10\bar{1}2\}$ planes. This results in an extension along the c-axis and so is activated by a tensile stress parallel to this axis or a compressive stress along $\langle 10\bar{1}0 \rangle$. T&K point out that twinning seems to be easier in compression along $\langle 10\bar{1}0 \rangle$ than in tension along $\langle 0001 \rangle$. The resolved stress required for twinning was found by Tuer and Kaufmann to be similar to that found by them for basal slip, i.e., about 55 MPa.

C-Axis Compression — LBT&K noted that the known ambient temperature deformation modes in beryllium would not allow contraction along the c-axis of the crystal. These authors reported that compression tests parallel to the c-axis shattered the

Fig. 21. Orientation that causes slip to be favored on one family of first order prism planes.

Fig. 22. Stereographic representation of reorientation of tensile axis, T , during extension.

crystals, without evidence of plastic deformation, at stresses exceeding 1380 MPa. This inability of beryllium to plastically contract along the c-axis has been most often cited as being the basic reason for the low temperature brittleness of polycrystalline beryllium.

More Recent Studies of Single Crystal Deformation — Studies of single crystal plasticity which followed the early work of LBT&K were mostly concerned with elucidating the effect of purity on the deformation processes. These later studies have shown that purification leads to a drop from 13.8 MPa to 1.7 MPa in critical resolved shear stress (CRSS) for basal plane glide.⁴⁸ The ductility in the basal glide mode was also found to increase to ~100% from the ~27% reported for commercial purity material as reported by Tuer and Kaufmann. The CRSS for prism glide was also found to decrease with purity; from ~69 MPa for commercial purity to ~55 MPa to distilled and zone refined material.⁴⁹ This drop is proportionately much less than found for basal glide.

Very carefully conducted c-axis compression tests on crystals of nominal ingot purity and high purity 12-pass zone refined materials have confirmed the earlier conclusion that no facile mode exists for plastic relaxation in compression along the c-axis at room temperature.⁵⁰ Fracture of these crystals at room temperature occurred at stresses exceeding 2070 MPa with strain at fracture less than $5 \times 10^{-4}\%$. Small amounts of pyramidal slip were, however, identified by slip trace analysis near the fracture surface of ingot purity and 12-pass zone refined crystals compressed to failure at room temperature.⁵¹

Fracture of Single Crystals — The early work of LBT&K showed that basal plane cleavage fracture was the most easily induced fracture mode. The stress for basal fracture was found by Tuer and Kaufmann⁴⁸ to be in the vicinity of 13.8 MPa if 2 to 3% prior shear occurred on the basal planes. In an orientation where the basal planes were unfavorably oriented for slip, fracture occurred at a normal stress of about 103 MPa. Tuer and Kaufmann also pointed out that basal fracture was always accompanied by prior (11 $\bar{2}$ 0) bend plane activity. Stroh later developed a model which he used to derive a quantitative relation between the fracture stress of a crystal and its orientation in certain instances where slip and cleavage fracture occur on the same set of crystallographic planes. The model, called the Stroh bend plane splitting mechanism is indicated in Fig. 23. A normal tensile stress, σ_n is developed across the basal plane as a consequence of the shear of the dislocation wall. It is assumed that some obstacle prevents the motion of the upper half of the wall. Catastrophic crack growth is assumed to occur when σ_n reaches a value necessary to satisfy the Griffith criterion ($\phi = 2\gamma$). Stroh's development as modified by Green and Sawkill⁵² was successfully used by Herman and Spangler⁵³ to account for the magnitude, orientation dependence, and effect of impurities on the basal fracture stress in beryllium single crystals.

Fig. 23. Stroh bend plane splitting mechanism, schematic.

Herman and Spangler's view of the phenomenological aspects of the fracture of single crystals oriented for basal slip is consistent with the earlier observations by Tuer and Kaufman.⁴⁸ Tensile elongation of the specimen produces rotation of the basal planes and this causes bending about an axis in the basal plane which results in the formation of a bend plane parallel to $\{11\bar{2}0\}$. Double bend planes or pink bands are also formed. Shear through the bend planes initiates a crack and once the Stroh criterion is met the crack becomes unstable and the crystal fractures.

The formation of bend planes near a grain boundary during extension of bi-crystals has been observed to lead to early failure by a bend plane splitting mechanism similar to that observed in single crystals.⁵⁴ Allen and Moore⁵⁵ show micrographs which suggest that polygonized walls act to nucleate basal fracture in a way similar to fracture nucleation by shear of walls formed by plastic deformation.

Cleavage fracture was observed, by Tuer and Kaufmann,⁴⁸ to occur on second order prism planes $\{11\bar{2}0\}$ in the case of crystals which were not favorably oriented for basal fracture. Normal stresses of ~25,000 psi were found to be necessary to initiate second order prism fracture. Fracture along the interface between matrix and $\{10\bar{1}2\}$ twins was also observed but this was apparently always preceded by fracture on $\{0001\}$ or $\{11\bar{2}0\}$ planes. This mode of fracture was considered by Tuer and Kaufman to be relatively rare in beryllium.

Fabrication and Preferred Orientation of Sheet

Beryllium sheet is commonly prepared by rolling vacuum cast ingot (ingot sheet) or by rolling block prepared by hot pressing of powder (powder sheet). In addition, sheet may be prepared by machining directly from hot pressed powder block (HPB sheet). The ingot and powder sheet materials develop varying degrees of preferred crystallographic orientation depending on the exact details of the thermo-mechanical history. The HPB sheet, on the other hand, will be relatively isotropic.

Preferred orientation has a strong effect on the flow and fracture characteristics of rolled sheet. A review of the development of preferred orientation and its effect on flow and fracture behavior will therefore be given. Finally, some consideration will be given to the possible effect of preferred orientation on fracture toughness.

A variety of preferred orientations have been reported⁵⁶⁻⁵⁸ depending on the degree of reduction, temperature of reduction, extent of cross rolling, and final annealing conditions. Neglecting differences in detail the nature of preferred orientation which develops during rolling is very qualitatively illustrated by the pole figures in Fig. 24. The tendency of the basal plane to become parallel to the rolling plane or sheet surface is a consequence of the tendency of $\langle 11\bar{2}0 \rangle$ to become parallel to the axis of greatest extension. The $\langle 10\bar{1}0 \rangle$ texture indicated in Fig. 24a is analogous to the single crystal orientation which leads to duplex slip on the first order prism planes (see Fig. 19). Cross rolling a suitable amount results in a relatively isotropic preferred orientation in the plane of the sheet (see Fig. 24b). Very severe cross rolling

Fig. 24. Effect of rolling on preferred orientation. R indicates random pole density. (a) Moderate unidirectional rolling, (b) cross rolled for balanced texture, (c) heavily rolled in cross direction.

apparently results in the tilting of the basal planes about an axis parallel to the cross rolling direction as indicated in Fig. 24c. The degree of preferred orientation indicated in Fig. 24 is similar to that developed in powder sheet products. For ingot sheet the intensity of the preferred orientation can be substantially less than for powder sheet.⁵⁷

The mechanical properties of rolled sheet reflect the effect of preferred orientations such as those indicated in Fig. 24 in a manner consistent with the properties of similarly oriented single crystals. This is illustrated by data reported in Ref. 57 and reproduced in Fig. 25. The high elongation (25a) and fracture strength (25b) in the extrusion direction for the as-extruded material (0% cross rolling) is a consequence of the high capacity of beryllium for prism slip and the relatively high resistance to fracture on 2nd order prism planes. This particular condition roughly corresponds to the pole figure in Fig. 24a. Testing in a direction perpendicular to this direction, however, results in about zero elongation and much lower fracture stress. It has been suggested that this results because the basal planes experience high resolved shear and normal stress in this orientation (see Fig. 24a) which leads to early failure. Cross rolling to produce a nearly isotropic in plane texture (see Fig. 24b) can result in greater than 40% elongation and nearly 690 MPa fracture strength in both the longitudinal and transverse directions. The greater loss in transverse properties with more severe reductions by cross rolling is apparently related to the fact that the basal planes begin to tilt about an axis parallel to the cross rolling direction (see Fig. 24c).

It has been recognized for some time that the high tensile elongation of cross rolled material is primarily due to width contraction associated with slip on $\langle 10\bar{1}0 \rangle$.⁴⁸ It has also been appreciated that the ability of highly textured beryllium to plastically contract through the thickness is very limited because of the lack of deformation modes which would allow contraction along the c-axis. This has led to the recognition that ductility measured in a biaxial stress state such as that produced in wide sheet bend test is a more significant measure of the ability of beryllium sheet to deform.⁵⁹ It is also recognized that this is a more useful parameter from an engineering viewpoint since engineering applications generally involve complex biaxial stress states.

In Fig. 26 it is indicated that balanced biaxial tension in the plane of a sheet with a perfect $\langle 10\bar{1}0 \rangle$ preferred orientation is equivalent to c-axis compression of a single crystal. This results because no plane shears can develop in balanced tension and the only mode of deformation which would allow plastic relaxation would be compression along the c-axis.

The lack of c-axis deformation should therefore result in a phenomenological response of material with an ideal $\langle 10\bar{1}0 \rangle$ texture in balanced biaxial tension similar to the high stress, shattering failure observed for crystals in c-axis compression.

The wide sheet bend test has been used to evaluate the ductility of beryllium sheet in a biaxial state of stress.⁵⁹ In this type of test the metal on the tension side of the neutral axis tends to contract through the width to maintain constant volume as the outer fibers lengthen. The compressed metal below the neutral axis, however, tends to

Fig. 25. Effect of degree of cross rolling; on uniaxial elongation, (b) on fracture strength. Based on data of Klien et al.⁵⁷

Fig. 26. Equivalency of balanced biaxial tension of sheet with perfect $[10\bar{1}0]$ texture and crystal under c-axis compression.

oppose width contraction on the tensile face. This leads to the development of stresses in the width direction and the inhibition of plastic contraction through the width. With width contraction completely suppressed, the longitudinal strain at failure would reflect only the contribution of through-thickness strain. This reasoning has led to use of the bend test for a more significant assessment of the ability of beryllium sheet to deform than is provided by the tensile test.^{59,60} Gieniza's studies⁵⁹ show that extremely wide sheets (width to thickness ratios up to 100) are still not sufficient to suppress width contraction and that the necessary width is a very sensitive function of the fabrication history of the beryllium sheet. Using very wide sheets, Gieniza found that the through-thickness strain of fracture was ~1.2%, ~1.1%, and ~0.18% for HPB sheet, hot upset sheet, and cross-rolled powder sheet, respectively. This drop in through-thickness ductility is in the order of increasing preferred orientation. In the case of the nearly isotropic HPB the through-thickness strain is similar to the tensile elongation, but in the other two cases the through-thickness strains are substantially less than the tensile elongations. In a plane stress fracture toughness test the capability for through-thickness contraction is expected to govern the level of fracture toughness measured. Equations (9) given in Ch. I, for example, show that the crack tip elastic field is balanced biaxial tension (assuming the z direction stress equal to zero). While the stress state in a plastic zone contained within the crack tip elastic field is not known it is apparent from the configuration that width contraction through the crack plane is highly constrained. Thus the contribution of width contraction to the ability of the crack tip metal to deform is probably negligible at least in cases where fracture occurs in the absence of general yielding. Thus, the plane stress fracture resistance of beryllium sheet would be expected to be less in cases of greater preferred orientation, other things being equal. It is possible, however, that the fracture toughness would not drop to very low levels even in cases of very perfect basal plane textures. The plane strain hinge mode of crack tip plasticity pictured in Fig. 13a could conceivably result in a fracture toughness (plane strain) in excess of that expected on the basis of plane stress [Eq. (33) with the effective elongation taken as the thickness contraction].

Another aspect of preferred orientation that might affect both the magnitude and anisotropy of fracture toughness is the orientation of fracture planes in the sheet. For a sheet with a strong $\langle 10\bar{1}0 \rangle$ preferred orientation, for example, the $\{11\bar{2}0\}$ fracture planes lie at about 60° to the rolling direction. Thus lower toughness values might be discovered with precracks oriented at 60° to the rolling direction, i.e., along $\{11\bar{2}0\}$ planes. For material with an isotropic in-plane texture such as that given in Fig. 24b it is not apparent that an orientation dependence of the toughness would exist. In the case of textures such as in Fig. 24c, the toughness would be expected to be lowest with crack parallel to the cross-rolling direction.

Grain Size and Substructure of Beryllium Sheet

Both the grain size^{57,61} and substructural details⁶⁰ of beryllium sheet are known to be sensitive functions of thermo-mechanical history and the source of the sheet. The

large range in structural detail possible might be expected to account for some of the observed variation in beryllium fracture toughness. Some discussion of the structural variation observed in commercial beryllium sheet and its possible effect on fracture toughness will, therefore, be considered.

Several studies⁶² have shown that both the yield and ultimate strengths of polycrystalline beryllium increase with a decrease in grain size. The difference between yield and ultimate is also found to increase with a decrease in grain size. This is accompanied by an increase in elongation to failure as would be expected if the strain hardening rates were not dramatically affected. The above has been found to be the case for both HPB material and textured material of both powder and ingot origin. It is possible that microstructural variation affects fracture toughness indirectly through the uniaxial flow properties. This is considered below.

The influence of uniaxial properties on fracture toughness is expected to be dependent on the stress state at failure. In a plane stress failure mode, through thickness yielding is unrestrained, and the fracture toughness would be expected to reflect the uniaxial tensile properties as discussed in the last chapter. The effect of ultimate and yield strengths, and elongation to failure on fracture toughness would be given by Eq. (33) or by a more refined version which improves on the approximation of linear strain hardening. An assumption in the development of Eq. (33) is that the elongation measured in a tensile test can be taken equal to the negative of the thickness contraction. This assumption is subject to varying degrees of error given even for isotropic sheet. In mechanically worked beryllium sheet, width contraction makes the largest contribution to elongation and so the use of Eq. (33) would be especially in error. A more meaningful measure of the ductility in this case would be thickness contraction since only this component of the elongation is able to contribute to crack opening displacement. The effect of microstructure on thickness contraction measured in a tensile test has not yet been the subject of experimental study. It might be anticipated, however, that thickness contraction would increase along with elongation if preferred orientation were held constant. An additional effect of preferred orientation is that the effective flow properties of the oriented metal at the crack tip would be expected to differ from the flow properties measured in uniaxial tension. This would not be expected, at least for initial yielding, for isotropic materials since both the Tresca and von Mises yield criterions⁶³ predict that flow would begin at the uniaxial yield stress in balanced biaxial tension. For anisotropic beryllium, however, superstressing beyond the uniaxial yield strength may be necessary to initiate flow in the biaxial stress state existing at the crack tip.

The above suggests that if failure of this beryllium sheet occurs in plane stress then the effect of microstructural variation will be exerted through the effect of microstructure on tensile properties via a relationship similar to Eq. (33) but with thickness contraction substituted for elongation. Since large variation in grain size and associated tensile properties is known to be achievable in beryllium sheet it is anticipated that grain size variation may account for quite substantial measured variation in

fracture toughness. The same conclusion is reached if it is assumed that failure occurs in plane strain and that the toughness is dependent on σ_F^*/σ_y as discussed in Ch. II.

The effect of substructure in the form of subgrains and dislocation networks on tensile, bend, or fracture properties has not been the subject of a significant amount of study. There have been instances, however, where the effect of polygonized walls or bend planes resolvable in the optical microstructure have been associated with lower ductility relative to specimens of the same grain size but free of bend planes.⁵⁵ In the latter reference it is also shown that microcracks perpendicular to polygonized walls form during deformation. These observations give some indication that the Stroh mechanism of bend plane splitting may initiate fracture after less deformation in the case of polycrystals containing bend planes than for polycrystals free of bend planes. The presence of bend planes might be expected, therefore, to result in low fracture toughness. It has also been observed that increasing subgrain size leads to greater bend ductility. This observation suggests that fracture toughness may be low in situations where subgrains are present.

Impurities and their Distributions

Impurities and their distribution are known to influence the mechanical behavior of commercially pure beryllium. Three principle effects attributable to impurities have been reported: 1) The ultimate strength and ductility can depend on the impurity content and disposition as influenced by heat treatment. 2) The fracture mode at room temperature can range from transgranular cleavage to intergranular, and 3) Beryllium may or may not exhibit an impurity induced yield drop. It is possible that these effects also influence the fracture toughness of commercial beryllium sheet. The current understanding of these effects and their possible influence on fracture toughness will be considered below.

The subject of impurity effects in beryllium has been considered in a recent review by Gelles.⁶⁴ Gelles points out that the disposition of the common impurities in commercial beryllium is best represented by the scheme first proposed by Fraikor, et al.⁶⁵ and reproduced in Fig. 27.

One of the most dramatic mechanical effects attributed to impurities is the large increase in high temperature ductility which occurs after aging of commercial purity beryllium initially in the solution treated condition. Gelles traced the evolution of the understanding of this effect to the current interpretation which relates high temperature ductility to the disposition of aluminum in the microstructure. The interpretation is consistent with Fig. 27. In the solution treated condition (900°C or above followed by fast cool) the elements Fe, Ni, Mn, and Cr, are dissolved in the matrix with free aluminum present in grain boundaries. Subsequent slow cooling or aging at temperatures below 800°C tends to result in the reaction of aluminum with the Fe, etc., and Be to form a high melting point ternary compound $AlMBe_4$ (where M is Fe, Ni, Mn, and/or

Fig. 27. Aging-solutionizing reaction in commercial purity beryllium. After Fraikor et al.⁶⁵

Cr). The low elevated temperature ductility in the solution treated condition is attributed to the presence of molten or soft aluminum in the grain boundaries. The effect of Fe, etc., in solid solution is thought to intensify the embrittling effect of the grain boundary aluminum by hardening the matrix and inhibiting stress relaxation which might otherwise prevent early intergranular rupture. The ductilizing effect of aging is then associated with formation of the ternary compound AlMBe_4 which removes free aluminum from the grain boundaries and reduces the amount of Fe in solid solution.

Various studies have indicated that heat treatment also affects room temperature mechanical properties and some of the changes have been attributed to similar impurity effects. As Gelles⁶⁴ pointed out, the observations are not consistent with one another and this might reflect the contribution of other factors. Gelles, *et al.*⁶⁶ for example, showed that slow cooling or aging at 600°C resulted in a dramatic increase in ductility and ultimate strength relative to extruded and solution treated bar (1100°C, water quench). Meredith and Sawkill,⁶⁷ on the other hand, observed a loss in ultimate tensile strength after aging of purer material. The latter authors also observed that a change in fracture mode from cleavage to intergranular was induced by the aging process.

It seems reasonable to expect changes in fracture toughness accompanying the redistribution of impurities as a function of heat treatment. In the case of plane stress fracture the changes in fracture toughness may simply reflect the dependence of uniaxial properties on the disposition of impurities. It is also possible that impurity induced effects exert unique influences on fracture toughness. The pattern and nature of precipitation might, for example, influence the size of the accommodation zone which affects fracture toughness by limiting the spatial extent of crack tip plasticity. In addition, a change in fracture mode in the fatigue precrack region might alter the effective acuity of the precrack and thereby cause a change in toughness not reflected by uniaxial properties.

Yield Point Effects

Besides affecting mechanical properties and fracture mode the disposition of impurities can also affect the yielding behavior of beryllium. Floyd,⁶⁸ for example, has shown that strong yield drops can occur in beryllium after slow cooling from the solution treating temperature. Floyd also found that solution treating or prior strain eliminated the yield drop. He attributed the yield drop to precipitate pinning (by probably AlFeBe_4) which reduces the density of glissile dislocations. Plastic flow in the pinned state requires high stress nucleation of mobile dislocations. The stress necessary to sustain their motion drops as the dislocation density increases. Floyd concluded that the type of model proposed by Gilman to account for yield point behavior in LiF and extended to BCC metals by Hahn,⁶⁹ best explained his observations.

There is no available model to predict the effect of yield drop behavior of fracture toughness. Qualitative considerations, however, suggest that the presence of a

yield drop may lead to lower fracture toughness. A drastic drop in ductility was found, for example, in the case of a strain aged steel tensile tested at a dynamic rate.⁷⁰ This was attributed to a high stress dependence of the Luders front velocity. It was postulated that the stress required to move the Luders fronts at a rate consistent with the imposed deformation velocity approached the ultimate strength of the yielded metal within the Luders bands at high velocity. This led to low ductility because neck down and rupture occurred before the bands could propagate over the entire gage length.⁷⁰ It is conceivable that the same effect could lead to low fracture toughness in the case of a material containing a yield point relative to the same material with the yield point removed. In the former situation the size of the accommodation zone might be limited by the extent of propagation of the Luders fronts at the crack tip.

IV. PROGRAM OF INVESTIGATION

Overview

The goal of the present program was to develop an understanding of the origin of differences in the fracture resistance of beryllium sheet. Relatively thin sheet was chosen for the study (0.80 mm thick) so that the high toughnesses associated with plane stress fracture might be observed. The approach taken in the program was to first empirically verify that the fracture test method and its interpretation in terms of elastic fracture mechanics provided results which could be used as a basis for comparison. Both linear and nonlinear load deflection behaviors had been observed in preliminary fracture tests. It was necessary, therefore, to consider both types of behavior so that comparisons of fracture resistance could be made over the widest possible range in microstructure. Second, a series of tests were planned to establish the effect of microstructure over the widest possible range using commercial ingot origin and powder origin beryllium sheet. The variables which were investigated include substructure, grain size, heat treatments designed to alter the disposition of aluminum and iron in the microstructure, and preferred orientation. Third, a series of experiments were planned which were intended to help develop an understanding of the factors which account for and limit the magnitude of fracture toughness.

Qualification of Fracture Test Methods

Linear Elastic Case — The single edge notched (SEN) fracture toughness specimen illustrated in Fig. 1 was chosen as the specimen type for the present investigation. This specimen was chosen because compliance measurements have been made from which the relationship between σ , applied load, crack length, and specimen dimensions have been derived using the methods discussed in Ch. I.⁴ Also, these experimental results have been shown to be in good agreement with K values derived from stress analysis.⁹ The relationship between K and the pertinent variables has already been given in Ch. I. Another important reason for selection of the SEN specimen is that the crack mouth compliance, v/F , normalized to elastic modulus, E , and thickness, h , has been determined as a function of crack length to width ratio, a/W . The experimentally determined relationship as reproduced from Ref. 9 is given in Fig. 10. One of the desirable features of the SEN specimen is that the rate of change of compliance with crack length (or a/W) is much greater than for the center notched or double edge notched specimen types. This means that crack mouth displacement would be more sensitive to changes in crack length or crack tip plasticity in the case of the SEN specimen relative to either the double edge cracked or center cracked panels.

The compliance calibration and crack mouth displacement results referred to above were determined using 7075-T6 aluminum alloy which is considered to be elastically isotropic. Preliminary experiments indicate that the two types of beryllium

sheet products (ingot and powder) being investigated exhibit the same crack mouth displacement per unit force, after normalizing for thickness and elastic modulus, as 7075-T6 at equal values of a/W . This indicates that the beryllium sheet is elastically isotropic in the plane of the sheet and that the results found for 7075-T6 can be used for the beryllium sheet.

As discussed in Ch. I, the concept of critical strain energy release rate, $\mathcal{G}_{crit}$, was developed on the basis of overall linear elastic behavior. In metals, however, a zone of nonlinear plasticity will develop at the crack tip as load is applied. The overall load-deflection response will tend to reflect the crack tip nonlinearity and the degree of overall nonlinearity will increase as the plastic zone size increases. This overall nonlinear behavior will affect the validity of a computation of $\mathcal{G}_{crit}$ based upon the use of a compliance calibration on stress analysis developed for linear behavior.

All metals will exhibit some crack tip plasticity prior to failure and therefore some degree of overall nonlinear response should always be present. For very brittle metals, however, it may be that extraordinary precision in the displacement measuring technique may be required to detect the nonlinear effect. In any event the problem of ascertaining what constitutes an acceptably small deviation from linearity must be considered. There is no purely theoretical basis for deciding this question and development of guidelines for maximum acceptable deviation is a problem which has, in the past, been approached empirically. In the present program the applicability of a linear elastic fracture analysis to beryllium sheet was evaluated by testing specimens containing widely different crack lengths. If the same values for $\mathcal{G}_{crit}$ and K_{crit} were obtained, the linear elastic method of analysis was considered suitable.

It is interesting to note that the ASTM standard for plane strain fracture toughness includes a requirement for minimum crack length, given previously in Ch. I, as follows:

$$a > 2.5 \left(\frac{K_{crit}}{\sigma_{ys}} \right)^2. \quad (18)$$

This criterion was developed semiempirically and is used in the case of plane strain fracture toughness testing to insure that linear elastic analysis is appropriate. It is reasonable to suppose that a similar requirement must be met in the case of thin beryllium sheet where fracture may occur in plane stress. Equation (18) can be used to estimate the range of crack lengths which may be required to insure linear elastic behavior. Using the lowest reported value for K_{crit} ($\sim 11 \text{ MPa}\cdot\text{m}^{1/2}$) together with a reasonably high value for yield strength ($\sim 414 \text{ MPa}$), Eq. (18) leads to a required crack length of $a > 1.78 \text{ mm}$. Using the highest reported value for K_{crit} ($\sim 44 \text{ MPa}\cdot\text{m}^{1/2}$) and a yield strength on the low end of the range ($\sim 172 \text{ MPa}$), Eq. (18) leads to the requirement that a must be $> 163 \text{ mm}$. This leads to the conclusion that for very brittle and high strength beryllium the specimen size requirements for linear behavior are reasonable but that for high toughness and low strength beryllium the specimen size

requirements become unreasonably large. For the present program, large specimens (95.2 mm width X 381 mm length) were used in an attempt to obtain linear behavior over a wide range of strength and toughness.

Nonlinear Case — The crack length estimated to be necessary for development of linear elastic behavior in the tougher forms of beryllium sheet leads to unreasonably large specimen size requirements. A crack length of 160 mm would lead to a specimen width of 480 mm and a length of nearly 1440 mm if the specimen geometry was chosen in conformance with recommendations.¹⁵ This is too large a specimen for exploration of the effect of metallurgical variables. Preliminary tests of the smaller (95.2 mm width X 381 mm length) specimens have led to nonlinear load-deflection response in some instances. The problem of comparison of the fracture resistance properties of such specimens with others which are linear elastic then depends on a method of analysis of nonlinear behavior.

The J-integral approach for analysis of nonlinear behavior was discussed in Ch. I. This method was chosen for the present investigation. In this approach it was planned to evaluate the force-deflection response of four specimens containing different crack lengths according to the methods discussed in Ch. I. It was hoped that this would lead to a value of J_{crit} for the particular metallurgical condition tested. In the case of a steel investigated by Begley and Landes¹⁷ it was shown that the experimentally determined value for J_{IC} was similar to a linear elastic value of $\mathcal{G}_{IC}$ determined with much larger fracture specimens. In the present case the specimen size required to measure a linear elastic value of $\mathcal{G}_{crit}$ will in general be unreasonably large. It was thus desirable to devise other methods of analyzing nonlinear behavior in order to verify and substantiate the significance of the value of J_{crit} measured.

Two new approaches were taken to the interpretation of nonlinear load-deflection response. These are possible partly because no stable crack growth was found to precede fracture during preliminary testing of rolled powder or ingot source sheet. Since no crack extension was observed prior to fracture, all of the nonlinearity in the force-deflection record could be attributed to the development of a plastic zone at the crack tip. In the first method of analysis, the effect of a plastic zone developing ahead of the plastic zone was assumed to have the same effect on compliance as an amount of crack extension of equal magnitude. It follows that the length of the plastic zone at fracture could be found by finding the value of a/W corresponding to the compliance at fracture from the correlation given in Fig. 10. The plastic zone length was then given by the corresponding value of crack length minus the original crack length. If the plastic zone length is some multiple of thickness then it seems appropriate to assume that crack tip deformation is occurring in plane stress and that the Dugdale model would be applicable. Using the average strength measured in uniaxial tension for the cohesive strength and the value of plastic zone length at fracture, it is then possible to derive from the Dugdale model (Eq. 24 in Ch. II) the applied stress which would have been necessary to develop this same plastic zone length in an infinite specimen containing

the same initial crack length. K_{crit} can then be computed using the stress intensity solution appropriate for the infinite SEN specimen⁸:

$$K_{crit} = \sigma_{inf} \sqrt{\pi a} \quad (37)$$

where σ_{inf} is the applied stress necessary to develop the critical plastic zone length in an infinite specimen containing an edge crack of length a .

It is possible that an error in the above method might occur because the length, shape, and strain intensity distribution in the plastic zone at fracture may depend on specimen size. In addition, the assumption that a plastic zone has the same effect on compliance as a crack of equal length would constitute a source of error. If the plastic zone length is a reasonably small fraction of the specimen width, however, it might be expected that the above sources of error would not be too significant. A good comparison between $\mathcal{G}_{crit}$ derived from the J testing and the compliance method discussed above would tend to substantiate these methods of interpretation.

A second new method for evaluation of $\mathcal{G}_{crit}$ results from consideration of the interpretation of $\mathcal{G}$ in terms of crack tip plasticity. Specifically from Ch. II:

$$\mathcal{G}_{crit} = \int_0^{\delta_t^*} \sigma(\delta) d\delta. \quad (29)$$

This is just the area under the cohesive stress vs displacement curve as illustrated in Fig. 28. The method of measurement which was planned is illustrated in Fig. 28b. It involves direct measurement of $\sigma(\delta)$ using a double edge notched DEN specimen chosen so the net section width is smaller than the plastic zone length at fracture. It was hoped that a comparison of $\mathcal{G}_{crit}$ from tests of a small DEN specimen with $\mathcal{G}_{crit}$ values from the other two methods might lead to a greater degree of confidence in the various methods of analysis.

Effects of Metallurgical Variation

Substructure and Grain Size — Having established a method of comparison of the fracture properties of beryllium sheet in various microstructural states the effects of metallurgical variation was investigated. The effect of substructure and grain size was evaluated by fracture tests of ingot sheet SEN specimens heat treated to produce a variety of microstructural states. Maximum response to heat treatment was obtained by finish rolling the ingot sheet at a temperature below its recrystallization temperature. Previous study has shown that the microstructure and uniaxial mechanical properties of this type of sheet can be varied over a wide range using this method.⁷¹ The range of heat treatment temperature over which this variation can be induced is from about 600 to 830°C. These temperatures lie below those necessary to produce

Fig. 28. Method for measuring G_{crit} .

significant resolution of impurities (see Fig. 27) and so the effect of variation in substructure and grain size is expected to outweigh effects due to redistribution of impurities in this temperature range. Other information leads to the conclusion that preferred orientation will not be significantly affected by the microstructural changes accompanying heat treatment.⁵⁸

Disposition of Iron and Aluminum – It is possible that changes in impurity distribution could account for some of the wide variation in reported fracture toughnesses. Commercial sheet contains significant amounts of iron and aluminum and so should be subject to changes in the disposition of impurities due to heat treatment. It was planned, therefore, to investigate the effects of heat treatments designed to modify the distribution of impurities on fracture toughness, tensile properties, and fracture mode. Using Fig. 27 as a guide, the heat treatments which were planned included: a) solution treatment to maximize iron in solution and free aluminum in the grain boundaries, b) a very slow cool after solution treatment to induce precipitation of the ternary compound AlMgBe_4 , and c) aging after solution treatment to cause precipitation of FeBe_{11} . It was not expected that microstructural changes or changes in preferred orientation would occur during slow cooling or aging below the solution treatment temperature. It was believed, therefore, that the effects of impurity distribution on fracture toughness could be fairly well isolated.

Effect of Preferred Orientation – It is not feasible to obtain commercial beryllium sheet having significantly different intensities of preferred orientation with other metallurgical factors held constant. Within the scope of the present program it was impossible, therefore, to isolate the variable of preferred orientation and evaluate its effect on fracture toughness.

Commercial ingot sheet generally has a significantly less well developed preferred orientation than rolled powder source sheet of about the same thickness. Powder sheet products also generally have a finer grain size and contain a significant amount of beryllium oxide. It appears possible to obtain some insight into the effect of preferred orientation on the fracture toughness of sheet by comparing the fracture properties of commercial ingot and powder sheet products. It was planned to accomplish this by relating the measured toughnesses to the capability for through thickness contraction measured in the tensile test. If fracture of the sheet occurs in plane stress then the fracture toughness should be predictable on the basis of the Dugdale model with the cohesive stress taken as that measured in uniaxial tension and the effective elongation taken as the through thickness contraction measured in uniaxial tension. In Ch. II, for example, it was shown that the Dugdale model successfully accounted for the magnitude of the plane stress fracture toughness for many metals. If fracture of the sheet did occur in plane stress then the effect of preferred orientation could be inferred. Increasing the intensity of preferred orientation while other factors are held constant would, for example, tend to decrease the toughness and vice versa.

It is possible, however, that fracture of one or both types of beryllium sheet would not occur in plane stress. In this circumstance it would not be possible to unequivocally establish the effect of preferred orientation on the fracture toughness.

Magnitude of Fracture Resistance

The fracture tests described above could not of themselves lead to an understanding of the factors that account for and limit the magnitude of fracture toughness. The following additional experimentation was planned to help develop this understanding.

Correlation with Uniaxial Properties - It is possible that many or all of the fracture tests that were planned would occur in plane stress in view of the thinness of the sheet which was to be evaluated (0.80 mm thick). To evaluate this possibility, uniaxial tensile properties were measured. It was planned to estimate the plane stress fracture toughness using the tensile properties and the assumption that the plastic zone height is given by the sheet thickness. In Ch. II it was shown that this results in calculated plane stress fracture toughnesses in good agreement with measured values for a variety of metals. In the case of thin beryllium sheet this method was expected to result in good estimates of plane stress toughness. Complications due to localized necking would be absent because only uniform thickness contraction occurs in ambient temperature tensile tests of beryllium.

A complication in the case of beryllium sheet arises because the elongation measured in uniaxial tension increases as the intensity of preferred orientation increases and this is due, as discussed in Ch. III, to an increasing contribution of width contraction. Width contraction cannot, however, contribute to the effective elongation capacity of the crack tip metal because of geometrical effects. It was therefore more appropriate to use thickness contraction as measured in the uniaxial tensile test for the plane stress fracture toughness estimate. Thickness contraction was determined together with the usual uniaxial properties for most of the conditions of heat treatment studied. This allowed estimates of the plane stress fracture toughness to be made for comparison with the experimental results.

If the measured fracture toughness is found to be similar to or in excess of that computed on the basis of the plane stress model, then this could be taken as good evidence that fracture occurs in plane stress. The problem of understanding the magnitude of fracture toughness would, in this circumstance, reduce to the understanding of the effect of metallurgical variables on uniaxial tensile properties. If on the other hand, the measured fracture toughness was found to be significantly below the estimated plane stress value then it would be concluded that fracture occurred in a combined plane stress-plane strain mode or in plane strain. The applicability of the empirical correlations between the plane strain value and σ_F^*/σ_{ys} could then be examined. In addition, it was thought that some understanding of the plane strain value might result from study of the effect of crack tip diameter as discussed below.

Effect of Crack Tip Geometry – Observed effects of crack tip sharpness on the value of σ_{crit} or K_{crit} were discussed in Ch. II. In the present study the primary pre-cracking method was fatigue because this was expected on the basis of studies of other materials to lead to the lowest and therefore most significant value of fracture resistance.

A study of the effect of crack root diameter, ρ , on the other hand, might lead to a better understanding of the factors that determine fracture toughness in situations where fracture occurs below the plane stress value. Inserting the crack root diameter times e_f for δ_t^* into Eq. (30) from Ch. II, for example, leads to the following:

$$K_{crit} = \sqrt{EY} e_f (\rho)^{1/2}. \quad (38)$$

The slope of a plot between K_{crit} and $(\rho)^{1/2}$ would, according to Eq. (38), depend on the effective cohesive strength and elongation. These can be compared with values measured in uniaxial tension (assuming reasonable constraint factors in the estimate for cohesive stress) to determine if a significant loss in e_f occurs ahead of the crack tip. Equation (38) would also lead to an estimate of the effective root diameter corresponding to tests on fatigue precracked specimens. It may be possible to relate this to a microstructural size parameter such as grain size to establish the effective gage length at the crack tip in cases where fracture does not occur in plane stress.

Direct Observation of Plastic Zone – It is possible that a successful correlation between measured plane stress fracture toughness with a value estimated from tensile properties would be coincidental. For example, it is conceivable that a plane strain value of fracture toughness would equal or exceed, in some circumstances, the plane stress value. It was, therefore, desirable to have some supporting evidence that fracture occurs in plane stress in case this was suggested by comparison with uniaxial tensile properties.

To accomplish this, some direct measure of plastic activity ahead of the crack tip was required. If it is assumed that a strip yield model is applicable then the plastic zone length displacement within the zone is predictable on the basis of the Dugdale model. It was planned, therefore, to measure the plastic strain at the crack tip directly, using foil resistance strain gages.

Attempts were made in preliminary experiments to observe plastic activity ahead of crack tips after fracture. Various polishing and etch pitting procedures were employed. All of these efforts were, however, unsuccessful and are not pursued further.

Fractography – Studies of the fracture surfaces using the scanning electron microscope were planned to establish the operative modes of fracture. It is known, for example, that the room temperature fracture mode in beryllium can be largely intergranular or largely transgranular. The effects of impurities may exert their influence through their effect on fracture mode so that a knowledge of the fracture mode may help

in the interpretation of impurity effects. In addition, it is possible that a different fracture mode may be induced in a precracked fracture toughness specimen relative to a smooth tensile specimen due to the stress intensifying effect of constraint. In the latter case, a fractographic analysis could provide an understanding of the microstructural factor that limits fracture toughness.

V. PROCESSING AND CHARACTERIZATION OF EXPERIMENTAL MATERIALS

Sheet Processing

Ingot Sheet — Cross rolled ingot sheet of commercial purity for the program was produced by the Rocky Flats Division of Dow Chemical Company using, except for the special deviations discussed below, their nominal production practice.⁷² To obtain ingot sheet with fine grain size, the last rolling passes were carried out at the lowest possible temperatures consistent with the need to produce relatively crack free sheets (254 mm × 508 mm × 0.80 mm). It was specified that the last rolling pass should occur below 690°C to cause a minimum of 10% thickness reduction and that the material not be given a final recrystallization anneal. The latter conditions were expected to result in unrecrystallized material containing substructure. It was also specified that hot flattening be carried out at 650°C to minimize microstructural changes during the flattening operation. Appendix A gives a summary of the thermo-mechanical history of the sheet.

Powder Sheet — Cross rolled powder sheet (380 mm × 380 mm × 1.02 mm) was purchased from Kawecki Beryllco Industries, Inc. (KBI).^{*} Material for the fracture tests was chemically thinned to 0.80 mm. It was specified that the sheet be cross rolled from HP-8 hot pressed block to obtain material with a relatively low BeO and iron content. KBI produced an "optimum" product within the scope of their proprietary production practice. Finish rolling and final annealing is, however, known to be done below 800°C. No special control over the thermo-mechanical history of the powder sheet was requested. Table 2 presents the room temperature mechanical properties of the finished sheet, as reported by KBI.

Table 2. Room temperature mechanical properties of cross rolled powder sheet beryllium.

	Parallel to last rolling direction	Transverse to last rolling direction
Ultimate tensile strength	524-531 MPa	511 MPa
Yield strength	460 MPa	404 MPa
Elongation	24-26%	24-33%

^{*}Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Atomic Energy Commission to the exclusion of others that may be suitable.

Impurity Content and Preferred Orientation

Impurity Content - As pointed out in Ch. III, the kind, amount and distribution of impurities are expected to influence the mechanical behavior of beryllium. The chemical analyses, as reported by the suppliers, are given in Table 3. As noted in Table 3 the ratio of iron to aluminum by weight for both of the materials is close to 2:1. This is close to the stoichiometric composition for the ternary compound AlFeBe_4 which is believed to be the phase favored by low temperature aging after solution treating. Since finish rolling for both sheet materials is carried out at temperatures below the solution range (see Fig. 27) it is expected that the iron and aluminum impurities would be tied up as the ternary compound and that heat treatments below $\sim 850^\circ\text{C}$ would not have a significant effect on the distribution of the iron and aluminum impurities.

Preferred Orientation - As pointed out in Ch. III, the degree of preferred crystallographic orientation is an important parameter affecting the mechanical behavior of rolled beryllium sheet. It was necessary, therefore, to characterize the preferred orientation of the present sheet. Since it was planned to vary microstructure by heat treatment it was also necessary to evaluate changes in preferred orientation that might also accompany heat treatment. The pole figure determinations were carried out at the Franklin Institute Research Laboratories. The results are plotted on a polar net as the ratio of the diffracted x-ray intensity to the diffracted intensity expected from a random

Table 3. Chemical analysis of beryllium sheet materials - wt%.

	Ingot sheet	Powder sheet
BeO	-	1.07
C	0.069	0.062
Fe	0.138	0.095
Al	0.072	0.048
Mg	<0.001	0.010
Si	0.061	0.009
Ca	<0.003	0.01
Cr	0.012	0.01
Co	<0.001	0.0005
Cu	0.007	0.003
Pb	<0.001	<0.0003
Mn	0.013	0.012
Mo	<0.003	<0.001
Ni	0.015	0.013
Ag	-	0.001
Fe/Al ratio	1.92	1.98

specimen. The latter was determined for each individual specimen by integrating the measured intensities over the entire polar net. The basal plane pole figures for ingot beryllium after heat treatments at 710°C, 780°C, and 830°C are given in Fig. 29. Metallographic characterization, discussed in following sections, showed that the heat treatments resulted in a recovered structure with subgrains (29a), a relatively fine grained fully recrystallized structure (29b), and a structure consisting of relatively large grains (29c). The pole figures indicate that a modest decrease in the intensity of the preferred orientation occurs as a consequence of recrystallization and grain growth. A pole figure for the powder sheet after heat treatment at 710°C for 1 h is given in Fig. 30. This pole figure shows that the powder sheet has a much more severe basal plane orientation than the ingot sheet.

The peak intensity measured for the powder sheet is 10X random while the peak intensity for the ingot sheet was only 4X random.

Microstructural Characterization of Ingot Sheet

As pointed out in the proposed program the mechanical properties are expected to reflect differences in microstructure induced by heat treatment. In order to characterize the microstructure for comparison with mechanical properties both optical microscopy and transmission electron microscopy was used.

Optical Microscopy — Specimens were prepared for optical microscopy using standard techniques. Briefly, the specimen is mounted in bakelite, rough ground, and hand sanded through 600 grit silicon carbide paper. The specimen is then automatically lapped on a vibratory polisher fitted with a cloth carrying 1 μm diamond paste for about 8 h. This is followed by a 4 h lap with 1/4 μm diamond. Finally, the specimen is hand lapped on a revolving wheel fitted with a cloth carrying 0.05 μm levitated alumina for about 2 min. The specimen is then illuminated with polarized light to view the microstructure.

Figure 31 shows photographs of the microstructures obtained after annealing the warm worked ingot sheet beryllium described in a previous section at a series of temperatures selected to produce a variety of microstructures. The grain sizes indicated in Fig. 31 were determined using an intercept method.

Linear segments separating regions of differing contrast within individual grains are visible in the microstructures produced by annealing at temperatures below 750°C. The number of these segments in any individual grain depends sensitively on the angular position of the polarizing lens. Slight adjustment of the polarizing lens angle resulted in the appearance of segments in some grains and the disappearance of segments in other grains. A meaningful measure of the density of segments cannot, therefore, be inferred from any single photograph. Observation of such segments in worked polycrystalline beryllium have been made previously^{61,71} and there has been an indication that the segments visible in the optical microstructure are associated with the

Fig. 29. Basal plane pole figures of ingot sheet after annealing for 1 h at various temperatures; (a) 710°C, (b) 780°C, (c) 830°C. R.D. = rolling direction.

Fig. 29 (continued)

Fig. 29 (continued)

Fig. 30. Basal plane pole figure of powder sheet after annealing for 1 h at 710°C.

Fig. 31. Microstructures of warm rolled ingot sheet after annealing at the indicated temperatures, all 100X.

presence of subgrains.⁷³ These subgrains become resolvable at the higher magnifications possible in transmission electron microscopy.⁷³

Transmission Electron Microscopy – Transmission electron microscopy was carried out to develop better characterization of microstructural features which were not resolvable by optical microscopy. Thin films were prepared by first chemical milling the 0.8 mm thick beryllium to about 0.1 mm thickness in a solution containing 70 vol% H_3PO_4 , 25 vol% H_2SO_4 , and 5 vol% HNO_3 . The specimens were then electrolytically thinned using the window technique described in Ref. 74. This technique generally resulted in foils that were more uniform in thickness over larger areas than foils prepared by jet thinning methods. The electrolytic thinning solution contained 600 cm³ methanol, 360 cm³ butyl cellosolve, and 60 cm³ perchloric acid. The bath was maintained at -25°C and the polishing voltage was 25 V. The area over which the foils were thinned was about 90 mm². The foils were cleaned in methanol, dried, and collected on a grid in preparation for viewing in the electron microscope. The foils were viewed in a Siemens Elmiskop I electron microscope at 100 kV accelerating voltage. A double tilt stage that allowed tilting the foil 20° about two perpendicular axes in its plane was used to maximize contrast and structural detail.

Transmission electron micrographs corresponding to various conditions of heat treatment of the warm rolled ingot sheet are given in Figs. 32-36. The most striking difference among the structures is the presence of fine subgrains in the specimens annealed below 750°C and the absence of subgrains in specimens annealed above 750°C. The absence of the subgrains seems to be coincident with complete recrystallization which occurred during anneals at temperatures above 750°C. The presence of the segments in the optical microstructures of the specimens annealed below 750°C is apparently a direct consequence of the presence of subgrains. The presence of segments separating regions of differing contrast in the grains seems to indicate that the subgrains are arranged to accommodate bending within grains in a manner similar to the bending produced by simple walls of edge dislocations. Figure 23 in Ch. III illustrates the latter effect. The subgrains apparently form to relieve bending stresses developed during the rolling process. In this connection it is noted that the formation of bend planes comprised of simple dislocation walls in deformed beryllium bi-crystals is well documented.⁵⁴

Microstructural Characterization of Powder Sheet

Limited microstructural characterization of the powder sheet was carried out. Figure 37 shows an optical photomicrograph of the powder sheet after a 1 h heat treatment at 710°C. The intercept grain size is noted in the figure.

5 000 ×

8 000 ×

10 000 ×

10 000 ×

10 000 ×

10 000 ×

13 000 ×

13 000 ×

20 000 ×

Fig. 32. Transmission electron micrographs of ingot sheet beryllium as received. Magnification as indicated.

10 000x

17 000x

20 000x

20 000x

20 000x

Fig. 33. Transmission electron micrographs of ingot sheet beryllium after annealing for 1 h at 680°C.

4 000 ×

5 000 ×

7 000 ×

8 000 ×

10 000 ×

10 000 ×

15 000 ×

16 000 ×

20 000 ×

Fig. 34. Transmission electron micrographs of ingot sheet beryllium after annealing for 1 h at 730°C.

7 000 x

8 000 x

8 000 x

10 000 x

10 000 x

10 000 x

20 000 x

20 000 x

20 000 x

Fig. 35. Transmission electron micrographs of ingot sheet beryllium after annealing for 1 h at 780°C.

Fig. 36. Transmission electron micrographs of ingot sheet beryllium after annealing for 1 h at 830°C.

Fig. 37. Grain structure of powder sheet beryllium after annealing for 1 h at 710°C. Note large range in grain sizes.

VI. MECHANICAL TEST PROCEDURES AND CHARACTERIZATION OF TENSILE DEFORMATION BEHAVIOR

Specimen Preparation

Tensile Specimens — The importance of removing cold worked metal from the surfaces of tensile specimens has been known for a long time.⁵⁷ Removal of this microcracked and twinned surface layer is necessary so that the ultimate tensile strength and ductility properties will not be influenced by premature fracture due the stress concentrating effect of the microcracks and twins. The mill cut depth was successively decreased (0.25, 0.13, 0.05, 0.02 mm) as the finished dimension was approached to minimize the depth of damage. After machining the width of the specimens was reduced by a minimum of 0.13 mm by chemical etching in 5% HF and water to remove the damaged layer. It had been previously demonstrated⁶⁶ that the mechanical properties became independent of further removal after etching of 0.05 mm from the machined surfaces of sheet beryllium. Heat treatment was always carried out after final machining and etching to further insure the absence of ductility limiting defects. The specimen configuration used is illustrated in Fig. 38. The larger of the two (38a) was used for the ingot sheet. Later in the program the smaller specimen configuration (38b) was used for the powder sheet for reasons of economy. Both types of specimens proved to be satisfactory.

The surfaces of the ingot sheet specimens were somewhat rough due to pitting which occurred during chemical milling after the final rolling pass. It was necessary to demonstrate, therefore, that the mechanical properties were not affected by the surface condition of the specimens. The peak surface pit depth, as determined by profilometer measurement in the standard surface, was found to be about 12.5 μm in the case of the standard surface and 6.2 μm in the case of the improved surface. The tensile ultimate strength and elongation after heat treatment at 710°C for 1 h was found to be 400.2 MPa and 8.5% for the standard surface condition and 391.2 MPa and 7.5% for the improved surface. This evaluation indicates that surface roughness did not influence the tensile results.

Fracture Toughness Specimens — Single edge notched (SEN) specimens of two sizes and a double edge notched specimen (DEN) were used for the fracture tests. The design of the SEN specimens shown in Figs. 39 (small SEN) and 40 (large SEN) is in accord with suggested geometry.¹⁶ DEN specimens were prepared using a small SEN specimen and extending opposing slots from both edges by electrical discharge machining to leave a 4.76 mm width ligament centered on the axis of the specimen. Standard shop practice was used for machining the specimens except that the machined notch was extended by electrical discharge machining (EDM) with a 0.08 mm diameter copper wire cutting electrode. This wire size produced an EDM slot with a root diameter between 0.11 and 0.13 mm. Larger diameter copper wires were used to produce roots

Notes:

1. Dimensioning and tolerancing are per USASI-Y14.5 in mm.
2. Positional and form tolerances apply R.F.S.
3. Surface texture per USAS-B46.1.
4. 63 finish all machined surfaces.
5. Tol. ± 0.25 unless otherwise noted.

Fig. 38. Design of tensile specimens for tests of beryllium sheet; (a) ingot, (b) powder. Note: the uneven dimensions reflect a conversion from English to SI units. All dimensions are in mm.

1. Dimensioning and tolerancing are per **USAS1-Y14.5** in mm.
2. Positional and form tolerances apply **R.F.S.**
3. Surface texture per **USAS-846.1**.
4. **63 finish** all machined surfaces.
5. **Tol. ± 0.25** unless noted.
6. Fabricate slot extension by electrical discharge method.

Notes:

1. Dimensioning and tolerancing are per USAS1-Y14.5 in mm.
2. Positional and form tolerances apply R.F.S.
3. Surface texture per USAS-B46.1.
4. 63 finish all machined surfaces.
5. Tol. ± 0.25 unless noted.
6. Fabricate slot extension by electrical discharge method.

Fig. 40. Design of large single edge notched (SEN) specimen.

with greater diameters. A specimen with a very small root diameter (0.03 mm) was prepared by hand sawing with a 0.025-mm diameter tungsten wire carrying a paste containing 3- μ m diamonds.

Heat Treatment – All heat treatments were carried out in a tungsten element vacuum furnace after the specimens were machined. Fatigue cracking of the fracture toughness specimens was accomplished after heat treatment. The normal cooling rate from the heat treatment temperatures generally used (varied from 680 to 950°C) was nonlinear but averaged about 0.5°Cs⁻¹ down to 450°C. Some specimens were slowly cooled at a rate of 0.0139°Cs⁻¹ by using a programmed cooling rate.

Characterization of Tensile Behavior

Test Procedure – Tensile tests of the beryllium sheet were determined in accord with the recommended practice for testing beryllium.⁷⁵ Two opposing extensometers spanning the specimen gage length were used to adjust alignment and measure strain. Proper alignment was assured by adjustment of the test fixtures until the output of the two extensometers was within 5% at an applied stress level of 34 MPa. A 196 kN capacity screw driven Instron testing machine was used for the tests.

Stress-Strain Behavior – The engineering stress-strain results for ingot sheet tensile specimens in eight different conditions of heat treatment are given in Fig. 41. Generally two or more specimens were tested for each condition of heat treatment. The results in Fig. 41 correspond to the specimens with the highest measured elongations because it was felt that these would have been least likely to have been affected by spurious defects in the gage length. It is noted in Fig. 41 that the yield strength spans a range of nearly a factor of 3, the ultimate strength spans a range of nearly a factor of 2, and there is 4-fold variation in elongation.

Examples of the engineering stress-strain results for powder sheet specimens in two conditions of heat treatment are given in Fig. 42. It is noted that the strength level and elongation are significantly higher for the powder sheet than for the ingot sheet, as expected. Most striking is the difference in the peak elongation for the ingot sheet (~9%) relative to the powder sheet (~34%). This difference is expected on the basis of the difference in preferred orientation reported in Ch. V and the fact that the more perfect basal texture of the powder sheet allows extension by width contraction due to slip on first order prism planes as discussed in Ch. III.

Measurement of Lateral Contractions – As pointed out in Ch. III, rolled beryllium sheet is expected to be plastically anisotropic and it is the capacity for through thickness contraction that is expected to influence the plane stress fracture toughness. It was desired, therefore, to experimentally measure the capacity for thickness contraction. This was done by measuring thickness before and after the tensile test to obtain

Fig. 41. Engineering stress-strain behavior for warm rolled ingot sheet beryllium annealed for 1 h at various temperatures.

Fig. 42. Engineering stress-strain behavior for powder sheet beryllium annealed for 1 h at two different temperatures.

a measure of the thickness strain at fracture. Width contraction and total elongation was also measured. The contractions are averages from three positions along the gage length and the elongation was that measured with the extensometer. Figure 43 gives these results for the ingot sheet beryllium after 1 h anneals at various temperatures. The precision of measurement of thickness contraction is deduced from the estimated sensitivity of the thickness measurement ($3\text{ }\mu\text{m}$) and the nominal pretest thickness (0.63 mm) as $\pm 0.5\%$. At this sensitivity the accuracy of the thickness contraction values should be better than 25% in the cases of contractions greater than 2%. In the cases of smaller contractions the possible error is greater. Constant volume deformation requires that the sum of the natural strains be zero or that the longitudinal natural strain (ϵ_l) should equal the sum of the lateral natural strains ($\epsilon_w + \epsilon_t$). This rule is also nearly obeyed by the engineering elongation (e_l) and the engineering lateral contractions (e_w and e_t) when the strains are small. At values of e_l less than 10%, the sum of the engineering lateral contractions ($e_w + e_t$) will be, for constant volume deformation, better than 90% of the value of e_l . The data in Fig. 43 are consistent, within this approximation and the sensitivity of the thickness contraction measurements, with the requirements of constant volume deformation.

Table 4 gives the elongation and lateral contractions measured for powder sheet in two conditions of heat treatment. The thickness contractions are considerably less

Table 4. Measurements of engineering elongation and lateral contractions for powder sheet beryllium.

Heat treatment	Elongation %	Thickness contraction ^a %	Width contraction ^b %
710°C, 1h	32.8	1.3	24.3
	25.6	1.2	26.0
780°C, 1h	20.4	0.74	18.5
	19.8	0.82	18.9

^a Average of measurements along gage length.

^b Measured at center of gage length.

than those found for the ingot sheet after similar heat treatments while the width contractions and tensile elongations are far greater.

As noted previously these differences are expected on the basis of the differences in preferred orientation reported in Ch. V and the fact that the more perfect basal texture of powder sheet allows extension by width contraction due to slip on the first order prism planes. At the 25% elongation level the value of $e_w + e_t$ should be, if deformation is homogeneous over the gage length, about 80% of e_l . Table 4 shows that this is not the case for the powder sheet. The reason for this is that the width contraction varied along the gage length to produce a relatively severe neck through the width. Since the

Fig. 43. Elongation and lateral contractions for ingot sheet beryllium as a function of annealing temperature.

extensometer which measures e_g spans the length of the neck it is apparent that the value of e_g is less than it would have been had the extensometer spanned a shorter length over a portion of the gage where the neck was more severe. The thickness contraction was, however, found to be relatively homogeneous over the gage length and the values given in Table 4 are averages of three measurements along the gage length.

Fracture Tests

Fatigue Precracking — All fracture specimens except as noted were fatigue precracked to develop a sharp crack front. This was expected to result in the lowest values of $\mathcal{G}_{crit}$ and K_{crit} on the basis of results obtained with other materials as discussed in Ch. I. A 196 kN capacity electrohydraulic machine was used to develop the fatigue cracks in tension-tension cycling. Generally, cycling was carried out at a calculated stress intensity [from Eq. (17)] range from 2 to 11 MPa-m^{1/2}. This generally caused between 0.13 and 0.25 mm of fatigue crack growth in fewer than 100 000 cycles. In certain cases peak tensile stresses as high as 16.5 MPa-m^{1/2} were necessary to develop fatigue cracks of reasonable length. The total amount of fatigue crack growth for each fracture specimen tested can be found by comparing the nominal crack length from Figs. 39 and 40 with the actual crack length for each specimen. The fatigue cracking was carried out with the machine cycling in position control. In this method, the position of the testing machine crosshead required to achieve the appropriate loading range was established and the crosshead cycled between these positions. The compliance of the specimen increases and the force drops as the crack lengthens so that more control over crack extension is possible than when fatigue cracking is carried out in force control.

Crack Mouth Displacement Measurement — In all cases except the J-integral tests the displacement in the fracture test was measured at the crack mouth as illustrated in Fig. 9. The crack mouth displacement gage which was used is fully described on pages 35-37 of Ref. 9. Briefly, the displacement is measured by following the output of a foil resistance strain gage bridge mounted on the arms of the displacement gage. The output of the gage is calibrated in a fixture incorporating a barrel micrometer. The sensitivity of the method is somewhat better than 2.5 μ m displacement.

Load Point Displacement Measurement — In the case of the J-integral tests it was necessary to measure the displacement along the line of loading in order to obtain a well defined energy input to the specimen. Figure 44 illustrates the overall displacement gage design and the method used to accomplish this. As indicated in this figure the gage is sprung between two stops cemented onto the specimen. The gage parameters were chosen to maximize linearity and sensitivity over the range of 0.25 mm displacement. The gage was designed for use with the small SEN specimen illustrated in Fig. 39. The length over which displacements were measured was chosen equal to

Fig. 44. Computed displacement gage calibration.

twice the specimen width (63.5 mm) because the work by Srawley, Jones and Gross⁴ indicated that the rate of change of compliance with crack length is independent of gage length at gage lengths of 2X width and greater. The displacement is measured by following the output of foil resistance strain gages bonded to the midsection of the gage. Two foil resistance gages are mounted on each side of the gage to form the full bridge. The output of two such gages on each side of a specimen are averaged to obtain the overall deflection. Figure 45 shows the displacement gages in place.

Fracture Test Procedure - All of the fracture tests except the J-integral tests were carried out on a 196 kN electrohydraulic machine at a constant cross-head velocity of $2.1 \mu\text{ms}^{-1}$. The J-integral tests were carried out on a screw driven, 588 kN capacity machine as a matter of convenience. Force and displacement were recorded over the course of the test. In preliminary studies it was established that no detectable stable crack growth occurred prior to fast fracture in the two types of sheet materials studied. This was concluded after microscopic examination of the crack tip during the course of the test. The result was confirmed by a later test in which the output of a strain gage mounted at the tip of the precrack indicated a gradual increase in strain with no indication of cracking. This particular test was carried out on ingot material in a condition of heat treatment which resulted in large nonlinear deflections during the test (to be discussed further in a following chapter). It was not necessary, therefore, to develop a sophisticated technique to monitor stable crack extension during the test. Visual observation was made during each test, however, to insure the absence of stable crack extension. In one condition of heat treatment of the ingot sheet pop-in behavior was observed. This particular result is discussed in a following chapter.

Fig. 45. Overall displacement gages in place on the small SEN specimen.

VII. RESULTS AND DISCUSSION - ANALYSIS OF NONLINEAR FRACTURE BEHAVIOR

It was discovered early in the program that certain heat treatments of initially warm rolled ingot sheet beryllium led to nonlinear load deflection behavior in the fracture test. This chapter is devoted to discussion of three independent methods of analysis of nonlinear fracture. This is necessary to develop a firm basis for comparison of the fracture toughness derived from the results of a nonlinear test with fracture toughnesses derived from linear elastic tests.

Compliance Technique

Figure 46 shows force-crack mouth deflection behavior observed for warm rolled ingot sheet heat treated for 1 h at 780°C. Records for both the large ($W = 95.25$ mm) and the small ($W = 31.75$ mm) SEN specimens are given. The elastic modulus and thickness compensated compliances (Evh/F) for the large specimen (4.87) and the small specimen (4.64) compare reasonably well with that corresponding to the same a/W ratio for 7076-T6 (4.15). The latter value, found from the correlation given in Fig. 10, was determined under essentially elastic conditions.⁴ The somewhat larger compliances observed in the present instance probably result from nonlinearity due to plastic deformation at even low applied force.

A method of analyzing these nonlinear records is obtained if it is assumed that all of the deviation from linearity is due to the development of a plastic zone ahead of the crack tip. This is reasonable since careful observation of the crack during the test revealed no crack growth. If it is assumed that the development of a plastic zone ahead of the crack tip has the same effect on compliance as an amount of crack extension of equal length then the plastic zone length can be deduced from the apparent compliance at the point of fracture and a prior correlation between compliance and a/W . As noted in Fig. 46 the apparent compliance is deduced from the reciprocal slope of the line drawn through the fracture point and the origin. A value of compliance compensated for modulus (taken as 290×10^9 Nm⁻² for beryllium) and thickness (0.80 mm) is then computed. A value of $a + \rho_{app}/W$, where ρ_{app} is the apparent length of plastic zone) is found from the correlation between compensated compliance and a/W given in Fig. 10. The apparent plastic zone lengths at fracture found using this method are 10.7 mm for the large SEN and 6.08 mm for the small SEN.

It is possible to compute the remote applied stress required to develop these same plastic zone lengths in infinite specimens if it is assumed that the Dugdale elastic plastic model for plane stress discussed in Ch. II is applicable. This seems reasonable in view of the suggestion by Hahn and Rosenfield that plane stress conditions prevail (for Si steel) in thicker sections at values of $\rho > 4$ h. The appropriate equation for this calculation, from Ch. II, is:

Fig. 46. Force-displacement behavior for SEN specimens of ingot sheet beryllium, annealed at 780°C.

$$\sec \frac{\pi \sigma}{2Y} = \frac{\rho}{a} + 1. \quad (24)$$

It is necessary to choose a value for the cohesive stress or the strength of the plastic metal within the zone for this calculation. A value for Y was taken as an average of the yield and ultimate strengths measured in uniaxial tension for the 780°C condition of heat treatment ($Y = 235$ MPa). The values for the remote stress so computed are 102 MPa ($\rho = 10.07$ mm, $a = 34.7$ mm) and 127 MPa ($\rho = 6.08$ mm, $a = 11.7$ mm). Values for fracture toughness can be found from these stresses using the solution for stress intensity for the infinite SEN specimen:

$$K_{crit} = \sigma \sqrt{\pi a}. \quad (37a)$$

Following a suggestion by McClintock and Irwin⁷⁶ a plastically corrected toughness is computed from:

$$K_{crit} (\text{corrected}) = \sigma \sqrt{\pi(a + \rho)}. \quad (37b)$$

The fracture toughness values so computed are summarized in Table 5.

Table 5. Summary of fracture toughness values from compliance method for ingot sheet beryllium heat treated at 780°C.

	Apparent plastic zone length — mm	K_{crit} MPa·m ^{1/2}	$K_{crit} (\text{corrected})$ MPa·m ^{1/2}
Large SEN a = 34.7 mm	10.7	33.7	38.5
Small SEN a = 11.7 mm	6.08	24.3	30.0

A notable feature of the data in Table 5 is that the plastic zone length and fracture toughness found from the tests on the smaller specimen are significantly lower than the results for the larger specimens. This difference probably results from the assumption that the plastic zone has the same effect on compliance as an equivalent amount of crack extension. Actually, because a cohesive force acts on the elastic-plastic boundary, the specimen will be less compliant than one with the plastic zone replaced by an equal increment of crack extension. Thus the actual plastic zone length will be greater than that found using the method described above. This effect is expected to be greater for smaller specimens in which the zone will occupy a greater fraction of the net ligament width.

The toughnesses recorded in Table 5 are much higher than those heretofore reported for ingot sheet. This, together with the facts that the analysis is based on an

assumed model and that the results are not independent of specimen size made it necessary to examine other independent methods of nonlinear analysis.

J-Integral Method

The J-integral method of nonlinear fracture analysis, first proposed and tested by Begley and Landes¹⁷ and reviewed in Ch. I, offers an independent method of analyzing nonlinear behavior. The approach to the J-integral tests taken in the present instance follows that described by Begley and Landes.

The force vs overall displacement behavior for four small SEN specimens (ingot sheet heat treated at 750°C), each containing a different crack length, was determined using the methods described in Ch. VII. The resulting force-displacement results are given in Fig. 47. The J-integral value corresponding to three different crack lengths was determined for a number of different deflections. To do this the area between adjacent curves up to the displacement of interest was measured with a planimeter to obtain a value for the pseudo energy release. This energy was then divided by the difference in crack length and thickness to obtain a value for J (a pseudo strain energy release rate) at each displacement. The resulting J vs displacement relationship for three different crack lengths is plotted in Fig. 48.

The critical value of J is obtained at the point of fracture and the critical values corresponding to the three crack lengths are indicated in Fig. 48. The three values have been converted to K_{crit} values assuming $\mathcal{G}_{crit} = J_{crit}$ and using the relationship $K = \sqrt{E\mathcal{G}}$. The values of K_{crit} from the J integral tests (39.6, 40.0, and 42.3 MPa·m^{1/2}) are in reasonable accord with the plastically corrected toughness value found using the large specimen compliance technique discussed in the previous section (38.5 MPa·m^{1/2}).

Begley and Landes¹⁷ found that the value of J_{crit} for the steel they investigated was the same within material variation as a linear elastic value of $\mathcal{G}_{crit}$ which had been determined for the same steel by a different investigator using much larger specimens. Begley and Landes concluded, therefore, that their J approach to nonlinear analysis yielded a result equivalent to a linear elastic test. They were careful to point out in subsequent discussion, however, that the specimen dimensions (including remaining ligament and crack length) required for a valid J result must be related to the dimensions over which the J parameter describes the crack tip stress field. It is necessary, therefore, to verify the validity of a J test by independent methods until sufficient experience and/or theoretical understanding is acquired to allow minimum specimen size requirements to be defined.

In the present instance it is not possible to verify the J results by testing a linear elastic specimen since the specimen size requirements estimated using (18) are larger than the sheets produced for the present study. Based upon a yield strength of 152 MPa and a K_{crit} of 40 MPa·m^{1/2} the length and width requirements are 1560 mm and 520 mm, respectively, for a SEN specimen. A third independent method of nonlinear analysis was developed to test the validity of the toughnesses derived from the J method and the compliance method.

Fig. 47. Force vs overall deflection results for J-integral specimens. Small SEN specimens annealed at 780°C.

Fig. 48. J vs displacement for ingot sheet beryllium fracture specimens 31.8 mm wide.

Plastic Work Method

The third method of fracture analysis is based on Rice's proof that:

$$\mathcal{G}_{\text{crit}} = \int_0^{\delta_t^*} \sigma(\delta) d\delta. \quad (29)$$

This proof applies strictly only to the Dugdale model in which the plastic zone is a constant height strip that extends directly ahead of the crack tip. Assuming that the fracture of the present sheet can be reasonably approximated by such a strip yield model it is apparent that the value of $\mathcal{G}_{\text{crit}}$ can be obtained by measuring the area under the cohesive stress-crack opening displacement curve. It was considered possible to measure the crack opening displacement, as defined in Fig. 49, directly at the crack mouth if the specimen could be made small enough so that the elastic contribution to crack mouth displacement was small compared to the contribution from crack tip plasticity. To accomplish this it was appropriate to choose a specimen with a net ligament smaller than the plastic zone size at fracture so that the plastic zone would span the entire ligament width. On this basis the net ligament width chosen was 4.57 mm which is considerably smaller than the critical plastic zone lengths deduced from the compliance technique (Table 5). A double edge notched specimen was chosen so that $\sigma(\delta)$ could be simply obtained from the applied force and net section area. The principle of the technique is indicated in Fig. 49. It is evident that the crack tip displacement δ indicated in this figure will be the same as the displacement of the upper and lower boundaries of the plastic zone and that this displacement, except for a small elastic contribution, will be equal to the displacement measured at the crack mouth. This is most easily visualized if the plastic zone is imagined to be a hole in the specimen.

It is pertinent to discuss the properties of the proposed test specimen somewhat further to illustrate the unique characteristics of this specimen design. In Fig. 50 the applied net section stress is plotted vs the crack length to width ratio, $2a/W$, for various applied stress intensity levels for a 31.75 mm width DEN specimen. The relationship between stress intensity, applied force, and specimen dimensions which was used is given on p. 10 of Ref. 9. Figure 50 shows that the net section stress will exceed both the yield and ultimate strengths of present material well below the value of force which would be required to cause linear elastic fracture at $40 \text{ MPa}\cdot\text{m}^{1/2}$. For the presently selected value of $a/W = 0.85$ the ultimate stress will be exceeded at a force which can only develop a stress intensity of less than half of the expected critical value. This means that the specimen should fail by overall plastic deformation even in the presence of the rather long and sharp cracks. This type of failure by overall plastic deformation has been termed a limit load failure and is the failure mode for small specimens of tough materials.¹⁷

The net section stress-displacement behavior observed for two specimens of the present design is shown in Fig. 51. The specimens were subjected to different fatigue

Fig. 49. Principle of plastic work method of fracture toughness testing.

Fig. 50. Characteristics of small double edge notch (DEN) specimen used for fracture analysis by the plastic work method.

Fig. 51. Net section stress vs displacement for narrow net section specimens used to determine fracture toughness by the plastic work method, Ingot sheet beryllium annealed at 780°C.

cracking histories. The area under the curve should, by (29), give a value for $\mathcal{G}_{crit}$. The areas were determined with a planimeter and the values of $\mathcal{G}_{crit}$ so obtained were converted to K_{crit} values using $K_{crit} = \sqrt{E\mathcal{G}_{crit}}$. As indicated in Fig. 51 this resulted in $39.1 \text{ MPa}\cdot\text{m}^{1/2}$ and $36.7 \text{ MPa}\cdot\text{m}^{1/2}$ for the two specimens tested. Of these, the first is considered to be the more correct value since it is evident that the extreme fatigue history is probably responsible for the lower second result. In this connection it is noted that it was necessary to fatigue precrack these specimens at stresses above the uniaxial yield stress in order to develop cracks in a reasonable number of cycles. Thus it is expected that some work hardening would result from the high fatigue loads and that the critical plastic displacement measured in a subsequent fracture test would be reduced.

The best results from each of the nonlinear methods of analysis are summarized in Table 6. It is evident that all three methods resulted in values which can, in view of

Table 6. Summary of K_{crit} values found using three methods of fracture analysis for 0.8 mm thick ingot sheet beryllium heat treated at 780°C.

	$K_{crit} - \text{MPa}\cdot\text{m}^{1/2}$
Compliance method - large SEN	38.5
J-integral method	42.3, 40.0, 39.6
Plastic work method	39.1

normally expected material variation, be considered the same. The correspondence is taken as evidence that the methods used result in the correct value of K_{crit} . It is expected that the same value would result from a linear elastic test of a very large specimen. The demonstration of this, however, was beyond the scope of the present program. The fracture toughness values found using the nonlinear methods are nevertheless used as a valid basis of comparison with linear elastic results found for ingot sheet given other heat treatments and for powder sheet material.

VIII. RESULTS AND DISCUSSION - THE EFFECT OF HEAT TREATMENT ON THE FRACTURE TOUGHNESS OF INGOT SHEET BERYLLIUM

Force-Deflection Behavior

A series of SEN specimens of warm rolled ingot sheet were given 1 h vacuum anneals at various temperatures from 680°C to 830°C. The optical and transmission electron microscope characterization of the microstructures corresponding to the various heat treatments have been given in Figs. 31 through 36. The specimens were fatigue precracked after heat treatment and fracture tested using the procedure described previously. The resulting force vs crack mouth deflection behavior are given in Fig. 52. It is noted that all the specimens annealed below 750°C exhibited essentially linear elastic behaviors while the specimens annealed at higher temperatures exhibited various amounts of nonlinearity.

The linear elastic results (first seven force-displacement records) were converted to K_{crit} values using the corresponding peak loads, specimen dimensions, and the stress intensity solution appropriate for the finite SEN specimen [Eq. (17), given in Ch. I]. The effect of specimen size was evaluated for the case of linear elastic behavior by testing both small and large SEN specimens after heat treatment at 710°C. One of the small specimen results ($K_{crit} = 15.5 \text{ MPa}\cdot\text{m}^{1/2}$) was similar to the two results from the larger SEN specimens ($K_{crit} = 13.7$ and $15.2 \text{ MPa}\cdot\text{m}^{1/2}$). This is taken as evidence that K_{crit} is independent of specimen size (and, consequently, of nominal stress at fracture), thus satisfying one of the basic tenets of elastic fracture mechanics. The second small SEN specimen fractured at a somewhat higher stress intensity ($K_{crit} = 19.8 \text{ MPa}\cdot\text{m}^{1/2}$) and this difference might be the result of material variation.

The nonlinear force-displacement results (last five records in Fig. 52) were analyzed to yield values for K_{crit} using the compliance method discussed in Ch. VII. The required cohesive strengths were taken as the averages of the yield and ultimate strengths measured in uniaxial tension for each case. The cohesive strength values were 297 MPa, 228 MPa, and 207 MPa for the 750°C, 780°C, and 830°C conditions of heat treatment, respectively.

The force-displacement record for the case of the 830°C condition of heat treatment shows several pop-ins, or sudden changes in slope, prior to the point of fracture. It is believed that these pop-ins were a result of a small change in the compliance of the specimen associated with stable fracture of individual grains or clusters of grains within the plastic zone. Such stable microcracking has been observed in tensile specimens. It is probable that pop-ins were not observed in the case of the specimens given lower temperature heat treatments because of the finer grain size of these specimens (less than 50 μm vs 72 μm for the 830°C condition). Cracking of the finer grains would have less of an effect on compliance and pop-ins would not be expected to be resolvable below a certain grain size. The pop-ins did not result in appreciable crack extension

Fig. 52. Force vs displacement behavior for ingot sheet beryllium fracture specimens annealed for 1 h at various temperatures.

(less than 0.25 mm) and the original crack length was used to calculate the fracture toughness. A fracture toughness corresponding to both the first pop-in indication and the point of fracture was computed for the specimen annealed at 830°C.

It is noted in Fig. 52 that three large SEN specimens, each containing a different crack length, were tested in the 750°C condition of heat treatment. The initial intent of these tests was to evaluate the effect of crack length on the fracture toughness for the case of linear elastic fracture. It was not known at the time that the records would turn out to be nonlinear. The large difference between K_{crit} found using the specimen containing the longest crack ($22.2 \text{ MPa}\cdot\text{m}^{1/2}$) and the other two specimens (35.7 and $38.6 \text{ MPa}\cdot\text{m}^{1/2}$) is attributed to metallurgical effects and not an effect of crack length. Some microstructural and fractographic support for this conclusion is given in following chapters.

Fracture Toughness vs Annealing Temperature

The fracture toughnesses derived from the data in Fig. 52 are summarized in Fig. 53. Also included in Fig. 53 are the fracture toughness values which were obtained using the J-integral and plastic work methods for the 780°C condition of heat treatment. The figure shows that a very sharp transition in behavior occurs at an annealing temperature of about 750°C. The two points at 830°C are connected by an arrow to indicate that both refer to the same specimen, as noted in the discussion of Fig. 52.

The toughness between the upper and lower plateau regions differs by about a factor of 2.5. This difference would lead to a factor of about 6 difference in critical crack length at any given stress in linear elastic fracture situations or of about 2.5 in the stress required for fracture at any given crack length. The reasons for this large effect of heat treatment on fracture toughness are the subject of the following chapter.

Fig. 53. Fracture toughness vs annealing temperature for ingot sheet beryllium.

IX. RESULTS AND DISCUSSION - ORIGIN OF FRACTURE TOUGHNESS OF INGOT SHEET BERYLLIUM

Comparison with Uniaxial Tensile Properties

The effect of annealing temperature on the engineering uniaxial tensile properties was determined for comparison with the fracture toughness results. These are necessary in order to test predictions of plane stress or plane strain fracture toughness using relationships based upon tensile properties. The through thickness contraction, e_t , was determined as well as the usual uniaxial properties of tensile elongation e_l , the 0.2% offset yield stress σ_{ys} , and the ultimate tensile stress σ_u . The resulting properties are shown in Fig. 54. A comparison between Figs. 53 and 54 shows that the dramatic increase in fracture toughness caused by annealing at temperatures above 750°C is not reflected in any obvious way by a similar increase in any tensile property. In fact, if one were to choose an optimum heat treatment on the basis of tensile properties one might choose 710°C since the yield and ultimate strengths, and tensile elongation are greater than after annealing at any of the higher temperatures. This would, however, entail a severe penalty in fracture toughness according to Fig. 53. The reasons for the abrupt increase in toughness caused by annealing above 750°C are discussed in the following sections.

Upper Plateau Toughness

Comparison with Plane Stress Estimate - An estimate of the plane stress value of fracture toughness was made using the estimating Eq. (33) developed in Ch. II and the tensile properties given in Fig. 54. The fractional elongation to fracture e_f given used in Eq. (33) is, however, replaced by the through thickness contraction e_t because it was postulated that the capacity for through thickness contraction will determine the magnitude of the plane stress toughness in the case of plastically anisotropic beryllium sheet. The estimating equation then becomes for the case of beryllium sheet:

$$K_{crit}(\text{plane stress estimate}) = \left[Eh \left(\frac{\sigma_{ys} + \sigma_u}{2} \right) e_t \right]^{1/2} \quad (39)$$

Where two values of e_t were measured, the greatest one was used for the toughness estimate. The estimated fracture toughness values are plotted together with the observed toughnesses in Fig. 55. The figure shows that the observed toughnesses are somewhat greater than the plane stress estimates for annealing temperatures above 750°C and similar in magnitude to the highest values corresponding to measurements after annealing at 750°C. This result gives a strong indication that fracture does occur in plane stress in the cases of specimens annealed above 750°C and that it is the through thickness contraction which determines the magnitude of the toughness. If one uses the plane stress estimating equation with the measured uniaxial elongation instead

Fig. 54. Effect of annealing temperature on tensile properties of warm rolled ingot sheet beryllium.

Fig. 55. Comparison of measured fracture toughness with predicted plane stress values for ingot sheet beryllium.

of thickness contraction one computes toughnesses which are far higher than any observed in the present program. The toughness estimated in this way for the 780°C condition of heat treatment is, for example, $64.6 \text{ MPa}\cdot\text{m}^{1/2}$. The fact that the estimated toughnesses are slightly less than the measured toughnesses for the 780 and 830°C conditions of heat treatment is probably due to the fact that e_t measured in the tensile test is a lower bound value. It might be anticipated, for example, that a higher e_t would result if the test were carried out in biaxial tension because large amounts of prism slip would be inhibited.

Below annealing temperatures of 750°C, however, the estimated plane stress toughnesses are at least a factor of two greater than the measured toughnesses. It is concluded that fracture in the cases of these lower annealing temperatures occurs in plane strain or in a combined plane stress-plane strain mode and possible explanations for the observed magnitude of the toughnesses are considered in a following section.

Direct Measurement of Crack Tip Strains - The comparison between the measured and estimated plane stress fracture toughnesses shown above gives a strong indication that fracture occurs in plane stress in the case of material annealed above 750°C. It was considered desirable, however, to develop a confirmation of this since strengthening of the crack tip metal, associated with the biaxial stress state and the plastic anisotropy of the beryllium, could have occurred. This would have the effect of reducing the effective ductility necessary to achieve a given toughness level.

A large SEN specimen in the 780°C condition of heat treatment was fatigue pre-cracked and foil resistance strain gages were applied at various distances ahead of the crack tip. A gage height (0.79 mm) nearly equal to the thickness (0.80 mm) was chosen because of the assumption that plane stress deformation at a crack tip involves shear deformation over a height about equal to thickness (see Ch. III). Each gage in the array was 1.78 mm long and the gages were 2.03 mm apart. The gages, mounted on a polyamide backing, were applied to the specimen by adhesive bonding. The leading edge of the first gage was applied within 0.03 mm of the fatigue crack tip. A photograph of a portion of the strain gaged specimen (after fracture) is given in Fig. 56.

The outputs of four strain gages in the array were monitored over the course of the fracture test. The strain output from each gage increased monotonically with the applied force until unstable fracture occurred. The strain indicated by each gage at the fracture point is plotted vs the distance of the gage center from the crack tip in Fig. 57. Also plotted for comparison is the strain calculated using displacements from the Dugdale model as given by Eq. (25) divided by thickness. A fracture toughness of $38.5 \text{ MPa}\cdot\text{m}^{1/2}$ was assumed for this calculation.

It is noted that the measured and computed strains near the crack tip are very similar but that significant differences between the two are evident at distances greater than about 1.5 mm ahead of the tip. Several factors can account for the differences. First, the Dugdale model assumes the metal within the zone is rigid-plastic. In reality this metal undergoes preyield elastic and microplastic strains. At the level of the

Fig. 56. Foil resistance strain gage instrumentation to measure crack tip strains in large SEN specimen.

Fig. 57. Comparison between measured and calculated (Dugdale model) crack tip strains for ingot sheet beryllium annealed at 780°C.

yield stress (151 MPa) the elastic strain is about 0.05% and the plastic strain is about 0.2%. The sum of these (0.25%) nearly accounts for the difference between the measured and computed strains near the end of the plastic zone (16 mm ahead of the tip). In the region between 2 mm and 10 mm ahead of the crack tip the computed strains are greater than the measured strains. To test the possibility that the effect of work hardening would account for this difference a strain was calculated assuming $Y = 276$ MPa (nearly the ultimate stress) at a distance of 3 mm ahead of the tip. The point plotted in Fig. 57 shows that work hardening probably cannot account for the difference between the measured and computed strains. The strains were computed assuming a constant plastic zone height equal to the sheet thickness. The computed strains are reduced if a larger effective height is assumed. It is possible that a bulging of the zone ahead of the crack tip could account for the differences in Fig. 57. The differences would suggest a zone shape about as indicated in Fig. 57.

The facts that the crack tip strains are nearly equal to those computed on the basis of the Dugdale plane stress model and that the differences between the calculated and measured strains are understandable in terms of a reasonable plastic zone shape is taken as confirmation that fracture in the 780°C condition of heat treatment occurs in plane stress. It was not within the scope of the present work to develop independent verification of the zone shape.

Lower Plateau Toughness

Lack of Correlation with Uniaxial Properties — As noted in Fig. 53, the ingot sheet specimens which were heat treated below 750°C fractured at similar values of fracture toughness. For this series of specimens the yield strength drops from about 360 MPa for the as-rolled specimen to about 245 MPa for the specimen annealed at 730°C (see Fig. 54). Figure 54 also shows that the ultimate tensile strength of tensile specimens heat treated below 750°C is about constant at 390 MPa. The drop in yield strength with increasing annealing temperature is qualitatively understandable on the basis of the increase in subgrain size and the decrease in dislocation density apparent in the transmission electron micrographs in Figs. 32 through 34. The relative constancy of the fracture stress is understandable, in view of the fact that the macroscopic grain size, as shown in Fig. 31, is about constant up to an annealing temperature of 750°C (38 to 47 μ m). The latter observation indicates that the grain size is the only factor that influences fracture stress over this range of annealing temperatures.

All known correlations of plane strain fracture toughness, K_{Ic} , with tensile properties predict that K_{Ic} will decrease as the ratio σ_F^*/σ_y or the difference $\sigma_F^* - \sigma_y$ decreases. σ_F^* is the so-called microcleavage fracture stress and is the stress that if achieved locally at the crack tip, would cause fracture to occur. Correlations of fracture toughness with uniaxial properties, taking σ_F^* equal to the uniaxial fracture strength, have been fairly successful in cases of cleavage fracture of mild steel.³⁸ One such correlation,³⁹ shows that $K_{Ic} \propto (\sigma_F^* - \sigma_y)$ would predict a marked decrease (by

more than a factor of four) in fracture toughness as the annealing temperature is reduced below 750°C. This is in contrast to the apparent insensitivity of the measured values to annealing temperature in this range. It is concluded that simple correlations between tensile properties and plane strain fracture toughness cannot account for the toughnesses measured after annealing at temperatures up to 750°C.

Interpretation in Terms of Microstructure and Constraint — The most striking difference between the high toughness specimens and the low toughness specimens annealed below 750°C is the absence of subgrains in the former and the presence of subgrains in the latter (see Figs. 32 through 36). Since bend plane splitting by shear of dislocation walls is known to be a principal mechanism of fracture of beryllium it is possible that the low fracture toughness values are associated with basal plane fracture due to the shear of low angle boundaries between subgrains. In addition to this it is probable that stress intensification due to plastic constraint played a role in the fracture of the low toughness specimens. If the plastic zone is not developed sufficiently in size to cause stress relaxation by through thickness shear, then superstressing beyond the uniaxial flow strength is expected at the crack tip. Hahn and Rosenfield suggested on the basis of experimental studies that the plastic zone length must be greater than about four times the thickness in order for the shear mode of plastic zone growth to be fully developed. In the case of the low toughness specimens, the plastic zone sizes calculated from Eq. (19) are less than the sheet thickness. This suggests that stress intensification due to plastic constraint might have been quite substantial in the case of the low toughness specimens.

Even in view of the above observations a basic analytical interpretation of the origin of the low fracture toughness values is not possible at present. Such an analytical treatment would require knowledge of the way in which the crack tip stress and strain varies with applied stress intensity and the conditions, in terms of a critical stress or strain, which must be met to initiate fracture. Since the critical stress for fracture probably depends on the prevailing strain, the relationship between the two over a large strain would have to be known to properly apply a failure criteria. Because of the absence of appropriate analytical insights into the above questions at the present time it would not seem possible to account for the toughness in terms of a precise analytical elastic-plastic analysis.

It is instructive, however, to consider a simple interpretation of the magnitude of the toughness in terms of the simple Dugdale model and several presumed effects of microstructure and constraint. The process of bend plane splitting requires a finite amount of shear strain and since nearly every grain contains subboundaries it is reasonable to postulate that rapid fracture will occur as soon as the applied stress intensity results in a small critical shear strain in the grains immediately adjacent to the crack tip. It has been suggested previously³¹ that the grain size is the lower limiting volume over which plastic shear can take place and that in the case of fracture due to initial yielding, the conditions which govern yielding (or shear) in the grains

immediately ahead of the crack tip control fracture. These considerations can be used as a basis for calculating the fracture toughness. Combining Eqs. (16) and (28) the following is obtained.

$$K = \sqrt{EY\delta_t}. \quad (40)$$

The postulated condition for fracture is:

$$\delta_t = \delta_t^* = e_t d, \quad (41)$$

where e_t is the through thickness contraction measured in the tensile test and d is the grain diameter.

Equation (41) implies that the plastic zone height is limited to the diameter of one grain. It also implies that the attainment of a small strain, similar to the through thickness strain required for fracture in a tensile test, is sufficient to cause bend plane splitting in these grains and this event triggers fracture. A constrained value for Y , the flow stress of the metal in the plastic zone, is taken equal to $3\sigma_{ys}$. The factor of 3 was used by Orowan²² in a similar application on the basis that slip line field solutions to similar plane strain plastic problems results (for ideal plastic materials) in a stress intensification factor of about 3. It is noted, however, that Rice¹⁸ has shown analytically that strain hardening can result in stress intensification factors appreciably above the factor of three. Combining (40) with (41) and the latter condition results in the following:

$$K_{crit} \text{ (flow toughness specimens)} = \sqrt{E(d)(3\sigma_y)(e_t)}. \quad (42)$$

Using (42) with the grain sizes and tensile properties reported previously results in the calculated toughnesses given in Table 7. The measured toughnesses are, for comparison, given in the same table.

Table 7. Calculated and observed fracture toughnesses for ingot sheet specimens annealed below 750°C, MPa·m^{1/2}.

Annealing temperature °C	Calculated ^a	Observed
As received	10.7	15.8
680	13.1	16.4
710	14.0	13.7, 15.5, 19.8, 15.2
730	13.2	15.6

^aFrom Eq. (42).

The relative constancy of the fracture toughness values calculated on the basis of Eq. (42) and the fact that the magnitudes compare favorably with the measured values is somewhat surprising in view of the imprecision inherent in the development of the equation. It is nevertheless apparent that the basic assumptions that the plastic zone height is limited to the grain size and that stress intensification plays a role in the fracture process may have some validity.

Effect of Crack Tip Root Diameter - A further test of the above development is made possible by evaluating the effect of crack tip root diameter on the fracture toughness of the low toughness specimens. Replacing the grain size d by the crack tip root diameter, ρ , in Eq. (42), results in the following:

$$K_{crit} = \sqrt{E(3\sigma_y)(e_t)} \rho^{1/2}. \quad (43)$$

Equation (43) suggests that a plot of K_{crit} vs $\rho^{1/2}$ should result in a slope of $\sqrt{E(3\sigma_y)(e_t)}$. Large SEN specimens containing various crack root diameters were fracture tested to test Eq. (43). A 710°C annealing treatment was chosen for this particular study. Figure 58 shows the force-crack mouth deflection behavior for the specimens containing the various root diameters. It is apparent from the figure that a small amount of non-linearity occurs at the high force levels necessary to fracture the specimens containing the large root radius cracks. Values for K_{crit} were nevertheless found using the peak load and the stress intensity solution for the finite SEN specimen given in Eq. (17). The resulting K_{crit} values are plotted vs $\rho^{1/2}$ in Fig. 59 together with K_{crit} values from fatigue cracked specimens. Also plotted in the figure are results for two specimens given the 730°C anneal. The horizontal line drawn from the ordinate corresponds to the magnitude of the toughness calculated from Eq. (42). The slope of the line beyond $\rho^{1/2} = 0.2$ was drawn so that it would be equal to the coefficient of $\rho^{1/2}$ in Eq. (43). A smooth connection was drawn between the two lines near their point of intersection. The calculated lines are seen to account quite well for the observed data. In addition, the break in the curve occurs at about $d^{1/2}$ which is expected if the grain diameter represents a lower limiting dimension over which yielding can occur and therefore a lower limit to $\rho^{1/2}$. The above is taken as good evidence that the interpretation of the magnitude of the lower plateau toughness in terms of a plastic zone height limited about to the grain diameter is correct. It may be fortuitous, however, that the use of a constraint factor of 3 and a fracture strain equal to e_t leads to about the right magnitude of toughness. Any combination of each which leads to the same product would also account for the observed data.

Toughness After Annealing at the Transition Temperature

Transmission electron microscopy was carried out on the highest and lowest toughness specimens which were annealed at 750°C. This was done to determine if a

Fig. 58. Force vs crack mouth deflection for large ingot sheet beryllium SEN specimens annealed at 710°C with various crack root diameters.

Fig. 59. Measured K_{crit} vs square root of crack tip root diameter.

difference in substructure could be found in these specimens. Several transmission electron micrographs from undeformed areas of the specimens are given in Fig. 60. Fine subgrains typical of those present in material annealed below 750°C were found in both specimens. In the case of the tougher specimen, however, a substantial number of large grains were found which did not contain substructure as shown in Fig. 60b. It is evident that the recrystallization process, while still incomplete, had progressed further in the case of the tougher specimen. It was not possible to obtain a statistically significant quantitative measure of the fraction of grains which were free of substructure because of the large size of the grains and the small area of the foil suitable for electron transmission. In the foil of the tough specimen, however, as much as 50% of the area seemed to be free of subgrains. The above observations support the conclusion that the presence of subgrains can account for the low fracture toughness of the ingot sheet specimens annealed below 750°C.

3 000 x

6 000 x

16 000 x

(a) Crack length, 48.92 mm; fracture toughness, $22.2 \text{ MPa} \cdot \text{m}^{1/2}$.

2 000 x

2 000 x

6 000 x

(b) Crack length, 35.36 mm; fracture toughness, $38.6 \text{ MPa} \cdot \text{m}^{1/2}$.

Fig. 60. Transmission electron micrographs of ingot sheet beryllium fracture specimens annealed for 1 h at 750°C .

X. RESULTS AND DISCUSSION - EFFECTS OF HEAT TREATMENT ON THE FRACTURE MODE OF INGOT SHEET BERYLLIUM

Effect of Heat Treatment on the Fracture Mode of Precracked Specimens

To gain further insight into the factors governing the fracture toughness of ingot sheet a fractographic study of the specimens on which fracture toughness measurements were made was carried out. Scanning electron microscopy of the fracture surfaces of the ingot sheet fracture specimens annealed at 680 and 780°C are given in Fig. 61. It is observed that the mode of fatigue cracking changes from cleavage or quasi-cleavage after heat treatment at 680°C to largely intergranular after the 780°C heat treatment. It might be argued that the two different precrack modes result in different effective notch acuities and that this could have led to the transition in behavior at 750°C. It is noted, however, that specimens annealed below 750°C containing finite root radius precracks fracture at about the same stress intensity as the prefatigue cracked specimens (see Fig. 59).

The mode of fracture in the fast fracture region is cleavage in both cases. In the case of the 680°C heat treatment, however, the fracture surface is more hackled. This is consistent with the expectation that passage of a crack through subgrain walls should lead to ledge formation and that a higher subgrain density should result in more ledges and hence a more hackled appearance. Figure 62 shows that both the fatigue and fast fracture regions contain large amounts of intergranular failure after heat treatment at 830°C. The above observations are summarized in Table 8.

Table 8. Effect of annealing temperature on fracture mode in the precrack and fast fracture regions.

Annealing temperature	Fracture mode	
	Precrack region	Fast fracture region
680°C	>95% cleavage	>95% cleavage
780	>95% intergranular	>95% cleavage
830	>95% intergranular	>50% intergranular

One possible reason for the observed change in fracture mode is the effect of heat treatment on the distribution of impurity elements (mainly Fe and Al). Figure 27 in Ch. III shows the effect of heat treatment on the distribution of impurity elements proposed by Fraikor *et al.* on the basis of transmission electron microscopic studies. This figure shows that annealing at temperatures above 850°C and rapid cooling results in free aluminum in the grain boundaries. It might be postulated that intergranular weakness observed in the present work is associated with grain boundary aluminum. The fact that the fracture mode was affected starting at lower temperatures

Fatigue precrack region

680°C anneal 20 ×

80 ×

200 ×

400 ×

780°C anneal 20 ×

80 ×

200 ×

400 ×

Fast fracture area

680°C anneal 20 ×

80 ×

200 ×

400 ×

780°C anneal 20 ×

80 ×

200 ×

400 ×

Fig. 61. Scanning electron micrographs of fracture surfaces of ingot sheet beryllium SEN specimens annealed at 680 and 780°C.

(intergranular failure in fatigue after annealing at 780°C and in fast fracture after annealing at 830°C) could reflect a particular sensitivity of the fracture mode to the presence of free aluminum in the boundaries. The fracture toughnesses of specimens annealed at 780 and 830°C were not substantially different even though the fast fracture mode was cleavage in the former case and largely intergranular in the latter case. This could mean that cleavage initiated fracture in both cases. Figure 62 shows that significant amounts of cleavage fracture are present in the case of the specimen heat treated at 830°C.

Effect of Various Cooling and Aging Treatments on the Fracture Mode

An additional heat treatment study was carried out to determine if the fracture properties are limited by the occurrence of intergranular fracture or by other impurity induced effects of heat treatment. The approach was to compare the tensile properties and fractographic appearance of tensile specimens given a variety of heat treatments designed to modify the distribution of impurities.

A summary of the effect of various heat treatments on the tensile properties is given in Table 9. The heat treatment previously used to develop various microstruc-

Table 9. Effect of various heat treatments on the tensile properties of 0.80 mm thick ingot sheet beryllium.

Heat treatment	0.2% yield stress MPa	Ultimate stress MPa	Elongation %	Thickness contraction %
780°C, 1 h, cooled 0.5°C s ⁻¹	157	297	6.7	2.4
	164	307	7.4	2.8
780°C, 1 h, cooled 0.5°C s ⁻¹ , +700°C, 2 h, +600°C, 2 h	152	313	9.0	1.5
	157	305	8.0	1.9
830°C, 1 h, cooled 0.5°C s ⁻¹	138	280	5.0	1.6
830°C, 1 h, cooled 0.014°C s ⁻¹	135	273	6.0	1.2
	135	274	5.5	2.1
880°C, 1 h, cooled 0.5°C s ⁻¹	141	228	2.4	0.7
880°C, 1 h, cooled 0.5°C s ⁻¹ , +700°C, 2 h, +600°C, 2 h	134	220	2.8	1.0
	128	202	2.4	1.1

tures for fracture toughness tests was one hour at temperature followed by a relatively fast cool ($\sim 0.5^\circ\text{C s}^{-1}$) to temperatures below 600°C. This treatment is hereafter referred to as a normal anneal. Annealing at temperatures above $\sim 800^\circ\text{C}$ might be

expected to result in some resolution of iron (among other elements) and the appearance of free Al and Si in grain boundaries according to the scheme suggested by Fraikor *et al.* The relatively fast cool of $\sim 0.50^\circ\text{C s}^{-1}$ might prevent substantial recombination of the elements to form compounds during cooling. The mechanical properties found after normal annealing at 780, 830, and 880°C are given in Table 9. The fractographic appearance of the fracture surfaces of tensile specimens given the normal annealing treatments are given in Fig. 63. It is noted that only cleavage appears on the fracture surfaces after anneals at 680 and 780°C but that a significant amount of intergranular separation occurred as a result of the anneals at 830 and 880°C. The fracture appearances of these are very similar to those found for the corresponding fracture toughness specimens. This is expected since no new mode of fracture should be developed during plane stress fracture of a precracked specimen. The change in fracture mode from transgranular cleavage to intergranular after normal anneals above 800°C might be anticipated from known effects of heat treatment on the distribution of impurities. The presence of aluminum in grain boundaries due to high temperature annealing might, for example, provide a mechanism for grain boundary separation. Very slow cooling or aging after solution annealing is expected to promote the recombination of Fe, etc., with Al and Si to form precipitates which might have different effects on the fracture mode. A fractograph corresponding to a specimen given a very slow cool after normal annealing at 830°C is given at d in Fig. 63. It is noted that there appears to be much less intergranular fracture in this specimen compared to the fractographs obtained after normal annealing (c). Aging for short times after normal annealing at 880°C, on the other hand, did not seem to affect the fracture mode (compare e and f) significantly. The tensile properties given in Table 9, however, indicate that no significant differences in mechanical properties occur between the normal anneal and the duplex heat treatments. Even in the case of the specimens annealed at 830°C where a noticeable increase in the amount of cleavage occurred, apparently due to slow cooling, no detectable change in mechanical properties was found.

It is possible that cleavage initiated fracture in the case of all heat treatments. As a test of this possibility the fracture stress for the various heat treatments was plotted vs $d^{-1/2}$. Several of the theories of cleavage fracture of polycrystals predict that the fracture data will fall on a straight line passing through the origin on such a plot.⁷⁷ This type of relationship has been observed in the case of low temperature cleavage of iron.⁷⁸ Figure 64 shows that the data for the ingot sheet beryllium seem to conform to these requirements. In the case of the specimens annealed at 780°C (the four bracketed points) a broader spectrum of grain sizes was present than for the other heat treatments (see Fig. 31). Since the larger grains should control fracture, the four bracketed points would lie farther left on the plot in Fig. 64 if this effect were accounted for.

The fact that the data in Fig. 64 falls on a straight line passing through the origin (except for the qualification made above for the 780°C anneal) suggests that a single mechanism controls fracture in all cases. The indicated mechanism is cleavage since

(a) 680°C, 1h, cooled $0.5^{\circ}\text{C s}^{-1}$

(b) 780°C, 1h, cooled $0.5^{\circ}\text{C s}^{-1}$

(c) 830°C, 1h, cooled $0.5^{\circ}\text{C s}^{-1}$

(d) 830°C, 1h, cooled $0.014^{\circ}\text{C s}^{-1}$

(e) 880°C, 1h, cooled $0.5^{\circ}\text{C s}^{-1}$

(f) 880°C, 1h + 700°C, 2h + 600°C, 2h

Fig. 63. Scanning electron micrographs of fracture surfaces of ingot sheet beryllium tensile specimens given various heat treatments. All 200X.

Fig. 64. Fracture stress vs reciprocal square root of grain size for ingot sheet beryllium given various heat treatments.

no intergranular fracture could be found on any of the fracture surfaces of specimens annealed below 750°C (all of those in the vicinity of $d^{-1/2} = 5.0$ in Fig. 64). In addition, the specimens annealed at 830°C which were cooled at different rates and which exhibited much different proportions of cleavage and intergranular fracture (see c and d in Fig. 63), fall very close together in Fig. 64.

It is concluded that the initiation of fracture was by the cleavage mechanism in all cases of heat treatment examined. While the presence of intergranular separation on the coarser grained specimen fracture surfaces was not accompanied by a loss in strength, the reason for its presence has not been established. The increase in the amount of cleavage on the fracture surface in the case of the more slowly cooled specimen (after annealing at 830°C), however, suggests that the effect is impurity induced. Another possible explanation is that slow cooling reduces intergrain microstresses due to anisotropy of the thermal expansion coefficient by allowing plastic relaxation to occur during cooling. This seems to be a less likely explanation because after a few percent tensile strain such microstresses would be expected to wash out.

XI. RESULTS AND DISCUSSION - COMPARISON OF FRACTURE TOUGHNESS OF THIN POWDER AND INGOT ORIGIN BERYLLIUM SHEET

Fracture Toughness of Powder Sheet Beryllium

Measurements - The fracture toughness of the 0.80 mm thick powder source beryllium sheet described in Ch. V was measured in two conditions of heat treatment for comparison with the toughness of the ingot sheet. Four large SEN specimens each containing a different crack length were fabricated and heat treated at 780°C for 1 h.

The four different crack lengths were used as a check on the compliance characteristics and to determine if the measured toughness would be independent of crack length. The force-deflection results of the fatigued precracked specimens are given in Fig. 65. It is noted that the behavior is essentially linear elastic. Also, no stable crack growth or pop-in behavior was observed prior to unstable fracture. The elastic modulus and thickness adjusted compliance (Evh/F) for each specimen are noted in the figure. These are within 6% of the values corresponding to equivalent a/W ratios found for 7075-T6.⁴ It is concluded that the powder source sheet can be considered elastically isotropic in its plane and that the equations of linear elastic fracture mechanics can be legitimately applied.

Values for K_{crit} corresponding to each crack length were calculated using (17) and these are recorded in Fig. 65. It is noted that the K_{crit} values are within $\pm 7\%$ of one another. It is concluded that the measured toughnesses are within normally expected material variation and that the values of K_{crit} measured represent values which are valid for the particular thickness tested. It is noted, however, that they are not valid K_{Ic} values because the test does not conform to the requirements of the ASTM E399 specification for plane strain fracture toughness testing. The thickness requirement for a valid K_{Ic} test is, using (18) above and the measured yield strength (350 MPa), >17.6 mm which is a factor of 22 greater than the thickness actually tested (0.80 mm). Also, it is possible that a different K_{crit} would result from tests on specimens with surface flaws because of the different constraints which might accompany the development of a plastic zone. The K_{crit} values measured for through-thickness flaws are nevertheless considered valid for the purpose of comparing the toughnesses of powder and ingot origin sheet tested using the same specimen type and thickness.

Origin of Powder Sheet Toughness - The fracture toughnesses determined for powder sheet in the 780°C condition of heat treatment and a result from a specimen annealed at 710°C are given in Table 10. Also included in the table are estimates of the plane stress toughnesses derived from Eq. (39) and the uniaxial tensile properties determined after the same heat treatments. The estimate for the plane stress toughness was made using the through thickness contraction ϵ_t measured in the tensile test. The table shows that the measured and estimated fracture toughnesses are within 15%

Fig. 65. Effect of crack length on load deflection behavior of powder sheet beryllium annealed for 1 h at 780°C.

Table 10. Comparison between measured fracture toughness and plane stress values estimated from tensile properties for cross rolled powder sheet beryllium 0.80 mm thick.

Annealing treatment	Uniaxial tensile properties				Fracture toughness	
	Lower yield MPA	Ultimate MPA	Elongation %	Thickness contraction %	Computed $\text{MPa}\cdot\text{m}^{1/2}$	Measured $\text{MPa}\cdot\text{m}^{1/2}$
710°C, 1 h cooled 0.5°C s ⁻¹	393	527	30	0.74	27.6	30.0
	384	483	27	0.81		
780°C, 1 h cooled 0.5°C s ⁻¹	350	491	22	1.30	32.8	29.4 (average)
	344	458	22	1.10		

for both conditions of heat treatment examined. It is concluded on this basis that fracture of the powder sheet in the two conditions tested occurred in plane stress. It is possible that the presence of subgrains or impurity effects could have led to lower fracture toughnesses for the powder sheet studied. The investigation of these was, however, beyond the scope of the present program.

Comparison of Powder Sheet and Ingot Sheet

Force-Deflection Behavior - A striking difference in the force-crack mouth deflection behavior was observed between the powder sheet and ingot beryllium in the 780°C condition of heat treatment although both failed in plane stress. For comparison note the force-deflection behavior for the powder sheet SEN with the 34.77 mm crack length in Fig. 65 and the behavior for the ingot sheet SEN with the 34.7 mm crack length in Fig. 46. The deflection at the fracture load associated with nonlinearity is $\sim 2.5 \mu\text{m}$ in the former case and $\sim 52 \mu\text{m}$ in the latter case. This large difference in force-deflection behavior can be understood in terms of the Dugdale plane stress model and the difference between the cohesive strength and fracture toughness properties of the two materials. Using Eq. (24) from the Dugdale model, the plastic zone length was computed for both powder and ingot sheet in the 780°C condition of heat treatment. From this a value of $(a + p)/w$ was deduced and the corresponding apparent value of thickness and modulus adjusted compliance found from the empirical correlation given in Fig. 10. A deflection was then calculated at the fracture load from this compliance. The elastic deflection was subtracted from the former value to give an apparent deflection which results because of crack tip plasticity. It is apparent that the deflection due to plasticity calculated this way will be greater than the actual deflection because of the restraining effect of the plastic zone. The difference between the calculated and observed deflections would be expected to be greater in the case of the powder material because of its higher flow stress.

Table 11. Calculated and observed values of crack mouth deflections due to plasticity for ingot and powder sheet beryllium heat treated at 780°C.

Sheet	Plastic deflection at fracture load	
	Observed μm	Calculated μm
Ingot	52	130
Powder	2.5	9

The calculated and observed deflections at the fracture load are summarized in Table 11. As expected the observed deflections are smaller than calculated deflections and this effect is greater for the powder sheet. It is apparent, however, that the large difference in the observed deflections for the ingot and powder sheet materials (factor of 21) is fairly well accounted for using the calculational procedure described above (factor of 14 difference predicted). It is concluded that the differences in force-deflection behavior observed for the two types of material can be accounted for by the difference in fracture toughness and yield strength levels of the materials and the assumption of a Dugdale plane stress fracture model.

Fracture Toughness - The fracture toughness of the ingot sheet beryllium ($\sim 14 \text{ MPa}\cdot\text{m}^{1/2}$ in brittle condition and $\sim 40 \text{ MPa}\cdot\text{m}^{1/2}$ in tough condition) and the powder sheet beryllium ($\sim 30 \text{ MPa}\cdot\text{m}^{1/2}$) span the entire range of values which have been reported previously for beryllium sheet near 1 mm thick (see Fig. 16). It is apparent from the interpretation of the origin of the toughness in the case of plane stress fracture that the use of processing parameters which result in an increase in strength or through-thickness contraction would lead to an increase in toughness. It is probable, therefore, that the use of processing practice designed to lessen the severity of the preferred orientation of the powder sheet in 0.80 mm thickness could lead to an increased capacity for through-thickness contraction and hence a higher fracture toughness. It is conceivable that toughness values as high as those observed in the present case for ingot sheet could be attained ($40 \text{ MPa}\cdot\text{m}^{1/2}$). It is also possible, however, that improved processing to refine the grain size of ingot sheet could result in both higher strengths and greater through-thickness contractions and these improvements would in turn lead to fracture toughnesses considerably greater than the $40 \text{ MPa}\cdot\text{m}^{1/2}$ level observed for the present ingot sheet. The refinement in grain size would have to be done, however, without developing a debilitating level of preferred orientation. It would appear that a large margin of improvement is possible in the case of ingot sheet. The grain size of the ingot sheet studied in the present instance is, for example, relatively coarse ($49 \mu\text{m}$) in comparison with the grain size of the powder sheet studied ($15 \mu\text{m}$).

It is apparent that the presence of subgrains reduces the fracture toughness of ingot beryllium to well below the values observed for powder sheet in the present study. It is possible that this effect can account for the low toughness reported in the review by Conrad *et al.*⁴⁵ for ingot sheet. It is not possible to check this possibility, however, because of the lack of suitable characterization of that material. It is apparent from the present study that generalizations alluding to the better toughness of one or the other class of beryllium sheet are inappropriate at this time because the flexibility in the processing practice could lead to a wide range of toughness in either class of materials. Also, it is possible that impurity effects, preferred orientation, or the presence of subgrains could lead to very low toughnesses in either class of materials.

One of the conclusions drawn in the review by Conrad *et al.* was that the fracture toughness of rolled beryllium sheet will increase with elongation measured in the tensile test. This would lead to the expectation that the powder sheet tested in the present program would be much tougher than the ingot sheet. The elongations of the former were ~30% and for the latter ~8% (in the 780°C condition of heat treatment). In fact the opposite was observed and the greater toughness of the ingot sheet was associated with its greater capacity for through thickness contraction. Greater elongations can result from increasing the preferred orientation of sheet at the expense of through thickness contraction and it is concluded that this method of increasing elongation would result, other things being equal, in a reduction in fracture toughness.

The in-plane anisotropy of the fracture toughness was not investigated in the present program. It is possible that subtle differences in the in-plane preferred crystallographic orientation could result in significant directional effects on the fracture toughness. It would be expected that such effects would be greatest in the more highly oriented powder sheet material. It was beyond the scope of the present program to examine the effect of in-plane orientation. This would have to be done, however, before the fracture toughness could be considered as a useful parameter in a design situation.

XII. SUMMARY AND CONCLUSIONS

The ambient-temperature fracture toughness of thin (0.80 mm) ingot and powder source beryllium sheet was measured for a variety of metallurgical conditions. Fracture test methods were developed and compared to an existing method (J-integral) for the large scale plastic yielding regime so that the toughnesses of high toughness-low yield strength conditions could be evaluated. Various experiments were carried out to develop an understanding of the factors which account for and limit the magnitude of the fracture toughness.

The principal conclusions of the study are:

1. Three independent methods, one developed previously by others (J-integral) and two new methods (based on a compliance technique and a plastic work approach), were found to give equivalent results. This allowed the toughness of the low yield strength and tough conditions of sheet, which resulted in nonlinear force-deflection behavior, to be measured and compared to results from linear elastic tests.
2. The presence of subgrains in ingot origin beryllium sheet was found to result in very low toughnesses ($\sim 15 \text{ MPa}\cdot\text{m}^{1/2}$).
3. A study of the effect of crack root diameter on the fracture toughness led to the conclusion that the conditions which lead to yielding in the grains immediately adjacent to the crack tip control the magnitude of the toughness in the case of the sheet containing subgrains. It was concluded that the low toughnesses of the sheet containing subgrains was due to bend plane splitting early in the development of the plastic zone.
4. Very high fracture toughnesses were found in completely recrystallized ingot source beryllium sheet ($\sim 40 \text{ MPa}\cdot\text{m}^{1/2}$).
5. The magnitude of the high toughnesses of the fully recrystallized ingot sheet was correctly predicted assuming a plane stress fracture model based on the Dugdale analysis and the flow strength and capacity for through-thickness contraction measured in uniaxial tension. Direct measurement of the strains ahead of the crack tip were found to be consistent with the plane-stress model and the assumption that the ability to contract through the thickness determines the magnitude of the fracture toughness.
6. The fracture toughness of ingot source sheet in the optimum condition of heat treatment ($\sim 40 \text{ MPa}\cdot\text{m}^{1/2}$) was found to be greater than the toughnesses measured for the powder sheet ($\sim 32 \text{ MPa}\cdot\text{m}^{1/2}$) in the two conditions of heat treatment examined. The lower toughness of the powder sheet was attributed to a smaller capacity for through-thickness contraction because of its more pronounced preferred orientation.
7. It is concluded that it would be erroneous to attribute to one or the other class of beryllium sheet materials a greater fracture toughness. Flexibility in the processing of either could lead to higher toughnesses than those measured

for either in the present study. It is also concluded, however, that ingot sheet has a greater propensity for development of higher toughness by refinement of its grain size. Grain size refinement would have to be accomplished, however, without increasing preferred orientation substantially beyond the levels observed in the present ingot sheet material.

8. It is concluded that increasing the elongation measured in uniaxial tension by increasing preferred orientation with the strength properties held about constant should result in a reduction in fracture toughness. Increasing the capacity for through-thickness contraction, however, should lead to an increase in toughness. These conclusions apply only to sheet with preferred orientation which fractures in plane stress.

9. Heat treatments designed to modify the distribution of impurities were not found to influence the tensile fracture properties. Increasing the grain size, however, was found to result in the development of increasing amounts of intergranular fracture in both the fatigue precracked and fast fracture regions of the fracture toughness specimens. The same effect was found in tensile specimens. It was concluded, however, that cleavage initiated fast fracture in all instances.

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APPENDIX A

INGOT SHEET FABRICATION HISTORY

Scope

The order consisted of supplying 60 beryllium sheets $254 \times 508 \times 0.80 \pm 0.05$ mm. Specifications included a minimum of 10% reduction on the last bare rolling pass at a temperature below 690°C . Control of rolling direction to produce a balanced cross rolled product was also a requirement.

Material Fabrication History

Four $216 \times 216 \times 89$ mm beryllium billets were used in filling this order. The billets were obtained from $229 \times 229 \times 508$ mm castings. The billets were canned in 16 mm thick 304 stainless steel and rolled per schedule No. 1. After can rolling each of the sheets was annealed for 16 h at 760°C . The sheets were etched, ground to remove surface defects, and bare rolled per schedule No. 2.

The bare rolled sheets ($0.5 \times 1.5 \text{ m} \times 2.16 \text{ mm}$) were vacuum annealed for 9 h at 760°C and furnace cooled. Each sheet was heated to 425°C , sheared into five pieces ($267 \times 508 \times 2.16 \text{ mm}$), acid etched to 2.11 mm thickness, and bare rolled per schedule No. 3 extending the 508 mm dimension. Sheet size after this operation was $267 \times 1067 \times 0.97 \text{ mm}$. The hot flattening operation consisted of weighting the sheets with approximately 25 kg and heating for 2 h in vacuum at 550°C followed by a furnace cool.

The next operation was to mill each strip in half. Some problems were encountered in this operation due to the worked condition of the sheets and their curl; clamping them in the mill induced some corner cracks.

All sheets were acid etched in a HNO_3 -HF solution to remove the surface oxide (approximately 0.1 mm from the thickness) and then chemically milled in a phosphoric acid solution to final thickness. The rolling schedules are summarized in Tables A1-A3.

Table A1. Canned beryllium billet rolling schedule No. 1.^a

Pass No.	Mill set	Rotation	Reheat	
			Time min	Temperature °C
1	113	90°		
2	106	90°		
3	93	90°		
4	90.4	90°		
			50	982
5	82.7	90°		
6	75.0	90°		
7	67.5	90°		
8	59.8	90°		
			45	982
9	52.2	90°		
10	45.8	90°		
11	39.5	90°		
			40	982
12	33.1	90°		
13	28.0	90°		
			60	870
14	22.9	90°		
15	19.1	90°		
			40	815
16	16.5	90°	40	815
17	14.0	90°		
18	11.4	180°		
			40	760
19	8.9	180°	40	760
20	7.39	180°	40	760
21	6.36	Decan and proceed to schedule 2 after anneal minimum of 16 h at 760°C.		

^aSolutionize at 1037°C minimum 4 h prior to rolling.

Table A2. Bare rolling from 5.21 mm to 2.16 mm thickness,^a schedule No. 2. Rolling direction 90° to last passes during can rolling.

Pass No.	Thickness mm	Reduction %	Anneal time ^b at 760°C min	Holding time at 727°C min
0	5.21		90	15
1	4.67	10.4	45	15
2	4.11	12.4	45	15
3	3.40	17.2	45	15
4	2.97	12.6	45	15
5	2.59	12.8	45	15
6	2.16	16.6		

^aTotal thickness reduction = 58.5%.

^bThe sequence for rolling involves a 45 min anneal at 760°C followed by an immediate transfer to a holding furnace at 727°C directly in front of the rolling mill. The sheets were held in this furnace for 15 min to allow temperature to stabilize. The sheets are then fed directly into the mill out of the furnace a distance of about 0.61 m.

Table A3. Bare rolling from 2.11 mm thickness to 0.97 mm,^a schedule No. 3.

Pass No.	Thickness mm	Reduction %	Anneal time at 760°C min	Holding time at 727°C min
0	2.11		60	15
1	1.88	10.9	40	15
2	1.70	10.6	40	15
3	1.47	13.5	40	15
4	1.27	13.6	40	15
5	1.14	10.2	40	15 ^b
6	0.97	14.9	None	

^aTotal thickness reduction = 54.21%.

^bThis hold was at 688°C instead of 727°C.