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HIGH-FLUX HEAT TRANSFER STUDIES:
AN ANALYTICAL INVESTIGATION OF LAMINAR FILM BOILING

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HIGH-FLUX HEAT TRANSFER STUDIES: AN ANALYTICAL INVESTIGATION OF LAMINAR FILM BOILING

P. W. McFadden and R. J. Grosh

ABSTRACT

An analysis was made of stable, laminar, free convection, film boiling from isothermal vertical plates and horizontal cylinders surrounded by a saturated liquid, where radiation was only of minor importance. The mathematical techniques of boundary layer theory were used and the boundary layer equations were reduced to ordinary differential equations by means of a transformation similar to those used in free convection and condensation. The equations were solved for: (1) compressible flow with variable specific heat, (2) variable specific heat and density variations considered only in the evaluation of the buoyant force, and (3) the case of constant properties. Numerical results were obtained for: (1) near critical water at 2800 and 3100 psia with wall to liquid temperature differences of 250, 500 and 1000 F; (2) for fluids with Prandtl numbers of $\frac{2}{3}$, 1 and 2; and (3) for mercury and methanol film boiling at one atmosphere, considering constant properties. The results obtained by assuming constant properties were compared to: (1) the results obtained by considering variable properties, (2) experimental results, and (3) the comparable case of laminar film condensation. It was shown that the method of considering density variations only in the evaluation of the buoyant force is not valid in film boiling. It was also shown that the constant-property solutions for heat transfer did not always agree with solutions obtained considering compressible flow and variable specific heat.

An approximate analysis of a nonisothermal wall, including the effects of radiation, was presented. It was shown that for high emissivity walls at high temperature, radiation is the controlling factor in film boiling heat transfer.

INTRODUCTION

The increased interest in film boiling in the past few years has been only partially academic. There exist now several areas where film boiling is of vital interest, and the number of these areas will no doubt increase in the future.

The high-performance heat exchangers utilized today necessitate a minimization of surface area. If boiling is depended upon to reduce this area and the liquid pressure approaches the critical pressure, the danger of film boiling increases due to a decrease in the maximum value of the nucleate boiling heat transfer. Some of the first film-boiling work was performed in the area of the quenching of steel, and this is still an area of film-boiling interest today. Additional factors which have helped focus attention on the phenomenon of film boiling are the increased use of liquid metals (which may film boil at wall to liquid temperature differences as low as 10 to 20 F) as heat transfer agents; the increased use of liquified gases such as nitrogen, oxygen, and hydrogen in the rocket and missile fields; and the vast potential of nuclear reactors. It has even been suggested that the drag on very hot torpedoes and underwater vehicles can be considerably reduced by film boiling.

The complete boiling curve of a typical liquid, where the heater surface is submerged in the stagnant liquid, is shown in Figure 1. The regions indicated on the curve are defined as follows:

FIG 1 TYPICAL BOILING CURVE

a. free convection, b. nucleate boiling, c. partial film boiling, and d. stable film boiling. A discussion of the first three regions is not of interest here, but may be found in the literature (see, for example, Zmola⁴⁰). In region d of Figure 1 the physical situation is characterized by a layer of vapor which separates the liquid from the heater surface. If a submerged wire or cylinder is used as the heater surface,

the thickness of the vapor film is observed to be more or less constant, except at the top of the heater surface. Here the vapor breaks away and rises through the liquid as irregularly shaped bubbles. The energy transfer through the vapor layer is by convection and radiation; thus much of the vapor formed is superheated. Stable film boiling can arise either by a sufficiently high heater surface temperature or a sufficiently high heat flux from the heater, as shown in Figure 1.

Although the first observation of the phenomenon of film boiling was made in 1746, an analytical development did not appear in the literature until 1950, when Bromley³ presented a theory of stable laminar film boiling. Bromley based his derivation on Nusselt's derivation³² for the heat transfer during laminar film condensation. In fact, throughout this work it will be seen that the phenomenon of laminar film boiling and laminar film condensation are remarkably similar. Therefore from time to time information will be drawn from and references made to the literature on laminar film condensation.

The papers which have followed Bromley's original work have amplified somewhat the mechanism of film boiling, but none have considered fluid inertia and convection terms, variable properties, or the effect of a nonisothermal wall. It was during a consideration of these deficiencies that the following six objectives of this investigation were proposed.

The first objective was to bring the literature survey of film boiling up to date, including the most important theoretical and experimental advancements, from the first observation of the phenomenon of film boiling to the present.

The second objective was to extend the available constant-property theoretical analyses to consider fluid inertia and convection, and to compare the approximate and exact analyses.

The third objective was to compare the heat transfer coefficients as calculated by laminar film condensation theory and laminar film boiling theory. This would give an insight as to the effect of liquid motion on the heat transfer coefficient in film boiling.

The fourth objective was to consider cases where fluid properties may no longer be considered constant. Pressures near the critical pressure were chosen to investigate the effects of extreme changes in constant pressure specific heat and density with temperature. The results of the variable property analysis would then be compared to the results obtained by assuming constant properties.

The fifth objective was to analyze approximately the case where the wall is nonisothermal, including the effects of radiation. It would be desirable to be able to predict a wall temperature variation, given a prescribed heat flux on the wall.

The sixth objective was to compare the results predicted by theory to the results obtained by experiment for laminar film boiling.

A model of the laminar film-boiling process was ascertained by means of the literature survey. This model was then used as the basis of the laminar film-boiling analytical theory which was developed in this work.

LITERATURE SURVEY

Film Boiling

There exist several excellent literature surveys on film boiling. Drew and Mueller,¹¹ as part of a general survey on boiling, surveyed the film boiling literature up to 1937. Westwater³⁸ reviewed the literature on film boiling up to 1955 and summarized the work done in terms of: descriptions from photographic studies; theoretical treatments; and experimental values where the effects of type of liquid, type of hot solid and surface texture, geometric arrangement, pressure, surface tension, agitation, and impurities were separately considered. For the sake of completeness the present authors reviewed the most important points of the previous surveys and, in particular, gave attention to the theoretical developments up to the present.

The first observation of the phenomenon of film boiling was made by Eller in 1746.* However, the first satisfactory description of film boiling was given by Leidenfrost** in 1756. Film boiling was described as the spheroidal state, defined by Drew and Mueller¹¹ as: "A non-equilibrium condition assumed by two or more bodies of solid or liquid when an attempt is made to bring them together while their temperatures differ by more than a determinant amount; it is characterized by the subsistence of a layer of vapor between the bodies, which resists and prevents their being brought into direct contact."

The early observations of the spheroidal state (film boiling) were concerned with droplets dancing on a hot plate. The most frequently noted observations of the early experimenters were: the slowness of evaporation of the drop, even on very hot surfaces; and the fact that allowing the surface to cool gives startling results, suddenly the drop flashes into vapor, sometimes with explosive force.*** This occurs when the surface temperature falls below that for which the spheroidal state is possible, and the drop loses its vapor blanket.

The research from 1756 until 1926 was for the most part unreliable and not reproducible. Data from one experimenter could not be compared with those of another. Results usually consisted of determining the minimum surface temperature at which the spheroidal state existed. The experimental method used was to heat a crucible or dish to a series of temperatures and, by various techniques, depositing droplets of liquid on the crucible or dish and observing whether or not the spheroidal state occurred. The effect of various kinds of surfaces on this phenomenon was also investigated. A typical presentation of data is given in Table 1.

*This is according to Chivolson.⁸

**See Ref. 11 in Bibliography.

***In 1827 Perkins attributed the explosion of boilers while cooling to the collapse of the vapor blanket in the cessation of the spheroidal state. Boutigny substantiated this also (see Ref. 11).

Table 1

THE EFFECT OF THE NATURE OF A HOT SURFACE ON
THE MINIMUM SURFACE TEMPERATURE NECESSARY
FOR THE SPHEROIDAL STATE

Data by Berger (see Ref. 11) for drops of ether on a hot dish

Hot Surface	Surface Temperature, C	
	Smooth Surface	Rough Surface
Iron	127	140
Lead	130	134
Zinc	108	116

Despite the difficulties encountered, several interesting qualitative results were determined. Baudrimont (see Ref. 14) showed that the liquid need not be at the saturation temperature, and von Hesehus (see Ref. 14) concluded that the drop did not touch the plate. Also Lang (see Ref. 11) in 1888 was apparently the first person to obtain a maximum of the overall heat transfer coefficient.

In 1930 Blaszkowska-Zakrzewak (see Ref. 11), and also Swietoslawski (see Ref. 11), obtained the curve shown in Figure 2. A similar effect had been noticed by Pilling and Lynch (see Ref. 20) in 1920 while investigating the quenching of steel.

FIG 2 DROP EVAPORATION CURVE

While the drop method of investigating film boiling was used as late as 1953,¹⁴ other methods of investigation were proving more fruitful. In 1926 Moscicki and Broder (see Ref. 11) had discarded the drop method as unreliable and adopted the technique of electrically heating a hot

wire. They observed the phenomenon of film boiling occurring on the outside of the wire. This latter method was adopted by Nukiyama³⁰ in 1934, who for the first time presented the curve shown in Figure 1. Nukiyama experienced difficulty with wire failure in trying to obtain regions c and d. This wire failure was due to the fact that the burnout temperature of the wire was exceeded when the maximum in the curve of Figure 1 was exceeded.

In 1937 Drew and Mueller¹¹ reproduced Figure 1 more reliably by using Lang's method of a constant-temperature heating device. They used steam condensing on the inside of a tube to film boil liquid on the outside of

the same tube. This method enabled the investigation of regions c and d in Figure 1 without the danger of burnout. The door was now open for a quantitative experimental investigation of film boiling, and many experiments were to follow. The next major advance came with the first analytical investigation.

The first suggestion concerning an analytical investigation of film boiling seems to have come from Colburn (see Ref. 38) in 1941. He suggested that Nusselt's derivation³² for film condensation be modified for use with film boiling. The heat transfer in the case of film boiling could then be expressed in terms of the Nusselt number as

$$N_{Nu,D} = C \left[\frac{g(\rho_L - \rho)\rho D^3 h_{fg}}{k\mu\theta g_0} \right]^{\frac{1}{4}}, \quad (1)$$

where C was 0.50 for film boiling outside a horizontal tube and C was 0.667 for film boiling on a vertical tube (with D^3 replaced by L^3).

In 1950 Bromley³ derived an equation similar to the above for free convection film boiling on the outside of an isothermal horizontal tube, considering the following assumptions: a vapor blanket exists between the liquid and the tube wall, heat is transferred through the film by conduction and radiation, the vapor rises due to buoyant forces, the liquid-vapor interface is smooth, viscous drag retards the rise of the vapor (that is, vapor inertia terms are not considered), the enthalpy of vaporization is the major energy supplied to the film, the kinetic energy of the film is negligible, the liquid is at rest and saturated, properties are evaluated at an average temperature, and the resulting theoretical equation will be modified with an experimentally determined constant in order to fit the physical situation. Bromley's results could be expressed as

$$N_{Nu,D} = 0.62 \left[\frac{g(\rho_L - \rho)\rho D^3 h_{fg}}{k\mu\theta g_0} \right]^{\frac{1}{4}}, \quad (2)$$

where 0.62 is empirical, a compromise between 0.724 (liquid moving with the same velocity as the vapor, zero shear stress at the interface) and 0.551 (liquid everywhere at rest, very large shear stress at the interface). Bromley also corrected his heat transfer coefficient for radiation by assuming infinite parallel plane plate radiation. A radiation coefficient can be expressed as

$$h_r = \frac{\sigma}{\left(\frac{1}{\epsilon_w} + \frac{1}{\epsilon_L} - 1 \right)} \frac{(T_w^4 - T_L^4)}{\theta}, \quad (3)$$

and a total convection coefficient can be expressed as

$$h_T = h_{co} \left(\frac{h_{co}}{h_r} \right)^{\frac{1}{3}} + h_r \quad . \quad (4)$$

Any radiation present will increase the vapor film thickness, and thus decrease h_{co} , which explains the form of the first term in the above equation.

Bromley⁴ later showed that the expression of heat transfer could be modified if the superheating of the vapor film was important. The modified expression for the Nusselt number then becomes

$$N_{Nu,D} = 0.62 \left[\frac{g(\rho_L - \rho) \rho D^3 h_{fg}}{k \mu \theta g_0} \left(1 + \frac{0.4 c_p \theta}{h_{fg}} \right) \right]^{\frac{1}{4}} \quad . \quad (5)$$

Bromley³ supported his theory with film boiling experimental data for six different liquids.

In 1954 Ellion¹³ analyzed an isothermal vertical flat plate with laminar stable film boiling. He used the same assumptions as Bromley³ and arrived at the result suggested by Bromley

$$N_{Nu,L} = 0.717 \left[\frac{g(\rho_L - \rho) \rho L^3 h_{fg}}{k \mu \theta g_0} \right]^{\frac{1}{4}} \quad . \quad (6)$$

The value 0.717 is arrived at theoretically by the assumption of zero vapor velocity at the liquid-vapor interface. Ellion also corrected his expression for radiation by assuming infinite plane plates as Bromley did, but combined the convection and radiation effects as follows:

$$h_T = h_{co} \left(\frac{q_{co}}{q_{co} + q_r} \right)^{\frac{1}{4}} + h_r \quad . \quad (7)$$

Ellion also reported experimental results for water film boiling on a vertical isothermal cylinder.

Bromley⁵ analyzed the case of stable laminar film boiling from a horizontal isothermal cylinder in the case of uniform vertical upward flow forced convection. For low water velocities his results reduced to Equation (2). For high water velocities his results can be expressed as

$$N_{Nu,D} = 2.7 \left[\frac{U \rho h_{fg} D}{k \theta} \right]^{\frac{1}{2}} \quad , \quad (8)$$

which is significantly different from free convection. Bromley corrected the convection coefficient for radiation as before. Data on four different liquids were presented for a range (up to 14 ft/sec) of velocities.

Cess⁶ also analyzed stable laminar film boiling from a vertical isothermal flat plate. His analysis differed from the Bromley analysis³ in that he considered convection within the film as well as a shear stress at the liquid-vapor interface. The heat transfer results of Cess could be expressed as

$$N_{Nu,x} = a_1 \left[\frac{g(\rho_L - \rho) \rho x^3 h_{fg}}{k \theta \mu g_0} \right]^{\frac{1}{4}}, \quad (9)$$

where a_1 is a number dependent upon $c_p \theta / h_{fg}$ and $\rho_L \mu_L / \rho \mu$.

In 1952 Marx and Davis²⁶ presented a theory on the termination of the film-boiling mechanism. They suggested that free molecular heat conduction provides the critical energy transfer that just precedes the collapse of the vapor blanket. They gave an expression for the extreme possible values of the heat transfer coefficient and numerically evaluated the coefficient for water.

Chang⁷ in 1958 presented a wave theory of heat transfer in film boiling for both horizontal and vertical isothermal surfaces. His results for the heat transfer can be expressed in the general form

$$N_{Nu,L} = \left[\frac{g(\rho_L - \rho) L^3}{8 \pi^2 \mu g_0 \alpha_c} \right]^{\frac{1}{3}}, \quad (10)$$

where the equivalent thermal diffusivity, α_c can be evaluated approximately (for slight subcooling of the liquid) by the expression

$$\alpha_c = \theta k / 2 \left(h_{fg} \rho + \theta L c_{p,L} \rho_L \right) \quad (11)$$

Chang also included a radiation correction which was similar to the correction used by Ellion.¹³

As was stated previously, the experimental techniques necessary for the investigation of film boiling were well established by 1937. Since then experimental investigators have attacked the problem by isolating and investigating variables one at a time. A summary of the variables investigated follows. The papers listed are typical and not all papers are mentioned.

Type of Liquid Boiled. Bromley³ performed film-boiling measurements using water, liquid nitrogen, carbon tetrachloride, ethyl alcohol, benzene, diphenyl ether, and n-pentane. Nukiyama³⁰ used water, McAdams²⁸ used water, and Westwater³⁹ used methanol. Monroe²⁹ performed tests with liquid oxygen, and Dean and Thompson⁹ performed tests with liquid

nitrogen. In addition, Lyon²³ has performed film-boiling experiments with mercury.

No liquids thus far have indicated a radical difference in heat transfer coefficient. The main difference between liquids seems to be the wettability of the liquid on a particular surface. When a liquid does not wet the surface, film boiling will occur at a smaller wall to liquid temperature difference than when the liquid wets the surface. This is indicated by the results of the experiments performed by Lyon.²³

Type of Surface on Which Boiling Occurs. Both Bromley³ and Bancho² found no effect due to the type of surface on which the film boiling occurred. This differs markedly from the case of nucleate boiling, and is due no doubt to the fact that the vapor layer completely covers the surface. However, as was pointed out by Westwater³⁸ there are two conditions where the surface would play an important role in film boiling. Very rough surfaces would change the character of the flow and high temperatures would cause the emissivity of the surface to be important in thermal radiation.

Type of Surface Geometry. The free-convection film-boiling theoretical values for the heat transfer coefficient on a horizontal cylinder show a $D^{-1/4}$ dependence on diameter. This relation fails for small diameters, as was shown by McAdams²⁸ and Bancho.² McAdams suggested

$$q = (\text{constant}) D^{\frac{1}{2}} \quad (12)$$

for water at 1 atmosphere and small (0.004 to 0.024 inch) diameters. Bancho states that the best correlation is

$$h = (\text{constant}) \left(\frac{1}{D} + C_1 \right) \quad , \quad (13)$$

where C_1 is a temperature-dependent constant. Nukiyama³⁰ and Rinaldo²⁸ have also investigated horizontal wires. The horizontal cylinder has been investigated by Bromley,³ Westwater,³⁹ Bancho,² and also Lyon.²³ The vertical cylinder was investigated by Ellion¹³ and Dean and Thompson.⁹ Monroe²⁹ investigated film boiling inside a vertical tube.

Pressure Effect. Bancho² has investigated the effect of pressure on free-convection stable film boiling on horizontal cylinders, and Ellion¹³ for free (very low velocity, 1 ft/sec) and forced convection on vertical cylinders. Dean and Thompson⁹ have also studied the effect of pressure on film boiling during forced convection on vertical cylinders. Monroe²⁹ investigated the effect of pressure on forced-convection flow inside a vertical tube. It was seen that increases in pressure yielded increases in the heat transfer coefficient.

Forced Convection Effect. Bromley⁵ investigated forced-convection stable film boiling on the outside of horizontal cylinders with vertical upward flow. Dean and Thompson⁹ and Ellion¹³ have investigated forced convection on the outside of vertical cylinders. In addition, Monroe²⁹ has investigated forced convection inside vertical tubes. The effect of increasing the liquid velocity is to increase the heat transfer coefficient.

From the literature a model of film boiling has been developed. There is a heated surface separated from the liquid by a thin film of vapor. The liquid-vapor interface may be smooth, as on the bottom half of a horizontal cylinder or near the bottom edge of a vertical plate, or contain waves. The heat transfer across the film is by convection and radiation, and the process is amenable to mathematical analysis.

Film Condensation

Since the phenomena of film boiling and film condensation are so similar, a very brief review of the literature of analytical laminar film condensation is given here. The first theory of laminar film condensation was given by Nusselt³² in 1916. This theory was modified by Rohsenow³⁴ in 1956 to take care of convection in the liquid film, and finally by Sparrow and Gregg,³⁷ who solved the complete boundary layer equations for laminar film condensation. In 1958 Kamal-Eldin Hassan¹⁶ extended Nusselt's theory to include condensation on nonisothermal surfaces.

FUNDAMENTAL EQUATIONS

The results of all experiments and theories set forth in the previous section indicated that the mechanism of heat transfer in film boiling is by convection and radiation. Also, situations exist where radiation is only of minor importance and, to a first approximation, only convection need be considered. Since it was also observed that the isothermal wall has been of most interest to date, stable, free-convection, laminar film boiling from a nonradiating isothermal surface was chosen for analysis.

The first step in the analysis of stable, free-convection, laminar film boiling was to apply the laws of conservation of mass, momentum, and energy. The resulting partial differential equations then required simplification. The technique of solution was to transform the simplified partial differential equations to ordinary differential equations by means of a similarity transformation. The resulting ordinary differential equations were then amenable to numerical solution. It should be noted here that the similarity transformations used in this work considered similarity of velocity and temperature profiles.³⁵ The justification for choosing this was the success of using this particular type of transformation in solving related boundary layer problems.^{17,35}

The fundamental equations which express the laws of conservation of mass, momentum, and energy were written as follows:³⁵

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \vec{V}) = 0 \quad (14)$$

$$\frac{\rho d\vec{V}}{d\tau} = \mu g_0 \nabla^2 \vec{V} - \left\{ \nabla P + \vec{G} \right\} g_0 + \frac{\mu}{3} g_0 \nabla (\nabla \cdot \vec{V}) \quad (15)$$

$$\rho c_p \frac{dT}{d\tau} = k \nabla^2 T + q''' \quad , \quad (16)$$

for the case of compressible flow and variable specific heat with constant viscosity and thermal conductivity.

The thermal conductivity and viscosity of the vapor were assumed constant in all cases. In the application of Equations (14) through (16) to water film boiling, the variations in thermal conductivity and viscosity were always at least one-half order of magnitude less than the variations of specific heat and density, as seen in Table 2.

Table 2

THE VARIATION OF PROPERTIES IN THE VAPOR FILM
FOR WATER VAPOR

P(psia)	$\Theta(^{\circ}\text{F})$	$\frac{(\rho_{\text{max}} - \rho_{\text{min}})}{\rho_{\text{min}}}$	$\frac{(c_{p,\text{max}} - c_{p,\text{min}})}{c_{p,\text{min}}}$	$\frac{(\mu_{\text{max}} - \mu_{\text{min}})}{\mu_{\text{min}}}$	$\frac{(k_{\text{max}} - k_{\text{min}})}{k_{\text{min}}}$
		ρ_{min}	$c_{p,\text{min}}$	μ_{min}	k_{min}
2800	250	1.5	7.5	0.1	0.1
	1000	3.4	9.5	0.7	0.7
3100	250	3.0	14.0	0.06	0.2
	1000	5.5	19.0	0.6	0.6

In order to simplify Equations (14) through (16), throughout the remainder of this thesis only steady, two-dimensional, stable, free-convection, boundary layer type laminar film boiling, as shown in Figure 3, was considered. Equations (14) through (16) then became

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (17)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \mu g_0 \frac{\partial^2 u}{\partial y^2} - \left\{ \left[\nabla P \right]_x + \left[\vec{G} \right]_x \right\} g_0 \quad (18)$$

FIG 3 PHYSICAL MODEL AND COORDINATE SYSTEM FOR A: PLANE WALL, B. HORIZONTAL CYLINDER

$$\rho c_p \frac{\partial T}{\partial x} + \rho c_p \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} \quad , \quad (19)$$

where the heat generation per unit volume has been neglected.

For boundary conditions it was assumed that the liquid was at rest and saturated, and the temperature of the vapor at the liquid-vapor interface was the saturation temperature of the liquid. Thus the boundary conditions

$$\left. \begin{aligned} u = v = 0 \quad , \quad T = T_w \quad \text{for} \quad y = 0 \\ u = 0 \quad , \quad T = T_{sat} \quad \text{for} \quad y = \delta \end{aligned} \right\} \quad (20)$$

were determined. Since δ was unknown, an additional condition was required. This was determined by noting that all the heat conducted at the liquid-vapor interface was used to vaporize additional liquid. Thus

$$q = -k \frac{\partial T}{\partial y} = h_{fg} \rho_v \quad \text{at} \quad y = \delta \quad (21)$$

gave a condition for determining the boundary layer thickness δ .

Equations (17) through (19) together with boundary conditions (20) and (21) completely described the process of free-convection, steady, two-dimensional, compressible, laminar film boiling on an isothermal wall with variable specific heat and no radiation. Application of these equations to specific situations is considered in the next section.

APPLICATION OF THE FUNDAMENTAL EQUATIONS TO AN ISOTHERMAL SURFACE

In order to satisfy the objectives of this work it was desirable to solve Equations (17) through (21) for the following three cases: (1) compressible flow with variable specific heat, (2) variable specific heat with density variations considered only in the evaluation of the buoyant force, and (3) all properties constant. The two geometries, vertical plane wall and horizontal cylinder, were chosen because of their great practicality as engineering geometries and their ease of experimental comparison. It was noted that the results of the vertical plate can also be applied to the vertical cylinder if $\delta/r < 0.03$. This should introduce an error in film thickness of about

one percent, according to Greenburg.¹⁵ It was also noted that the heat transfer from a vertical plane wall in film boiling can be related to the heat transfer from a horizontal cylinder in film boiling, as will be shown in a later section of this thesis.

Vertical Plane Wall

The vertical plane wall was chosen for analysis first since it was the simpler of the two geometries investigated. Equations (17) through (21) were applied to the vertical plane wall three separate times in order to satisfy the objectives listed above. The method of reduction of Equations (17) through (21) follows.

Compressible Flow with Variable Specific Heat. The first step in the reduction of Equations (17) through (21) to ordinary differential equations was the introduction of a stream function. To this end a stream function, defined as

$$\rho u = \rho_{\text{sat}} \frac{\partial \psi}{\partial y} \quad , \quad \rho v = -\rho_{\text{sat}} \frac{\partial \psi}{\partial x} \quad , \quad (22)$$

was introduced. Substitution of Equations (22) into Equations (17), (18) and (19) had the advantage in that Equation (17) was eliminated, but the disadvantage that the order of Equations (18) and (19) was increased. Equations (18) and (19) thus became

$$\begin{aligned} & \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - \frac{\mu g_0}{\rho_{\text{sat}}} \frac{\partial^3 \psi}{\partial y^3} - \frac{\rho g_0}{\rho_{\text{sat}}} \left\{ [\nabla P]_x - [\vec{G}]_x \right\} \\ &= \frac{1}{\rho} \frac{\partial \rho}{\partial x} \left(\frac{\partial \psi}{\partial y} \right)^2 - \frac{1}{\rho} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \frac{\partial \rho}{\partial y} - \frac{2\mu g_0}{\rho \rho_{\text{sat}}} \frac{\partial \rho}{\partial y} \frac{\partial^2 \psi}{\partial y^2} \\ & \quad - \frac{\mu g_0}{\rho \rho_{\text{sat}}} \frac{\partial \psi}{\partial y} \left\{ \frac{\partial^2 \rho}{\partial y^2} - \frac{2}{\rho} \left(\frac{\partial \rho}{\partial y} \right)^2 \right\} \quad ; \end{aligned} \quad (23)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho_{\text{sat}} C_p} \frac{\partial^2 T}{\partial y^2} \quad , \quad (24)$$

and the boundary conditions, Equations (20) and (21), became

$$\left. \begin{aligned} & \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = 0 \quad , \quad T = T_w \quad \text{for } y = 0 \\ & \frac{\partial \psi}{\partial y} = 0 \quad , \quad T = T_{\text{sat}} \quad , \quad \frac{\partial T}{\partial y} = 3 \frac{\mu g_0}{k} \frac{h_{fg}}{\theta} \frac{\partial \psi}{\partial x} \quad \text{for } y = \delta \end{aligned} \right\} \quad (25)$$

Equations (23), (24) and (25) were simplified by expressing the pressure gradient and body force in terms of the specific weights of vapor and liquid and by introducing a similarity transformation which reduced the partial differential equations to ordinary differential equations. These steps were performed as follows.

The pressure gradient in the x-direction was the hydrostatic pressure gradient:

$$[\nabla P]_x = -(g/g_0)\rho_L \quad , \quad (26)$$

and the body force in the x-direction was the gravity force on the vapor

$$[\vec{G}] = -(g/g_0)\rho \quad . \quad (27)$$

The sum of the pressure gradient and body force,

$$-(g/g_0) (\rho_L - \rho) \quad , \quad (28)$$

gave the buoyant force.

The similarity transformation replaced the independent variables x and y by a single independent variable, η , and gave the form of the stream function as

$$\left. \begin{aligned} \psi &= 4 \frac{\mu g_0}{\rho_{\text{sat}}} c_1 x^{\frac{3}{4}} \zeta(\eta) \\ \eta &= c_1 y / x^{\frac{1}{4}} \end{aligned} \right\} \quad , \quad (29)$$

where

$$c_1 = \left[g (\rho_L - \rho_{\text{sat}}) \rho_{\text{sat}} / 4 \mu^2 g_0^2 \right]^{\frac{1}{4}} \quad . \quad (30)$$

Also, at a given pressure, density is a function only of temperature:

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial x} \quad ; \quad \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial y} \quad . \quad (31)$$

Equation (31) was introduced into Equations (23) and (24), and yielded

$$\zeta''' + 3\zeta\zeta'' - 2(\zeta')^2 + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} = \frac{1}{\rho} \frac{d\rho}{d\theta} \theta' [3\zeta\zeta' + 2\zeta'']$$

$$+ \frac{1}{\rho} \zeta' (\theta')^2 \left[\frac{d^2\rho}{d\theta^2} - \frac{2}{\rho^2} \left(\frac{d\rho}{d\theta} \right)^2 \right] + \frac{1}{\rho} \frac{d\rho}{d\theta} \zeta' \theta'' \quad ; \quad (32)$$

$$\theta'' + \left(3\mu g_0 c_p / k \right) \zeta \theta' = 0 \quad , \quad (33)$$

where the primes on ζ and θ represented differentiation with respect to η . The boundary conditions on this set of equations were obtained from Equations (25), (29) and (30), which yielded

$$\left. \begin{array}{ll} \zeta = \zeta' = 0 \quad , \quad \theta = 1 & \eta = 0 \\ \zeta' = 0, \quad \theta = 0 \quad , \quad \theta' = -3\mu g_0 h_{fg} \zeta / k \theta & \eta = \eta_\delta \end{array} \right\} \quad (34)$$

as the transformed boundary conditions.

The simultaneous solution of Equations (32) and (33) with the boundary conditions given by Equation (34) completely described stable, free-convection, laminar film boiling from a vertical plane wall where the fluid is considered compressible with a variable specific heat, and radiation is only of minor importance. Equations (32) through (34) were solved numerically, as indicated on page 30. The results of the solution are presented in Table 3 (see page 39) for water film boiling.

Variable Specific Heat with Density Variations Considered Only in the Evaluation of the Buoyant Force. The reduction of the partial differential equations for this case was very similar to the reduction in the case of compressible flow with variable specific heat. As before, a stream function was introduced as the first step:

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = - \frac{\partial \psi}{\partial x} \quad (35)$$

was first introduced. As before, introduction of Equation (35) into Equations (17) through (19) eliminated Equation (17), but increased the order of Equations (18) and (19). Substitution of Equation (35) in Equations (18) and (19) yielded

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{\mu g_0}{\rho} \frac{\partial^3 \psi}{\partial y^3} + \frac{g_0}{\rho} \left\{ [\nabla P]_x - [\vec{G}]_x \right\} \quad ; \quad (36)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho} \frac{1}{c_p} \frac{\partial^2 T}{\partial y^2} \quad (37)$$

as the new equations. The sum of the hydrostatic pressure gradient and the gravity force on the vapor yielded the buoyant force as given by Equation (28). The similarity transformation given by Equations (29) and (30) replaced the independent variables x and y in Equations (36) and (37) by the single independent variable η . Thus Equations (36) and (37) became

$$\zeta''' + 3\zeta\zeta'' - 2(\zeta')^2 + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} = 0 \quad ; \quad (38)$$

$$\theta'' + 3 \frac{\mu g_0}{k} c_p \zeta \theta' = 0 \quad , \quad (39)$$

where the primes on ζ and θ represent differentiation with respect to η . The boundary conditions on Equations (38) and (39) became

$$\left. \begin{aligned} \zeta = \zeta' = 0 \quad , \quad \theta = 1 & \quad \eta = 0 \\ \zeta' = 0 \quad , \quad \theta = 0 \quad , \quad \theta' = -3 \frac{\mu g_0}{k} \frac{h_{fg}}{\Theta} \zeta & \quad \eta = \eta_\delta \end{aligned} \right\} \quad , \quad (40)$$

as obtained from Equations (25), (29) and (30).

The simultaneous solution of Equations (38) and (39) with the boundary conditions as given by Equation (40) completely described stable, free-convection, laminar film boiling from a vertical plane wall where density variations are considered only in the evaluation of the buoyant force, specific heat of the vapor is variable, and radiation is only of minor importance. Equations (38) through (40) were solved numerically, as indicated on page 30. The results of the solution are presented in Table 3 for water film boiling.

All Properties Constant. The first step in the reduction of Equations (17) through (19) to ordinary differential equations in the case of constant properties was the introduction of the stream function as defined by Equation (35). Equation (17) was thus eliminated. Since all properties were considered constant, the mean buoyant force was expressed as

$$g(\rho_L - \rho_m) / g_0 \rho_m \quad , \quad (41)$$

where ρ_m was evaluated at T_m . Substitution of Equations (35) and (41) into Equations (18) and (19) then yielded

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{\mu}{\rho_m} g_0 \frac{\partial^3 \psi}{\partial y^3} + \frac{g(\rho_L - \rho_m)}{\rho_m} \quad ; \quad (42)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho_m c_p} \frac{\partial^2 T}{\partial y^2} \quad (43)$$

The similarity transformations which made Equations (42) and (43) ordinary differential equations were taken as

$$\left. \begin{aligned} \psi &= 4 \frac{\mu g_0}{\rho_m} c_2 x^{\frac{3}{4}} \zeta(\eta) \\ \eta &= c_2 y / x^{\frac{1}{4}} \end{aligned} \right\} \quad (44)$$

where

$$c_2 = \left[\frac{g(\rho_L - \rho_m) \rho_m}{4 \mu^2 g_0^2} \right]^{\frac{1}{4}} \quad (45)$$

Introduction of Equations (44) and (45) into Equations (42) and (43) gave

$$\zeta''' + 3 \zeta \zeta'' - 2(\zeta')^2 + 1 = 0 \quad ; \quad (46)$$

$$\theta'' + 3 N_{pr} \zeta \theta' = 0 \quad ; \quad (47)$$

where the primes on ζ and θ represent differentiation with respect to η . The boundary conditions applicable to Equations (46) and (47) were

$$\left. \begin{aligned} \zeta = \zeta' = 0 \quad , \quad \theta = 1 & \quad \eta = 0 \\ \zeta' = 0 \quad , \quad \theta = 0 \quad , \quad \theta' = -3 N_{pr} \frac{h_{fg}}{c_p \theta} \zeta & \quad \eta = \eta_\delta \end{aligned} \right\} \quad (48)$$

as was obtained from Equations (20) and (21).

It was noted here that the simultaneous solution of Equations (46) and (47) with boundary conditions as prescribed by Equation (48) were completely general. The results depended only upon the Prandtl number of the vapor and the value of the dimensionless quantity $c_p \theta / h_{fg}$. The solutions were obtained numerically as explained on page 30. The results of the solutions are presented in Table 3 for water film boiling, and in Figures 7 through 10 for fluids with vapor Prandtl number equal to one.

Horizontal Cylinder

While the horizontal cylinder was considered the more complicated of the two geometries investigated, it was observed that the equations describing free-convection laminar film boiling on a horizontal cylinder reduce to the

corresponding equations for the plane vertical wall. Equations (17) through (21) were applied to the horizontal cylinder three times in order to satisfy the objectives of the work. The method of reduction of Equations (17) through (21) follows.

Compressible Flow with Variable Specific Heat. The first step in the reduction of Equations (17) through (19) for the case of the horizontal cylinder was the introduction of a stream function. The stream function used was defined by Equation (22). Introduction of Equation (22) into Equations (17) through (19) eliminated Equation (17).

The pressure gradient in the x-direction was the hydrostatic pressure gradient, as in the case of the vertical plane wall. This was expressed as

$$[\nabla P]_x = -\frac{g}{g_0} \rho_L \sin \frac{x}{r} \quad (49)$$

in the case of the horizontal cylinder. The body force in the x-direction,

$$[\vec{G}]_x = -\frac{g}{g_0} \rho \sin \frac{x}{r} \quad , \quad (50)$$

was the gravity force on the vapor. The sum of the body force and the gravity force yielded the buoyant force:

$$-\frac{g}{g_0} (\rho_L - \rho) \sin \frac{x}{r} \quad . \quad (51)$$

Substitution of Equations (22) and (51) into Equations (18) and (19) gave

$$\begin{aligned} & \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - \frac{\mu}{\rho_{\text{sat}}} g_0 \frac{\partial^3 \psi}{\partial y^3} - \frac{\rho}{\rho_{\text{sat}}^2} g (\rho_L - \rho) \sin \frac{x}{r} \\ &= \frac{1}{\rho} \frac{\partial \rho}{\partial x} \left(\frac{\partial \psi}{\partial y} \right)^2 - \frac{1}{\rho} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \frac{\partial \rho}{\partial y} - \frac{2 \mu g_0}{\rho \rho_{\text{sat}}} \frac{\partial \rho}{\partial y} \frac{\partial^2 \psi}{\partial y^2} \\ & \quad - \frac{\mu g_0}{\rho \rho_{\text{sat}}} \frac{\partial \psi}{\partial y} \left\{ \frac{\partial^2 \rho}{\partial y^2} - \frac{2}{\rho} \left(\frac{\partial \rho}{\partial y} \right)^2 \right\} \end{aligned} \quad (52)$$

and

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho_{\text{sat}}} \frac{1}{c_p} \frac{\partial^2 T}{\partial y^2} \quad . \quad (53)$$

The next step in reducing Equations (52) and (53) to ordinary differential equations for the horizontal cylinder was to introduce the following

$$\left. \begin{aligned} \beta &= x/r \\ \phi &= c_3 y/r \\ \Psi &= \psi \rho_{\text{sat}} / \mu c_3 g_0 \end{aligned} \right\} , \quad (54)$$

where

$$c_3 = \left[\frac{g r^3 (\rho_L - \rho_{\text{sat}}) \rho_{\text{sat}}}{\mu^2 g_0^2} \right]^{\frac{1}{4}} . \quad (55)$$

In this way Equations (52) and (53) became

$$\begin{aligned} \frac{\partial \Psi}{\partial \phi} \frac{\partial^2 \Psi}{\partial \beta \partial \phi} - \frac{\partial \Psi}{\partial \beta} \frac{\partial^2 \Psi}{\partial \phi^2} - \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} \sin \beta - \frac{\partial^3 \Psi}{\partial \phi^3} &= \frac{1}{\rho} \frac{\partial \rho}{\partial \beta} \left(\frac{\partial \Psi}{\partial \phi} \right)^2 \\ - \frac{1}{\rho} \frac{\partial \Psi}{\partial \phi} \frac{\partial \Psi}{\partial \beta} \frac{\partial \rho}{\partial \phi} - \frac{2}{\rho} \frac{\partial \rho}{\partial \phi} \frac{\partial^2 \Psi}{\partial \phi^2} - \frac{\partial \Psi}{\partial \phi} \left\{ \frac{1}{\rho} \frac{\partial^2 \rho}{\partial \phi^2} - \frac{2}{\rho^2} \left(\frac{\partial \rho}{\partial \phi} \right)^2 \right\} & \end{aligned} \quad (56)$$

and

$$\frac{\partial \Psi}{\partial \phi} \frac{\partial \theta}{\partial \beta} - \frac{\partial \Psi}{\partial \beta} \frac{\partial \theta}{\partial \phi} = \frac{k}{g_0 \mu c_p} \frac{\partial^2 \theta}{\partial \phi^2} . \quad (57)$$

In order to simplify Equations (56) and (57),

$$\left. \begin{aligned} \xi &= \phi H(\beta) \\ \Psi(\beta_1 \phi) &= p(\xi) f(\beta) \\ \theta(\beta_1 \phi) &= t(\xi) \end{aligned} \right\} \quad (58)$$

were introduced. Thus Equations (56) and (57) became

$$\begin{aligned} p''' \{f H^3\} + p p'' \{f f' H^2\} - (p')^2 \{f^2 H H' + H^2 f f'\} + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} \sin \beta \\ = \frac{1}{\rho} \frac{\partial \rho}{\partial \xi} \{H^2 f f' p p' + 2 H^3 f p''\} + \frac{1}{\rho} \left\{ \frac{\partial^2 \rho}{\partial \xi^2} - \frac{2}{\rho} \left(\frac{\partial \rho}{\partial \xi} \right)^2 \right\} H^3 f p' \end{aligned} \quad (59)$$

and

$$t''H + \mu g_0 c_p f' p t' / k = 0 \quad , \quad (60)$$

where primes on t and p represent differentiation with respect to ξ and primes on f and H represent differentiation with respect to β .

It was noted that if

$$\left. \begin{aligned} f'(\beta) &= aH(\beta) \\ f^2(\beta)H(\beta)H'(\beta) &= b \sin \beta \\ f(\beta)H^2(\beta)f'(\beta) &= c \sin \beta \\ f(\beta)H^3(\beta) &= d \sin \beta \end{aligned} \right\} , \quad (61)$$

Equations (59) and (60) could be written as

$$\begin{aligned} p'''d + pp''c - (p')^2 \{b+c\} + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} = \frac{1}{\rho} \frac{\partial \rho}{\partial \xi} \{cpp' + 2p''d\} \\ + \frac{1}{\rho} p'd \left\{ \frac{\partial^2 \rho}{\partial \xi^2} - \frac{2}{\rho} \left(\frac{\partial \rho}{\partial \xi} \right)^2 \right\} \end{aligned} \quad (62)$$

and

$$t'' + a\mu g_0 c_p p t' / k = 0 \quad . \quad (63)$$

Fortunately, Equations (61) have been solved by Hermann,¹⁷ who gives graphs of H and fH versus β , as shown in Figure 4. Hermann also gives values for the constants as follows: $a = 3$, $b = 1$, $c = 3$ and $d = 1$.

FIG. 4 THE FUNCTIONS H AND fH
(TAKEN FROM FIG. 17, REFERENCE 17)

Finally, noting that density was a function only of temperature at a given pressure, and using values for the constants a through d, Equations (62) and (63) became

$$p''' + 3pp'' - 2(p')^2 + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{sat})} \frac{\rho}{\rho_{sat}} = \frac{1}{\rho} \frac{d\rho}{dt} t' [3pp' + 2p'']$$

$$+ \frac{1}{\rho} p' (t')^2 \left[\frac{d^2\rho}{dt^2} - \frac{2}{\rho} \left(\frac{d\rho}{dt} \right)^2 \right] + \frac{1}{\rho} p' \frac{d\rho}{dt} t'' \quad (64)$$

and

$$t'' + 3\mu g_0 c_p p t' / k = 0 \quad (65)$$

The boundary conditions on Equations (64) and (65) were

$$\left. \begin{array}{ll} p = p' = 0 & , \quad t = 1 \\ p' = 0 & , \quad t = 0 \quad , \quad t' = -3\mu g_0 h_{fg} p / k \theta \end{array} \right\} \begin{array}{l} \xi = 0 \\ \xi = \xi_\delta \end{array} \quad (66)$$

as obtained from Equations (20) and (21).

Thus it was seen that Equations (64) and (65), together with boundary conditions (66), were the same differential equations derived for the case of compressible flow with variable specific heat, laminar film boiling on a vertical plane wall. The solution of the corresponding equations for the vertical plane wall also represent solutions for Equations (64) through (66).

Variable Specific Heat with Density Variations Considered Only in the Evaluation of the Buoyant Force. As was the situation with the vertical plane wall, the reduction of the partial differential equations for this case was very similar to the reduction in the case of compressible flow with variable specific heats. A stream function defined by Equation (35) was introduced into Equations (17) through (19), eliminating Equation (17). The sum of the pressure gradient and the body force gave the buoyant force as given by Equation (51). Thus Equations (18) and (19) became

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{\mu g_0}{\rho} \frac{\partial^3 \psi}{\partial y^3} + g \frac{(\rho_L - \rho)}{\rho} \sin \frac{x}{r} \quad (67)$$

and

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (68)$$

The next step in reducing Equations (67) and (68) to ordinary differential equations for the horizontal cylinder was to introduce Equations (54) and (55). In this way Equations (67) and (68) became

$$\frac{\partial \Psi}{\partial \phi} \frac{\partial^2 \Psi}{\partial \beta \partial \phi} - \frac{\partial \Psi}{\partial \beta} \frac{\partial^2 \Psi}{\partial \phi^2} - \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} \sin \beta - \frac{\partial^3 \Psi}{\partial \phi^3} = 0 \quad (69)$$

and

$$\frac{\partial \Psi}{\partial \phi} \frac{\partial \theta}{\partial \beta} - \frac{\partial \Psi}{\partial \beta} \frac{\partial \theta}{\partial \phi} = \frac{k}{\mu c_p} \frac{\partial^2 \theta}{\partial \phi^2} \frac{1}{g_0} \quad (70)$$

As before, in order to simplify the above equations, Equations (58) were introduced, and Equations (69) and (70) could be expressed as

$$p''' \{fH^3\} + pp'' \{ff'H^2\} - (p')^2 \{f^2HH' + H^2ff'\} + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} \sin \beta = 0 \quad (71)$$

and

$$t''H + (\mu g_0/k) c_p p t' f' = 0 \quad (72)$$

where primes on t and p represent differentiation with respect to ξ and primes on f and H represent differentiation with respect to β .

From the reduction of the comparable equations for the case of compressible flow with variable specific heats, it was proved that Equations (71) and (72) became

$$p''' + 3pp'' - 2(p')^2 + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} = 0 \quad (73)$$

and

$$t'' + 3(\mu g_0/k) c_p p t' = 0 \quad (74)$$

The boundary conditions for Equations (73) and (74) became

$$\left. \begin{aligned} p = p' = 0 \quad , \quad t = 1 & \quad \xi = 0 \\ p' = 0 \quad , \quad t = 0 \quad , \quad t' = -3(\mu g_0/k)(h_{fg}/\theta)p & \quad \xi = \xi_\delta \end{aligned} \right\} \quad (75)$$

as obtained from Equations (20) and (21).

These are the same equations and boundary conditions that were derived for the comparable case of film boiling on a vertical plane wall.

All Properties Constant. As was the situation with the vertical plane wall, the equation reduction was simplified for the constant-property case. The introduction into Equations (17), (18) and (19) of the stream function defined by Equations (35) eliminated Equation (17). The buoyant force for the constant-property case was defined as

$$(g/g_0)(\rho_L - \rho_m)/\rho_m, \quad (76)$$

where ρ_m was evaluated at T_m . The substitution of Equations (35) and (76) into Equations (18) and (19) yielded

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{\mu g_0}{\rho} \frac{\partial^3 \psi}{\partial y^3} + g \frac{\rho_L - \rho_m}{\rho_m} \quad (77)$$

and

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (78)$$

for the constant-property case.

The next step in reducing Equations (77) and (78) to ordinary differential equations for the horizontal cylinder was to introduce

$$\left. \begin{aligned} \beta &= x/r \\ \phi &= c_4 y/r \\ \Psi &= \psi \rho_m / g_0 \mu c_4 \end{aligned} \right\}, \quad (79)$$

where

$$c_4 = \left[\frac{g r^3 (\rho_L - \rho_m) \rho_m}{\mu^2 g_0^2} \right]^{\frac{1}{4}}. \quad (80)$$

In this way Equations (77) and (78) became

$$\frac{\partial \Psi}{\partial \phi} \frac{\partial^2 \Psi}{\partial \beta \partial \phi} - \frac{\partial \Psi}{\partial \beta} \frac{\partial^2 \Psi}{\partial \phi^2} - \sin \beta - \frac{\partial^3 \Psi}{\partial \phi^3} = 0 \quad (81)$$

and

$$\frac{\partial \Psi}{\partial \phi} \frac{\partial \theta}{\partial \beta} - \frac{\partial \Psi}{\partial \beta} \frac{\partial \theta}{\partial \phi} = \frac{k}{g_0 \mu c_p} \frac{\partial^2 \theta}{\partial \phi^2} \quad (82)$$

By the introduction of Equations (58) into Equations (81) and (82),

$$p''' + 3pp'' - 2(p')^2 + 1 = 0 \quad (83)$$

and

$$t'' + 3N_{pr} p t' = 0 \quad (84)$$

were obtained, where the primes on t and p represent differentiation with respect to ξ . The boundary conditions applicable to Equations (83) and (84) were

$$\left. \begin{aligned} p = p' = 0, \quad t = 1 & \quad \xi = 0 \\ p' = 0, \quad t = 0, \quad t' = -3N_{pr} \left(\frac{h_{fg}}{c_p \Theta} \right) p & \quad \xi = \xi_\delta \end{aligned} \right\}, \quad (85)$$

as obtained from Equations (20) and (21).

Equations (83) and (84) with boundary conditions (85) are the same equations derived for the case of film boiling on a plane vertical wall considering constant properties.

Relation of Heat Transfer on Plane Vertical Wall and Horizontal Cylinder

Since the reduced differential equations were the same for the plane vertical wall and the horizontal cylinder, it was to be expected that the heat transfer in each case was related.

For the vertical wall the local wall heat transfer could be expressed as

$$q = -k \left. \frac{\partial T}{\partial y} \right|_w = -k \Theta c x^{-\frac{1}{4}} \left. \frac{d\theta}{d\eta} \right|_w = -h \Theta \quad (86)$$

and an average heat transfer coefficient could be found by integrating the local coefficient over the entire plate. Thus,

$$h_{avg} = \frac{1}{\Theta L} \int_0^L k \Theta c \left. \frac{d\theta}{d\eta} \right|_w x^{-\frac{1}{4}} dx = \frac{4kc}{3L^{\frac{1}{4}}} \left. \frac{d\theta}{d\eta} \right|_w \quad (87)$$

The heat transfer could also be expressed in terms of a dimensionless group,

$$N_{Nu,L} = \frac{4c}{3} \left. \frac{d\theta}{d\eta} \right|_w L^{\frac{3}{4}}, \quad (88)$$

known as the Nusselt number.

The horizontal cylinder was next considered. In this case the wall heat transfer could be expressed as

$$q = -k \left. \frac{\partial T}{\partial y} \right|_w = -k\theta C_c \frac{1}{r} \left. \frac{\partial \theta}{\partial \phi} \right|_w = -k\theta C_c \frac{1}{r} H(\beta) \left. \frac{dt}{d\xi} \right|_w = -h_c \theta, \quad (89)$$

and an average heat transfer coefficient was found by integrating the local coefficient over the entire cylinder. Thus,

$$h_{c,avg} = \frac{1}{\theta\pi} \int_0^\pi \frac{kC_c \theta}{r} \left. \frac{dt}{d\xi} \right|_w H(\beta) d\beta = \frac{(1.91)kC_c}{\pi r} \left. \frac{dt}{d\xi} \right|_w, \quad (90)$$

which follows from Hermann's graphical integration,¹⁷ shown in Figure 4, where

$$\int_0^\pi H(\beta) d\beta = 1.91. \quad (91)$$

The heat transfer could also be expressed in terms of

$$N_{Nu,D} = 1.025 C_c \left. \frac{dt}{d\xi} \right|_w, \quad (92)$$

where the Nusselt number was based on the diameter of the cylinder.

The similarity between Equations (88) and (92) was then apparent. For a given fluid surrounding a plate or cylinder at the same pressure and with the same wall to liquid temperature difference, it followed that

$$\left. \frac{d\theta}{d\eta} \right|_w = \left. \frac{dt}{d\xi} \right|_w. \quad (93)$$

Therefore, for identical characteristic lengths,

$$h_{c,avg}/h_{avg} = 0.77 \quad (94)$$

and

$$Q_c/Q = 2.42 \quad . \quad (95)$$

Thus, once the vertical plate problem was solved, heat transfer results could be transferred directly to the horizontal cylinder, or vice versa.

SOLUTION OF EQUATIONS

The differential equations derived in the preceding sections were solved numerically^{18,33} by the Runge-Kutta method on a Datatron 204 digital computer. The methods of solution for a particular set of equations are indicated here; the methods of solution for the other equations differ only in detail.

Consider the simultaneous ordinary differential equations

$$\begin{aligned} \xi''' + 3\xi\xi'' - 2(\xi')^2 + \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} = \frac{1}{\rho} \frac{d\rho}{d\theta} \theta' [3\xi\xi' + 2\xi''] \\ + \frac{1}{\rho} \frac{d\rho}{d\theta} \xi'\theta'' + \frac{1}{\rho} \xi'(\theta')^2 \left[\frac{d^2\rho}{d\theta^2} - \frac{2}{\rho} \left(\frac{d\rho}{d\theta} \right)^2 \right] \end{aligned} \quad (32)$$

and

$$\theta'' + 3(\mu g_0/k)c_p \xi \theta' = 0 \quad , \quad (33)$$

with the boundary conditions

$$\left. \begin{aligned} \xi = \xi' = 0 \quad , \quad \theta = 1 \quad \eta = 0 \\ \xi' = 0 \quad , \quad \theta = 0 \quad , \quad \theta' = -3(\mu g_0/k)(h_{fg}/\theta)\xi \quad \eta = \eta_\delta \end{aligned} \right\} \quad , \quad (34)$$

derived previously.

In these equations, ρ_L , ρ_{sat} , μ and k were given constants and the quantities c_p , ρ , $d\rho/d\theta$ and $d^2\rho/d\theta^2$ were considered known and were tabulated for each 0.01 interval of θ and stored in the computer. A linear interpolation was used for property values required between the tabulated points. An explanation of how the properties and property gradients were evaluated is provided in Appendix A.

The computer program used was originally devised to numerically solve the following system of simultaneous first-order ordinary differential equations:³³

$$\left. \begin{aligned} \dot{y}^1 &= f_1(y^1, y^2, \dots, y^n, \eta) \\ \dot{y}^2 &= f_2(y^1, y^2, \dots, y^n, \eta) \\ &\vdots \\ \dot{y}^n &= f_n(y^1, y^2, \dots, y^n, \eta) \end{aligned} \right\}, \quad (96)$$

where the dots on y represent differentiation with respect to the independent variable η and the superscripts on y represent the particular dependent variables

Given the values $y_j^1, y_j^2, \dots, y_j^n$ at η equal to η_j , where the subscript j refers to the value of η being considered, the values $y_{j+1}^1, y_{j+1}^2, \dots, y_{j+1}^n$ at η equal to η_{j+1} were estimated by the formula³³

$$y_{j+1}^1 = y_j^1 + (1/6)[K_0^1 + 2K_1^1 + 2K_2^1 + K_3^1] \quad ;$$

$$i = 1, 2, \dots, n \quad , \quad (97)$$

where

$$\left. \begin{aligned} K_0^1 &= hf_1(y_j^1, y_j^2, \dots, y_j^n, \eta_j) \\ K_1^1 &= hf_1\left(y_j^1 + \frac{1}{2}K_0^1, y_j^2 + \frac{1}{2}K_0^2, \dots, y_j^n + \frac{1}{2}K_0^n, \eta_j + \frac{1}{2}h\right) \\ K_2^1 &= hf_1\left(y_j^1 + \frac{1}{2}K_1^1, y_j^2 + \frac{1}{2}K_1^2, \dots, y_j^n + \frac{1}{2}K_1^n, \eta_j + \frac{1}{2}h\right) \\ K_3^1 &= hf_1\left(y_j^1 + \frac{1}{2}K_2^1, y_j^2 + \frac{1}{2}K_2^2, \dots, y_j^n + \frac{1}{2}K_2^n, \eta_j + \frac{1}{2}h\right) \end{aligned} \right\} \quad (98)$$

and

$$h = \eta_{j+1} - \eta_j = \Delta\eta \quad (99)$$

gave the desired interval of the independent variable. The truncation error³³ introduced by these approximations was of the order of h^5 .

In order to reduce Equations (32) and (33) to a system of simultaneous first-order ordinary differential equations, the substitutions

$$\left. \begin{aligned} y^1 &= \zeta \\ y^2 &= \zeta' \\ y^3 &= \zeta'' \\ y^4 &= \zeta''' \\ y^5 &= \theta \\ y^6 &= \theta' \\ y^7 &= \theta'' \end{aligned} \right\} \quad (100)$$

were made. In this way it was seen that

$$\left. \begin{aligned} \dot{y}^1 &= y^2 = f_1 \\ \dot{y}^2 &= y^3 = f_2 \\ \dot{y}^3 &= y^4 = f_3 \\ y^4 &= -3y^1y^3 + 2(y^2)^2 - \frac{(\rho_L - \rho)}{(\rho_L - \rho_{\text{sat}})} \frac{\rho}{\rho_{\text{sat}}} + \frac{1}{\rho} \frac{d\rho}{d\theta} y^2y^7 \\ &\quad + \frac{1}{\rho} \frac{d\rho}{d\theta} y^6 [3y^1y^2 + 2y^3] + \frac{1}{\rho} y^2(y^6)^2 \left[\frac{d^2\rho}{d\theta^2} - \frac{2}{\rho} \left(\frac{d\rho}{d\theta} \right)^2 \right] \\ \dot{y}^5 &= y^6 = f_5 \\ \dot{y}^6 &= y^7 = f_6 \\ y^7 &= 3(\mu g_0/k)c_p y^1 y^6 \end{aligned} \right\} \quad (101)$$

where y^4 and y^7 were obtained from Equations (32) and (33).

Given the values of y^1 , y^2 , y^3 , y^4 , y^5 , y^6 and y^7 at η equal to η_1 , the value of y^1 at η equal to $(\eta_1 + \Delta\eta)$ was found as follows. First,

$$\left. \begin{aligned} K_0^1 &= \Delta\eta(y^1) \\ K_1^1 &= \Delta\eta[y_1^2 + \frac{1}{2}(\Delta\eta)y_1^3] \\ K_2^1 &= \Delta\eta[y_1^2 + \frac{1}{2}(\Delta\eta)\{y_1^3 + \frac{1}{2}(\Delta\eta)y_1^4\}] \\ K_3^1 &= \Delta\eta\left\{y_1^2 + \Delta\eta\left[y_1^3 + \frac{1}{2}(\Delta\eta)y_1^4\right]\right\} \end{aligned} \right\} \quad (102)$$

as seen from Equation (97). This yielded

$$y_{\eta_1 + \Delta\eta}^1 = y_{\eta_1}^1 + (\Delta\eta)\dot{y}_{\eta_1}^1 + \frac{(\Delta\eta)^2}{2}\ddot{y}_{\eta_1}^1 + \frac{(\Delta\eta)^3}{3!}\dddot{y}_{\eta_1}^1 \quad (103)$$

as the value of y^1 at η equal to $(\eta_1 + \Delta\eta)$. It was interesting to note that the result for $y^1(\xi)$ represented the first few terms of a Taylor Series expansion of y^1 .

In a similar manner the values of y^2 , y^3 , y^5 and y^6 (ξ' , ξ'' , θ and θ') were calculated at η equal to $(\eta_1 + \Delta\eta)$ from the values given at η_1 . The values of $y^4(\xi''')$ and $y^7(\theta''')$ were found according to Equation (101). This entire process was then repeated to find the values of y^1 , y^2 , y^3 , y^4 , y^5 , y^6 and y^7 at η equal to $(\eta_1 + 2\Delta\eta)$, η equal to $(\eta_1 + 3\Delta\eta)$, and so forth. Any desired accuracy within reason may be obtained by choosing appropriately small values of $\Delta\eta$. Since each iteration as outlined above required approximately five minutes, a $\Delta\eta$ of 0.01 was used for constant-property solutions and for all solutions for water at 2800 psia; a $\Delta\eta$ of 0.05 was used for all solutions for water at 3100 psia. It was observed that solutions for θ' obtained using a $\Delta\eta$ of 0.10 always differed by less than 5 per cent from solutions for θ' obtained using a $\Delta\eta$ of 0.01.

Unfortunately, the conditions to be satisfied by the solution of Equations (32) and (33) were specified on both ends of the interval over which the solution was desired. The computer program required that the conditions be given at η equal to zero. Thus since only ξ , ξ' and θ were known at η equal to zero, it was necessary to estimate the values of ξ'' and θ' at η equal to zero. This then specified ξ''' and θ''' as well, and the computer calculated successive values of ξ , ξ' , ξ'' , ξ''' , θ , θ' and θ'' at increasing values of η . Adjustments in the initial values of ξ'' and θ' were then made until the boundary conditions at η_δ were satisfied.

The boundary conditions at η_δ were considered satisfied when ξ' and θ went to zero at the same value of η , and at the same time

$$\theta'_\delta = -3(\mu g_0/k)(h_{fg}/\theta)\xi_\delta \quad (104)$$

was satisfied to within ± 5 per cent. A ± 5 per cent variation in Equation (104) gave a variation to θ' at η equal to zero of less than 2 per cent. Since η_δ was originally unknown, this process also served to determine η_δ . Figure 5 illustrates typical variations of ξ' and θ across the vapor film.

The computer was programmed to print out the values of ξ , ξ' , ξ'' and θ at specified intervals of η . Thus the velocity and temperature profiles across the vapor film were obtained, along with the heat transfer at any point.

FIG 5 TYPICAL VARIATIONS OF ζ'
AND θ WITH η

A COMPARISON OF FILM BOILING AND FILM CONDENSATION THEORIES AND THE EFFECT OF LIQUID MOTION ON FILM BOILING

An investigation of the phenomena of laminar film condensation and film boiling revealed that the two are remarkably similar. In Figure 6A laminar film condensation, on an isothermal vertical plane wall, of a vapor at rest is pictured; and in Figure 6B laminar film boiling, on an isothermal vertical plane wall, of a liquid at rest is pictured. The driving force of the phenomenon in each case was due to the force of gravity and to a wall-to-liquid temperature difference. In the case of laminar film condensation the

FIG 6 A COMPARISON OF LAMINAR FILM
BOILING (B) AND LAMINAR FILM
CONDENSATION (A)

temperature difference caused condensation on the surface; the force of gravity on the condensate removed the condensate so additional condensation could occur. In the case of film boiling the temperature difference caused vaporization at the surface; the buoyant force on the vapor removed the vapor so additional vaporization could occur. In each case the film thickness increased proportionally to $x^{\frac{1}{4}}$, where x is measured as shown in Figure 6.

Here, however, the similarity ended. In laminar film condensation the vapor

exerts a negligible drag on the falling liquid film. Thus it is assumed that at the liquid-vapor interface in film condensation there is a zero shear stress and the velocity distribution in the liquid film is as shown in Figure 6A. In laminar film boiling the liquid exerts a considerable drag on the rising vapor. Thus it is usually assumed in film boiling that at the liquid-vapor interface there is zero velocity in the x direction, and the velocity distribution in the vapor film is as shown in Figure 6B.

It was observed that the simultaneous Equations (46) and (47), derived to describe laminar film boiling for the case of constant properties, are the same equations which describe laminar film condensation for the case of constant properties. Except for the boundary condition on velocity at the liquid-vapor interface as mentioned above, the remaining boundary conditions are the same for laminar film boiling and laminar film condensation. It is known that this difference in boundary conditions would yield a different solution of Equations (46) and (47) for each case.

Figure 7 represents the solutions of Equations (46) and (47) for the two cases considered for a Prandtl number equal to one, and in terms of the dimensionless number $c_p \theta / h_{fg}$.

FIG 7 A COMPARISON OF SOLUTIONS OF THE COMPLETE BOUNDARY LAYER EQUATIONS FOR LAMINAR FILM BOILING AND LAMINAR FILM CONDENSATION, CONSIDERING AN ISOTHERMAL VERTICAL PLANE WALL AND A FLUID WITH $N_{Pr} = 1$
(CONDENSATION CURVE TAKEN FROM REFERENCE 37)

At all times the dimensionless heat transfer, as represented in Figure 7, was larger for the case of laminar film condensation than for the case of laminar film boiling. This is due to the fact that the film thickness for the case of zero interface shear stress is less than the film thickness for the case of zero interface velocity, and thus offers less resistance to heat transfer.

It was interesting to note that, while the similarity transformations used in deriving Equations (46) and (47) were not valid for the general case of considering a moving liquid in film boiling, there was one particular liquid motion that could be handled. This is the case where the liquid velocity is such that the shear stress at the liquid-vapor interface is zero. This film boiling situation is exactly satisfied

by the equations and boundary conditions used to describe laminar film condensation, and the results for a vapor of Prandtl number equal to one are also shown in Figure 7. It should be noted that the liquid velocity required

to satisfy the condition of zero shear stress at the liquid-vapor interface could be large. For example, for water at atmospheric pressure, film boiling on a vertical 1500° F wall, this velocity would be of the order of 100 ft/sec.

Thus it was seen at least qualitatively that theory predicts an increase in heat transfer in laminar film boiling with an increase in liquid velocity in the vertical direction. This expected increase in heat transfer has been found experimentally.^{5,9}

A COMPARISON OF EXACT AND APPROXIMATE FILM BOILING THEORIES

There are now several methods available to calculate the heat transfer in laminar film boiling when radiation is only of minor importance. These methods were compared here. The approximate constant-property analyses were first compared to the exact constant-property analysis developed previously. Then the exact constant-property analysis was compared to the variable-property analyses developed in this thesis. The solutions presented represent application of film boiling theories to a plane vertical plate.

FIG. 8 HEAT TRANSFER RESULTS FROM THE SOLUTION OF THE COMPLETE BOUNDARY LAYER EQUATIONS AS COMPARED TO APPROXIMATE THEORIES WITH $N_{Pr} = 1$

Constant Properties

Before going on to the more general problem of considering variable properties, it was desirable to look at the results for the constant-property analysis. Figure 8 represents a comparison between: (1) Bromley's 1950 analysis³ (as applied to a vertical plane plate) neglecting vapor inertia and convection, (2) Bromley's 1953 analysis⁴ which approximately considers vapor convection but neglects vapor inertia, and (3) the present analysis which considered both vapor convection and inertia exactly, and more specifically the solution of Equations (46) and (47) with boundary conditions (48).

As was seen in Figure 8, Bromley's 1953 analysis and the exact analysis agree within 2.5 per cent up to a $c_p \theta / h_{fg}$ value of 2. Beyond this range the curves start to diverge

considerably. Both of the aforementioned analyses gave significantly different results than the 1950 Bromley analysis. The curves of Figure 8 illustrated that the effect of convection on heat transfer in laminar film boiling is greater than the effect of consideration of inertia terms.

A very interesting Prandtl number effect was observed in the above analyses. As can be seen directly from Bromley's analysis, for the same value of $c_p \theta / h_{fg}$,

$$\left\{ \frac{N_{Nu,x} \left[\frac{g(\Delta\rho)\rho x^3}{4\mu^2 g_0^2} \right]^{\frac{1}{4}}}{N_{Nu,x} \left[\frac{g(\Delta\rho)\rho x^3}{4\mu^2 g_0^2} \right]^{\frac{1}{4}}} \right\}_2 = \left[\frac{N_{Pr,2}}{N_{Pr,1}} \right]^{\frac{1}{4}} \quad (105)$$

This relation was also found to hold true in the solution of the complete boundary layer equations in the range of Prandtl Numbers (from $N_{Pr} = \frac{2}{3}$ to $N_{Pr} = 2$) studied. Also, both exact and approximate theories predicted an $x^{\frac{1}{4}}$ dependence of film thickness on x . In all three cases, for the same value of $c_p \theta / h_{fg}$,

$$\frac{\delta_2 \left[\frac{g(\Delta\rho)\rho}{4\mu^2 g_0^2 x} \right]^{\frac{1}{4}}}{\delta_1 \left[\frac{g(\Delta\rho)\rho}{4\mu^2 g_0^2 x} \right]^{\frac{1}{4}}} = \left[\frac{N_{Pr,1}}{N_{Pr,2}} \right]^{\frac{1}{4}} \quad (106)$$

where the subscripts 1 and 2 refer to the vapor under consideration. Thus, as the vapor Prandtl number increases the film thickness decreases and the heat transfer increases.

FIG 9 VARIATION OF DIMENSIONLESS FILM THICKNESS FOR CONSTANT PROPERTIES, $N_{Pr} = 1$

Figure 9 represents the growth of the vapor layer as a function of $c_p \theta / h_{fg}$ and Figure 10 gives the dimensionless temperature distribution through the vapor layer, both for a vapor with unit Prandtl number.

Figure 11 presents the velocity distribution across the vapor layer as a function of $c_p \theta / h_{fg}$.

FIG 10 DIMENSIONLESS TEMPERATURE DISTRIBUTION IN THE VAPOR LAYER FOR CONSTANT PROPERTIES, $N_{Pr} = 1$

FIG 11 VELOCITY DISTRIBUTION IN THE VAPOR FILM FROM THE SOLUTION OF THE COMPLETE BOUNDARY LAYER EQUATIONS, $N_{Pr} = 1$

Variable Properties

The effect of considering variable properties is now considered. Variable property results were obtained for water film boiling from: (1) the solution of Equations (38) through (40), where variable specific heats and density variations considered only in the evaluation of the buoyant force were considered; and (2) the solution of Equations (32) through (34) where variable specific heat and compressible flow were considered. The significant results of the variable-property analyses were presented in Table 3. Table 4 presents a comparison of variable-property to constant-property results. The constant-property results represent the solution of the complete boundary layer equations.

Table 3

SIGNIFICANT RESULTS

		Compressible			Incompressible			All Properties Constant				
P, psia	θ , F	$\eta \delta$	$\zeta'_{\max}$	$\left. \frac{d\theta}{d\eta} \right _w$	$\eta \delta$	$\zeta'_{\max}$	$\left. \frac{d\theta}{d\eta} \right _w$	$\frac{c_p \theta}{h_{fg}}$	N_{Pr}	$\eta \delta$	$\zeta'_{\max}$	$\left. \frac{d\theta}{d\eta} \right _w$
2800	250	1.39	0.17	0.76	0.96	0.25	1.14	0.92	1.26	1.27	0.181	0.88
	500	1.64	0.19	0.67	1.04	0.35	1.09	1.32	1.0	1.43	0.233	0.79
	1000	1.84	0.21	0.62	1.14	0.47	1.04	2.3	0.90	1.60	0.285	0.73
3100	250	2.35	0.093	0.48				1.8	1.45	1.37	0.214	0.84
	500	2.83	0.103	0.41				2.3	1.0	1.56	0.271	0.75
	1000	3.15	0.113	0.39				3.8	0.88	1.69	0.302	0.71

Table 4

A COMPARISON OF RESULTS

		Compressible			Incompressible	
P, psia	θ , F	$\frac{q}{q_{cp}}$	$\frac{u_{\max}}{u_{\max, cp}}$	$\frac{\delta}{\delta_{cp}}$	$\frac{q}{q_{cp}}$	$\frac{\delta}{\delta_{cp}}$
2800	250	0.96	0.61	0.98	1.66	0.66
	500	0.98	0.47	0.99	1.6	0.62
	1000	1.04	0.37	0.94	1.63	0.59
3100	250	0.65	0.23	1.55		
	500	0.64	0.17	1.57		
	1000	0.63	0.145	1.53		

The comparisons of heat transfer, maximum velocity, and film thickness were made according to the following. Since

$$q = -k \frac{\partial T}{\partial y} = -k\theta \frac{\partial \theta}{\partial y} = -k\theta \frac{C}{x^{\frac{1}{4}}} \frac{d\theta}{d\eta} \quad (107)$$

for the same wall-to-liquid temperature difference and the same value of x ,

$$\left[\frac{q}{C \frac{d\theta}{d\eta}} \right]_{\text{variable properties}} = \left[\frac{q}{C \frac{d\theta}{d\eta}} \right]_{\text{constant properties}} \quad (108)$$

was used to compare the heat transfer as calculated by constant-property and variable-property analyses. Also, since

$$u = (4\mu g_0/\rho) C^2 x^{\frac{1}{2}} \zeta' \quad (109)$$

and

$$\eta_\delta = C\delta/x^{\frac{1}{4}} \quad (110)$$

for the same wall-to-liquid temperature difference and the same value of x ,

$$\left[\frac{u_{\max} \rho}{C^2 \zeta'_{\max}} \right]_{\text{variable properties}} = \left[\frac{u_{\max} \rho}{C^2 \zeta'_{\max}} \right]_{\text{constant properties}} \quad (111)$$

and

$$\left[\frac{\delta C}{\eta_\delta} \right]_{\text{variable properties}} = \left[\frac{\delta C}{\eta_\delta} \right]_{\text{constant properties}} \quad (112)$$

were used to compare the maximum velocity and film thickness, respectively, as calculated by the two methods.

FIG 12 DIMENSIONLESS VELOCITY DISTRIBUTION FOR WATER VAPOR, $P = 3100$ psia

It was of interest to note that if, at a given pressure, $u/u_{\max}$ was plotted against y/δ , the data fell on a single curve no matter what the wall temperature, as shown in Figure 12. If data for different pressures were compared in this manner, the position of the maximum velocity was seen to vary, as shown in Figure 13. For constant properties the maximum velocity occurs at $y/\delta = \frac{1}{2}$, as seen in Figure 13. The shifting of the velocity maximum was not easy to explain. However, if inertia terms were neglected in Equation (18), if $(\rho_L - \rho)/\rho$ was assumed to decrease linearly from the wall to the liquid-vapor interface, and if all other properties were assumed constant, it was seen that the velocity maximum should shift in the direction indicated.

FIG 13 COMPARISON OF DIMENSIONLESS VELOCITY DISTRIBUTIONS

As seen in Table 6, the effect of considering compressible flow was to decrease the maximum velocity in the film. This would tend to offset the increase in shear stress at the liquid-vapor interface caused by the shifting of the maximum velocity.

The temperature profiles at three different wall-to-liquid temperature differences at 2800 and 3100 psia are shown in Figures 14 and 15, respectively. Note that if the compressible flow temperature profiles for the same value of $c_{pm}\theta/h_{fg}$ are plotted against y/δ , the curves are identical, as shown in Figure 16. The

effect on the temperature profile of considering a variable specific heat and buoyant force but otherwise incompressible flow is shown in Figure 17. The profiles for only one wall-to-liquid temperature difference is shown, the effect being the same for all investigated.

The effect of considering the vapor as incompressible while taking into account variations of specific heat and buoyant force was to increase the heat transfer by about sixty per cent over that calculated by a constant-property analysis at 2800 psia, as shown in Table 4. When compressible flow was considered as well, the variable-property analysis gave no significantly different results from the constant-property analysis at 2800 psia, provided the constant properties were evaluated at the average temperature, T_m .

The situation was not so clear at 3100 psia. Here the decrease in film velocity due to considering compressible flow increased the film thickness considerably. The effect was to decrease the heat transfer compared to that expected with a constant-property analysis.

It was interesting to note that at the two pressures considered

$$(q\delta)_{\text{variable properties}} = (q\delta)_{\text{constant properties}} \quad (113)$$

within at most an error of 6 per cent.

FIG 14 DIMENSIONLESS TEMPERATURE DISTRIBUTION IN THE VAPOR LAYER CONSIDERING VARIABLE PROPERTIES, WATER AT 2800 psia

FIG 15 DIMENSIONLESS TEMPERATURE DISTRIBUTION IN THE VAPOR LAYER CONSIDERING VARIABLE PROPERTIES, WATER AT 3100 psia

FIG 16 COMPARISON OF TEMPERATURE DISTRIBUTIONS FOR $\frac{C_p \theta}{h_{fg}} = 2.3$

FIG 17 THE EFFECT OF AN INCOMPRESSIBLE FLUID WITH VARIABLE BUOYANT FORCE AND SPECIFIC HEAT ON TEMPERATURE DISTRIBUTION, WATER AT 2800 psia, $\theta = 1000$ F

A COMPARISON OF THEORY WITH EXPERIMENT

Since the theory developed in the preceding chapters considered only heat transfer by conduction and convection in the vapor film, and not radiation through the film, the applicable experimental data were extremely limited. If one wishes to apply these theories to situations where radiation is important, an approximate correction must be made such as that given by Bromley.³ This correction is so approximate, however, that the exact theory developed here is not required. However, there are situations where radiation is only of minor importance, and the exact theory developed here should apply.

Figure 18 shows a comparison between the predicted and measured heat transfer for mercury film boiling on the outside of a horizontal tube.

FIG 18 OBSERVED AND CALCULATED FILM-BOILING HEAT TRANSFER VALUES FOR MERCURY AT 1 ATMOSPHERE
(EXPERIMENTAL DATA TAKEN FROM REFERENCE 23)

Predicted results represent the solution of Equations (83) and (84), with boundary conditions as given by Equation (85). On first inspection it seemed that agreement was poor. The experimental data were correlated by measuring the tube surface temperature on the top and 90 degrees from the top, averaging these two temperatures and using this average temperature to correlate with the measured heat transfer. As is seen from Equation (89) or by reference to Bromley,³ there is very little heat transfer on the top of the cylinder and thus this method of specifying a representative surface temperature is misleading. A much better method of specifying a representative surface temperature is to use the temperature 90 degrees from the top, or better yet an average between temperatures measured 90 degrees from the top and on the bottom.

If, then, the representative surface temperature was measured as just discussed, and since the surface temperature decreases as the bottom of the cylinder is approached,³ it was seen that the experimental data should be shifted to a lower representative wall temperature and towards the theoretical curve. At higher surface temperatures the theoretical expression would not be expected to hold because of radiation.

Figure 19 shows a comparison between the predicted and measured heat transfer of methanol film boiling on the outside of a horizontal tube.

FIG 19 OBSERVED AND CALCULATED FILM-BOILING HEAT TRANSFER VALUES FOR METHANOL AT 1 ATMOSPHERE

(EXPERIMENTAL DATA TAKEN FROM REFERENCE 39)

Again the agreement between theory and experiment did not seem good, but the method of specifying a representative surface temperature was misleading. The outside tube wall temperature was calculated using the resistance concept (see for example Jakob,²⁰ page 14) of heat transfer. Since the calculations were based on the total tube area and the major portion of the heat was transferred on the bottom half of the tube, the outside tube wall temperature as calculated was too high. A more reasonable calculation of outside tube wall temperature would move the experimental data given in Figure 19 to a lower value of ($T_w - T_{sat}$), and towards the curve predicted by the present theory. Again, the theory would not be expected to hold where surface temperatures are such that radiation is significant.

The quantity of film-boiling heat transfer experimental data in the present literature is far too meagre, and the methods of correlation of such data far too inexact, to provide a good basis for extensive correlation of theory and experiment. It is hoped that this can be rectified in the near future.

CONCLUSION

A method of analyzing stable, free-convection, laminar film boiling on an isothermal surface where radiation is only of minor importance and considering variable fluid properties has been developed. The method, of course, includes the special case where properties may be considered constant. This method was used to extend the previously available analytical investigations of laminar film boiling to include vapor inertia and convection terms, as well as property variations.

The results of the investigation showed that the heat transfer as calculated by the constant-property method did not always agree with the heat transfer as calculated by the variable-property methods. At 2800 psia for water, consideration of compressible flow and variable specific heat did not appreciably change the value of the heat transfer from that calculated by assuming constant properties, if the constant properties were evaluated at

the arithmetic mean film temperature. At 3100 psia, however, the heat transfer as calculated by considering constant properties was 54 to 59 per cent greater than the heat transfer as calculated by considering compressible flow and variable specific heat.

The method of considering variable specific heat while density variations were considered only in the evaluation of the buoyant force was not valid. The consideration of variable specific heat tends to increase the wall heat transfer; the consideration of variable density thickens the vapor layer and thus tends to decrease the heat transfer. At 2800 and 3100 psia radiation plays a more important role in film boiling than does the consideration of variable properties.

Actually, 3100 psia is still very far from the critical pressure. It is felt that, as the critical pressure is approached even closer, the role that variable properties play in film-boiling heat transfer will become increasingly important. The work required near the critical pressure can not be completed until more accurate property values in this region are available.

The situation when the wall is nonisothermal and radiation is important is not as clearly understood. From the results of Appendix B it was seen that for common engineering materials with emissivities of 0.8 or more, a constant heat flux wall approaches an isothermal wall if sections removed from the leading edge are considered. Most film-boiling experimental work tries to simulate an isothermal wall. There is a definite need for nonisothermal wall experimental data, along with the measurement of wall temperatures to test the theory presented here.

It was also concluded that liquid motion can increase the heat transfer in laminar film boiling, though the liquid velocities required to get an appreciable effect may be quite large.

APPENDIX A

EVALUATION OF PROPERTIES

The properties used for numerical calculations were chosen, after a careful search of available property data, as the most representative. For solutions requiring an average property value, the actual properties used are listed in Tables 5, 6 and 7 for water, methanol and mercury vapors, respectively. In all cases where a property was considered constant, it was evaluated at

$$T_m = \frac{1}{2} (T_w + T_{\text{sat}}) \quad , \quad (114)$$

where T_m is the mean temperature. The references should be consulted for values of properties considered variable. In all cases a linear interpolation was used for property values between points listed in the references.

Water Vapor. Saturation and density data for water were taken from Ref. 21. For density data beyond the limits of the reference, the tabulated data were connected with the ideal gas equation of state. This would introduce at most an error of 3 per cent beyond the limits of accuracy of the table at the worst case, 3100 psia. The values of $d\rho/d\theta$ and $d^2\rho/d\theta^2$ were obtained numerically, based upon the above density values. Values of $d\rho/d\theta$ and $d^2\rho/d\theta^2$ at saturation were extrapolated. Numerical differentiation such as this is not accurate, but the numbers obtained are the best approximation to the actual values.

Specific heat data was also taken from Ref. 21. The given data were extrapolated according to the curves given in the reference for temperatures above the table limits. Values of specific heat near the saturation curve at high pressure were obtained numerically from enthalpy values.

Viscosity and thermal conductivity data for water was obtained from Ref. 12. The data were extrapolated for temperatures beyond the limits of the reference according to the curves given by the author.

Mercury Vapor. Saturation data for mercury vapor was obtained from Ref. 22 and 36. Vapor densities were calculated using the ideal gas equation of state. The values of density calculated in this manner were found to differ by 6 per cent at saturation from saturation vapor densities given in Ref. 36. The specific heat was obtained by numerical methods from enthalpy data given in Ref. 36. This method was checked by calculating the specific heat assuming a monatomic ideal gas, and the value thus obtained agreed with values given in Ref. 36.

Table 5

WATER VAPOR PROPERTIES

P = 2800 psia; T _{sat} = 685 F; h _{fg} = 285 B/lb _m ; ρ _L = 9.46 lb _m /ft ³				
T(F)	ρ(lb _m /ft ³)	μ(kg sec/m ²)	c _p (B/lb _m F)	k(kcal/m hr C)
810	5.0	2.91 x 10 ⁻⁶	1.05	8.54 x 10 ⁻²
935	4.05	3.15 x 10 ⁻⁶	0.75	8.4 x 10 ⁻²
1185	3.12	3.7 x 10 ⁻⁶	0.64	9.37 x 10 ⁻²
P = 3100 psia; T _{sat} = 700 F; h _{fg} = 168 B/lb _m ; ρ _L = 13.26 lb _m /ft ³				
T(F)	ρ(lb _m /ft ³)	μ(kg sec/m ²)	c _p (B/lb _m F)	k(kcal/m hr C)
825	5.59	3.0 x 10 ⁻⁶	1.2	10.34 x 10 ⁻²
950	4.48	3.22 x 10 ⁻⁶	0.78	8.79 x 10 ⁻²
1200	3.44	3.76 x 10 ⁻⁶	0.64	9.67 x 10 ⁻²

Table 6

METHANOL VAPOR PROPERTIES

P = 1 atm; T_{sat} = 148 F; h_{fg} = 472 B/lb_m; ρ_L = 47.1 lb_m/ft³

T(F)	ρ(lb _m /ft ³)	μ(lb _f sec/ft ²)	c _p (B/lb _m F)	k (B/ft sec F)
248	0.0625	2.69 x 10 ⁻⁷	0.431	4.40 x 10 ⁻⁶
258	0.0616	2.73 x 10 ⁻⁷	0.436	4.52 x 10 ⁻⁶
268	0.0608	2.77 x 10 ⁻⁷	0.441	4.62 x 10 ⁻⁶
278	0.0600	2.81 x 10 ⁻⁷	0.446	4.74 x 10 ⁻⁶
288	0.0591	2.85 x 10 ⁻⁷	0.451	4.85 x 10 ⁻⁶

Table 7

MERCURY VAPOR PROPERTIES

P = 1 atm; T_{sat} = 675 F; h_{fg} = 125.5 B/lb_m; ρ_L = 796 lb_m/ft³

T(F)	ρ(lb _m /ft ³)	μ(lb _f sec/ft ²)	c _p (B/lb _m F)	k (B/ft sec F)
680	0.241	1.30 x 10 ⁻⁶	0.025	1.56 x 10 ⁻⁶
685	0.240	1.31 x 10 ⁻⁶	0.025	1.58 x 10 ⁻⁶
700	0.237	1.33 x 10 ⁻⁶	0.025	1.61 x 10 ⁻⁶
725	0.232	1.38 x 10 ⁻⁶	0.025	1.66 x 10 ⁻⁶
775	0.222	1.46 x 10 ⁻⁶	0.025	1.77 x 10 ⁻⁶

The viscosity of mercury vapor was taken from Ref. 19. The viscosity values thus obtained were used to calculate the thermal conductivity according to the method given on page 71 of Ref. 20. No experimental values of the thermal conductivity of mercury vapor were found.

Methanol Vapor. Saturation data for methanol vapor were taken from Ref. 31. Liquid density data were taken from Ref. 1. Vapor densities were calculated using the van der Waal equation of state, with critical constants taken from Ref. 10. The methanol vapor specific heat was calculated according to an equation given in Ref. 25.

The viscosity of methanol vapor was taken from Ref. 19 and 27. Both sources agreed. The thermal conductivity of methanol vapor was calculated according to the method given on page 71 of Ref. 20. The calculated values were checked at one point with an experimental value given in Ref. 27. The calculated value was 13 per cent higher than the experimental value. Even though there was a discrepancy, the calculated thermal conductivity values were used because of the lack of something better.

APPENDIX B

THE NONISOTHERMAL WALL AND THE EFFECTS OF RADIATION

The similarity transformations considered previously were not amenable to solutions of problems where a nonisothermal wall and/or radiation must be considered. It was observed, however, that approximate analyses gave very representative results. An approximate analysis of laminar film boiling in a saturated liquid from a plane vertical wall, considering constant properties and including the effects of radiation and wall temperature variation, is presented here.

The vapor film shown in Figure 20 was considered. The velocity distribution across the vapor film was found from Equation (18) by making an additional assumption that vapor inertia terms were negligible. Thus Equation (18) became

$$0 = \frac{\mu g_0}{\rho_m} \frac{d^2 \mu}{dy^2} + \frac{g(\rho_L - \rho_m)}{\rho_m} \quad (115)$$

Since the velocity in the x-direction was considered zero at the liquid-vapor interface, the velocity distribution in the film was given by

$$u = \frac{g(\rho_L - \rho_m)}{2 g_0 \mu} (\delta y - y^2) \quad (116)$$

FIG. 20 PHYSICAL MODEL AND COORDINATE SYSTEM

A mean velocity was found by integrating Equation (116) over the vapor film:

$$u_m = \frac{1}{\delta} \int_0^{\delta} u \, dy = \frac{g(\rho_L - \rho_m)}{12 \mu g_0} \delta^2 \quad (117)$$

where δ was a function of x . The mass flow rate at a particular value of x was then expressed as

$$\dot{m} = \frac{g(\rho_L - \rho_m) \rho}{12 \mu g_0} \delta^3 \quad (118)$$

An energy balance was performed on the element shown in Figure 20 by assuming a temperature distribution through the film. From the results of the isothermal wall analysis, for example Figure 16, it was

assumed that the temperature distribution in the film could be expressed as

$$T - T_{\text{sat}} = (T_w - T_{\text{sat}}) \left(1 - \frac{y}{\delta}\right) \quad (119)$$

For steady state an energy balance on the element yielded

$$\text{Energy In} = \text{Energy Out} \quad (120)$$

Now

$$\text{Energy In} = \int_x^{(x + \Delta x)} q_w dx + \text{Energy Convected In} \quad , \quad (121)$$

where

$$\begin{aligned} \text{Energy Convected In} &= \int_0^\delta \rho_m u c_p (T - T_{\text{sat}}) dy \\ &= \frac{g(\rho_L - \rho_m) \rho_m c_p \delta^3}{24 \mu g_0} (T_w - T_{\text{sat}}) \end{aligned} \quad (122)$$

and q_w represented the total heat transferred at the wall as a function of x . In a similar manner,

$$\text{Energy Convected Out} = \frac{g(\rho_L - \rho_m) \rho_m c_p}{24 \mu g_0} (\delta + \Delta\delta)^3 (T_w + \Delta T_w - T_{\text{sat}}) \quad , \quad (123)$$

the energy converted out of the element was found. The only other energy leaving the element was that used to vaporize additional liquid. This was expressed as

$$(\Delta m) h_{fg} = \frac{g(\rho_L - \rho_m) \rho_m h_{fg}}{12 \mu g_0} [(\delta + \Delta\delta)^3 - \delta^3] \quad , \quad (124)$$

which followed from Equation (118).

It was further assumed that

$$q_w = q_r + q_{co} \quad (125)$$

and

$$q_{co} = k(T_w - T_{\text{sat}})/\delta \quad (126)$$

where the subscript r represented radiation and the subscript co represented conduction. Thus the film thickness could be expressed in terms of the

thermal conductivity of the vapor, the wall-to-liquid temperature difference, and the heat transfer.

In this manner a complete energy balance on the element shown in Figure 20 became

$$\begin{aligned} & \int_x^{(x + \Delta x)} q_w dx + \frac{g(\rho_L - \rho_m) \rho_m^2 c_p k^3 [T_w(x) - T_{sat}]^4}{24 \mu g_0 [q_{co}(x)]^3} \\ &= \frac{g(\rho_L - \rho_m) \rho_m^2 c_p k^3 [T_w(x + \Delta x) - T_{sat}]^4}{24 \mu g_0 [q_{co}(x + \Delta x)]^3} \\ &+ \frac{g(\rho_L - \rho_m) \rho_m^2 h_{fg} k^3}{12 \mu g_0} \left\{ \frac{[T_w(x + \Delta x) - T_{sat}]^3}{[q_{co}(x + \Delta x)]^3} - \frac{[T_w(x) - T_{sat}]^3}{[q_{co}(x)]^3} \right\}. \quad (127) \end{aligned}$$

Finally, by defining

$$\theta_x = \frac{T_w(x) - T_{sat}}{T_w(0) - T_{sat}} = \frac{T_w(x) - T_{sat}}{\theta_0}, \quad (128)$$

Equation (127) was rearranged to give

$$\begin{aligned} \theta_x^4(x + \Delta x) + 2 \frac{h_{fg}}{c_p \theta_0} \theta_x^3(x + \Delta x) &= \frac{24 \mu g_0}{g(\rho_L - \rho_m) \rho_m^2 c_p k^3 \theta_0^4} \left(\frac{q_{co}}{q_w} \right)^3 \\ &\int_x^{(x + \Delta x)} q_w dx \theta_x^4(x) + 2 \frac{h_{fg}}{c_p \theta_0} \theta_x^3(x), \quad (129) \end{aligned}$$

where

$$(q_{co}/q_w)^3 = (1 - q_r/q_w)^3. \quad (130)$$

Equation (129) could than be solved by trial and error for the following cases: (1) given a specified wall temperature variation, the wall heat transfer could be determined; and (2) given a specified wall heat flux variation, the wall temperature variation could be determined. It should be noted that in order to solve case (2), the wall temperature at x equal to zero must be known. Also, since it was assumed in the development of Equation (129) that all the radiation emitted in the interval Δx was also absorbed in the interval Δx , the increment Δx must be chosen carefully.

Figure 21 represents the solution of Equation (129) for the case of a constant heat flux wall immersed in water at one atmosphere, where the leading edge of the wall is at 1000 F. The heat flux considered was

WATER AT 1 ATMOSPHERE BOILING, $q_w = 0.1 \frac{B}{in^2 sec}$, $T_{w0} = 1000 F$

FIG. 21 VARIATION OF WALL TEMPERATURE WITH DISTANCE FROM THE LEADING EDGE FOR A CONSTANT HEAT FLUX, VERTICAL WALL

$0.1 B/in^2 sec$, and Equation (129) was solved for three wall emissivities as shown. All properties were evaluated at 650 F. The radiation-only temperature limit represents the wall temperature at which the entire $0.1 B/in^2 sec$ could be dissipated by radiation.

An examination of Figure 21 revealed two interesting facts: (1) for large wall emissivities the wall temperature approaches the radiation limit very rapidly; and (2) for low wall emissivities the wall temperature increases very rapidly, thus increasing the possibility of wall failure.

While the above analysis was approximate, it did point out qualitatively the effect of considering nonisothermal walls and the effects of radiation. The authors have not been able to locate any quantitative experimental data to substantiate the above results.

Comparison to the Isothermal Wall. Ellion¹³ investigated film boiling of water at one atmosphere on an isothermal wall with an emissivity of about 0.8. In order to dissipate $0.1 B/in^2 sec$, a wall temperature of 1500-1600 F was required. Examination of Figure 21 revealed that with the particular wall chosen, in order to dissipate $0.1 B/in^2 sec$ at a wall emissivity of 0.8, a wall temperature of 1800-1900 F is soon reached. While the two cases were not directly comparable, it was noted that in the case of the isothermal wall the $0.1 B/in^2 sec$ represented an average value of the wall. The local value after 3 inches would be $0.075 B/in^2 sec$.

APPENDIX C

NOMENCLATURE

The following nomenclature was used in this thesis:

a,b,c,d = dimensionless constants.

a_1 = function defined by Equation (9).

A = area, ft^2 .

c_1 = dimensional constant, $\left[\frac{g(\rho_L - \rho_{\text{sat}})\rho_{\text{sat}}}{4\mu^2 g_0^2} \right]^{\frac{1}{4}}, \frac{1}{\text{ft}^{\frac{3}{4}}}$

c_2 = dimensional constant, $\left[\frac{g(\rho_L - \rho_m)\rho_m}{4\mu^2 g_0^2} \right]^{\frac{1}{4}}, \frac{1}{\text{ft}^{\frac{3}{4}}}$

c_3 = dimensionless constant, $\left[\frac{gr^3(\rho_L - \rho_{\text{sat}})\rho_{\text{sat}}}{\mu^2 g_0^2} \right]^{\frac{1}{4}}$

c_4 = dimensionless constant, $\left[\frac{gr^3(\rho_L - \rho_m)\rho_m}{\mu^2 g_0^2} \right]^{\frac{1}{4}}$

c_p = constant pressure specific heat, $\text{B}/\text{lb}_m \text{ F}$

C = a constant

C_1 = a constant

D = diameter, ft

f = function of β defined by Equation (58).

$g, \vec{g}$ = acceleration or acceleration vector due to gravity,
 32.174 ft/hr^2

g_0 = dimensional constant, $32.174 \text{ ft lb}_m/\text{lb}_f \text{ hr}^2$

$\vec{G}$ = body force per unit volume, lb_f/ft^3

H = function of β defined by Equation (58).

h = local convection heat transfer coefficient, $\text{B}/\text{ft}^2 \text{ hr } ^\circ\text{F}$

h_{co} = local convection heat transfer coefficient based on conduction, $B/ft^2 \text{ hr } F$

h_{fg} = enthalpy of evaporation, B/lb_m

h_r = local convection heat transfer coefficient based on radiation, $B/ft^2 \text{ hr } F$

h_T = total local convection heat transfer coefficient, $B/ft^2 \text{ hr } F$

k = thermal conductivity, $B/hr \text{ ft } F$

K = functions defined by Equations (98).

L = length of vertical plate, ft

m = mass flow rate, $lb_m/hr \text{ ft}$

$N_{Nu, \lambda}$ = Nusselt number based on characteristic dimension

N_{Pr} = Prandtl number

p = function of ξ , dimensionless

P = pressure, lb_f/ft^2

q = local heat transfer rate, $B/hr \text{ ft}^2$

q_{co} = local heat transfer rate based on conduction, $B/hr \text{ ft}^2$

q_r = local heat transfer rate based on radiation, $B/hr \text{ ft}^2$

q''' = heat generation per unit volume, $B/ft^3 \text{ hr}$

Q = total heat transferred, B/hr

r = cylinder radius, ft

t = dimensionless temperature defined by Equation (58).

T = temperature, F

T_L = liquid temperature, F

T_m = mean temperature, $\frac{1}{2} (T_w + T_{sat})$, F

T_{sat} = saturation temperature, F

T_w = wall temperature, F

u = velocity component in the x -direction, ft/hr

U = liquid velocity, ft/hr

v = velocity component in the y -direction, ft/hr

$\vec{V}$ = velocity vector, ft/hr

x = coordinate surface, ft

y = coordinate normal to surface, ft

y_j^i = functions defined by Equation (96)

α_c = equivalent thermal diffusivity defined by Equations (11)

β = angular coordinate, radians

δ = thickness of vapor film, ft

ϵ = emissivity

ξ = dimensionless function defined by Equation (29)

η = dimensionless independent variable, $c y/x^{1/4}$

θ = dimensionless temperature, $(T - T_{\text{sat}})/(T_w - T_{\text{sat}})$

θ_x = dimensionless temperature, $[T_w(x) - T_{\text{sat}}]/[T_w(x=0) - T_{\text{sat}}]$

$\Theta = T_w - T_{\text{sat}}$, F

$\Theta_L = T_L - T_{\text{sat}}$, F

$\Theta_0 = T_w(x=0) - T_{\text{sat}}$, F

μ = viscosity, $\text{lb}_f \text{ hr}/\text{ft}^2$

ξ = dimensionless independent variable defined by Equation (58)

ρ = vapor density, lb_m/ft^3

ρ_L = liquid density, lb_m/ft^3

ρ_m = mean vapor density, evaluated at T_m , lb_m/ft^3

ρ_{sat} = saturated vapor density, lb_m/ft^3

σ = radiation constant, $0.174 \times 10^{-8} \text{ B/hr ft}^2 \text{ R}^4$

τ = time, hr

ϕ = dimensionless independent variable, $c \ y/r$

ψ = stream function, ft^2/hr

Ψ = dimensionless stream function defined by Equation (54)

∇ = vector operator used for the gradient of a scalar

∇^2 = Laplace operator

Subscripts:

c = horizontal cylinder

co = conduction

L = liquid

r = radiation

sat = saturation

T = total

w = wall

x = x-direction

δ = interface

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