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Error Analysis of a ^{252}Cf Fuel-Rod-Assay System

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R. A. Forster

D. B. Smith

H. O. Menlove

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ERROR ANALYSIS OF A ^{252}Cf FUEL-ROD-ASSAY SYSTEM

by

R. A. Forster, D. B. Smith, and H. O. Menlove

ABSTRACT

This report is an error analysis of a nondestructive assay system designed to measure rapidly and accurately the total fissile content of homogeneous, unpoisoned, low-enrichment (< 5% fissile content by weight), nuclear-reactor fuel rods or small samples. The assay system uses thermal-neutron interrogation to induce fissions in the sample. The resultant high-energy fission neutrons are counted using energy-biased neutron detectors, and the count rate is converted into total fissile mass in the material by using known standards. The contributions of several possible sources of error to the uncertainty in the total fissile-assay result are considered: (1) uniqueness of the measured response to the fissile content of the rod, (2) system calibration, (3) precision, and (4) effects of geometric and loading variations in the fuel rods. These general considerations are applicable to any similar assay system that produces an absolute measurement by comparison to known standards. The overall conclusion is that, with suitable standards and proper calibration, the fuel-rod-assay system can assay a large number of rods so that the uncertainty in the assayed total fissile mass is < 0.5% (2σ).

I. INTRODUCTION

The ^{252}Cf fuel-rod-assay system is designed to measure rapidly and accurately the total fissile content of homogeneous, low-enrichment (< 5% fissile content by weight) nuclear-reactor fuel rods or small samples using thermal-neutron interrogation. The fissile content of any unpoisoned fuel rod suitable for use in a nuclear reactor in which most of the fissions are produced by thermal neutrons can, in principle, be measured using this instrument, if the rod's gamma-ray dose rate at contact is less than 1 R/h. Following a discussion of the design criteria and a description of the system, we examine the contribution of several possible sources of error to the uncertainty

in the total fissile assay result: (1) uniqueness of the measured response to the fissile content of the rod, (2) system calibration, (3) precision, and (4) effects of geometric and loading variations in the fuel rods. Detailed examination of these topics is absolutely necessary for correct understanding and interpretation of the assay results. Note that these general considerations are applicable to any similar fuel-rod-assay system that produces an absolute measurement by comparison to known standards.

II. THE ^{252}Cf FUEL-ROD-ASSAY SYSTEM

A. Design Considerations

Neutron interrogation has been used extensively during the past few years for nondestructive

assay of fissionable materials. Its primary advantages are the relatively high penetrability of the neutrons and the uniqueness of the neutron-fission reaction to the fissionable material(s) of interest (e. g. ^{235}U).

In applying radioactive sources to a given measurement problem, one must consider three fundamental components: (1) the type of neutron source, (2) the moderator and shield for the source, and (3) the detector to count the induced neutrons and/or gamma rays. There is considerable flexibility in each of these areas, and a good assay system combines the three components in an optimum manner for a given problem.

One application of a radioactive neutron source is to the assay and quality control of fabricated reactor fuels for which the sample geometry is well known. Specifically, thermalized neutrons can be used to interrogate the fuel sample; the resulting fission neutrons born in the sample can be counted using an energy-biased neutron detector and compared with similar measurements on standards. Such a system is limited to material with low fissile density because of self-absorption of thermal neutrons in the fuel.

Investigations¹ have been performed to evaluate the merits of various combinations of sources and energy-biasable detectors for assay systems that use thermal neutrons. The results have shown that a ^{252}Cf neutron source (8.36×10^7 s half-life and $\sim 2.3 \times 10^{12}$ n/sec/g) and ^4He -gas recoil detectors provide a high signal rate and a large signal-to-background ratio for such a system.

One must know the effects of various moderators on ^{252}Cf neutrons in detail to design tailoring assemblies for particular assay applications. To demonstrate the ability of various moderators to tailor the neutron flux, Fig. 1 shows the calcu-

Fig. 1. Calculated thermal-neutron fluxes (neutron energies < 0.1 eV) vs radius produced by a ^{252}Cf source in a spherical composite moderator assembly with various moderators in its midregion.

lated* thermal-neutron flux as a function of radius from a ^{252}Cf source. The source is at the center of nested spheres (0.3-m-outer radius) of composite moderator materials CH_2 , D_2O , Be, and C, which have large moderating ratios, chosen to give a maximum thermal-to-fast flux ratio near the outer regions of the moderators. The thermal-to-fast neutron flux ratio is indicated in Fig. 2, which shows the calculated ^{235}U : ^{238}U fission ratio as a function of radius for the same moderator configurations. Replacing CH_2 with ZrH_2 in the center region yields similar curves in both figures. These large ^{235}U : ^{238}U ratios ($\sim 10^5$) show that it is possible to interrogate the fissile component (^{235}U)

* We used the results of one-dimensional discrete-ordinate calculations with 25-group cross sections containing two thermal groups. Three-dimensional Monte Carlo calculations agreed very well with the one-dimensional results.

Fig. 2. Calculated ^{235}U : ^{238}U fission ratios (equal masses of both isotopes) vs radius for a ^{252}Cf source in a spherical composite moderator assembly with various moderators in its midregion.

in low-enrichment or natural uranium preferentially, using a relatively high-energy neutron (2-MeV) source such as ^{252}Cf . In addition, the signal-to-background ratio for a detector biased near the ^{238}U fission threshold is proportional to the ^{235}U : ^{238}U ratio (the signal comes from fissions induced in the ^{235}U sample). Therefore, such composite assemblies represent a significant improvement over a one-region moderator for certain assay applications.

B. The ^{252}Cf Fuel-Rod-Assay System

We used the preceding moderator calculations to optimize design of a ^{252}Cf moderator system for assaying the fissile content of low-enrichment uranium or plutonium fuel rods and other small samples. Thermalized neutrons from a 100- μg ^{252}Cf source interrogate the fuel material, and the resultant high-energy (~ 2 -MeV) neutrons from the induced fission reactions are counted using energy-biased (~ 0.5 -MeV) ^4He -gas recoil detectors.

Figure 3 shows a cut-away model of the moderator and shield assembly with the ^{252}Cf

Fig. 3. Cut-away model showing the composite moderator assembly for the four-channel version of the ^{252}Cf fuel-rod-assay system.

source at the center and the ^4He -filled fast-neutron detectors near the sample holes. The ^{252}Cf source is surrounded by a 20-mm-thick tungsten* shell to take advantage of its relatively large inelastic-scattering cross section and also to partially remove the gamma rays emitted in the spontaneous fission of the ^{252}Cf . The adjacent layer of ZrH_2 (40-mm-thick) further moderates the source neutrons by virtue of its large slowing-down power ($\xi\Sigma_s \approx 1.4$); the ZrH_2 performs the same moderator function as the CH_2 core in Fig. 1. The zirconium also helps to reduce the fast-neutron flux by inelastic scattering and to partially remove source gamma rays. The central hydrogenous core (ZrH_2 plus $\sim 20\%$ CH_2) is held to a 60-mm outer radius because capture of thermal neutrons by the hydrogen becomes excessive if more hydrogen is added to the central region. The surrounding 140-mm-thick D_2O further moderates the source neutrons without significant capture of the thermalized neutrons because of the large value of its moderating

* The tungsten was an alloy containing 90% tungsten, 6% nickel, and 4% copper.

ratio ($\xi \Sigma_s / \Sigma_a \approx 2000$ for 99.8% enriched heavy water). The outer layer of CH_2 serves primarily as a neutron reflector but also provides additional neutron shielding at the exterior of the system. A thin layer of polyvinyl chloride with lithium outside of the CH_2 captures $> 80\%$ of the thermal neutrons before they leak into the outer shield. This reduces the exterior gamma-ray dose, because, unlike neutron capture in boron, capture of neutrons in lithium does not produce any appreciable secondary gamma rays.

Irradiation channels to accommodate the fuel pins or small samples to be assayed are located in the outer cylindrical layer of CH_2 at a radius of 205 mm (Fig. 3) where the signal-to-background ratio reaches its maximum, as indicated in Fig. 2. The calculated average neutron spectrum in the fuel channel, shown in Fig. 4, is predominantly thermal. The neutron spectrum is not sensitive to nominal 5°-C variations in room

Fig. 4. Calculated neutron spectrum in the center of an irradiation channel of the ^{252}Cf fuel-rod-assay system.

temperature (20°C) because the heavy-water density changes by less than 0.1%, and the CH_2 density remains virtually unchanged. The calculations predict a cadmium ratio of 192 for ^{235}U at the center of the irradiation channel, which agrees with the experimental value of 188 ± 13 (1σ).

The fabricated ^{252}Cf moderator and shield is shown in Fig. 5, and the complete assay system optimized for power-reactor fuel rods is shown in Fig. 6. This system includes a 4- or 6-channel rod-handling mechanism, as well as data-acquisition, reduction, and printout equipment. The programmable calculator and interface unit, shown in Fig. 7, make it possible to reduce the raw data, rapidly print out the assay value, and automate the measurement procedure for real-time in-plant applications. Because the high-energy neutrons from the ^{252}Cf source which reach the detector have nearly the same spectrum as the signal neutrons from fissions in the sample, we used

Fig. 5. Moderator and shield assembly for ^{252}Cf fuel-rod-assay system. A pressurized-water reactor fuel rod is shown protruding from one of the six assay channels.

Fig. 6. Complete ^{252}Cf fuel-rod-assay system with six-channel rod-handling mechanism, and automated data acquisition, reduction, and printout. Assay precision $\sim 1\%$ for fissile content of power-reactor fuel rods; assay time ~ 40 sec for six simultaneous channels.

the background rate in a feedback loop to automatically stabilize the system response to source decay and long-term electronic drifts.

C. In-Plant Operation

The complete ^{252}Cf fuel-rod-assay system shown in Fig. 6 is controlled by a Hewlett-Packard 9100B calculator that receives microswitch signals through an interface unit. The assay starts as the

rods enter the moderator and shield assembly and ends after the rods leave the assembly. At this time, the calculator software accumulates the data from all channels, reduces the data, and prints the results from each channel. In addition to the assay results, the ratio of the current ^{252}Cf background to the initial background in the ^4He detectors at the time of calibration, which provides a check on system stability, is printed for each channel following the measurement of the rods. The date and/or batch identification number is then printed from a manual data-entry unit, thus concluding the printout from the assay.

For in-plant applications of the ^{252}Cf assay system, one must determine the external neutron and gamma-ray radiation levels from the assay system. Table I lists the measured dose levels with a $100\text{-}\mu\text{g}$ ^{252}Cf source in the assay unit. As the normal working area exceeds 1 m from the ^{252}Cf shield, the shield was designed to reduce the total dose rate in this area to < 2 mrem/h as shown in Table I.

Fig. 7. Programmable calculator, interface, and printout unit of ^{252}Cf fuel-rod-assay system. The electronics include modular amplifiers and scalars for six independent data channels.

TABLE I

NEUTRON AND GAMMA-RAY DOSE RATES
NEAR THE ^{252}Cf FUEL-ROD-ASSAY SYSTEM

Distance From Shield (m)	Measured Dose (100 μg ^{252}Cf)	
	Neutrons (mrem/h)	Gamma Rays (mrem/h)
0.3	3	7
0.6	2	4
0.9	0.5	1
1.5	0.4	0.8

Fuel-rod-assay systems (either the 4- or 6-channel model) have been introduced into both a boiling-water reactor (BWR) and a pressurized-water reactor (PWR) fuel-fabrication plant for evaluation. Calibration and operation of each assay system were as described in the following sections. Assay of a group of four or six fuel rods required ~ 37 sec, with an additional 40 sec for background standardization. This is equivalent to assaying a maximum of 180 rods/h in the four-channel system. During in-plant operation, rod handling and identification decreases the effective throughput to 400-600 rod assays in an 8-h shift, or about 1 rod assay/min. During the evaluation period, over 20 000 BWR and PWR fuel-rod assays were performed, and the calibration remained stable for periods exceeding six months.

III. UNIQUENESS AND DEPENDENCE OF THE BACKGROUND-CORRECTED RESPONSE TO THE FISSILE CONTENT OF THE FUEL ROD

Several aspects of the assay must be considered to ensure that the increase in the measured response above the background is due only to the fissile components in the fuel rod. This section will show that the background-corrected response is, indeed, unique to the fissile content of the fuel rod.

A. Response to Natural or Induced Gamma Rays

One possible response or signal source from materials other than the fissile isotopes is the natu-

ral or induced gamma-ray activity in the fuel rod. Experiments have shown that there is no significant increase in response rate from gamma-ray sources in either the fuel (Pu or U) or cladding. This is not surprising because the ^4He detectors operate in a gamma-ray field of about 250 mrem/h from a 100- μg ^{252}Cf source with negligible gamma-ray response above the operating bias. Figure 8 shows that the ^4He detectors could operate in a gamma-ray field of more than 1 R/h.

B. Neutron Backgrounds from the Fuel Rod

The neutron background from a low-enrichment uranium fuel rod has been undetectable in the presence of the normal ^{252}Cf background. This is to be expected, because the spontaneous fission rate of ^{238}U is roughly 7 fissions/kg/sec. Thus, fuel rods that contain roughly 4 kg of

Fig. 8. Measured gamma-ray sensitivity of ^4He detectors (9-atm pressure) for different gamma-dose levels at the detector. Also shown for comparison is a fission-neutron source pulse-height distribution.

low-enrichment uranium emit only about 70 spontaneous-fission neutrons/sec/rod, and only 1/10 of these are in the detector region at any one time. A comparable number of neutrons arise from oxygen (α, n) reactions in this fuel. The total is still roughly five orders of magnitude less than the induced signal. Because the detector efficiency times the solid angle subtended at the fuel rod is $\sim 0.09\%$, the rod contributes ~ 0.01 counts/sec to the net signal rate (~ 1000 counts/sec). In an attempt to measure this effect, we removed the ^{252}Cf source and placed a BWR fuel rod in the assay unit. We observed no significant increase (< 0.1 counts/sec) in the ^4He detectors.

To examine this effect with plutonium fuel, we placed a BWR-type plutonium-recycle fuel rod containing 2.34% Pu ($\sim 12\%$ ^{240}Pu) in the system with no ^{252}Cf present; the background rate increased to ~ 2 counts/sec, caused by the spontaneous fission of ^{240}Pu [~ 1040 n/sec ($g^{240}\text{Pu}$)]. We estimate that the neutron-emission rate from this plutonium rod in the detector region was ~ 2000 n/sec. Thus, the expected counting rate in the detector of ~ 1.8 counts/sec agrees well with the measured value. This rate is only 0.1% of the net signal rate from the same rod with 100 μg of ^{252}Cf in the assay unit and is included as part of the calibration. Thus, the system is not sensitive to the spontaneous fission neutrons of ^{240}Pu in a plutonium-recycle fuel rod.

Other sources of naturally occurring high-energy neutrons in a fuel rod are the delayed neutrons from ^{17}N formed by $^{17}\text{O}(n, p)$ and $^{18}\text{O}(n, d)$ fast-neutron reactions in oxide fuel. These reactions have 7.9- and 13.8-MeV thresholds, respectively, higher than at least 99% of fission-neutron energies. Also, the abundances of ^{17}O and ^{18}O are 0.037 and 0.204%, respectively, in normal oxygen; therefore, the delayed-neutron contribution is eight orders of magnitude lower than the signal in oxide fuel and is thus totally negligible.

C. Response from Fissions in the Fertile Component(s)

Neutrons from fissions in the fertile component(s) could produce a significant response, depending upon the interrogating neutron spectrum. Fertile materials have fission thresholds at about 0.8 MeV; therefore, one important criterion for the ^{252}Cf moderator configuration is maximization of the fissile:fertile fission ratio. Calculations (see Figs. 2 and 4) predict that a $^{235}\text{U}:^{238}\text{U}$ fission ratio of 10^5 is obtained at the fuel-rod channel position when heavy water is used as the moderator in the middle region. This was confirmed by measurements at the center of a channel which produced a value of $7 \pm 4 \times 10^4$ (1 σ). Thus, only 0.05% of the fissions are from the ^{238}U in a 2% enriched rod. (Even for a natural-uranium rod, ^{238}U contributes only 0.14% of the response, and this increment would be included in the standard rod responses.) This result is insensitive to moderator temperature, because the density of heavy water changes by less than 0.1% for normal room-temperature variations.

The fission neutrons born in the rod will cause fast-neutron multiplication in both the fissile and fertile species, resulting in a small amplification of the fission-neutron signal (1.0-1.5% for light-water reactor (LWR) fuel). This amplification is reasonably independent of rod enrichment, because ^{235}U and ^{238}U have comparable fission-neutron-production cross sections $\bar{\nu}\sigma_f$. The thermal-neutron flux around the rod caused by thermalization of the fission neutrons emitted from the rod increases the fission response by less than 5%. Thus, the fission-neutron response is proportional to the fuel rod's fissile content alone.

D. Neutron Interaction with the Cladding

Variations in cladding thickness within manufacturing tolerances will change the assay because of neutron interaction with the cladding. Figure 9 shows the calculated relative change in

Fig. 9. Calculated fractional change in response of a 3% enriched fuel rod as a function of thickness of zircaloy-2 and Type 304 stainless steel cladding.

the response for 0- to 1-mm-thick cladding (with maximum impurities) of zircaloy-2 and Type 304 stainless steel. This change is linear over the range examined. Zircaloy-2 changes the response by $-1.14\%/mm$ ($-0.029\%/mil$) of cladding, whereas stainless steel, which has greater thermal-neutron absorption because of its iron content, changes the response by $-12.2\%/mm$ ($-0.31\%/mil$). Most LWR fuel rods are clad with zircaloy-2; therefore, for typical production variations in cladding thickness which are random about the thickness of the standard rod's cladding and ~ 0.03 mm, the absolute error should be a small fraction of the $-1.14\%/mm$ error.

E. Neutron Absorptions by Impurities in the Fuel

Impurities in the fuel could affect the measured fission-neutron response because of changes in the thermal-neutron absorption. By considering the effect of typical impurity concentrations on the thermal-absorption cross section of a 3% enriched LWR fuel rod, we found the change in the fission-neutron response to be less than 0.05%.

This change may be much greater in plutonium recycle fuel because the isotopes of plutonium

have large absorption cross sections. Variations in the isotopic composition of plutonium for a specified fissile loading would cause significant changes in the macroscopic absorption cross section of the fuel and thus in the measured response. Therefore, the standard fuel rods must have an isotopic composition that very closely approximates the unknown plutonium recycle rods for accurate assay results in a thermal neutron interrogation system.

The fissile content of LWR fuel rods that contain thermal neutron poisons cannot be measured directly because of the change in the thermal neutron absorption cross section; i.e., there is no unique combination of poison and fissile material to yield a given response. The response could, however, be used as a measure of the reactivity worth of a poisoned fuel rod in a LWR.

The presence of neutron absorption resonances in materials could affect the measurement by changing either the number of resonance fissions in the fissile component(s) in the fuel and/or the number of epithermal neutrons near the rod which will thermalize and cause fissions in the fuel. The measured cadmium ratio of ^{235}U at the center of the channel is ~ 190 ; therefore, $< 0.5\%$ of the fission neutron response from ^{235}U occurs in the resonance region. This is confirmed by the fact that the neutron flux in the resonance region is at least three orders of magnitude lower than in the thermal neutron region as shown in Fig. 4. Thus, resonance absorbers cannot measurably affect the epithermal ^{235}U response because few of the resonances will overlap.

A simple argument can be used to show that resonance absorptions from impurities do not change the magnitude of the interrogating thermal neutron flux. The total number of neutrons per cubic centimeter with energies between E and $E+dE$ is $n(E)dE = \Phi(E)dE/v(E)$, where $v(E)$ is the velocity of neutrons with energy E . The largest thermal neutron flux that could be produced by $n(E)dE$ occurs if all the neutrons slow down to

thermal energy E_T . This slowing down produces a thermal neutron flux of $\phi_T = [v(E_T)/v(E)] [\phi(E)dE]$. Examination of Fig. 4 shows that the contribution to the thermal neutron flux from neutrons with energies greater than 1 eV is at least three orders of magnitude lower than the thermal flux derived using the above expression. Thus, the reduction in magnitude of the thermal neutron flux from resonance absorbers will be a small fraction of this conservative estimate of 0.1%.

F. Effect of Crosstalk between Fuel Rods in Adjacent Channels

It is reasonable to expect that assay of a fuel rod in a given channel will be affected by the presence of fuel rods in other channels. This crosstalk between channels can occur in two ways in this system: (1) by a direct contribution from fission neutrons born in adjacent channels and counted in the given channel, and (2) by moderation of the fission neutrons from rods in adjacent channels which, in turn, cause fissions in a rod in the given channel. Measurements on the four-channel system showed that virtually all of the crosstalk resulted from the second mechanism; direct contributions to a channel from fission neutrons born in other channels were found to be insignificant. This is to be expected because of the excellent moderating materials (CH_2) between fuel channels. The crosstalk contribution to the response of a channel from its neighbors was < 1%. Thus, each channel can be considered independent, because the small crosstalk effects can be removed during calibration, as discussed in Sec. V-B.

IV. CALIBRATION

We have shown that an increase of the measured response above background caused by a fuel rod in an irradiation channel is due only to the rod's fissile content. Standard rods of known fissile content whose characteristics approximate the unknowns to be assayed as closely as possible are required to calibrate the increase in the measured response of an unknown rod. The calibration-curve

approach in which the mean response(s) of the standard(s) is used to determine the calibration parameter(s) is discussed here. The hypothesis-testing approach is discussed in Appendix C.

A. Types of Calibration

Two basic types of calibration can be performed on the ^{252}Cf fuel-rod-assay system, depending on its intended use. The first assumes that rods of the same nominal enrichment will occupy all channels during an assay. The second assumes that fuel rods of different enrichments will be assayed randomly. The major difference in the two approaches involves crosstalk: in the second case, the crosstalk to each rod is not known exactly because the enrichments of the neighboring rods can differ.

The first type of calibration is the most accurate and is preferred. The second is useful for inventory of a large number of rods that cannot be conveniently grouped by enrichment. The total fissile mass of a group of rods would be known to essentially the same accuracy in either case (the average crosstalk for the second case can be computed); however, the assay results for the fissile content of an individual rod would be more accurate using the first type of calibration, unless the crosstalk is routinely measured and the data are corrected.

B. The Standard Rods

The accuracy of the assay is a function both of the uncertainties in the fissile content of the standards (assuming that the expected value of the fissile mass of the standards is not biased with respect to the true fissile content) and of how well the standards approximate the unknowns to be assayed. The uncertainty in a standard rod's mass can be determined by propagating the uncertainties in quantities required to compute the fissile mass: the weight of the fuel column, grams of fuel per gram of sample, and the enrichment(s) of the fissile isotope(s). Clearly, the uncertainties in the values of these components must be minimized to

produce the most precisely known standard mass. A simple check of whether the standards form a consistent set is to be sure that the ratio of fissile mass-to-enrichment is the same for all standards to within their respective uncertainties.

In addition to an accurately known mass, the standards must have the same characteristics as the unknowns to be assayed. Specifically, the standards should have the same fuel-column length, cladding material and thickness, fuel-pellet shape (dished or undished) and diameter, fuel density, and matrix material. Preferably, the enrichment of the fissile isotope(s) will be the same in the standards as in the unknowns; however, because a functional form that adequately describes the change in response with fissile mass is available (see the next section and Appendix A), perfectly matched enrichments are not essential. As a minimum requirement, the enrichments should bracket those to be assayed, so that extrapolation is not required. These points are discussed more fully in the next section.

C. The Response Function of the ^{252}Cf Fuel-Rod-Assay System

Assaying an unknown rod requires a mathematical function or model that adequately describes the behavior of the mean background-corrected response as a function of fissile mass (enrichment); no other property of the rod is allowed to change. This model must interpolate correctly between standard-rod mean-response points to assay unknowns whose enrichments differ from those of the standards. The mathematical expression used to convert the background-corrected response R (counts/sec) of the thermal ^{252}Cf fuel-rod-assay system to the fissile content U (grams) of a fuel rod is

$$R = a(1 - e^{-bU}) \quad (1)$$

where a and b are the calibration parameters. This function provides a simple semiempirical means of accounting for thermal-neutron absorption in the fuel rod, which is dependent on U . We investigated the theoretical basis for this function

(see Appendix A) to determine its validity for interpolation between, and extrapolation from, calibration points obtained with standard rods and to search for an improved close-form function. The study showed that the following calibration curve should be used when four or more standard enrichments are available:

$$R = a(1 - ce^{-bU}) \quad (2)$$

The exponential term reflects the thermal-neutron self absorption within a rod. The parameter a is the proportionality constant incorporating neutron-source strength, absolute detection efficiency, multiplication (thermal and fast) effects, rod-surface area, and the fraction of the absorptions in the fuel which are in the fissile species. The parameter b is proportional to the product of the fuel's macroscopic absorption cross section and radius.^{*} The parameter c , which represents the probability of absorption in the nonfissile species of the fuel, is assumed to be constant because it does not vary appreciably over the range of 1 to 5% enrichments of the fissile component(s). From physics considerations, c should be 0.9-1.0 for typical LWR fuel (see Appendix A). Both expressions have the correct asymptotic behavior for high and low masses: a linear relationship for small U and a constant for large U . The slope of each decreases monotonically as U increases.

Note that a quadratic or higher-order polynomial, while having no theoretical basis, could be used as an approximation to the more proper mathematical expression, Eq. (1) or (2); however, there are some drawbacks to this approach. The polynomial does not have the proper asymptotic behavior at high masses. In addition, all

^{*} Note that b is also a function of the flux depression in the moderator caused by the presence of the fuel rod; i. e., as the diffusion length of thermal neutrons in the moderator increases, both the flux depression and b decrease. For example, the calibration curve for fuel rods measured in graphite would be more nearly linear than that for measurements in CH_2 . (See Appendix A.)

polynomials of order higher than a quadratic may not have a monotonically decreasing slope as U increases; thus, the higher order polynomials would not be expected to interpolate and extrapolate well or to predict the slope properly at a specified mass. In other words, the shape of a polynomial beyond a quadratic is not fixed as is the shape of either Eq. (1) or (2); therefore, such a polynomial could mask a "bad" standard point used for calibration by adequately fitting the data.

Both Eqs. (1) and (2) require more than one standard mass. If only one standard is available for a group of unknowns, neither equation can be used. As a first approximation, a straight line through the origin could be used when the sample is very dilute and $R \approx abU$. If this is not the case, the straight line's slope will be too large and the spread in the measured masses will be underestimated. A better approach is to estimate one of the parameters (e. g., b might be approximately known from past experience) and use the standard to determine the other. Generally, the number of standards should at least equal the number of parameters plus one to assess the adequacy of the functional model.

To estimate the ability of Eqs. (1) and (2) to interpolate between, and extrapolate from, the standard points, we performed several weighted least-squares fits to BWR standards data, omitting one or more of the enrichments. The resulting curves for each equation were compared to the same curve fit to all of the data. Tables II and III show the results for Eqs. (1) and (2), respectively. The nominal enrichments of 0.71, 1.70, 2.10, and 2.79% are denoted by the numbers 1, 2, 3, and 4, respectively. The four responses represent the measured responses of the four standards in channel one, so the reference curve obtained from a fit to all four sets (1-2-3-4) should be accurate in predicting masses of ^{235}U at these responses. When either of the two middle enrichments (set 2 or 3) is deleted, Eq. (1) interpolates to within 0.2% of its reference curve. In comparison, Eq. (2) interpolates to within 0.04% of its reference curve.

The errors caused by extrapolation are predictably larger than those caused by interpolation, as is verified by Tables II and III. If the extrapolation trend exhibited in the tables is typical, then Eqs. (1) and (2) can be expected to overestimate a mass falling outside the range of the

TABLE II
ERRORS IN INTERPOLATION AND EXTRAPOLATION BY
 $R = a(1 - e^{-bU})^*$

Response (counts/sec)	^{235}U Mass (g)	Standard Enrichment Sets Used in Fit (% Error in ^{235}U Mass Compared to 1-2-3-4 Fit)**							
		1-2-3-4	1-2-4	1-3-4	1-4	1-2-3	1-2	2-3-4	3-4
450	28.408		-0.12	-0.18	-0.49	-0.37	-0.50	0.62	0.78
900	66.696		-0.11	-0.12	-0.37	0.07	0.22	0.32	0.41
1050	83.093		-0.11	-0.08	-0.31	0.29	0.60	0.17	0.22
1250	109.579		-0.11	-0.01	-0.19	0.71	1.30	-0.11	-0.12
Total: 287.776			Interpolation				Extrapolation		

* $a = 1991.49$ and $b = 0.00901613$ for the 1-2-3-4 data set.

** % Error = $100(\text{Mass}_{x-x-x} - \text{Mass}_{1-2-3-4}) / \text{Mass}_{1-2-3-4}$.

TABLE III
ERRORS IN INTERPOLATION AND EXTRAPOLATION BY
 $R = a(1 - ce^{-bU})^*$

Response (counts/sec)	²³⁵ U Mass (g)	Standard Enrichment Sets Used in Fit (% Error in ²³⁵ U Mass Compared to 1-2-3-4 Fit) **			
	<u>1-2-3-4</u>	<u>1-2-4</u>	<u>1-3-4</u>	<u>1-2-3</u>	<u>2-3-4</u>
450	28.265	-0.01	0	0.01	-0.03
900	66.878	-0.04	0.02	-0.01	0.01
1050	83.251	-0.04	0.01	0.03	0.01
1250	109.441	-0.03	-0.01	0.14	0.01
<u>Total: 287.835</u>		<u>Interpolation</u>		<u>Extrapolation</u>	

* $a = 2040.13$, $b = 0.00861543$, and $c = 0.994332$ for the 1-2-3-4 data set.

** % Error = $100(\text{Mass}_{x-x-x} - \text{Mass}_{1-2-3-4}) / \text{Mass}_{1-2-3-4}$.

standard masses on the high side; e. g., extrapolating from 83.1 to 109.6 g of ²³⁵U using Eq. (1) leads to an assay error of 0.71%; the error is 1.30% for extrapolation from 66.7 g. For comparison, the Eq. (2) error in extrapolation from 83.1 to 109.6 g of ²³⁵U is 0.14%, a significant improvement.

The data from all four sets were also fitted with two four-parameter functions: a cubic equation and Eq. (A-4) in Appendix A. The ²³⁵U masses predicted at the four responses by both four-parameter functions differed by < 0.02% from the ²³⁵U values shown in Table III. Assuming that these values are correct, Eq. (1) differs from the correct masses by up to 0.5%. If the spread in the standard masses is reduced by considering the 1-2-3 data set, the maximum error from Eq. (1) at responses of 450, 900, and 1050 counts/sec shrinks to 0.2%. If we consider the 2-3-4 data set which reduces the standard mass range even further, the maximum deviation of the predicted mass at responses of 900, 1050, and 1250 counts/sec derived using Eq. (1) from the correct mass derived using Eq. (2) is only 0.04%. This deviation indicates that Eq. (1) does have some difficulty in predicting the correct mass at low mass values

(enrichments), as mentioned in Appendix A. The parameter c in Eq. (2) has its largest effect in the low mass value region which explains the success of Eq. (2) in predicting the correct mass throughout the range.

Both Eqs. (1) and (2) fit the standard mass points satisfactorily and provide a good model for interpolation and extrapolation. Although the two-parameter function, Eq. (1), accounts for thermal-neutron absorption remarkably well, when four or more enrichment standards are available, Eq. (2) can be used to provide an even better response function.

To compare the ability of Eqs. (1) and (2) to interpolate and extrapolate with that of a quadratic equation, we made a similar investigation by fitting a quadratic equation to the data. The results are shown in Table IV. The predicted masses at the four responses agree to within 0.25% of the values shown in Table III. The interpolation errors (relative to the four-parameter fits) of this three-parameter function are an order of magnitude larger than the error for Eq. (2), also a three-parameter function. The errors for extrapolation are even worse. Therefore, we highly recommend that Eq. (2) be used instead of a quadratic equation

TABLE IV
 ERRORS IN INTERPOLATION AND EXTRAPOLATION BY
 $R = aU^2 + bU + c$ *

Response (counts/sec)	²³⁵ U Mass (g)	Standard Enrichment Sets Used in Fit (% Error in ²³⁵ U Mass Compared to 1-2-3-4 Fit)**				
		1-2-3-4	1-2-4	1-3-4	1-2-3	2-3-4
450	28.236		0.04	0.02	0.07	-5.75
900	67.036		-0.27	0.27	-0.25	-0.23
1050	83.098		-0.25	0.21	0.21	0.20
1250	109.440		-0.10	-0.04	1.78	0
Total: 287.810		Interpolation		Extrapolation		

* $a = -0.0411838$, $b = 15.5217$, and $c = 44.5641$ for the 1-2-3-4 data set.

** % Error = $100(\text{Mass}_{x-x-x} - \text{Mass}_{1-2-3-4}) / \text{Mass}_{1-2-3-4}$.

when a three-parameter function is used as the calibration curve for a thermal-neutron interrogation system. Note that all three functions yield essentially the same value for the total mass at the four responses for the 1-2-3-4 fit. This fact indicates that the total assayed mass of a range of unknown masses is likely to be more accurate than the total for a single unknown mass.

If the range of standard rod enrichments is so large that neither Eq. (1) or (2) provides an adequate fit to the data, two or more calibration curves of their form could be used to describe various parts of the standards' enrichment range.

D. Calibration Procedure

To calibrate the assay system, one must determine the mean response of each standard rod as precisely as possible. To this end, the response of each standard j is measured in each channel i often enough for the statistically estimated standard deviation of the mean response to be reduced to 0.1% or less of the mean. These mean responses R_{ij} , along with the mass of each standard U_j and the corresponding estimated variance of each, s_R^2 and s_U^2 , form the data set from which the calibration parameters are determined; i. e., $R_{ij} = a_i(1 - c_i e^{-b_i U_j})$. At least three standard enrichments are required. One should use at least one more standard than there are calibration

parameters to better estimate the uncertainties in the parameters and to assess the adequacy of the function (see Sec. V-D). This procedure produced a different calibration curve for each channel. Because the irradiation channels are symmetric about the source (the neutron spectrum is the same in each), the expected values of parameters b and c should be the same for all channels. (This is not a good assumption if the D_2O tank is not full.) Thus, more precise values of these parameters can be obtained by solving the system of equations $R_{ij} = \frac{a}{k_i}(1 - c e^{-b U_j})$ where j ranges from 1 to the number of standards, i ranges from 1 to the number of channels, and the k_i s are channel-normalization parameters. The channel with the highest count rate has $k = 1$. Determination of the k_i s by fitting requires a minimum of $i + 2$ distinct standard masses, but is more precise and is the preferred method. We will fully describe the method for finding the $i + 2$ parameters in the second technique, because the first approach is merely a simplification.

The parameters a , b , and c and the k_i s are obtained by fitting the calibration curve, Eq. (1) or (2), to the data points (U_j, R_{ij}) using standard least-squares techniques; however, the procedure depends on the number of standards and the computing capability available. Because both Eqs. (1)

and (2) are nonlinear in the parameter b , an iterative technique is required to obtain a solution to the least-squares equations. If only two standards are used, the channel-normalization factors k_i must be obtained "by hand" by normalizing the mean response of the highest mass standard in each channel to its response in the channel with the largest count rate. The function $R = a(1 - e^{-bU})$ is then fitted to the means of the normalized responses $\overline{k_i R_{ij}}$ to obtain the shape (parameters a and b) of the calibration curve for all channels. This technique, of course, yields a "perfect" fit of the curve to the resulting two data points but gives no information about the quality of the fit. If three standard masses are available, one can use the function $R_{ij} = \frac{a}{k_i}(1 - e^{-bU_j})$, obtaining the scaling factors k_i by hand normalization.

If 1 + 2 or more distinct standard masses are available, one can perform a weighted fit of the system of equations $R_{ij} = \frac{a}{k_i}(1 - e^{-bU_j})$ to the data to obtain the parameters a , b , and c and the k_i s simultaneously. This can be accomplished by treating the channel number as a second independent variable used to determine with which k_i a given data point is associated. Also, the uncertainties in both the responses and the masses can be included in the weights used in the fit.² This technique will produce the "best" calibration curve and can give information about how precisely the various parameters are known; however, access to a computer and a general least-squares fitting code³ is required.

If the scaling factors k_i are obtained by normalizing the data before fitting, the fitting can be done using a programmable desk-top calculator, particularly if an unweighted fit is acceptable. If the weights of all the data points are approximately equal (if the variances of both responses and

masses are equal for all standards), an unweighted fit is probably sufficient.

To investigate how variations in the fitting procedure affect the calibration curve, we used the

following 5 possibilities to fit the data from a set of 16 BWR fuel-rod standards (four rods at each nominal enrichment).

- I. $R = \frac{a}{k_i}(1 - e^{-bU})$; fit for a , b , c , and the k_i s; weights include s_R^2 and s_U^2 , the estimated variances of R and U , respectively.
- II. $R = \frac{a}{k_i}(1 - e^{-bU})$; fit for a , b , and the k_i s; weights include s_R^2 and s_U^2 .
- III. $R = \frac{a}{k_i}(1 - e^{-bU})$; fit for a and b ; the k_i s obtained by normalizing the data before fitting; weights include s_R^2 and s_U^2 .
- IV. $R = \frac{a}{k_i}(1 - e^{-bU})$; fit for a and b ; the k_i s obtained by normalizing the data before fitting; weights include only s_R^2 (i. e., determination of standard mass is assumed to be error-free).
- V. $R = \frac{a}{k_i}(1 - e^{-bU})$; fit for a and b ; the k_i s obtained by normalizing the data before fitting; weights = 1.0.

The above are listed in decreasing order of preference.

The values of each calibration parameter obtained by these five procedures are compared in Table V. Also given for each of the resulting calibration curves is the average of the deviation between each standard mass and the curve, $\bar{\Delta}$. This deviation is defined as

$$\Delta = \left| \frac{U - U_j}{U_j} \right|,$$

where U_j is the mass of standard j , and U is the mass computed from the mean response $\bar{R}$ using the inverse relation

$$U = -\frac{1}{b} \ln \left[\frac{1}{c} \left(1 - \frac{k_i \bar{R}}{a} \right) \right],$$

or

$$U = -\frac{1}{b} \ln \left(1 - \frac{k_i \bar{R}}{a} \right), \text{ if } c = 1.$$

This average deviation is only one measure of the quality of the calibration curve; however, because it is the average amount by which the curve "misses" the standard mass of a well-known

response and because the curve will be used to deduce an unknown mass by measuring its response, it is a quantity of particular interest. Note that the average deviation is approximately equal to or less than the uncertainty (one standard deviation) for any one of the standard masses (0.2-0.5%). So, this data-point spread is probably primarily due to the uncertainties in the standards themselves, not to the function and/or fitting procedure.

Another possible measure of the quality of the calibration curve is the error the curve introduces into the total ^{235}U mass in the standard rods, i. e., the difference between the sum of the assay values for each rod in each channel and the total mass "known" to be in the standards. This difference, expressed as a fraction of the total "known" standard mass and labeled ΔT , is also given in Table V. It is always less than 0.1%, but this excellent result is not unexpected, because the calibration curve is obtained by techniques designed to minimize the square of the differences for this combination of standard masses. The dif-

ference between the assay and "true" total ^{235}U mass for a different combination of rod masses would, of course, be expected to be different. It is difficult, if not impossible, to predict either the magnitude or sign of this difference without detailed knowledge of the combination of rod masses to be assayed; the difference, however, is expected to be less than a few tenths of a percent. See Sec. IV-C for information on interpolation and extrapolation.

The final six entries in Table V give the "assayed" ^{235}U mass at four representative responses in the mass range of interest for BWR fuel rods, the total mass at the four responses, and the initial slope for each case. The difference between fitting procedures for any given response is small and, again, is of the same order as the uncertainty in the standard masses. The good agreement of the total mass at the four responses for all five cases emphasizes that the total assayed mass for a range of unknown masses is likely to be more accurate than the total at one unknown mass.

TABLE V
PARAMETERS OBTAINED BY VARIOUS FITTING PROCEDURES

	Fitting Procedure				
	I	II	III	IV	V
a	2053.69	1997.56	2000.72	2007.40	1999.43
b	0.008 529 13	0.008 985 55	0.008 971 86	0.008 917 18	0.008 973 85
c	0.993 577	1.0	1.0	1.0	1.0
k_1	1.0	1.0	1.0	1.0	1.0
k_2	1.104 206	1.104 229	1.104 755	1.104 755	1.104 755
k_3	1.155 815	1.155 816	1.156 544	1.156 544	1.156 544
k_4	1.099 239	1.099 247	1.099 559	1.099 559	1.099 559
$\bar{\Delta}$ (g)	0.17	0.23	0.23	0.22	0.23
ΔT (%)	0.97	0.03	0.06	0.05	0.01
U (1250)	109.242	109.383	109.256	109.306	109.351
U (1050)	83.187	82.999	82.931	83.028	82.992
U (900)	66.857	66.645	66.602	66.706	66.646
U (450)	28.243	28.406	28.399	28.399	28.413
Total U	287.529	287.433	287.188	287.439	287.402
Initial slope	17.4037	17.9492	17.9502	17.9003	17.9426

Fig. 10. Calibration curve for the ^{252}Cf fuel-rod-assay system. The solid curve has the form $R = a(1 - ce^{-bU})$.

The calibration curve is shown in Fig. 10. Only one curve is given because the five are indistinguishable on a plot with the scale used. Also shown are four of the calibration data points: only four points are shown because of overlap of the three remaining points at each nominal enrichment. The estimated uncertainties (two standard deviations) in both the ordinate and abscissa of each point lie well within the plot symbol. It is emphasized that the calibration curve allows only one physical characteristic, the enrichment, to vary. Thus, the curve will be invalid for fuel with a different length, density, diameter, etc. (see Sec. VI).

V. PRECISION CONSIDERATIONS

A. Effect of Variation in Drive-Motor Speed

Small variation in drive-motor speed should have no influence on the result of a rod

Fig. 11. Measured crosstalk to a channel from the adjacent channels as a function of the ^{235}U enrichment of the fuel rods in all channels. The solid curve has the form $R = a(1 - e^{-bU})$.

assay, assuming that the rod contains pellets of identical enrichment, because the response is constant per unit of fuel length, neglecting end effects. The scaler counts are divided by the time of the rod assay to obtain a count rate independent of small variations in the drive-motor speed (see Appendix B). Small errors would be introduced in the assay of a mixed-enrichment rod because the response per unit length may not then be constant during speed variation.

B. Crosstalk Corrections

Because the magnitude and sign of crosstalk (see Sec. III-F) can be measured, it is possible to make an appropriate correction in the final data analysis. This correction is required only if the system is used to assay rods of different enrichments simultaneously. Because the crosstalk originates from the neutrons induced in the fuel-rod irradiation channels (fission reactions in the rods), it varies with enrichment in the same manner as the induced fission rate; i. e., crosstalk $\propto (1 - ce^{-bU})$. We measured the total crosstalk to a channel from the two adjacent channels (the other channels make a negligible contribution) for enrichments of 1.99% and 3.03%, and fitted the curve shown in Fig. 11 to the data. As an example of the use of this figure, assume that the average

standard-rod enrichment is 1.85% as compared with an average of 2.51% for the above two enrichments of reactor rods. Using the curve, the crosstalk effect is 0.65% for the standards and 0.81% for the reactor rods. Thus, the average bias in the rod measurement is $(0.16 \pm 0.02)\%$ where the error (1 σ) was determined from the uncertainty in the crosstalk curve. As all of the rods have been measured with four rods in the channels, it would be appropriate to reduce the cumulative ^{235}U total by 0.16%.

An assumption implicit in this argument is that all the channels used during calibration contain a fuel rod. If one or more of these channels are empty during an assay, the expected assay value will be low by the amount of crosstalk which is not present. A figure similar to Fig. 11 can be used to estimate the amount. The net signal rate in each detector predicts the magnitude of the crosstalk correction from that channel to each of the other channels. With careful calibration and sufficient data-handling capacity, the crosstalk correction could be made on-line during data collection and reduction. The mean response R_{ij} mentioned in Sec. IV-D would then be corrected for both background and crosstalk.

C. System-Stability Measurements

The assay-system stability can be investigated by means of the ^{252}Cf background in the ^4He detectors which involves two components, the decay of the ^{252}Cf neutron source and the instability of the electronics. The half-life of ^{252}Cf is 8.36×10^7 s; hence, the background decreases about 1% every 2 weeks (23%/yr). Operating experience has shown that electronic drifts are small ($< 1\%$), and backgrounds over a several-week period have fluctuated in the expected statistical manner. The system also has proved virtually independent of ambient temperature (4-40°C). The next section describes a method of tracking any long-term electronic drifts, as well as the natural decay of the source, and removing their effect on the assay.

D. Propagation of Counting Statistics

The mass of fissile material in any fuel rod is computed from the observed counting rates by means of

$$R = \frac{B_0}{B} (C - B) \quad (3)$$

and

$$U = -\frac{1}{b} \ln \left[\frac{1}{c} \left(1 - \frac{kR}{a} \right) \right] \quad (4)$$

where

R = background- and drift-corrected response rate,

B_0 = background rate at time of calibration,

B = "current" background rate,

C = observed count rate from the assay (scaler counts/time),

(a, b, c, k) = calibration parameters,

U = fissile content of the rod.

The parameter k is channel-dependent; a , b , and c are common to all channels (see Sec. IV-D).

The factor B_0/B is a scaling factor applied to $(C - B)$ and is included to correct for long-term drifts in the electronics and decay of the source. Because Eq. (4), which is the inverse of the calibration curve, Eq. (2), is smooth and not highly nonlinear in the region of interest, * the usual formula for the "propagation of error" in a single measurement may be used:⁴

$$\begin{aligned} \sigma_U^2 = & \left(\frac{\partial U}{\partial R} \right)^2 \sigma_R^2 + \left(\frac{\partial U}{\partial a} \right)^2 \sigma_a^2 + \left(\frac{\partial U}{\partial b} \right)^2 \sigma_b^2 + \left(\frac{\partial U}{\partial k} \right)^2 \sigma_k^2 \\ & + \left(\frac{\partial U}{\partial c} \right)^2 \sigma_c^2 + 2 \text{Cov}(a, b) \left(\frac{\partial U}{\partial a} \right) \left(\frac{\partial U}{\partial b} \right) + 2 \text{Cov}(a, k) \left(\frac{\partial U}{\partial a} \right) \\ & \times \left(\frac{\partial U}{\partial k} \right) + 2 \text{Cov}(b, k) \left(\frac{\partial U}{\partial b} \right) \left(\frac{\partial U}{\partial k} \right) + 2 \text{Cov}(a, c) \left(\frac{\partial U}{\partial a} \right) \left(\frac{\partial U}{\partial c} \right) \\ & + 2 \text{Cov}(b, c) \left(\frac{\partial U}{\partial b} \right) \left(\frac{\partial U}{\partial c} \right) + 2 \text{Cov}(c, k) \left(\frac{\partial U}{\partial c} \right) \left(\frac{\partial U}{\partial k} \right). \quad (5) \end{aligned}$$

* The more definitive requirement that the product of the second-order (and higher) derivatives and higher moments of the distribution of each parameter be small compared to the product of the square of the first derivative and the variance is also satisfied.

In this equation, σ_i^2 represents the variance of the i^{th} parameter and $\text{Cov}(i, j)$ is the covariance of parameters i and j . The first term on the right-hand side of Eq. (5) represents the uncertainty due to counting statistics, and the remaining terms account for the uncertainty in the assay due to the calibration curve. Equation (5) assumes that the properties of standard rods are representative of the unknowns so that there are no bias terms to be included.

Taking the partial derivatives of Eq. (4) and inserting the results into Eq. (5) yields the general expression for the variance of the fissile mass for a single assay. (See Appendix C for the general calibration-curve case.)

$$\begin{aligned} \sigma_U^2 = & \frac{k^2 \sigma_R^2}{b^2 (a - kR)^2} + \frac{k^2 R^2 \sigma_a^2}{a^2 b^2 (a - kR)^2} + \frac{U^2}{b^2} \sigma_b^2 \\ & + \frac{R^2 \sigma_k^2}{b^2 (a - kR)^2} + \frac{\sigma_c^2}{b^2 c^2} + \frac{2kRU \text{Cov}(a, b)}{ab^2 (a - kR)} - \frac{2kR^2 \text{Cov}(a, k)}{ab^2 (a - kR)^2} \\ & - \frac{2RU \text{Cov}(b, k)}{b^2 (a - kR)} - \frac{2kR \text{Cov}(a, c)}{ab^2 c (a - kR)} - \frac{2U \text{Cov}(b, c)}{b^2 c} \\ & + \frac{2R \text{Cov}(c, k)}{b^2 c (a - kR)}. \end{aligned} \quad (6)$$

The uncertainty in the total assayed mass of a large number of rods is treated in Appendix C.

The variances and covariances in Eq. (6) must be evaluated to obtain σ_U^2 . Estimates of the variances and covariance of the parameters are obtained in the fitting procedure.³

σ_R^2 is found from an equation similar to Eq. (5). In Eq. (3), B and C are independent (uncorrelated), so

$$\sigma_R^2 = \left(\frac{\partial R}{\partial B} \right)^2 \sigma_B^2 + \left(\frac{\partial R}{\partial C} \right)^2 \sigma_C^2. \quad (7)$$

B_0 is mathematically merely a reference point, and is defined to be known exactly. Performing the required partial derivatives of Eq. (3) yields

$$\sigma_R^2 = \left(\frac{B_0 C}{B^2} \right)^2 \sigma_B^2 + \left(\frac{B_0}{B} \right)^2 \sigma_C^2. \quad (8)$$

σ_R^2 involves the variances σ_B^2 and σ_C^2 , which can be estimated as follows from the scaler counts, assuming Poisson statistics.

The "current" background is a weighted average of the past and present backgrounds; i. e.,

$$B_i = \frac{1}{5} \left(4B_{i-1} + \frac{S_i}{\tau_B} \right),$$

where S_i/τ_B is the background rate measured after the i^{th} assay and B_i is the weighted average of the background to be used for the $i + 1$ assay. Use of this weighted average permits following a long-term drift in the electronics, but reduces the effect of a sudden large fluctuation in the background count. If a single background measurement is precise enough, however, the previous background rate S_i/τ_B can be used instead of a weighted average. If the S_i are all approximately equal to S , B_i may be represented by

$$B_i = \frac{S}{5\tau_B} \sum_{j=1}^i \left(\frac{4}{5} \right)^{i-j} + \left(\frac{4}{5} \right)^i B_0.$$

An estimate of the variance of S is S from Poisson statistics, so an estimate of the variance $\sigma_{B_i}^2$ is

$$\sigma_{B_i}^2 = \frac{S}{\tau_B^2} \left[\frac{1}{5} \sum_{j=1}^i \left(\frac{4}{5} \right)^{i-j} \right]^2.$$

As i increases, the quantity in brackets approaches unity; therefore, the variance of the "current" background may be estimated as

$$s_B^2 \approx \frac{S}{\tau_B^2} = \frac{B}{\tau_B}.$$

The count rate from a rod assay is given by

$$C = \frac{S_A}{\tau_A},$$

where S_A is the total counts accumulated from the rod during an assay plus the ^{252}Cf background in assay time τ_A . The estimated variance of C , assuming no error in the assay time τ_A , is

$$s_C^2 = \frac{S_A}{\tau_A^2} = \frac{C}{\tau_A}.$$

Equation (6) can be numerically evaluated to obtain s_U by using the above expressions, once the values of the operating characteristics are known.

If the precision of a single assay is dominated by the propagated counting statistics, Eq. (6) simplifies to

$$\sigma_U^2 = \frac{k^2 \sigma_R^2}{b^2 (a - kR)^2} = \frac{\sigma_R^2}{\left(\frac{ab}{k} - bR\right)^2}.$$

Note that if different calibration curves are compared on the basis of equal initial slopes (ab/k , assuming $c \approx 1$), the second form of the above expression shows that the curve with the smallest value of b will have the least uncertainty due to counting statistics for a given R ; i. e., the less linear the curve, the greater the uncertainty in the assay result. Substituting the expressions for s_R^2 and R , Eqs. (8) and (3), into the above equation, dividing both sides by U^2 , and taking the square root yields

$$\frac{s_U}{U} = \frac{k B_0 \sqrt{C \left(\frac{C}{\tau_B B} + \frac{1}{\tau_A} \right)}}{U b [aB - k B_0 (C - B)]}.$$

This expression for the fractional uncertainty in the fissile mass involves only the contribution from the counting statistics. Figure 12 shows the calculated fractional uncertainty for each of the four channels used for assaying PWR fuel rods in the six-channel system. This curve can also be generated experimentally by assaying each standard of mass U a number of times, estimating the standard deviation of the assays from the data, and plotting the s_U/U ratio. Figure 13 shows the results of such an experiment and calculations for one channel of the four-channel system measuring BWR fuel rods. The excellent agreement between the measured points and the computed curve indicates that the precision of the rod counter is determined effectively by counting statistics alone; i. e., no system instability increases the standard deviation appreciably.

We investigated the fractional uncertainty s_U/U in the PWR assay due to counting statistics in some detail, as a function of ^{252}Cf source decay (Fig. 14) and of assay time (Fig. 15). Figure 14 shows that the precision of a single assay does not

Fig. 12. Calculated fractional uncertainty (1σ) at the time of calibration due to counting statistics only, for the four channels of the fuel-rod-assay system used at a PWR fuel-fabrication facility (35-sec assay, 40-sec background count).

Fig. 13. Fractional uncertainty (1σ) in ^{235}U mass due to propagation of counting statistics vs ^{235}U content of the rod. The solid curve is computed for channel 1. The points represent the measured standard deviation of ~ 30 measurements of each standard in channel 1.

Fig. 15. Calculated fractional uncertainty (1σ) due to counting statistics only, for several assay times in the fuel-rod-assay system at a PWR fuel-fabrication plant (channel 1, 40-sec background count).

deteriorate seriously over a half-life of the ^{252}Cf source. As shown in Fig. 15, the assay precision depends strongly on assay time for times up to ~ 60 sec; however, little is gained by using times longer than ~ 120 sec for a fixed source size. A larger ^{252}Cf source would correspondingly decrease the assay time required to obtain a desired

precision. The assay time could be increased periodically to offset source decay and thus keep the precision due to counting statistics approximately constant--this should not require system recalibration because count rate, not total counts, is used (see also Appendix B).

To obtain a measure of the accuracy of the ^{252}Cf fuel-rod-assay system, the variances of the standard masses must be estimated and included in the estimates of the variances and covariances of the calibration parameters.² This can be done as follows. Consider the function D defined as

$$D \equiv R - \frac{a}{k} (1 - ce^{-bU}) \quad (9)$$

Using the "propagation of error" equation (e. g., see Eq. (5)), one can estimate the variance of D using

$$s_D^2 = s_R^2 + \left(\frac{abc}{k} e^{-bU} \right)^2 s_U^2 \quad (10)$$

An unweighted least-squares fit of D/s_D to the data can be performed to obtain the calibration parameters a , b , c , and k . The estimated variances and covariances of the parameters now include the uncertainty in the standard masses. When Eq. (6) is

Fig. 14. Calculated fractional uncertainty (1σ) due to counting statistics only, for several time increments after calibration at the fuel-rod-assay system at a PWR fuel-fabrication plant (channel 1, 35-sec assay, 40-sec background count).

evaluated, the square root of the resulting variance is interpreted as the standard deviation for the accuracy, not just precision, of the assay result. This, of course, assumes that the grams of fissile material listed for each standard is the actual amount present in the rod to within the stated uncertainties (i. e., there is no bias in the given standard-mass values), and the properties of the standards represent those of the unknowns.

To investigate how the precision with which the masses of the standards are known affects the precision of a single assay, we used the measured responses of four BWR standards (each having a different enrichment) to obtain a calibration curve for these rods in a single channel. The uncertainty in each standard (i. e., one estimated standard deviation, s_U) was varied from $s_U = 0$ to $s_U = 0.05 U$, where U is the mass of the standard. The results, shown in Fig. 16, vividly illustrate that the assay

Fig. 16. Calculated fractional uncertainty (1σ) in assayed ^{235}U mass vs ^{235}U content of the rod as a function of the fractional uncertainty s_U/U in the mass of the standard rods.

values become more uncertain as the standard masses are known with less precision. The standard masses should have a small enough standard deviation that the uncertainty in a single assay is due mostly to the propagated counting statistics. If the converse is true, the instrument is too precise for the standards, and one should try to obtain more precise standards. The standards used for the data in Fig. 12 had an $s_U \leq 0.005 U$ which is quite adequate for this assay system.

VI. GEOMETRIC AND LOADING VARIATIONS IN THE FUEL

A. Effect of Fuel-Length Variation

There are several geometric and loading variations in the fuel which cause assay errors. Consider, first, the variation in length of the fuel column in the rod when the position of measurement region remains unchanged. The fixed-length, variable-enrichment calibration curve generated for unknown rods is based upon standard rods of the same length, i. e., the nominal length of the unknowns. Therefore, in the strictest sense, the calibration curve is valid only for those rods that have a fuel region of this nominal length. The response from a rod of uniform enrichment is directly proportional to its length (see Appendix B); i. e., the response R_L of a uniformly enriched rod as a function of fuel length L is $R_L = R_0(L/L_0)$ where L_0 is the fuel length of a standard rod and R_0 is the response of a rod of fissile mass m_0 and length L_0 with the same uniform enrichment. This equation is represented by the dashed straight line in Fig. 17. In contrast, the fixed-length, variable-enrichment calibration curve (solid curve in Fig. 17) actually used for the assay is nonlinear because of thermal-neutron self-shielding in the fuel and flux depression in the moderator, which is a function of fuel enrichment, not fuel-column length.

Consider cases in which the fuel length is less than and greater than that of the rod with fissile mass m_0 . For fuel length $L_1 < L_0$, the response is defined to be $R_1 = R_0(L_1/L_0)$, as shown

Fig. 17. Response of hypothetical fuel rods of length l_0 , l_1 , and l_2 vs ^{235}U content m_0 , $m_0 l_1/l_0$, and $m_0 l_2/l_0$ of the rods.

in Fig. 17. The true mass of ^{235}U in this rod is $m_0(l_1/l_0)$, but the calibration curve predicts m_1 grams, $m_1 \leq m_0(l_1/l_0)$. The equality applies only for low ^{235}U enrichment where the calibration curve is linear. The smaller the slope of the calibration curve at the mass of interest (in this case, m_0), the greater the inequality between the actual and predicted mass of ^{235}U in the rod for a given fractional change in fuel length. Also illustrated in Fig. 17 is the response $R_2 = R_0(l_2/l_0)$ for a fuel length $l_2 > l_0$. The correct mass is then $m_0(l_2/l_0)$ as compared to m_2 grams predicted by the calibration curve. In this case, $m_2 \geq m_0(l_2/l_0)$. Again, the inequality increases as the slope of the curve decreases at the mass of interest, m_0 .

As an example, assume that the fuel length in a BWR production rod deviates from the length

of the standard by $\pm 1\%$. Using the point on a typical BWR calibration curve corresponding to a production rod with an enrichment of 3.03% and an assumed mass of 122 g of ^{235}U causes an overestimate of 0.88% (+1.07 g of ^{235}U) for a 1% increase in the fuel length. A 1% decrease in fuel length yields an assay error of -0.86% (-1.05 g of ^{235}U). The typical 0.4% production-length changes would reduce these errors proportionately. Similarly, the errors in the assayed value for a +1% and -1% change in the fuel length of 1.99% enriched rods containing 79 g of ^{235}U are +0.49% (+0.39 g of ^{235}U) and -0.48% (-0.38 g of ^{235}U), respectively. Thus, a 33% decrease in enrichment reduces the relative error caused by fuel-length variations by almost one-half, because of the increased slope of the calibration curve.

A 5% change in the fuel length of a 3.03% enriched rod would have ~96% chance of being detected in a single measurement, assuming a statistical uncertainty of 2.4% (2σ). A variation of $\pm 1\%$ would probably not be detected in a single assay; however, if the length variations are $\sim 0.4\%$ and random about the nominal length, the net assay error for many rods should be small, e.g., $< 0.01\%$ for the 3.03% enriched rods and $< 0.005\%$ for 1.99% enriched rods. Thus, small random fuel-length variations should have little effect on the assay of rods in a bundle or an inventory.

B. Effect of Mixed Enrichments in a Fuel Rod

We have discussed why the calibration curve is valid for only one fuel-column length, and the magnitude of the error involved if the length varies from that of the standards. A similar discussion is required to show that although the calibration curve is strictly valid only for a rod having uniform enrichment over its entire length, the error introduced into an assay is small for a rod containing two enrichments.

Assume that a rod has the nominal (standard) fuel-column length, but that its first third contains pellets of enrichment Z which, in a normal

uniformly enriched rod, would yield a mass of z grams of ^{235}U . The rest of the rod is filled with pellets of enrichment Y , which in a rod uniformly filled with these pellets would contain y grams of ^{235}U . This hypothetical mixed-enrichment rod would contain $(2y + z)/3$ g of ^{235}U with response $R_{\frac{1}{3}(2Y + Z)}$. The response per unit length of each fuel-column enrichment section is linear, neglecting end effects (see Appendix B); therefore, two-thirds of the response is composed of the slope associated with the y -mass rod and one-third is composed of the slope of the z -mass rod. The sum of the two responses of different slopes, which determines the point $[(2y + z)/3, R_{\frac{1}{3}(2Y + Z)}]$ on the dashed line between the points (y, R_Y) and (z, R_Z) , is depicted in Fig. 18. Because the sum of two linear functions is linear and the limiting points are (y, R_Y) and (z, R_Z) , the dashed line between these two points in Fig. 18 is the calibration for segregated mixtures of the two

Fig. 18. Response of a hypothetical fuel rod containing pellets having enrichment Y and Z vs ^{235}U content of the rod.

enrichments. Figure 18 shows that the calibration curve predicts g grams of ^{235}U for a response of $R_{\frac{1}{3}(2Y + Z)}$. Thus, the assay error is $g - (2y + z)/3$ grams of ^{235}U . This discussion neglects the end effects caused by the junction of differently enriched fuel regions. Such effects are negligible because there is only one junction for the two segregated enrichments. When the Z -enrichment pellets are intermixed with the Y pellets, the response per unit length of the Z pellets increases, because there is less thermal-neutron self-shielding at the end of any Z pellet adjoining a Y pellet. Similarly, the response per unit length of the Y -enrichment pellets decreases because of increased self-shielding resulting from adjoining Z pellets. The net effect of intermixing the pellets is to increase the response relative to that of segregated pellets (as in the above example), which decreases the assay error. If the two pellet enrichments were perfectly blended, the response would be R_M (see Fig. 18), which would yield the correct mass from the actual calibration curve. Thus, segregating pellets of a given enrichment represents the worst case in terms of assay error. Further, as the difference in the calibration-curve slope at the two enrichments Y and Z increases, the assay error increases.

A mixture of three or more segregated enrichments can be treated similarly; the linear response of each region based on its length can be summed to find the total response of the fuel rod. The mass predicted by the calibration curve can then be compared with the true mass, and an error can be assigned to the assay.

As an example of a mixture of two enrichments, assume that the first half of a production rod is loaded with 3.03% pellets (a full 3.03% rod is assumed to contain 122 g of ^{235}U) and the second half with 1.99% pellets (a full rod is assumed to contain 79 g of ^{235}U). (A half-and-half distribution of two enrichments will give essentially the maximum error in the total fissile-mass assay.) The

correct total mass of ^{235}U is 100.5 g, but the calibration curve predicts 98.35 g, resulting in a -2.14% error in the assay. However, because the 98.35-g assay result differs greatly from both 79 and 122 g, the bad rod is readily identified. If the pellets are intermixed, the assay error will decrease.

A more probable occurrence is the insertion of a few pellets of another enrichment. If a 3.03% rod contains 1% of 1.99% pellets, the actual mass is 121.57 g of ^{235}U . The predicted mass from the calibration curve is 121.47 g of ^{235}U , which is in error by -0.08%. The error increases linearly up to a loading of about 10% of 1.99% pellets. Consider a 3.03% rod containing 12% of the 1.99% pellets. The true and measured masses of ^{235}U are 116.84 and 115.83 g, respectively, whereas the mass of a normal 3.03% rod is 122 g. It is quite likely that this rod would be detected in a single assay having a statistical uncertainty of 2.4% (2σ), because for these two values the statistical uncertainties do not intersect at the $2\text{-}\sigma$ level (i. e., they are more than 4σ apart).

The results are similar for a 1.99% rod containing some 3.03% pellets. For example, if 10% of the pellets are 3.03%, the assay error is -0.84% (-0.70 g of ^{235}U). A 1.99% rod containing 11% of 3.03% pellets should be detected as a bad rod in a single assay. The probability of detection is increased in this case if the two varieties of pellets are intermixed in some manner.

C. Effect of Variations in Fuel-Pellet Density and Diameter

There is some small variation in the pellet density which could introduce error into the assay. Comparison of calculations made using the method described in Sec. II with a typical calibration curve shows that for a $\pm 2\%$ change in the density of a 3% enriched BWR rod (with constant radius, as the value of the radius is well-controlled), the assay is in error by $\pm 0.15\%$. The error would decrease for lower enrichments. Increased density causes more self-absorption, and the response

per gram of fissile material decreases. Thus, assuming that the density variations are random in a single rod, the error introduced into the assay should be a small fraction of the $\pm 0.15\%$ error.

Small changes in the fuel-pellet diameter could affect the assay result. Figure 19 shows the calculated fractional change in the response, assuming that the diameter is altered identically for all pellets in a rod with 0.8-mm cladding, but the mass of uranium in the rod remains unchanged. Thus, the maximum error for a fuel rod clad with zircaloy-2 is 1.3%/mm change in diameter. Because pellet variations are random, the error per rod should be, on the average, considerably lower than this upper limit. (See also Sec. VI-E for further discussion of fuel-pellet diameter variations.)

D. Effect of Dished and Undished Pellets

Production rods may contain either flat-ended (undished) pellets or dished pellets. Typical dishing removes $\sim 2\text{--}3\%$ of the pellet volume. The response per unit mass at the ends of a dished pellet of a given enrichment is greater than that at the ends of a flat pellet because of reduced self-absorption in the dished pellet. If the pellets were to have no significant self-absorption (i. e., a

Fig. 19. Calculated fractional change of the response (fission neutrons) from ^{235}U vs change in the diameter of pellets of constant mass (3% enrichment).

linear calibration curve), the response per unit mass would be identical for dished and undished pellets. Because self-absorption is significant, the most accurate assay would require a separate calibration curve for each type of rod (two sets of standard rods) and segregation of the rods to be assayed. If an inventory containing rods of dished and undished pellets is to be assayed, a compromise calibration curve could be generated using both dished pellet standards and flat-ended pellet rods. The error introduced by the use of a flat-pellet calibration curve to assay dished-pellet rods should be larger than the error introduced by using the compromise curve for either type of rod. The effect of using such a compromise calibration curve must be included to determine the final assay error.

The magnitude of this effect can be estimated in the following manner. If the density of the material in a dished pellet is reduced so that the material fills the volume of the undished pellet, the effect can be calculated. For example, consider a pellet with 2 vol% dishing. As mentioned in the previous section, a -2% change in density of a 3% enrichment BWR fuel would cause an assay error of 0.15%. This estimate of the fractional change in response caused by dishing would be smaller for a rod of less enrichment or smaller diameter.

E. Assay of Rods of Different Diameter from the Standards

We considered whether a fuel rod containing pellets of a different diameter from those in the standards could be properly assayed. We investigated the effect of variation in rod diameter using neutron-transport calculations. The number of fission neutrons born per thermal source neutron was calculated for various diameters of an infinitely long fuel rod inside a fixed-size channel in a CH_2 moderator. We assumed the UO_2 fuel density to be 10 Mg/m^3 , and computed the ^{235}U content by assuming the ^{235}U mass: ^{238}U enrichment ratio to be 40, for a fuel diameter of 12.37 mm.

Fig. 20. Calculated fission-neutron response of typical LWR fuel rods from a thermal-neutron source as a function of ^{235}U mass content for various fuel-pellet diameters.

Figure 20 shows the results of the calculations for fuel diameters of 9.0, 11.0, and 13.0 mm, which span the range of current LWR fuel. For ^{235}U masses below $\sim 80 \text{ g}$, the differences are rather small. At small masses ($< 40 \text{ g}$ of ^{235}U), the smallest diameter rod has the largest response per unit mass because of its higher relative enrichment and, hence, lower ^{238}U absorption. As the ^{235}U mass increases, this effect becomes relatively small and the geometry effects of the rod dominate; i. e., the larger the rod-surface area, the more neutrons that interrogate the rod per unit mass of ^{235}U . So the response at higher mass is proportional to the diameter of the fuel pellets. For example, a 3% reduction in the nominal diameter of 3% BWR fuel pellets would produce an $\sim 0.5\%$ assay error. Thus it is clear that, in general, a fuel rod containing pellets of a different nominal diameter from those of the standards should not be assayed, particularly at ^{235}U masses greater than $\sim 80 \text{ g}$, because errors of one to several percent would be introduced into the assay.

F. Heterogeneities in Fissile Content in the Fuel Rods

One can assume that the fuel material in a LWR fuel rod is radially homogeneous in fissile content; however, the assay-system design should try to flatten the response from all radial positions in the fuel rod at a specified axial location. There is a thermal-neutron flux gradient across the fuel rod as illustrated in Fig. 1; i. e., the part of the rod nearest the source is being interrogated with a larger neutron flux than is the rest of the rod. The effect of the resultant fission-neutron gradient generated across the fuel rod can be removed by positioning the detectors so as to cancel the gradient effect. Figure 3 shows the positioning of the two ^4He detectors, which preferentially detect neutrons from the rear (with respect to the source) side of the fuel rod because of $1/r$ considerations. Therefore, the measured rod response is reasonably uniform across the fuel rod, producing an assay that is not sensitive to the position, relative to the source, of any small inhomogeneities in the fuel.

Any heterogeneities in the fuel, such as lumping, may cause errors in the assay because of the self-shielding of the thermal neutrons by the particles.⁵ Thus, the assay of fuel for a high-temperature gas-cooled reactor (HTGR) that contains fissile kernels is more a measurement of the number of particles of a specified size and reactivity than of the fissile content. The total fissile assay of this type of fuel by thermal neutron interrogation is difficult because the density and diameter of the particles can be changed to yield the same measured response with a different fissile content. Clearly, standard fuel for this case is particularly important.

VII. SUMMARY OF ERRORS AND UNCERTAINTIES

A. Errors in Operational Procedure

This study has shown that the system must be operated as calibrated. Also, the standards

should approximate the unknowns to be assayed as closely as possible and should both bracket and span their enrichments. There should be at least one more standard than the number of calibration parameters. Our considerations of fuel-column length, diameter, density, and pellet shape have shown that sizable errors can be introduced into the assay of unknowns that differ from the standards in one or more of these properties; i. e., the standards must represent the nominal values of every physical property of the unknowns that will affect the measurement, with the possible exception of enrichment. Cladding differences can also be important; e. g., stainless-steel-clad standards should not be used for zircaloy-2-clad unknowns in thermal-neutron interrogation systems because of differences in thermal-neutron absorption.

This system can be only as accurate as the standards used for calibration. The uncertainty in each standard mass must be minimized to minimize the uncertainty in the assay result. A simple check of whether the standards form a consistent set is to be sure that the fissile mass: enrichment ratio is the same for all standards, to within their respective uncertainties.

B. Uncertainties and Biases

We found the following effects to be small ($< 0.10\%$ error altogether) for single-rod assays, assuming that the standards were representative of the unknowns: (1) response to gamma-ray activity from the fuel rod; (2) response changes caused by normal tolerances in the cladding (especially zircaloy) thickness and impurities in the fuel material; (3) neutron background from the fuel rod; (4) response of fissions in the ^{238}U from ^{252}Cf source neutrons; (5) normal fuel-pellet density and diameter variations; and (6) fluctuations in the drive-motor speed. The effects of possible long-term electronic drifts and the decay of the radioactive neutron source are effectively eliminated by using the current background from the ^{252}Cf source to normalize the measured response.

Sources of possible error in the assay of a single rod, in order of increasing estimated magnitude, are the following: (1) calibration curve ($\sim 0.2\%$, 1σ); (2) fuel-length variations ($< 0.4\%$ for a 0.4% change in fuel length); and (3) propagated counting statistics (1.1 - 1.3% at the 1σ level). The standard-mass uncertainty also contributes to the assay uncertainty; e. g., if the relative standard deviation of the standard masses is $\sim 0.5\%$, the propagated uncertainty increases to 1.3 - 1.6% at the 1σ level. If the pellets in the unknowns are dished and those in the standards are not, an error of $\sim 0.2\%$ will occur (and vice versa). Finally, if the calibration is not done batchwise, crosstalk between channels could introduce a bias of $\sim 0.2\%$, which can be removed (see Sec. V-B).

After assay of ~ 100 unknowns, any biases will begin to dominate the uncertainties in the total assayed fissile mass. The biases, assuming representative standards, are the following: (1) the calibration curve; (2) the uncertainty in the standard masses; and (3) dished and undished pellets (not a bias for proper standards). The minimum bias occurs for batch calibration and proper standards. For this situation and more than 500 assays, the total assayed fissile mass should have a relative uncertainty of $< 0.5\%$ (2σ).

Note that the fractional uncertainty in the total ^{235}U mass contained in a large number of rods would be expected to be very small ($< 0.1\%$) if all of the calibration parameters were known exactly and the model were correct. Because all of these parameters can never be known perfectly, the measurements on individual rods are correlated through the calibration curve, and the resulting covariance terms must be included in any estimate of the uncertainty of the total mass (see Appendix C).

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APPENDIX A THE RESPONSE FUNCTION OF THE ^{252}Cf FUEL-ROD-ASSAY SYSTEM

The mathematical expression used to convert the response R of the thermal ^{252}Cf fuel-rod-assay system to the ^{235}U content (in grams) of an LWR fuel rod is

$$R = a(1 - e^{-bU}) \quad (\text{A-1})$$

This function provides a simple semiempirical means of accounting for thermal-neutron self-shielding by the fuel rod. We investigated the theoretical basis for this function to determine its validity for interpolation between, and extrapolation from, calibration points obtained with standard rods and to search for an improved closed-form function.

We modeled a single channel of the assay system as a two-region, infinitely long cylinder. The inner region, defined by radius r_f , contains fuel and is surrounded by the second region, an infinite moderator. This model can be used to estimate the neutron absorption in the rod, assuming the following:⁵

- one-group diffusion theory is valid in the moderator,
- the angular flux distribution incident on the rod is isotropic, and
- the first-flight transport model is applicable in the absorber (fuel rod).

The last assumption is modified using Stuart's work⁶ to include the effects of scattering in the rod.

The resulting solution using the above model for neutron absorption in the ^{235}U in the rod leads to the following expression for R .

$$R = gF\beta \{1 - 1/[1 + S(2 - \beta)/\beta]\} \quad (\text{A-2})$$

where

$$F = \Sigma_{a25}/\Sigma_{af} \quad ,$$

$$\beta = \frac{\Sigma_{af}}{\Sigma_{Tf}} \beta_{PA} \left/ \left[1 - \frac{\Sigma_{sf}}{\Sigma_{Tf}} (1 - \beta_{PA}/2\Sigma_{Tf}r_f) \right] \right. ,$$

$$\beta_{PA} = 1 - \frac{2}{p} [1 - (p+1)e^{-p}] \quad ,$$

$$p = 3\Sigma_{af}r_f \quad ,$$

$$S = 2D_M k K_1(kr_f)/K_0(kr_f) \quad ,$$

$$k = \sqrt{\Sigma_{aM}/D_M} \quad ,$$

where D_M is the diffusion coefficient of the moderator, $K_i(kr_f)$ is the modified Bessel function of the second kind of order i , β_{PA} is the fraction of neutrons absorbed by a pure absorber, F is the fraction of neutrons in the fuel absorbed by the ^{235}U , Σ_i is the macroscopic cross section for the i^{th} process (absorption, scattering, etc.), and g is the proportionality constant incorporating source strength, rod-surface area, multiplication effects, and absolute detection efficiency. In Eq. (A-2), β is the probability that a neutron incident on the fuel rod will be absorbed, and the term in parentheses is the flux depression in the moderator caused by the fuel.

The product

$$\frac{\beta}{\beta_{PA}} \{1 - 1/[1 + S(2 - \beta)/\beta]\}$$

is nearly a constant for up to ~4% ^{235}U enrichment and various moderators and rod properties. Thus, Eq. (A-2) may be simplified to

$$R = aF\beta_{PA} \quad (\text{A-3})$$

By expanding β_{PA} into a series and collecting terms, R can be further simplified to

$$\begin{aligned} R &= aF(1 - e^{-bU} e^{-2\Sigma_{a28}r_f}) \\ &\equiv \frac{aU}{h+U} (1 - ce^{-bU}) \quad , \end{aligned} \quad (\text{A-4})$$

where

$$bU + 2\Sigma_{a28}r_f \approx 2p/3 \quad , \quad c = e^{-2\Sigma_{a28}r_f} \quad ,$$

and h represents the non- ^{235}U absorption.

Fitting Eq. (A-4) to LWR fuel-rod-standards data by using the general least-squares code PACKAGE³ showed that $h \approx 1$ and $c \approx 0.994$. Thus, for large U Eq. (A-4) can be reduced to the original function, Eq. (A-1).

Goodness-of-fit checks of Eqs. (A-4) and (A-1) showed that including either c or h in the model significantly improves the fit over that of Eq. (A-1), whereas including the second parameter gains little. Hence, we suggest the following calibration curve as an alternative for Eq. (A-1):

$$R = a(1 - ce^{-bU}) \quad (A-5)$$

[Another form suggested by Eq. (A-4) has the nonlinearity in U in the denominator:

$R = (a + bU)/(1 + cU)$.] Equation (A-5) does not intersect the origin when $c \neq 1$. In the strictest

sense, Eqs. (A-1) and (A-5) would not be valid for very low enrichments where $R \approx 0$; however, because the assay system would produce a small response in a rod with no ^{235}U from fissions in the ^{238}U , Eq. (A-5) is an excellent mathematical function for the ^{252}Cf fuel-rod-assay system.

Note that if Eq. (A-5) is forced through the origin by subtracting $a(1 - c)$ from the right-hand side, then Eq. (A-5) simplifies to the form of Eq. (A-1); that is,

$$R = ac(1 - e^{-bU})$$

We also used a linearly anisotropic angular-flux distribution incident on the fuel as a model. The improvement in fit over that of the above model was not enough to warrant using the more complicated model.

APPENDIX B

SOME PROPERTIES OF THE BACKGROUND-CORRECTED RESPONSE

Assume that the count-rate profile in the detector (net count rate of Q counts/sec, where Q is a function of enrichment) of a rod being assayed is as shown in Fig. B-1: (1) for a time t_1 (determined by the microswitch position) after the beginning of the assay, the rod has not yet entered the active region of the detector; (2) at time t_1 , the fuel column enters the active region; (3) at time

$t_1 + t_2$, the rod completely fills the detector region until time $t_1 + t_2 + t_3$; (4) at time $t_1 + 2t_2 + t_3$, the rod is no longer in the detector region; and (5) the assay ends at time $t_1 + 2t_2 + t_3 + t_4$, determined by the position of the last microswitch. This assumes that the rod is longer than the detector region and that the detector region is axially symmetric. As shown in the figure, the time required for the count rate to increase from B to $Q+B$ (the rod totally fills the detector region) is the same that it takes to decrease from $Q+B$ to B . Recall from Sec. V-D that the background-corrected response was defined in Eq. (3) as

$$R = \frac{B_0}{B} (C - B) \quad (B-1)$$

From Fig. B-1, the average observed count rate, C , from the assay can be written as

$$C = \frac{B(t_1 + t_4) + (Q + 2B)t_2 + (Q + B)t_3}{T} \quad (B-2)$$

Fig. B-1. Count-rate profile of a rod in the detector during an assay.

where $T = t_1 + 2t_2 + t_3 + t_4$. Inserting the above equation into Eq. (B-1) and combining terms yields

$$R = \frac{B_0 Q(t_2 + t_3)}{BT} \quad (B-3)$$

The total time T for the assay is assumed to be a constant for specified microswitch positions and rod-feed rate; note, however, that if the drive-motor speed increases or decreases for an entire assay, Eq. (B-3) shows that this will have no effect on R , because all of the t_i s will be scaled by the same factor, which will cancel. Small variations in drive-motor speed during an assay will

have no effect as long as the rate of travel maintains its average value through each of the time zones in Fig. B-1.

Consider a system with a sensitive detector length of d m, a fuel-column length of L m, and a rod-feed rate of $\dot{x}$ m/sec. The time t_2 would be given by $d/\dot{x}$, and the time t_3 would be given by $(L - d)/\dot{x}$. Thus, Eq. (B-3) can be written as

$$R = \frac{B_0 Q L}{B T \dot{x}} \quad (B-4)$$

Equation (B-4) shows that the background-corrected response is directly proportional to the fuel-column length L .

APPENDIX C

ANALYSIS OF THE UNCERTAINTIES OF NONDESTRUCTIVE-ASSAY-SYSTEM RESULTS⁷

There are two primary ways in which to use the response of an instrument for comparison with the mean response of known standard(s):

(1) the response of an unknown rod is compared with the mean response of a standard rod to test the hypothesis that the responses are the same (and, therefore, so is the content of fissile material), and (2) the response of an unknown rod is transformed by using a calibration curve, determined from the standard rods, to produce an assay. (This is the approach generally used in a ²⁵²Cf fuel-rod-assay system.)

I. COMPARISON BY HYPOTHESIS TEST

For the hypothesis test, only the random-error component and the bias caused by changes in instrument operation need be included, because no transformation of the response is required. With the estimated standard deviation of a measured response known, the probabilities of rejecting or not rejecting the hypothesis of equal fissile-mass quantities can be determined by fundamental statistical techniques. Note that any nonlinear behavior of the response with varying enrichment

must be included in this analysis. For example, if the response is saturated, unknowns with masses much greater than the standard will yield the same response and hence pass the test that the standard and the unknown contain the same fissile mass.

II. ASSAY USING A CALIBRATION CURVE

Assaying the rod by transforming the response R into a quantity of fissile material U in the rod requires a function of the form

$$U = F(R, a_1, a_2, \dots, a_m) \quad (C-1)$$

where the a_i s are the values of the calibration constants determined from a weighted least-squares fit of the equation

$$\bar{R} = H(U, a_1, a_2, \dots, a_m) \quad (C-2)$$

to the standard-rod mean-response points $(U, \bar{R})$. (At least m standard rods of different fissile masses are required.) Equation (C-1) is the inverse function of Eq. (C-2), so Eq. (C-2) must be strictly monotonic in the region of interest. It is desirable to have a physical basis for the functional

form in Eq. (C-2) used to correlate the independent variable U to the dependent variable $\bar{R}$.

Equation (C-1) allows one physical characteristic directly associated with the fissile-mass content of a rod (e. g., enrichment, fuel density, or one fuel dimension) to be varied in the standard rods. If Eq. (C-2) adequately describes the behavior of the variation, then Eq. (C-2) can be used to interpolate between the standard rod points to produce an assay. Extrapolation outside the range of the standards using Eq. (C-1) should be avoided because of the possible large error in the assay result. Normally, the characteristic varied in the standard rods is the enrichment, as is assumed in this report. The standards need not have the same enrichments as the unknown rods in this case.

III. A SINGLE ASSAY OF ONE ROD

Measurement of a rod yields a response R having an estimated standard deviation of s_R . The assay of the total fissile content is U from Eq. (C-1) with an estimated standard deviation of $(\partial F / \partial R) s_R$. This assumes that the calibration constants $a_1, a_2, \dots, a_m$ are known to be true and that the model is perfect. Because the a_i s were estimated from standard data that are not known exactly and the model is perhaps not perfect, each calibration constant has an associated uncertainty. The standard deviation of U including both the response and calibration error components is estimated by

$$s_U = \left[\left(\frac{\partial F}{\partial R} \right)^2 s_R^2 + \sum_{i=1}^m G(R, a_i) \right]^{\frac{1}{2}},$$

where

$$G(R, a_i) = \left(\frac{\partial F}{\partial a_i} \right)^2 s_{a_i}^2 + \sum_{\substack{k=1 \\ k \neq i}}^m \text{Cov}(a_i, a_k) \left(\frac{\partial F}{\partial a_i} \right) \left(\frac{\partial F}{\partial a_k} \right),$$

in which $\text{Cov}(a_i, a_k)$ is the estimated covariance of a_i and a_k , and $s_{a_i}^2$ is the estimated variance (standard deviation squared) of a_i . The above expression for s_U with

$$F = -\frac{1}{b} \ln \left[\frac{1}{c} \left(1 - \frac{kR}{a} \right) \right]$$

results in Eq. (6) in Sec. V-D. The calibration-parameter variances and covariances can be estimated during weighted least-squares fitting techniques, but the functional model's adequacy can be assessed only if at least $m + 1$ standard rods with different fissile masses are used in the calibration. Generally, the more standard masses available, the better the estimates of the calibration parameters and their uncertainties.

This estimate of the standard deviation of the assay result does not include variations from the nominal values of a particular unknown rod's physical characteristics produced by manufacturing tolerances. Without additional measurements, it is impossible to perform any corrections to the assay result; however, the measurement capability of an acceptable rod-assay instrument should be insensitive to such variations in the unknowns.

IV. MULTIPLE ASSAYS OF A SINGLE ROD

For a rod measured M times resulting in assay values of $U_1, U_2, \dots, U_M$, the average assay value $\bar{U}$ is

$$\bar{U} = \sum_{j=1}^M U_j / M,$$

with an estimated standard deviation of

$$s_{\bar{U}} = \left[\left(\frac{\partial F}{\partial R} \right)^2 \frac{s_R^2}{M} + \sum_{i=1}^m G(\bar{R}, a_i) \right]^{\frac{1}{2}}.$$

This assumes that the $R_j \approx \bar{R}$ and the $s_{R_j} \approx s_R$. Thus, even as M tends toward infinity, the uncertainty due to the calibration curve does not vanish. See the second paragraph of the previous section.

V. MULTIPLE ASSAYS OF ROD AGGREGATES

For assay of N different rods in which the j th rod is measured M_j times, the mean assay value of the fissile mass in the j th rod is denoted by $\bar{U}_j$ with an estimated standard deviation of

$$s_{\bar{U}_j} = \left[\left(\frac{\partial F}{\partial R} \right)^2 \frac{s_{R_j}^2}{M_j} + \sum_{i=1}^m G(\bar{R}_j, a_i) \right]^{\frac{1}{2}}.$$

The total assayed fissile mass of the N rods is given by

$$T = \sum_{j=1}^N \bar{U}_j$$

with an estimated standard deviation of

$$s_T = \left\{ \sum_{j=1}^N \left[\left(\frac{\partial F}{\partial \bar{R}} \right)^2 \frac{s_{R_j}^2}{M_j} + \sum_{i=1}^m G(\bar{R}_j, a_i) \right] + \sum_{j=1}^N \sum_{\substack{\ell=1 \\ j \neq \ell}}^N \sigma_{U_j U_\ell} \right\}^{\frac{1}{2}}$$

where

$$\sigma_{U_j U_\ell} = \sum_{i=1}^m \sum_{k=1}^m \frac{\partial F(\bar{R}_j, a_1, \dots, a_m)}{\partial a_i} \sigma_{ik} \times \frac{\partial F(\bar{R}_\ell, a_1, \dots, a_m)}{\partial a_k},$$

$$\sigma_{ik} = \text{Cov}(a_i, a_k) \text{ and } \sigma_{ii} = s_{a_i}^2.$$

If the average response of each of the N rods is approximately the same, so that the $\partial F / \partial a_i$ are nearly the same for each rod, $i = 1, \dots, m$, and $\partial F / \partial \bar{R}$ are all essentially equal, then the expression for the estimated standard deviation of the total can be simplified to

$$s_T = \left[\left(\frac{\partial F}{\partial \bar{R}} \right)^2 \sum_{j=1}^N \frac{s_{R_j}^2}{M_j} + N^2 \sum_{i=1}^m G(\bar{R}, a_i) \right]^{\frac{1}{2}}.$$

These estimates of the standard deviation of the assay results assume that the standard rods represent the nominal values of all of the physical characteristics of the N rods assayed, with the possible exception of enrichment.

REFERENCES

1. H. O. Menlove, Los Alamos Scientific Laboratory report LA-4605-MS (1970), pp. 12-13.
2. A technique for including the uncertainty in the independent variable as well as the uncertainty in the dependent variable in a least-squares fit is described in W. Edwards Deming, Statistical Adjustment of Data (John Wiley & Sons, New York, 1943); see also R. H. Moore, "Linear Functions, Differing Weights, and Non-Linear Least Squares," Los Alamos Scientific Laboratory report LA-5116 (1973).
3. For example, see R. H. Moore and R. K. Zeigler, "The Solution of the General Least-Squares Problem with Special Reference to High-Speed Computers," Los Alamos Scientific Laboratory report LA-2367 (1960).
4. Yardley Beers, Theory of Error (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1958), second printing.
5. R. V. Meghreblian and D. K. Holmes, Reactor Analysis (McGraw Hill, Inc., New York, N. Y., 1960), pp. 244-260.
6. G. W. Stuart, "Multiple Scattering of Neutrons," Nucl. Sci. Eng. 2, 617 (1957).
7. D. B. Smith, L. A. Forster, and R. H. Moore, "Calibration Procedures and Error Propagation for Nondestructive Assay Instrumentation," to be published.