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EDDY CURRENT EFFECTS IN THE ISA VACUUM CHAMBER

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ABSTRACT

The eddy current effects associated with a changing magnetic multipole field applied to a thin-wall cylindrical chamber are studied in some detail. We specialize the results to the ISABELLE vacuum envelope and obtain the power loss, the time constant, and the multipole field perturbation within the chamber.

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I. INTRODUCTION

The eddy currents produced in the ISABELLE vacuum chamber when the applied magnetic fields are changing will result in perturbing fields within the chamber. The fact that the chamber is a thin cylindrical envelope and that the time rate of change of the fields is such that the skin depth is very much greater than the wall thickness makes the calculation of these eddy current effects straightforward. In CRISP Note 72-17, J. Bittner has considered eddy current effects and applied the results to a chamber maintained at a low temperature. In this note, we shall consider this problem in more detail and apply the results to a room temperature chamber.

II. EDDY CURRENTS AND FIELDS

The applied magnetic field can be written as an expansion in terms of multipole components. Thus, at a field point (r, θ) [see Fig. 1] we have

$$\vec{B}(r, \theta) = \sum_{n=1} B(n) \left(\frac{r}{a}\right)^{n-1} \left[\vec{I}_r \sin n\theta + \vec{I}_\theta \cos n\theta \right], \quad (1)$$

where a is the radius of the thin-wall cylinder, and $B(1)$, $B(2)$, $B(3)$, etc. are the amplitudes of the dipole, quadrupole, sextupole, etc. component fields. If this field is changing, the time rate of change is then

$$\frac{d\vec{B}}{dt} = \sum_{n=1} \dot{B}(n) \left(\frac{r}{a}\right)^{n-1} \left[\vec{I}_r \sin n\theta + \vec{I}_\theta \cos n\theta \right]. \quad (2)$$

When we apply Faraday's and Ohm's law, that is (in Gaussian units),

$$\vec{\nabla} \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad \text{and} \quad (3)$$

$$\vec{J} = \sigma \vec{E}, \quad (4)$$

we find that the resulting current flowing longitudinally in the cylinder wall is

$$\frac{di}{d\phi} = \frac{\sigma a^2 \tau}{c} \sum_{n=1}^{\infty} \dot{B}(n) \frac{\cos n\phi}{n}, \quad (5)$$

where:

- σ = conductivity of material (Gaussian units)
- τ = thick of cylinder in centimeters
- c = velocity of light in cm/sec
- $\dot{B}(n)$ = multipole field at cylindrical surface (gauss/sec)
- $di/d\phi$ = current per unit angle (statamperes).

Corresponding to these eddy currents in the chamber, the vector potential for field points within the radius a is

$$A_z(r, \theta) = \frac{2\sigma a^2 \tau}{c^2} \sum_{n=1}^{\infty} \frac{\dot{B}(n)}{n} \int_0^{2\pi} d\phi \cos n\phi \ln \left(\sqrt{(r \cos \theta - a \cos \phi)^2 + (r \sin \theta - a \sin \phi)^2} \right). \quad (6)$$

By taking the curl of Eq. (6) one obtains the field caused by the eddy currents

$$\vec{B}_e = - \frac{2\pi \sigma a^2 \tau}{c^2} \sum_{n=1}^{\infty} \frac{\dot{B}(n)}{n} \left(\frac{r}{a} \right)^{n-1} \left[\vec{I}_r \sin n\theta + \vec{I}_\theta \cos n\theta \right]. \quad (7)$$

III. POWER LOSS AND TIME CONSTANT

The eddy current distribution given by Eq. (5) is accompanied by a power loss per unit axial length of cylinder. This is

$$\frac{P}{l} = \frac{\pi \sigma a^3 \tau}{c^2} \sum_{n=1}^{\infty} \left(\frac{\dot{B}(n)}{n} \right)^2. \quad (8)$$

Since the energy density expressed in Gaussian units is

$$U = \frac{B^2}{8\pi}, \quad (9)$$

we can readily calculate the energy stored in the eddy current field given by Eq. (7). Thus, the magnetic energy per unit length of chamber is

$$w_1 = \frac{\pi a^2 \tau^2}{2c^4} \sum_{n=1}^{\infty} \frac{1}{n^3} [\dot{B}(n)]^2 \quad (10)$$

Corresponding to this energy storage and the power dissipated Eq. (8), the time constant (λ) is given approximately by

$$\lambda \approx \frac{w_1}{P/L} = \frac{\pi a^2 \tau}{2c^2} \quad (11)$$

IV. TOTAL FIELD INSIDE CHAMBER

The total magnetic field inside the chamber is that due to the applied field and that due to the eddy currents. Combining Eq. (1) and (7) therefore yields

$$\vec{B} + \vec{B}_e = \sum_{n=1}^{\infty} \left(B(n) - \frac{2\pi a^2 \tau}{c^2} \frac{\dot{B}(n)}{n} \right) \left(\frac{r}{a} \right)^{n-1} \left[\vec{I}_r \sin n\theta + \vec{I}_\theta \cos n\theta \right] \quad (12)$$

This result can best be expressed in terms of the median plane parameters b_n (see page 122 of BNL 18891). We then have

$$(\vec{B} + \vec{B}_e)_y = B(1) \left\{ (1 - \eta(1)) + b_1 \left(1 - \frac{\eta(1)}{2} \right) x + b_2 \left(1 - \frac{\eta(1)}{3} \right) x^2 + \dots \right\} \quad (13)$$

Here the shielding factor which reduces the b_n coefficient is $\left[1 - \frac{\eta(1)}{n} \right]$ with

$$\eta(1) = \frac{2\pi a^2 \tau}{c^2} \frac{\dot{B}(1)}{B(1)} \quad (14)$$

In writing Eq. (13) we have assumed that the changing multipole field components all vary in proportion, or specifically that

$$\eta(1) = \frac{\dot{B}(1)}{B(1)} = \frac{\dot{B}(2)}{B(2)} = \dots \quad (15)$$

V. NUMERICAL RESULTS

a) Power Loss/Unit Length of Chamber in the Dipoles

In SI units, Eq. (8) takes the form

$$P/l = \frac{\pi a^3 \tau}{\rho} \left[\dot{B}(1) \right]^2, \quad (16)$$

where:

- a = radius of cylinder in meters
- τ = thickness of cylinder wall in meters
- ρ = resistivity in ohm meters
- $\dot{B}$ = dipole field change in Tesla per second
- P/l = power loss in watts per meter.

For ISABELLE:

- a = 0.04 meter
- τ = 0.002 meter
- ρ = 4×10^{-8} ohm meter (AL 6061 - T6 at 20°C)
- $\dot{B}$ = 0.0333 tesla/sec
- P/l = 0.011 watts/meter.

b) Time Constant

In SI units, Eq. (11) becomes

$$\lambda = \frac{\pi}{2} \frac{a \tau}{\rho} \times 10^{-7}. \quad (17)$$

Using the above numerical constants, we find that the time constant associated with changes in the applied field is

$$\lambda = 0.3 \times 10^{-3} \text{ seconds}. \quad (18)$$

c) Eddy Current Field Perturbation

As shown in Eq. (13), the magnetic multipole perturbation produced by eddy currents in the chamber is characterized by the factor $\eta(1)$. For the injection field of 5.9 kilogauss and the numerical values given above, $\eta(1)$ assumes the value 7×10^{-5} . Hence the total field at injection is expressible as:

$$(\vec{B} + \vec{B}_e)_y = B(1) \left[(1 - 7 \times 10^{-5}) + b_1 (1 - 3.5 \times 10^{-5}) + b_2 (1 - 2.3 \times 10^{-5}) + \dots \right]. \quad (19)$$

d) The normal lattice of ISABELLE has a chromaticity of about -40. It is desired that this chromaticity be corrected to about +3. Therefore, the added sextupole must be capable of producing a chromaticity of +43. Since

$$\Delta \nu = b_2 \frac{\beta_{av}^2}{\nu} \frac{\Delta p}{p}, \quad (20)$$

it is necessary that the average sextupole coefficient b_2 be such that

$$b_2 = 43 \frac{\nu}{\beta_{av}^2}. \quad (21)$$

For a ν of about 20 and a β_{av} of 2000 cm, Eq. (21) yields

$$b_2 = 0.2 \times 10^{-3} / \text{cm}^2. \quad (22)$$

It follows, then, that the eddy currents at injection will produce a change in such a b_2 coefficient of only

$$\delta b_2 = 5 \times 10^{-9} / \text{cm}^2. \quad (23)$$

This value is well within the tolerance given by George Parzen of about $1.5 \times 10^{-6} / \text{cm}^2$.

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Fig. 1. Notation for cylindrical chamber.