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NOTES ON AN AZIMUTHALLY VARYING
FIELD (AVF) CYCLOTRON

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Notes on an Azimuthally Varying Field (AVF) Cyclotron

Electronuclear Research Division

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Edited by F. T. Howard

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INTRODUCTORY COMMENTS

The use of an azimuthally varying magnetic field to provide for phase compensation in the acceleration of relativistic particles in a fixed-frequency cyclotron was first proposed by Thomas⁽¹⁾ in 1938. The subsequent development of cyclotrons continued, however, to follow the path of increasing the dee voltage, and later the use of frequency modulation, to obtain higher energies.

The much reduced ion currents obtained in the high energy frequency-modulated cyclotrons led to a renewed interest in increasing the energy of the fixed-frequency machines. An electron model F-F cyclotron with clover-leaf pole pieces to produce the Thomas-type field was test operated at Berkeley⁽²⁾. It demonstrated that the azimuthally varying field system was entirely practicable; electrons were accelerated to 70 keV, where $v/c \approx 0.5$, with a threshold dee voltage of less than 25 volts.

A magnetic field configuration with much greater gradients, placed alternately, was developed as the alternating gradient (A-G) focusing system⁽³⁾; it is used in several proton synchrotrons now being designed. In studying the A-G system, the MURA group (Midwestern Universities Research Associates) found that a constant-current magnet (FFAG Mark I) could be constructed with A-G sectors designed to accommodate stable orbits for particles of a wide energy range in a small radial aperture. This design is closely related to

1. Thomas, L. H., "The Paths of Ions in the Cyclotron. I. Orbits in the Magnetic Field. II. Paths in the Combined Electric and Magnetic Fields," Phys. Rev. 54, 580-98 (1938).
2. Pyle, R. V., The Second Electron Model Phase-Compensated C-W Cyclotron, Report UCRL-2435 (December 1953).
3. Courant, E. D., M. S. Livingston, and H. S. Snyder, "The Strong-Focusing Synchrotron-A New High Energy Accelerator," Phys. Rev. 88, 1190-1196 (1952).

the Thomas-type field for fixed-frequency cyclotrons. They later found that introducing a spiral in the azimuthal field variations reduced the magnitude of the required field variations and, thus, the size of the magnet; the spiral feature is incorporated in the FFAG Mark V proposal⁽⁴⁾. In this case the "FF" is used for fixed field rather than for fixed frequency. As with other proton synchrotrons, the proposed FFAG machine uses frequency modulation to accomodate the increase in relativistic mass; the field configuration produced by the spirally ridged poles of the constant-current magnet provides strong focusing forces for ions of a wide range of energies in a narrow toroid.

Interest in the use of an azimuthally varying field (AVF) to obtain phase compensation for relativistic particles in a fixed-frequency cyclotron first arose at ORNL in 1950 in connection with the design of pole pieces for the ORNL 86-Inch Cyclotron. The theory was re-examined⁽⁵⁾, model magnet tests were made⁽⁶⁾, and possible design changes for the 86-in. cyclotron were investigated⁽⁷⁾. After the successful acceleration of protons to 25 Mev in this cyclotron, with a conventional field, there was little motivation for modifying the magnetic field of this machine. Interest in azimuthally varying magnetic fields has since been directed toward the development of a phase-compensated fixed-frequency cyclotron to operate at relatively high energies. The present goal is an AVF cyclotron which will operate at about 1 bev.

4. Kerst, D. W., K. M. Terwilliger, K. R. Symon, and L. W. Jones, "Fixed-Field Alternating-Gradient Accelerator with Spirally Ridged Poles," Phys. Rev. 98, 1153 (1955).
5. Y-760, B. L. Cohen, Theory of Phase-Compensated Fixed-Frequency Cyclotron, (May 1951).
6. Y-795, Electromagnetic Research Division Progress Report, pp 29-34, (July 16, 1951).
7. ORNL-1270, Electromagnetic Research Division Progress Report, pp 13-16, (January 15, 1952).

The Physics Division and the Electronuclear Research Divisions are now collaborating in the design and development of this proposed AVF cyclotron. The Physics Division is studying magnetic field designs through theoretical investigations which involve extensive use of the Oracle; from their studies it now appears possible to achieve proton energies of over 1 bev with such a machine. In the Electronuclear Research Division preliminary design studies and cost estimates are being prepared, major theoretical and constructural problems are being identified and a cyclotron analogue is being constructed in which electrons will be used in studying ion-transmission problems that may be encountered in accelerating protons to high energies through azimuthally varying magnetic fields.

This informal collection of excerpts from ORNL progress reports and of abstracts for papers presented at technical meetings has been assembled to make immediately available an outline of the development of the project and a summary of the concepts currently being considered.

ACCELERATOR STUDIES AT 1 bev

Studies of the feasibility of a fixed-frequency accelerator in the 1-bev energy range are being made in collaboration with the Physics Division. A fixed-frequency machine should yield sufficiently large proton currents at this energy to be very useful in meson physics research. The theory of an azimuthally varying magnetic field cyclotron, first proposed by Thomas in 1938, is being extended to several different magnetic field configurations; calculations are being made with the Oracle. It is planned to make preliminary designs for magnets appropriate for the most promising configurations. The effects of spiraling the axes of the hills and valleys, as suggested by the Midwestern Universities Research Associates (MURA) group, are being investigated.

The radius and the magnetic field patterns were calculated for machines having 4, 5, 6, 8, 10, and 12 hills and valleys (poles). Some calculations were made for the design requirements of an eight-pole machine. The machine would have eight wedge-shaped pole pieces, with the coils wound in such a manner that the magnetic field in the valleys would be reversed for the larger radii (see Fig. 1). The magnet specifications for a 1-bev cyclotron of this design are given in Table 1.

An important problem encountered in the theoretical work is the effect of the "imperfection"

TABLE 1. MAGNET SPECIFICATIONS
FOR A 1-bev CYCLOTRON

	Closed Yoke	Open Yoke	The Bevatron
Excitation, 10^4 amp-turns	65	242	
Current density, amp/in. ²	1000	1000	
Power, Mw	2.5	8.5	3.5
Copper, tons	100	400	350
Iron, tons	9700	6200	9500

resonances. The ions, as they gain energy, are continually undergoing focusing oscillations about an equilibrium orbit. The frequency of these oscillations is a function of the energy - equal to the ion frequency at zero energy and increasing in frequency with increasing ion energy. Therefore, at certain energies the focusing oscillation will complete an integral number of cycles in either one or two ion revolutions. When this condition exists, any slight imperfection in the fields will cause the radial focusing oscillation to increase in amplitude, and, hence, the ions may be lost. Resonance energy levels for different pole configurations are given in Table 2.

The success with which ions can be accelerated through these energy levels is not known. It is thought that by careful construction the resonances can be passed without excessive loss of ions. Plans are being made for an electron-model accelerator to be used in assessing the importance of imperfection resonances and the feasibility of their penetration. These resonances serve one useful purpose, however, in that the radial beam spread at these radii can probably be used to an advantage in deflecting the beam.

The MURA group has suggested the use of spiral poles, the hills and valleys spiraling outward from the magnet center. This has two advantages: the imperfection resonances occur at higher energy levels, and a more compact magnet structure is possible. Some data have been calculated for an eight-pole machine which indicate that the first and second resonances would occur at 456 and 880 Mev, instead of at the 415 and 720 Mev listed

Fig. 1. An Eight-Pole, 1-bev, Fixed-Frequency Cyclotron, Plan View.

TABLE 2. RESONANCE ENERGY LEVELS FOR FOUR POLE CONFIGURATIONS

Pole Configuration	Resonance Energy Levels (Mev)			
	First Resonance	Second Resonance	Third Resonance	Fourth Resonance
4-pole	275	360		
6-pole	370	570	750	800
8-pole	415	720	960	
12-pole	440	810	1120	1380

in Table 2. Also, the variation in the magnetic field from hill to valley would be less, with the result that the average magnetic field would be higher, and the over-all magnet structure would be reduced in size by a factor of approximately 2.

The practical problems encountered are the design of the coils and pole faces and the design

of the r-f accelerating system. The large magnetic field variations and the accuracy with which the field must be constructed add to the difficulty. For a machine in the bev energy range, the conventional cyclotron dee system is unsatisfactory. The requirements for a practical r-f electrode system are being studied.

STUDY OF A 1-bev PROTON CYCLOTRON

A study is being made of the practicability of a fixed-frequency cyclotron designed to accelerate protons to about one billion electron volts. At this energy range the large currents of protons usually associated with a fixed-frequency machine would be extremely useful in meson research.

As reported earlier, calculations performed with the Oracle indicate that it is feasible to accelerate protons well into the 1-bev energy range in a fixed-frequency, fixed-magnetic-field machine. The relativistic mass effect which has previously limited the energy obtainable on fixed-frequency cyclotrons is overcome by the use of an unusual magnetic-field configuration. The average magnetic field increases radially, and thus the ions remain in phase, even though their mass may increase over 100%. The defocusing forces produced by this increasing average field are compensated for by periodic azimuthal field variations which tend to restore the ions to the median plane.

The azimuthally varying field can be produced by radial "hill and valley" contours in the pole faces, by special pole-face windings, or by both. Calculations show that the required field variations are substantially reduced if the axes of the hills and valleys are spiraled in the direction of the orbital path. The optimum number of azimuthal cycles and the optimum amount of spiral were investigated. Interest is presently centered on a

magnetic-field configuration which has eight azimuthal cycles with about 72 deg of spiral, Fig. 2.

This study is being made in collaboration with the Physics Division. The theoretical problems in optimizing the design are chiefly the concern of the Physics Division, while the Electronuclear Research Division is more directly concerned with the conceptual design of the equipment in relation to costs and with the design of preliminary experiments required for solving certain theoretical and design problems. The work in this Division has involved the equivalent of one full-time staff member and, during the summer months, the co-operative efforts of four research participants from southern universities.

The Theoretical Data

The data which served as the basis for an evaluation of conceptual designs were calculated with the Oracle. Calculations were made for configurations having 4, 6, 8, 12, and 16 azimuthal cycles and for several different degrees of spiral.

The orbits are of the form

$$q = \text{normalized radius} = q_0 [1 + q_1 \cos (p\theta + \phi)] ,$$

where

p = number of azimuthal cycles in the magnetic field,

ϕ = spiral parameter = $\alpha_1 q + \alpha_2 q^2 + \dots$

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Fig. 2. Artist's Sketch of Conceptual Design of a Fixed-Frequency Cyclotron with an Eight-Cycle Azimuthally Varying Magnetic Field (cut away to show some internal details).

The quantities q_0 and q_1 are calculated to produce constant-frequency orbits of a selected vertical-focusing frequency. Since the orbits have a sinusoidal variation about a circle, the magnetic-field variation is only approximately sinusoidal azimuthally.

The information about each magnet configuration consists in the vertical- and horizontal-focusing frequencies for some 60 orbits, with energies taken in steps from zero to at least the third resonance energy. Along approximately 12 of these orbits, the Oracle is made to calculate and print out normalized values for the radius and for the magnetic field.

Design Specifications

The feasibility of accelerating particles to high energies in a fixed-frequency cyclotron equipped

with an azimuthally varying magnetic field was demonstrated at Berkeley⁹ with a test cyclotron which accelerated electrons to $v/c = 0.51$. Theoretical studies at ORNL indicate that it should be possible to accelerate protons into the 1-bev energy range with such a machine. The many factors affecting the cost of the large magnet that will be required are being examined carefully in order to arrive at a design that is both economical and practicable.

Design Energy. — In the selection of the design energy, resonance between the frequency of the radial oscillations about the equilibrium orbit and the frequency of ion revolution in the orbits must be considered. The resonances occur at the frequency ratios of 1/1, 3/2, 2/1, etc. The first

⁹R. Pile, *The Second Electron Model Phase-Compensated C-W Cyclotron*, UCRL-2435 rev (March 1955).

resonance, which occurs in the initial accelerations in all cyclotrons, causes no great loss of ions if the magnetic field is adequately shimmed. The second, $3/2$, resonance may cause some loss of ions, but it is expected that an appreciable number of ions can be transmitted through this zone. The third resonance, $2/1$, which can be made to occur at about 900 Mev, may be a serious barrier to further acceleration. The increase in beam width at this resonance should prove helpful, however, in deflecting the ions. Consequently, the design energy was set at the $2/1$ resonance level.

Number of Azimuthal Cycles. – The number of azimuthal cycles of variation to be introduced in the magnetic field is, of course, limited by practical considerations. More important, however, is the effect of the number of cycles on the energy levels at which resonance occurs. The energy for the $2/1$ resonance for an eight-cycle field is 300 Mev greater than that for a four-cycle field, but rather little is gained in going to a more expensive sixteen-cycle field. Therefore, an eight-cycle machine was specified for further study.

Effect of Spiral. – The energy levels for the resonances are also found to increase rapidly as the hill and valleys are made to spiral but a few degrees from the straight, radial configuration; further tightening of the spiral increases the energy only slightly. As the spiral is made tighter, the field variation from hill to valley along an orbit decreases, the width across a hill decreases, and the length of the hill from the center of the magnet to the periphery increases. If the hill-and-valley field configuration is produced by pole-face windings, this lengthening and narrowing causes the power requirement to be a function of the degree of spiral. Cost estimates show an optimum spiral to exist.

Magnet Design and Cost Estimates

The magnet design problem was that of determining the details of the iron pole tips, or the proper combination of iron and pole-face windings, which would give the required field shape at the lowest cost. For the cost estimate the field was converted to an approximately two-dimensional one. The simplification was considered to be accurate enough for the preliminary cost study, and the time and effort were considerably less than would have been required by the more exhaustive three-dimensional analysis. A path in the median plane

of the machine to be studied was plotted so as to always be perpendicular to the spirals of the hills and valleys (see Fig. 3). The required field along this path was plotted vs the length of path.

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Fig. 3. Configuration of Eight-Cycle Azimuthally Varying Magnetic Field.

The problem was approached in three steps:

1. Magnetic equipotential surfaces needed to produce a given sinusoidal distribution of flux in the median plane were calculated. From these data the required contours of the pole faces, used without pole-face windings, could be calculated.
2. The magnitude of the current required in flat coils, placed above and below the median plane, used without iron to produce the desired field, was calculated as a function of the flux magnitude, width of a flux cycle, and the gap between coils.
3. Flat, iron pole faces were combined with the flat coils for a more economical design.

Pole Tips Without Pole-Face Windings. – The location of the iron for pole tips with contoured pole faces was determined from the data for magnetic equipotential surfaces. The result was that for the eight-cycle machine the minimum gap, which occurs at the outside edge of the field,

could be no more than about 4 in. Since this gap does not permit the insertion of any accelerating structures, the distribution of the iron and the pole-face windings required for machines having larger gaps were investigated.

Pole Tips with Pole-Face Windings. — For a given field variation and geometry and with the gap as a variable, the current required in coils backed by iron pole pieces was found to vary as $\sinh kb/2$, where π/k is the width of a hill and b is the magnetic gap. This rapid variation of current with b and k was found to be the controlling factor in the magnet design. In preparing the cost estimate, the iron pole tips are assumed to supply the average field and the pole-face windings to superimpose the azimuthally varying field.

In the first calculation the pole tips were de-

signed to provide the average field, with a large gap at the center and a narrow gap at the periphery; the pole-face windings were to be attached directly to the pole faces. The current in the pole-face windings reached extremely large values near the center of the magnet, and therefore the design was considered impractical.

As a second attempt, flat pole faces were assumed, with the gap held constant at all radii to reduce the current and power in the pole-face windings; these windings are attached directly to the flat faces of the pole tips (see Fig. 4). To obtain the correct average field in this case, the reluctance of the iron in the pole faces must be controlled. This adjustment may be made by pruning the iron on the top of the upper pole and, symmetrically, on the bottom of the lower pole. This scheme, whose

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Fig. 4. A 90-deg-Arc Section Through the Cyclotron, Suggesting an Arrangement of Pole Tips, Pole-Face Windings, and an Eight-Electrode Accelerating System.

practicality has not been thoroughly investigated, would certainly result in a minimum amount of iron in the magnet. In addition, it would reduce the current required in the pole-face windings, since the iron gap is everywhere the minimum value and the current in the hill-and-valley coils decreases as the coils spiral in from the outside toward the center of the magnet.

The pole-face windings would consist of 32 identical coils, 2 for each hill and 2, with reversed current, for each valley. Because of the rapid variation of the current with gap, a thickness of winding exists for which the power requirement is least. The power needed in the windings for various central flux densities was calculated (see Fig. 5).

Iron and Copper Required. - The most important items in the cost estimate are the iron in the pole pieces and return yoke, the copper for the pole-face windings and for the large coil which produces the average field, the total power for both the windings and the large coil, and the motor-generator capacity. A flux density of 18,000 gauss in the iron was used in all calculations. Because the flux density at the center of this magnet was lower than that at the outside, the iron required for the pole pieces is much less than that needed in a comparable frequency-modulated cyclotron. The thickness of the iron should vary somewhat as a parabola from the center to the outside edge of the magnet, instead of being constant. To provide a basis of cost comparisons for the several magnet designs, which use varying amounts of iron and copper, the cost of the power required for 50,000 hr of operation is included in the total estimate.

Cost Estimates. - The following unit costs were assumed in the calculation of total magnet costs:

	Cost
Iron, erected (per ton)	\$500
Copper, erected (per ton)	\$3000
Power (mills/kwhr)	6
Motor-generator capacity, installed (per kw)	\$100

Because of the large power requirements for the pole-face windings of high-flux magnets with even quite small gaps, the cost of the magnets was examined as a function of orbit diameter, with the assumption that the beam space in the gap should be directly proportional to the orbit diameter. This analysis was prompted by the fact that, for a given

Fig. 5. Power Required in Pole-Face Windings to Produce Magnetic Field for Accelerating Protons to 850 Mev.

amount of power in the pole-face windings, the magnet gap increases more than linearly as a function of diameter. The magnet gap must provide space both for the beam and for the placing of accelerating electrodes. The effects of the magnet gap and of the diameter on the total magnet cost are illustrated in Fig. 6.

The Accelerating System

The large size and the unusual shape of the cyclotron being considered introduce some difficult problems in the design of a suitable accelerating system. Three possible systems have been considered briefly: conventional dees, an eight-electrode system, and a cavity system.

Fig. 6. Magnet Cost Estimates. Specifications: Proton energy, 850 Mev; azimuthal cycles in magnetic field, eight; spiral constants, $\alpha_1 = 0$ and $\alpha_2 = 32$. Total cost includes cost of power for 50,000 hr of operation.

Conventional-Dee System. — There are rather serious disadvantages in the use of a conventional-dee system. The space it would occupy in the magnet gap would greatly increase the cost, and because of the size of the dee its mechanical design would be very difficult. Also, because of the space required for inserting and removing the dee, it would be necessary to replace the several small magnet yokes with a large yoke; this conventional design would require more magnet iron than would be required with the other systems.

Eight-Electrode Accelerating System. — Since this type of magnet may be designed with large gaps in the valleys, it would appear reasonable to concentrate the accelerating structures in these spaces. The valleys are $22\frac{1}{2}$ deg wide, azimuthally, and the eight electrodes could be from 20 to 40 deg wide. To make full use of the voltage on the electrode, the particles should pass through the field-free region inside the electrode in the time required for the voltage to reverse. Therefore, an electrode which is $22\frac{1}{2}$ deg wide should be driven at a frequency eight times the ion frequency. Such an electrode could not be driven through a stem at the edge of the machine, as in a conventional cyclotron, because the voltage standing wave along the electrode would pass through zero inside

the radius of maximum orbit. The electrodes could, however, be driven and supported from above and below by stems through the valleys (see Fig. 2). The applied frequency would then be four or eight times the ion frequency, depending upon the width of the electrodes. This system is believed to be worthy of further consideration.

Cavity System. — A square cavity operating in the TE_{011} mode seems to offer the simplest accelerating system. This cavity could be made very thin, be curved to fit the spiral, and be placed through the center of the machine in two opposite valleys, separating the machine into two identical halves. A slit in the cavity would permit the beam to pass through an electric field much like that between two dees. The size of this cavity is such that operation at a frequency twice the ion frequency would best fit the dimensions of the magnet.

Two cavities positioned at right angles to each other should probably be used. The cavities would be driven with a 180-deg phase difference to provide the properly phased voltages. The radio-frequency equipment for such a system of cavities would be much simpler than that for a multielectrode structure. Suitable cavities would be quite efficient; most important, they would require no space in the magnetic gap, and savings in magnet cost would be quite large (see Table 3). A possible disadvantage is that the magnet would have to be constructed in four identical sectors for a two-cavity accelerating system.

Test-Model Electron Cyclotron

No adequate experimental evidence exists on which to base an estimate of the difficulties to be expected in accelerating large currents of protons through the $3/2$ resonance. The electron cyclotron used at UCRL to demonstrate the effectiveness of the Thomas-type magnetic field was not designed to accelerate electrons through their $3/2$ resonance energy level. Consequently, an electron cyclotron was designed to be operated as a test model.¹⁰ It is to be the simplest machine which will yield the required information about the $3/2$ resonance. Knowledge is needed concerning the effects of small variations in the magnetic field on beam

¹⁰ Research participants working with the Division on this problem during the summer were B. P. Foster, North Texas State College; A. W. Hanson, The Citadel; D. L. Lafferty, Vanderbilt University; and H. M. Moseley, Texas Christian University.

TABLE 3. COMPARISON OF THE TWO-CAVITY AND THE EIGHT-ELECTRODE ACCELERATING SYSTEMS

	Accelerating System	
	Two-Cavity	Eight-Electrode
Design Features		
Magnet gap (in.)	4	23
Orbit diameter, maximum (ft)	21	52.5
Flux density, central (kilogauss)	7.9	3.2
Flux density, maximum (kilogauss)	20	9.8
Applied radiofrequency (megacycles/sec)	24.4	19.5
Requirements		
Iron, total (tons)	1850	8500
Copper, main field coil (tons)	82	400
Copper, pole-face windings (tons)	16	143
Power, main field coil (Mw)	0.75	1.5
Power, pole-face windings (Mw)	1.8	2.1
Magnet Cost, from Fig. 12	\$2,200,000	\$8,000,000

attenuation and on threshold voltage, especially in the region of this resonance.

Several cyclotron and magnet designs were considered before a design was selected. A four-cycle magnetic field was found to be sufficient to accelerate electrons through their $3/2$ resonance, which occurs at about 150 kev. For simplicity, in this test machine the hills and valleys of the four-leaf-clover magnet were not spiraled. To accelerate electrons to 220 kev, which is the $2/1$ resonance, the central field was fixed at 40 gauss; this makes the maximum orbit radius 12.2 in. and the ion frequency 112.2 megacycles/sec.

Dish-shaped iron pole faces are used to establish the average field in the magnetic plane. The azimuthal field variations are produced by 16 coils spaced at a fixed distance above and below the median plane. The hill coils and valley coils are identical and will carry equal currents, but, since the currents will be in opposite directions, the coils will have very little effect on the average field. Also, there are circular coils which can produce small changes in the average field without affecting the hill-and-valley flux. Because of the iron pole faces, the calculation of the field produced by the hill-and-valley coils is somewhat complex; the problem is being studied with the Oracle.

The resonant system will consist of two dees driven by a self-excited oscillator. To prevent any unbalance in the r-f voltage, which would have the

effect of introducing undesirable components in the accelerating field, the two dees are to be driven symmetrically.

Magnetic Phase Compensation*

It is generally accepted that the fixed-frequency cyclotron cannot provide for the resonant acceleration of particles traveling with a velocity which is a large fraction of that of light. It is obvious, of course, that voltages on dee systems cannot be raised indefinitely and that there will be a practical energy limit, perhaps of the order of 100 Mev for deuterons, for a conventional fixed-frequency cyclotron, even with very high dee voltage. It is also well known that if the magnetic field is caused to rise as the radius increases, thereby magnetically correcting for the phase loss which the increased-mass particle would experience, a defocusing magnetic field results.

In 1938 L. H. Thomas calculated the proper conditions for providing a rising magnetic field which would still yield radial and vertical focusing. Focusing was obtained by introducing essentially a series of wedge magnets azimuthally about the poles, or described in another way a series of pie-shaped hills and valleys. The Radiation Laboratory at Berkeley was interested in this unique idea as Ernest O. Lawrence writing in 1939 in a preface for a book by W. B. Mann acknowledged, but had some question as to the degree of difficulty the development would entail. The invention of the frequency-modulated cyclotron and the discovery of the phase-stable features thereof postponed the acute need for such development until rather recently.

As was reported by E. O. Lawrence at the Geneva Conference, August 1955, and at the meeting of the American Physical Society in Mexico City, August 1955, electron models of azimuthally varying cyclotrons were built a few years ago at the University of California

* Excerpt from a paper, Trends in the Design of Cyclotrons after 25 Years, presented by Robert S. Livingston at the Nuclear Engineering and Science Congress, Cleveland, Ohio, December 11-17, 1955.

and, indeed, it was found entirely practical to obtain an appropriate degree of focusing. It was recently pointed out, at the New York Meeting of the American Physical Society, January 1955, by Kerst and others of the Midwestern Universities Research Association (MURA) accelerator study group that adding a spiral to the axis of the hills and valleys of the azimuthally varying field will have important advantages in reducing the necessary amplitude variations in the magnetic field required for satisfactory focusing.

At the spring meeting of the Southeastern Section of the American Physical Society, T. A. Welton of our laboratory outlined a program underway at Oak Ridge to make design studies of a fixed-frequency machine which would achieve particles in the vicinity of 1000 Mev. Calculations to determine focusing conditions and to insure that one avoids dangerous resonances have in the past proven tedious. Adroit use of high speed computing machines (the Oracle at Oak Ridge) has made possible, however, some degree of understanding of conditions which must be met. The following four figures illustrate a conceptual design for an 850-Mev accelerator which we believe could deliver approximately 100 μ a of high energy protons: Figs. 7, 8, 9, 10, and 11.

The machine has the following characteristics: the maximum orbit diameter 21 feet, the central flux density 8 kilogauss, and the maximum flux about 20 kg. The clear gap between magnetic poles is 4 inches and the frequency is 24.4 megacycles/second. The magnet is composed of eight sectors, which provide for eight hills and valleys in the magnetic field. The degree of twist is indicated by the hills spiraling 72 degrees. A double curved resonant cavity is proposed, instead of the conventional dees, for accelerating the protons.

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Fig. 7. Artist's Sketch of an Eight-Sector Magnet with Azimuthally Varying Field (ORNL Design).

Fig. 8. Section through Installed Magnet.

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Fig. 9. Artist's Sketch for a Double Curved Accelerating Cavity (ORNL Design).

Fig. 10. Plan View of Azimuthally Varying Field Cyclotron (ORNL Design).

SECTION A A

Fig. 11. Section View A-A of Preceding Figure.

The biggest engineering problem for such a machine is, of course, the design of the magnet. In order to arrive at an optimum economic design, it has been necessary to take into account the cost of operation of the magnet. This follows because, for example, higher power will reduce the amount of copper and steel required. We have decided somewhat arbitrarily to include the cost for 50,000 hours of electricity as part of the capital cost. A major technical problem is that of achieving an adequately large gap and still obtaining suitable field variation amplitudes. The cost of current for the pole face windings goes up rapidly with gap and also with absolute value of the magnetic field. The present estimated cost for the magnet for this machine, including the cost of electricity for 50,000 hours, is approximately 2.2 million dollars. We still hope to reduce this by further refinements and new ideas.

It will be necessary for the protons to pass through the first so-called imperfection resonance, which occurs in this design at 460 Mev. It is planned to measure the beam loss at this first resonance shortly on an electron model. If indeed the attenuation here is not too large, it is possible in principle to go to much higher energies, perhaps even 2300 Mev. We feel, however, that the proposed design energy represents the largest prudent extrapolation from present knowledge.

I would like to close by suggesting that laboratories which find themselves with early model unimproved cyclotrons can expect to obtain rich rewards in stimulating research, from moving toward incorporation of one or more of the design trends I have discussed today.

Fixed-Frequency Electron Cyclotron with Azimuthally
Varying Magnetic Fields. I. General Description*

Previous experimental work on fixed-frequency electron cyclotrons which accelerate well into the relativistic region was reported at recent meetings of the Society in Mexico City and Los Angeles by E. O. Lawrence and his co-workers. At the New York meeting last year D. W. Kerst and his colleagues described proposals for fixed-frequency accelerators incorporating spiralled azimuthal variations in the magnetic field.

Today we will give a brief description of an electron machine intended to carry to higher energies the work done at Berkeley. The primary purpose of this electron cyclotron is to obtain verification or evaluation of the calculations of ion orbits and of the focusing properties for the machine, particularly in the vicinity of some important resonances.

The proton accelerator which has received the most attention at our laboratory to date is an eight-sector spiral-pole cyclotron which can reach 850-900 Mev. The electron cyclotron which has been selected is, however, a four-sector non-spiralled machine which should produce approximately 220-kv electrons, equivalent relativistically to about 400-Mev protons. This appears to be the simplest and least expensive device for studying the region of interest, which should include at least the first imperfection resonance. It is hoped that the method of design will permit later extension of the work on the same machine, perhaps to eight or more sectors with the appropriate amounts of spiral. The spiral feature appears to have important advantages in reducing the required amplitude of magnetic field variation and in increasing the energy at which the resonances appear.

* A paper presented by R. E. Worsham and R. S. Livingston at the New York meeting of the American Physical Society, February 1956.

The prescribed magnetic field shape is shown in Fig. 12. The average field is 40 gauss at the center and increases with radius the appropriate amount to compensate for the increase in electron mass - 220-kv electrons at about 12 inches. The choice of the central field is somewhat arbitrary but represents a compromise between the ease of measuring fields or minimizing stray fields and, on the other hand, keeping the apparatus large enough for convenience of experimentation.

The hill fields go up to about 120 gauss, the valley fields to -25 gauss. The $3/2$ resonance, which occurs when the betatron radial oscillation frequency is $3/2$ of the electron orbital frequency, occurs at 156 kv or 9.7-in. average radius. The $2/1$ resonance occurs at 220 kv or 10.0-in. average radius. This orbit swings out to about 12 in. at the hills.

For threshold voltages of the order of 50 kv, in the equivalent proton machine for 400-Mev protons, an accuracy of $\sim 0.05\%$ is required in the average field. The azimuthal field amplitudes need, however, be accurate to only $\sim 5\%$. Because of this large difference it was decided to produce the two fields separately.

The general layout of the electron cyclotron is shown in Fig. 13. Features to be noted are:

1. A solid supporting structure is used which isolates the magnet from the vacuum forces.
2. The removable lid and side ports provide easy access; all leads and connections come through the bottom.
3. The dish-shaped 30-in. poles produce the average field.
4. Adjustments of the ion source can be made from the outside.
5. The radio frequency is 112 Mc/sec from a 5786 self-excited oscillator which provides up to about 1-kv acceleration per turn.
6. The pumping speed is about 500 l/sec.

A top view of the system is shown, Fig. 14. Here may be seen the method of supporting the upper portion of the magnet with six stanchions. The viewing and probe ports and the locations of access holes through the lower base are also shown.

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Fig. 12.

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Fig. 13.

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SCALE IN FEET

Fig. 14.

A section near the edge of the gap is shown in Fig. 15. In sequence are the copper dee wall, the insulators, the copper ground-plane liner, the sector coils for producing increased or decreased fields at the appropriate azimuthal angles, the water-cooled copper plate for cooling the sector coils, the circular trimming coils for correcting the average field, and the iron pole piece. The clear gap for experimental work is $1\frac{1}{4}$ inches. The magnet gap is $3\frac{1}{4}$ inches.

It is hoped that after an appropriate period of study, sufficient data will be available from this electron cyclotron regarding magnetic field errors, shimming problems, transport of ions through resonances, and detailed characteristics of the deflected beam, so that one can proceed with confidence to design a large cyclotron of this general type.

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Fig. 15.

A Fixed-Frequency Electron Cyclotron with Azimuthally Varying
Magnetic Fields, II. Magnet Design*

As Dr. Welton has pointed out in a preceding paper, the data for the design of the accelerator are obtained by assuming a succession of equilibrium orbits at progressively larger energies. The equation of the orbits is shown in Fig. 16. The Oracle is used in adjusting q_0 and q_1 for the fixed-frequency condition and for axial focusing. The magnetic field obtained from the orbits is shown here (Fig. 16) along those orbits as a function of q_1 . We need, then, to construct a magnetic field having these values along the orbits: average, $\cos p \theta$, and $\cos 2 p \theta$.

The fundamental component of azimuthal field varies directly with q_1 , and the average field has a small term involving q_1^2 . In this four-pole electron cyclotron, q_1 may be as large as 0.1. The value of q_1 may vary over 10%, and the particles would still have axial focusing. A 5% variation of the azimuthal field, 5% variation in q_1 , results in less than 1% variation of the average field. It was decided, therefore, to design the magnet so that the average and the azimuthal fields are produced as independently as possible.

With the dish-shaped iron pole pieces for the average field and with the sector coils (described in the preceding paper) for the azimuthal field, the interaction of the two fields is dependent upon the linearity of the iron and the accuracy of adjusting the currents and the positions of the coils.

* A paper presented by H. M. Moseley and R. E. Worsham at the New York meeting of the American Physical Society, February 1956.

The average field produced by the iron pole pieces must be circularly symmetrical. Therefore it was necessary to convert the magnetic field data long orbits to similar data along circles. After some labor, the results shown in Fig. 17 were obtained. It should be noted that this is the field to produce orbits having a sinusoidal variation about the circles q_0 , and q_1 is the peak fractional variation.

These fields are shown, Fig. 18, for the four-pole, zero-spiral magnet. This is the same data as Dr. Livingston's first slide, in slightly different form. The phase of the 80 component is such that the 40 and 80 components subtract on the hill center lines and add along the valley center lines. Introduction of an 80 component in the orbits can reduce the 80 component of the field to zero.

The average field in the median plane of the magnet is shown in Fig. 19. Here, ϕ is the potential of the magnetic equipotential surfaces. The surface $\phi = 0.1389$ gives a 4.690-in. gap at the center of the magnet and is matched in gap, slope, and curvature at the 12-1/4-in. radius to a surface which gives a 3 1/4-in. gap at the 15-in. radius. The maximum orbit radius under a hill is about 12 inches. The dashed lines indicate the plane of the sector coils which produce the azimuthal field. The coils are most effective in varying the field when they are closest to the median plane. Notice that the iron surfaces are nearly conical.

In an effort to find out exactly what coils and currents are needed for the azimuthal field, an attempt was made to calculate the field of the sector coils by using the field of the coils plus the field of approximate images which are formed in the iron. The amount of labor involved in the calculations proved to be rather great even with the use of the Oracle. Also, since the accuracy of the method was questionable, the process is now being held in reserve awaiting measured data on the coil field to serve as a check.

A plan view of the coils of one sector is shown in Fig. 20. Placement of the coils is fixed by pins located in the corners of the triangles or trapezoids. Because of the smaller iron gap near the edge of the machine, the field tends to be too high there, so that a second layer of coils to obtain greater field variation near the center was necessary. The cost of a complete set of these coils, including the circular windings which provide correction for the average field, is quite nominal. The sector we have wound for tests is held together with a resin, "Laminac." The resin in turn bonds the windings to a water-cooled copper sheet which also provides support. Each set of 16 series windings will be controlled by a current-regulated power supply. Provision is made for controlling, independently, the current in any coil or group of coils.

The magnetic field will be measured with a flux-gate magnetometer to an accuracy of about 5 milligauss or 0.05%. The magnetometer pickup will rotate along circles of various radii. Knowledge of the Fourier components of field produced by each coil will permit determination of currents required in each coil to set the field to prescribed values in the median plane.

ORBIT EQUATION:

$$\frac{r}{a} = q(\beta, \theta) = q_0(\beta) \left[1 + q_1(\beta) \cos p\theta \right]$$

$$z = 0$$

$$a = \frac{c}{\omega}$$

MAGNETIC FIELD:

$$\begin{aligned} \frac{B_z(\beta, \theta)}{B_0} = F_{z_0}(\beta, \theta) = & \frac{1 - \frac{1}{2}(\rho^2 - 1)q_1^2}{\sqrt{1 - \beta^2}} \\ & + \frac{(\rho^2 - 1)q_1}{\sqrt{1 - \beta^2}} \cos p\theta \\ & - \frac{\frac{5}{4}(\rho^2 - \frac{2}{5})q_1^2}{\sqrt{1 - \beta^2}} \cos 2p\theta \\ & + \dots \end{aligned}$$

WHERE:

$$q_1 = q_1(\beta)$$

Fig. 16.

MAGNETIC FIELD:

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$$\begin{aligned}
 F_{z_0}(q_0, \theta) = & \frac{1 - \frac{\rho^2 - 1}{2} \left\{ \frac{q_1^2}{1 - q_0^2} + q_0 q_1 \frac{dq_1}{dq_0} \right\} + \frac{q_0^2}{4(1 - q_0^2)} \left\{ \rho^2 q_1^2 + \frac{3q_1^2}{1 - q_0^2} + 2q_0 q_1 \frac{dq_1}{dq_0} \right\}}{\sqrt{1 - q_0^2}} \\
 & + \frac{\left\{ \rho^2 - \frac{1}{1 - q_0^2} \right\} q_1}{\sqrt{1 - q_0^2}} \cos p\theta \\
 & - \frac{\frac{5}{4} \left(\rho^2 - \frac{2}{5} \right) q_1^2 + \frac{\rho^2 - 1}{2} q_0 q_1 \frac{dq_1}{dq_0} - \frac{q_0^2}{4(1 - q_0^2)} \left\{ \frac{3q_1^2}{1 - q_0^2} + 2q_0 q_1 \frac{dq_1}{dq_0} - 2(\rho^2 - 1) q_1^2 \right\}}{\sqrt{1 - q_0^2}} \cos 2p\theta \\
 & + \dots
 \end{aligned}$$

WHERE

$$q_1 = q_1(q_0)$$

Fig. 17.

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Fig. 18.

$$F_{z_0} = \frac{1 - 1.891q_0^4 + 6.628q_0^6 - 7.126q_0^8}{\sqrt{1 - q_0^2}}$$

$$\phi = zF_{z_0} + \frac{z^3}{6} \mathcal{L}F_{z_0} - \frac{z^5}{120} \mathcal{L}^2 F_{z_0} + \dots$$

$$\text{where: } \mathcal{L} = \frac{\partial^2}{\partial q_0^2} + \frac{1}{q_0} \frac{\partial}{\partial q_0}$$

Fig. 19.

EACH COIL CONSISTS OF 55 TURNS OF NO. 24 WIRE. COIL CROSS-SECTION: $\frac{1}{4} \times \frac{1}{8}$ in.

FIRST LAYER OF SECTOR COILS

SECOND LAYER OF SECTOR COILS

Fig. 20.