C00-2200-5

NUMERICAL SIMULATIONS OF PLASMAS
WITH SMOOTHING IN PHASE SPACE
AND FILTERING IN TIME

Progress Report

J. Denavit

Northwestern University

Evanston, Illinois

October 1, 1973 - September 30, 1974

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Prepared for the U. S. Atomic Energy Commission
Under Contract No. AT(11-1)-2200

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

leg

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

The research is directed toward the development and testing of new numerical methods for plasma simulation and their application to physical problems of current interest in Controlled Thermonuclear Research. During the past year, research on the project has been concerned with the following specific problems.

(i) Computer simulations of whistler wave propagation have been carried out using a particle code developed earlier in the program. These computations have shown the existence of a whistler sideband instability and of secondary emissions from large-amplitude wavepackets caused by phase-correlated electrons.

(ii) The study of particle-pushing algorithms applicable on a time scale larger than ω_{pe}^{-1} has been continued. Such an algorithm has been tested on simple problems such as Landau damping and the trapping of an energetic electron beam.

(iii) A code to simulate collisional effects in plasmas has been developed and applied to the study of weak collisional effects in anomalous absorption and heating. This code is based on a one-dimensional Fokker-Planck operator and allows a velocity dependence of the collision frequency.

(iv) Fluid codes have been developed to compute the dynamics of one- and two-dimensional Langmuir solitons. Plane and annular solitons have been shown to be unstable to perturbations in the transverse direction and the collapse of annular solitons in the azimuthal direction is faster than their radial collapse.

RESEARCH ACCOMPLISHED SINCE LAST RENEWAL

The proposed research for the present period included the following problems (1) a study of the effect of trapped electrons on whistler wave propagation, (2) the development of long-time-step algorithms, and (3) the study of weak collisional effects on turbulent absorption in plasmas. The progress accomplished on these problems during the past year is described below. In addition, new research has been initiated on the use of fluid codes in dynamical studies of one- and two-dimensional Langmuir solitons. This research has been undertaken because of its growing significance to turbulent absorption and heating in plasmas and the possibility to combine such fluid codes with particle simulations of the wave-particle interactions.

1. Trapping Effects in Whistler Propagation. Analytical and computational studies of the whistler sideband instability and of secondary emissions from whistler wavepackets caused by phase-correlated electrons have been continued. These studies include computer simulations of the propagation of whistler wavepackets in loss-cone unstable plasmas showing the wave growth and saturation, and the particle drift into the loss-cone. These results have been presented in two paper co-authored with R. N. Sudan, "Whistler Sideband Instability" (to be published in The Physics of Fluids) and "Effect of Phase-Correlated Electrons on Whistler Wavepacket Propagation" (submitted for publication). The simulation code used in the above studies is applicable to other problems, such as cyclotron heating, requiring self-magnetic field computations.

2. Long-Time-Scale Algorithm. The study of numerical methods applicable to long time scales has been continued. The time steps in conventional particle simulation methods are limited by electron dynamics to values $\Delta t \sim 0.2 \omega_{pe}^{-1}$ (ω_{pe} : electron plasma frequency). Thus, particle simulations involving ion dynamics or other phenomena varying at a time scale much longer than ω_{pe}^{-1} are often expensive or unfeasible. A promising approach is the use of hybrid codes in

which the dynamics of the main plasma is followed using a fluid model, with particle simulation limited to resonant particle interactions which cannot be included into a fluid formulation. We have tested such an algorithm in terms of simple problems such as Landau damping and the trapping of an energetic electron beam. This method is summarized in the Appendix and a more complete paper is in preparation.

3. Collisional Effects on Turbulent Absorption in Plasmas. A numerical algorithm has been developed to simulate collisional effects in plasmas in the regime where plasma instabilities are dominant but are modified by weak collisions. This algorithm is based on the method of periodic smoothing in phase space* in which collisions are introduced in terms of a one-dimensional Fokker-Planck operator. This method is presented in the paper "Simulation of Collisional Effects in Plasmas" which has been recently submitted for publication.

Computer simulations of collisional effects on electron heating due to parametric instabilities have been carried out using the above method. These simulations show that weak collisions, with collisions frequencies $\nu \sim 0.001 \omega_{pe}$ (ω_{pe} : electron plasma frequency), and driving fields E_D corresponding to $\eta = E_D / (4 \pi n_e T_e)^{1/2} \approx 0.25$ can indeed channel a significant fraction of the absorbed energy into the main body of the electron distribution, rather than in the formation of suprathermal tails as occurs in the collisionless case. These results have been presented at the 4-th Annual Anomalous Absorption Conference (Livermore, Cal. April 1974).

4. Dynamics and Stability of One- and Two-Dimensional Langmuir Solitons. This study, done on cooperation with the Laboratory of Plasma Studies, Cornell

* J. Denavit, Journal of Computational Physics 9, 75 (1972).

University, was carried out using fluid codes, which solve numerically the fundamental equations^{*}, including the effect of ion inertia, in one- and two-dimensions in Fourier transformed space.

In one dimension our investigation of the interaction of two Langmuir solitons show that (i) if ion inertia is neglected then the two solitons pass through each other and emerge without change in shape or amplitude; (ii) inclusion of ion inertia allows the possibility of energy being either radiated or absorbed as ion sound waves. Thus two solitons can coalesce under certain conditions to form a single soliton with the excess energy being radiated away. In two dimensions we have studied the stability of plane Langmuir solitons to perturbations in the transverse direction and of cylindrically symmetric solitons to azimuthal perturbations. Both cases are unstable and the collapse of annular solitons in the azimuthal direction is faster than their radial collapse.

The results of these studies will be presented in two papers (co-authored with N. R. Pereira and R. N. Sudan) to be presented at the forthcoming of the APS Division of Plasma Physics (Albuquerque, Oct. 28-31, 1974, papers 3G1 and 3G2).

^{*} V. E. Zakharov, Sov. Phys., JETP 35, 908 (1972).

PERSONNEL

During the present period (1 February 1974 to 31 January 1975) research on the project is carried out by the following personnel:

1. Principal investigator:

Professor J. Denavit, 10% of time from 1 February 1974 to 15 June 1974

15 September 1974 to 31 January 1975

Full time from 1 September 1974 to 14 September 1974

2. Research assistants:

Carl E. Rathmann

1 May 1974 to 30 September 1974

John Vomvoridis

1 May 1974 to 31 December 1974

3. Student assistant (Programmer):

John E. Holm

15 June 1974 to 9 August 1974

PUBLICATIONS

(See Table I, last page of report)

APPENDIX

LONG-TIME-SCALE PARTICLE SIMULATIONS

The time scale of conventional particle simulation methods⁽¹⁾ is limited by electron dynamics which require the time step Δt to satisfy the conditions $\omega_{pe} \Delta t \ll 1$, $kv_{Te} \Delta t \ll 1$ where ω_{pe} is the electron plasma frequency, v_{Te} the electron thermal velocity and k the wavenumber. The first condition requires that the time step be small compared to the plasma period and the second condition states that a thermal electron travels only a small fraction of the wavelength during a time step. For most applications these limitations yield values of Δt in the range from $0.1 \omega_{pe}^{-1}$ to $0.4 \omega_{pe}^{-1}$ and the simulation of phenomena varying at a time scale much slower than ω_{pe}^{-1} can become unfeasible because of computer limitations.

In the study of resonant wave-particle interactions, which has been an important application of particle simulation methods, it is sometimes possible to separate the fluid-like behavior of the plasma from resonant wave-particle interactions. This approach has been used in analytical studies of the damping of linear and nonlinear plasma oscillations⁽²⁾ in which the wave amplitude is allowed to vary according to the energy transferred from the waves to the resonant particles.

In the present algorithm the plasma is represented in terms of a fluid model with an electric field, in the one-dimensional case, of the form $E(x,t) = \sum_k E_k \sin(kx - \omega_k t + \phi_k)$ where the mode amplitudes E_k and phase angles ϕ_k vary at a slow time scale compared to the high frequencies ω_k . This fluid model includes damping (or excitation) terms, with time dependent coefficients $\nu_k(t)$ accounting for resonant wave-particle interactions, which are updated after each time step Δt in terms the motions of a set of test particles.

The velocities and positions of the test particles are advanced over the time interval Δt according to their linear response to the electric field, given as a sum of increments due to each mode,

$$v_n(t + \Delta t) = v_n(t) + \sum_k \delta v_{nk} + O(E^2),$$

$$x_n(t + \Delta t) = x_n(t) + v_n(t)\Delta t + \sum_k \xi_{nk} + O(E^2),$$

where

$$\delta v_{nk} = \frac{eE_k \Delta t}{m} \left\{ \left[(\cos \theta_{nk} - 1) / \theta_{nk} \right] \cos \psi_{nk} - \left[\sin \theta_{nk} / \theta_{nk} \right] \sin \psi_{nk} \right\},$$

and

$$\xi_{nk} = \frac{eE_k \Delta t^2}{m} \left\{ \left[(\sin \theta_{nk} - \theta_{nk}) / \theta_{nk}^2 \right] \cos \psi_{nk} + \left[(\cos \theta_{nk} - 1) / \theta_{nk}^2 \right] \sin \psi_{nk} \right\}.$$

Here $\psi_{nk} = kx_n(t) + \varphi_k$ is the phase of particle n relative to wave k and $\theta_{nk} = k[v_n(t) - \omega_k/k]\Delta t$ is a resonance parameter. Note that the increments δv_{nk} and ξ_{nk} corresponding to each mode are proportional to resonance functions which fall as θ_{nk}^{-1} or faster. Since resonant wave-particle interactions only are of interest here, these resonance functions may be cut off, and the increments applied only to resonant particles for which $\theta_{nk} \lesssim 2\pi$.

The time interval Δt must be small compared to the slow time scale of E_k and φ_k and to the trapping frequencies $\alpha_k = (eE_k/m)^{1/2}$. Computations of trapped and untrapped electron trajectories using this algorithm indicate that values of $\Delta t \sim 0.5 \alpha_k^{-1}$ are suitable. In the time evolution of Langmuir solitons, for example, the high-frequency electric field amplitude varies on a time scale

$\sim (m_e/m_i) \omega_{pe}^{-1}$ and the trapping frequency is $\sim (m_e/m_i)^{1/2} \omega_{pe}^{-1}$ (3). The present algorithm would therefore be well suited to include resonant wave-particle interactions into existing fluid computer models of Langmuir turbulence and to follow the resulting turbulent heating of the plasma on a long time scale.

The results of a simple test of this algorithm involving the nonlinear damping of plasma oscillations is shown in Fig. 1. A single mode with $k\lambda_D = 0.25$ is considered with an initially Maxwellian distribution function and an initial amplitude corresponding to a trapping frequency $\alpha = \omega_{pe}/40$. This computation was carried out with $\Delta t = 25 \omega_{pe}^{-1}$. The expected behavior of the electric field is observed with an initial damping close to the theoretical value, $\gamma_L = -0.0033 \omega_p$, followed by decaying amplitude modulations. The phase-plane plots of the resonance region show the formation of the familiar trapping vortex associated with this phenomenon.

REFERENCES

1. In this Appendix we include under "particle simulations" any method such as finite differences or Fourier or Hermite transforms, to solve the Vlasov equation with a detailed representation of the distribution function.
2. John Dawson, Phys. Fluids 4, 869 (1961); T. O'Neil, Phys. Fluids 8, 2255 (1965).
3. V. E. Zakharov, Sov. Phys. JETP 35, 908 (1972).

Figure 1

List of Publications

(Table I)

1) Publications submitted or in preparation during the past year

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percent AEC Support</u>
"Whistler Sideband Instability"	J. Denavit and R. N. Sudan	Submitted to <u>The Physics of Fluids</u>	25%
"Effect of Phase-Correlated Electrons on Whistler Wavepacket Propagation"	J. Denavit and R. N. Sudan	Submitted to <u>The Physics of Fluids</u>	25%
"Simulation of Collisional Effects in Plasmas" (AEC Technical Report C00-2200-6)	C. E. Rathmann and J. Denavit	Submitted to <u>Journal of Computational Physics</u>	50%
"Two-Dimensional Stability of Langmuir Solitons"	J. Denavit, N. R. Pereira and R. N. Sudan	Submitted to <u>Physical Review Letters</u>	33%
"Long-Time-Scale Particle Simulations"	J. Denavit	In Preparation	50%

2) Publications submitted during previous year

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percent AEC Support</u>
"Theory of Triggered VLF Emissions from the Magnetosphere"	R. N. Sudan and J. Denavit	Published in <u>Physics Today</u> , Vol. 12, Dec. 1973	25%
"Discrete Particle Effects in Whistler Simulation"*	J. Denavit	Published in <u>Journal of Computational Physics</u> , Aug. 1974	50%

* Note: This paper was presented under the title "Numerical Simulation of Whistler Wave Propagation" at 6-th Conference on Numerical Simulation of Plasmas, Berkeley, July 1973.