UNIVERSITY OF ILLINOIS
GRADUATE COLLEGE
DIGITAL COMPUTER LABORATORY

TECHNICAL PROGRESS REPORT

- PART I - HIGH-SPEED COMPUTER PROGRAM
PART II - MATHEMATICAL METHODS
PART III - SWITCHING THEORY
PART IV - ILLIAC USE AND OPERATION -
GENERAL LABORATORY INFORMATION

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PART I

HIGH-SPEED COMPUTER PROGRAM

This work is supported in part through Contract No. AT(11-1)-415 of the Atomic Energy Commission. The contract is supported jointly by the Atomic Energy Commission and the Office of Naval Research. In addition, this work is supported through Contracts N6ori-07130 and Nonr-1834(15), both from the Office of Naval Research.

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1. Transistor Circuits

The investigation of the emitter-base crosscoupled flipflop with flow-gating was continued throughout the month. Several alterations of the basic circuit shown in last month's progress report were considered. Approximate designs were calculated by hand, and the computer was used in a three-to-four step improvement process. From the considerable amount of data, it was determined that the alpha requirements for the flipflop were much higher than anticipated. As for the standard flow-gating flipflop one must have $\alpha \geq .98$. Under these circumstances flow-gating for this type of flipflop was abandoned, especially since for double-gating its advantages are minor.

A new C or $\bar{C}$ element was developed. This is a storage element with two inputs. If the inputs are different, the last state is retained; if they coincide, the output follows the input in the case of the C element, or gives the complement in the case of the $\bar{C}$ element. Many circuits can be designed which have this property (see July, 1956, Progress Report), but the tolerance conditions are usually hard to satisfy. Figure 1 gives the layout of the new design which is essentially a Schmitt trigger preceded by two switching inverters having diode ANDs and ORs between the collectors. The output of the AND and OR feed the input of the Schmitt trigger through two Zener diodes, their voltage difference providing the necessary hysteresis. Tolerance calculations show that the new circuit is easy to design for worst case conditions. Preliminary tests gave operation times of the order of 30 μ s for either one of the outputs. It should be remarked that the C output is not truly a last moving point.

Figure 1
 C or $\bar{C}$ ELEMENT

A plug-in package was designed for the driver circuit discussed in last month's progress report. Some circuit modifications were made in order to comply with the standard supply voltages. Figure 2 shows the redesigned circuit, and Figure 3 gives typical waveforms obtained with a logical oscillator using the output of the driver to provide the reply-back signal. More speed could be obtained by using a free-running logical oscillator.

Figure 2
 PLUG-IN DRIVER CIRCUIT
 (100 ma, Rise Time < 20 µs)

Figure 3

APPROXIMATE WAVEFORMS OF OUTPUTS OF LOGICAL OSCILLATOR
USING PLUG-IN DRIVERS

(N. Wiseman, S. Ray, N. Johnson and J. Muerle)

2. Arithmetic Unit

A proof was devised confirming several features of the multiplier recording described in Report No. 80. Using the following notation for the k^{th} step of a binary multiplication,

w_{n-k+1} mode digit sensed during k^{th} step

w_{n-k} new mode digit resulting from k^{th} step

y_{n-k} least significant of multiplier digits sensed during k^{th} step

z_{n-k} recoded multiplier digit (each $z_i = -1, 0, \text{ or } +1$),

it can be shown that

$$z_{n-k} = y_{n-k} + w_{n-k+1} - 2w_{n-k}.$$

The recoded multiplier y , after $n+1$ steps, is then

$$\begin{aligned} y &= \sum_{i=0}^n 2^{-i} z_i = \sum_{i=0}^n 2^{-i} y_i + \sum_{i=1}^{n+1} 2^{-i+1} w_i - \sum_{i=0}^n 2^{-i+1} w_i \\ &= y_0 + \sum_{i=1}^n 2^{-i} y_i + 2^{-n} w_{n+1} - 2w_0. \end{aligned}$$

For a two's complement representation of y ,

$$y = -y_0 + \sum_{i=1}^n 2^{-i} y_i, \text{ implying that } w_{n+1} = 0 \text{ and } w_0 = y_0.$$

$$\text{For one's complement, } y = -y_0(1-2^{-n}) + \sum_{i=1}^n 2^{-i} y_i,$$

$$\text{implying that } w_0 = w_{n+1} = y_0.$$

It should be noted that $w_0 = y_0$ can be accomplished by setting $y_{-1} = y_0$

(J. Robertson)

Calculation of Carry Chain Lengths. A Monte-Carlo program for determining lengths of carry-carry completion chains was run. Results are summarized in the table below, in which the average chain length c is given as a function of the number of digital positions n . 4096 points were used to calculate each entry in the table.

n	c	n	c
40	4.69	52	5.06
41	4.74	53	5.09
42	4.74	54	5.17
43	4.75	55	5.14
44	4.75	56	5.15
45	4.84	57	5.25
46	4.88	58	5.16
47	4.95	59	5.24
48	4.89	60	5.28
49	4.95	61	5.29
50	5.00	62	5.34
51	5.04	63	5.35
		64	5.38

The construction of the Monte Carlo program lead to the realization that certain operations are really nothing more than Boolean-homomorphisms of the Boolean algebra which the various digit combinations give rise to. Examples are:

- Logical right and left shifts of A, circular shifts of A (maps 53 digits of A onto the 53 digits of A).
- Logical right and left shifts of AQ (maps 106 digits).
- When x is a double length number in A, Q, taking most and least significant parts of x, $m(x)$, $L(x)$, which maps AQ onto an algebra of 53 digits. Note that

$$\begin{aligned} m(x \vee y) &= m(x) \vee m(y) & L(x \vee y) &= L(x) \vee L(y) \\ m(\bar{x}) &= \overline{m(x)}, \text{ etc.} \end{aligned}$$

These considerations lead to a formal characterization of addition and subtraction by functional equations:

$$\begin{aligned} \text{Addition: } a \times b &= (a + b) \times \varphi(ab) \\ \text{Subtraction: } a \circ b &= (a + b) \circ \varphi(\bar{a}b) \end{aligned}$$

These relations have been completely analyzed.

(R. Farrell)

3. Control Studies

Floating Point Arithmetic. It is proposed that a floating point number in the core memory should consist of a 45-bit fractional part and an 8-bit base 4 exponent. This is equivalent to a range of about 10^{+77} , and a minimum precision, for standardized numbers, of one part in 4×10^{12} .

Non-standardized multiplication may be performed by first comparing the two fractional parts, standardizing the one of larger absolute value, and noting whether its absolute value then lies in the range $1/4$ to $1/2$. The unrounded product of the two fractional parts (one standardized) is formed, and if the magnitude of the standardized factor was between $1/4$ and $1/2$, the product is shifted left one place (multiplied by 4), provided this does not cause overflow. This means, in effect, that the less significant number is multiplied by a number between $1/2$ and 2 in magnitude, and the result scaled only if overflow occurs.

For a base 4 computer, this minimizes the maximum change in significance (one binary place left or right), but does not have the property that the expected value of the fractional part of the product equals that of the smaller of the fractional parts of the factors. If non-zero standardized fractions are assumed to be distributed as $1/x$ (the unique continuous density invariant under multiplication by an arbitrary non-zero factor and change of base), then the expected absolute value of the product is about 8% higher than the smaller factor. Other assumed distributions give 10% or 15% increase. It is felt that if significance changes during multiplication, it is better for it to increase than decrease.

Data on the inversion of a 6×6 Hilbert matrix by non-standardizing floating point was obtained and sent to Professor Metropolis. This appears to be a case in which the non-standardizing arithmetic is not appropriate for the back substitution since rounding errors are not independent.

(R. Farrell and D. Gillies)

Trial Programming

Eight sets of ordinary differential equations, which arose at MURA, were studied as examples of miscellaneous algebra. They were sufficiently complex and different that they probably provide a fair test for this type of problem for the

proposed computer. Exclusive of functional subroutines, the time for arithmetic was between one and two times the memory time, and the majority took nearly twice as long for arithmetic. Except for a short delay of the arithmetic unit at the start of most functional subroutines, overlap of memory time with arithmetic time was very nearly perfect. The reasons for the surprisingly small use of the core memory appear to be

1. the high frequency of such operations as squaring, which require only one memory reference per multiplication,
2. the high density of multiplications and divisions,
3. the large number of temporary results which can be held in fast registers rather than stored in the main memory
4. the fact that nearly all instructions can be made short by holding addresses in B-lines, and
5. the absence of arithmetic-conditional jump instructions.

(W. Gear)

Over-all Design. From considerations of the duty cycle of the core memory, it is felt that if sufficiently cheap slow circuitry can be built, the input-output control should be able autonomously to transfer blocks of data between the drum and core memory and between any two selected tape units and the core memory, word by word, using only one core memory cycle per word transferred, and only using the program interrupt system in the event of an error or at the end of the block. An interruption system requires that advanced control be halted and arithmetic control allowed to catch up before the interruption takes place. When a short loop is executed in $O_1 O_2$, the control counter should count up and down to correspond to the actual core address of the word being executed, and a fairly simple interlock is required to enable advanced control to read out the first word of a loop if it happened to execute first the second word of that loop. This is essential for resuming the program after an interrupt, and desirable for normal operation too.

If $O_1 O_2 X Y R_2 R_3$ and the 12 B-lines are realized as one flow-gating memory which is shared by the core memory, arithmetic control, advanced control, and possibly the input-output control, then the duty cycle of this memory can be very high. If this causes a slow-down in speed, then the B-lines could be held in a second flow-gating memory.

(R. Farrell, W. Gear and D. Gillies)

4. Core Memory

A 1,000-core-test board was modified to allow switching of bit cores at various distances from the driven end. The effect of the discontinuity introduced by a switching core into the transmission line formed by the wired cores can be studied. The possibilities of using core drivers having constant voltage instead of constant current characteristics can be investigated simultaneously.

(K. Smith)

5. Test Equipment

A transistor-aging tester (Dwg. M778) was built and tested. This device subjects 10 transistors simultaneously to 100 mw dissipation (10 ma, 10 v) while monitoring V_{eb} fwd. It is possible to set a threshold on a neon-bulb alarm which indicates that a transistor V_{eb} fwd. has exceeded some desired limit.

The tester uses only one power supply (Illiac + 100) and to this extent needs no special attention. Certain critical circuits have been made sufficiently redundant to prevent destruction of transistors due to tester internal failures.

(K. Smith)

6. Input-Output and Auxiliary Storage

Two versions of magnetic head driving circuits were tested using dummy heads; one was judged suitable for further testing when sample recording heads are received. Two sets of sample heads were ordered: Brush and Davies. Possible means of measuring tape skew (interchannel time-displacement errors) and of reducing its effect on maximum pulse packing density are currently under consideration.

Writing Circuit Tests. Two writing circuits were tested by observing the resulting current waveform through a dummy recording head. The more successful of the two circuits could handle frequencies up to 75 kc (corresponding to approximately 500 pulses/inch packing density at 150 inches/sec tape speed). Since the dummy head was a 10-mh choke, performance of this driver with a low-inductance head should be considerably improved. Heads are available with inductance as low as 1 mh, according to the Ampex local representative. Furthermore, the writing current required for tape saturation with some pulse recording heads is as low as 1 ma, in comparison with the 6 to 10 ma used in the tests.

The following sample heads have been ordered, mainly for use in further driving circuit tests: Brush BK-3202 and BK-3302, and Davies 2001AP and 2001DP.

Tape Skew and Packing Density. In magnetic-tape-recorder operations using a clock track to synchronize the data tracks, the maximum reliable packing density in any channel may be determined by the maximum interchannel displacement error on playback. Packing density may also be limited by such things as dust and oxide build-up, but apparently these effects are not a serious problem at densities of 200 pulses/inch, which seems to be a standard for the industry at present. At any event, skew effects would become the limiting factor under sterile operating conditions, and so are worth considering.

A pertinent question concerns the maximum timing error that can be tolerated between tracks--by what fraction of a bit period or wavelength (on the tape) may the clock pulse be displaced from the nominal bit position, without resulting in erroneous readings? This fraction must be determined experimentally, based on our own standard for reliable operation. Various data available from Ampex literature indicate that the answer will be something less than 0.4 bit period and perhaps something more than 0.1 bit period.

It may be possible to use more than one clock track on one tape, effectively dividing the tape width into small groups of independently clocked channels. This would tend to reduce the maximum interchannel displacement error between channels in any one group, provided that the interchannel displacement error follows some fairly smooth pattern across the tape. (Best results would be obtained if the displacement were linear across the tape, but some improvement would result just so long as the maximum interchannel displacement does not occur between adjacent channels.) If the displacement should be linear, for example, and if some value has been determined for the fraction discussed in the previous paragraph, the packing density in any channel can be increased by a factor n , where n is the number of independently clocked groups of adjacent tracks. Allowing for the increased number of tracks used for clocks and, therefore, not available for data, it is still reasonable to increase the number of bits recorded per inch of tape by a factor of three. For other than linear distributions, this factor is reduced.

The practicality of using several clock tracks will thus depend in part upon the shape of the interchannel displacement curve, which must be determined experimentally. The most promising scheme for determining this involves generating an electrical analog of the displacement of each channel from a single reference channel, smoothing, and recording all such signals simultaneously on a recording oscillograph.

Another important question will be the amount of extra circuitry required by multiple clock tracks. Data recorded simultaneously on several tracks will not be read at the same time. A separate operation is required to resynchronize each row of data before transmitting it on to the receiving equipment. An estimate of the amount of extra equipment required is being prepared at this time.

This multiple clock arrangement leads naturally to another consideration: let each track be self-clocking¹. Possible complications of this arrangement, including equipment costs, will be investigated in the near future.

There is one recording head arrangement available which puts up to 47 channels on a tape 1.25 inches wide. This involves two separate head stacks, displaced along the tape, with heads of the second writing on the spaces between heads of the first. The same resynchronization system needed for the multiple clock system described above might be of use in this instance. Possible variations in tape length between the two head stacks could be ironed out in the resynchronization system.

(R. Cummins)

A Proposed Method for Coding on Magnetic Tapes and Drums. A number of systems of coding binary information for storage on magnetic tapes and drums are in use. In most of these systems, including both the return-to-zero (RZ) and the non-return-to-zero (NRZ) schemes, the length of time between reversals of current in the writing heads varies over wide limits, and is determined by the number being stored. For example, the direction of writing current as a function of time for the binary number 11010000011, in the system now used for the Illiac drum, is shown in Figure 4(a).

1. Richards, R. K., Digital Computer Components and Circuits, D. Van Nostrand, 1957, pp. 332-334.

A negative current records a 1; a positive current records a 0. The shortest current pulse is one bit length, b , and the longest is nb for a sequence of n 0's or n 1's. It is difficult to maintain a uniform writing current for a long string of 0's or 1's, even at modest writing speeds. If the current falls off appreciably, one or more erroneous 1's or 0's may be read at the end of the string.

Similar arguments apply to the system illustrated in Figure 4(b), in which a change of direction of magnetizing current represents a 1; no change represents a 0. In this case the longest pulse length is $(n+1)b$ for a sequence of n 0's.

When one attempts to increase the density of bits by increasing the writing speed, the problem becomes more acute. From a circuit point of view, it would be possible to speed up the current reversal by overdriving, provided the ratio of the longest pulse length to the shortest one could be held to a small value. Several methods of achieving this have been proposed. For example, the bits can be arranged in small blocks, with an arbitrary current reversal at the end of each block², but this increases the complexity of coding and does not seem to be practical for restricting the ratio of pulse lengths to, say, a factor of two or three. Another suggestion is the double-pulse method, in which the direction of the changes is used to distinguish between a 0 and a 1.³ As illustrated in part (c) of Figure 4, positive changes represent 1's and negative changes represent 0's. Reading is done only at the center of the bit cell; the signals generated at the cell edge between two 1's or two 0's are disregarded. Gating of the reading circuit must be done to within at least $b/2$, but this does not seem to be a serious limitation. The double-pulse method has the distinct advantage of a ratio of only two between the longest and the shortest pulse length. However, the fact that the frequency response of the writing head must be twice that used for the systems of Figure 4 (a) and (b) is a serious disadvantage.

The scheme proposed here is illustrated in part (d) of Figure 4. A change of magnetization at the center of the cell is used to represent a 1; no

2. Ibid. p. 326.

3. Ibid. p. 332.

change to represent a 0, just as for the system of part (b). In addition, an arbitrary change at the bit cell edge is introduced whenever a 0 is to be followed by another 0. Thus, pulse lengths of b , $1.5b$ and $2b$ are present, for a maximum ratio of 2, but the frequency response required of the head is the same as for the system of (a).

Figure 4

A possible method of writing is shown schematically in Figure 5. Bits to be recorded on a given tape channel are gated into the control circuit at intervals of b , as set by a clock signal, C . A bit, A_1 , arriving at time t_0 is compared with the previous bit, A_0 , and stored in the register R . At $(t_0 + b)$, A_1 is gated out of the register and compared with A_2 , the newly arrived bit. If A_1 is 1, the OR circuit is actuated and the writing current reverses. If A_1 is 0 and A_2 is 1, nothing happens. If A_1 and A_2 are both 0, the OR circuit is actuated $b/2$ later, and the writing circuit reverses at $(t_0 + 1.5b)$. In addition to the clock pulses, C , another set, C' , will be required spaced halfway between the C pulses. Both sets could be derived from a sine wave used to regulate the timing of all channels being recorded, and to produce the clock track recorded on the tape or drum.

Figure 5

The writing current is shown again in Figure 6(a), and the voltage signals as read under ideal conditions are shown in (b). If the density is increased considerably, by increasing the speed, the voltage pulses are likely to become spread out, as suggested in (c), where solid vertical lines show the voltages that occur during the reading interval. Two smaller voltages, indicated by arrows might cause errors. Methods of eliminating them (besides the obvious idea of using a threshold) might be based upon some of the following characteristics of the errors:

1. A zero may be misread as a one, but the reverse error is not likely. More specifically, a 100.... sequence might be misread as 110...., or a001 sequence as011.
2. An erroneous signal at a cell center is always preceded and/or followed by a larger signal in the same direction at the cell edge. A correct signal at a cell center is always preceded and/or followed by either a smaller signal in the same direction or no signal at the cell edge.
3. If all center and edge pulses are read, a voltage of one polarity is always followed by a voltage of the opposite polarity.
4. If the reading strobe should get out of phase so far that cell edges were being read instead of cell centers, a partial check

could be based upon the fact that a pulse of length $2b$ can occur only between cell centers (a 101 sequence), never between cell edges.

Figure 6

(T. Murrell)

PART II

MATHEMATICAL METHODS

1. Iterative Methods for Solving Systems of Linear Equations

(Supported in part by the Office of Naval Research under Contract N6ori-07130)

The definite symmetric $N \times N$ matrix A in the equations $Ax = y$ has Young's property A if there exist two disjoint subsets S and T of the set of first N integers, such that the union of the subsets is equal to this set and if $a_{ij} \neq 0$, then either $i = j$ or i is in S and j is in T . If the matrix A has this property, a coordinate system may be chosen so that

$$A = \left\| \begin{array}{c|c} I_1 & S^t \\ \hline S & I_2 \end{array} \right\|$$

where I_1 and I_2 are unit matrices of M and $N - M$ dimensions.

In this coordinate system approximants to the solution of the system of equations $Ax = y$ may be written as

$$\eta^k = \left\| \begin{array}{c} \eta_1^k \\ \eta_2^k \end{array} \right\|$$

It has been shown that the vectors

$$\xi^k = \left\| \begin{array}{c} \eta_1^k \\ \eta_{k+1}^k \end{array} \right\|$$

where the η^k are defined by the recursive relations given by von Neumann (cf. February Monthly Progress Report) can be computed with the use of memory space for the storage of ξ^{k-1} alone and storage for ξ^{k-1} and ξ^{k-2} is not needed. Further, the ξ^k converge faster to the solution of the equations than do the η^k .

(G. Golub)

2. Hydrodynamic Flows with Shocks (Supported in part by the National Science Foundation under Grant G2794.)

A new program was written for solving the system of difference equations

$$mN_k^{n+1} = (1 - \theta_k^n) mN_k^n + \theta_{k-1}^n mN_{k-1}^n$$

$$mN_k^{n+1} u_k^{n+1} = (1 - \theta_k^n) mN_k^n u_k^n + \theta_{k-1}^n mN_{k-1}^n u_{k-1}^n + r(p_{k-1}^n - p_k^n)$$

$$mN_k^{n+1} E_k^{n+1} = (1 - \theta_k^n) mN_k^n E_k^n + \theta_{k-1}^n mN_{k-1}^n E_{k-1}^n + r(p_{k-1}^n u_{k-1}^n - p_k^n u_k^n),$$

where

$$p_k^n = (\gamma - 1) mN_k^n \left(E_k^n - \frac{1}{2} u_k^{n2} \right)$$

$$\theta_k^n = \frac{\frac{1}{2} r_1 (u_{k+1}^n + u_k^n)}{1 + r(u_{k+1}^n + u_k^n)}$$

$$r = \frac{\Delta t}{\Delta x},$$

for the approximate solution of one-dimensional problems of compressible flow.

The problem considered was that of a steady-state shock. For the true solution, the width of the shock is zero. A measure of the width of the shock, as represented by the difference equations at the n^{th} step of the calculation, is a quantity denoted by $2\sigma_n$ (cf. March Monthly Progress Report). The original purpose of the program was to test an upper bound for σ_n^2 which had previously been obtained analytically—that is, to test how well this upper bound approximated σ_n^2 .

While the program was being written, a lower bound and another "estimate" can be obtained which are sharper than those furnished by the upper and lower bounds of σ_n^2 . Both the upper and lower bounds for σ_n^2 are proportional to n , and another question of interest was whether σ_n^2 is also strictly proportional to n .

In the numerical example which was run, σ_n^2 was found to be very nearly (within 2%), but not exactly, proportional to n . If we write $\sigma_n^2 = \alpha n (\Delta x)^2$, the following is a summary of the numerical results:

upper bound α	=	.230
lower bound α	=	.125
"estimate" α	=	.200
true α	=	.196 $\pm$ 2%

(C. Farrington)

3. Determination of Flows Behind Shocks (Supported in part by the National Science Foundation under Grant G2794.)

Computations were performed to determine the flow behind a pseudo-stationary shock with an essential singularity. The shock was taken to have an inclination given by

$$\begin{aligned}\alpha(S) &= \alpha_0 & 0 \leq S \leq S_* \\ \alpha(S) &= \alpha_0 + K(S-S_*)^{1/2} & S_* \leq S \leq 2S_*\end{aligned}$$

That is, the parametric equations of the shock are

$$\frac{da_1}{dS} = -\cos \alpha(S) \quad \frac{da_2}{dS} = -\sin \alpha(S)$$

where the constants of integration of these equations are chosen so that

$$\begin{aligned}a_1(S_x) &= \frac{V \cos \chi}{\sin(\alpha_0 - \chi)} \\ a_2(S_x) &= \frac{V \sin \chi}{\sin(\alpha_0 - \chi)}\end{aligned}$$

In these equations V is the normal velocity of the shock into still air and χ is the angle the line connecting the origin and the singular point on the shock makes with the a_1 axis. We further determine K and S_* so that the shock is perpendicular to the a_1 axis at $S = 2S_*$. Thus, K and S_* are determined by

$$\alpha_0 + KS_*^{1/2} = \frac{\pi}{2}$$

$$\frac{V \sin \chi}{\sin (\alpha_0 - \chi)} = \frac{2}{K^2} (1 - \sin \alpha_0),$$

The calculations were performed for $\chi = 1^{\circ} 34''$ and $\alpha_0 = 62^{\circ} 6'$ and a V corresponding to a shock with an inverse pressure ratio of $p_0/p_1 = .811$. These are observed values occurring in a particular Mach reflection. It is the purpose of the calculations to see if they lead to pressure and density distributions similar to those observed. In particular, the loci, $\xi_2(S,T) = \text{constant}$, are being studied where the quantity ξ_2 is the inclination of the streak-lines behind the shock relative to the a_1 axis.

(Y. Chow)

4. Program Library (Supported by Office of Naval Research under Contract N6ori-07130)

F-6 Integration of a System of Ordinary Differential Equations with Automatic Control of Integration Interval. This program uses the classical Runge-Kutta method to integrate a system of ordinary differential equations. An estimate of the truncation error introduced during each step of the process is made during the step by comparing the results of the numerical integration with those obtained from a lower order process based on some of the intermediate calculations. The step interval is then shortened or lengthened appropriately so as to keep the truncation error just within a pre-assigned bound subject to the restriction that the step interval should never be made greater than a given maximum value.

(A. Nordsieck)

M-21 Matrix Multiplication. This new matrix multiplication program is an automatic routine which will handle larger matrices than previous programs by virtue of the fact that it uses the magnetic drum for temporary storage of data. It has been written so that no special scaling of the matrices is required.

(R. Polivka)

M-22 Eigenvalues and Eigenvectors of a Symmetric Matrix Using the Methods of Givens and Wilkinson: The method of Givens for finding eigenvalues of symmetric matrices, which was used in routine M-20, has been extended so as to yield the eigenvectors, as well, by use of a technique suggested by Wilkinson.

(C. Gear)

P-17 Maximum Speed Fraction Print to Twelve or Fewer Places Closed with One Program Parameter: This subroutine uses the same entry conventions as P-16 for punching correctly rounded fractions. The punching takes place at the maximum speed possible with the high-speed punch.

(R. Flenner)

PART III

SWITCHING THEORY

1. Distributive Circuit Theory

Three principal results have been obtained in the theory of distributive circuits which do not reach equilibrium. The cyclical behavior of such circuits can best be described in terms of certain elements of the change chart which are called "cyclically spanned". These are elements (α, i) , corresponding to join-irreducible elements $\underline{a}$ of the lattice of C states such that there is a cycling vector w having non-zero i th component w_i , and $(\alpha + w_i, i)$ is also an element of the change chart whose corresponding join-irreducible element is $\underline{a} + w$ which has the same terminal state as $\underline{a}$. Sets of cyclically-spanned changes describe the cycles that occur in the change chart in a way which is analogous to the description of cyclical behavior of the lattice of C states which was provided by the equivalence sets.

It may be shown that in a binary circuit if (α, i) is cyclically spanned and (β, j) is any element such that $(\alpha, i) \leq (\beta, j)$, then (β, j) is also cyclically spanned. Furthermore, the corresponding w_i and w_j are components of the same cycling vector w and $(\alpha + w_i, i) \leq (\beta + w_j, j)$. In such cases there is a (γ, i) such that $(\beta, j) \leq (\gamma, i)$.

(W. Bartky)

2. Time Tolerance Theory

A study is being made of the behavior of circuits under the assumption that the elementary components have time tolerances upon their actions. For each element i two times δ_i and M_i are specified and the assumption is made that if the element becomes excited at some time t , and remains excited, it will act at a time in the interval $t + \delta_i$ to $t + M_i$.

Circuits are classified as being of one of two types:

1. those for which there exists a number r , where $0 < r \leq 1$, such that a unique specified behavior can be achieved only if $\delta_i/M_i \geq r$ for some i , and

2. those for which no such r exists, or in other words, those for which we may make $\delta_i/M_i < r$ for all i and for any $r > 0$ which we may choose.

Asynchronous circuits which are of greatest interest to the designer are of type 2, and a way of designing circuits of this type is being sought.

(J. Shelly)

3. Logical Programming

An idealized computer was developed in order to study logical systems capable of expressing their own syntax. This computer was chosen as one which can easily be described, and yet possesses properties which simplify programming.

A memory location is regarded as containing a symbol (word) and is addressable by an ordered pair of symbols. A single instruction is provided which copies the symbol in one specified memory location into another and jumps to the next instruction. This instruction, therefore, consists of five symbols: two to specify the location copied from, two to specify the location which altered, and one for the jump.

A formal equivalence between this idealized computer and any universal computer (such as a Turing machine) was shown to exist, provided the memory is assumed to contain an infinity of symbols, initially. An investigation is being made of the possibility of writing programs describing the properties of the computer. A theorem has been obtained which applies to programs in which the data are not used to modify the instructions. Such programs may be described by a specification of their action upon data consisting of only unlike symbols. This theorem provides the link between the notation of variables used in mathematics and the more cumbersome notation used in programming.

(D. Muller)

PART IV

ILLIAC USE AND OPERATION - GENERAL LABORATORY INFORMATION

Machine Use

During April, specifications were presented for 13 new problems. This list does not indicate how the Illiac was used because large amounts of machine time may have been consumed by problems with numbers less than 1137.

1137 Chemistry. Reaction Kinetics of Polymers. A least squares fit will be made to determine the molecular weight and chain transfer constant from measured quantities.

1138 Anthropology. Analysis of Kinship Patterns of American Indian Tribes. The problem is a test of the major statistical description of kinship traits of American tribes reported by Driver and Massey in Comparative Studies of North American Indians. These authors used an inspectional technique for arranging a series of phi coefficients describing the presence or absence of kinship traits in various tribes. By this method they derived a scale of kinship-associated traits.

The test of the Driver-Massey conclusions will be through securing a factor analysis of the phi coefficients of the same data. The hypothesis is that the Driver-Massey scale will not be confirmed either because internal inadequacies of their data or because of bias introduced by their inspectional technique. However, if their results are confirmed, it will support one of the first major applications of statistical techniques to anthropological data.

1139 Computer. Partial and Ordinary Differential Equations. This problem consists of two parts: The first part consists of finding a first order solution of the two dimensional Poisson equation $\Delta \psi = \rho$. The second part is an attempt to find an improved forward facing integration formula of the Runge-Kutta type which permits automatic step modification.

1140 T Economics. Market Factors Influencing the Sale of Gasoline. A factor analysis will be made of the market factors influencing the sale of gasoline.

1141 Aviations Psychology. Statistical Analysis of Human Engineering. This program will be used to calculate frequency distributions, means, absolute sums, sum X^2 , and variances from data to be collected at the Aviation Psychology Laboratory in the near future. This data will be produced automatically in punched tape form suitable for direct Illiac input. It will be derived from human engineering experiments in aircraft instrumentation.

1142 T Psychology. A Methodological Analysis of "Assumed Similarity" Scores. Previous research has indicated significant relationships between certain "assumed similarity" (D) scores and certain behavioral and personality characteristics. The present investigation is an attempt to re-analyze these data, using components of the D score rather than the global D measure as previously used. The components needed would be two means, two standard deviations, and the intercorrelations for each pair of scale sheets for which D scores were previously computed. In addition, the scale sheets used will be factor analyzed over all subjects to determine whether there is more than one dimension present. If so, the above components will be computed for the dimensions obtained, rather than for the entire scale sheet as a unit. As in the previous studies, each of the components will be correlated with the available behavioral and personality criteria to determine what relationships exist.

1143 Computer. Flow Behind Shock Front. The problem consists of finding numerical solutions of a flow behind a shock front and then to predict the configuration of some shock fronts.

1144 Psychology. Objective Measurement of Children's Interests. This study is on the objective measurement of children's interests, i.e., an attempt to develop "unfakeable" interest tests for children. The first study is intended to clarify some scoring questions. The second study aims to develop additional tests and to clarify the factors involved in children's responses to such tests. The third study will aim to investigate the "structure of children's interests".

1145 Psychology. Analysis of Depth Perception Tests. The major purpose of this study will be to analyze the interrelationships among 29 measures of depth perception. In the course of previous research, the question arose as to whether or not the wide variety of tests were actually measuring a single homogeneous function. For example, in the perception of depth and distance which is possible with one eye only, the important monocular cues may include relative size, interposition, linear perspective, relative rate of movement, light and shade, and accommodation. These conditions become increasingly important as the discriminations involved occur at greater and greater distances from the subject. A total of 228 unselected college students were tested on 29 visual perception measures. Approximately one week later, these tests were repeated on the same 228 students.

1146 T Electrical Engineering. Network Approximation. Rational polynomials are generally used for approximation of certain magnitude or phase characteristic of realizable electrical networks. The problem concerning the approximation of both magnitude and phase characteristics will be investigated. Illiac can be used to solve the poles and zeros, and also magnitude and phase shift of a network function, especially, in the case of the function with higher order polynomials.

1147 Statistical Service Unit. Product Moment Correlation. The problem is to develop a product moment correlation program which will handle incomplete data problems.

1148 Bureau of Educational Research. Redundancy Problem. This problem is concerned with assessing the amount of overlap between two test batteries in terms of the common variance. The method consists essentially in the generalization of the multiple correlation. Each battery is orthogonalized by the square root method of factor analysis and intercorrelations of the resulting orthogonal variates are obtained. Each of the resulting orthogonal variates in battery A is then predicted by its multiple correlation with all the orthogonal variates in battery B. The average of these multiple R's is a measure of the extent to which battery B can account for the variance in battery A. Similarly, each of the orthogonal variates in battery B is then predicted from A by the multiple correlation; the average of

these represents the extent to which battery A can account for battery B's variance.

1149 Psychology: Personality and Perception: This is a study to determine relationships between perceptual and personality variables, the data being derived from tests involving illusions.

Table I shows the distribution of machine time for the month of April:

TABLE I

	Hrs:Min
Regular Maintenance	36:50
Unscheduled Maintenance	5:56
Drum Engineering	36:32
R.A.R.	5:23
Leapfrog	37:32
Wasted	:00

Use by Departments

Computer	36:51
Physics	13:38
Control Systems Lab.	96:15
Structural Research	34:43
Struct. Res. (AF 464)	17:32
Theor. and Ap. Mech. (ORD 593 IC)	2:54
Psychology	33:20
Psychology (I.D.P.W. 44-32-66-312)	3:10
Electrical Engineering	6:42
Elect. Eng. (Boeing Aircraft)	:48
Elect. Eng. (AF 3220)	:03
Elect. Eng. (NOBSR 64723)	1:55
Chemistry	66:43
Agriculture	22:59
State Water Survey	4:21
Inst. of Com. Res.	:21
Inst. of Com. Res. (PH 9067C)	2:46
Mech. Eng. (CONVAIR)	:22
U. of Utah (Psychology)	3:02
U. of Iowa (Psychology)	:26
U.S. Navy (Personnel Research)	1:02
Classes	29:47
Demonstrations	2:42
Miscellaneous	28:05
	<hr/> 532:40

Error Frequency and Analysis

The machine is normally used for "engineering" and maintenance between 7 a.m. and 11 a.m., and for a check of its performance between 5:30 p.m. and 6:30 p.m. of each weekday. Since the periods between 7 a.m. and 11 a.m., together with certain irregular periods, such as Saturdays and Sundays, are devoted to a heterogeneous group of functions, it is more instructive, from an error standpoint, to look at the periods between 11 a.m. and 7 a.m. of the next day in order to make an observation of the error frequency in the machine. This is the actual period when the machine is designated for use. With this in mind, a summary table has been prepared using the period between 11 a.m. and 7 a.m. of the next day. This table lists the running time when the machine was operating, the amount of time devoted to repairs because of breakdowns, and a number of failures while the machine was listed as running. During the 5:30-6:30 period (when the machine is checked) if no errors are found, the time is given to the "running" column. Each failure was considered to have terminated a running period and was followed by a repair period in preparing this table. Since the leapfrog code is our most significant machine test, the length of time which it has been used on the machine is listed separately, together with the number of errors associated with that particular code. This information for the month is presented in Table II.

It is important to notice that any interruption of machine time that was not planned is considered a failure in this table. In rare cases where the failure is not known until a later time, it is possible that no repair period is associated with the failure. This over-all system has been adopted because it makes it possible for a machine user to estimate directly the probability that the machine will be "running" at any instant of time and the probability of a failure during any given interval of running time.

Table III presents a summary of errors or interruptions for April.

TABLE III

Punch Error	3
Control Error	3
Arithmetic Error	1
Memory Error	5
Drum Failure	5
Reader error	3
Unknown	<u>2</u>
Total	22

DATE	RUNNING OK TIME	REPAIR TIME	INTERRUPTIONS OR FAILURES STOPPING OK TIME	TYPES OF INTERRUPTIONS OR FAILURES CAUSING REPAIR TIME	WASTED	LEAPFROG	FAILURES STOPPING LEAPFROG
4/1/58	19:10	:50	1	(1) Arithmetic error. 2^{-24} 2^{-20} adder	0	1:08	0
4/2/58	19:41	:19	2	(1) Drum Failure (2) Drum Failure	0	:26	0
4/3/58	19:29	:31	2	(1) Leapfrog Failure $1 \rightarrow 0$ 2^{-8} (memory) (2) Punch #1 failed light on B Gate out	0	:37	1
4/4/58	20:00	:00	0		0	:39	0
4/7/58	19:59	:01	1	(1) Bulb in "A" gate punch #1 out	0	:50	0
4/8/58	19:30	:30	2	(1) Unknown (2) Unknown	0	:43	0
4/9/58	19:30	:30	1	(1) Reader control chassis causing error	0	:26	0
4/10/58	20:00	:00	0		0	:48	0
4/11/58	19:40	:20	1	(1) Punch #1 jammed	0	:40	0
4/14/58	19:33	:26	1	(1) Drum Failure	0	:51	0
4/15/58	20:00	:00	0		0	:44	0
4/16/58	19:58	:02	1	(1) Memory error 2^{-4} $0 \rightarrow 1$	0	1:01	0
4/17/58	19:08	:52	3	(1) Clutch on reader "G" loose (2) Clutch on reader "K" seems to catch (3) Memory failure 2^{-8}	0	1:00	1
4/18/58	20:00	:00	0		0	:55	0
4/21/58	19:57	:03	1	(1) Memory error 2^{-8} $0 \rightarrow 1$	0	6:33	1
4/22/58	20:00	:00	0		0	:43	0
4/23/58	19:59	:01	1	(1) Reader "K" light burned out	0	:40	0
4/24/58	19:15	:45	1	(1) Control error	0	:58	0
4/25/58	18:03	1:57	2	(1) Control error (2) Drum Failure	0	1:13	0

[illegible]

Reports and Seminars

Seminars

"On the Geometry of Tschebyscheff-Approximation", by Dr. E. Stiefel, Institut fur angewandte Mathematik, Zurich, Switzerland, April 15, 1958.

"On Automatic Tschebyscheff-Approximation", by Dr. E. Stiefel, Institut fur angewandte Mathematik, Zurich, Switzerland, April 17, 1958.

"Gaseous Equilibrium in Stars", by Hokee Minn, April 22, 1958.

"The Synthesis of a Semi-Automatic Data Processing System for Public Utility Revenue Accounting", by Dr. Gerard Salton, Computation Laboratory, Harvard University, Cambridge, Massachusetts, April 29, 1958.

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