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A FORTRAN Program for Elastic Deuteron Scattering

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ABSTRACT

A FORTRAN program has been written to calculate the elastic scattering of deuterons by complex nuclei. The scattering is described by an optical model potential which includes some tensor forces.

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1. Theory

The elastic scattering of deuterons by a complex nucleus of mass number A and charge Z is assumed to be described in terms of an optical potential of the following form

$$v(r) = C - Vg(V) - iWf(W) + r^{-1} (S+iT) \frac{dg(S)}{dr} L \cdot S + (M+iN)f(R)T_R + (Q+iR)f(L)T_L \quad (1)$$

where $T_R = [(S \cdot r)^2 r^{-2} - 2/3]$, $T_L = [(L \cdot S)^2 + \frac{1}{2} (L \cdot S) - \frac{2}{3} L^2]$,

$$g(i) = g(r, r_i, a_i) = \{1 + \exp[(r - r_i A^{1/3})/a_i]\}^{-1},$$

$$f(i) = f(r, r_i, a_i) = 4[g(i)]^2 \exp[(r - r_i A^{1/3})/a_i]$$

and C is the Coulomb potential for a uniform charge distribution of radius $R_c = r_c A^{1/3}$.

The general solution of the Schrodinger equation for this interaction when the incoming spin component is m may be written as a partial wave expansion

$$\psi_m = \sum_{JLL'M} A_{JLMm} R_{JLL'}(r) F_{JL'M} \quad (2)$$

where

$$F_{JL'M} = \sum_{m'} (L' 1 J | M - m' \ m' \ M) Y_{L'}^{M-m'}(\theta, \phi) x_1^{m'}$$

are spin-angle functions, $(j_1 j_2 j | m_1 m_2 m)$ is the Clebsch-Gordan coefficient as defined by Rose,¹ $R_{JLL'}(r)$ are radial wave functions, J is the total

angular momentum with component M and L and L' are the incoming and

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outgoing orbital angular momenta, respectively. For large values of r ,

$$\begin{aligned} \psi_m \sim & \left\{ \left[1 + \frac{\eta^2}{i(kr - \underline{k} \cdot \underline{r})} \right] e^{i[\underline{k} \cdot \underline{r} + \eta \log(kr - \underline{k} \cdot \underline{r})]} \right\} x_1^m \\ & + \sum_{m'} M_{m m'}(\theta, \phi) r^{-1} e^{i[kr - \eta \log 2kr + 2\sigma_0]} x_1^{m'} \end{aligned} \quad (3)$$

where $\eta = Ze^2/\hbar v$ and $\sigma_0 = \arg \Gamma(1 + i\eta)$ are the usual Coulomb parameters and $M_{m m'}$ is the scattering matrix.

The A_{JLMm} of eq. (2) which satisfy the boundary condition (3) are given by

$$A_{JLMm} = 4\pi i^L Y_L^{M-m*}(\theta_k, \phi_k) (LLJ | M-m \ m \ M) \quad (4)$$

where θ_k, ϕ_k refer to the vector $\underline{k}$ along the incident direction.

If the z -axis is chosen to be along the incident direction, M only takes the value m and if the y -axis is taken parallel to $\underline{k} \times \underline{k}'$ where $\underline{k}'$ is the final momentum so that $\phi = 0$

$$\psi_m = \sum_{JLL'm'} [4\pi(2L+1)]^{\frac{1}{2}} i^L (LLJ | 0mm) (L' LJ | m-m' \ m' \ m) R_{JLL'}(r) Y_L^{m-m'}(\theta, 0) x_1^{m'} \quad (5)$$

The scattering matrix for these axes is given by

$$\begin{aligned} M_{mm'}(\theta) = & \delta_{mm'} f_c(\theta) + \frac{1}{2ik} \sum_L [4\pi(2L+1)]^{\frac{1}{2}} \\ & \times \left\{ \sum_{JL'} \Delta_{JLL'} (LLJ | 0mm) (L' LJ | m-m' \ m' \ m) Y_L^{m-m'}(\theta, 0) i^{L-L'} - \delta_{mm'} \delta_{LL'} e^{2i\omega_L} Y_L^0(\theta, 0) \right\} \end{aligned} \quad (6)$$

where
$$\omega_L = \sum_{L''=1}^L \tan^{-1} (\eta/L''),$$

$$f_c = -\frac{\eta}{2k} \operatorname{cosec}^2 \frac{\theta}{2} e^{-2i\eta \log \sin \frac{1}{2} \theta},$$

and $\Delta_{JLL'}$ is given by the asymptotic relations

$$R_{JLL'} \underset{r \rightarrow \infty}{\sim} (kr)^{-1} [e^{-2i\omega_{L'}} \Delta_{JLL'} A_{L'} + \delta_{LL'} B_L] e^{i\omega_{L'}} \quad (7)$$

with $A_L = \frac{1}{2}(F_L - iG_L)$ and $B_L = \frac{1}{2}(F_L + iG_L)$ where F_L and G_L are the standard Coulomb functions. The elements of the scattering matrix satisfy the relation

$$M_{-m-m'}(\theta) = (-1)^{m-m'} M_{m m'}(\theta) \quad (8)$$

so that M may be written in the form

$$M = \begin{pmatrix} A & D & C \\ B & E & -B \\ C & -D & A \end{pmatrix} \quad (9)$$

Only four of these amplitudes are independent and it may be shown that

$$C = (A-E) - \sqrt{2} \cot \theta (B+D) \quad (10)$$

Explicit formulae for the amplitudes A, B, C, D and E are given in the Appendix. The amplitudes are functions of the $\Delta_{JLL'}$, which are determined by solving the radial equations involving $R_{JLL'}$ for the boundary conditions of eq. (7). Writing

$$X_{JLL'} = \rho R_{JLL'}, \quad \rho = kr, \text{ the radial equations are}$$

$$\frac{d^2 x_{JLL'}}{d\rho^2} + \left\{ 1 + \alpha V_{JL'} - \frac{L'(L'+1)}{\rho^2} \right\} x_{JLL'} = \beta \alpha V_J^t x_{JLL'}, \quad (11)$$

where

$$L'' = L' + 2(J-L), \quad \beta = [\delta_{L'L+1} + \delta_{L'L-1}], \quad \alpha = E_{\text{relative}}^{-1},$$

$$V_{JL} = -C + V g(V) + iWf(W) - \frac{(S+iT)}{r} \frac{dg(S)}{dr} K_{JL}(S)$$

$$- (Q+iR)f(L)K_{JL}(L) - (M+iN)f(R)K_{JL}(R),$$

$$V_J^t = (M+iN)f(R)K_J(R)$$

$$\left. \begin{aligned} K_{JL}(S) &= L \quad \text{for } J = L+1 \\ &= -1 \quad J = L \\ &= -L-1 \quad J = L-1 \end{aligned} \right\}, \quad \left. \begin{aligned} K_{JL}(L) &= \frac{1}{6} L(2L-1) \quad \text{for } J = L+1 \\ &= -\frac{1}{6} (2L+3)(2L-1) \quad J = L \\ &= \frac{1}{6} (2L+3)(L+1) \quad J = L-1 \end{aligned} \right\},$$

$$\left. \begin{aligned} K_{JL}(R) &= -\frac{L}{3(2L+3)} \quad \text{for } J = L+1 \\ &= \frac{1}{3} \quad J = L \\ &= -\frac{(L+1)}{3(2L-1)} \quad J = L-1 \end{aligned} \right\}, \quad K_J(R) = \frac{\sqrt{J(J+1)}}{(2J+1)}.$$

Another useful choice of axes is to take the z-axis along $\underline{k}'$ and the y-axis parallel to $\underline{k}' \times \underline{k}$. It may be shown that the amplitudes referred to these axes, $\hat{M}_{m m'}$, are related to the amplitudes $M_{m m'}$ by the relation

$$\hat{M}_{m m'}(\theta) = \hat{M}_{m', m}(\theta) \quad (12)$$

[N.B. Both the relations of eq. (10) and eq. (12) hold even when the T_R tensor force is included.]

The uncoupled radial equations for X_{LLL} and the special case of X_{Oll} are solved by numerical integration of the equations from near the origin, where the solutions may be adequately approximated apart from an overall normalization factor, to large values of ρ where the boundary conditions of eq. (7) are applied. Method VII of Fox and Goodwin² is used for the integration. The equations are of the form $y'' = f(x)y$ and the step-by-step recurrence formula is

$$(1 - \frac{1}{12} h^2 f_{n+1})y_{n+1} = (2 + \frac{5}{6} h^2 f_n)y_n - (1 - \frac{1}{12} h^2 f_{n-1})y_{n-1} \quad (13)$$

where $h = \Delta x$ is the interval. The functions Δ_{LLL} (and Δ_{Oll}) and normalization constants are found by matching the calculated wave functions to the appropriate asymptotic solutions at two large values of ρ .

The coupled equations involving either $X_{L+1 LL}$ and $X_{L+1 L L+2}$ or $X_{L-1 LL}$ and $X_{L-1 L L-2}$ are of the form

$$\left. \begin{aligned} y'' &= f(x)y + \tilde{g}(x)\tilde{y} \\ \tilde{y}'' &= \tilde{f}(x)\tilde{y} + g(x)y \end{aligned} \right\} \quad (14)$$

An extension of the Fox-Goodwin method is used to numerically integrate these equations and the step-by-step recurrence relations are

$$\begin{aligned} \left\{ (1 - \frac{1}{12} h^2 f_{n+1})y_{n+1} - \frac{1}{12} h^2 \tilde{g}_{n+1}\tilde{y}_{n+1} \right\} &= \left\{ (2 + \frac{5}{6} h^2 f_n)y_n + \frac{5}{6} h^2 \tilde{g}_n\tilde{y}_n \right\} \\ &- \left\{ (1 - \frac{1}{12} h^2 f_{n-1})y_{n-1} - \frac{1}{12} h^2 \tilde{g}_{n-1}\tilde{y}_{n-1} \right\} \end{aligned}$$

and

$$\begin{aligned} \left\{ (1 - \frac{1}{12} h^2 \tilde{f}_{n+1})\tilde{y}_{n+1} - \frac{1}{12} h^2 g_{n+1}y_{n+1} \right\} &= \left\{ (2 + \frac{5}{6} h^2 \tilde{f}_n)\tilde{y}_n + \frac{5}{6} h^2 g_n y_n \right\} \\ &- \left\{ (1 - \frac{1}{12} h^2 \tilde{f}_{n-1})\tilde{y}_{n-1} - \frac{1}{12} h^2 g_{n-1}y_{n-1} \right\} \end{aligned}$$

(15)

In this case we have two unknown normalizations. An arbitrary choice for the approximate solutions near the origin will not generally satisfy the required boundary conditions. It is necessary to make two linearly independent selections and then to find a linear combination of the two "solutions" which give the desired asymptotic forms. In this way

$\Delta_{L+1 \text{ LL}}, \Delta_{L+1 \text{ LL}+2}$ or $\Delta_{L-1 \text{ LL}}, \Delta_{L-1 \text{ L L-2}}$ are determined.

The state of polarization of a beam of deuterons may be described in terms of the nine quantities $\langle T_{AB} \rangle$ (ref. Lakin³)

where

$$T_{00} = 1,$$

$$T_{10} = \sqrt{\frac{3}{2}} \sigma_z,$$

$$T_{11} = -\frac{1}{2} \sqrt{3} (\sigma_x + i\sigma_y),$$

$$T_{20} = \frac{1}{\sqrt{2}} (3 \sigma_z^2 - 2),$$

$$T_{21} = -\frac{1}{2} \sqrt{3} [(\sigma_x + i\sigma_y)\sigma_z + \sigma_z(\sigma_x + i\sigma_y)],$$

$$T_{22} = \frac{1}{2} \sqrt{3} (\sigma_x + i\sigma_y)^2,$$

and

$$T_{A-B} = (-)^B T_{A \ B}^\dagger \quad (16)$$

$\sigma_x, \sigma_y, \sigma_z$ are the usual spin matrices for spin-1 particles

$$\sigma_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \sigma_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (17)$$

The $\langle T_{A \ B} \rangle$ are given by

$$\langle T_{A B} \rangle = \text{trace}[\rho_D T_{AB}] / \text{trace}[\rho_D] \quad (18)$$

where ρ_D is the density matrix of the deuteron beam. If the incident beam is unpolarized and the z-axis is chosen to be along $\underline{k}'$ and the y-axis is taken parallel to $\underline{k}' \times \underline{k}$ (we refer to this set of axes as AXES 1), $\rho_D = \frac{1}{3} \hat{M} \hat{M}^\dagger$ where

$$\hat{M} = \begin{pmatrix} A & B & C \\ D & E & -D \\ C & -B & A \end{pmatrix} \quad (19)$$

The differential cross section is given by

$$\sigma = \frac{1}{3} [2|A|^2 + 2|B|^2 + 2|C|^2 + 2|D|^2 + |E|^2] \quad (20)$$

The vector polarizations are

$$\langle T_{10} \rangle = 0 \quad (21)$$

$$\langle T_{11} \rangle = -i3 \sqrt{6} \text{Im} [A^* D + B^* E + D^* C] / \sigma \quad (22)$$

and the tensor polarizations are

$$\langle T_{20} \rangle = 3 \sqrt{2} [|A|^2 + |B|^2 + |C|^2 - 2|D|^2 - |E|^2] / \sigma \quad (23)$$

$$\langle T_{21} \rangle = -3\sqrt{6} \text{Re} [A^* D + B^* E - D^* C] / \sigma \quad (24)$$

$$\langle T_{22} \rangle = 3\sqrt{3} [2 \text{Re}(A^* C) - |B|^2] / \sigma \quad (25)$$

Thus the deuteron polarization is determined by the four real quantities

$\langle T_{11} \rangle$, $\langle T_{20} \rangle$, $\langle T_{21} \rangle$ and $\langle T_{22} \rangle$.

If the z-axis is along $\underline{k}$ and the y-axis parallel to $\underline{k} \times \underline{k}'$ (we refer to this set of axes as AXES 2) the deuteron polarization is given by

the same four real quantities except that the amplitudes B and D are interchanged in the expressions (22)-(25).

2. Use of Program

Input Data

The following quantities are card input data:

TI	the initial angle θ_i in degrees,
DT	the increment angle $\Delta\theta$ in degrees,
JC	the number of angles for which σ , $\langle T_{11} \rangle$, $\langle T_{20} \rangle$, $\langle T_{21} \rangle$, $\langle T_{22} \rangle$ and σ/σ_R (ratio to Rutherford) are to be calculated,
NNN	defines the number of sets of optical model well depths to be read in on cards 6 and 7, cards 8 and 9, ... etc.; if only one set is required put $NNN = 1$, for two sets put $NNN = 2$, etc.,
MMM	defines the axes to be used, AXES 1 or AXES 2 are used for $MMM = 1$ or 2, respectively,
ELAB	the deuteron bombarding energy (LAB) in MeV,
RD	the radius parameter r_v in f,
RC	the Coulomb radius parameter in f,
AD	the diffuseness parameter a_v in f,
RW	the radius parameter r_w in f,
AW	the diffuseness parameter a_w in f,
RT	the radius parameter r_R in f,
AT	the diffuseness parameter a_R in f,
RSO	the radius parameter r_S in f,
ASO	the diffuseness parameter a_S in f,

RL the radius parameter r_L in f,
 AL the diffuseness parameter a_L in f,
 LMAX the number of orbital angular momenta to be used,
 $L < LMAX$,
 JMAX the number of integration steps to be used, JMAX should
 be given the value of $(JJ + JJB)$,
 JJ defines the radii at which the wave functions are
 matched, $r = JJh$ and $r = (JJ + JJB)h$,
 JJB defines the interval between the two joining radii,
 H the interval h to be used in the radial integration,
 AMT the target mass number A ,
 ZT the target charge number Z ,
 V, W, S, T, Q, R, AM and AN denote V, W, S, T, Q, R, M, N of
 the deuteron optical model.

Arrangement of Data on Cards

<u>Card</u>	<u>Quantities</u>	<u>Format</u>
1	TI, DT, JC, NNN, MMM	1Hb, 2F7.2, 3I3
2	ELAB, RD, RC, AD, RW, AW, RT, AT	1Hb, 8F7.3
3	RSO, ASO, RL, AL	1Hb, 4F7.3
4	LMAX, JMAX, JJ, JJB, H	1Hb, 4I4, F5.1
5	AMT, ZT	1Hb, 2F5.0
6	V, W, S, T	1Hb, rF7.3
7	Q, R, AM, AN	1Hb, 4F7.3

(The letter b stands for blank.)

Output

<u>Line</u>	<u>Quantities</u>
1	TI, DT, JC, NNN
2	ELAB, RD, RC, AD, RW, AW, RT, AT
3	RSO, ASO, RL, AL
4	LMAX, JMAX, JJ, JJB, H
5	AMT, ZT
6	V, W, S, T
7	Q, R, AM, AN
8	the heading DELTAS followed by a list of $(L'+1)$, $\Delta_{JLL'}$, real and imaginary parts, the order is Δ_{110} , Δ_{102} , Δ_{011} , Δ_{111} , Δ_{211} , Δ_{213} , Δ_{122} , Δ_{120} , Δ_{222} , Δ_{322} , Δ_{324} , ... etc.

The next line is the heading AMPLITUDES followed by a tabulation of angle, and the complex amplitudes A, B, C, D, E. Finally appear the cross section, polarizations and the ratio of the cross section to Rutherford scattering as a function of angle.

Appendix

Explicit Formulae for Amplitudes A, B, C, D and E

$$\begin{aligned}
 A(\theta) &= f_c(\theta) + \frac{\pi}{2ik} \sum_L' \left\{ - \left[\frac{(L+1)(L+2)}{(2L+5)} \right]^{\frac{1}{2}} Y_{L+2}^o \Delta_{L+2 \text{ LL}+2} - \left[\frac{L(L-1)}{(2L-3)} \right]^{\frac{1}{2}} Y_{L-2}^o \Delta_{L-1 \text{ LL}-2} \right. \\
 &\quad \left. + [(L+2)\Delta_{L+1 \text{ LL}} + (2L+1)\Delta_{\text{LLL}} + (L-1)\Delta_{L-1 \text{ LL}} - 2(2L+1)e^{2i\omega_L}](2L+1)^{-\frac{1}{2}} Y_L^o \right\} \\
 B(\theta) &= \frac{[2\pi]}{2ik} \sum_L' \left\{ - \left[\frac{(L+1)(L+3)}{(2L+5)} \right]^{\frac{1}{2}} Y_{L+2}^{-1} \Delta_{L+1 \text{ L L}+2} + \left[\frac{L(L-2)}{(2L-3)} \right]^{\frac{1}{2}} Y_{L-2}^{-1} \Delta_{L-1 \text{ L L}-2} \right. \\
 &\quad \left. + [\Delta_{L+1 \text{ LL}} - \Delta_{L-1 \text{ LL}}] \left[\frac{L(L+1)}{(2L+1)} \right]^{\frac{1}{2}} Y_L^{-1} \right\} \\
 C(\theta) &= \frac{\pi}{2ik} \sum_L' \left\{ - \left[\frac{(L+3)(L+4)}{(2L+5)} \right]^{\frac{1}{2}} Y_{L+2}^{-2} \Delta_{L+1 \text{ L L}+2} - \left[\frac{(L-2)(L-3)}{(2L-3)} \right]^{\frac{1}{2}} Y_{L-2}^{-2} \Delta_{L-1 \text{ L L}-2} \right. \\
 &\quad \left. + [L\Delta_{L+1 \text{ LL}} - (2L+1)\Delta_{\text{LLL}} + (L+1)\Delta_{L-1 \text{ LL}}] \left[\frac{(L-1)(L+2)}{L(L+1)(2L+1)} \right]^{\frac{1}{2}} Y_L^{-2} \right\} \\
 D(\theta) &= \frac{[2\pi]}{2ik} \sum_L' \left\{ - \left[\frac{(L+1)(L+3)}{(2L+5)} \right]^{\frac{1}{2}} Y_{L+2}^{-1} \Delta_{L+1 \text{ L L}+2} + \left[\frac{L(L-2)}{(2L-3)} \right]^{\frac{1}{2}} Y_{L-2}^{-1} \Delta_{L-1 \text{ L L}-2} \right. \\
 &\quad \left. + [-L(L+2)\Delta_{L+1 \text{ LL}} + (2L+1)\Delta_{\text{LLL}} + (L+1)(L-1)\Delta_{L-1 \text{ LL}}][L(L+1)(2L+1)]^{-\frac{1}{2}} Y_L^{-1} \right\} \\
 E(\theta) &= f_c(\theta) + \frac{[4\pi]}{2ik} \sum_L' \left\{ \left[\frac{(L+1)(L+2)}{(2L+5)} \right]^{\frac{1}{2}} Y_{L+2}^o \Delta_{L+1 \text{ L L}+2} + \left[\frac{L(L-1)}{(2L-3)} \right]^{\frac{1}{2}} Y_{L-2}^o \Delta_{L-1 \text{ L L}-2} \right. \\
 &\quad \left. + [(L+1)\Delta_{L+1 \text{ LL}} + L\Delta_{L-1 \text{ L L}} - (2L+1)e^{2i\omega_L}][2L+1]^{-\frac{1}{2}} Y_L^o \right\}
 \end{aligned}$$

where $Y_L^M = Y_L^M(\theta, 0)$. $\sum'$ means that the summation ignores all terms for which $|M| > L$.

References

1. M. E. Rose, Elementary Theory of Angular Momentum, New York (Wiley) 1957.
2. L. Fox and E. T. Goodwin; Proc. Cambridge Phil. Soc. 45, 373 (1949).
3. W. Lakin, Phys. Rev. 98, 139 (1955).

**FTN,L,P,E.

```

PROGRAM DD
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION DP(40),DPA(40),DZ(40),DZA(40),DM(40),DMA(40)
  DIMENSION F(91),FA(91),A(91),AA(91),B(91),BA(91),C(91),CA(91),D(91)
  1),DA(91),E(91),EA(91),SG(91),T11(91),T20(91),T21(91),T22(91),SGR(9
  21),DPP(40),DPPA(40),DMM(40),DMMA(40)
  1 FORMAT(1H ,62H ANGLE CROSS SECTION      T11      T20      T21      T22
  1      RATIO)
  2 FORMAT(1H ,F7.2,D15.8,4F8.4,D15.8)
  3 FORMAT(1H ,2F7.2,3I3)
  4 FORMAT(1H ,8F7.3)
  5 FORMAT(1H ,4I4,F5.1)
  6 FORMAT(1H ,2F5.0)
  8 FORMAT(1H ,F7.2,10D12.5)
  12 FORMAT(1H ,10HAMPLITUDES)
  16 FORMAT(5H AXES,I3)
  11 READ 3,TI,DT,JC,NNN,MMM
  PRINT3,TI,DT,JC,NNN
  TI=TI/57.2958
  DT=DT/57.2958
  READ 4,ELAB,RD,RC,AD,RW,AW,RT,AT
  PRINT4,ELAB,RD,RC,AD,RW,AW,RT,AT
  READ 4,RSO,ASO,RL,AL
  PRINT4,RSO,ASO,RL,AL
  READ 5,LMAX,JMAX,JJ,JJB,H
  PRINT5,LMAX,JMAX,JJ,JJB,H
  READ 6,AMT,ZT
  PRINT6,AMT,ZT
  CUB=AMT**0.33333333
  RD=RD*CUB
  RC=RC*CUB
  RW=RW*CUB
  RT=RT*CUB
  RSO=RSO*CUB
  RL=RL*CUB
  ECM=AMT/(AMT+2.0)*ELAB
  AK=C.310399*AMT/(AMT+2.0)*ELAB**0.5
  RHOU=AK*RD
  DEE=1.0/(AK*AD)
  RHODW=AK*RW
  DEEW=1.0/(AK*AW)
  RHOOT=AK*RT
  DEET=1.0/(AK*AT)
  DEESO=1.0/(AK*ASO)
  DEEL=1.0/(AK*AL)
  RHUOSO=AK*RSO
  RHUOL=AK*RL
  ALPH=1.0/ECM
  DEL=AK*H
  RHUC=AK*RC
  ETA=0.7197*ZT*AK*ALPH
  9 READ 4,V,W,S,T
  PRINT4,V,W,S,T
  READ 4,Q,R,AM,AN
  PRINT4,Q,R,AM,AN
  AK2=AK*AK
  CALL DELTA(RHUC,ETA,RHOU,DEE,V,W,S,T,Q,R,AM,AN,ALPH,DEL,JMAX,JJ,JJ
  18,DP,DPA,DZ,DZA,DM,DMA,LMAX,RHODW,DEEW,RHOOT,DEET,AK2,DPP,DPPA,DMM
  2,DMMA,RHOUUSU,DEESO,RHUOL,DEEL)
  CALL FC(AK,ETA,TI,DT,JC,F,FA)

```

```
CALL MATRIX(DP,DPA,DZ,DZA,DM,DMA,OPP,UPPA,DMM,UMMA,F,FA,A,AA,B,BA,
1C,CA,D,DA,E,EA,TI,DT,JC,LMAX,ETA,AK2)
IF(MMM-1)14,14,15
15 DO 13 M=1,JC
    WSP=B(M)
    B(M)=D(M)
    D(M)=WSP
    WSP=BA(M)
    BA(M)=DA(M)
13 DA(M)=WSP
14 CALL SIGPUL(A,AA,B,BA,C,CA,D,DA,F,EA,SG,T11,T20,T21,T22,JC)
    TI=TI*57.2958
    DT=DT*57.2958
    TA=TI
    PRINT 12
    DO 10 M=1,JC
        PRINT 8,TA,A(M),AA(M),B(M),BA(M),C(M),CA(M),D(M),DA(M),E(M),EA(M)
10 TA=TA+DT
    TA=TI
    PRINT 16,MMM
    PRINT 1
    DO 7 M=1,JC
        SGR(M)=SG(M)/3.0/(F(M)**2+FA(M)**2)
        SG(M)=SG(M)*10.0/3.0
        PRINT 2,TA,SG(M),T11(M),T20(M),T21(M),T22(M),SGR(M)
7 TA=TA+DT
    TI=TI/57.2958
    DT=DT/57.2958
    NNN=NNN-1
    IF(NNN)11,11,9
END
```

```

SUBROUTINE DELTA(RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,ALPHA,DEL,JMA
1X, JJ, JJB, DP, DPA, DZ, DZA, DM, DMA, LMAX, RHOOW, DEEW, RHOOT, DEET, AK2, DPP, D
2PPA, DMM, DMMA, RHOOSO, DEESO, RHOOL, DEEL)
IMPLICIT DOUBLE(A-H,O-Z)
DIMENSION DP(40),DPA(40),DZ(40),DZA(40),DM(40),DMA(40),FS(2),GS(2)
1,FC(2,2),GC(2,2),F1(80),F2(80),G1(40),G2(40),XXS(2),YYS(2),XXC(2,2
2),YYC(2,2),WFR(302,2),WFC(302,2),ALD(400),ALP(400),AJD(400),AJP(40
30),XXAC(2,2),YYAC(2,2),DPP(40),DPPA(40),DMM(40),DMMA(40)
COMMON ALD,ALP,AJD,AJP,WFR,WFC
14 FORMAT(1H ,13,2D12.5)
15 FORMAT(1H ,6HDELTAS)
PRINT 15
AJJ=JJ
RHOJ1=AJJ*DEL
AJJ=JJ+JJB
RHOJ2=AJJ*DEL
LMAX=LMAX+2
CALL COULMB(ETA,RHOJ1,LMAX,F1,G1)
CALL COULMB(ETA,RHOJ2,LMAX,F2,G2)
LMAX=LMAX-2
DO 1 K=1,LMAX
AL=K-1
IF(AL)6,6,7
6 N=3
GO TO 8
7 N=1
8 DO 2 I=N,3
AJ=K+I-3
IF(AJ-AL)3,4,5
3 IF(AL-1.0)2,4,5
4 CALL SWAVES(RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,DEL,JM
1AX, JJ, XXS, YYS, JJB, RHOOW, DEEW, RHOOT, DEET, AK2, RHOOSO, DEESO, RHOOL, DEE
2L)
FS(1)=F1(K)
GS(1)=G1(K)
FS(2)=F2(K)
GS(2)=G2(K)
CALL SMATCH(XXS,YYS,FS,GS,A,B,ALAMB,AMU,ETA,AL)
PRINT 14,K,ALAMB,AMU
GO TO 9
5 CALL CWAVES(RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,DEL,JM
1AX, JJ, XXC, YYC, XXAC, YYAC, JJB, RHOOW, DEEW, RHOOT, DEET, AK2, RHOOSO, DEESO
2, RHOOL, DEEL)
KP=K+2*(I-2)
ALPA=KP-1
FC(1,1)=F1(K)
GC(1,1)=G1(K)
FC(1,2)=F1(KP)
GC(1,2)=G1(KP)
FC(2,1)=F2(K)
GC(2,1)=G2(K)
FC(2,2)=F2(KP)
GC(2,2)=G2(KP)
CALL CMATCH(XXC,YYC,XXAC,YYAC,FC,GC,A,B,C,D,ALAMB,AMU,ALAMBP,AMUP,
1AL,ALPA,ETA)
PRINT 14,K,ALAMB,AMU
PRINT 14,KP,ALAMBP,AMUP
9 IF(I-2)10,11,12
10 DM(K)=ALAMB
DMA(K)=AMU

```

```
DMM(K)=ALAMP  
DMA(K)=AMUP  
GO TO 2  
11 DZ(K)=ALAMB  
DZA(K)=AMU  
GO TO 2  
12 DP(K)=ALAMB  
DPA(K)=AMU  
DPP(K)=ALAMP  
DPPA(K)=AMUP  
2 CONTINUE  
1 CONTINUE  
RETURN  
END
```

```
SUBROUTINE FC(AK,ETA,TI,DT,J,F,FA)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION F(91),FA(91)
  A=-ETA/(2.0*AK)
  T=TI
  DO 1 I=1,J
    S=DSIN(0.5*T)
    B=-2.0*ETA*DLOG(S)
    F(I)=A*DCOS(B)/(S*S)
    FA(I)=A*DSIN(B)/(S*S)
1  T=T+DT
  RETURN
  END
```

```

SUBROUTINE MATRIX(DP,UPA,DZ,DZA,DM,DMA,DPP,DPPA,DMM,DMMA,F,FA,A,AA
1,B,BA,C,CA,D,DA,E,EA,TI,DT,J,LMAX,ETA,AK2)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION DP(40),UPA(40),DZ(40),DZA(40),DM(40),DMA(40),DPP(40),DPP
1A(40),DMM(40),DMMA(40),F(91),FA(91),A(91),AA(91),B(91),BA(91),C(91
2),CA(91),D(91),DA(91),E(91),EA(91)
  AB=AK2**0.5
  T=TI
  DZ(1)=0.0
  DZA(1)=0.0
  DM(1)=0.0
  DMA(1)=0.0
  DMM(1)=0.0
  DMMA(1)=0.0
  DMM(2)=0.0
  DMMA(2)=0.0
  DO 1 I=1,J
    A(I)=F(I)
    AA(I)=FA(I)
    B(I)=0.0
    BA(I)=0.0
    C(I)=0.0
    CA(I)=0.0
    D(I)=0.0
    DA(I)=0.0
    E(I)=F(I)
    EA(I)=FA(I)
    X=DCOS(T)
    DO 2 K=1,LMAX
      EM=-2.0
      EL=K-1
      IF(EL-2.)12,13,13
12 U=0.0
      V=0.0
      GO TO 14
13 CALL ASSLEG(EM,EL,X,TLM,PM)
      H=-TLM*((EL+2.)*(EL-1.)/(EL*(EL+1.)*(16.*EL+8.)))*0.5/AB
      W=0.0
      Y=EL*UP(K)-(2.*EL+1.)*DZ(K)+(EL+1.)*DM(K)
      Z=EL*UPA(K)-(2.*EL+1.)*DZA(K)+(EL+1.)*DMA(K)
      CALL COMPLX(W,H,Y,Z,3,U,V)
14 EM=-2.0
      EL=K+1
      CALL ASSLEG(EM,EL,X,TLMA,PM)
      H=TLMA/AB
      Y=((EL+1.)*(EL+2.)/(16.*EL+8.))*0.5
      Z=Y*DPPA(K)
      Y=Y*DPP(K)
      CALL COMPLX(W,H,Y,Z,3,S,TA)
      EM=-2.0
      EL=K-3
      IF(EL-2.)15,16,16
15 Q=0.0
      R=0.0
      GO TO 17
16 CALL ASSLEG(EM,EL,X,TLMB,PM)
      H=TLMB/AB
      Y=(EL*(EL-1.)/(16.*EL+8.))*0.5
      Z=Y*DMMA(K)
      Y=Y*DMM(K)

```

```

      CALL COMPLEX(W,H,Y,Z,3,Q,R)
17 C(I)=C(I)+U+S+Q
      CA(I)=CA(I)+V+TA+R
      EM=-1.0
      EL=K-1
      IF(EL-1.)7,8,8
7  U=0.0
      D=0.0
      V=0.0
      P=0.0
      GO TO 6
8  CALL ASSLEG(EM,EL,X,TLM,PM)
      H=-TLM*(EL*(EL+1.)/(8.*EL+4.))*0.5/AB
      Y=DP(K)-DM(K)
      Z=DPA(K)-DMA(K)
      CALL COMPLEX(W,H,Y,Z,3,U,V)
      H=-TLM/(EL*(EL+1.)*(2.*EL+1.))*0.5/2./AB
      Y=EL*(EL+2.)*DP(K)-(2.*EL+1.)*DZ(K)-(EL*EL-1.)*DM(K)
      Z=EL*(EL+2.)*DPA(K)-(2.*EL+1.)*DZA(K)-(EL*EL-1.)*DMA(K)
      CALL COMPLEX(W,H,Y,Z,3,O,P)
6  EM=-1.0
      EL=K+1
      CALL ASSLEG(EM,EL,X,TLMA,PM)
      H=TLMA/AB
      Y=((EL-1.)*(EL+1.)/(8.*EL+4.))*0.5
      Z=Y*DPPA(K)
      Y=Y*DPP(K)
      CALL COMPLEX(W,H,Y,Z,3,S,TA)
      EM=-1.0
      EL=K-3
      IF(EL-1.)9,10,10
9  Q=0.0
      R=0.0
      GO TO 11
10 CALL ASSLEG(EM,EL,X,TLMB,PM)
      H=-TLMB/AB
      Y=(EL*(EL+2.)/(8.*EL+4.))*0.5
      Z=Y*DMMMA(K)
      Y=Y*DMM(K)
      CALL COMPLEX(W,H,Y,Z,3,Q,R)
11 B(I)=B(I)+U+S+Q
      BA(I)=BA(I)+V+TA+R
      D(I)=D(I)-O+S+Q
      DA(I)=DA(I)-P+TA+R
      EM=0.0
      EL=K-1
      CALL ASSLEG(EM,EL,X,TLM,PM)
      H=-TLM/(16.*EL+8.))*0.5/AB
      Y=(EL+2.)*DP(K)+(2.*EL+1.)*DZ(K)+(EL-1.)*DM(K)
      Z=(EL+2.)*DPA(K)+(2.*EL+1.)*DZA(K)+(EL-1.)*DMA(K)
      CALL COMPLEX(W,H,Y,Z,3,U,V)
      H=-TLM/(4.*EL+2.))*0.5/AB
      Y=(EL+1.)*DP(K)+EL*DM(K)
      Z=(EL+1.)*DPA(K)+EL*DMA(K)
      CALL COMPLEX(W,H,Y,Z,3,O,P)
      CALL OMEGA(EL,ETA,OM)
      OM=2.*UM
      OMR=DSIN(OM)*TLM/AB/2.82842*(2.*EL+1.))*0.5
      OMC=-DCOS(OM)*TLM/AB/2.82842*(2.*EL+1.))*0.5
      EM=0.0
      EL=K+1
      CALL ASSLEG(EM,EL,X,TLMA,PM)

```



```
H=TLMA/AB
Y=((EL-1.)*EL/(16.*EL+8.))*0.5
Z=Y*DPPA(K)
Y=Y*DPP(K)
CALL COMPLX(W,H,Y,Z,3,S,TA)
EM=0.0
EL=K-3
IF(EL)4,3,3
4 Q=0.0
R=0.0
GO TO 5
3 CALL ASSLEG(EM,EL,X,TLMB,PM)
H=TLMB/AB
Y=((EL+2.)*(EL+1.)/(16.*EL+8.))*0.5
Z=Y*DMMA(K)
Y=Y*DMM(K)
CALL COMPLX(W,H,Y,Z,3,Q,R)
5 A(I)=A(I)+U+S+Q-2.*QMR
AA(I)=AA(I)+V+TA+R-2.*OMC
E(I)=E(I)+U-2.*(S+Q)-2.*QMR
EA(I)=EA(I)+P-2.*(TA+R)-2.*OMC
2 CONTINUE
1 T=T+DT
RETURN
END
```

```
SUBROUTINE SIGPOL(A,AA,B,BA,C,CA,D,DA,E,EA,S,V,T20,T21,T22,J)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION A(91),AA(91),B(91),BA(91),C(91),CA(91),D(91),DA(91),E(91),EA(91),S(91),V(91),T20(91),T21(91),T22(91)
  DO 1 I=1,J
    F=A(I)*A(I)+B(I)*B(I)+C(I)*C(I)+AA(I)*AA(I)+BA(I)*BA(I)+CA(I)*CA(I)
    G=2.0*(D(I)*D(I)+DA(I)*DA(I))+E(I)*E(I)+EA(I)*EA(I)
    S(I)=2.0*F+G
    T20(I)=1.41421*(F-G)/S(I)
    V(I)=2.44949*(DA(I)*(A(I)-C(I))+D(I)*(CA(I)-AA(I))+B(I)*EA(I)-BA(I)*E(I))/S(I)
    T21(I)=2.44949*(DA(I)*(AA(I)-CA(I))+D(I)*(A(I)-C(I))+B(I)*E(I)+BA(I)*EA(I))/S(I)
    T21(I)=-T21(I)
    T22(I)=1.73205*(2.0*(A(I)*C(I)+AA(I)*CA(I))-B(I)*B(I)-BA(I)*BA(I))/S(I)
1  CONTINUE
  RETURN
  END
```

```

SUBROUTINE COULMB(ETA,RHO,LMAX,FA,GA)
IMPLICIT DOUBLE(A-H,O-Z)
DIMENSION B(3),SIGMA(4),FA(80),GA(40),GAP(40)
IF(RHO-1.1)9,10,10
9 M=0
R=RHO
GO TO 100
10 M=(RHO-1.0)*10.0
R=1.0
ZM=M
H=(RHO-1.0)/ZM
GO TO 100
101 F=CDE*R*BS
FP=CDE*(BS+SS)
L=1
12 IF(M)11,11,200
201 F=SUM
FP=SUMP/H
R=R+H
M=M-1
GO TO (12,17),L
11 IF(ETA-4.0)64,64,65
65 G=1.0-0.0088888889/(ETA**2)-0.00091089581/(ETA**4)
GG=0.049595702/ETA+.0024551992/(ETA**3)+.0002534684/(ETA**5)
G=G+GG/(ETA**0.33333333)
G=G*1.223404*ETA**0.16666667
GP=1.0+0.00031746032/(ETA**2)+0.00031178247/(ETA**4)
GPG=.17282604+.0035812149/(ETA**2)+.00090739664/(ETA**4)
GP=GP-GPG/(ETA**0.66666667)
GP=GP*(-0.70788177)/(ETA**0.16666667)
IF(RHO-2.0*ETA)66,14,68
66 M=(2.0*ETA-RHO)*10.0+1.0
69 ZM=M
H=(RHO-2.0*ETA)/ZM
R=2.0*ETA
GO TO 301
68 M=(RHO-2.0*ETA)*10.0+1.0
GO TO 69
64 IF(RHO-5.1)13,14,15
13 IF(RHO-4.9)40,14,14
14 M=0
GO TO 300
15 M=(RHO-5.0)*10.0
ZM=M
H=(RHO-5.0)/ZM
RHO=5.0
GO TO 300
40 M=(5.0-RHO)*10.0
ZM=M
H=(RHO-5.0)/ZM
RHO=5.0
GO TO 300
301 FO=F
FPO=FP
F=G
FP=GP
L=2
17 IF(M)19,19,200
100 CDE=6.2831853*ETA
CDE=(CDE/(EXP(CDE)-1.0))*0.5

```

```

B(1)=1.0
B(2)=ETA*R
BS=B(1)+B(2)
BK=2.0
SS=B(2)
IA=0
IB=0
120 B(3)=(2.0*ETA*B(2)-R*B(1))*R/(BK*(BK+1.0))
    IF(IA)107,107,108
107 IF(1.0E-8-B(3)/BS)109,109,110
110 IF(1.0E-8+B(3)/BS)109,109,111
111 IA=1
109 BS=BS+B(3)
108 SA=BK*B(3)
    IF(IB)112,112,113
112 IF(1.0E-8-SA/SS)114,114,115
115 IF(1.0E-8+SA/SS)114,114,116
116 IB=1
114 SS=SS+SA
113 IF(IA+IB-2)117,101,101
117 BK=BK+1.0
    DO 119 J=1,2
119 B(J)=B(J+1)
    GO TO 120
200 SIGMA(1)=F
    SIGMA(2)=H*FP
    SIGMA(3)=-0.5*H*H*(1.0-2.0*ETA/R)*F
    SUM=SIGMA(1)+SIGMA(2)+SIGMA(3)
    SUMP=SIGMA(2)+2.0*SIGMA(3)
    ANN=2.0
    IC=0
    ID=0
    E=2.0*ETA
213 SIGMA(4)=H*(H*(H*SIGMA(1)+(R-E)*SIGMA(2))+(ANN*ANN-ANN)*SIGMA(3))
    SIGMA(4)=-SIGMA(4)/(R*(ANN*ANN+ANN))
    IF(IC)204,204,205
204 IF(1.0E-8-SIGMA(4)/SUM)211,211,202
202 IF(1.0E-8+SIGMA(4)/SUM)211,211,203
203 IC=1
211 SUM=SUM+SIGMA(4)
205 IF(ID)206,206,207
206 IF(1.0E-8-ANN*SIGMA(4)/SUMP)208,208,209
209 IF(1.0E-8+ANN*SIGMA(4)/SUMP)208,208,210
210 ID=1
208 SUMP=SUMP+(ANN+1.0)*SIGMA(4)
207 IF(IC+ID-2)212,201,201
212 ANN=ANN+1.0
    DO 214 J=1,3
214 SIGMA(J)=SIGMA(J+1)
    GO TO 213
300 R=RHO
    A=(2.0*ETA+22.0)/RHO
    SDE=0.0
    SP=0.0
    T=0.0
    TP=0.0
    BB=-1.0
    N=2
    DELTY=0.11764706
    DELTZ=A/102.0
    Y=0.11764706
    Z=DELTZ

```

```

305 BD=0.0
    IF(N) 302,302,303
302 N=3
    BD=BD+1.0
303 N=N-1
    THANY=EXP(2.0*Y)
    THANY=(THANY-1.0)/(THANY+1.0)
    ZZ=RHO*THANY-2.0*ETA*Y
    SB=(1.0-THANY**2)*SIN(ZZ)
    SPB=THANY*COS(ZZ)*SB/SIN(ZZ)
    SA=EXP(-RHO*7+2.0*ETA*ATAN(Z))
    SPA=Z*SA
    BB=-BB
    BC=2.0*BB+3.0+BD
    SDE=SDE+BC*SA
    SP=SP+BC*SPA
    T=T+BC*SB
    TP=TP+BC*SPB
    Z=Z+DELTZ
    Y=Y+DELTY
    IF(A-Z) 304,304,305
304 SDE=(SDE+1.0-SA)*DELTZ*0.3
    SP=(SP-SPA)*DELTZ*0.3
    T=(T-SB)*DELTZ*0.3
    TP=(TP-SPB)*DELTZ*0.3
    A=EXP(-3.1415926*ETA)/CDE
    G=A*RHO*(SDE-T)
    GP=-A*RHO*(SP+TP)+G/RHO
    GO TO 301
19 GA(1)=F
    GAP(1)=FP
    DO 20 J=1,LMAX
    AL=J
    U=AL*AL/R+ETA
    V=(AL*AL+ETA*ETA)**0.5
    GA(J+1)=(U*GA(J)-AL*GAP(J))/V
20 GAP(J+1)=(V*GA(J)-U*GAP(J))/AL
    LP=10
    IR=R+1.0
    IF(IR-LMAX) 24,24,25
24 LP=LP+LMAX
    GO TO 26
25 LP=LP+IR
26 FA(LP+2)=0.0
    FA(LP+1)=1.0
    DO 27 J=1,LP
    K=LP+1-J
    AL=K
    U=(AL*AL+ETA*ETA)**0.5*(AL+1.0)
    V=(2.0*AL+1.0)*(ETA+AL*(AL+1.0)/R)
    W=AL*((AL+1.0)**2+ETA*ETA)**0.5
27 FA(K)=(V*FA(K+1)-W*FA(K+2))/U
    ALPHA=F0/FA(1)
    DO 29 J=1,LMAX
29 FA(J+1)=ALPHA*FA(J+1)
    FA(1)=F0
    RETURN
    END

```

```

SUBROUTINE SWAVES(RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,
1 DELTA,JMAX,JJ,XX,YY,JJB,RHOOW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHO
200L,DEEL)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION WFR(302,2),WFC(302,2),SFR(3),SFC(3),XA(3),YA(3),X(2),Y(2
1),XX(2),YY(2)
  DIMENSION ALD(400),ALP(400),AJD(400),AJP(400)
  DIMENSION WFPR(302,2),WFPC(302,2)
  COMMON ALD,ALP,AJD,AJP,WFR,WFC
  JJA=JJ
  RHO=2.0*DELTA
  DD=DELTA*DELTA/12.0
  K=1
  J=4
  M=1
  CALL START(RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,DELTA,X
1,Y,RHOOW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
  DO 1 I=1,2
    XA(I)=X(I)
    YA(I)=Y(I)
    WFR(I+1,1)=XA(I)
1 WFC(I+1,1)=YA(I)
200 DO 101 I=K,3
  CALL POTENS(RHO,RHOC,ETA,RHOO,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,VPR,VPC,
1 VTR,VTC,ALPHA,RHOOW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
  SFR(1)=(-1.0-VPR+AL*(AL+1.0)/(RHO*RHO))*DD
  SFC(1)=-VPC*DD
101 RHO=RHO+DELTA
  A=2.0+10.0*SFR(2)
  B=10.0*SFC(2)
  C=XA(2)
  D=YA(2)
  CALL COMPLEX(A,B,C,D,3,VPR,VPC)
  A=1.0-SFR(1)
  B=-SFC(1)
  C=XA(1)
  D=YA(1)
  CALL COMPLEX(A,B,C,D,3,VTR,VTC)
  CALL COMPLEX(VPR,VPC,VTR,VTC,2,A,B)
  C=1.0-SFR(3)
  D=-SFC(3)
  CALL COMPLEX(A,B,C,D,4,VPR,VPC)
  XA(3)=VPR
  YA(3)=VPC
  DO 100 I=1,2
    XA(I)=XA(I+1)
    YA(I)=YA(I+1)
    SFR(I)=SFR(I+1)
100 SFC(I)=SFC(I+1)
  WFR(J,1)=XA(3)
  WFC(J,1)=YA(3)
  IF(J-JJ)302,301,302
301 IF(M-2)303,304,302
303 JJ=JJ+JJB
304 XX(M)=XA(3)
  YY(M)=YA(3)
  M=M+1
302 J=J+1
  K=3
  IF(J-JMAX)200,200,2

```

2 JJ=JJA
RETURN
END

```

SUBROUTINE SMATCH(XX,YY,F,G,ALPHA,ALPHAA,ALAMB,AMU,ETA,AL)
IMPLICIT DOUBLE(A-H,O-Z)
DIMENSION XX(2),YY(2),F(2),G(2),A(2),B(2),AA(2),BA(2)
DO 1 I=1,2
  A(I)=0.5*F(I)
  B(I)=A(I)
  AA(I)=-0.5*G(I)
1 BA(I)=-AA(I)
  W=B(1)
  X=BA(1)
  Y=A(1)
  Z=AA(1)
  CALL COMPLX(W,X,Y,Z,4,U,V)
  W=B(2)
  X=BA(2)
  Y=A(2)
  Z=AA(2)
  CALL COMPLX(W,X,Y,Z,4,S,T)
  ALPHA=U-S
  ALPHAA=V-T
  W=XX(1)
  X=YY(1)
  Y=A(1)
  Z=AA(1)
  CALL COMPLX(W,X,Y,Z,4,U,V)
  W=XX(2)
  X=YY(2)
  Y=A(2)
  Z=AA(2)
  CALL COMPLX(W,X,Y,Z,4,S,T)
  U=U-S
  V=V-T
  CALL COMPLX(ALPHA,ALPHAA,U,V,4,S,T)
  CALL OMEGA(AL,ETA,OM)
  OMR=COS(OM)
  OMC=SIN(OM)
  CALL COMPLX(S,T,OMR,OMC,3,ALPHA,ALPHAA)
  CALL COMPLX(ALPHA,ALPHAA,W,X,3,S,T)
  W=B(2)
  X=BA(2)
  CALL COMPLX(W,X,OMR,OMC,3,U,V)
  U=S-U
  V=T-V
  W=A(2)
  X=AA(2)
  CALL COMPLX(OMR,OMC,W,X,4,S,T)
  CALL COMPLX(U,V,S,T,3,ALAMB,AMU)
  RETURN
END

```



```

SUBROUTINE CWAVES(RHOC,ETA,RHOU,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,
1 DELTA,JMAX,JJ,XX,YY,XXA,YYA,JJB,RHOUW,DEEW,RHOOT,DEET,AK2,RHOOSO,D
2 EESO,RHOOL,DEEL)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION WFR(302,2),WFC(302,2),SFR(3,2),SFC(3,2),SGR(3,2),SGC(3,2
1),XA(3,2),YA(3,2),X(2),Y(2),BN(2),XX(2,2),YY(2,2),XXA(2,2),YYA(2,2
2),WFPR(302,2),WFPC(302,2)
  DIMENSION ALD(400),ALP(400),AJD(400),AJP(400)
  COMMON ALD,ALP,AJD,AJP,WFR,WFC
  JJA=JJ
  DO 2 KA=1,2
  RHO=2.0*DELTA
  DD=DELTA*DELTA/12.0
  K=1
  J=4
  M=1
  BN(1)=KA
  BN(2)=3-KA
  DO 1 IA=1,2
  AI=IA-1
  AA=AL+2.*(AJ-AL)*AI
  CALL START(RHOC,ETA,RHOU,DEE,V,W,S,T,Q,R,AM,AN,AA,AJ,ALPHA,DELTA,X
1,Y,RHOUW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
  DO 1 I=1,2
  XA(I,IA)=X(I)*BN(IA)
1 YA(I,IA)=Y(I)*BN(IA)
  DO 6 I=1,2
  WFPR(I+1,KA)=XA(I,2)
  WFPC(I+1,KA)=YA(I,2)
  WFR(I+1,KA)=XA(I,1)
  6 WFC(I+1,KA)=YA(I,1)
200 DO 101 I=K,3
  DO 4 IA=1,2
  AI=IA-1
  AA=AL+2.*(AJ-AL)*AI
  CALL POTENS(RHO,RHOC,ETA,RHOU,DEE,V,W,S,T,Q,R,AM,AN,AA,AJ,VPR,VPC,
1 VTR,VTC,ALPHA,RHOUW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
  SFR(I,IA)=(-1.0-VPR+AA*(AA+1.0)/(RHO*RHO))*DD
  SFC(I,IA)=-VPC*DD
  SGR(I,IA)=VTR*DD
  4 SGC(I,IA)=VTC*DD
101 RHO=RHO+DELTA
  A=2.0+10.0*SFR(2,1)
  B=10.0*SFC(2,1)
  C=XA(2,1)
  D=YA(2,1)
  CALL COMPLX(A,B,C,D,3,VPR,VPC)
  A=10.0*SGR(2,2)
  B=10.0*SGC(2,2)
  C=XA(2,2)
  D=YA(2,2)
  CALL COMPLX(A,B,C,D,3,VTR,VTC)
  CALL COMPLX(VPR,VPC,VTR,VTC,1,SA,TA)
  A=1.0-SFR(1,1)
  B=-SFC(1,1)
  C=XA(1,1)
  D=YA(1,1)
  CALL COMPLX(A,B,C,D,3,VTR,VTC)
  CALL COMPLX(SA,TA,VTR,VTC,2,VPR,VPC)
  A=SGR(1,2)

```

```
B=SGC(1,2)
C=XA(1,2)
D=YA(1,2)
CALL COMPLX(A,B,C,D,3,VTR,VTC)
CALL COMPLX(VPR,VPC,VTR,VTC,1,AR,AC)
A=2.0+10.0*SFR(2,2)
B=10.0*SFC(2,2)
C=XA(2,2)
D=YA(2,2)
CALL COMPLX(A,B,C,D,3,VPR,VPC)
A=10.0*SGR(2,1)
B=10.0*SGC(2,1)
C=XA(2,1)
D=YA(2,1)
CALL COMPLX(A,B,C,D,3,VTR,VTC)
CALL COMPLX(VPR,VPC,VTR,VTC,1,SA,TA)
A=1.0-SFR(1,2)
B=-SFC(1,2)
C=XA(1,2)
D=YA(1,2)
CALL COMPLX(A,B,C,D,3,VTR,VTC)
CALL COMPLX(SA,TA,VTR,VTC,2,VPR,VPC)
A=SGR(1,1)
B=SGC(1,1)
C=XA(1,1)
D=YA(1,1)
CALL COMPLX(A,B,C,D,3,VTR,VTC)
CALL COMPLX(VPR,VPC,VTR,VTC,1,BR,BC)
A=1.0-SFR(3,2)
B=-SFC(3,2)
C=1.0-SFR(3,1)
D=-SFC(3,1)
CALL COMPLX(A,B,C,D,3,DR,DC)
A=SGR(3,1)
B=SGC(3,1)
CALL COMPLX(A,B,C,D,3,DAR,DAC)
CALL COMPLX(DR,DC,DAR,DAC,2,SA,TA)
A=1.0-SFR(3,2)
B=-SFC(3,2)
CALL COMPLX(A,B,AR,AC,3,VPR,VPC)
A=SGR(3,1)
B=SGC(3,1)
CALL COMPLX(A,B,BR,BC,3,VTR,VTC)
CALL COMPLX(VPR,VPC,VTR,VTC,1,DR,DC)
CALL COMPLX(DR,DC,SA,TA,4,VPR,VPC)
XA(3,1)=VPR
YA(3,1)=VPC
A=SGR(3,1)
B=SGC(3,1)
CALL COMPLX(A,B,AR,AC,3,VPR,VPC)
A=1.0-SFR(3,1)
B=-SFC(3,1)
CALL COMPLX(A,B,BR,BC,3,VTR,VTC)
CALL COMPLX(VPR,VPC,VTR,VTC,1,DR,DC)
CALL COMPLX(DR,DC,SA,TA,4,VPR,VPC)
XA(3,2)=VPR
YA(3,2)=VPC
DO 100 IA=1,2
DO 100 I=1,2
XA(I,IA)=XA(I+1,IA)
YA(I,IA)=YA(I+1,IA)
SFR(I,IA)=SFR(I+1,IA)
```

```
SFC(I,IA)=SFC(I+1,IA)
SGR(I,IA)=SGR(I+1,IA)
100 SGC(I,IA)=SGC(I+1,IA)
WFR(J,KA)=XA(3,1)
WFC(J,KA)=YA(3,1)
WFPR(J,KA)=XA(3,2)
WFPR(J,KA)=YA(3,2)
IF(J JJ)302,301,302
301 IF(M-2)303,304,302
303 JJ=JJ+JJB
304 XX(M,KA)=XA(3,1)
YY(M,KA)=YA(3,1)
XXA(M,KA)=XA(3,2)
YYA(M,KA)=YA(3,2)
M=M+1
302 J=J+1
K=3
IF(J-JMAX)200,200,2
2 JJ-JJA
RETURN
END
```

```

SUBROUTINE CMATCH(XX,YY,XXA,YYA,F,G,ALPHA,ALPHAA,BETA,BETAA,ALAMB,
1AMU,ALAMB,AMUP,AL,ALP,ETA)
  IMPLICIT DOUBLE(A-H,O-Z)
  DIMENSION XX(2,2),YY(2,2),XXA(2,2),YYA(2,2),F(2,2),G(2,2),C(2,2),C
1A(2,2),E(2),EA(2),A(2,2),B(2,2),AA(2,2),BA(2,2)
  DO 1 I=1,2
  DO 1 J=1,2
    A(I,J)=0.5*F(I,J)
    B(I,J)=A(I,J)
    AA(I,J)=-0.5*G(I,J)
1 BA(I,J)=-AA(I,J)
    W=XX(1,1)
    X=YY(1,1)
    Y=A(1,1)
    Z=AA(1,1)
    CALL COMPLX(W,X,Y,Z,4,U,V)
    C(1,1)=U
    CA(1,1)=V
    W=XX(2,1)
    X=YY(2,1)
    Y=A(2,1)
    Z=AA(2,1)
    CALL COMPLX(W,X,Y,Z,4,U,V)
    C(1,1)=C(1,1)-U
    CA(1,1)=CA(1,1)-V
    W=XXA(1,1)
    X=YYA(1,1)
    Y=A(1,2)
    Z=AA(1,2)
    CALL COMPLX(W,X,Y,Z,4,U,V)
    C(2,1)=U
    CA(2,1)=V
    W=XXA(2,1)
    X=YYA(2,1)
    Y=A(2,2)
    Z=AA(2,2)
    CALL COMPLX(W,X,Y,Z,4,U,V)
    C(2,1)=C(2,1)-U
    CA(2,1)=CA(2,1)-V
    W=B(1,1)
    X=BA(1,1)
    CALL OMEGA(AL,ETA,OM)
    OMR=COS(OM)
    OMC=SIN(OM)
    CALL COMPLX(W,X,OMR,OMC,3,S,T)
    Y=A(1,1)
    Z=AA(1,1)
    CALL COMPLX(S,T,Y,Z,4,U,V)
    E(1)=U
    EA(1)=V
    W=B(2,1)
    X=BA(2,1)
    CALL COMPLX(W,X,OMR,OMC,3,SA,TA)
    Y=A(2,1)
    Z=AA(2,1)
    CALL COMPLX(SA,TA,Y,Z,4,U,V)
    E(1)=E(1)-U
    EA(1)=EA(1)-V
    W=XX(1,2)
    X=YY(1,2)

```

```
Y=A(1,1)
Z=AA(1,1)
CALL COMPLEX(W,X,Y,Z,4,U,V)
C(1,2)=U
CA(1,2)=V
W=XX(2,2)
X=YY(2,2)
Y=A(2,1)
Z=AA(2,1)
CALL COMPLEX(W,X,Y,Z,4,U,V)
C(1,2)=C(1,2)-U
CA(1,2)=CA(1,2)-V
W=XXA(1,2)
X=YYA(1,2)
Y=A(1,2)
Z=AA(1,2)
CALL COMPLEX(W,X,Y,Z,4,U,V)
C(2,2)=U
CA(2,2)=V
W=XXA(2,2)
X=YYA(2,2)
Y=A(2,2)
Z=AA(2,2)
CALL COMPLEX(W,X,Y,Z,4,U,V)
C(2,2)=C(2,2)-U
CA(2,2)=CA(2,2)-V
W=C(2,2)
X=CA(2,2)
Y=E(1)
Z=EA(1)
CALL COMPLEX(W,X,Y,Z,3,U,V)
ALPHA=U
ALPHA=V
W=C(2,1)
X=CA(2,1)
Y=F(1)
Z=EA(1)
CALL COMPLEX(W,X,Y,Z,3,U,V)
BETA=-U
BETA=V
W=C(1,1)
X=CA(1,1)
Y=C(2,2)
Z=CA(2,2)
CALL COMPLEX(W,X,Y,Z,3,U,V)
D=U
DA=V
W=C(2,1)
X=CA(2,1)
Y=C(1,2)
Z=CA(1,2)
CALL COMPLEX(W,X,Y,Z,3,U,V)
D=D-U
DA=DA-V
CALL COMPLEX(ALPHA,ALPHA,D,DA,4,U,V)
ALPHA=U
ALPHA=V
CALL COMPLEX(BETA,BETA,D,DA,4,U,V)
BETA=U
BETA=V
W=ALPHA
X=ALPHA
```

```
Y=XX(1,1)
Z=YY(1,1)
CALL COMPLX(W,X,Y,Z,3,D,DA)
W=BETA
X=BETAA
Y=XX(1,2)
Z=YY(1,2)
CALL COMPLX(W,X,Y,Z,3,U,V)
W=D+U-S
X=DA+V-T
Y=A(1,1)
Z=AA(1,1)
CALL COMPLX(OMR,OMC,Y,Z,4,U,V)
CALL COMPLX(U,V,W,X,3,ALAMB,AMU)
Y=XXA(1,1)
Z=YYA(1,1)
CALL COMPLX(ALPHA,ALPHA,Y,Z,3,D,DA)
Y=XXA(1,2)
Z=YYA(1,2)
CALL COMPLX(BETA,BETAA,Y,Z,3,U,V)
W=D+U
X=DA+V
Y=A(1,2)
Z=AA(1,2)
CALL OMEGA(ALP,ETA,OM)
OMR=COS(OM)
OMC=SIN(OM)
CALL COMPLX(OMR,OMC,Y,Z,4,U,V)
CALL COMPLX(U,V,W,X,3,ALAMP,AMUP)
RETURN
END
```

```
SUBROUTINE OMEGA(AL,ETA,S)
  IMPLICIT DOUBLE(A-H,U-Z)
  S=0.0
  A=1.0
3  IF(AL-A)1,2,2
2  S=S+ATAN(ETA/A)
  A=A+1.0
  GO TO 3
1  RETURN
  END
```

```
SUBROUTINE ASSLEG(EM,EL,X,DM,PM)
  IMPLICIT DOUBLE(A-H,O-Z)
498 FORMAT(5H ER 3)
570 FORMAT(5H ER 4)
  IF(ABS(X)-1.0)550,550,499
499 PRINT 498
  CALL EXIT
550 IF(EM)554,554,552
552 DN=EM
553 DN=DN-2.0
  IF(DN)569,568,553
554 EM=-EM
568 DN=1.0
569 IF(EL-EM)555,556,556
555 DM=0.0
  PM=0.0
  RETURN
556 A=0.0
559 IF(EL-EM-A-2.0)557,558,558
558 A=A+2.0
  DN=-DN*A
  GO TO 559
557 Q=EL-EM-A
  DM=(1.0-Q+Q*X)/DN
  DN=2.0*EL-A-1.0
561 DM=DM*DN
  DN=DN-2.0
  IF(DN)560,560,561
560 PM=DM
562 Q=Q+2.0
  DM=(-DM)*X*X*(2.*EL+1.-A)*A/(Q*(Q-1.))
  B=0.0
  IF(EL)563,584,563
584 IF(EM)563,590,563
590 B=2.0
563 PM=PM+DM+B
  A=A-2.0
  IF(A)564,564,562
564 Q=0.0
  DN=1.0
567 IF(2.0*EM-Q)555,565,566
566 DN=(EL+EM-Q)*DN
  Q=Q+1.0
  GO TO 567
565 IF(EM)571,572,571
571 EMMM=0.5*EM
  PM=PM*(1.0-X*X)**EMMM
572 DM=PM*SQR((EL+0.5)/DN)
  RETURN
END
```



```
SUBROUTINE COMPLEX(A,B,C,D,N,R,S)
  IMPLICIT DOUBLE(A-H,O-Z)
  CA=C
  DA=D
  IF(N-2)1,2,3
2  C=-C
  D=-D
1  R=A+C
  S=B+D
6  C=CA
  D=DA
  RETURN
3  IF(N-4)4,5,5
5  T=C*C+D*D
  C=C/T
  D=-D/T
4  R=A*C-B*D
  S=B*C+A*D
  GO TO 6
END
```

```

SUBROUTINE START(RHOC,ETA,RHOD,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,ALPHA,D
IELTA,X,Y,RHOOW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
IMPLICIT DOUBLE(A-H,O-Z)
DIMENSION X(2),Y(2),BESWS(51)
RHO=2.*DELTA
DO 2 J=1,2
CALL POTENS(RHO,RHOC,ETA,RHOD,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,VPR,VPC,
1VTR,VTC,ALPHA,RHOOW,DEEW,RHOOT,DEET,AK2,RHOOSO,DEESO,RHOOL,DEEL)
AG=1.+VPR
BG=VPC
UA=((AG+(AG*AG+BG*BG)**0.5)**0.5)*RHO*0.70710678
VA=RHO*RHO*BG*0.5/UA
LM=AL+11.0
N=LM
11 ANN=N
AK=1.0
SA=1.0
TA=1.0
5 TA=(-UA*UA*TA)/(2.0*AK*(2.0*(ANN+AK)-1.0))
IF(TA+1.0E-8)3,3,4
4 IF(TA-1.0E-8)6,3,3
3 SA=SA+TA
AK=AK+1.0
GO TO 5
6 D=1.0
DA=2.0*ANN-1.0
9 IF(DA-2.0)7,7,8
8 D=DA*D
DA=DA-2.0
GO TO 9
7 BESWS(N)=1.0/D
QQQ=ANN-1.0
BESWS(N)=(UA**QQQ*SA)*BESWS(N)
N=N-1
IF(N-LM+2)10,10,11
10 BESWS(N)=(2.0*ANN-1.0)*BESWS(N+1)/UA-BESWS(N+2)
IF(N-LM+10)12,12,13
13 N=N-1
ANN=ANN-1.0
GO TO 10
12 RRS=BESWS(N)
RCS=0.0
TA=VA/UA
PA=(VA*TA)*0.5
QA=-VA
PPA=PA
QPA=QA
DO 32 L=1,10
K=N+L
RRS=RRS+PA*BESWS(K)
RCS=RCS+QA*BESWS(K)
ALA=L
PPA=PPA*ALA/(ALA+1.0)
QPA=QPA*ALA/(ALA+1.0)
PW=PA
PA=PPA*PA-QPA*QA
32 QA=QPA*PW+PPA*QA
X(J)=RRS*UA-RCS*VA
Y(J)=RCS*UA+RRS*VA
2 RHU=RHO+DELTA
RETURN
END

```

```

SUBROUTINE POTENS(RHO,RHOC,ETA,RHOU,DEE,V,W,S,T,Q,R,AM,AN,AL,AJ,VP
1R,VPC,VTR,VTC,ALPHA,RHODW,DEEW,RHOOT,DEET,AK2,RHOOS,DEES,RHOOL,D
2EEL)
  IMPLICIT DOUBLE(A-H,O-Z)
  CALL CRHO(RHO,RHOC,ETA,C)
  CALL SAXON(RHO,RHOU,DEE,G)
  CALL DSAXON(RHO,RHOOS,DEES,DG)
  CALL USAXON(RHO,RHODW,DEEW,DGW)
  CALL USAXON(RHO,RHOOT,DEET,DGT)
  CALL DSAXON(RHO,RHOOL,DEEL,DGL)
  DGW=-DGW*4.*RHO/DEEW
  DGT=-DGT*4.*RHO/DEET
  DGL=-DGL*4.*RHO/DEEL
  DG=AK2*DG
  VPR=-C+V*G*ALPHA
  VPC=W*DG*ALPHA
  VTR=0.0
  VTC=0.0
  IF(AJ-AL-0.75)1,1,3
1  IF(AJ-AL-0.25)11,11,2
11 IF(AJ-AL+0.25)12,12,6
12 IF(AJ-AL+0.75)4,4,5
2  AK=AL
  GO TO 7
5  AK=-AL-1.0
7  VPR=VPR-S*DG*AK*ALPHA
  VPC=VPC-T*DG*AK*ALPHA
  RETURN
4  AK=-AL-1.0
  AKL=-(2.*AL+3.)*AK/6.0
  AKR=AK/(2.*AL-1.)/3.0
  GO TO 8
3  AK=AL
  AKL=AL*(2.*AL-1.)/6.0
  AKR=-AL/(2.*AL+3.)/3.0
  GO TO 8
6  AK=-1.0
  AKL=-(2.*AL+3.)*(2.*AL-1.)/6.0
  AKR=1./3.0
8  VPR=VPR-(S*AK*DG+Q*AKL*DGL+AM*AKR*DGT)*ALPHA
  VPC=VPC-(T*AK*DG+R*AKL*DGL+AN*AKR*DGT)*ALPHA
  IF(AL-AJ)9,10,9
9  VTC=(AJ*(AJ+1.))**0.5/(2.*AJ+1.)*DGT*ALPHA
  VTR=AM*VTC
  VTC=AN*VTC
10 RETURN
  END

```

```
SUBROUTINE CRHO(RHO,RHOC,ETA,C)
  IMPLICIT DOUBLE(A-H,O-Z)
  IF(RHO-RHOC)1,2,2
2 C=2.*ETA/RHO
  RETURN
1 C=ETA*(3.-(RHO/RHOC)**2)/RHOC
  RETURN
END
```

```
SUBROUTINE SAXON(RHO,RH00,DEE,G)
  IMPLICIT DOUBLE(A-H,O-Z)
  G=1.+EXP((RHO-RH00)*DEE)
  G=1./G
  RETURN
END
```

```
SUBROUTINE DSAXON(RHO,RHOO,DEE,DG)
IMPLICIT DOUBLE(A-H,O-Z)
IF(RHO-0.1*RHOO)1,2,2
2 CALL SAXON(RHO,RHOO,DEE,G)
DG=-G*G*DEE*EXP((RHO-RHOO)*DEE)/RHO
RETURN
1 DG=0.0
RETURN
END
```

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