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FUEL COSTS IN BATCH-
AND CONTINUALLY-PROCESSED HOMOGENEOUS REACTORS

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Abstract

The fuel requirements of a heavy-water moderated, homogeneous, power reactor were estimated for a variety of initial loadings, for both batch and continuous methods of fuel removal. A 12-foot diameter, spherical reactor was considered, loaded initially with specified concentrations of U^{235} and U^{238} . The fuel feed rates were obtained for the "batch" cases as a function of operating time, and for the "continuous" cases as a function of feed enrichment and fuel processing cycle time. The associated fuel costs were estimated as a function of operating time. This study considered a reactor temperature of 250°C , 500 Mw thermal power, 125 Mw electrical power capability, 0.8 load factor, and 4%/year inventory charges for uranium and D_2O . The fuel shipping-and-processing charges were assumed to be \$1/gm of fissionable fuel for the "batch" processed reactors, and \$0.37/gm for the "continuous" processed reactors. Under these conditions, the minimum fuel costs associated with a 10-year "batch" operating period were about 1.8 or 3.1 mills/kw-hr, if highly enriched uranium cost \$15/gm or \$30/gm, respectively. The analogous costs for the "continuous" processed reactor were about 1.6 and 2.6 mills/kw-hr, respectively. For these cases, the optimum U concentration was about 500 gm/liter.

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If fuel processing costs were \$3.70/gm of fissionable fuel, the fuel costs for the "optimum" batch operated reactors would not be significantly changed. However, the fuel costs for the continuously processed reactor would now be about 2.1 or 3.2 mills/kw-hr based on \$15/gm or \$30/gm, respectively, for highly enriched uranium.

Introduction:

In aqueous homogeneous reactor systems, radiation damage to the fuel itself does not appear to limit fuel exposure time. This, in combination with the system flexibility² which permits large changes in fuel concentration and removal of fission gases such as Xe¹³⁵, indicates that homogeneous reactors may be operated for long periods of time without any fuel processing necessary. Under these circumstances nuclear power stations could be built and operated many years without the need of a fuel processing facility. Centralized processing facilities to recover the fuel at the end of the operating period could be designed and built to meet the needs of these nuclear power plants as required.

Previous results³ for homogeneous U²³⁵ burner reactors indicated that no nuclear limitation should limit the operating period, and that the economic penalty associated with batch operation could be small. One purpose of this paper was to extend the above study to include reactor systems which are initially loaded with a fuel of relatively low enrichment, and to determine whether fuel costs in these systems could be appreciably lower than those in pure burner reactors. The other purpose was to determine whether it would be necessary from an economic-power viewpoint to process the fuel continuously.

Cases Studied:

The particular cases studied are given in Table 1. The choice of reactor diameter was arbitrary and may not be the best one. However, the chosen 12-ft diameter core size implies low neutron leakage, and also appears within the realm of fabrication feasibility. The uranium was assumed to be present as UO_3 , since this compound has a low parasitic capture cross-section, and shows promise as a reactor fuel.⁴ Heavy water was assumed to be the moderator because of its excellent moderating ratio. The total power was assumed to be 500 Mw, based on an average core power density of about 20 kw/liter (the latter being the average power density of the HRE¹). The power removal capability was assumed to be equivalent to removing 20 kw/(liter of fuel solution) based on the external fuel system volume (piping and heat exchangers), so that the total fuel system volume was twice the core volume. The average reactor temperature was assumed to be 250°C, corresponding to an outlet core fluid temperature which would permit a reasonable thermal efficiency insofar as electrical power generation is concerned.

TABLE 1
SPECIFIC CASES INVESTIGATED*

Case	$\frac{\text{gm U}^{238}}{\text{kg D}_2\text{O}}$	$\frac{\text{gm U}^{238}}{\text{liter}}$	p(28)	D, cm
1	250	220	.770	.966
2	500	440	.680	.923
3	750	660	.615	.896
4	1000	880	.562	.870

*General Specifications: Reactor Temperature, 250°C; Core Diameter 12 ft; Core Volume, 25,500 liters; Reactor Fuel-System Vol, 51,000 liters; Moderator-Coolant, D₂O; Power, 500 Mw; Ave. Power Density in reactor core, 20 kw/l; Equivalent removal-power-density in fuel system external to core volume, 20 kw/l; Uranium as UO₃.

The cases presented in Table 1 were studied for both "batch" and "continuous" fuel processing, and case numbers will be designated, e.g., as Case 1B or Case 1C when referring to Case 1 of Table 1, for "batch" or "continuous" processing, respectively.

Cost Factors and Assumptions:

In making economic calculations the following assumptions were made:

1. Initial inventory of D_2O was 50,000 kg.
(All other initial inventories were based on this number and initial concentrations.)
2. Cost of D_2O = \$28/lb.
3. Cost of natural U as UO_3 = \$40/kg U^{235}
4. Value of U^{235} and Pu^{249} assumed to be either \$15/gm or \$30/gm for 90% enriched material⁵ (value of fuel having lower enrichments was calculated on the same bases used by Lane⁷).
5. Interest on investment in D_2O , U^{238} and U^{235} assumed to be 4%/year.
6. D_2O losses taken as 2%/year.
7. Cost of fuel processing = \$0.37/gm of U^{235} and Pu^{239}
(Based on estimate of Ohlgren et al⁷ for one 500 Mw plant processing fuel of low enrichment).
8. Cost of shipping fuel = 0 for continuous processing; assumed to equal \$0.63/gm of 25 and 49 for batch operation, i.e., cost of shipping and processing was \$1/gm fissionable material for "batch" cases.
9. Plant load factor = 0.80
10. Net thermal efficiency = 25%; corresponding to 125 Mw of electrical power.

Total fuel costs were estimated for reactor operating times of 10 and 25 years.

BATCH OPERATION STUDIES

Mathematical Formulation:

The fuel costs are a function of the required fuel concentrations. The time behavior of the fuel concentration is controlled by the rate material balances in combination with the critical equation. Assuming that the U^{238} concentration was independent of time, that the fuel fluid was exposed to an average neutron flux while within the reactor system, that fluid transport times were negligible in comparison to times of interest, that U^{239} decayed virtually instantly to Pu^{239} , and that U^{236} and Pu^{240} had essentially zero neutron absorption cross sections, the appropriate equations were

$$\frac{d N(25)}{dt} = F(25) - \sigma(25) N(25) \phi \quad (1)$$

$$\frac{d N(49)}{dt} = \Sigma(28) \phi + \frac{1 - p(28)}{1 + B^2 \tau} \left[\eta(25) \Sigma(25) + \eta(49) \Sigma(49) \right] \phi - \Sigma(49) \phi \quad (2)$$

$$\eta \in pf = (1 + B^2 \tau) (1 + B^2 L^2) \quad (3)$$

$$\Sigma(p) = \Sigma(p_1) + \Sigma(p_2) \quad (4)$$

$$\frac{d N(p_1)}{dt} = c_1 \left[\Sigma_f(25) + \Sigma_f(49) \right] \phi - \Sigma(p_1) \phi \quad (5)$$

$$\frac{d N(p_2)}{dt} = c_2 \left[\Sigma_f(25) + \Sigma_f(49) \right] \phi - \Sigma(p_2) \phi \quad (6)$$

Eqs. 1 and 2 are rate material balances on U^{235} and Pu^{239} , Eq. 3 is the critical equation, Eq. 4 relates the total neutron poison cross section in terms of two effective poison cross sections, as approximated by Robb et al;⁸ Eqs. 5 and 6 are rate material balances on these effective neutron poisons. Defining $dT = \phi dt$, Eqs. 1 - 6 can be written (after manipulating the critical equation) as

$$\frac{d N(25)}{dT} = \frac{F(25)}{\phi} - \sigma(25) N(25) \quad (7)$$

$$\begin{aligned} \frac{d N(49)}{dT} = & \Sigma(28) + \frac{1 - p(28)}{1 + B^2 \tau} \left[\eta(25) \sigma(25) N(25) + \eta(49) \sigma(49) N(49) \right] \\ & - \sigma(49) N(49) \end{aligned} \quad (8)$$

$$\beta(25) N(25) + \beta(49) N(49) = m_1 + \sigma(p_1) N(p_1) + \sigma(p_2) N(p_2) \quad (9)$$

$$\frac{d N(p_1)}{dT} = c_1 \left[\sigma_f(25) N(25) + \sigma_f(49) N(49) \right] - \sigma(p_1) N(p_1) \quad (10)$$

$$\frac{d N(p_2)}{dT} = c_2 \left[\sigma_f(25) N(25) + \sigma_f(49) N(49) \right] - \sigma(p_2) N(p_2) \quad (11)$$

where $T = \int_0^t \phi dt$

$$\beta(25) = \left[\frac{\eta(25) (1 - p(28))}{1 + B^2 \tau} - 1 - (fp)_0 \right] \sigma(25)$$

$$\beta(49) = \left[\frac{\eta(49) (1 - p(28))}{1 + B^2 \tau} - 1 - (fp)_0 \right] \sigma(49)$$

$$m_1 = B^2 D + \Sigma(mod) + \Sigma(28)$$

(f.p.)_o = fraction poisoning due to high absorption-cross-section poisons, based upon fuel absorption-cross-section

Eqs. 7 - 11 were solved simultaneously and the results written in the following form:

$$N(49) = A_1 e^{r_1 T} + A_2 e^{r_2 T} + A_3 e^{r_3 T} + A_4 \quad (12)$$

$$N(25) = \frac{1}{\beta(25)} \left[m_1 + D_4 + D_1 e^{r_1 T} + D_2 e^{r_2 T} + D_3 e^{r_3 T} \right] \quad (13)$$

$$\begin{aligned} \frac{F(25)}{\phi} = \frac{1}{\beta(25)} & \left\{ (D_4 + m_1) \sigma(25) + D_1 e^{r_1 T} \left[\sigma(25) + r_1 \right] \right. \\ & \left. + D_2 e^{r_2 T} \left[\sigma(25) + r_2 \right] + D_3 e^{r_3 T} \left[\sigma(25) + r_3 \right] \right\} \quad (14) \end{aligned}$$

where r_1 , r_2 and r_3 are the roots of the equation

$$s^3 + E_1 s^2 + E_2 s + E_3 = 0 \quad (15)$$

and A_i , D_i and E_i are functions of input parameter values and are defined in the Nomenclature.

Since $\Sigma(28)$ was assumed constant, the feed rate of U^{238} required to satisfy this condition was obtained from the equation

$$\frac{F(28)}{\phi} = \Sigma(28) + \frac{1 - p(28)}{1 + B^2 \tau} \left[\eta(25) \Sigma(25) + \eta(49) \Sigma(49) \right] \quad (16)$$

The enrichment of the uranium fed to the system was then obtained from the relation

$$\gamma = \frac{F(25)}{F(25) + F(28)} \quad (17)$$

Separation of t and T could be obtained since $\Sigma_f \phi$ is a constant for constant power operation. Thus,

$$t = \int_0^t \frac{\phi dt}{\phi} = \int_0^T b [\Sigma_f(T)] dT \quad (18)$$

where b is a constant, and $\Sigma_f(T) = \sigma_f(49) N(49) + \sigma_f(25) N(25)$.

The value of $\int_0^t F dt$ was found analytically from the relation

$$\int_0^t F dt = \int_0^t \frac{F}{\phi} \phi dt = \int_0^T \frac{F}{\phi}(T) dT \quad (19)$$

Results and Discussion:

Eqs. 12 - 19 were used to obtain the concentration behavior and feed requirements as a function of operating time for the various cases. These results were used in conjunction with the cost factors given previously to obtain fuel costs in terms of mills/kw-hr. The total fuel costs for the four cases under study are summarized in Table 2 and plotted in Fig. 1 as fuel costs versus U^{238} concentration. The results can be compared to a minimum fuel cost of about 4 or 8 mills/kw-hr for pure burner reactors³ if highly enriched fuel-feed cost 15 or \$30/gm, respectively (the same cost factors were used in the "burner" calculations). Thus, an appreciable savings in fuel costs could be achieved by the addition of fertile material to the reactor. Because of the conversion of U^{238} to Pu^{239} , the fuel costs are not as sensitive to the cost of enriched uranium as in the case of a "burner." Little savings in fuel costs appear to be effected by operating for 25 years rather than 10 years.

TABLE 2

TOTAL FUEL COSTS FOR BATCH REACTOR^a

Operating Period		10 yrs		25 yrs	
Cost of 90% enriched U, \$/gm	15		30	15	30
Case ^b	Total Fuel Cost, mills/kw-hr				
1B	2.21		4.04	2.26	4.23
2B	1.80		3.17	1.79	3.27
3B	1.92		3.31	1.72	2.92
4B	2.70		4.11	2.21	3.66

a. Based on reactor temperature of 250°C; 500 Mw thermal power; 125 Mw electrical power; 0.8 load factor, 4%/year inventory charges; and shipping and processing costs = \$1/gm fissionable fuel.

b. The B after the case number specifies batch fuel processing.

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A breakdown of the total fuel costs for Case 2B is given in Table 3, and illustrates the relative importance of the various costs. It can be seen that feed cost was the most important item contributing to the total fuel cost. The inventory charge for the initial fuel loading was also an important item, and this increased with case number since the initial fuel enrichment and the total fuel mass would be higher for reactors with higher fertile material concentrations. This charge was partially offset by the value of the U^{235} recovered at the end of operation. The variation obtained indicated that recovery of the fuel would become more important as the concentration of fertile material were increased, and that fuel processing costs would also increase. However, the fuel shipping and processing charges did not become an important item, which indicated that appreciably higher charges could be permitted for these items than assumed here, without significantly affecting the total fuel costs.

A summary of the concentration and feed data for Cases 1B - 4B is given in Table 4. The feed rate can be compared to a minimum feed rate of $660 \frac{\text{gm } U^{235}}{\text{day}}$ required in a pure burner reactor. An effective conversion ratio, $(C.R.)_e$, can be considered, which would be

$$(C.R.)_e = 1 - \frac{\text{feed requirements}}{\text{feed requirements in burner}} \quad (20)$$

Thus, a feed rate of $400 \frac{\text{gm } U^{235}}{\text{day}}$ would correspond to a $(C.R.)_e \approx 0.4$.

TABLE 3

BREAKDOWN OF FUEL COSTS FOR CASE 2B^a

Operating Period			10 yrs		25 yrs	
D ₂ O Inventory chg	mills/kw-hr		0.32		0.21	
D ₂ O Operating chg	mills/kw-hr		0.07		0.07	
Cost of 90% enriched U,	\$/gm	15	30	15	30	
Value of Initial Fuel Loading	mills/kw-hr	0.46	0.72	0.18	0.29	
Inventory Charge for Initial Loading		0.18	0.29	0.18	0.29	
Feed Cost	mills/kw-hr	1.55	3.10	1.78	3.56	
Value of Recovered fuel	mills/kw-hr	0.86	1.41	0.69	1.20	
Cost of fuel recovery plus Cost of fuel shipping*	mills/kw-hr		0.07		0.05	
Fuel losses †		0.009	0.015	0.007	0.012	
Total Fuel Costs	mills/kw-hr	1.80	3.17	1.79	3.27	

a. Reactor temperature 250°C; 500 Mw thermal power; 125 Mw electrical power; 0.8 load factor; 4%/yr inventory chgs; 220 gm U²³⁸/liter

* Assumed to equal \$1/gm fuel (25 + 49). This compares with \$0.37/gm assumed for "continuous" processing charge.

† Assumed to be 1% of final fuel value

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TABLE 4

SUMMARY OF CONCENTRATION DATA FOR BATCH OPERATED REACTORS*

Case	1B		2B		3B		4B	
Initial Critical Conc. gm U ²³⁵ /liter	2.68		7.40		17.64		49.22	
U ²³⁸ concentration, gms/liter	220		440		660		880	
Operating Time, yrs	10	25	10	25	10	25	10	25
N(25), gms/liter	4.13	8.21	7.03	13.2	15.4	26.3	71.8	112.4
N(49), gms/liter	1.93	3.21	5.56	9.06	13.3	22.2	28.3	56.4
N(40) + N(26) (approx), gm/liter	9	30	12	40	20	50	20	50
feed enrichment, %	48.7%	52.9%	37.3%	40.2%	35.7%	37.2%	53.0%	52.4%
U ²³⁵ feed rate, gm/day	403	429	335	354	354	367	753	735
∫ ₀ ^{Op. Time} F(25) dt, gm U ²³⁵	1.30x10 ⁶	3.60x10 ⁶	1.02x10 ⁶	2.92x10 ⁶	1.07x10 ⁶	3.06x10 ⁶	2.85x10 ⁶	6.91x10 ⁶

* Reactor temperature, 250°C; 500 Mw thermal power

CONTINUOUSLY PROCESSED REACTORS

Mathematical System:

The equations used here were also based on the assumptions given prior to Eqs. 1 - 6; however, it was assumed that steady-state conditions apply during the period of operation (deviations from this condition are considered later). It was also assumed fission product poisons were removed in the fuel processing cycle, and that the fuel and fertile material were returned to the reactor at the same rate they were removed (since the reactor had a conversion ratio less than 1, no difficulty was encountered because of this assumption). The mathematical system was then given by

$$F(28) = \Sigma(28) \phi + \frac{1 - p(28)}{1 + B^2 \tau} \left[\eta(25) \Sigma(25) + \eta(49) \Sigma(49) \right] \phi \quad (21)$$

$$F(28) = \Sigma(49) \phi \quad (22)$$

$$F(25) = \Sigma(25) \phi \quad (23)$$

$$\beta(25) N(25) + \beta(49) N(49) = m_1 + \Sigma(p) \quad (24)$$

$$\Sigma(p) \phi + \frac{N(p)}{\theta} = cP \quad (25)$$

where c = constant, $\frac{\text{atoms of fission product poisons produced}}{\text{vol-time-fission power}}$

P = reactor thermal power

$N(p)$ = concentration of fission product poisons

$\Sigma(p) = N(p) \sigma(p)$ = effective macroscopic cross-section of fission product poisons

θ = fuel processing cycle time

All other symbols have their previous meaning.

In solving the above system, the method of evaluating $\Sigma(p)$ should consider the effect of fuel processing. This was done on the following basis: If a reactor were operated at a constant fuel loading and constant flux, the value for $\Sigma(p)$ at time t after startup would be given by

$$\Sigma(p)_b = c_3 \Sigma_f \phi \int_0^t \sigma_p(t - \tau) d\tau \quad (26)$$

where c_3 = atoms of fission product poisons produced/fission

$\Sigma(p)_b$ = effective macroscopic fission-product-poison-cross-section for batch operation

The value for $\sigma_p(t)$ would then be given by

$$\sigma_p(t) = \frac{1}{c_3 \Sigma_f \phi} \frac{d \Sigma(p)_b}{dt} \quad (27)$$

For the above conditions, $\Sigma(p)_b$ has been approximated by Robb et al⁸ as

$$\Sigma(p)_b = c_1 \left(1 - e^{-\sigma(p)_1 \phi t} \right) + c_2 \left(1 - e^{-\sigma(p)_2 \phi t} \right) \quad (28)$$

so that $\sigma_p(t)$ can be evaluated. For a continuously processed reactor, the effective poison cross section will be lowered due to the fuel processing cycle which would remove fission product poisons. Under these circumstances the $\Sigma(p)$ of Eq. 24 or 25 would be given by

$$\Sigma(p) = c_3 \Sigma_f \phi \int_{-\infty}^t \sigma_p(t - \tau) e^{-\frac{t - \tau}{\theta}} d\tau \quad (29)$$

Combining Eqs. 27-29, $\Sigma(p)$ was obtained as

$$\Sigma(p) = \Sigma_f \left[\frac{c_1 \sigma(p)_1 \phi}{\sigma(p)_1 \phi + \frac{1}{\theta}} + \frac{c_2 \sigma(p)_2 \phi}{\sigma(p)_2 \phi + \frac{1}{\theta}} \right] \quad (30)$$

The mathematical system then consisted of Eqs. 21 - 24 and Eq. 30, and was solved to give feed rates and element concentrations for various process cycle times θ . This was most easily done by assuming that $\Sigma(p)$ was some fraction of $\Sigma(25) + \Sigma(49)$, and solving Eqs. 21 - 24 for feed rates and fuel concentrations. Since $\Sigma_p \phi$ would be a constant for constant power operation, the above results were used to evaluate ϕ . With ϕ known, the value of θ which produced the assumed value of $\Sigma(p)$ was obtained from Eq. 30. The above procedure was repeated until θ covered the desired range.

Results and Discussion:

The steady-state results obtained are summarized in Table 5, and include feed enrichment, feed rate, and fuel concentration as a function of fuel processing cycle time. These results were combined with previous cost factors to give the total fuel costs. Table 6 gives the total fuel cost as a function of enriched uranium cost, fuel processing cycle time, fertile material concentration, and operating period. These results are plotted in Figs. 2 and 3 for a 10-year operating period. A minimum fuel cost occurred for each case, which was dependent upon the processing cycle time. For θ greater than this optimum, the fuel inventory charge became excessive due to the increased poison level, while for smaller values of θ the fuel processing cost became excessive. The results given in Figs. 2 and 3 were combined and are presented in Fig. 4, which plots the minimum fuel cost versus the fertile-material concentration. The cycle time at specific points is also indicated.

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TABLE 5

SUMMARY OF STEADY-STATE RESULTS FOR CASES OF CONTINUOUS FUEL PROCESSING

Case	(f.p.)-%	θ -Days	γ -%	F(25) $\frac{\text{gm}}{\text{day}}$	N(25) - gm/l	N(49) gm/l	N(49)+N(25) gm/l
1C	0	0	23.62	189.4	0.648	0.874	1.522
	14.14	211.4	30.57	240.4	1.070	1.014	2.084
	21.00	397.3	34.05	265.2	1.383	1.118	2.501
	27.73	676.5	37.53	289.5	1.816	1.261	3.077
	34.33	1115	41.01	313.3	2.452	1.472	3.924
2C	0	0	17.56	143.4	1.423	2.787	4.210
	2.92	91.56	19.13	155.5	1.672	2.949	4.621
	11.53	535.6	23.84	191.1	2.725	3.634	6.359
	14.35	771.6	25.40	202.7	3.237	3.967	7.204
	21.28	1752	29.32	231.4	5.236	5.268	10.504
3C	0	0	15.68	128.7	3.508	7.876	11.384
	0.74	58.94	16.08	131.9	3.731	8.124	11.855
	2.20	194.4	16.90	138.3	4.230	8.679	12.909
	4.40	455.2	18.13	147.8	5.146	9.698	14.844
	7.29	959.9	19.76	160.3	6.832	11.58	18.412
	11.58	2407	22.22	178.9	11.41	16.67	28.08
4C	0	0	14.84	122.1	17.41	41.70	59.11
	0.74	347.7	15.26	125.4	20.88	48.40	69.28
	1.18	632.7	15.55	127.7	24.05	54.51	78.56
	1.62	967.6	15.76	129.3	26.94	60.10	87.04
	2.94	2861	16.51	135.2	45.07	95.11	140.18

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TABLE 6

Total Fuel Cost for Continuous Fuel Processing, mills/kwhr^a

Case ^b →		1C		2C		3C		4C	
Cost of 90% enriched U, \$/gm →		15	30	15	30	15	30	15	30
Operating Time, ↓	T Processing Cycle Time, days ↓	Total Fuel Cost, mills/kwhr							
10 yrs.	50	1.81	2.97	1.98	2.91	3.17	4.25	10.58	12.60
	100	1.78	3.02	1.74	2.71	2.47	3.55	6.86	8.89
	200	1.84	3.19	1.67	2.70	2.13	3.24	5.02	7.05
	400	1.97	3.47	1.69	2.81	2.00	3.15	4.10	6.14
	800	2.13	3.80	1.76	2.99	1.97	3.18	3.66	5.72
	1200	2.22	3.98	1.83	3.14	1.99	3.24	3.34	5.41
25 yrs.	50	1.70	2.86	1.87	2.80	3.05	4.13	9.41	12.43
	100	1.67	2.91	1.63	2.60	2.35	3.43	6.68	8.71
	200	1.73	3.08	1.56	2.59	2.01	3.12	4.83	6.86
	400	1.86	3.36	1.58	2.70	1.88	3.03	3.91	5.95
	800	2.02	3.69	1.64	2.87	1.84	3.05	3.45	5.51
	1200	2.11	3.87	1.71	3.02	1.86	3.11	3.11	5.18

a. Based on a reactor temperature of 250°C; 500 Mw thermal power; 125 Mw electrical power, 0.8 load factor, and 4%/year inventory charges. The reactor diameter was 12 ft, the system volume 51,000 liters, and the cost of D₂O was taken as \$28/lb.

b. The C after the Case number specifies continuous fuel processing.

Fig 4

Minimum Fuel Costs For Continuous
Fuel Processing vs U-238 Concentration

The results show that as the U^{238} concentration increased, the optimum value of θ also increased, this behavior being due to the decreasing effect that fission product poisons would have upon fuel concentration. Also, as the cost of feed increased, the optimum θ decreased, since a shorter θ would increase the conversion ratio and tend to decrease fuel costs. However, if the processing costs were increased (with a given feed cost), the optimum θ would be greater than given here.

A breakdown of the total fuel costs is given in Table 7 for Case 2C, based on steady-state calculations for a 10-year operating period. This case corresponds to about the optimum economic system. It is seen that the fuel processing cost decreased with increasing θ , but that fuel-feed cost increased under the same condition. Changing the operating period from 10 to 25 years lowered the fuel costs only slightly.

In calculating the fuel costs, the initial fuel loading was assumed to be of low-enrichment uranium, corresponding to that for the "batch" reactors. Under this condition, the fuel feed rate would not be given by the steady-state result; rather, it would be determined from the time-dependent equations, taking into consideration the poison-removal rate. However, if the operating period were long, the final fuel concentrations would be adequately given by steady-state results. To indicate the approach to steady-state conditions, the time-dependent equations were solved for Case 2 assuming that no fission-product poisons were present ($\theta = 0$). The results are given in Fig. 5, and consist of the U^{235} feed rate, total U^{235} feed, feed enrichment, and fissionable-fuel concentration as a function of reactor operating time. The results indicate that steady-state conditions would exist for times greater than about two years.

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TABLE 7

Breakdown of Total Fuel Costs for Case 2C, for 10-Year Operating Time^a

D ₂ O Inventory chg,	mills/kw-hr													.316	
D ₂ O Operating chg,	mills/kw-hr													.070	
Fuel Processing Cycle Time,	θ, days	50		100		200		400		800		1200			
"Continuous" Processing Cost	mills/kw-hr	.56		.293		.159		.092		.057		.045			
Final Processing Cost	mills/kw-hr	.009		.010		.011		.012		.016		.019			
Cost of 90% enriched U,	\$/gm	15	30	15	30	15	30	15	30	15	30	15	30	15	30
Steady-state U feed cost	mills/kw-hr	.835	1.67	.868	1.736	.927	1.854	1.015	2.03	1.13	2.26	1.21	2.42	1.21	2.42
Inventory chg for initial loading of U	mills/kw-hr	.185	.287	.185	.287	.185	.287	.185	.287	.185	.287	.185	.287	.185	.287
Total Fuel Cost	millw/kw-hr	1.98	2.91	1.74	2.71	1.67	2.70	1.69	2.81	1.76	2.99	1.83	3.14	1.83	3.14

a. Based on a reactor temperature of 250°C; 500 Mw thermal power; 125 Mw electrical power; 0.8 load factor, 4%/yr inventory charges, and steady-state calculations.

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Fig. 5
Feed Rate, Total Feed, Amount of
Fissionable Material Present and
Enrichment versus Time for Case 2,
with no poisons
present

after start-up for this particular case. Thus, it appears that steady-state calculations should adequately give the final fuel concentrations after 10 and 25 years operating time.

With all poisons removed, the total U^{235} feed given by the steady-state results would be slightly higher than the actual feed required, since during the initial two years of operation the actual feed rate would be less than the steady-state value. This is a result of the conversion of U^{238} to Pu^{239} , and is due to the lower critical mass of a Pu^{239} -fueled-reactor relative to a U^{235} -fueled-reactor. Based on the results of Fig. 5, the steady-state results for $F(25)$ would be about 10% high after 10 years operation and about 4% high after 25 years.

If not all the fission-product poisons were removed ($\theta > 0$), the U^{235} feed-rate curve would vary in the manner indicated in Fig. 6. Under such conditions the overestimate of feed costs given by steady-state results would not be as great as indicated above. [With $\theta = 0$, the overestimate of the feed cost would be equivalent to about 0.08 mills/kw-hr after 10 years operation for Case 2C (\$15/gm for highly enriched uranium), while with $\theta = 200$ days the overestimate would be equivalent to about 0.04 mills/kw-hr (\$15/gm for highly enriched uranium)].

With finite θ (some poison buildup), the actual feed rate could be appreciably greater than the steady-state value. However, it was assumed that this "extra" fuel would be recovered by fuel processing at the end of the operating period.

FIG. 6. EFFECT OF POISON-REMOVAL
UPON U^{235} FEED RATE.

The fuel processing costs were based on the steady-state fuel concentrations; also fuel processing was assumed to be carried out throughout the operating period. However, it would not be necessary to operate the fuel processing plant during the period of initial reactor operation, for times less than about θ ; also, the steady-state fuel concentrations would not be applicable over the entire period of operation, and so it might be argued that the "continuous" fuel processing costs were overestimated. However, because of the relatively rapid approach to steady-state conditions, and the short processing cycle time relative to the assumed operating period, this overestimate would be small; also, it could exist only if the amortization costs of the processing plant were small in comparison to operating costs.

Based on the previous discussion, it appears that the curves given in Fig. 4 are not exactly the correct shape; however, the minimum fuel cost would still occur near the conditions of Case 2, but be lower than indicated by about 0.05 or 0.10 mills/kw-hr if highly enriched uranium cost \$15 or \$30/gm, respectively.

The cost of fuel processing itself has not as yet been firmly established. Lewis⁵ arbitrarily assigned \$10/gm of fissionable material) as the cost for processing low enriched fuel, which is much higher than the estimates of Ohlgren et al.⁷ If the fuel processing cost assumed here were increased by a factor of 10 (from \$0.37/gm fissionable fuel to \$3.70/gm) the fuel costs would be increased, as illustrated in Table 8.

TABLE 8
EFFECT OF FUEL PROCESSING COSTS
ON MINIMUM FUEL COSTS[#]

Case	1C				2C				3C			
Cost of Fuel Processing, \$/gm fissionable material	0.37		3.70		0.37		3.70		0.37		3.70	
Cost of enriched uranium, \$/gm	15	30	15	30	15	30	15	30	15	30	15	30
Approx. θ associated with min. fuel cost, days	100	50	200	200	200	200	1200	800	800	400	>1200	>1200
Min. Fuel Cost,* mills/kw-hr	1.8	3.0	2.4	3.6	1.7	2.7	2.2	3.4	2.0	3.2	2.9	4.1

[#] Based on results of steady-state calculations and 10-year operating period.

* These costs are high by about 0.05 - 0.20 mills/kw-hr (for Case 2C) if deviations from steady state conditions are considered.

COMPARISON OF RESULTS

In comparing total fuel costs over a ten-year period in reactors operating either on a continuous or a batch fuel processing cycle, the minimum fuel costs would be 0.2 to 0.5 mills/kw-hr lower for continuous processed reactors if fuel processing costs were \$0.37/gm of fissionable material; however, if processing costs were a factor of ten higher, the minimum fuel costs for batch processed reactors would be 0.1 - 0.3 mills/kw-hr lower. Thus, the fuel cost for these two systems depends upon fuel processing charges in a different way, with the cost of processing having a greater effect upon the continuous fuel-processed reactor. This indicates that batch operated reactors have an economic advantage, from the viewpoint of generating nuclear power some time before relatively inexpensive fuel processing is available. However, it would be economically necessary to recover the fuel at the end of the operating period, so that processing facilities would eventually be needed. As inexpensive fuel processing facilities became available, reactors processed on a continuous basis would have the lower fuel cost.

Comparing these results with "burner" fuel costs,³ it is seen that the addition of a fertile material to a "burner" type system could halve the total fuel costs, and that this does not result in any nuclear difficulties during a 25-year operating period. However, the technology associated with a "burner" type reactor has been developed to a greater degree than that associated with reactor systems involving slurries or highly concentrated solutions.

In these studies the inventory charge was based on the initial fuel loading of the reactor; it is thus tacitly assumed that any later accumulation of fuel within the reactor would be paid for out of station income. However, since the fuel feed-rate would be relatively high during the initial period of operation, the above assumption may not be correct. The comparative results should still be valid, though, since the above condition would not control fuel costs and would affect comparative reactors to nearly the same degree.

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TABLE OF NOMENCLATURE

(Values are for average temperature of 250°C)

$$A_1 = \frac{p_1 r_1^2 + r_1 J_1 + J_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$A_2 = \frac{p_1 r_1^2 + r_2 J_1 + J_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$A_3 = \frac{p_1 r_3^2 + r_3 J_1 + J_2}{r_3(r_3 - r_1)(r_3 - r_2)}$$

$$A_4 = \frac{-J_2}{r_1 r_2 r_3}$$

$$b_x = -c_1 q_1$$

$$b_y = \sigma(p_1) \left[1 - \frac{c_1 \sigma_f(25)}{\beta(25)} \right]$$

$$b_z = \frac{-q_2}{m_1} \sigma(p_2)$$

$$B^2 = \text{buckling of reactor; } B^2 = \left(\frac{\pi}{R} \right)^2 \text{ where } R \text{ is the physical reactor radius plus an effective extrapolation distance (this extrapolation distance was taken as 3 inches); cm}^{-2}$$

$$B_L^2 = \text{Ratio of thermal neutron leakage to absorption, dimensionless}$$

$$B^2 \tau = \text{Ratio of fast neutron leakage to moderation; dimensionless}$$

$$B_1 = \frac{q_2 r_1^2 + K_1 r_1 + K_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$B_2 = \frac{q_2 r_2^2 + K_1 r_2 + K_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$B_3 = \frac{q_2 r_3^2 + K_1 r_3 + K_2}{r_3(r_3 - r_2)(r_3 - r_1)}$$

$$B_4 = \frac{-K_2}{r_1 r_2 r_3}$$

$$c = \frac{\text{atoms of fission product poisons produced}}{\text{unit vol} - \text{unit time} - \text{fission power}}$$

$$c_1 = \text{fraction of atoms produced having effective cross section } \sigma(p_1) \text{ per fission}$$

$$c_2 = \text{fraction of atoms produced having effective cross section } \sigma(p_2) \text{ per fission}$$

$$c_3 = c_1 + c_2 = \text{atoms of fission product poisons produced/fission}$$

$$c_x = -c_2 q_1$$

$$c_y = -q_3 \sigma(p_1)/m_1$$

$$c_z = \sigma(p_2) \left[1 - c_2 \frac{\sigma_f(25)}{\beta(25)} \right]$$

$$(C.R.)_e = 1 - \frac{\text{feed requirements}}{\text{feed requirements in "burner"}}$$

$$C_1 = \frac{q_3 r_1^2 + r_1 L_1 + L_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$C_2 = \frac{q_3 r_2^2 + r_2 L_1 + L_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$C_3 = \frac{q_3 r_3^2 + r_3 L_1 + L_2}{r_3(r_3 - r_1)(r_3 - r_2)}$$

$$C_4 = \frac{L_2}{r_1 r_2 r_3}$$

D = thermal diffusion coefficient for reactor core, cm

$$D_1 = \sigma(p_1) B_1 + \sigma(p_2) C_1 - \beta(49) A_1$$

$$D_2 = \sigma(p_1) B_2 + \sigma(p_2) C_2 - \beta(49) A_2$$

$$D_3 = \sigma(p_1) B_3 + \sigma(p_2) C_3 - \beta(49) A_3$$

$$D_4 = \sigma(p_1) B_4 + \sigma(p_2) C_4 - \beta(49) A_4$$

$$E_1 = b_y + c_z + v_x$$

$$E_2 = b_y c_z - b_z c_y + v_x (b_y + c_z) - v_y b_x - v_z c_x$$

$$E_3 = v_x (b_y c_z - b_z c_y) - v_y (b_x c_z - c_x b_z) + v_z (b_x c_y - c_x b_y)$$

f = thermal utilization, $\frac{\Sigma(\text{fuel})}{\Sigma_T}$, dimensionless

f.p. = fraction poisons, $\frac{\Sigma(p)}{\Sigma(\text{fuel})}$

(fp)₀ = fraction poisons due to high cross section poisons; 0.02

F = feed rate of uranium to reactor system, $\frac{\text{atoms}}{\text{barn-cm-sec}}$ or $\frac{\text{gms}}{\text{day}}$
for system. F(25) = U²³⁵ feed rate; F(28) = U²³⁸ feed rate

$$j(x) = \frac{1 - p(28)}{1 + B^2 \tau} \eta(x) \sigma(x)$$

$$J_1 = p_1 (b_y + c_z) - v_y q_2 - v_z q_3$$

$$J_2 = p_1 [b_y c_z - b_z c_y] - v_y [q_2 c_z - b_z q_3] + v_z [q_2 c_y - q_3 b_y]$$

$$K_1 = q_2 (c_z + v_x) - b_z q_3 - p_1 b_x$$

$$K_2 = [q_2 c_z - b_z q_3] v_x - [b_x c_z - b_z c_x] p_1 + v_z [b_x q_3 - q_2 c_x]$$

$$L^2 = \text{square of thermal diffusion length: } \frac{D}{\Sigma_T} ; \text{ cm}^2$$

$$L_1 = q_3 (v_x + b_y) - c_y q_2 - p_1 c_x$$

$$L_2 = (q_3 b_y - c_y q_2) v_x - (b_x q_3 - c_x q_2) v_y + p_1 (b_x c_y - c_x b_y)$$

$$m_1 = B^2 D + \Sigma(\text{mod}) + \Sigma(28) ; \text{ cm}^{-1}$$

$$N = \text{concentration of particular element; } \frac{\text{atoms}}{\text{barn} - \text{cm}} \text{ or } \frac{\text{gms}}{\text{liter}} ;$$

N(25) refers to U^{235} concentration; N(28) refers to U^{238}

concentration; N(49) refers to Pu^{239} concentration

$$N_0 = \text{concentration of particular element at zero time; atoms/barn-cm or gms/liter}$$

$$p(28) = p = \text{resonance escape probability as determined by } U^{238} \text{ concentration}$$

$$p_1 = \Sigma(28) + \frac{j(25) m_1}{\beta(25)}$$

$$q_1 = \sigma_f(49) - \frac{\sigma_f(25)}{\beta(25)} \beta(49)$$

$$q_2 = \frac{c_1 \sigma_f(25) m_1}{\beta(25)}$$

$$q_3 = \frac{c_2 \sigma_f(25) m_1}{\beta(25)}$$

$$r_1, r_2, r_3; \text{ roots of cubic equation: } s^3 + E_1 s^2 + E_2 s + E_3 = 0$$

$$t = \text{time, sec or yrs.}$$

$$T = \int_0^t \phi dt, \text{ barn}^{-1}$$

$$v_x = \sigma(49) - j(49) + j(25) \frac{\beta(49)}{\beta(25)}$$

$$v_y = \frac{j(25)}{\beta(25)} \sigma(p_1)$$

$$v_z = - \frac{j(25)}{\beta(25)} \sigma(p_2)$$

$$\beta(25) = \left[\frac{\eta(25) \epsilon p}{1 + B^2 \tau} - 1 - (fp)_0 \right] \sigma(25); \text{ barns}$$

$$\beta(49) = \left[\frac{\eta(49) \epsilon p}{1 + B^2 \tau} - 1 - (f.p.)_0 \right] \sigma(49); \text{ barns}$$

$$\gamma = \text{fraction enrichment of feed uranium}$$

$$\epsilon = \text{fast effect; taken to be 1}$$

$$\eta = \text{fast neutrons per thermal neutron absorbed in fuel,}$$

$$\eta(25) = 2.08; \eta(49) = 1.95$$

θ = fuel processing cycle time, days

$\sigma(25)$ refers to absorption cross section of U^{235} ;

$\sigma(49)$ refers to absorption cross section of Pu^{239} ;

$\sigma(p_1)$ refers to absorption cross section of effective fission product poison p_1 ; etc ;

$\sigma(25) = 435$; $\sigma(28) = 1.82$; $\sigma(p_1) = 132$; $\sigma(p_2) = 13.9$; $\sigma(49) = 1060$

σ_f = microscopic fission cross section of fuel; barns;

$\sigma_f(25) = 368$, $\sigma_f(49) = 685$

Σ = macroscopic absorption cross section; cm^{-1} ; $\Sigma = N\sigma$

Σ_f = macroscopic fission cross section, cm^{-1} ;

$\Sigma_f(25) = \sigma_f(25) N(25)$; $\Sigma_f(49) = \sigma_f(49) N(49)$; $\Sigma_f = \Sigma_f(25) + \Sigma_f(49)$

$\Sigma(mod)$ = macroscopic absorption cross-section of moderator; cm^{-1} .

$\Sigma(D_2O) = 4.49 \times 10^{-5}$ (99.75% D_2O);

$\Sigma(p)$ = fission-products macroscopic absorption cross section; cm^{-1} ;

Σ_T = total macroscopic absorption cross section; cm^{-1} ;

$\Sigma_T = \Sigma(25) + \Sigma(28) + \Sigma(49) + \Sigma(p) + \Sigma_o(p) + \Sigma(mod)$

τ = Fermi Age; cm^2 ; $\tau(D_2O) = 187$

ϕ = neutron flux; neutrons/barn-sec

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