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### CONTROLLABILITY OF A STEADY-STATE TOKAMAK FUSION REACTOR

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## CONTROLLABILITY OF A STEADY-STATE TOKAMAK FUSION REACTOR

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Several modes of feedback stabilization of the power level of a steady-state thermonuclear reactor have been suggested to control plasma density and temperature fluctuations during reactor operation.<sup>(1-3)</sup> In this work we examine the controllability of a steady-state fusion power reactor by some of the proposed modes of control; namely by variations in the tritium enrichment, the injection rate, and the leakage rate of the fuel. Changes in the tritium enrichment is assumed to be achieved by proper control of the fuel composition during injection but independent of other control modes. Variation in the leakage rate can be done through the control of magnetic field.<sup>(3)</sup>

The fusion device is taken to be a tokamak with flat plasma density and temperature profiles similar to those proposed by Carruthers.<sup>(4)</sup> Fluctuations in temperature are not considered here and the fuel is assumed to contain D, T and  $^3\text{He}$  isotopes of densities  $n_D$ ,  $n_T$  and  $n_3$  respectively. By setting up a material balance over the plasma volume for each species, three spatially independent nonlinear kinetic equations are obtained. For slow time variation of the parameters involved, the steady-state continuity equations are obtained from the three time-dependent kinetic equations. By assuming that the variations in the density of the species and the controlled changes in the leakage rate, the injection rate, and the enrichment are only in the form of small perturbations, the plasma state can be represented by the linearized set of time-dependent equations

$$\dot{n}(t) = A n(t) + B U(t) \quad (1)$$

where  $\dot{\eta}(t)$  is the rate of change of the plasma three-dimensional state vector  $\eta(t)$  which is defined by

$$\eta(t) = \begin{bmatrix} \delta n_D \\ \delta n_T \\ \delta n_3 \end{bmatrix} \quad (2)$$

where the symbol  $\delta$  is applied to the densities to notate linear perturbations in such densities. The variable  $U(t)$  is the control vector,

$$U(t) = \begin{bmatrix} \delta I \\ \delta L \\ \delta r \end{bmatrix} \quad (3)$$

where  $\delta I$ ,  $\delta L$ , and  $\delta r$  are the linear perturbations in the injection rate per particle,  $I(= \frac{1}{\tau_{inj}})$ ; the leakage rate per particle,  $L(= \frac{1}{\tau})$ ; and the enrichment  $r(= \frac{n_D}{n_T})$  respectively. Here  $\tau_{inj}$  and  $\tau$  are the mean time of injection and the confinement time respectively. The system matrix  $A$  and the control matrix  $B$  are explicitly given by

$$A = \begin{bmatrix} -\langle \sigma v \rangle_{DD} n_{D0} & -\langle \sigma v \rangle_{DT} n_{D0} & -\langle \sigma v \rangle_{D3} n_{D0} \\ \frac{1}{4} \langle \sigma v \rangle_{DD} n_{D0} + r_0 L_0 & -(\langle \sigma v \rangle_{DT} n_{D0} + L_0) & 0 \\ \frac{1}{2} \langle \sigma v \rangle_{DD} n_{D0} - \langle \sigma v \rangle_{D3} n_3 & 0 & -(L_0 + \langle \sigma v \rangle_{D3} n_{D0}) \end{bmatrix} \quad (4)$$

and

$$B = \begin{bmatrix} n_{D0} & -n_{D0} & 0 \\ n_{D0} r_0 & -n_{T0} & n_{D0} I_0 \\ 0 & -n_{30} & 0 \end{bmatrix} \quad (5)$$

where the subscript 0 is designated to parameters evaluated at steady state and  $\langle \sigma v \rangle_{DD}$ ,  $\langle \sigma v \rangle_{DT}$ , and  $\langle \sigma v \rangle_{D3}$  are the products of the fusion cross sections

times the relative velocities averaged over the two Maxwellian velocity distributions of the two interacting species for D-D, D-T, and D-<sup>3</sup>He reactions respectively.

Rewriting Eq. (1) in the form

$$\dot{n}(t) = An(t) + b_1 n_{D0} \delta I + b_2 n_{D0} \delta L + b_3 n_{D0} \delta r \quad (6)$$

where

$$b_1 = \begin{bmatrix} 1 \\ r_0 \\ 0 \end{bmatrix}, \quad b_2 = - \begin{bmatrix} 1 \\ r_0 \\ \frac{n_{30}}{n_{D0}} \end{bmatrix} \quad \text{and} \quad b_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (7)$$

it can be shown that the 3x3 matrices

$$(A \mid Ab_i \mid A^2 b_i) \quad i = 1, 2, 3 \quad (8)$$

are non-singular for all the three control variables. Thus, the origin of the  $n_1$ - $n_2$ - $n_3$  phase plane can be reached in a finite period of time and the reactor is controllable by variation in tritium enrichment, injection rate, and leakage rate of the fuel.

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