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EFFECT OF A DENSITY GRADIENT ON TAYLOR INSTABILITY

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February 21, 1955

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Livermore, California

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ABSTRACT

Taylor's derivation of the instability of an accelerated surface of density discontinuity is extended to the case in which the density has a smooth transition.

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The model considered is an infinite incompressible fluid subjected to an acceleration, g , in the y direction.⁽¹⁾ The initial density is of the form

$$(1) \quad \rho_1 - \frac{1}{2} \delta \rho e^{-Ky}, \quad y > 0$$

$$(2) \quad \rho_2 + \frac{1}{2} \delta \rho e^{Ky}, \quad y < 0$$

$$(3) \quad \text{where } \delta \rho = \rho_1 - \rho_2 > 0$$

An initial perturbation of the form $\cos kx$ will be considered in the region of the $y = 0$ plane. The amplitude of the perturbation is assumed small compared to its wave length. Let $y = \eta(x, t)$ be the equation of the surface of density $\frac{1}{2}(\rho_1 + \rho_2)$.

The linearized hydrodynamical equations which will be used are:

$$(4) \quad u_x + v_y = 0$$

$$(5) \quad \rho u_t + P_x = 0$$

$$(6) \quad \rho v_t + P_y - \rho g = 0$$

Take a velocity potential

$$(7) \quad \phi = e^{-ky} \cos kx f(t) \quad y > 0$$

$$= e^{ky} \cos kx f(t) \quad y < 0$$

$$(8) \quad \text{To first order } \rho = \rho_t = 0 + \int \dot{\rho}_t = 0 \quad dt$$

$$\text{where } \dot{\rho}_t = 0 \approx -v \rho_y$$

Integrating (6) from L to η and from $-L$ to η where L is some constant large compared to $\frac{1}{K}$ and $\frac{1}{k}$, dropping terms of second order and higher in f , and $\dot{f}$, and equating the two expressions for $P_y = \eta$, one obtains

$$\dot{f}(t) \cos(kx)(\rho_1 + \rho_2) - g \frac{kK}{k+K} \delta \rho \cos kx \int f(t) dt + \text{const.} = 0$$

$$\text{or } (\rho_1 + \rho_2) \ddot{f} - g \frac{kK}{k+K} \delta \rho f = 0$$

The rate of growth is thus exponential with a growth constant

$$\frac{1}{t} = \sqrt{\frac{gkK \delta \rho}{(k+K)(\rho_1 + \rho_2)}}$$

At the short wave length limit $k \rightarrow \infty$ $t \rightarrow \text{const.}$ rather than diverging as in the case of a discontinuity in density. (2)

REFERENCES

- (1) This is an extension of the work done by G. I. Taylor, Proc. Roy. Soc., A201, p 192, (1950).

See also R. H. Pennington and R. Bellman, PNJ-LA-1.

- (2) The possibility of this behavior was suggested to us by E. Teller.

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