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THREE-DIMENSIONAL POWER
DISTRIBUTION ANALYSIS

by

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Three-Dimensional Power Distribution Analysis

H. A. Hassan, W. A. Wittkopf

Key Words: Analysis, Power Distribution, 3-D, PWR

ABSTRACT

B&W's Physics Laboratory conducted critical experiments to determine the reliability of the PDQ07 analytical model in predicting detailed radial and axial power distributions for PWR cores. It is concluded that the PDQ07 analytical model can be applied to conservatively calculate all power operating conditions.

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1. INTRODUCTION

The reliability and the degree of conservatism of an analytical model should be established before it is used to design a nuclear reactor core. The model is usually verified by comparing analytical results with experimental data. This report considers tests conducted to establish the validity of an analytical model using the PDQ07¹ computer code. The capability of a model to predict each major core characteristic that can affect the ultimate design of the core should be established. In this respect, critical experiments have an important role in establishing a high level of confidence in an analytical model and are directly related to the verification of the overall analytical model. The results of applying the PDQ07 model presented here are intended to verify the analytical model for predicting detailed radial and axial power distributions. The model is tested under various conditions of fuel and control rod positions, and the capability of the model to represent each of these conditions is determined.

The following sections describe the critical experiments, the analytical model, and the results of several comparison tests.

2. SUMMARY

1. The calculated average eigenvalue for 25 critical assemblies was 0.9977 with a standard deviation of 0.00095. Assuming that the deviations follow a normal error curve, there is a 95% probability that the calculated eigenvalue of a critical core will be within 0.0019 of the average value, and a 99.7% probability that it will be within 0.0029.

2. Analysis of two independent sets of power peaking data taken on the same core indicates an average deviation of 0.92% from their mean values. Assuming a normal error curve, this would result in a standard deviation of 1.15% for the experimental values.

3. Considering all of the 22,000 power points in 26 cores, the average deviation between calculation and experiment is 3.66%, indicating a standard deviation of 4.57%. These points include many near the top, bottom, and periphery of the core, where diffusion theory loses accuracy near the boundaries of dissimilar materials. The deviations at these locations are larger than average, but the power values are low and of less concern to the reactor designer.

4. Considering only the peak power values in the 26 cores, the average deviation is 1.20% and the standard deviation is 1.44%.

5. Considering the four hottest pins in each of the 26 cores, a total of 110 points, the average deviation between calculation and experiment is 1.53% and the standard deviation is 1.90%. Assuming a normal error curve, this indicates a 95% probability that the calculated power peak is within 3.8% of the measured value, and a 99% probability that the calculated peak is within 5.0% of the measured value.

3. CRITICAL EXPERIMENTS

3.1. General Description

Critical experiments are performed in B&W's Physics Laboratory at the Lynchburg Research Center. The series of experiments used as a basis for the analysis described herein are described in reference 2. The critical assembly is erected in a core tank 5 feet in diameter by 6 feet high with aluminum walls 0.5 inch thick. The fuel rods are 0.475 inch OD by 61.59 inches long. The cladding is 0.032 inch thick, type 6061 aluminum, and the UO_2 fuel is enriched to 2.46 wt % ^{235}U . The physical properties of the experimental fuel rod and a typical fuel rod of B&W design are as follows:

<u>Parameter</u>	<u>Rods used in critical experiments</u>	<u>Rods used in typical B&W plant</u>
Outside diameter, in.	0.4738 ± 0.0006	0.430
Wall thickness, in.	0.032 ± 0.001	0.0265
Wall material	6061 aluminum	Zircaloy-4
Pellet (fuel) diameter, in.	0.4054 ± 0.0005	0.370
Total length, in.	61.59 ± 0.16	153.7
Active (fuel) length, in.	60.37 ± 0.35	144.0
Weighted fuel, g/rod	1305.5 ± 1.0	2229.0
U/ UO_2 , wt %	88.13 ± 0.01	88.15
Enrichment, $^{235}\text{U}/\text{U}$, wt %	2.459 ± 0.002	$\sim 2.0 - 3.5$
Pellet (fuel) density, g/cc	10.24 ± 0.04	10.14

The fuel rods used in the critical experiments are very representative of power reactor design even though they have different cladding material, a slightly larger diameter, and a shorter active fuel length than those used in the typical B&W plant.

The poison pins are an alloy of silver, indium, and cadmium. The 0.400-inch-diameter pins consist of 80 wt % silver, 15 wt % indium, and 5 wt % cadmium. Full-length and partial-length poison pins are used in the assembly. The full-length pins (62 inches long) span the full core height; they are clad in type 304 stainless steel tubes which are 0.441 inch in diameter by 0.018 inch wall. The partial-length pins (15.12 inches long) rest in the bottom of stainless steel tubes and span approximately one-quarter of the core height. (The type-321 SS tubes are 66 inches long, 0.438-inch in diameter, and have an 0.016-inch-wall.)

Important features of the full-length poison pins are as follows:

<u>Parameter</u>	<u>Control rods used in critical experiments</u>	<u>Control rods used in typical B&W plant</u>
Length, in.		
Full	62.03 ± 0.00	134.0
Partial	15.12 ± 0.01	36.0
Diameter, in.	0.400 ± 0.000	0.392
Weight, g	1296 ± 5	2695
Analysis, wt %		
Silver	79.68	~80
Indium	15.09	15 ± 0.3
Cadmium	5.2	5 ± 0.5
Impurities	0.067	0.002

The control pins used in the critical experiments are very representative of power reactor design.

Fuel rods are aligned in the core tank by top and bottom eggcrate-type grid plates consisting of 1-inch-wide slotted aluminum strips interlocked to form a square matrix. The strips are 0.016 inch thick and are spaced on 0.644 ± 0.001 -inch centers. As shown in Figure 1, the core used in these measurements is loaded to approximately cylindrical geometry and is arranged on a square lattice pitch of 0.644 inch. The core contains 4910 fuel rods and three poison pin clusters.

The safety blades are 8-inch-wide boral, and each is thin enough to pass between rows of fuel rods to maintain uniform lattice spacing. During measurement, the safety blades are held in a cocked position to minimize their contribution to core perturbations. Criticality is achieved by raising the moderator level. Boric acid dissolved in the moderator controls excess reactivity.

3.2. Power Distribution Measurements

Axial power distributions were measured on selected fuel rods within the core of the critical assembly. The relative power distribution within the core was determined by counting collimated fission product gammas from activated fuel rods with an NaI(Tl) scintillation counter. Photons with less than 200 keV energy were discriminated out. The collimator consisted of a 1-inch-diameter hole in 2-inch-thick lead shielding placed between the fuel rod and the counter. Four inches of lead above and below the detector and 2 inches along the sides provided shielding from other portions of the fuel rod.

In order to perform all fuel rod activations at a constant moderator height, the boron concentration was adjusted to make the assembly critical at a moderator height of about 139 cm (all fuel measurements are in centimeters). Activations were then made on an exponential period obtained by raising the moderator height to 145 cm. The integrated exposure was approximately 1 kW-minute. The counting

time varied from 8 to 50 seconds, and the total count collected at each rod position varied from 60,000 to 80,000 counts. The background activity was kept low by loading essentially unirradiated fuel rods in the positions to be studied.

Decay corrections were obtained from a "master" fuel rod, which was irradiated simultaneously with the rods to be scanned and then counted in a second counting system. The second system was identical to the first in collimation and electronics. A standard fuel rod, recounted at frequent intervals, provided data to correct for any small changes in either system.

All power density data were normalized to a reference point on the standard fuel rod, 74.4 cm from the bottom of the fuel region.

3.3. Core Loading

The two core loadings have been chosen to mock up typical control rod cluster configurations in PWRs. The effects of both full-length poison control rods and partial-length axial power shaping rods have been investigated. Core IV-H (Figure 1) has three poison pin clusters along the north-south diameter. The north and south clusters contain full-length pins, while the central cluster contains partial-length pins. The vacant positions at the center of each poison pin cluster mock up an instrument hole. The standard fuel rod is located 20 pin pitches to the west of the center of the core.

The core IV-J loading shown in Figure 2 is identical to that of core IV-H except that the full-length rods are relocated along the northwest-southeast axis. A composite detector string occupied the center hole of core IV-J during all measurements. The standard fuel rod for core IV-J is located $15 \times \sqrt{2}$ pin pitches to the northeast of the center of the core.

3.4. Loading Designation

The position of the tip of the partial- and full-length control rods from the bottom of the core is specified by a loading designation consisting of a letter followed by two digits. The partial-length position is designated by the letters A, B, C, D, or E, which indicate that the rod is 0.0 cm, 30.7 cm, 61.4 cm, 92.1 cm, or fully withdrawn, respectively. The full-length rod positions are indicated by either a 0 or a 1, which designates that they are either fully inserted (for 0) or 72.5 cm withdrawn (for 1) from the bottom of the core.

Partial- and full-length rod positions and boron concentrations for the just-critical condition are tabulated below for the 24 experiments analyzed.

<u>Experiment</u>	<u>Boron concentration, ppm</u>
1. IV-H,A00	1412
2. IV-H,A11	1427
3. IV-H,B00	1381
4. IV-H,B11	1399
5. IV-H,C00	1370
6. IV-H,C11	1392
7. IV-H,D00	1400
8. IV-H,D11	1418
9. IV-H,E00	1419
10. IV-H,E11	1446
11. IV-H,A10	1358
12. IV-H,B10	1389
13. IV-H,C10	1378
14. IV-H,D10	1408
15. IV-H,E10	1435
16. IV-J,A00	1335
17. IV-J,A11	1375
18. IV-J,B00	1305
19. IV-J,B11	1348
20. IV-J,C00	1294
21. IV-J,D00	1308
22. IV-J,D11	1371
23. IV-J,E00	1341
24. IV-J,E11	1388

Figure 1. Core IV-H Loading Diagram

Figure 2. Core IV-J Loading Diagram

4. ANALYTICAL MODEL

4.1. General

The power distributions for the various core geometries were obtained using the PDQ07 computer code in three-dimensional calculations. The geometries used in the calculation simulated the experimental conditions as follows: The compositional overlay distinctly described the fuel, the water holes, and the poison pins in the control rod region. As shown in Figure 3, water occupied the volumes below poison pins of the full- and partial-length control rod clusters when they were moved up from the bottom of the core. However, dry air occupied the volumes above the active part of the partial-length poison pins. The conditions above and below the critical core were represented by homogenized regions as shown in Figure 3. In the figure, the aluminum base and boric water were combined to form a single composition below the fuel region. However, three regions—dry fuel combined with air, dry fuel combined with aluminum, and dry fuel combined with air—were used consecutively above the moderator level of 145 cm to simulate conditions above the critical core length. It is felt that the use of these various compositional overlays in this manner mocked up the experimental setup as closely as practicable. Thermal feedback was not considered since the temperature variation from room temperature was negligible.

Solution points were placed along the boundary, as well as in the center, of the cells in the poison pin regions. A mesh interval of 5 cm was used for most of the axial direction; one or two intervals were set equal to 10 cm. The location of 10-cm intervals was changed from case to case depending on the position of the partial-length rod relative to the bottom of the core.

4.2. Cross Section Calculations

4.2.1. Fuel Rods

The LIFET program was used to determine the coefficients for the fuel cells based on an infinite-medium fuel array. The fuel temperature (22C) used in the calculation is the same as that used for the moderator. Table 1 lists the atom concentration of pure materials (heterogeneous number densities) of the fuel cell, the homogeneous number densities, and fuel cell volume fractions used in the calculation. The cross sections were calculated at various boron concentrations, and the values were found to be fairly linear with the variation in boron concentration. The cross-section values used in the PDQ07 calculation were thus fit versus boron concentration. For each cross-sectional type, three cross-section values (1446, 1421, and 1393 ppm boron concentration) were input to the HARMONY system.³ Linear interpolation or extrapolation was used to obtain the cross-section values at given boron concentrations used in the calculation of the various cores. LIFET is described briefly in paragraphs 4.2.1.1 and 4.2.1.2.

4.2.1.1. Calculation of Epithermal Cross Sections

The epithermal neutron energy spectrum for each composition region is calculated using the P_1 transport approximation with the Fermi age slowing-down treatment for heavy nuclides and the Greuling-Goertzel⁴ treatment for the light nuclides. For nuclides exhibiting resonances in the epithermal energy range, the effective resonance integral, corrected for shielding effects and Doppler broadening, is calculated for each epithermal multigroup by either the narrow resonance model or the narrow resonance-infinite absorber model. To account for heterogeneous lattice effects, the Dancoff factor is calculated by the method of Sauer.^{5, 6}

The resonance integral calculation in the LIFET program is based on the work of Dresner⁷ and of Adler, Hinmann, and Nordheim.⁸ Effective resonance integrals are calculated at each resolved and unresolved peak for each resonance nuclide. The effective resonance integral for each energy interval in the multigroup structure is then the sum of the integrals over all peaks in the multigroup. These multigroup

resonance integrals are subsequently used in the epithermal spectrum calculation. In computing the epithermal spectrum, both inelastic scattering and (n, 2n) reactions are taken into account. After calculating the neutron spectrum, macrogroup (three or less groups) diffusion constants are obtained by averaging over the spectrum.

4.2.1.2. Calculation of Thermal Cross Sections

A thermal spectrum calculation is performed for each composition region of a reactor. Since these regions are not strictly homogeneous, a space-energy calculation is performed to account for the heterogeneous configuration. For uniformly distributed fuel rods in a moderating medium, the space-energy flux may be described in terms of separate space- and energy-dependent terms. The heterogeneous effects on the microscopic cross sections are calculated for each nuclide. These cross sections reflect the geometric configuration of the fuel rods, cladding, and moderator. A P_1 spectrum calculation is then used to determine the energy dependence of the flux for an equivalent homogeneous region. Finally, the cross sections are averaged over the energy spectrum to obtain a single thermal cross section set for use in the multigroup diffusion calculation.

4.2.2. Control Rods

The LIFET program and a mini-PDQ07 calculation were used to calculate cross-section values for the control rods. The volume fractions and homogeneous and heterogeneous number densities for the control rod cell are shown in Table 2. The standard B&W Lumped Burnable Poison Model for fuel cycle analysis was used to calculate the attenuation factor as required to reduce the thermal absorption cross section.

4.2.3. Reflectors and Dry Fuel

Reflector and dry fuel cross sections were calculated for boron concentrations of 1446, 1421, and 1393 ppm. Cross sections for the various boron concentrations were obtained by linear interpolation or by extrapolation between the three values given above. Cross sections were also calculated for the following conditions:

1. A mixture of aluminum and water for the base plate.
2. A mixture of aluminum and fuel for the top grid.
3. Dry air above partial-length poison pins.
4. Dry fuel above moderator level and above the top grid.

For the dry fuel pins, a series of LIFET runs was made with the water density varying from 5 to 20%. By extrapolating to 0% water density, coefficients for dry pins were obtained.

The number densities used in the calculations are as follows:

<u>Material</u>	<u>Isotope</u>	<u>ID</u>	<u>N</u>	<u>Comment</u>
Water	Hydrogen	01	0.06671	Varies with ppm
	Oxygen	08	0.033355	
	Boron, ppm	05	$0.55707 \times \text{ppm} \times 10^{-7}$	

4.3. Normalization to Experiments

To properly compare the calculated power distribution (within the critical core) with the available experimental data, the calculated relative power density in the core must be normalized to the experimental data. To do this, an integral normalization procedure has been selected wherein the total power integral from the calculation (for a given core configuration) is set equal to the total power integral from the experiment. Mathematically, the normalization factor NF is calculated from the equation

$$NF = \frac{\sum_i \sum_j P_{ij}(\text{Exp})}{\sum_i \sum_j PC_{ij}(\text{Cal})} \quad (1)$$

where

NF = power normalization factor,

$P_{ij}(\text{Exp})$ = experimental relative power density at height
j for fuel rod i,

$PC_{ij}(\text{Cal})$ = calculated relative power density at height j
for fuel rod i.

Note that the integration is performed on identical points from calculation and from experiments. The summation over identical points from experiments and from calculations makes equation 1 volume independent.

After the normalization factor NF is computed, it is used to obtain the normalized calculated power density as follows:

$$P_{ij}(\text{Cal}) = \text{NF} \times PC_{ij}(\text{Cal}) \quad (2)$$

where $P_{ij}(\text{Cal})$ = normalized calculated relative power density at height j for fuel rod i.

The integral method of normalization was adopted for the analysis because all the experimental points are utilized to compute the normalization factor and thus provide a more accurate factor based on statistics.

Table 1. Atomic Concentrations and Volume Fractions
for Fuel Cell

<u>Volume Fraction</u>				
	UO ₂		0.3112	
	Gap		0.0084	
	Cladding		0.1073	
	Water		0.5731	
<u>Atom Concentration of Pure Materials, atom/b-cm</u>				
<u>Material</u>	<u>Isotope</u>	<u>ID</u>	<u>10⁻⁴ N</u>	<u>Comment</u>
Fuel oxide	²³⁵ U	35	5.68	
	²³⁸ U	38	222.43	
	Oxygen	08	456.21	
	Nat boron	05	0.0039	Impurity boron equivalent
Fuel cladding	Aluminum	13	671.6	Same absorptions as 6061 aluminum.
<u>Homogeneous Number Density, 10⁻⁴ atom/b-cm</u>				
	Hydrogen		382.3	
	Oxygen		333.1	
	Aluminum		72.1	
	²³⁵ U		1.777	
	²³⁸ U		69.23	
	Boron		10 ⁻² [0.12 + 0.0320 (ppm)]	

Table 2. Atomic Concentrations and Volume Fractions
for Control Rod Cell

<u>Volume Fraction</u>				
	Cd-In-Ag		0.302997	
	Gap		0.005392	
	Cladding		0.059157	
	Water		0.632455	
<u>Atom Concentration of Pure Materials, atom/b-cm</u>				
<u>Material</u>	<u>Isotope</u>	<u>ID</u>	<u>N</u>	<u>Comment</u>
Cd-In-Ag rod	Silver	47	0.045134	
	Cadmium	48	0.0027833	
	Indium	49	0.0080303	
Control rod cladding	Steel	23	0.08436	Same absorptions as actual 304-SS used at laboratory.

Homogeneous Number Density, atom/b-cm

Hydrogen	0.042191
Oxygen	0.021096
304 SS	0.004990
Silver	0.013675
Cadmium	0.00084333
Indium	0.0024332
Boron	$0.03526 \text{ (ppm)} \times 10^{-6}$

Figure 3. Schematic Representation of Homogeneous Region – Axial Direction

5. RESULTS AND DISCUSSION

5.1. General

This section compares the results from the three-dimensional calculation, i.e., the eigenvalue, the relative power distribution, and the peak power values for the various cores studied, with the corresponding experimental values. The uncertainties associated with the peak power values for the cores are discussed, and an average uncertainty to be assigned to the peak power in the large PWR core is derived. A total of 22,002 individual power points located in 26 different critical assemblies was analyzed.

5.2. Eigenvalues

The calculated eigenvalues for the various rod positions of core loadings IV-H and IV-J are as follows:

EIGEN VALUES FOR CRITICAL EXPERIMENTS		
CRITICAL ASSEMBLY	EIGEN VALUE	PERCENT DIF.
IV-H, A00	.997230	-4.379418E-02
IV-H, A11	.997430	-2.374740E-02
IV-H, B00	.997389	-2.785699E-02
IV-H, B11	.997124	-5.441836E-02
IV-H, C00	.996895	-7.737252E-02
IV-H, C11	.996765	-9.040292E-02
IV-H, D00	.996008	-1.662799E-01
IV-H, D11	.996439	-1.230792E-01
IV-H, E00	.996865	-6.037953E-02
IV-H, E11	.995600	-2.071754E-01
IV-H, A10	.998200	5.343266E-02
IV-H, B10	.998251	5.854459E-02
IV-H, C10	.998156	4.922284E-02
IV-H, D10	.997219	-4.489675E-02
IV-H, E10	.996674	-9.952420E-02
IV-J, A00	.998352	6.866821E-02
IV-J, A11	.998923	1.259017E-01
IV-J, B00	.998587	9.222316E-02
IV-J, B11	.998375	7.097359E-02
IV-J, C00	.998282	6.165184E-02
IV-J, C11	.998101	4.350951E-02
IV-J, D00	.999508	1.845385E-01
IV-J, D11	.998530	8.650983E-02
IV-J, E00	.998285	6.195254E-02
IV-J, E11	.998483	8.179884E-02

AVERAGE EIGEN VALUE = .997667
 SQUIRE ROOT OF SIGMA = 9.4659E-04

Twenty-five different rod positions were studied, and the eigenvalues shown above are for the just-critical condition of the experimental setup. The average eigenvalue for the 25 cores is 0.99767 with an uncertainty of ± 0.00095 . The average eigenvalue $\bar{\lambda}$ and its uncertainty $\pm \Delta\lambda$ were calculated as follows:

$$\bar{\lambda} = \left(\sum_{i=1}^{25} \lambda_i \right) \div 25, \quad (3)$$

$$\Delta\lambda = \pm \sqrt{\left[\sum_{i=1}^{25} (\lambda_i - \bar{\lambda})^2 \right] \div 25}, \quad (4)$$

where λ_i is the eigenvalue for core loading i .

The just-critical condition from experiments yields an eigenvalue of 1.0. It is generally assumed that calculated eigenvalues in the range of 1.003 to 0.997 represent a just-critical condition. The core IV eigenvalue was calculated in the same manner as for cores IV-H and IV-J. Since core IV was a uniform fuel lattice of 4961 fuel rods, and there were no poison pins or water holes in the core, the calculation gave a base for the just-critical condition without having the uncertainty associated with the poison pin cross sections. The calculated eigenvalue for core IV (0.997900) compares favorably with the average eigenvalue from the 25 different loadings of cores IV-H and IV-J (0.9977 ± 0.0095). Thus, it is felt that the integral combined effect of the cross-section values of the poison pins, the water holes, and the dry air above partial-length poison pins is fairly accurate. Also, this comparison shows that the use of the average eigenvalue as a reference to calculate the deviation of each eigenvalue from a just-critical condition is justified. The largest deviation of an eigenvalue from the reference value was -0.21% for core loading IV-H, E11.

5.3. Power Distribution

5.3.1. General

The analytical power values (as discussed previously) were properly normalized using the power normalization technique rather than the standard single-point normalization used for the experimental data. The analytical results were then compared with the experimental data for each individual point of each fuel rod.

5.3.2. Experimental Data

Inspection of the experimental data reported in reference 2 revealed some inconsistencies. Therefore, all experimental data were plotted point by point for visual inspection. A computer code was written to plot the radial power values at each core height: 5, 10, 15, ... 135, and 140 cm from the bottom of the core. The power values at each level were then connected by straight lines between each two consecutive points to give a first-order approximation of radial power shapes at the various levels. Each plot was carefully examined, and the points that were obviously incorrect were identified. These points were rechecked and corrected through linear interpolation from the four neighboring power values based on the general trend of the power shapes and the authors' judgment. Figure 4 is an example of these plots for core loading IV-J, A11. In the figure, the power value for the pin located 7, 7 pin pitches northeast from the core center and at a height of 10 cm from the core bottom is reported to be 0.216. From the examination of the figure, rod locations, and core general properties, it has been concluded that there was no physical reason for a dip in power value at that point. Therefore, a linear interpolation was carried out based on power values at the surrounding points; the results indicated that 0.316 is a more accurate value. Thus, this value, rather than the reported value of 0.216, was used in the power normalization and power comparison. In order to conserve the statistical nature of the data, this type of correction was applied only to points that were clearly inconsistent with neighboring values.

Careful examination of the plots of each core loading indicated that the experimental data for core loading IV-J, C11 are far from what would be expected. Since there was no good explanation for

the large deviation in power shapes, it was felt that this core loading should be dropped from the analysis. Therefore, loading IV-J, C11 is not considered in any of the data comparison presented here except for the calculation of the average eigenvalue.

5.3.3. Power Shapes

The normalized power values from the calculation were compared with the experimental values for each core studied. Figures 5, 6, and 7 are typical examples of the calculated power shapes as compared to the experimental data. In the figures, selected radial power distributions from three different core loadings are shown for different core heights. The solid lines are the calculated shapes, and the points represent the experimental values. The results shown in these figures are typical of the good agreement between the calculated power profiles and the experimental power profiles.

However, in the calculation there was a tendency to underestimate the power values near the edge of the core. Also, the power values in the fuel pins next to poison pins were generally underestimated, especially for rods located in zones with low power values. A good example of the low-power zone is near the top and bottom of the core.

5.3.4. Confidence Level and Average Error

A confidence level has been assigned to each core loading calculation. The level was calculated from the difference between the experimental and calculated power values as follows:

For each fuel pin i at a height j , there is a calculated power value denoted by P_{ij} (Cal) and a corresponding experimental power value denoted by P_{ij} (Exp). The fractional difference between experimental and calculated values σ_{ij} is given by

$$\sigma_{ij} = [P_{ij}(\text{Cal}) - P_{ij}(\text{Exp})] \div P_{ij}(\text{Cal}) . \quad (5)$$

Now for all fuel rods I and all levels J , a total of $I \times J$ points is used in the calculation. The confidence level (CL) percentage is then calculated for a given core loading from the expression.

$$CL = \sqrt{\left\{ \left[\sum_{i=1}^I \sum_{j=1}^J (\sigma_{ij})^2 \times P_{ij}(\text{Cal}) \right] \div \sum_{i=1}^I \sum_{j=1}^J P_{ij} \right\}} \times 100. \quad (6)$$

Effectively, this is the square root of the average of the sum of the square of the fractional difference weighted by the power values. The choice of the power values as weighting factors is to further emphasize the importance of the difference between the experimental and calculated power values in the high-power zones of the reactor. The confidence levels for all core loadings studied are as follows:

<u>Core loading</u>	<u>No. of points used</u>	<u>CL, %</u>	<u>$\bar{\sigma}$, %</u>
IV-H,A00	896	4.57	4.93
IV-H,A11	864	3.65	3.53
IV-H,B00	832	4.25	4.47
IV-H,B11	832	4.05	4.05
IV-H,C00	832	3.85	3.96
IV-H,C11	832	4.07	3.90
IV-H,D00	832	3.48	3.61
IV-H,D11	832	3.81	3.97
IV-H,E00	896	3.70	3.97
IV-H,E11	896	3.21	3.32
IV-H,A10	864	3.18	3.19
IV-H,B10	1, 274	4.01	3.95
IV-H,B01	1, 274	4.01	4.42
IV-H,C10	832	3.85	3.66
IV-H,D10	1, 274	3.08	2.94
IV-H,D01	1, 274	3.06	3.15
IV-H,E10	896	2.99	3.19
IV-J,A00	616	4.23	4.18
IV-J,A11	594	3.07	2.96
IV-J,B00	1, 040	4.34	4.19
IV-J,B11	572	4.11	3.31
IV-J,C00	572	4.29	3.95
IV-J,D00	572	4.42	3.74
IV-J,D11	572	3.66	3.02
IV-J,E00	616	3.09	3.22
IV-J,E11	616	2.69	2.41
Avg values:	22, 002	3.72	3.66

The confidence level can be thought of as a measure of how, on the average, the calculated power values deviate from experimental power values considering that more importance is given to differences in the high-power zones than in the low-power zones. Note that the smaller the confidence factor, the better the agreement between calculated and experimental values.

The average CL of all cores analyzed is 3.72%; a maximum value of 4.57 was calculated for core loading IV-H,A00, and a minimum of 2.69% was calculated for core loading IV-J,E11.

A second value of interest in analyzing integral experimental and calculated results of this type is the average of the sum of the absolute value of the fractional deviations ($\bar{\sigma}$). This value is a measure of the average deviation (%) that can be assigned to any given point in the core analyzed; it is calculated from the expression

$$\bar{\sigma}\% = \left[\left(\sum_{i=1}^I \sum_{j=1}^J \left| \sigma_{ij} \right| \right) \div (I \times J) \right] \times 100.$$

As shown in the foregoing tabulation, the average deviation for all cores analyzed is 3.66%; a maximum average deviation of 4.93% is calculated for core loading IV-H,A00, and a minimum of 2.41% is calculated for core IV-J,E11. It is of interest that the values of CL are very close to those of $\bar{\sigma}$.

An attempt was made to calculate the confidence level and the average deviation to reproduce an experimental data set. Two sets of experimental data for core loading IV-J,D00 were used to check the accuracy of the mapping method. The results revealed a confidence level of 2.235% and an average deviation of 1.847%. Thus, the average deviation from the mean values of the two experiments is about 0.9%. The experimental average deviation from the average varied from about 0.5 to 1.0%.

5.3.5. Peak Power Values

The experimental peak power values and locations are listed in Table 3 for each core analyzed. The calculated power value at the location of the experimental peak and the percentage difference between calculated and experimental peak power values are also listed. A letter (b) next to the calculated power value indicates that the calculated peak was at a different location from that of the experimental values. For these cases, the location of the calculated power peak is also given. Note that the variations between the locations of experimental and calculated power peaks are within one axial mesh interval.

Considering that the fractional deviation of the experimental values from the calculated values for peak power values is of statistical nature, the precision of determining the experimental peak through calculational procedure can be indicated by a graphical plot of the distribution of the deviations. When the number of points is large and the deviations are random, this plot will approach the shape of a normal error curve. Thus, an average fractional deviation and a fractional standard deviation can be defined for the number of events used to calculate each individual fractional deviation.

From Table 3, an average fractional deviation of 1.204% and a fractional standard deviation of 1.444% are calculated for the 26 cores analyzed. A maximum fractional deviation of 3.420% was calculated for core loading IV-J,B00, and a minimum fractional deviation of -0.126 was calculated for core loading IV-J,D00. Note that a positive fractional deviation indicates that the calculated peak power value is overestimated, while a negative deviation indicates the opposite.

To obtain a better estimate of the average fractional deviation and the fractional standard deviation of the calculated peak power values, a different approach was considered. The peak powers from the four hottest pins measured in the core were used in the calculation rather than the single peak power value in the core. Naturally, the peak power value will be one of the four hottest pins. Tables 4, 5, and 6 show the experimental peak power values and calculated values for the four hottest pins at the same location. The percentage difference between calculated values and experimental data, as well as the location of the experimental value, is also given. For core loadings IV-H, the fuel pins located at one and three pin pitches in the north and west directions from the core center were the four hottest pins. Likewise, for core loadings IV-J, the pins $\sqrt{2}$ and $3\sqrt{2}$ pin pitches in the northwest and northeast directions from the core center were used. In addition, for asymmetrical loadings IV-H,B10, IV-H,D10, and IV-H,D01, additional pins at three pin pitches in the south and east directions from the core center were used.

Thus, a total of 110 points is used for the fractional standard deviation and the average fractional deviation for the 26 core loadings analyzed. The average fractional deviation was found to be 1.527%, and the fractional standard deviation was 1.90%. This produces a ratio of 1.243 for standard-to-average deviations. For a normal error curve,⁹ this value is 1.253, which provides some assurance that the deviations are random.

Table 3. Comparison of Experimental and Calculated Power Peaks at Same Locations

Core loading	Experimental power peak	Location from core bottom, cm	Calculated power value	% diff ^(a)	Location of power peak if different from experimental
IV-H,A00	1.426	80	1.429	0.218	
IV-H,A11	1.458	80	1.436(b)	-1.528	75
IV-H,B00	1.355	90	1.392	2.669	
IV-H,B11	1.347	90	1.375	2.053	
IV-H,C00	1.253	45	1.233(b)	-1.658	50
IV-H,C11	1.319	15	1.304(b)	-1.173	50
IV-H,D00	1.401	65	1.395	-0.428	
IV-H,D11	1.452	60	1.441	-0.799	
IV-H,E00	1.434	70	1.413(b)	-1.495	75
IV-H,E11	1.416	70	1.430	-1.122	
IV-H,A10	1.449	75	1.434	-1.029	
IV-H,B10	1.367	90	1.385	1.268	
IV-H,B01	1.364	90	1.378	1.001	
IV-H,C10	1.304	50	1.284	-1.551	
IV-H,D10	1.433	60	1.421	-0.837	
IV-H,D01	1.430	60	1.416	-1.005	
IV-H,E10	1.436	70	1.412	-1.694	
IV-J,A00	1.193	75	1.215(b)	1.793	80
IV-J,A11	1.250	70	1.254	0.313	
IV-J,B00	1.157	90	1.198	3.420	
IV-J,B11	1.131	85	1.158	2.347	
IV-J,C00	1.117	50	1.112	-0.434	
IV-J,D00	1.256	65	1.254	-0.126	
IV-J,D11	1.372	55	1.366	-0.458	
IV-J,E00	1.245	75	1.253	0.644	
IV-J,E11	1.335	60	1.332	-0.253	

Avg deviation: 1.204

(a) % diff = $P(\text{Cal}) - P(\text{Exp}) \times 100.0 \div P(\text{Cal})$.

(b) Calculated peak power occurred at a different location as indicated; note that they are within one axial mesh interval.

Table 4. Calculated and Experimental Values for Four Hottest Fuel Pins of Symmetrical Core Loading IV-H

Loading	North 3	Location ^(b)	% diff ^(c)	North 1	Location	% diff	West 01	Location	% diff	West 03	Location	% diff	Cal location
IV-H, A00	1.338/1.355 ^(a)	80	1.263	1.426/1.429	80	0.218	1.412/1.430	80	1.238	1.347/1.357	75	0.747	80
IV-H, A11	1.338/1.362	80	1.793	1.421/1.434	70	0.923	1.458/1.436	80	-1.528	1.338/1.373	75	2.595	75
IV-H, B00	1.266/1.320	90	4.103	1.353/1.391	90	2.772	1.355/1.392	90	2.669	1.285/1.325	90	3.052	90
IV-H, B11	1.255/1.306	90	3.865	1.347/1.375	90	2.053	1.335/1.373	85	2.795	1.268/1.310	90	3.188	90
IV-H, C00	1.178/1.186	50	0.689	1.253/1.233	45	-1.658	1.245/1.243	50	-0.190	1.178/1.191	50	1.121	50
IV-H, C11	1.237/1.250	50	1.063	1.307/1.303	45	-0.277	1.319/1.304	45	-1.173	1.227/1.252	50	2.014	50
IV-H, D00	1.323/1.328	60	0.375	1.397/1.394	65	-0.183	1.401/1.395	65	-0.428	1.300/1.333	65	2.508	65
IV-H, D11	1.361/1.374	60	0.962	1.452/1.441	60	-0.799	1.448/1.441	60	-0.500	1.351/1.377	60	1.863	60
IV-H, E00	1.332/1.346	75	1.035	1.434/1.413	70	-1.495	1.421/1.414	75	-0.489	1.334/1.351	70	1.239	75
IV-H, E11	1.368/1.363	70	-0.328	1.428/1.426	65	-0.104	1.446/1.430	70	-1.122	1.365/1.364	65	-0.108	70

(a) Experimental power value/calculated power value.

(b) cm from core bottom.

(c) $\% \text{ diff} = \frac{\text{Cal} - \text{Exp}}{\text{Cal}} \times 100.$

Table 5. Calculated and Experimental Values for
Asymmetrical Core Loadings IV-H

<u>Loading</u>	<u>North 3</u>	<u>Location</u> ^(a)	<u>% diff</u>	<u>West 01</u>	<u>Location</u>	<u>% diff</u>	<u>West 01</u>	<u>Location</u>	<u>% diff</u>	<u>West 05</u>	<u>Location</u>	<u>% diff</u>	<u>Cal location</u>
IV-H,A10	1.370/1.365	75	-0.298	1.444/1.434	75	-0.692	1.444/1.432	75	-0.847	1.362/1.363	75	0.041	75
IV-H,B10	1.274/1.318	90	3.372	1.355/1.386	90	2.220	1.367/1.385	90	1.268	1.265/1.318	90	4.055	90
IV-H,B01	1.271/1.302	90	2.412	1.344/1.376	90	2.304	1.364/1.378	90	1.001	1.258/1.312	90	4.118	90
IV-H,C10	1.223/1.236	50	1.067	1.304/1.284	50	-1.551	1.299/1.280	50	-1.485	1.217/1.227	50	.819	50
IV-H,D10	1.336/1.362	60	1.930	1.433/1.421	60	-0.837	1.415/1.418	60	0.179	1.340/1.355	60	1.089	60
IV-H,D01	1.339/1.342	60	0.199	1.430/1.416	60	-1.005	1.419/1.420	60	2.607	1.318/1.353	65	2.607	60
IV-H,E10	1.339/1.345	75	0.465	1.436/1.412	70	-1.694	1.428/1.410	70	-1.302	1.356/1.347	70	-0.639	70
	<u>South 03</u>	<u>Location</u>	<u>% diff</u>	<u>East 03</u>	<u>Location</u>	<u>% diff</u>							
IV-H,B10	1.259/1.309	90	3.806	1.274/1.318	90	3.372							
IV-H,D10	1.303/1.339	60	2.698	1.339/1.351	65	0.867							
IV-H,D01	1.353/1.365	60	0.871	1.335/1.353	65	0.087							

(a) cm from core bottom.

Table 6. Calculated and Experimental Values for Four Hottest Fuel Pins of Symmetrical Core Loadings IV-J

Loading	Northwest 03,03	Location ^(a)	% diff	Northwest 01,01	Location	% diff	Northeast 01,01	Location	% diff	Northeast 03,03	Location	% diff	Cal location
IV-J,A00	1.190/1.208	80	1.499	1.175/1.213	80	3.152	1.193/1.215	75	1.793	1.191/1.239	75	3.880	80
IV-J,A11	1.250/1.254	70	0.313	1.215/1.247	70	2.566	1.234/1.249	70	1.235	1.246/1.268	65	1.744	70
IV-J,B00	1.150/1.165	90	1.267	1.123/1.169	90	3.930	1.147/1.173	90	2.227	1.157/1.198	90	3.420	90
IV-J,B11	1.115/1.132	85	1.503	1.086/1.129	85	3.784	1.112/1.132	85	1.746	1.131/1.158	85	2.347	85
IV-J,C00	1.073/1.083	50	0.918	1.051/1.057	50	0.614	1.041/1.061	50	1.904	1.117/1.112	50	-0.434	50
IV-J,D00	1.168/1.223	65	4.507	1.184/1.202	60	1.506	1.213/1.206	65	-0.609	1.256/1.254	65	-0.126	65
IV-J,D11	1.339/1.348	50	0.659	1.296/1.318	55	1.672	1.329/1.313	60	-1.255	1.372/1.366	55	-0.458	55
IV-J,E00	1.242/1.223	75	-1.589	1.184/1.202	70	1.488	1.191/1.206	70	1.206	1.245/1.253	75	0.644	70/75
IV-J,E11	1.335/1.332	60	-0.253	1.284/1.292	60	0.626	1.294/1.288	65	-0.435	1.334/1.339	60	0.394	60

(a) cm from core bottom

Figure 4. Relative Power Vs Pin Pitch (Core Loading IV-J11)

Figure 5. Comparison of Calculated Relative Power Values and Experimental Data (Core Loading IV-H, A00)

Figure 6. Comparison of Calculated Relative Power Values and Experimental Data (Core Loading IV-H, C11)

Figure 7. Comparison of Calculated Relative Power Values and Experimental Data (Core Loading IV-H, D11)

6. CONCLUSIONS

This analysis has demonstrated that there is a 99% probability that the uncertainty in the calculated power peak for large power reactor cores is no more than 5% when using PDQ7.

Since all of the experiments were performed at room temperature, one might question the application of the uncertainty of 5.0% to the peak power value for the hot operating conditions. From the theoretical standpoint of diffusion theory, the local flux peaking is more severe at room temperature than it is at operating core temperatures because of the greater flux-trapping capability of cold water. Because of the longer neutron mean free path at higher temperatures, the core is more "homogeneous" than at room temperature. Consequently, the results of this study are expected to apply conservatively to the hot operating core conditions.

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