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THE ANATOMY OF A FREEWAY

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FOREWORD

This paper was adapted from a joint Mechanical Engineering — Electrical Engineering Colloquium given at LLL on January 11, 1974.

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THE ANATOMY OF A FREEWAY

ABSTRACT

A new theory, called the Headway Control theory, is offered to explain the behavior of vehicular traffic. It presumes that driver behavior is based upon time headways and that the value of the desired headway is determined by the combined perception time and reaction time of the individual driver. A mathematical model based upon this theory is developed and examined. The characteristics of the model are explored and simulation studies are run comparing the behavior of the model to real traffic. Model behavior is also examined when an automatic traffic control is impressed upon it.

INTRODUCTION

California freeways are paradoxes cast in concrete. Their outstanding characteristic is the fact that the only time we can go fast is when nobody else is using the freeway, and under these conditions one doesn't need a freeway. Any old two lane highway would do just fine. The most irritating part about driving on the freeway during the rush hour was sitting there with the gear selector lever in park, staring at a sign post that said "speed limit 65 mph." It was this inconsistency that got one of us (Collins) interested in freeway traffic some years ago. It wasn't so much being trapped by the system that bothered him, it was the fact that he was trapped by a system which he did not understand. He could never figure out why a roadway three lanes wide and without traffic lights could jam up to the point where traffic was absolutely stopped in all lanes over a considerable length of this roadway.

In the past this kind of stop and go traffic has been an irritant; but with the current energy crunch, the fact that a car in stop and go traffic uses up to 2.3 times as much fuel as the same car uses in high speed traffic makes this irritant an intolerable burden. For this reason the Energy Conservation Group at LLL has become interested in traffic behaviour. In the short term, energy conservation is the only reasonable approach to a solution of the energy crisis. We can conserve energy in two ways: reducing demand and using existing equipment more efficiently. Since the only way we can achieve a reduction in transportation demand is by fostering a recession, the increased efficiency approach appears to be the more viable.

Roadway design has always been "cut and try." Up to the present time there have been no useful models of traffic to aid the highway engineer in his task. The rules used in highway construction are analogous to the rules used in hydraulic engineering prior to the introduction and understanding of the Reynolds number effect and Bernoulli's law. Each new roadway is an extension of the philosophy of the last roadway in what is hoped to be a more favorable direction. Since Mr. Murphy is still the engineer's guardian angel, the results of this exercise haven't been too good.

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The limited access highway, or freeway as it is called in this state, represents the most advanced attempt to build a roadway which should permit free-flowing traffic.

Freeways have the following characteristics:

- Wide lanes: typical lane width in the State of California and in the Federal highway system is 12 feet, twice the width of the average American automobile.
- Easy grades: on the new interstate highway system grades in excess of 3% are rarely found.
- High radius turns: curves are built and banked so that a passenger automobile can take them very easily at 60 to 80 mph.
- One-way traffic: multi-lane roadways are separated by solid median barriers; motorists need not be concerned with vehicles traveling in the opposite direction.
- Acceleration lanes for entering and deceleration lanes for leaving the freeway.
- No cross traffic.
- No traffic lights.
- High speed limits.

This set of design parameters should result in a roadway on which one could drive easily and rapidly; but, unfortunately, it doesn't always work out that way. However, the freeway does represent a roadway stripped to its theoretical essentials, and as such gives us a roadway which should be amenable to analysis.

The total effort to analyze traffic behavior from a theoretical standpoint has been fairly small. No serious attempts were made before about 1950, and none of the few models proposed have met with any overwhelming success. However, certain characteristics of vehicular traffic have emerged. It has been established and verified that driver reaction time is real. You can't get on the brakes instantly. It takes about 0.75 seconds to recognize that something is happening, decide what to do about it, and then put the decision into operation.

It has also been found that inter-car spacing increases with increasing speed. People trail at short distances only in slow traffic. A third item which has been a source of frustration to highway designers, is that traffic flow, a measure of the number of cars passing a point in a given length of time, decreases with increasing traffic density. The fourth fact of freeway life is that there AIN'T NO SUCH THING AS THE FAST LANE. In moderate and heavy traffic there is no speed stratification from lane to lane. Cars in the left lane have the same average speed as cars in the right lane.

HEADWAY CONTROL THEORY

The traffic flow theory which we shall discuss is called the Headway Control Theory. Its basic premise is that drivers try to keep a constant time separation between themselves and the car ahead, independent of speed. In other words, if a driver is traveling 30, 40, or 60 mph, he will stay a distance behind the car ahead such that if the lead car passes a pole at a given time, he will pass the same pole 1.5 seconds later.

This time separation is called a time headway. Passing is also accomplished by paying attention to time headways. A driver won't move into an adjacent lane unless there will be an adequate time separation between his vehicle and the vehicles in that lane. A more complete statement of this behavioral theory is given below.

A driver tries to travel at his desired speed. If he finds himself closing on the car ahead, he will attempt to pass when his headway falls below 3 seconds. He will change lanes to pass if his trailing headway in the passing lane is adequate, and he is closing on the car ahead more quickly than the trailing car in the passing lane would be closing on him. If he cannot pass he will follow by attempting to keep his headway at a value of 1.5 seconds.

Time Headway Distributions

Let us examine the premise that drivers prefer to travel at a constant time separation. Figure 1 shows headway distributions as a function of traffic volume measured on the Eisenhower Expressway in Chicago.¹ When few people are using the freeway, the average headway is fairly large. As traffic volume begins to increase, the headway distribution starts to peak in the vicinity of 1.5 seconds. At fairly high volumes this peak is dominant. Figure 2 shows data measured by Dr. Richard Stuart² for Highway 24 between Berkeley and Walnut Creek, California. This was essentially automobile commuter traffic. Stuart's data show a distinct and very sharp peak in the vicinity of 1.5 seconds. The actual data is clustered more closely around that median value of 1.5 seconds than is the normal fit.

Measurements made by Collins of mixed car and truck traffic on the Nimitz Freeway in Oakland, California are shown in Fig. 3. The solid line represents traffic in the two left lanes which contained no heavy trucks. The dashed line represents the headway distribution found in the two right lanes which contained about 25% heavy trucks. The general shape of the headway distribution curve is independent of the traffic mix. Both car-only traffic and mixed car-truck traffic have that same modal value of about 1.5 seconds. In this particular case, the average concentration was about 13 vehicles per mile, and traffic was traveling at about 50 mph. Figure 4 shows Collins' measurements of the headway distribution for heavy traffic on the Nimitz. The average vehicle concentration has increased to about 60 vehicles per mile and the average speed has decreased to about 20 mph. Again, we have the same modal peak appearing at 1.5 seconds for both pure automobile traffic and traffic containing heavy trucks.

Measurements made in various parts of the country, by various investigators, on various types of traffic all show the same kind of headway distribution: a distinct modal peak at 1.5 seconds, followed by a very sharp decrease in the number of headways at any other time. This leads us to believe that the basic tenet of the headway control theory, that there is a constant time separation, is valid.

Fig. 1. Headway distributions as a function of traffic volume (Vpm — vehicles per minute) determined by May¹ for the Eisenhower Expressway in Chicago, Illinois. The average headway value is shown as $\bar{x}$.

Fig. 2. Headway distribution data as measured by Stuart² on Highway 24 between Berkeley and Walnut Creek, California.

Fig. 3. Light traffic headway distribution, Nimitz freeway, Oakland, California. The average concentration is 13 vehicles per mile.

Fig. 4: Heavy traffic headway distribution, Nimitz freeway, Oakland, California. The average concentration is 60 vehicles per mile.

The Car-Following Mode

As traffic builds up on a given roadway, the opportunities for passing will decrease, as the headway requirements for passing will no longer be met. This will cause a higher proportion of the vehicle population to remain in the car-following mode. Thus, the car-following mode governs the behavior of the majority of vehicles in heavy traffic situations. Since heavy traffic is where traffic problems exist, then the behavior of a vehicular stream whose members are constrained to follow is worth examining. Headway Control Theory says that if a driver cannot pass, he will follow by attempting to keep his headway at a value of 1.5 seconds. Because a driver has a finite reaction time, he cannot respond instantaneously to changes in the behavior of the car ahead. When the vehicle ahead speeds up or slows down, there is an operating delay time before the car following can attempt to mimic the leader's behavior. In trying to maintain the proper separation, a driver responds both to a deviation from his desired headway and the rate at which that headway is changing. His acceleration is governed by the following equation:

$$\ddot{X} = C_1(H - H_0) + C_2\dot{H},$$

where H_0 = desired headway,
 H = actual headway,
 $\dot{H}$ = time rate of change of headway.

In certain cases the contributions from the two portions of the equation can cancel each other. If a car suddenly cuts in front of you but is pulling away at a rapid rate, the first half of the equation says that your headway is much less than it should be, and urges you to step on the brakes. However, the second half of the equation says that the rate of change of headway is positive and urges you to step on the gas. The sum of these two inputs is zero, and you in fact do sit in your car unable to either step on the brake or the gas. On the other hand, if the same car cuts closely in front of you and brakes sharply, both sides of the equation tell you to slow down as rapidly as possible.

This equation is the heart of the car-following mode. For the lack of any better information, the simulation studies which we ran were based upon the assumption that both headway deviation and rate of change of headway would have approximately equal influence on the driver. The numerical constants selected were designed to keep the two contributions approximately equal.

As a first check, we assumed a line of identical cars following each other at the desired headway of 1.5 seconds. We then let the lead car decelerate from 50 to 30 mph and hold this new speed. Figure 5 shows the response of the 10th car in that line of cars. The top curve is the velocity-time profile and the bottom curve is the headway profile. The deceleration level for the lead car was 1.61 ft/sec^2 , just about the level you would achieve by taking your foot off the gas. Note that the system behaves stably, with the 10th car obtaining the desired speed after a fairly small oscillation.

Fig. 5. Deceleration response characteristics of the 10th car in a line of cars following.

Figure 6 shows the behavior of the 20th car in that line of cars. The system's response is still stable, but the amplitude of the velocity oscillations has increased. The initial undershoot is now closer to 23 mph.

Another feature of note is the fact that the delay time — the time lag between the time the lead car slows down and the 20th car starts to slow down — is on the order of 30 seconds. This is twice the cumulative reaction times for those 20 drivers. The average response time of each vehicle is 1.5 seconds, even though the reaction time of each driver is 0.75 seconds. The response time is coincident with the desired headway separation.

This happens because we set a threshold value in the computer program for the recognition of the fact that the headway had changed. The following car does not realize that his headway has changed until it changes by 0.01 seconds. In other words, the program does not recognize deviations from the desired headway which are less than 0.01 seconds. For the deceleration levels and speeds chosen here, the recognition threshold is not crossed until about 0.75 seconds after each driver takes his foot off the gas. It seems reasonable that real drivers will also have some recognition threshold. Although no actual measurements have been made, we suspect that the number selected for the program is fairly close to real-life values. We believe this to be true because the operating delay time (which is the sum of the recognition time and reaction time) is coincident with the desired headway.

Figure 7 shows the response of the 10th car to an acceleration of the lead car. Again we see a stable, damped oscillatory response. The response of the 20th car in line to the same acceleration of the lead car is shown in Fig. 8. We have the same damped oscillatory response. The model studies show:

- A total response time on the order of the desired headway separation.
- A stable, damped oscillatory response for each succeeding car in a line of cars.
- Increasing initial oscillatory amplitude with increasing car number.

SIMULATION STUDIES

This model is based on human behavior and reactions. After studying the behavior of real-life traffic we have come to the conclusion that this behavior is analogous to human group behavior. M. Sherif³, a social psychologist, defines a group as follows:

"A group is a social unit which consists of individuals who stand in definite status and role relationships to one another and which possesses a set of values and norms of its own which regulates the behavior of individual members."

A "platoon" of vehicles on a freeway traveling in the interactive mode fits this definition very closely. We would expect this, since the drivers of our vehicles in the real-life situation are after all human beings. When a number of cars are forced into the interactive mode because of the heavy traffic conditions, their behavior follows group behavior.

Fig. 6. Deceleration response characteristics of the 20th car in a line of cars following.

Fig. 7. Acceleration response characteristics of the 10th car in a line of cars following.

Fig. 8. Acceleration response characteristics of the 20th car in a line of cars following.

In examining the literature on traffic modeling, we found that most of the previous work has been mechanistic in nature and the mathematics used to describe the behavior of traffic has become rather complex and involved. The Headway Control approach to modeling represents a significant departure from past methodologies. The validity of this observation is when we examine the analysis of our simulation runs.

Before describing the structure of our simulation, we would like to review the steps in the development of a model for simulation. The first step is to develop a comprehensive definition of the problem. In the case presented here, the problem was to develop a comprehensive and flexible model which can be applied to a broad range of problems in the analysis of vehicular roadways.

The second step is the formulation of a mathematical model. The derivation of the basic equation used in this model has been described. The third task is to construct a computer program for the model. The results of that effort were presented in the preceding section. The same basic program was used as the foundation for the various programs we used in our simulation runs.

The fourth step is to validate the model. The validation process involves taking a running model and plugging in data from real-life observations to determine if the model will reproduce the results of the real-life experiments.

The fifth step in this process is to design experiments to use the model as a tool in the analysis of significant problems. The sixth and final task is to run the model and analyze the results.

Figure 9 shows the plots of data collected by Treiterer and Nemeth⁴ of Ohio State University in a traffic study they conducted in 1967. These data were collected from commute traffic on Interstate 71, north of Columbus, Ohio on July 25, 1967. The data plotted represent the tracks of vehicles in lane 1 (the fast lane) and each trace on the plot depicts the track of one vehicle for the duration of the observations. These data were collected by taking photographs at timed intervals from helicopters flying on parallel courses with the traffic flow. From these timed photos, data points were derived on the position of each car, and all of this was used to create files from which this Calcomp plot was generated.

This type of plot is what we call a "time-space diagram." Time is plotted along the abscissa, and distance is shown along the ordinate. The slope of the vehicle tracks shown in Fig. 9 is actually the velocity. In the middle of the diagram some of the tracks are horizontal, constant with time. Thus, we know that these cars have stopped.

Let us now observe the behavior of this string of vehicles caused by a deceleration of the leader followed by an acceleration of the leader a short time later. On the left side of the diagram we see this deceleration followed by an acceleration. Following this to the right and curving slightly downward we see the build up of a shock wave. We see how this shock wave builds up in amplitude until one of the cars in the line finally comes to a stop. The first car to come to a stop remains stopped for a short time (about 5 seconds here). As the wave regresses further down the line, succeeding vehicles come

to a stop and remain stopped for a longer period of time. Some of the cars in the lower part of this diagram actually remain stopped for about 15 seconds. In the lower right hand portion of the diagram the line of cars are still being affected, but are no longer coming to a stop.

The condition that finally broke up the jam and let the late arrivals through without stopping, was the arrival of several cars in this area with longer headways. Cars entering the area of the jam arrive with rather long headways; but at start up when leaving the jam, they depart with short headways, indicating that most of them are in the interactive mode. After restarting we can observe the damped oscillatory behavior of some vehicles described earlier.

Figure 10 shows the same time-space diagram with the track of one vehicle identified by a heavy line. We have chosen this vehicle to be the lead car in our simulation. Data points were derived from this plot to determine the position of this car for each 5 seconds of its displayed run. The scale on this plot is 10 seconds per division on the abscissa and 600 feet per division on the ordinate. Headways for each of the 52 following cars were determined from this same plot by reading horizontally along the abscissa starting at the zero point of the lead car.

Figure 11 shows the results of one of our simulation runs. At the bottom of this plot it should be noted that we have approximated the original headway distributions of real-life plot.

Note the behavior of the following cars in the simulation in the area of the "knock-down." Here we see the shock wave regressing back the chain with increasing stopping times. Figure 10 shows that the model approximates the real-life case very closely.

Figure 10 shows the breakup of the jam at about the same point in the real-life plot. Note the increased headways of several cars in the area where the breakup of the jam occurs. Figure 11 shows the same increased headways of several cars has led to the breakup of the jam in the simulation.

Let us now examine the area of the simulation above the shock wave. Here we observe that when the cars start up after the jam they are in an interactive mode with 1.5-second headways. In this same area we see the damped oscillatory start-up reaction with a little higher amplitude than that occurring in the real-life plot.

In Fig. 10 the cars are in the interactive mode above the shock wave, and there is just a slight indication of the damped oscillatory start-up reaction. We probably do not see more of a reaction here because in the area where we would expect to see it, there are some random headways greater than 3 seconds that break the interactive mode in this area.

The simulation shown in Fig. 11 runs for 2.5 minutes and plots 53 cars for 1.2 miles. The initial velocity of the cars is 54 mph, and the velocity after leaving the jam is 44 mph.

We feel that the simulation runs we made using this model are sufficiently close to the real-life data that we have demonstrated validation of the model.

Fig. 10. Time-space diagram of traffic on Interstate 71 near Columbus, Ohio⁴; the track of one vehicle chosen as the lead car is outlined.

Fig. 11. Computer simulation of a line of real traffic subjected to nonsteady behavior of the lead car.

A logical question might be what are the possible applications of this type of work? This model is basically simple in concept and flexible enough in structure so that it can be applied to a broad range of roadway problems. New design concepts for freeways can be analyzed before final building is undertaken to determine the performance under different input conditions. This model could also prove valuable as a tool in the analysis of existing roadways to discover problem areas.

CONTROL

It was apparent in the simulation studies that the behavior of the lead car is amplified by the following cars. This was implied in the 10th- and the 20th-car deceleration profiles shown in Figs. 6 and 7, but it wasn't really apparent until the simulation study was made. To take a little closer look at the response of a line of cars to an impulse by the lead driver, we ran some studies where the lead car decelerated from 40 to 15 mph, held this new speed for 30 seconds, and then accelerated to 40 mph. The cars following the lead car were given initial headways selected from the distribution which Stuart measured. Thus, these studies more closely approximated real traffic. Figure 12 shows the results. We see the same kind of thing shown in the Treiterer and Epps studies: as the car number increases, the initial velocity change also increases, until we see that these cars are actually forced to halt. We tried to restart the system by giving the 50th car a 5-second headway, but this was inadequate.

Figure 13 shows the same simulation with the 50th car being given a 10-second headway. We were still unable to break the jam. We were able to clear up the first halt, but not the second one. Figure 14 shows the same problem with the 50th car being given a 20-second headway. This was enough to get the system rolling again. It seems clear from these studies that any disturbance initiated by the leader in heavy traffic will be amplified by each successive car in line, and if the line is long enough, we can drive traffic to a halt. Because of the forward reaction time and braking reaction time effects, heavy traffic is not smooth but is characterized by stop-and-go driving. We have seen that the only way we can break the stop-and-go pattern is to leave a large enough gap in traffic at some specific interval so that the system can start flowing again.

Since one of the reasons for developing this model was to try the implementation of optimal control schemes, we decided to see if we could add a specific control function to the model. We decided to try to model a line of single lane high speed traffic traveling on a road containing one stop light. We called this the Novato Project. Figure 15 is a time-space diagram of the signalized intersection. The traffic signal is at the top where you can see that the cars have been stopped. The total period of the signal, the red-to-red time, is 40 seconds. The difference between Fig. 15a and Fig. 15b is the green fraction of the total period. In Fig. 15a, the green fraction amounts to 50% of the signal period, and in Fig. 15b the green fraction amounts to 67% of the total period. The inflow traffic has the same starting headway distribution as in the

Fig. 12. Traffic impulse study: decelerate from 40 to 15 mph, 30-second hold; accelerate to 40 mph. Fiftieth car had a 5-second headway.

Fig. 13. Traffic impulse study: decelerate from 40 to 15 mph, 30-second hold; accelerate to 40 mph. Fiftieth car had a 10-second headway.

Fig. 14. Traffic impulse study: decelerate from 40 to 15 mph, 30-second hold; accelerate to 40 mph. Fiftieth car had a 20-second headway.

Fig. 15. Traffic control study: the traffic signal at 10,000 feet has a period of 40 seconds and (a) a green fraction of 50%, (b) a green fraction of 67%.

preceding figures. We found out that on this 40-second period, a 50% green fraction caused traffic to bunch up at the signal, whereas a 67% green fraction allowed traffic to flow fairly freely. Figure 16 shows the same investigation with the traffic signal period increased to 60 seconds. Infeed speeds and headway distributions are the same. Even though the total period of the traffic signal has been increased by 50%, the results are still the same as for the shorter period. A 50% green fraction causes traffic to back up, whereas a 67% green fraction allows traffic to flow. Our last run is shown in Fig. 17. Here we increased the signal period to 90 seconds, again experimenting with a 50% green fraction and a 67% green fraction. The results were the same as before, the 50% green fraction caused traffic to move reasonably freely. It appears that for a given traffic volume, the effect of a traffic signal on traffic flow is independent of the total signal period and depends only on the percentage of time devoted to the green light.

CONCLUSIONS

Even though the Headway Control traffic model is far from complete, it has enabled us to isolate and identify certain traffic factors. First, traffic is bistable. Things steady down only when the freeway is empty or when it is jammed solid with unmoving cars. The impulse studies showed us that if we perturb traffic slightly, the perturbation tends to drive the freeway to a halt. The only way to reverse this perturbation is to place a gap in traffic which drives the system back to its free-flowing state. Second, we have found that the flow of traffic is controlled by the passage time of the individual vehicle. Since cars try to follow one another at constant headway separations, independent of speed, it would seem that traffic flow should be speed-independent. It is not, simply because cars are not point masses. Flow decreases with decreasing speed because at lower speeds it takes more time to push 17 feet of car past a point than it does at higher speeds. This is why traffic is bistable. When traffic starts to slow down, the time required to process each successive car in line increases with increasing car number, thus driving the system stably to a halt. When traffic starts to speed up, the time required to process each subsequent car in a line of cars decreases slightly with increasing car number, causing the system to move back to its free speed.

Our third major finding is a rationale for the fact that the desired headway of 1.5 seconds is about twice the average reaction time. In order to successfully mimic the actions of a car ahead of us, we must trail at a time which is the sum of our perception and reaction times. Reaction time seems to be built into the human system, but the perception time is a threshold recognition time, the time required for us to recognize that a change in headway has indeed occurred.

Fig. 16. Traffic control study: the traffic signal at 10,000 feet has a period of 60 seconds and (a) a green fraction of 50%, (b) a green fraction of 67%.

Fig. 17. Traffic control study: the traffic signal at 10,000 feet has a period of 90 seconds and (a) a green fraction of 50%, (b) a green fraction of 67%.

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