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LAWRENCE LIVERMORE LABORATORY

University of California/Livermore, California

**DESTABILIZATION OF LOW-FREQUENCY DRIFT WAVES IN THE
BASEBALL II EXPERIMENT**

A. Futch

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DESTABILIZATION OF LOW-FREQUENCY DRIFT WAVES IN THE BASEBALL II EXPERIMENT

J. F. Clarke has discussed a mechanism for coupling between a low-frequency drift wave and an ion-cyclotron instability.* He assumes that a high-frequency ion-cyclotron instability results in both electron losses and a fluctuating potential ϕ_{rf} having a perturbed component ϕ'_{rf} . This perturbed potential modulates the confining potential well for the electrons. The resulting perturbed electron loss rate renders one branch of the flute unstable.

Clarke states that "the lower branch of the drift wave is unstable when coupled to electron loss by the microinstability in the manner postulated." In going through the algebra, I have concluded that the upper branch (ion wave) is unstable instead of the lower branch. The purpose of this note is to reproduce the algebra so that it may be checked by others in hopes of resolving the discrepancy.

Clarke assumes that the saturated amplitude of the cyclotron-frequency instability is proportional to some power of the local ion density: i.e.,

$$\phi_{rf} = a(n_+)^p. \quad (1)$$

Substituting

$$n_+ = n_+^0 + n_+^1, \quad (2)$$

where n_+^1 is the perturbed ion density resulting from the drift wave amplitude, gives

$$\begin{aligned} \phi_{rf} &= a(n_+^0 + n_+^1)^p = a(n_+^0)^p \left[1 + p \frac{n_+^1}{n_+^0} + \dots \right], \\ &= \phi_{rf}^0 + \phi'_{rf} \end{aligned} \quad (3)$$

where

$$\phi'_{rf} = \phi_{rf}^0 p \frac{n_+^1}{n_+^0}. \quad (4)$$

The perturbed electron loss rate is

$$S_-^1 = \beta n_+^0 \frac{\phi'_{rf}}{\phi_p}. \quad (5)$$

* J. F. Clarke, Phys. Rev. Lett. **23**, 1215 (1969).

Substituting ϕ'_{rf} from (4) into (5) gives

$$S'_- = \beta_p \frac{\phi'^0_{rf}}{\phi_p} n'_+ = \nu n'_+, \quad (6)$$

where β is the electron bounce frequency and ϕ_p is the plasma potential. The linearized continuity equation becomes

$$\begin{aligned} \frac{\partial n'_+}{\partial t} + \vec{v}_0 \cdot \nabla n'_+ + \vec{v}' \cdot \nabla n_0 + n_0 \nabla \cdot \vec{v}' &= 0 \quad (\text{ions}) \\ &= -\nu n'_+ \quad (\text{electrons}) \end{aligned} \quad (8)$$

where

$$\vec{v}'(r, \phi) = \frac{\vec{E} \times \vec{B}}{B^2} = \frac{-e^{i(\omega t + m\phi)}}{B} \left[-\frac{\partial \psi}{\partial r} \vec{r}_0 + \frac{im\psi}{r} \vec{r}_0 \right]. \quad (9)$$

Since $\vec{E} = -\nabla \Psi$, where $\Psi = \psi e^{i(\omega t + m\phi)}$,

$$\begin{aligned} n'_+ e^{i(\omega t + m\phi)} i\omega + \frac{v_0}{r} n'_+ e^{i(\omega t + m\phi)} im + v'_r \frac{dn_0}{dr} + n_0 \nabla \cdot \vec{v}' &= 0 \quad (\text{ions}) \\ &= -\nu n'_+ e^{i(\omega t + m\phi)} \quad (\text{electrons}). \end{aligned}$$

$$n'_+ = \frac{ie^{-i(\omega t + m\phi)}}{(\omega + m\Omega_D)} \left(v'_r \frac{dn_0}{dr} + n_0 \nabla \cdot \vec{v}' \right), \quad (10)$$

but

$$n'_- = \frac{ie^{-i(\omega t + m\phi)}}{(\omega - i\nu)} \left(v'_r \frac{dn_0}{dr} + n_0 \nabla \cdot \vec{v}' \right) \quad (11)$$

since Ω_D is negligible for electrons. If we neglect the $\nabla \cdot \vec{v}'$ term then

$$n'_+ = \frac{m}{(\omega + m\Omega_D)B} \frac{dn_0}{dr} \frac{\psi}{r} \quad (12)$$

$$n'_- = \frac{m}{(\omega - i\nu)B} \frac{dn_0}{dr} \frac{\psi}{r} \quad (13)$$

Substituting into Poisson's equation gives

$$\nabla^2 \psi = \frac{-4\pi ecm}{B} \left[\frac{1}{\omega + m\Omega_D} - \frac{1}{\omega - i\nu} \right] \frac{dn_0}{dr} \frac{\psi}{r} \quad (14)$$

$$= \frac{4\pi ecm}{B} \left[\frac{m\Omega_D + i\nu}{(\omega + m\Omega_D)(\omega - i\nu)} \right] \frac{dn_0}{dr} \frac{\psi}{r} \quad (15)$$

or

$$\frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} + \left(\alpha^2 - \frac{m^2}{r^2} \right) \psi = 0, \quad (16)$$

where

$$\alpha^2 = D \left[\frac{m\Omega_D + i\nu}{(\omega + m\Omega_D)(\omega - i\nu)} \right] \quad (17)$$

and

$$D = \left(\frac{\omega_{pi}}{\omega_{ci}} \right)^2 \omega_{ci} \frac{m}{Nr} \frac{dN}{dr} \approx - \left(\frac{\omega_{pi}}{\omega_{ci}} \right)^2 \omega_{ci} \frac{2m}{r_o^2}. \quad (18)$$

For a parabolic plasma profile of

$$N = N_o \left(1 - \frac{r^2}{r_o^2} \right), \quad (19)$$

$$\frac{dN}{dr} = \frac{-N}{(1 - r^2/r_o^2)} \frac{2r}{r_o^2}$$

and

$$\frac{1}{Nr} \frac{dN}{dr} = \frac{-2}{r_o^2 (1 - r^2/r_o^2)} \approx \frac{-2}{r_o^2} \quad \text{for } r \ll r_o.$$

Therefore,

$$\frac{4D}{\alpha^2} \frac{1}{m\Omega_D} = -8 \left(\frac{\omega_{pi}}{\omega_{ci}} \right)^2 \frac{\omega_{ci}}{(\alpha r_o)^2 \Omega_D}. \quad (20)$$

The solution of Eq. (16) for α independent of r is

$$\Psi(r) = A J_m(\alpha r).$$

Rewriting Eq. (17)

$$(\omega + m\Omega_D)(\omega - i\nu) - \frac{D}{\alpha^2} (m\Omega_D + i\nu) = 0.$$

Then

$$\omega^2 + (m\Omega_D - i\nu)\omega - i\nu m\Omega_D - D/\alpha^2 (m\Omega_D + i\nu) = 0$$

and

$$\begin{aligned}
\omega &= -\frac{(m\Omega_D - i\nu)}{2} \pm \sqrt{\frac{(m\Omega_D + i\nu)^2}{4} + \frac{D}{\alpha^2} (m\Omega_D + i\nu)} , \\
&= -\frac{(m\Omega_D - i\nu)}{2} \pm \frac{(m\Omega_D + i\nu)}{2} \left[1 + \frac{4D}{\alpha^2} (m\Omega_D + i\nu)^{-1} \right]^{1/2} , \\
&= -\frac{(m\Omega_D - i\nu)}{2} \pm \frac{(m\Omega_D + i\nu)}{2} \sqrt{1 + \frac{4D}{\alpha^2 m\Omega_D} \left(1 - \frac{i\nu}{m\Omega_D} \right) \dots} . \quad (21)
\end{aligned}$$

Equation (21) differs from that of Clarke by additional imaginary terms outside the radical and a different sign for the imaginary term under the radical.

Re-writing Eq. (21),

$$\begin{aligned}
\omega &= -\frac{m\Omega_D - i\nu}{2} + \frac{m\Omega_D + i\nu}{2} \left[1 + \frac{4D}{\alpha^2 m\Omega_D} - \frac{4D i\nu}{(\alpha m\Omega_D)^2} \right]^{1/2} \\
&= -\frac{(m\Omega_D - i\nu)}{2} \pm \frac{(m\Omega_D + i\nu)}{2} \left\{ \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} \left[1 - \frac{4D i\nu}{(\alpha m\Omega_D)^2} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1} \right]^{1/2} \right\} .
\end{aligned}$$

For $\nu \ll \Omega_D$, the second radical can be expanded giving

$$\omega = -\frac{(m\Omega_D - i\nu)}{2} \pm \frac{(m\Omega_D + i\nu)}{2} \left\{ \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} - \frac{2D i\nu}{\alpha^2 m^2 \Omega_D^2} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1/2} \right\} . \quad (22)$$

If the negative sign is chosen,

$$\begin{aligned}
\omega &= -\frac{m\Omega_D}{2} \left[1 + \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} \right] - \frac{\nu^2 D}{(\alpha m\Omega_D)^2} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1/2} \\
&\quad + \frac{i\nu}{2} \left[1 - \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} + \frac{2D}{\alpha^2 m\Omega_D} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1/2} \right] ,
\end{aligned}$$

and

$$\omega_{\text{imag.}} = \frac{\nu}{2} \left[1 - \frac{1 + \frac{2D}{\alpha^2 m\Omega_D}}{\sqrt{1 + \frac{4D}{\alpha^2 m\Omega_D}}} \right] . \quad (23)$$

If the positive sign is taken,

$$\omega = -\frac{m\Omega_D}{2} \left[1 - \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} \right] + \frac{\nu^2 D}{(\alpha m\Omega_D)^2} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1/2} + \frac{i\nu}{2} \left[1 + \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{1/2} - \frac{2D}{\alpha^2 m\Omega_D} \left(1 + \frac{4D}{\alpha^2 m\Omega_D} \right)^{-1/2} \right],$$

and

$$\omega_{\text{imag.}} = \frac{\nu}{2} \left[1 + \frac{1 + \frac{2D}{\alpha^2 m\Omega_D}}{\sqrt{1 + \frac{4D}{\alpha^2 m\Omega_D}}} \right]. \quad (24)$$

The second term in both Eqs. (23) and (24) is of the form

$$\frac{1+x}{\sqrt{1+2x}}$$

which is ≥ 1 for any nonzero value of $x > -1/2$.

The ion wave corresponds to taking the negative sign since $\omega_{\text{real}} \rightarrow -m\Omega_D$ at low densities. The wave corresponding to the ion branch of the dispersion equation has a negative imaginary component and is therefore unstable for all values of x in a min-B field and for $-1/2 < x < 0$ for a mirror field. The ion wave would be expected to be stabilized by a broadened distribution, since Ω_D does not have a unique value.

The electron drift wave corresponds to taking the positive sign in the dispersion equation. The imaginary part of ω is positive and the electron drift wave is stable for both a mirror field and a min-B field. Table 1 summarizes these conclusions.

Table 1. Drift wave stability in different magnetic field configurations.

	Electron drift wave	Ion drift wave
Mirror field	Stable	Unstable
Min-B field	Stable	Unstable

Don Pearlstein* found an expansion error in the original dispersion equation which had the effect of reversing the stability conditions for a min-B field. It should also be noted again that these conclusions are different from those of Clarke.

*L. D. Pearlstein, Lawrence Livermore Laboratory, private communication, October, 1973.

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