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**A Transient, Two-Dimensional, Discrete-
Element, Far-Field Model for Thermal
Impact Analysis of Power Plant
Discharges in Coastal and Offshore
Regions. Part I: General Description
of the Mathematical Model and the
Results of an Application**

Arsev H. Eraslan

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ENERGY DIVISION

**A TRANSIENT, TWO-DIMENSIONAL, DISCRETE-ELEMENT, FAR-FIELD
MODEL FOR THERMAL IMPACT ANALYSIS OF POWER PLANT DISCHARGES
IN COASTAL AND OFFSHORE REGIONS. PART I: GENERAL DESCRIPTION
OF THE MATHEMATICAL MODEL AND THE RESULTS OF AN APPLICATION**

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A TRANSIENT, TWO-DIMENSIONAL, DISCRETE-ELEMENT, FAR-FIELD MODEL FOR THERMAL IMPACT ANALYSIS OF POWER PLANT DISCHARGES IN COASTAL AND OFFSHORE REGIONS. PART I: GENERAL DESCRIPTION OF THE MATHEMATICAL MODEL AND THE RESULTS OF AN APPLICATION

Arsev H. Erizlan

ABSTRACT

A far-field mathematical model is presented for numerical simulation of short-time (within tidal cycle) transient, two-dimensional temperature distributions in large coastal and offshore regions resulting from the condenser cooling water discharges of power plants.

The Eulerian FLIDE (fluid-in-discrete-element) formulation employs the integral forms of the conservation principles for mass and thermal energy in variable-sized discrete elements that span the specified flow region.

The contributions of vertical variations of the velocity components and temperature are rigorously incorporated in the development of depth-averaged, two-dimensional energy transport fluxes by spatially integrating the conservation equations over the enclosure surfaces of the discrete elements.

The general mathematical formulation considers completely arbitrary, transient oceanic flow conditions, which include periodic tidal, geostrophic and wind-induced currents, as locally specified inputs to the model. The fundamental differences, both from physical and mathematical considerations, between meso-scale and micro-scale eddy turbulence, shear flow dispersion, and stratification are discussed in detail to establish a realistic formulation of depth-averaged, nonconvective thermal energy fluxes associated with the general turbulent transport phenomenon.

The surface heat-transfer effect is formulated according to a short-time model (hourly variations) which approximately incorporates solar irradiation, black body back radiation, evaporation, and convection modes for the conditions of the ocean surface in coastal and offshore regions.

The resulting mathematical system of weakly coupled, nonlinear, ordinary differential equations is numerically integrated, from arbitrarily specified initial conditions, by the Runge-Kutta-Gill method with a time step size based on a stability criterion for explicit methods.

The general mathematical model is applied to the assessment of the thermal impact of a hypothetical, multiunit generating station in a coastal region where the oceanic flow conditions are assumed to be strictly periodic tidal currents without any appreciable net drift of sufficient duration to remove the heated effluent out of the far-field region.

The numerical simulation indicates that the periodic flow conditions cause considerable variations in the temperature distributions during the day and the tidal cycles, which result in severe recirculation and re-entrainment of the heated water between the intakes and the discharges of the different units. This thermal interaction effect ultimately leads to a gradual, long-term increase of the temperatures in the immediate vicinity of the discharge structures and also in the far-field zone, which could cause adverse environmental impact in the coastal region.

It is concluded that steady-state near-field studies of the individual discharges are definitely inadequate, and a transient far-field analysis which considers the variations in oceanic flow conditions is essential for the realistic assessment of the thermal impact of a large nuclear power plant in a coastal or offshore region.

1. INTRODUCTION

It is already decided that the solution of the predicted energy crisis of the next decade will include the construction of new nuclear power plants and the expansion of the existing ones. The locations of some of these plants will be at coastal zones and possibly at offshore regions. In view of the observed thermal effects of some of the already operational installations and the magnitudes of the power output (e.g., 2-unit, 2400-MWe) of some proposed nuclear generating stations, the siting of these plants will require careful assessment of their possible impacts to minimize their detrimental effect on the ecology.

The fundamental reason for siting these plants in coastal regions is the assumption that large quantities of heated water from the outfalls of once-through cooling systems can be readily discharged into a limitless receiving water body where the oceanic phenomena naturally eliminate the detrimental thermal effects of

the effluents. However, as indicated in various references, for example, [1]*, this assumption may not be valid if one considers the details of the oceanic phenomena which include the tidal and geostrophic currents.

Since the net rate of surface cooling to the atmosphere is considerably less than the rate of heat input from the discharge, the thermal plume can travel large distances and it can grow into geographically significant dimensions. Hence, the entrainment water and the intake temperatures become higher than the natural ambient conditions due to the recirculation created by the oceanic flow conditions. This could invalidate the assumption about the existence of infinitely large oceanic receiving water bodies for the discharges of power plants in coastal and offshore regions (see Fig. 1).

The cyclical flow due to combined tidal and ocean currents can cause the thermal plume to remain in the vicinity of a coastal or offshore power plant, where it will be partially recirculated through the condenser to result in discharge temperatures considerably in excess of expected values without recirculation. Furthermore, and more significantly, it raises the temperature of the ambient water entrained by the near-field momentum jet; hence, it severely reduces the rapid decrease of temperature in the immediate vicinity of the discharge. This in turn increases the temperature of the far-field plume and hence the thermal impact of the power plant.

In the case where the thermal plume of a coastal or offshore power plant extends to large distances parallel or toward the shoreline, it can alter the thermal impact conditions of other power plants. Particularly, if it reaches the mouth of a large estuary, for example, the Hudson, where the cold oceanic conditions during flood flow is essential to cool the estuary, the thermal interaction could alter the existing

*Numbers in brackets represent similarly numbered references at the end of the report.

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Fig. 1. Schematic representation of power plants with possible thermal interaction in coastal and offshore regions.

thermal equilibrium and, hence, the overall thermal impact of the other power plant discharges in the estuary.

In view of the above discussions, it is evident that the assessment of the thermal impact of a large nuclear generating station in a coastal or offshore region is a critically important task. The analysis of the thermal problem must be based on valid mathematical models which incorporate the significant effects of the oceanic flow conditions.

At present, the assessment of the thermal impact of power plant discharges is almost exclusively based on near-field models: see, for example, Edinger and Polk [2], Koh and Fan [3], Motz and Benedict [4], etc., which are modified extensions of the classical, steady-state, turbulent jet studies (see Schlichting [5], Abramovich [6], and Harsha [7]). Some recent excellent studies by Stolzenbach and Harleman [8] and by Hirst [9] have presented mathematical models which can incorporate the effects of three-dimensional approximations, cross flows, and stratified ambient conditions for various discharge locations and orientations relative to the surface or bottom of the receiving water body. In a state-of-the-art study by Policastro and Tokar [10], an extensive survey of the presently available mathematical models and the discussion of their particular modeling characteristics were presented in relation to thermal discharges into large lakes.

In a later paper, Policastro and Paddock [11] compared the results of the above-referenced mathematical models in application to actual thermal discharge conditions with available experimental field data. Although this study indicated that there exists more confrontation than comparison between the results of various well-established models and experimental measurements, the majority of the above-mentioned near-field models are acceptable as valid analysis methods for a steady, initial-momentum-dominated jet problem into an infinitely large receiving body of water.

Unfortunately, the severe restrictions on physical conditions associated with the simplifying assumptions on the mathematical systems of the near-field models are frequently overlooked in their applications to various power plant discharges. The fundamental reason for the popularity of near-field models is that the governing steady-state partial differential equations constitute parabolic systems which can be solved with simple pseudo-analytical or numerical marching techniques starting from specified positions along the main flow direction of the jets. However, it should be noted that these systems are at best only valid approximations to the physically correct mathematical forms of conservation equations which are spatially elliptic under steady-state conditions. Since either analytical or numerical solution of an elliptic system is inherently more difficult than a parabolic one, a near-field jet mathematical model is usually reduced from the elliptic conservation equations by neglecting certain terms based on seemingly valid and plausible physical arguments about the magnitudes of various diffusive and convective effects. Unfortunately, the gain in mathematical simplicity is paid for by the involuntary loss of certain other physical effects which never directly appear in the reduction procedure.

As the overall consequence of its parabolic property, a near-field jet model, regardless of the level of sophistication in its mathematical formulation, can never account for its own influence on the surroundings. Furthermore, the discharge momentum of the jet is the fundamental characteristic which controls the development of the complete flow field in the near field. Indeed, for specified turbulent Reynolds stress conditions in the fluid, it uniquely determines the complete distribution of entrainment flow across the jet boundary. Consequently, the physical boundary conditions of the surrounding body of receiving water can never have any effect on the entrainment conditions of the jet. The dimensions of the near-field plume, that is, the axial distance from the discharge where the momentum of the jet becomes negligible and the width of the jet boundary, are also determined by the Reynolds stress and discharge momentum conditions. Hence, the mathematical model of the near-field jet completely determines its own

size without taking into account the dimensions of the surroundings. Finally, the near-field models with spatially parabolic systems are strictly steady-state models. The modification of these systems to transient forms directly from the steady equations is completely unacceptable, since such an approach invalidates the fundamental physical assumptions of the reduction procedure.

The above-listed restrictive characteristics of the steady, parabolic mathematical system of a near-field model are definitely acceptable for an aerodynamic jet engine discharging into a practically limitless and motionless atmosphere. However, considering the realistic physical conditions of the receiving water body in a coastal or offshore region, the same restrictions on the near-field model may be unrealistic for the assessment of the thermal impact of a power plant with a continuous large discharge rate.

First, the discharge is always into a region which is definitely limited in depth and possibly in overall volume, as in the case of a cove. Consequently, in view of the high values of heat release rate by the outfall, it is unacceptable to assume that entrainment water to the plume is never affected by the discharge.

Second, the receiving body of water in a coastal or offshore region is almost never motionless or in uniform motion; but it frequently moves with possibly periodic, meso-scale eddy or vortex-type motions due to geostrophic and tidal currents in the ocean. Since these motions could periodically return at least a portion of the heated discharge water into the vicinity of the near-field plume, the assumption that ambient entrainment water is never affected by the discharge again becomes unacceptable.

In all near-field models, dilution of the discharge water by cooler ambient entrainment flow is the major mechanism (surface cooling is approximately 10%) for the temperature drop in the plume. Since the parabolic mathematical systems of these models cannot directly incorporate the temperature rise in the ambient fluid, resulting from above-discussed physical conditions, all near-field models with specified ambient temperatures necessarily underpredict the thermal impact of a power plant. Attempts to manipulate the values of the controlling parameters, for example, entrainment coefficients, etc., to verify experimental field measurements for a few particular cases result in more confusion, since the universal applicability of the physical parameters is thereby destroyed.

Recent mathematical models by Barry and Hoffman [12], with higher-order approximations to investigate the recirculation from discharges to inlets in regions between river cross sections, and numerical treatment of the full steady-state elliptic equations by Trent [13], for vertical jets in shallow coastal waters, still cannot incorporate the transient variations of the ambient conditions in the analyses.

In conclusion, a near-field by itself is definitely inadequate for the assessment of the thermal impact of a power plant unless it is preceded or supplemented by an analysis which can account for the effect of the discharge on the ambient receiving body of water.

- From a truly environmental point of view, the far-field assessment of the thermal impact of a discharge is more important than the near-field, since it considers long-time transient changes in the natural conditions of a large region of water which must act as an intermediate sink for the continuous rate of heat release from a power plant. Unfortunately, the state-of-the-art of far-field models for coastal and offshore regions is considerably less developed than for near-field models, since they are not commonly employed for the assessment of the thermal impact of power plants.

The majority of existing studies associated with transport phenomenon in coastal and offshore regions is concerned with the problem of dispersion of matter according to the turbulent flow conditions in the ocean.

The mathematical formulations of all these models are based on the original study by Taylor [14] of the dispersion of matter in turbulent pipe flow. The results of this excellent study conclude that an instantaneous release of matter (salt), initially restricted to a volume of infinitesimal width in the axial

direction, if observed from a Lagrangian frame of reference moving with the constant, cross-section mean velocity of the fluid, diffuses along the axial direction according to an apparent axial diffusion coefficient which is approximately 200 times greater than the dominant turbulent diffusion coefficient in the radial direction. Elder [15] applied Taylor's approach to study the dispersion problem in an open channel under a limited range of turbulent flow conditions. In two later studies, Fischer [16, 17] further verified the mathematical results of Elder's study, with various improvements and modifications, by data from laboratory experiments under a wide range of turbulent flow conditions and also by data from field measurements in natural streams.

The Eulerian formulation of long-time, tidal-averaged dispersion of matter in an estuary under strictly periodic tidal flow conditions was presented by Bowden [18]. The dependence of the meso-scale apparent diffusion coefficient along the estuary on various theoretical and empirical forms of vertical variations in velocity profiles was also considered by Bowden [18, 19, 20]. These studies indicated that additional density flows resulting from stratified conditions could appreciably affect the dispersion rates in seas and estuaries under predominant tidal flow conditions. It is interesting to note that the apparent diffusion coefficient obtained by Bowden [18], although similar in form and magnitude, is significantly different, from conceptual considerations, than the one presented by Elder [15]. Although the vertical turbulent diffusion mechanism is the fundamental effect which causes the horizontal dispersion in both studies, Elder's [15] analysis is based on a Lagrangian formulation which represents short-time variations in a frame of reference moving with the mean flow velocity, along the general motion of the initial instantaneous release, whereas, Bowden's [18] result is based on an Eulerian formulation which represents long-time, tidal-averaged dispersion of matter as observed from a fixed frame of reference.

The Lagrangian formulation was extensively employed by Okubo [21, 22, 23] for the study of dispersion of matter under generalized oceanic turbulent flow conditions in unbounded regions, with dispersion coefficients for turbulent transport based on Kolmogoroff's [24] hypotheses on isotropic turbulence, turbulent energy dissipation, and the length scale of turbulence. In a later study, Okubo and Ozmidov [25] presented empirical results for the dependence of apparent coefficient of horizontal diffusion on the scale of turbulent phenomenon, based on extensive field data observed under tidal-averaged, meso-scale turbulence conditions. A survey study of the mathematical models for the dispersion of matter in the ocean and coastal regions was presented by Okubo and Pritchard [26], including the discussions and comparisons of the forms employed for the modeling of the apparent horizontal diffusion coefficients.

An application of these dispersion models to the study of an introduced contaminant in an estuary under periodically transient conditions superimposed on a net flow rate was considered by Pritchard, Okubo, and Mehr [27]. The mathematical results severely under predict concentrations for high numbers of tidal cycles, corresponding to long-time behavior, indicating possible overestimation in the modeling of the apparent longitudinal diffusion coefficient for the one-dimensional analysis of the flow conditions in the estuary.

The discussion of various other aspects of the dispersion problems in oceans, estuaries, and coastal regions, for example, wind and wave effects, density currents, etc., are presented in texts by Ippen [28] and Wiegand [29] for further consideration.

A Lagrangian mathematical model for the dispersion of an initial instantaneous release of matter cannot be readily modified, as suggested by Okubo [23], for the far-field analysis of the general continuous thermal discharge problems. The proposition about approximating a continuous discharge condition by successive intermittent "patches" with initial instantaneous levels of thermal energy, with each "patch"

individually undergoing a cooling process according to Gaussian-type solutions of the dispersion equation in unbounded regions with apparent horizontal diffusion coefficients determined from meso-scale turbulent conditions, is not acceptable as a valid far-field model under periodic tidal flow conditions in shallow (less than 40 ft) coastal regions.

The superposition concept for determining the excess temperature at a point by summing the local excess temperatures of all the overlapping individual "patches" which contain the point is approximately acceptable, provided surface heat transfer is negligible, since it is fundamentally based on the principle of conservation of matter or energy in an infinitely large region with impermeable and insulated boundary. It is not necessarily based on the linear mathematical property of the governing partial differential equations associated with the dispersion models. However, the formal method is not acceptable as a valid far-field model for general thermal discharges because of the two fundamental reasons inherent to its formulation.

First, the apparent horizontal diffusion coefficients employed in the solution of the dispersion equation in the Lagrangian frame of reference of each moving "patch" is not valid for continuous thermal discharges under general flow conditions, particularly with periodic characteristics, in a coastal region. The mathematical problems considered by Taylor [14], Elder [15], and Fischer [16, 17] for the dispersion of an instantaneous release lead to apparent longitudinal diffusion coefficients in a Lagrangian frame of reference moving with the cross section mean velocity in pipes and open channels. In the mathematical derivation of these apparent diffusion coefficients from a system which considers the vertical and/or lateral diffusion fluxes directly and the longitudinal one only indirectly, it is essential to make certain restrictive assumptions as instantaneous release, and steady and longitudinally uniform turbulent velocity distribution for the only component along the axial direction. Since in a coastal region the flow conditions are definitely transient and at least two-dimensional with nonuniform distributions varying with the bottom topography, the necessary assumptions inherent to the development of these theoretical coefficients are not valid.

Kolmogoroff-type [24] dispersion coefficients, which are indeed valid for instantaneous release problems in unbounded regions in the ocean and atmosphere, are also not acceptable in coastal regions, since two fundamental hypothesis of the Kolmogoroff's model for the length scale of the turbulence phenomena are not valid in the shallow-water regions near the shoreline.

Second, the Gaussian-type distributions in the "patches" are associated with the solutions of the dispersion equation, which considers strictly transportive effects without any convection, for instantaneous releases in completely unbounded regions. The necessary boundary condition which must be imposed for these solutions is that at infinitely large distances from the center of the "patch" the conditions tend to be ambient in the ocean. In the important shallow-water regions near the shoreline this boundary condition, which is inherent to the Gaussian distribution solutions, is definitely unrealistic, since the shoreline actually represents an impermeable and insulated boundary with conditions definitely not ambient. Hence the Gaussian-type distributions in the Lagrangian coordinates of the "patches" are not valid in regions near the shoreline.

Hence, a Lagrangian mathematical model for the dispersion of an instantaneous release of matter in unbounded ocean regions cannot be modified, as proposed by Okubo [23], to develop a valid far-field mathematical model for continuous thermal discharges in coastal regions under general transient flow conditions.

The assessment of the environmental impact of a power plant in a coastal region requires accurate information about the temperature distributions in the vicinity of the discharges and in the shallow water regions near the shoreline where its effect on the ecology can be detrimental. Furthermore, additional information may also be required about the duration of the maximum thermal conditions in the region, controlled by various combinations of tidal and geostrophic currents, to determine the behavioral

characteristics of certain aquatic organisms under short-time elevated-temperature conditions. Hence, an acceptable far-field model for the assessment of the thermal impact of power plants in coastal and offshore regions must be transient and at least two-dimensional, and it must fully incorporate both convection and turbulent transport of thermal energy in the mathematical formulation of its governing system of equations. There exists in the literature various thermal analysis models which satisfy the above criteria.

The early water-quality models developed by Thomann [30] and O'Connor [31] are based on the simple application of constituent mass conservation principles to the "segments" or "boxes" representing various parts of estuaries and bay areas. Numerous successful applications of these and other conceptually similar mathematical models are extensively discussed by Ward and Espey in a survey report [32]. Two fundamental weaknesses of these Eulerian far-field models are their simplistic formulation of the two-dimensional turbulent transport fluxes and their lack of rigorous mathematical consideration of the stability and accuracy of their solutions.

The general water-quality simulation model presented by Leendertse [33], and its associated computer codes developed by Leendertse and Gritton [34, 35], is based on rigorous finite difference formulations of the differential equations for continuity, momentum, energy, and constituent concentrations; and it is capable of simultaneously obtaining solutions for the complete transient flow fields and temperature distributions in two-dimensional regions. The details of the numerical methods employed in this model are also discussed in the survey report by Ward and Espey [32]. The thermal model developed by Sonnichsen and Kolesar [36, 37] only considers the finite difference formulation of the energy equation for specified two-dimensional flow and constant depth conditions. The general mathematical modeling of heated discharges, including the momentum effects of the near-field jets, was extensively studied by Wada [38, 39] (see also [10] for discussions) based on finite difference formulations which also considered the case of two-layer conditions existing in severely stratified seas.

These models can be employed, with proper modifications, for the assessment of the thermal impact of power plants in coastal and offshore regions. However, their classical finite difference formulations with constant grid size become computationally inefficient for obtaining accurate temperature solutions in only small, specified subregions of a large coastal region with open boundaries.

A recent study by Loznuk, Anderson, and Belytschko [40] employs a finite-element method for the solution of transient temperature distributions, under specified, steady, circulating flow conditions, in a small lake used as a cooling pond for several power plants. Their results agree very well with the field-measured temperatures at various locations in the lake. Although the finite-element formulation of this study is very efficient for computations in small regions with closed boundary and steady flow conditions, it cannot be readily employed in large coastal regions with open boundaries and transient, periodic tidal flow conditions.

The detailed solution of the flow field created by tidal, geostrophic, and wind currents in a coastal or offshore region with complicated shoreline and bottom geometry is extremely difficult and usually prohibitive from computational cost considerations. Although there exists recent studies by Bryan [41] and Bryan and Cox [42] in numerical modeling of ocean currents, from thermal impact considerations, the solution of the hydrodynamic problem is not essential, since transient depth and velocity data can be obtained in a coastal or offshore region before the construction of a power plant. Hence, in the far-field analysis of the thermal impact, the field-measured data for the oceanic currents can be directly incorporated in the mathematical model, since the momentum of the intakes and the discharges cannot influence the flow field except in the near-field zone, in the immediate vicinity of the outfall structures.

This study presents the rigorous mathematical development of a computationally efficient numerical simulation model for predicting transient, two-dimensional, far-field temperature distributions which result

from the condenser cooling water discharges of power plants in large coastal and offshore regions with completely arbitrary shoreline and open boundaries, and under specified transient depth and oceanic flow conditions.

2. MATHEMATICAL FORMULATION OF GOVERNING EQUATIONS

2.1 General Discussion of Numerical Simulation Model

The formulation of the present model is based on the control volume approach, which highlights the direct numerical simulation process without necessitating continuum limit differential equations for conservation principles. It is best exemplified by the famous computer codes developed at Los Alamos Scientific Laboratory by Evans and Harlow [43] for PIC (particle-in-cell), and by Gentry, Martin, and Daly [44] for FLIC (fluid-in-cell) formulations for transient fluid flow problems. The details of these Eulerian methods and their numerous successful applications are available in reports by Harlow [45], Harlow and Amsden [46, 47], and in a recent excellent text in computational fluid mechanics by Roache [48].

Both PIC and FLIC methods directly formulate the necessary computational system of equations by applying the macroscopic conservation forms of mass momentum and energy principles for a cell of rectangular enclosure surfaces by assuming uniform values over the surface areas. The flow region is divided into a sufficient number of cells which span the flow region; and the computation procedure is based on the approximate satisfaction of the conservation equations for each cell at every time step.

For the present formulation of the far-field thermal model, equal cell division will not be computationally practical. The model requires high resolution in the vicinity of the discharges, and possibly in shallow water regions near the shoreline, to predict accurate temperatures for the assessment of the ecological impact. The analysis must also consider sufficiently large areas in the coastal regions to include the growth and travel of the thermal plume under oceanic currents and periodic tidal flow conditions. Consequently, if high resolution, equal cell division is employed throughout the region, there will be an excessive number of computational cells which will require excessive storage and computer time for the model to be of practical use. On the other hand, if the number of cells is decreased for computational efficiency, the results will not be sufficiently accurate in the vicinity of the discharges for the model to be a reliable assessment method for the thermal impact of power plants.

The "segment" or "box" water-quality models suggested by Thomann [30] and O'Connor [31] are indeed conceptually similar to the FLIC method; and they are extremely efficient from computational considerations for the solution of the constituent conservation equations in large flow regions in estuaries and coastal bays.

In the present formulation, the fundamental concepts of the FLIC method and "segment" or "box" models are combined to develop a computationally efficient and mathematically rigorous numerical simulation model for the transient far-field analysis of thermal discharges in coastal and offshore regions. The formulation is based on the relaxation of the conditions of the FLIC method in its application of the macroscopic, integral forms of the conservation principles to the equal-sized computational cells. The two necessary modifications are the following:

1. Divide the flow region into arbitrarily sized discrete elements, preferably with rectangular enclosure surfaces except at the boundaries, such that the finite number of discrete elements completely spans the region.
2. Employ the integral forms of the physical conservation laws to each discrete element, without assuming uniform values for flow properties over the surface areas; and integrate the necessary volume and surface integrals in formulating the governing equations.

This new advanced version of the FLC method will be referred to as the FLIDE (fluid-in-discrete-element) method in the subsequent development of the far-field thermal model.

In view of the complete generality of the FLIDE method in selecting the size distribution of the discrete elements, any specified coastal and/or offshore region of interest can be divided into subregions with different number and size of discrete elements. Hence, large numbers of small discrete elements can be specified in the vicinities of the discharges for high resolution in the solutions; and at the same time large areas of the flow region can be covered by a few large discrete elements to include the areas of the growth and travel of the far-field plumes in the analyses. As an example, the coastal and offshore region as shown in Fig. 1 can be divided into 12 subregions and approximately 500 discrete elements, with different numbers and different size distributions in different subregions, as shown in Fig. 2. It should be stated that the development of the mathematical model does not require the definitions of subregions. However, for geometrically complicated shorelines, for example, with numerous coves, islands, etc., the computational bookkeeping can be extremely simplified by appropriate definitions of various subregions in the analysis.

In Fig. 2, the intake and discharge locations of the power plants are represented according to the particular discrete elements which contain them. Consequently, the far-field thermal model cannot specify the locations of intakes and discharges with more accuracy than the dimensions of their associated discrete elements.

Fig. 2. Schematic representation of the division of a coastal or offshore region into subregions and discrete elements according to the FLIDE method.

2.2 Integral Forms of Conservation Principles

As discussed in the previous section, the mathematical development of the far-field thermal model will be based on the FLIDE numerical simulation method, which employs the integral forms of the physical conservation principles. Since the model will consider known depth and velocity field data, the momentum equation need not be considered in the analysis. Hence, conservation of mass and conservation of energy are the necessary and sufficient physical principles for the formulation of the mathematical system.

Conservation of Mass (Auxiliary Equation):

$$\frac{\partial}{\partial t} \iiint_V \rho \, dV + \iint_A \rho \mathbf{V} \cdot \mathbf{n} \, dA = 0. \quad (1)$$

Conservation of Energy (Principal Equation):

$$\begin{aligned} \frac{\partial}{\partial t} \iiint_V \rho e \, dV + \iint_A \rho e \mathbf{V} \cdot \mathbf{n} \, dA = & \iiint_V \dot{q}_v \, dV \\ & + \iiint_V \mathbf{f}_b \cdot \mathbf{V} \, dV + \iint_A \mathbf{q} \cdot (-\mathbf{n}) \, dA + \iint_A \boldsymbol{\sigma} \cdot \mathbf{V} \, dA. \end{aligned} \quad (2)$$

The density of the fluid can readily be assumed as constant, since the vertical variations due to stratification are only appreciable in the vertical component of the momentum equation, which is not included in the analysis. Hence, the conservation of mass equation (1) can be simplified as

$$\frac{\partial \mathbf{V}}{\partial t} + \iint_A \mathbf{V} \cdot \mathbf{n} \, dA = 0. \quad (3)$$

Since the vertical component of velocity resulting from the periodic tidal variations of depth is definitely negligible in comparison to the horizontal velocity components caused by the oceanic flow conditions, the velocity vector can be assumed as two dimensional; that is,

$$\mathbf{V} = \mathbf{V}(t, x, y, z) = u(t, x, y, z) \mathbf{i} + v(t, x, y, z) \mathbf{j}. \quad (4)$$

Since the energy contribution of the flow work due to Coriolis and gravitational body forces is definitely negligible in comparison to the thermal energy transport effects,

$$\iiint_V \mathbf{f}_b \cdot \mathbf{V} \, dV = 0. \quad (5)$$

The contribution of viscous stresses to flow work is definitely negligible. Hence, the surface force $\boldsymbol{\sigma}$ can be approximated by pressure force. Consequently, the surface work integral becomes

$$\iint_A \boldsymbol{\sigma} \cdot \mathbf{V} \, dA = \iint_A P(-\mathbf{n}) \cdot \mathbf{V} \, dA = - \iint_A P \mathbf{V} \cdot \mathbf{n} \, dA. \quad (6)$$

Substituting the simplifying assumptions, Eqs. (5) and (6), in Eq. (2) and rearranging, the energy conservation equation becomes

$$\frac{\partial}{\partial t} \iiint_V e \, dV + \iint_A \left(e + \frac{P}{\rho} \right) \mathbf{V} \cdot \mathbf{n} \, dA = \frac{1}{\rho} \iiint_V \dot{q}_v \, dV + \frac{1}{\rho} \iint_A \mathbf{q} \cdot (-\mathbf{n}) \, dA. \quad (7)$$

The contribution of kinetic energy to the specific internal energy e of the fluid is definitely negligible in comparison with thermal energy; hence,

$$e = e_0^* + \int_{T_0}^T C_v \, dT; \quad h = e + \frac{P}{\rho} = h_0^* + \int_{T_0}^T C_p \, dT. \quad (8)$$

where e_0^* and h_0^* are internal energy and enthalpy of formation at arbitrarily assigned reference temperature T_0^* .

For the incompressible fluid, the specific heats at constant volume and pressure are equal; that is,

$$C_v = C_p = C^*, \quad e = (e_0^* - C^* T_0^*) + C^* T, \quad h = (h_0^* - C^* T_0^*) + C^* T. \quad (9)$$

Since the reference temperature T_0^* is arbitrarily assigned, the first terms in the definitions of specific internal energy and enthalpy can be eliminated. Hence, specific enthalpy and internal energy can be expressed as

$$e = h = C^* T. \quad (10)$$

Substituting Eq. (10) in Eq. (7) and rearranging, the temperature form of the energy equation becomes

$$\frac{\partial}{\partial t} \iiint_V T \, dV + \iint_A TV \cdot \mathbf{n} \, dA = \frac{1}{\rho C^*} \iiint_V \dot{q}_v \, dV + \frac{1}{\rho C^*} \iint_A \mathbf{q} \cdot (-\mathbf{n}) \, dA. \quad (11)$$

where $\dot{q}_v$ represents the rate of generation of energy per unit volume, and $\mathbf{q}$ is the general enclosure surface heat flux vector consisting of molecular diffusion in the horizontal directions in the fluid and rates of heat transfer from the ocean surface and the bottom area of the control volume.

It should be immediately noted that since the flow field is predominantly turbulent, the velocity vector $\mathbf{V}$ and temperature T are instantaneous quantities which include the contributions of random turbulent fluctuations. Furthermore, since the model is two-dimensional, the velocity components and temperature must be averaged over depth in the vertical direction, based on the appropriate integration of volume and surface integrals in Eqs. (3) and (11). Hence, there are two averaging processes associated with the formulation. The first one is the mean time averaging of the flow properties to separate the turbulent fluctuations; and the second one is the spatial depth averaging to reduce the mathematical system into two dimensions.

At this point, considering that ultimately two averaging processes must be employed, each flow property can be separated into three parts:

$$\begin{aligned} u(t, x, y, z) &= \bar{U}(t, x, y) + \bar{u}^*(t, x, y, z) + u'(t, x, y, z), \\ v(t, x, y, z) &= \bar{V}(t, x, y) + \bar{v}^*(t, x, y, z) + v'(t, x, y, z), \\ T(t, x, y, z) &= \bar{T}(t, x, y) + \bar{T}^*(t, x, y, z) + T'(t, x, y, z). \end{aligned} \quad (12)$$

where

$$\left. \begin{array}{l} \bar{U}(t, x, y) \\ \bar{V}(t, x, y) \\ \bar{T}(t, x, y) \end{array} \right\} = \text{mean time averaged and depth averaged principal variables of the formulation,}$$

$$\left. \begin{array}{l} \bar{u}^*(t, x, y, z) \\ \bar{v}^*(t, x, y, z) \\ \bar{T}^*(t, x, y, z) \end{array} \right\} = \text{differences between actual mean time averaged vertical distributions and depth averaged values of principal variables,}$$

$$\left. \begin{array}{l} u'(t, x, y, z) \\ v'(t, x, y, z) \\ T'(t, x, y, z) \end{array} \right\} = \text{contributions of random turbulent fluctuations.}$$

A schematic representation of the terms defined in Eq. (12) is given in Fig. 3 for the x direction.

The definition of the terms in Eq. (12) is slightly different from the commonly employed form in turbulent analyses. The first two terms, consisting of the mean time averaged and depth averaged principal variable and the difference between actual mean time averaged vertical distribution and depth averaged value, represent the classical form of the mean time averaged quantity. Hence, the present formulation is equivalent to the classical form used in turbulent analysis.

Fig. 3. Schematic representation of the parts of the flow variables in the x direction.

As consequences of their definitions in Eq. (12), the turbulent fluctuations and the mean time averaged differences obey the following relations for time and space averaging:

$$\begin{aligned}\overline{u(t, x, y, z)} &= \overline{U(t, x, y)} + \overline{u^*(t, x, y, z)}, & \overline{u^*(t, x, y, z)} &= 0, \\ \overline{v(t, x, y, z)} &= \overline{V(t, x, y)} + \overline{v^*(t, x, y, z)}, & \overline{v^*(t, x, y, z)} &= 0, \\ \overline{T(t, x, y, z)} &= \overline{T(t, x, y)} + \overline{T^*(t, x, y, z)}, & \overline{T^*(t, x, y, z)} &= 0.\end{aligned}\quad (13)$$

$$\int_0^{\bar{D}} \overline{u^*(t, x, y, z)} dz = \int_0^{\bar{D}} \overline{v^*(t, x, y, z)} dz = \int_0^{\bar{D}} \overline{T^*(t, x, y, z)} dz = 0, \quad (14)$$

where depth $\bar{D}$ is independent of turbulent fluctuations in the flow field.

The final integral forms of the conservation equations, (3) and (11), can also be employed in mean time averaged forms, since they are macroscopic representations of the physical laws. Hence, applying the mean time averaging on the equations and noting the distributive property of the operation, Eqs. (3) and (11) become

$$\frac{\partial \overline{V}}{\partial t} + \overline{\iint_A \mathbf{V} \cdot \mathbf{n} dA} = 0, \quad (15)$$

$$\frac{\partial}{\partial t} \overline{\iiint_V T dV} + \overline{\iiint_A T \mathbf{V} \cdot \mathbf{n} dA} = \frac{1}{\rho C^*} \overline{\iiint_V \dot{q}_v dV} + \frac{1}{\rho C^*} \overline{\iint_A \mathbf{q} \cdot (-\mathbf{n}) dA}. \quad (16)$$

It should be noted that time derivatives were taken out of the mean time averaging of the equations. This simplification is based on the fundamental assumption about the comparison of the large time scales of the mean time averaged phenomena and the small time scales of random turbulent fluctuations.

2.3 Formulation of FLIDE Equations

At this point, the standard FLIC method [44, 46, 47] employs the integral forms of the conservation equations on the control volume cells of equal size throughout the flow region by assuming uniform values for flow properties on all rectangular enclosure surfaces of the cells.

In the formulation of the two-dimensional FLIDE method, the discrete elements, which replace the cells of the FLIC method, are generally of unequal size in all subregions of the flow region, as shown in Fig. 4. Since the formulation is strictly Eulerian, the size of each discrete element remains constant at all times. However, the depth of the fluid in each element can vary with time according to physical conservation principles.

Each discrete element has a constant cross-sectional area normal to the vertical direction z with dimensions $\Delta x_{i,j}^{(1)}$ and $\Delta y_{i,j}^{(1)}$ along $x^{(1)}$ and $y^{(1)}$ respectively. The subscripts i, j represent the discrete element position in the $x^{(1)}, y^{(1)}$ plane of the subregion (r) . The flow properties, for example, temperature $T_{i,j}^{(1)}$, associated with the discrete element are assumed to be evaluated at the centroid of the cross-sectional area, which represents the mean values of the properties over the discrete elements. The common boundaries between two adjacent discrete elements are represented by half-point subscripts, for example, $(i + \frac{1}{2}, j)$, for the boundary between the elements (i, j) and $(i + 1, j)$, and the fluid properties are assumed to be uniform along the boundary lines in any horizontal plane, but definitely varying in the vertical direction according to the turbulent distributions given by Eq. (12) and Fig. 3.

Fig. 4. Geometrical configuration of discrete elements for the FLIDE method.

Fig. 5. Schematic representation of the flow properties for a discrete element.

With the control volume V and the enclosure surface A well defined for each discrete element, the integrals of the macroscopic conservation equations, (15) and (16), can be evaluated in terms of the flow properties inside and on the boundaries of the element (see Fig. 5), to obtain the necessary numerical simulation system of equations for the FLIDE method.

Considering the auxiliary mass conservation principle, Eq. (15), for an arbitrary discrete element (i, j) in subregion (r) , and evaluating the terms in the equation,

$$\frac{\partial \bar{V}}{\partial t} = \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \frac{\partial \bar{D}_{i,j}^{(r)}}{\partial t} \quad (17)$$

Noting that the unit normal vectors $\mathbf{n}$ are constants on all faces of the element,

$$\begin{aligned} \iint_A \bar{V} \cdot \mathbf{n} \, dA &= \iint_A \bar{V} \cdot \mathbf{n} \, dA = -\Delta y_{i,j}^{(r)} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \bar{u}_{i-1/2,j}^{(r)} \, dz \\ &+ \Delta y_{i+1/2,j}^{(r)} \int_0^{\bar{D}_{i+1/2,j}^{(r)}} \bar{u}_{i+1/2,j}^{(r)} \, dz - \Delta x_{i,j-1/2}^{(r)} \int_0^{\bar{D}_{i,j-1/2}^{(r)}} \bar{v}_{i,j-1/2}^{(r)} \, dz + \Delta x_{i,j+1/2}^{(r)} \int_0^{\bar{D}_{i,j+1/2}^{(r)}} \bar{v}_{i,j+1/2}^{(r)} \, dz \\ &- \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[(\hat{g}_b^*)_{i,j}^{(r)} + (\hat{g}_s^*)_{i,j}^{(r)} \right] - \dot{G}_m \delta_{i,j,m}^{(r)} - \dot{G}_n \delta_{i,j,n}^{(r)} \quad (18) \end{aligned}$$

where

$(\hat{g}_b^*)_{i,j}^{(r)}$ = volumetric flux per unit area (velocity) of fluid inflow from the bottom.

$(\hat{g}_s^*)_{i,j}^{(r)}$ = volumetric flux per unit area (velocity) of fluid inflow on the ocean surface (e.g. rainfall).

$\dot{G}_m$ = volumetric flow rate of intake m ,

$\dot{G}_n$ = volumetric flow rate of discharge n .

(19)

$\delta_{i,j,m}^{(r)} = \begin{cases} 1: & \text{intake } m \text{ in element } (i, j) \text{ in subregion } (r), \\ 0: & \text{intake } m \text{ not in element } (i, j) \text{ in subregion } (r). \end{cases}$

$\delta_{i,j,n}^{(r)} = \begin{cases} 1: & \text{discharge } n \text{ in element } (i, j) \text{ in subregion } (r), \\ 0: & \text{discharge } n \text{ not in element } (i, j) \text{ in subregion } (r). \end{cases}$

Substituting the turbulent distributions along the vertical direction from Eq. (12) and employing the distributive property of mean time averaging with constraint Eqs. (13) and (14), the integrals in Eq. (18) become

$$\begin{aligned} \int_0^{\bar{D}_{i+1/2,j}^{(r)}} \bar{u}_{i+1/2,j}^{(r)} \, dz &= \int_0^{\bar{D}_{i+1/2,j}^{(r)}} \left[\overline{u_{i+1/2,j}^{(r)}} + \overline{u_{i+1/2,j}^{(r)}} + \overline{u_{i+1/2,j}^{(r)}} \right] \, dz \\ &= \bar{D}_{i+1/2,j}^{(r)} \bar{U}_{i+1/2,j}^{(r)} \quad (20) \end{aligned}$$

$$\begin{aligned} \int_0^{\bar{D}_{i,j+1/2}^{(r)}} \bar{v}_{i,j+1/2}^{(r)} \, dz &= \int_0^{\bar{D}_{i,j+1/2}^{(r)}} \left[\overline{v_{i,j+1/2}^{(r)}} + \overline{v_{i,j+1/2}^{(r)}} + \overline{v_{i,j+1/2}^{(r)}} \right] \, dz \\ &= \bar{D}_{i,j+1/2}^{(r)} \bar{V}_{i,j+1/2}^{(r)} \quad (21) \end{aligned}$$

Substituting Eqs. (20) and (21) in Eq. (18), and the resulting equation and Eq. (17) in Eq. (15) and rearranging, the mass conservation equation for the discrete element becomes

$$\begin{aligned} \frac{\partial \bar{D}_{i,j}^{(r)}}{\partial t} = \frac{1}{\Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)}} & \left\{ \left[\Delta y_{i-1/2,j}^{(r)} \bar{D}_{i-1/2,j}^{(r)} \bar{U}_{i-1/2,j}^{(r)} - \Delta y_{i+1/2,j}^{(r)} \bar{D}_{i+1/2,j}^{(r)} \bar{U}_{i+1/2,j}^{(r)} \right] \right. \\ & \left. + \left[\Delta x_{i,j-1/2}^{(r)} \bar{D}_{i,j-1/2}^{(r)} \bar{V}_{i,j-1/2}^{(r)} - \Delta x_{i,j+1/2}^{(r)} \bar{D}_{i,j+1/2}^{(r)} \bar{V}_{i,j+1/2}^{(r)} \right] \right\} \\ & + \frac{1}{\Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)}} \left[\bar{G}_m \delta_{i,j,m}^{(r)} + \bar{G}_n \delta_{i,j,n}^{(r)} \right] + (\bar{g}_s^*)_{i,j}^{(r)} + (\bar{g}_s^*)_{i,j}^{(r)}. \end{aligned} \quad (22)$$

Considering the principal energy conservation equation, (16), for the arbitrary discrete element (i,j) in subregion (r), and evaluating the terms in the equation,

$$\frac{\partial}{\partial t} \iiint_V T dV = \frac{\partial}{\partial t} \left[\Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \int_0^{\bar{D}_{i,j}^{(r)}} \bar{T}_{i,j}^{(r)} dz \right]. \quad (23)$$

Substituting the turbulent distributions along the vertical direction from Eq. (12) and employing the distributive property of mean time averaging with constraint Eqs. (13) and (14), the integral in Eq. (23) becomes

$$\int_0^{\bar{D}_{i,j}^{(r)}} \bar{T}_{i,j}^{(r)} dz = \int_0^{\bar{D}_{i,j}^{(r)}} \left[\bar{T}_{i,j}^{(r)} + \bar{T}_{i,j}^{(r)} + \bar{T}_{i,j}^{(r)} \right] dz = \bar{D}_{i,j}^{(r)} \bar{T}_{i,j}^{(r)}. \quad (24)$$

Substituting Eq. (24) in Eq. (23) and rearranging,

$$\begin{aligned} \frac{\partial}{\partial t} \iiint_V T dV &= \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \frac{\partial}{\partial t} \left[\bar{D}_{i,j}^{(r)} \bar{T}_{i,j}^{(r)} \right] \\ &= \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[\bar{D}_{i,j}^{(r)} \frac{\partial \bar{T}_{i,j}^{(r)}}{\partial t} + \bar{T}_{i,j}^{(r)} \frac{\partial \bar{D}_{i,j}^{(r)}}{\partial t} \right]. \end{aligned} \quad (25)$$

$$\begin{aligned} \iint_A \overline{TV \cdot n} dA &= \iint_A \bar{T} \bar{V} \cdot n dA \\ &= \Delta y_{i-1/2,j}^{(r)} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \bar{T}_{i-1/2,j}^{(r)} \bar{u}_{i-1/2,j}^{(r)} dz \\ &+ \Delta y_{i+1/2,j}^{(r)} \int_0^{\bar{D}_{i+1/2,j}^{(r)}} \bar{T}_{i+1/2,j}^{(r)} \bar{u}_{i+1/2,j}^{(r)} dz \\ &- \Delta x_{i,j-1/2}^{(r)} \int_0^{\bar{D}_{i,j-1/2}^{(r)}} \bar{T}_{i,j-1/2}^{(r)} \bar{v}_{i,j-1/2}^{(r)} dz \\ &- \Delta x_{i,j+1/2}^{(r)} \int_0^{\bar{D}_{i,j+1/2}^{(r)}} \bar{T}_{i,j+1/2}^{(r)} \bar{v}_{i,j+1/2}^{(r)} dz \\ &- \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[(\bar{g}_s^*)_{i,j}^{(r)} (\bar{T}_s^*)_{i,j}^{(r)} + (\bar{g}_s^*)_{i,j}^{(r)} (\bar{T}_s^*)_{i,j}^{(r)} \right] - \bar{G}_m \delta_{i,j,m}^{(r)} \bar{T}_m - \bar{G}_n \delta_{i,j,n}^{(r)} \bar{T}_n. \end{aligned} \quad (26)$$

where

$(T_{b,i,j}^{(r)})$ = temperature of the fluid inflow from the bottom.

$(T_{s,i,j}^{(r)})$ = temperature of the fluid inflow on the ocean surface (e.g., rainfall).

T_m = temperature of the fluid at the intake m (e.g., local temperature).

T_n = temperature of the discharge n .

(27)

The evaluation of the integrals in Eq. (26) requires special attention, since the integrands contain product forms of instantaneous quantities. Substituting the turbulent distributions along the vertical direction from Eq. (12), the first two integrals in Eq. (26) become

$$\begin{aligned} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)} u_{i-1/2,j}^{(r)}} dz \\ = \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \left[\overline{T_{i-1/2,j}^{(r)}} + \overline{T_{i-1/2,j}^{(r)}} + \overline{T_{i-1/2,j}^{(r)}} \right] \cdot \left[\overline{U_{i-1/2,j}^{(r)}} + \overline{u_{i-1/2,j}^{(r)}} + \overline{u_{i-1/2,j}^{(r)}} \right] dz. \end{aligned} \quad (28)$$

where the bar representing the mean time averaging is over the complete integrand in the equation. Expanding the integrand in Eq. (28) and employing the distributive property of mean time averaging,

$$\begin{aligned} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)} u_{i-1/2,j}^{(r)}} dz &= \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{U_{i-1/2,j}^{(r)}} dz \\ &+ \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz \\ &+ \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{U_{i-1/2,j}^{(r)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz \\ &+ \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{U_{i-1/2,j}^{(r)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz \\ &+ \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{U_{i-1/2,j}^{(r)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz. \end{aligned} \quad (29)$$

Rearranging the integrals according to the definitions of the parts of the instantaneous quantities,

$$\begin{aligned} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)} u_{i-1/2,j}^{(r)}} dz &= \bar{D}_{i-1/2,j}^{(r)} \overline{T_{i-1/2,j}^{(r)}} \overline{U_{i-1/2,j}^{(r)}} \\ &+ \overline{T_{i-1/2,j}^{(r)}} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz + \overline{T_{i-1/2,j}^{(r)}} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz \\ &+ \overline{U_{i-1/2,j}^{(r)}} \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(r)}} \overline{T_{i-1/2,j}^{(r)}} \overline{u_{i-1/2,j}^{(r)}} dz \end{aligned}$$

$$\begin{aligned}
& + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \overline{u_{i-1/2,j}^{(n)}} dz + \bar{U}_{i-1/2,j}^{(n)} \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} dz \\
& + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{u}_{i-1/2,j}^{(n)} \overline{T_{i-1/2,j}^{(n)}} dz + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \overline{u_{i-1/2,j}^{(n)}} dz. \quad (30)
\end{aligned}$$

Employing the constraint conditions, Eqs. (13) and (14), six of the integrals vanish in Eq. (30); hence,

$$\begin{aligned}
\int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \overline{u_{i-1/2,j}^{(n)}} dz & = \bar{D}_{i-1/2,j}^{(n)} \bar{U}_{i-1/2,j}^{(n)} \bar{T}_{i-1/2,j}^{(n)} \\
& + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \bar{u}_{i-1/2,j}^{(n)} dz + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \overline{u_{i-1/2,j}^{(n)}} dz. \quad (31)
\end{aligned}$$

Similarly, the last two integrals in Eq. (26) become

$$\begin{aligned}
\int_0^{\bar{D}_{i,j-1/2}^{(n)}} \bar{T}_{i,j-1/2}^{(n)} \overline{v_{i,j-1/2}^{(n)}} dz & = \bar{D}_{i,j-1/2}^{(n)} \bar{V}_{i,j-1/2}^{(n)} \bar{T}_{i,j-1/2}^{(n)} \\
& + \int_0^{\bar{D}_{i,j-1/2}^{(n)}} \bar{T}_{i,j-1/2}^{(n)} \bar{v}_{i,j-1/2}^{(n)} dz + \int_0^{\bar{D}_{i,j-1/2}^{(n)}} \bar{T}_{i,j-1/2}^{(n)} \overline{v_{i,j-1/2}^{(n)}} dz. \quad (32)
\end{aligned}$$

The integrals in Eqs. (31) and (32) are extremely important, since they represent the overall thermal energy transfer across the surfaces of the discrete element. The dominant turbulent transport phenomenon is included in the definitions of the integrals according to mean time averaged products of random turbulent fluctuations.

The proper evaluation of these integrals requires the knowledge of vertical distributions of turbulent velocity and temperature profiles. This important problem will be considered in detail later in the development of the mathematical model.

At this point, the integrals can be represented as transport flux terms; that is,

$$(\bar{q}_T^{(n)})_{i-1/2,j} = \frac{1}{\bar{D}_{i-1/2,j}^{(n)}} \left[\int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \bar{u}_{i-1/2,j}^{(n)} dz + \int_0^{\bar{D}_{i-1/2,j}^{(n)}} \bar{T}_{i-1/2,j}^{(n)} \overline{u_{i-1/2,j}^{(n)}} dz \right]. \quad (33)$$

$$(\bar{q}_T^{(n)})_{i,j-1/2} = \frac{1}{\bar{D}_{i,j-1/2}^{(n)}} \left[\int_0^{\bar{D}_{i,j-1/2}^{(n)}} \bar{T}_{i,j-1/2}^{(n)} \bar{v}_{i,j-1/2}^{(n)} dz + \int_0^{\bar{D}_{i,j-1/2}^{(n)}} \bar{T}_{i,j-1/2}^{(n)} \overline{v_{i,j-1/2}^{(n)}} dz \right].$$

Hence, substituting Eq. (33) in Eqs. (31) and (32), and the resulting equation in Eq. (26),

$$\begin{aligned}
\iint_A \overline{TV \cdot n} dA & = -\Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{U}_{i-1/2,j}^{(n)} \bar{T}_{i-1/2,j}^{(n)} \\
& + \Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{U}_{i-1/2,j}^{(n)} \bar{T}_{i-1/2,j}^{(n)} \\
& - \Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{V}_{i,j-1/2}^{(n)} \bar{T}_{i,j-1/2}^{(n)} \\
& + \Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{V}_{i,j-1/2}^{(n)} \bar{T}_{i,j-1/2}^{(n)} \\
& \Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} (\bar{q}_T^{(n)})_{i-1/2,j} + \Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} (\bar{q}_T^{(n)})_{i-1/2,j}
\end{aligned}$$

$$\begin{aligned}
& - \Delta x_{i,j-1/2}^{(r)} \bar{D}_{i,j-1/2}^{(r)} (\bar{q}_T^{(r)} f_{i,j-1/2}^{(r)} + \Delta x_{i,j+1/2}^{(r)} \bar{D}_{i,j+1/2}^{(r)} (\bar{q}_T^{(r)} f_{i,j+1/2}^{(r)} \\
& - \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[(\bar{q}_b^{(r)} f_{i,j}^{(r)} (T_b^{(r)} f_{i,j}^{(r)} + (\bar{q}_s^{(r)} f_{i,j}^{(r)} (T_s^{(r)} f_{i,j}^{(r)} \right] \\
& - \dot{G}_m \delta_{i,j,m}^{(r)} T_m - \dot{G}_a \delta_{i,j,a}^{(r)} T_a. \quad (34)
\end{aligned}$$

Evaluating the integrals on the right-hand side of Eq. (16),

$$\iiint_V \dot{q}_v dV = \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \bar{D}_{i,j}^{(r)} (\dot{q}_v f_{i,j}^{(r)}). \quad (35)$$

where the rate of heat generation per unit volume $\dot{q}_v$ is assumed to be uniform in depth.

The heat flux vector $\mathbf{q}$ consists of molecular diffusion, which depends linearly on temperature gradients in the horizontal planes and the heat transfer effects from the ocean surface and the bottom. Hence, the last integral in Eq. (16) becomes

$$\begin{aligned}
\iint_A \mathbf{q} \cdot (-\mathbf{n}) dA &= \Delta y_{i-1/2,j}^{(r)} \bar{D}_{i-1/2,j}^{(r)} (\bar{q}_m^{(r)} f_{i-1/2,j}^{(r)} - \Delta y_{i+1/2,j}^{(r)} \bar{D}_{i+1/2,j}^{(r)} (\bar{q}_m^{(r)} f_{i+1/2,j}^{(r)} \\
&+ \Delta x_{i,j-1/2}^{(r)} \bar{D}_{i,j-1/2}^{(r)} (\bar{q}_m^{(r)} f_{i,j-1/2}^{(r)} - \Delta x_{i,j+1/2}^{(r)} \bar{D}_{i,j+1/2}^{(r)} (\bar{q}_m^{(r)} f_{i,j+1/2}^{(r)} \\
&+ \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[(\bar{q}_b^{(r)} f_{i,j}^{(r)} + (\bar{q}_s^{(r)} f_{i,j}^{(r)} \right]. \quad (36)
\end{aligned}$$

where

$(\bar{q}_m^{(r)} f_{i-1/2,j}^{(r)})$ = heat flux due to molecular diffusion along $x^{(r)}$.

$(\bar{q}_m^{(r)} f_{i,j+1/2}^{(r)})$ = heat flux due to molecular diffusion along $y^{(r)}$.

$(\bar{q}_b^{(r)} f_{i,j}^{(r)})$ = heat flux through bottom area.

$(\bar{q}_s^{(r)} f_{i,j}^{(r)})$ = heat flux through ocean surface.

(37)

Substituting Eqs. (25), (34), (35), and (36) in Eq. (16), and rearranging,

$$\begin{aligned}
\frac{\partial \bar{T}_{i,j}^{(r)}}{\partial t} + \frac{\bar{T}_{i,j}^{(r)}}{\bar{D}_{i,j}^{(r)}} \left[\frac{\partial \bar{D}_{i,j}^{(r)}}{\partial t} \right] &= \frac{1}{\Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \bar{D}_{i,j}^{(r)}} \\
&\cdot \left\{ \left[\Delta y_{i-1/2,j}^{(r)} \bar{D}_{i-1/2,j}^{(r)} \bar{U}_{i-1/2,j}^{(r)} \bar{T}_{i-1/2,j}^{(r)} - \Delta y_{i+1/2,j}^{(r)} \bar{D}_{i+1/2,j}^{(r)} \bar{U}_{i+1/2,j}^{(r)} \bar{T}_{i+1/2,j}^{(r)} \right] \right. \\
&+ \left[\Delta x_{i,j-1/2}^{(r)} \bar{D}_{i,j-1/2}^{(r)} \bar{V}_{i,j-1/2}^{(r)} \bar{T}_{i,j-1/2}^{(r)} - \Delta x_{i,j+1/2}^{(r)} \bar{D}_{i,j+1/2}^{(r)} \bar{V}_{i,j+1/2}^{(r)} \bar{T}_{i,j+1/2}^{(r)} \right] \\
&+ \Delta y_{i-1/2,j}^{(r)} \bar{D}_{i-1/2,j}^{(r)} \left[\frac{(\bar{q}_m^{(r)} f_{i-1/2,j}^{(r)})}{\rho C^*} + (\bar{q}_T^{(r)} f_{i-1/2,j}^{(r)}) \right] \\
&- \Delta y_{i+1/2,j}^{(r)} \bar{D}_{i+1/2,j}^{(r)} \left[\frac{(\bar{q}_m^{(r)} f_{i+1/2,j}^{(r)})}{\rho C^*} + (\bar{q}_T^{(r)} f_{i+1/2,j}^{(r)}) \right] \\
&+ \Delta x_{i,j-1/2}^{(r)} \bar{D}_{i,j-1/2}^{(r)} \left[\frac{(\bar{q}_m^{(r)} f_{i,j-1/2}^{(r)})}{\rho C^*} + (\bar{q}_T^{(r)} f_{i,j-1/2}^{(r)}) \right] \\
&- \Delta x_{i,j+1/2}^{(r)} \bar{D}_{i,j+1/2}^{(r)} \left[\frac{(\bar{q}_m^{(r)} f_{i,j+1/2}^{(r)})}{\rho C^*} + (\bar{q}_T^{(r)} f_{i,j+1/2}^{(r)}) \right] \\
&+ \Delta x_{i,j}^{(r)} \Delta y_{i,j}^{(r)} \left[\frac{(\bar{q}_b^{(r)} f_{i,j}^{(r)})}{\rho C^*} + (\bar{q}_s^{(r)} f_{i,j}^{(r)}) \right] \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& + \Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \left[\frac{(\bar{q}_m^*)_{i,j-1/2}^{(n)}}{\rho C^*} + (\bar{q}_T^*)_{i,j-1/2}^{(n)} \right] - \Delta x_{i,j+1/2}^{(n)} \bar{D}_{i,j+1/2}^{(n)} \left[\frac{(\bar{q}_m^*)_{i,j+1/2}^{(n)}}{\rho C^*} + (\bar{q}_T^*)_{i,j+1/2}^{(n)} \right] \\
& + \left\{ \bar{G}_m \delta_{i,j,m}^{(n)} T_m + \bar{G}_n \delta_{i,j,n}^{(n)} T_n \right\} + \frac{(\bar{q}_v^*)_{i,j}^{(n)}}{\rho C^*} + \frac{1}{\bar{D}_{i,j}^{(n)}} \left[(\bar{q}_b^*)_{i,j}^{(n)} (T_b^*)_{i,j}^{(n)} + (\bar{q}_s^*)_{i,j}^{(n)} (T_s^*)_{i,j}^{(n)} \right] \\
& + \frac{1}{\rho C^* \bar{D}_{i,j}^{(n)}} \left[(\bar{q}_b^*)_{i,j}^{(n)} + (\bar{q}_s^*)_{i,j}^{(n)} \right] \quad (38)
\end{aligned}$$

Molecular and turbulent diffusion fluxes can be combined as:

$$\begin{aligned}
\bar{q}_{i,j-1/2}^{(n)} &= \frac{(\bar{q}_m^*)_{i,j-1/2}^{(n)}}{\rho C^*} + (\bar{q}_T^*)_{i,j-1/2}^{(n)} \\
\bar{q}_{i,j+1/2}^{(n)} &= \frac{(\bar{q}_m^*)_{i,j+1/2}^{(n)}}{\rho C^*} + (\bar{q}_T^*)_{i,j+1/2}^{(n)}
\end{aligned} \quad (39)$$

To eliminate the time derivative of depth from the energy conservation equation, (38), one can employ the mass conservation equation, (22). Hence, substituting Eq. (22) in Eq. (38), the final form of the energy conservation equation for a discrete element becomes

$$\begin{aligned}
\frac{\partial T_{i,j}^{(n)}}{\partial t} &= \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \left[\Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{U}_{i-1/2,j}^{(n)} \left[\bar{T}_{i-1/2,j}^{(n)} - T_{i,j}^{(n)} \right] \right. \right. \\
& - \Delta y_{i+1/2,j}^{(n)} \bar{D}_{i+1/2,j}^{(n)} \bar{U}_{i+1/2,j}^{(n)} \left[\bar{T}_{i+1/2,j}^{(n)} - T_{i,j}^{(n)} \right] \left. \right\} + \left\{ \Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{V}_{i,j-1/2}^{(n)} \left[\bar{T}_{i,j-1/2}^{(n)} - T_{i,j}^{(n)} \right] \right. \\
& + \Delta x_{i,j+1/2}^{(n)} \bar{D}_{i,j+1/2}^{(n)} \bar{V}_{i,j+1/2}^{(n)} \left[\bar{T}_{i,j+1/2}^{(n)} - T_{i,j}^{(n)} \right] \left. \right\} + \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \left[\Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{q}_{i-1/2,j}^{(n)} \right. \right. \\
& - \Delta y_{i+1/2,j}^{(n)} \bar{D}_{i+1/2,j}^{(n)} \bar{q}_{i+1/2,j}^{(n)} \left. \right] + \left[\Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{q}_{i,j-1/2}^{(n)} - \Delta x_{i,j+1/2}^{(n)} \bar{D}_{i,j+1/2}^{(n)} \bar{q}_{i,j+1/2}^{(n)} \right] \left. \right\} \\
& + \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \bar{G}_m \delta_{i,j,m}^{(n)} \left[T_m - \bar{T}_{i,j}^{(n)} \right] + \bar{G}_n \delta_{i,j,n}^{(n)} \left[T_n - \bar{T}_{i,j}^{(n)} \right] \right\} + \frac{(\bar{q}_v^*)_{i,j}^{(n)}}{\rho C^*} + \frac{1}{\bar{D}_{i,j}^{(n)}} \left\{ \frac{1}{\rho C^*} \left[(\bar{q}_b^*)_{i,j}^{(n)} \right. \right. \\
& \left. \left. + (\bar{q}_s^*)_{i,j}^{(n)} \right] + (\bar{q}_b^*)_{i,j}^{(n)} \left[(T_b^*)_{i,j}^{(n)} - \bar{T}_{i,j}^{(n)} \right] + (\bar{q}_s^*)_{i,j}^{(n)} \left[(T_s^*)_{i,j}^{(n)} - \bar{T}_{i,j}^{(n)} \right] \right\} \quad (40)
\end{aligned}$$

Equation (40) represents the final simulation form of the energy conservation equation for the arbitrary discrete element according to the FLIDE method. The mathematical terms on the right-hand side of the

equation can be grouped into four parts representing the four fundamental types of physical phenomena which control the time rate of change of temperature in the discrete element.

a. Convection effect of mean time averaged and depth averaged flow field:

$$(F_c)_{i,j}^{(n)} = \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \left[\Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{U}_{i-1/2,j}^{(n)} \left[\bar{T}_{i-1/2,j}^{(n)} - \bar{T}_{i,j}^{(n)} \right] \right. \right. \\ \left. \left. \Delta y_{i+1/2,j}^{(n)} \bar{D}_{i+1/2,j}^{(n)} \bar{U}_{i+1/2,j}^{(n)} \left[\bar{T}_{i+1/2,j}^{(n)} - \bar{T}_{i,j}^{(n)} \right] \right] + \left[\Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{V}_{i,j-1/2}^{(n)} \left[\bar{T}_{i,j-1/2}^{(n)} - \bar{T}_{i,j}^{(n)} \right] \right. \right. \\ \left. \left. \Delta x_{i,j+1/2}^{(n)} \bar{D}_{i,j+1/2}^{(n)} \bar{V}_{i,j+1/2}^{(n)} \left[\bar{T}_{i,j+1/2}^{(n)} - \bar{T}_{i,j}^{(n)} \right] \right] \right\} \quad (41)$$

It is interesting to note that the convective terms in Eq. (41) are arranged in a special form based on temperature differences between the half-point values, which represent the location of the enclosure surfaces, and the temperatures in the discrete element. This slightly unconventional form has a distinct computational property which minimizes the effect of possible errors in the specified flow field. Since the mathematical model cannot guarantee the flow field to satisfy the mass conservation and momentum equations, the standard forms for convective terms based on the half-point values of temperature can cause artificial temperature variations due to the flow field errors starting from equal temperature conditions throughout the flow region. The particular forms of the convective terms in Eq. (41) avoid this computational problem, since equal temperatures automatically eliminate convective effects regardless of the velocity values at the half-point surfaces.

b. Overall diffusion effect of depth averaged molecular and turbulent transport phenomena:

$$(F_D)_{i,j}^{(n)} = \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \left[\Delta y_{i-1/2,j}^{(n)} \bar{D}_{i-1/2,j}^{(n)} \bar{q}_{i-1/2,j}^{(n)} - \Delta y_{i+1/2,j}^{(n)} \bar{D}_{i+1/2,j}^{(n)} \bar{q}_{i+1/2,j}^{(n)} \right] \right. \\ \left. + \left[\Delta x_{i,j-1/2}^{(n)} \bar{D}_{i,j-1/2}^{(n)} \bar{q}_{i,j-1/2}^{(n)} - \Delta x_{i,j+1/2}^{(n)} \bar{D}_{i,j+1/2}^{(n)} \bar{q}_{i,j+1/2}^{(n)} \right] \right\} \quad (42)$$

The formulation of the diffusion flux terms will be discussed in detail later in the development.

c. Thermal effect of power plant intakes and discharges:

$$(\bar{F}_P)_{i,j}^{(n)} = \frac{1}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} \left\{ \dot{G}_m \delta_{i,j,m}^{(n)} \left[T_m - \bar{T}_{i,j}^{(n)} \right] + \dot{G}_n \delta_{i,j,n}^{(n)} \left[T_n - \bar{T}_{i,j}^{(n)} \right] \right\} \quad (43)$$

It is important to note that the heat release by the power plants into the flow region is not represented as heat source terms. It is more realistic to employ the present form, Eq. (43), which considers the heating effects based on the flow rates and temperatures at the intakes and discharges. For discharge temperature T_n equal to the local temperature $\bar{T}_{i,j}^{(n)}$, according to present formulation the heating effect of the discharge completely vanishes, which is correct from physical considerations. However, according to source formulations, the heating effect would continue; and it would result in unrealistic increases in temperature unless the conservation of mass principle is also identically satisfied according to the specified flow conditions.

Futhermore, if the discharge temperature is lower than the local temperature, the present form, Eq. (43), correctly introduces the cooling effect; hence, the equation can be readily used for the study of cold-shock problems on biological organisms.

Under normal operational conditions, particularly in shallow regions, it is realistic to assume that the intake temperature T_m is equal to the local mean temperature $\bar{T}_{i,j}^{(r)}$. This assumption eliminates the heating or cooling effect of the intake, according to Eq. (43), which is definitely correct from physical considerations. However, in certain thermal impact analyses, it is usually assumed that, due to severe stratified conditions, the intake temperature T_m is equal to the ambient temperature, which may be considerably lower than the local mean temperature $\bar{T}_{i,j}^{(r)}$ if conditions for the recirculation of heated water between the discharges and the intakes exist. Considering the present form, Eq. (43), since the intake flow rate $\dot{G}_m$ is negative, this assumption immediately produces a heating effect in the discrete element. This result is definitely correct from physical considerations, since the assumption forces the intake to eliminate the colder stratified water in the element. Consequently, the warmer portion of the water which remains in the element causes the unavoidable effect of increasing the mean temperature of the water in the discrete element.

Hence, in the application of the present mathematical model to the assessment of the thermal impact of a power plant, it is more realistic to assume the intake temperature T_m to be equal to the local mean temperature $\bar{T}_{i,j}^{(r)}$. If stratification effects are to be imposed on the intake conditions, the modifications on the values of the intake temperatures T_m must be based on careful physical considerations.

d. Overall effect of surface and bottom conditions, and heat generation:

$$(\bar{F}_S)_{i,j}^{(r)} = \frac{(\dot{q}_v)_{i,j}^{(r)}}{\rho C^*} + \frac{1}{\bar{D}^{(r)}} \left\{ \frac{1}{\rho C^*} \left[(\dot{q}_b^*)_{i,j}^{(r)} + (\dot{q}_s^*)_{i,j}^{(r)} \right] + (\dot{g}_b^*)_{i,j}^{(r)} \left[(T_b^*)_{i,j}^{(r)} - \bar{T}_{i,j}^{(r)} \right] + (\dot{g}_s^*)_{i,j}^{(r)} \left[(T_s^*)_{i,j} - \bar{T}_{i,j}^{(r)} \right] \right\} \quad (44)$$

The internal heat generation term $(\dot{q}_v)_{i,j}^{(r)}$ is retained in the formulation for the sake of generality and for modeling special heating effects. The thermal effects of the intakes and discharges should not be included in the definition of this term.

The terms representing the heat flux $(\dot{q}_b^*)_{i,j}^{(r)}$ and inflow $(\dot{g}_b^*)_{i,j}^{(r)}$ from the bottom can be neglected in usual applications, which consider the total depth. However, for special modeling, particularly in very deep regions, where the thermal plume may remain stratified, these terms can be employed to approximately incorporate the vertical entrainment and heat transfer to the plume from the lower stratified region.

The surface heat flux $(\dot{q}_s^*)_{i,j}^{(r)}$ and inflow $(\dot{g}_s^*)_{i,j}^{(r)}$ depend on the atmospheric conditions, for example, solar irradiation, back radiation, wind, evaporation, rainfall, etc. The appropriate formulations of these terms will be discussed in detail later in the development of the model.

Substituting the mathematical forms of the above discussed physical effects, Eqs. (41), (42), (43), and (44), in Eq. (40), the final simulation form of the energy conservation equation can be expressed as

$$-\frac{\partial \bar{T}_{i,j}^{(r)}}{\partial t} = \bar{F}_{i,j}^{(r)} = (\bar{F}_C)_{i,j}^{(r)} + (\bar{F}_D)_{i,j}^{(r)} + (\bar{F}_P)_{i,j}^{(r)} + (\bar{F}_S)_{i,j}^{(r)} \quad (45)$$

which represents a first-order, ordinary differential equation for the unknown mean temperature $\bar{T}_{i,j}^{(r)}$ in

discrete element i, j in region r . Hence, provided the terms in the right-hand function $F_{ij}^{(r)}$ of Eq. (45) can be formulated in terms of the principal unknown variables, mean temperatures $\bar{T}_{e,k}^{(r)}$, the mathematical system of the FLIDE method consists of a system of coupled, first-order, ordinary differential equations which can be integrated from a given set of initial conditions.

2.4 Formulation of Half-Point Values on Discrete-Element Surfaces

The first necessary step to render consistent the governing system of equations of the FLIDE method, Eq. (40), is to eliminate the half-point values for velocity components $\bar{U}_{i+1/2,j}^{(r)}$, $\bar{V}_{i,j+1/2}^{(r)}$ and temperature $\bar{T}_{i+1/2,j}^{(r)}$, $\bar{T}_{i,j+1/2}^{(r)}$ in terms of their mean values in the discrete elements.

From the solution point of view this is a very crucial problem, since the stability of the numerical methods, particularly the explicit ones, critically depends on the forms of the convective terms. An excellent discussion of the stability of various forms associated with conservation equations is given by Roache [48].

The most successful form, from the computational point of view, for eliminating the half-point values in terms of the principal unknown values, is the "second upwind differencing" [48] or "donor cell" method, which was first presented by Gentry, Martin, and Daly [44] within the formulation of the FLIC method. This particular form, which is based more on physical reasoning about fluid flow conditions than on rigorous mathematical derivations, possesses both conservative and transportive properties (see [48]) for the explicit finite difference formulations of conservation equations. Hence, in numerous applications it has been proven to be extremely dependable from numerical stability considerations.

The standard "second upwind differencing," or "donor cell," forms of the FLIC method must be slightly modified to incorporate the unequal dimensions of the discrete elements of the present FLIDE method in evaluating the half-point values of depth, $\bar{D}_{i+1/2,j}^{(r)}$, $\bar{D}_{i,j+1/2}^{(r)}$, and velocity components $\bar{U}_{i+1/2,j}^{(r)}$, $\bar{V}_{i,j+1/2}^{(r)}$. Hence, the modified forms based on linear averaging between two discrete elements become

$$\begin{aligned}\bar{D}_{i+1/2,j}^{(r)} &= \frac{|\Delta x_{i,j}^{(r)} \bar{D}_{i+1,j}^{(r)} + \Delta x_{i+1,j}^{(r)} \bar{D}_{i,j}^{(r)}|}{|\Delta x_{i,j}^{(r)} + \Delta x_{i+1,j}^{(r)}|}, \\ \bar{D}_{i,j+1/2}^{(r)} &= \frac{|\Delta y_{i,j}^{(r)} \bar{D}_{i,j+1}^{(r)} + \Delta y_{i,j+1}^{(r)} \bar{D}_{i,j}^{(r)}|}{|\Delta y_{i,j}^{(r)} + \Delta y_{i,j+1}^{(r)}|}, \\ \bar{U}_{i+1/2,j}^{(r)} &= \frac{|\Delta x_{i,j}^{(r)} \bar{U}_{i+1,j}^{(r)} + \Delta x_{i+1,j}^{(r)} \bar{U}_{i,j}^{(r)}|}{|\Delta x_{i,j}^{(r)} + \Delta x_{i+1,j}^{(r)}|}, \\ \bar{V}_{i,j+1/2}^{(r)} &= \frac{|\Delta y_{i,j}^{(r)} \bar{V}_{i,j+1}^{(r)} + \Delta y_{i,j+1}^{(r)} \bar{V}_{i,j}^{(r)}|}{|\Delta y_{i,j}^{(r)} + \Delta y_{i,j+1}^{(r)}|}.\end{aligned}\tag{46}$$

The standard forms of the FLIC method for the transported property of the fluid, represented by the temperature T , are retained in the FLIDE method as:

$$\begin{aligned}
\bar{T}_{i-1/2,j}^{(r)} &= \begin{cases} \bar{T}_{i,j}^{(r)} & \text{for } \bar{U}_{i-1/2,j}^{(r)} < 0, \\ \bar{T}_{i-1,j}^{(r)} & \text{for } \bar{U}_{i-1/2,j}^{(r)} > 0, \end{cases} \\
\bar{T}_{i+1/2,j}^{(r)} &= \begin{cases} \bar{T}_{i+1,j}^{(r)} & \text{for } \bar{U}_{i+1/2,j}^{(r)} < 0, \\ \bar{T}_{i,j}^{(r)} & \text{for } \bar{U}_{i+1/2,j}^{(r)} > 0, \end{cases} \\
\bar{T}_{i,j-1/2}^{(r)} &= \begin{cases} \bar{T}_{i,j}^{(r)} & \text{for } \bar{V}_{i,j-1/2} < 0, \\ \bar{T}_{i,i-1}^{(r)} & \text{for } \bar{V}_{i,j-1/2} > 0, \end{cases} \\
\bar{T}_{i,j+1/2}^{(r)} &= \begin{cases} \bar{T}_{i,j+1}^{(r)} & \text{for } \bar{V}_{i,j+1/2} < 0, \\ \bar{T}_{i,j}^{(r)} & \text{for } \bar{V}_{i,j+1/2} > 0. \end{cases}
\end{aligned} \tag{47}$$

In Eq. (47), the special cases when the normal components of the velocity field identically vanish on the surfaces need not be considered, since under these conditions the convective transfer terms also vanish, regardless of the values of the temperature on the element surfaces.

The physical significance of the particular forms, Eq. (47), from fluid flow considerations is clearly evident. The evaluation of a half-point value is based on the flow direction: that is, as the fluid crosses a surface area normal to its flow direction, it always has the temperature of the discrete element upstream of the surface. The generalized mathematical expressions for this physically realistic argument are represented by Eq. (47) for the evaluation of the half-point values on all enclosure surfaces of a discrete element.

It should be noted that the "second upwind differencing" or "donor cell" forms could cause undesirable inaccuracies in the FLIDE method due to the unequal dimensions of the adjacent elements. Hence, in the division of a region into discrete elements, the sizes of the adjacent elements should not be excessively different from each other.

The mathematical consequences of these forms will be more evident in later discussions of the stability of the FLIDE equations.

2.5 Formulation of the Heat Flux Terms

From the mathematical point of consistency of the system of equations of the FLIDE method, Eq. (40), it is essential to eliminate the half-point values of the diffusion flux terms $\bar{q}_{i-1/2,j}^{(r)}$ and $\bar{q}_{i,j+1/2}^{(r)}$ in terms of the principal unknown values of the temperature $\bar{T}_{i,k}^{(r)}$ in the discrete elements. This is by no means an easy task, since it requires careful consideration of the turbulence phenomena which control the diffusion of thermal energy in coastal and offshore regions. A detailed theoretical development of an accurate horizontal turbulent diffusion model for the general FLIDE formulation is definitely beyond the scope of this study. Hence, in the present mathematical model, the formulation of the diffusion heat flux terms will be based on the existing state-of-the-art in classical and oceanic turbulence theories. However, special emphasis will be placed on the selection of the proper diffusion model, consistent with the eddy phenomenon, which is considered as turbulence in the mathematical formulation.

The molecular diffusion fluxes $(\bar{q}_m^* \bar{T}_{i-1/2,j}^{(r)})$, $(\bar{q}_m^* \bar{T}_{i,j+1/2}^{(r)})$ are readily formulated according to the Fourier conduction law, based on the coefficient of thermal conductivity κ ; that is,

$$(q_m^*)_{i-1/2,j}^{(r)} = \kappa \left[\frac{\partial \bar{T}}{\partial x} \right]_{i-1/2,j}^{(r)},$$

$$(q_m^*)_{i,j-1/2}^{(r)} = \kappa \left[\frac{\partial \bar{T}}{\partial y} \right]_{i,j-1/2}^{(r)},$$
(48)

where the right-hand sides in Eq. (48) represent the derivatives of the temperature evaluated at half-point surfaces between the discrete elements.

Considering the dimensions of the adjacent discrete elements, the derivatives in Eq. (48) can be approximated by their finite forms as:

$$(q_m^*)_{i-1/2,j}^{(r)} = \frac{2\kappa}{[\Delta x_{i,j}^{(r)} + \Delta x_{i-1,j}^{(r)}]} [\bar{T}_{i,j}^{(r)} - \bar{T}_{i-1,j}^{(r)}],$$

$$(q_m^*)_{i,j-1/2}^{(r)} = \frac{2\kappa}{[\Delta x_{i,j}^{(r)} + \Delta x_{i,j-1}^{(r)}]} [\bar{T}_{i,j}^{(r)} - \bar{T}_{i,j-1}^{(r)}],$$
(49)

$$(q_m^*)_{i,j-1/2}^{(r)} = \frac{2\kappa}{[\Delta y_{i,j}^{(r)} + \Delta y_{i,j-1}^{(r)}]} [\bar{T}_{i,j}^{(r)} - \bar{T}_{i,j-1}^{(r)}],$$

$$(q_m^*)_{i,j+1/2}^{(r)} = \frac{2\kappa}{[\Delta y_{i,j}^{(r)} + \Delta y_{i,j+1}^{(r)}]} [\bar{T}_{i,j+1}^{(r)} - \bar{T}_{i,j}^{(r)}].$$

In general, molecular diffusion can be neglected in comparison with turbulent diffusion under most oceanic flow conditions. However, since the present mathematical model considers fully transient tidal motions, molecular diffusion flux terms will be retained in the analysis to include their effects during local slack water conditions when turbulent diffusion effects are minimum.

Formulation of the turbulent heat flux terms, Eq. (33), requires very careful consideration of the turbulence phenomenon which generates the eddy mechanism of turbulent transport. They cannot be directly approximated by a straightforward Boussinesq [49] form in terms of a constant mixing coefficient.

First, it should be realized that the physical and mathematical problems associated with continuous thermal discharge conditions do not belong to the class of dispersion problems. The virtual coefficients of horizontal diffusion, derived by Taylor [14] and Elder [15] are based on strict mathematical assumptions for the integration of conservation of matter equation along the vertical direction. These assumptions are inherently dependent on the behavior of the initial, instantaneous release type of concentration in a Lagrangian frame of reference moving with the mean velocity. For the present Eulerian system with continuous discharge conditions, which violate the conditions of the instantaneous release, the essential mathematical assumptions of the dispersion studies definitely do not apply.

Second, since the present mathematical model considers fully transient tidal flow conditions in the convection terms, oceanic diffusion coefficients, which are based on meso-scale eddies created by tidal motions, for example, Wada [38], Okubo and Ozmidov [25], etc., according to Kolmogoroff's theory [24], cannot be employed in the formulation of the turbulent transport flux terms in small time scale

applications within a tidal period. Similarly, the dispersion analysis by Bowden [18] for periodic tidal flow conditions in an estuary does not apply since the formulation considers, in a sense, a mean-scale eddy turbulence coefficient based on averaging over the period of the tidal cycle.

However, considering the relatively shallow water conditions in coastal and offshore regions, it is physically realistic to assume that the fundamental mechanism of turbulent diffusion is based on the turbulent eddies associated with the vertical distributions of the turbulent velocity profiles due to oceanic flow conditions. Hence, the formulation of the turbulent transport flux terms of the present model can employ the velocity distributions which were considered by Elder [15] and Bowden [18].

To simplify the arithmetic operations, the subscripts and superscripts in Eq. (32) will be discarded in the formulation of the turbulent flux terms; that is,

$$\begin{aligned} q_{T,x}^* &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{u}^* dz + \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{u}^* dz, \\ q_{T,y}^* &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{v}^* dz + \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{v}^* dz. \end{aligned} \quad (50)$$

Considering the terms separately,

$$\begin{aligned} (q_{T,x}^*)_1 &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{u}^* dz, & (q_{T,x}^*)_2 &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{u}^* dz, \\ (q_{T,y}^*)_1 &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{v}^* dz, & (q_{T,y}^*)_2 &= \frac{1}{D} \int_0^{\bar{D}} \bar{T}^* \bar{v}^* dz. \end{aligned} \quad (51)$$

such that

$$q_{T,x}^* = (q_{T,x}^*)_1 + (q_{T,x}^*)_2, \quad q_{T,y}^* = (q_{T,y}^*)_1 + (q_{T,y}^*)_2. \quad (52)$$

The mean time averaged velocity difference profiles u^* and v^* can be approximated as similarity-type distributions along the vertical direction based on defect type, phenomenological universal turbulent distribution laws; that is,

$$\begin{aligned} \bar{u}^* &= \frac{u_r^*}{k_n} [R(\xi) - \bar{r}], & \xi &= z/\bar{D}, \\ \bar{v}^* &= \frac{v_r^*}{k_n} [R(\xi) - \bar{r}] & \xi &= z/\bar{D}, \end{aligned} \quad (53)$$

where

$$\begin{aligned} u_r^* &= \sqrt{\tau_{w,x}/\rho} = \text{friction velocity along } x, \\ v_r^* &= \sqrt{\tau_{w,y}/\rho} = \text{friction velocity along } y, \\ \tau_{w,x} &= \text{shear stress along } x \text{ on the bottom}, \\ \tau_{w,y} &= \text{shear stress along } y \text{ on the bottom}, \\ k_n &= 0.41 = \text{von Karman's constant}, \end{aligned} \quad (54)$$

$$\bar{r} = \int_{z_0}^1 r(z) dz, \quad (55)$$

in which z_0 represents the laminar sublayer.

It should be realized that the assumed profiles, Eq. (53), are only very approximate representations of the actual transient, three-dimensional turbulent velocity distributions.

Assuming that the temperature distribution along the vertical direction has a maximum deviation from the mean as ΔT_s , a similarity form for the mean time averaged temperature difference distribution can be defined as

$$T = \Delta T_s f(z), \quad 0 \leq z \leq 1 \quad (56)$$

In view of the similarity of the profiles, the friction velocities u_τ^* , v_τ^* and the stratification temperature difference ΔT_s must be independent of z , hence, substituting Eqs. (53) and (56) in Eqs. (51), the first parts of the turbulent transport flux terms become

$$(q_{T,x}^*)_1 = \frac{u_\tau^*}{k_0} \Delta T_s \int_0^1 |R(z) - \bar{r}| f(z) dz, \quad (57)$$

$$(q_{T,y}^*)_1 = \frac{v_\tau^*}{k_0} \Delta T_s \int_0^1 |R(z) - \bar{r}| f(z) dz$$

Hence, defining the constant value of the integral as β ,

$$\beta = \int_0^1 |R(z) - \bar{r}| f(z) dz, \quad (58)$$

and the first parts of the transport flux terms become

$$(q_{T,x}^*)_1 = \frac{\beta}{k_0} u_\tau^* \Delta T_s, \quad (59)$$

$$(q_{T,y}^*)_1 = \frac{\beta}{k_0} v_\tau^* \Delta T_s$$

Friction velocities u_τ^* and v_τ^* can be expressed in terms of bottom friction coefficients λ_x and λ_y according to their definitions, that is,

$$\rho u_\tau^{*2} = \tau_{w,x} = \lambda_x \rho \bar{U}^2, \quad \rho v_\tau^{*2} = \tau_{w,y} = \lambda_y \rho \bar{V}^2 \quad (60)$$

Hence, substituting Eq. (60) in Eq. (59),

$$(q_{T,x}^*)_1 = \frac{\beta}{k_0} \sqrt{\lambda_x} \bar{U} \Delta T_s, \quad (61)$$

$$(q_{T,y}^*)_1 = \frac{\beta}{k_0} \sqrt{\lambda_y} \bar{V} \Delta T_s$$

The expressions for the first parts of the transport flux terms, Eq. (61), which result from Eq. (51), have the following very important properties:

a. Nonuniform temperature and velocity distributions in the vertical direction contribute to horizontal transport fluxes. However, if the flow conditions create sufficient vertical mixing such that thermal stratification is negligible, that is, $\overline{\Delta T_z^*}$ is approximately zero, then this effect cannot contribute to the transport fluxes, regardless of the nonuniform vertical distributions of the velocity profiles.

b. The transport flux contribution of the vertically nonuniform temperature profile cannot be considered as a truly diffusion type of phenomenon, since in general it does not conform with the forms which consider the flux proportional to the gradient of temperature as in Fourier and Boussinesq type formulations.

These two properties clearly indicate that the additional horizontal transport flux due to thermal stratification cannot be compensated for by increasing the effective value of the turbulent eddy diffusion coefficients. Furthermore, since thermal stratification conditions can be considerably different from dye or salinity stratifications, and the transport does not depend on micro-scale random eddies, Reynold's analogy definitely does not hold true. Hence, experimentally determined values of diffusion coefficients from dye or salinity observations cannot be directly employed in a thermal analysis.

Unfortunately, the stratification effects can indeed result in large transport effects over long time averaged observations for many tidal cycles if the periodic tidal phenomena are considered as meso-scale turbulent eddies in a purely diffusion type of analysis. Consequently, turbulent eddy diffusion coefficients determined from these analyses have extremely high values, particularly for estuaries with time-wise discontinuous salinity conditions at their ocean ends. For the present mathematical model, which considers tidal motions as short time convective phenomena, these coefficients are definitely not valid. Erroneous use of the coefficients can result in mathematical predictions severely on the unconservative side from environmental thermal impact considerations.

It should be noted that the expressions, Eq. (51), for the transport flux terms $(q_{Tx}^*)_1$ and $(q_{Ty}^*)_1$, due to vertical variations, are indeed the general forms which lead to the classical virtual horizontal diffusion flux expressions obtained by Taylor [14], Elder [15], and Bowden [18] in their studies of the dispersion problems. However, as stated earlier, the dispersion flux expressions can only be derived from the general expressions, Eq. (51), under very rigid mathematical and physical assumptions which are only valid for the dispersion phenomenon but definitely not valid for continuous discharge conditions in a transient flow field.

Hence, in the application of the present short time transient mathematical model, it is safer to neglect the stratification effect, particularly in shallow water regions. If it can be established that stratification definitely exists in a flow region, then an appropriate distribution in the region for $\overline{\Delta T_z^*}$ must be specified, based on other parameters, for example, Froude number [50], etc., for the evaluation of β and the transport flux terms $(q_{Tx}^*)_1$ and $(q_{Ty}^*)_1$.

The second parts of the transport flux terms $(q_{Tx}^*)_2$ and $(q_{Ty}^*)_2$, Eq. (51), represent the true turbulent diffusion phenomenon due to micro-scale random turbulent eddies. Hence, they can be represented in Boussinesq forms as:

$$\begin{aligned}\overline{T'u'} &= \epsilon_{Tx} \left(\frac{\partial \overline{T}}{\partial x} + \frac{\partial \overline{T}^*}{\partial x} \right), \\ \overline{T'v'} &= \epsilon_{Ty} \left(\frac{\partial \overline{T}}{\partial y} + \frac{\partial \overline{T}^*}{\partial y} \right).\end{aligned}\tag{62}$$

where ϵ_{Tx} and ϵ_{Ty} represent thermal eddy diffusivities along x and y directions respectively. Assuming that isotropic turbulence is valid, particularly for shallow water

$$\epsilon_{Tx} = \epsilon_{Ty} = \epsilon_{Tz} = \epsilon_T \quad (63)$$

where ϵ_{Tz} represents the vertical thermal eddy diffusivity.

Since the particular diffusion transport is solely due to the turbulent eddy phenomenon, Reynold's analogy definitely holds true; that is,

$$\epsilon_T = \epsilon_{Tz} = \epsilon_{vz} = \frac{\tau}{(\partial V^*/\partial z)} \quad (64)$$

$$V^* = |V| = \sqrt{(U + u^*)^2 + (V + v^*)^2} \quad (65)$$

where

τ = local shear stress on a horizontal plane.

ϵ_{vz} = momentum eddy diffusivity.

V^* = magnitude of horizontal velocity.

Assuming linear variation for the shear stress from its bottom value τ_w^* along the z direction, and substituting the universal turbulent distribution law in Eq. (64),

$$\epsilon_T = \frac{\overline{D} \tau_w^* \zeta}{\rho V_f^* f'(\zeta)} = \overline{D} V_f^* \frac{\zeta}{f'(\zeta)} \quad (66)$$

where V_f^* represents the friction velocity due to the bottom stress τ_w^* caused by the horizontal velocity with magnitude V^* .

Hence, substituting Eq. (66) in Eq. (62), and the resulting equation and Eq. (56) in Eq. (51), the turbulent diffusion flux terms become

$$\begin{aligned} (q_{Tx}^*)_2 &= \overline{D} V_f^* \left\{ \left| \int_0^1 \frac{\zeta}{f'(\zeta)} d\zeta \right| \frac{\partial \overline{T}}{\partial x} + \left| \int_0^1 \frac{\zeta f(\zeta)}{f'(\zeta)} d\zeta \right| \frac{\partial}{\partial x} (\Delta \overline{T}_s^*) \right\} \\ (q_{Ty}^*)_2 &= \overline{D} V_f^* \left\{ \left| \int_0^1 \frac{\zeta}{f'(\zeta)} d\zeta \right| \frac{\partial \overline{T}}{\partial y} + \left| \int_0^1 \frac{\zeta f(\zeta)}{f'(\zeta)} d\zeta \right| \frac{\partial}{\partial y} (\Delta \overline{T}_s^*) \right\} \end{aligned} \quad (67)$$

The first integral in Eq. (67) was evaluated by Elder [15] for a logarithmic velocity profile; and defining the second integral as a constant,

$$\int_0^1 \frac{\zeta}{f'(\zeta)} d\zeta = \frac{k_0}{6}, \quad \int_0^1 \frac{\zeta f(\zeta)}{f'(\zeta)} d\zeta = \gamma \quad (68)$$

Substituting Eq. (68) and eliminating the friction velocity V_f^* in terms of bottom friction coefficient λ , similar to Eq. (60), for the magnitude of velocity V^* in Eq. (67),

$$(q_{T,x}^*) = D\sqrt{\lambda} V^* \left| \frac{k_0}{6} \frac{\partial T}{\partial x} + \gamma \frac{\partial}{\partial x} (\Delta T_s^*) \right| \quad (69)$$

$$(q_{T,y}^*) = D\sqrt{\lambda} V^* \left| \frac{k_0}{6} \frac{\partial T}{\partial y} + \gamma \frac{\partial}{\partial y} (\Delta T_s^*) \right|$$

The turbulent heat flux terms $(q_{T,x}^*)$ and $(q_{T,y}^*)$ in Eq. (69) also depend on stratification conditions in the region. However, noting that ΔT_s^* is considerably smaller than the mean temperature T , the terms containing the effects of stratification can possibly be neglected.

Substituting Eqs. (61) and (69) in Eq. (50), and replacing V^* with the magnitude of the mean velocity vector $\bar{V}$, the combined turbulent heat flux terms become

$$q_{T,x}^* = \left| \frac{k_0 \sqrt{\lambda}}{6} D|\bar{V}| \right| \frac{\partial \bar{T}}{\partial x} + \left| \frac{\beta \sqrt{\lambda}}{k_0} \bar{U} \Delta T_s^* - \gamma \sqrt{\lambda} D|\bar{V}| \frac{\partial}{\partial x} (\Delta T_s^*) \right| \quad (70)$$

$$q_{T,y}^* = \left| \frac{k_0 \sqrt{\lambda}}{6} D|\bar{V}| \right| \frac{\partial \bar{T}}{\partial y} + \left| \frac{\beta \sqrt{\lambda}}{k_0} \bar{V} \Delta T_s^* - \gamma \sqrt{\lambda} D|\bar{V}| \frac{\partial}{\partial y} (\Delta T_s^*) \right|$$

Defining the parts which contain the stratified temperature difference ΔT_s^* as the turbulent heat flux due to stratification,

$$Q_{sx} = \frac{\beta \sqrt{\lambda}}{k_0} \bar{U} \Delta T_s^* - \gamma \sqrt{\lambda} D|\bar{V}| \frac{\partial}{\partial x} (\Delta T_s^*) \quad (71)$$

$$Q_{sy} = \frac{\beta \sqrt{\lambda}}{k_0} \bar{V} \Delta T_s^* - \gamma \sqrt{\lambda} D|\bar{V}| \frac{\partial}{\partial y} (\Delta T_s^*)$$

Substituting Eq. (71) in Eq. (70),

$$q_{T,x}^* = \left| \frac{k_0 \sqrt{\lambda}}{6} D|\bar{V}| \right| \frac{\partial \bar{T}}{\partial x} + Q_{sx} \quad (72)$$

$$q_{T,y}^* = \left| \frac{k_0 \sqrt{\lambda}}{6} D|\bar{V}| \right| \frac{\partial \bar{T}}{\partial y} + Q_{sy}$$

which represent the general form of the turbulent heat flux terms.

Evaluating the right-hand sides of Eq. (72) to obtain the half-point values according to Eq. (39),

$$(q_{T,x}^*)_{i,j+1/2}^{(n)} = \left| \frac{k_0 \sqrt{\lambda}}{6} \bar{U}_{i,j+1/2}^{(n)} |\bar{V}_{i,j+1/2}^{(n)}| \right| \left| \frac{\partial \bar{T}}{\partial x} \right|_{i,j+1/2}^{(n)} + [Q_{sx}]_{i,j+1/2}^{(n)} \quad (73)$$

$$(q_{T,y}^*)_{i,j+1/2}^{(n)} = \left| \frac{k_0 \sqrt{\lambda}}{6} \bar{V}_{i,j+1/2}^{(n)} |\bar{V}_{i,j+1/2}^{(n)}| \right| \left| \frac{\partial \bar{T}}{\partial y} \right|_{i,j+1/2}^{(n)} + [Q_{sy}]_{i,j+1/2}^{(n)}$$

Considering the dimensions of the adjacent discrete elements, the derivatives in Eq. (73) can be approximated by their finite forms as

$$\begin{aligned}
 (\bar{q}_T^{(n)})_{i-1/2,j} &= -2 \left[\frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i-1/2,j}^{(n)} |\bar{V}_{i-1/2,j}^{(n)}| \right] \frac{|\bar{T}_{i,j}^{(n)} - \bar{T}_{i-1,j}^{(n)}|}{|\Delta x_{i,j}^{(n)} + \Delta x_{i-1,j}^{(n)}|} + |Q_{sx}|_{i-1/2,j}^{(n)} \\
 (\bar{q}_T^{(n)})_{i+1/2,j} &= 2 \left[\frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i+1/2,j}^{(n)} |\bar{V}_{i+1/2,j}^{(n)}| \right] \frac{|\bar{T}_{i+1,j}^{(n)} - \bar{T}_{i,j}^{(n)}|}{|\Delta x_{i,j}^{(n)} + \Delta x_{i+1,j}^{(n)}|} + |Q_{sx}|_{i+1/2,j}^{(n)} \\
 (\bar{q}_T^{(n)})_{i,j-1/2} &= -2 \left[\frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i,j-1/2}^{(n)} |\bar{V}_{i,j-1/2}^{(n)}| \right] \frac{|\bar{T}_{i,j}^{(n)} - \bar{T}_{i,j-1}^{(n)}|}{|\Delta y_{i,j}^{(n)} + \Delta y_{i,j-1}^{(n)}|} + |Q_{sy}|_{i,j-1/2}^{(n)} \\
 (\bar{q}_T^{(n)})_{i,j+1/2} &= 2 \left[\frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i,j+1/2}^{(n)} |\bar{V}_{i,j+1/2}^{(n)}| \right] \frac{|\bar{T}_{i,j+1}^{(n)} - \bar{T}_{i,j}^{(n)}|}{|\Delta y_{i,j}^{(n)} + \Delta y_{i,j+1}^{(n)}|} + |Q_{sy}|_{i,j+1/2}^{(n)}
 \end{aligned} \tag{74}$$

The molecular and turbulent transport flux terms from Eqs. (49) and (74) can be combined according to Eq. (32) in terms of equivalent diffusion coefficients $\bar{\epsilon}_{i-1/2,j}^{(n)}$ and $\bar{\epsilon}_{i,j-1/2}^{(n)}$ to obtain the net diffusion transport flux terms on the discrete element surfaces as:

$$\begin{aligned}
 \bar{q}_{i-1/2,j}^{(n)} &= -\bar{\epsilon}_{i-1/2,j}^{(n)} \frac{|\bar{T}_{i,j}^{(n)} - \bar{T}_{i-1,j}^{(n)}|}{|\Delta x_{i,j}^{(n)} + \Delta x_{i-1,j}^{(n)}|} + |Q_{sx}|_{i-1/2,j}^{(n)} \\
 \bar{q}_{i+1/2,j}^{(n)} &= \bar{\epsilon}_{i+1/2,j}^{(n)} \frac{|\bar{T}_{i+1,j}^{(n)} - \bar{T}_{i,j}^{(n)}|}{|\Delta x_{i,j}^{(n)} + \Delta x_{i+1,j}^{(n)}|} + |Q_{sx}|_{i+1/2,j}^{(n)} \\
 \bar{q}_{i,j-1/2}^{(n)} &= -\bar{\epsilon}_{i,j-1/2}^{(n)} \frac{|\bar{T}_{i,j}^{(n)} - \bar{T}_{i,j-1}^{(n)}|}{|\Delta y_{i,j}^{(n)} + \Delta y_{i,j-1}^{(n)}|} + |Q_{sy}|_{i,j-1/2}^{(n)} \\
 \bar{q}_{i,j+1/2}^{(n)} &= \bar{\epsilon}_{i,j+1/2}^{(n)} \frac{|\bar{T}_{i,j+1}^{(n)} - \bar{T}_{i,j}^{(n)}|}{|\Delta y_{i,j}^{(n)} + \Delta y_{i,j+1}^{(n)}|} + |Q_{sy}|_{i,j+1/2}^{(n)}
 \end{aligned} \tag{75}$$

where the equivalent diffusion coefficients are defined as:

$$\begin{aligned}
 \bar{\epsilon}_{i-1/2,j}^{(n)} &= \frac{\kappa}{\rho C^*} + \frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i-1/2,j}^{(n)} \left\{ |\bar{U}_{i-1/2,j}^{(n)}|^2 + |\bar{V}_{i-1/2,j}^{(n)}|^2 \right\}^{1/2} \\
 \bar{\epsilon}_{i,j-1/2}^{(n)} &= \frac{\kappa}{\rho C^*} + \frac{k_0 \sqrt{\lambda}}{6} \bar{D}_{i,j-1/2}^{(n)} \left\{ |\bar{U}_{i,j-1/2}^{(n)}|^2 + |\bar{V}_{i,j-1/2}^{(n)}|^2 \right\}^{1/2}
 \end{aligned} \tag{76}$$

The additional half-point values for the velocity components $\bar{U}_{i,j-1/2}^{(n)}$, $\bar{V}_{i-1/2,j}^{(n)}$ are evaluated according to linear approximations as in Eq. (46); that is,

$$\begin{aligned}\bar{U}_{i,j-1/2}^{(n)} &= \frac{[\Delta y_{i,j}^{(n)} \bar{U}_{i,j}^{(n)} + \Delta y_{i,j-1}^{(n)} \bar{U}_{i,j-1}^{(n)}]}{[\Delta y_{i,j}^{(n)} + \Delta y_{i,j-1}^{(n)}]} \\ \bar{V}_{i-1/2,j}^{(n)} &= \frac{[\Delta x_{i,j}^{(n)} \bar{V}_{i,j}^{(n)} + \Delta x_{i-1,j}^{(n)} \bar{V}_{i-1,j}^{(n)}]}{[\Delta x_{i,j}^{(n)} + \Delta x_{i-1,j}^{(n)}]}\end{aligned}\quad (77)$$

From mathematical considerations, Eqs. (75)–(77) complete the elimination of heat flux terms $\bar{q}_{i,j-1/2}^{(n)}$, $\bar{q}_{i-1/2,j}^{(n)}$ in terms of the principal unknown values of temperature $\bar{T}_{i,j-1,j}^{(n)}$, $\bar{T}_{i,j}^{(n)}$, $\bar{T}_{i,j+1}^{(n)}$ in the discrete elements. Hence, the system of first-order differential equations of the FLIDE method, Eq. (40) or Eq. (45), becomes a consistent system except for the discrete elements on the boundaries of the flow region.

However, from physical considerations, the evaluation of the equivalent eddy diffusivities $\bar{\epsilon}_{i,j-1/2}^{(n)}$, $\bar{\epsilon}_{i-1/2,j}^{(n)}$ and the transport flux terms due to stratification $[Q_{xy}]_{i,j-1/2,j}^{(n)}$, $[Q_{xy}]_{i,j-1/2}^{(n)}$ require considerable insight to the actual flow conditions in the coastal and offshore regions. In comparison with previous far-field studies, the equivalent eddy diffusivities in Eq. (76) represent the most mathematically consistent forms in accordance with the classical turbulent flow considerations. For short time transient, Eulerian mathematical models which consider continuous discharge conditions, Boussinesq-type relations for transport flux terms must necessarily employ the horizontal eddy diffusivity coefficients based on vertical velocity distributions. Particularly in shallow water regions with high vertical mixing conditions, these eddy diffusivity coefficients would be the most appropriate ones to employ in the application of the present model.

Additional transport effects resulting from possible stratified flow conditions must be formulated according to the expressions for Q_{xx} and Q_{yy} , Eq. (71), to supplement the net transport flux terms $\bar{q}_{i,j-1/2}^{(n)}$, $\bar{q}_{i-1/2,j}^{(n)}$ in Eq. (75). As clearly illustrated in this section, stratification effects should not be included in the Boussinesq forms by arbitrarily increasing the value of the eddy diffusivity coefficients.

Kolmogoroff [24] type turbulent diffusion coefficients [21, 22, 23, 25] could only be used in very deep ocean regions where the minimum dimension of the flow region within the solid boundaries, including the bottom, is significantly larger than the characteristic eddy scale dimension in the four-thirds power law.

2.6 Initial and Boundary Half-Point Conditions

Since the mathematical system of the FLIDE method, Eq. (40) or Eq. (45), represents a system of coupled, ordinary, first-order differential equations in time, the initial conditions are straightforward as specified values at any arbitrarily assigned time t_0 for all discrete elements at positions i, j in all regions r ; that is,

$$\bar{T}_{i,j}^{(n)}(t_0) = [\bar{T}_0]_{i,j}^{(n)}, \quad \text{for all } i, j, r. \quad (78)$$

These are necessary and sufficient initial conditions for the solution of the system.

According to the formulation of the FLIDE method there are no boundary conditions, in the sense of boundary value problems, since the mathematical system is based on the integration of conservation laws over the volumes and enclosure surfaces of the discrete elements. The effects of the boundaries are imposed

on the half-point surfaces of the discrete elements located on the physical or hypothetical boundaries of the flow regions in the horizontal $x^{(n)}, y^{(n)}$ plane.

There are four classes of boundary half-point conditions associated with the thermal problems in coastal and offshore regions. In the discussion of these conditions, the discrete elements which have common surfaces with the boundaries will be identified by I, J, K, L to distinguish them from interior elements.

a. **Specified boundary half-point conditions.** This class represents the cases when velocity components and temperature are specified as time-dependent values on the enclosure surfaces of boundary discrete elements. For example, a large river with known flow and temperature conditions discharging into a coastal region. The general conditions can be expressed as

$$\begin{aligned} \bar{U}_{I-1/2,1}^{(n)} &= \bar{U}_V^{(n)}(t), & \bar{V}_{I-1/2,1}^{(n)} &= \bar{V}_V^{(n)}(t), & \bar{T}_{I-1/2,1}^{(n)} &= \bar{T}_V^{(n)}(t), \\ \bar{U}_{L,J-1/2}^{(n)} &= \bar{U}_V^{(n)}(t), & \bar{V}_{L,J-1/2}^{(n)} &= \bar{V}_V^{(n)}(t), & \bar{T}_{L,J-1/2}^{(n)} &= \bar{T}_V^{(n)}(t), \end{aligned} \quad (79)$$

where the subscripts x and y represent the axes which are parallel to the lines that terminate at the discrete element surfaces corresponding to the boundaries (see Fig. 6).

The velocity conditions in Eq. (79) can be omitted, since according to the general assumption the complete flow field is specified in the region; however, they are retained in the formulations for the sake of completeness of the boundary half-point values on the enclosure surfaces.

This class of conditions can also be employed to approximate horizontal intake and discharge conditions of power plants on the shorelines. In such applications, the oceanic flow field may require

Fig. 6. Schematic representation of boundary half-point values for specified flow and temperature conditions.

modification, since the near-field conditions can appreciably influence the flow conditions in the vicinity of the intakes and discharges.

b. **Shoreline boundary half-point conditions.** One of the fundamental advantages of the FLIDE method is its complete flexibility in considering discrete elements of any arbitrary geometry. Consequently, although the general formulation employs a Cartesian index system, (i, j) , the method does not require the approximation of the shoreline geometries by rectangular elements.

The flow region in the immediate vicinity of any shoreline can always be represented by boundary discrete elements of trapezoidal shapes, as shown in Fig. 7. Each boundary element has an enclosure surface exactly corresponding to the section of the shoreline bounded by the arbitrarily specified dimension of the discrete element.

Since the shoreline is assumed to be impermeable and insulated, there is no mass or energy transfer across the enclosure surface of the element which corresponds to the section of the shoreline; that is,

$$\begin{aligned} \bar{U}_{i+1/2,j}^{(r)} = 0, \quad q_{i+1/2,j}^{(r)} = 0 &\Rightarrow \bar{T}_{i+1/2,j}^{(r)} = \bar{T}_{i,j}^{(r)} \\ \bar{V}_{i,j+1/2}^{(r)} = 0, \quad q_{i,j+1/2}^{(r)} = 0 &\Rightarrow \bar{T}_{i,j+1/2}^{(r)} = \bar{T}_{i,j}^{(r)} \end{aligned} \quad (80)$$

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Fig. 7. Schematic representation of boundary discrete elements and half-point values on shorelines.

The temperature conditions in Eq. (80) are consequences of the velocity and heat flux conditions, and they are not essential for the solution of the mathematical system of equations, Eq. (40) or Eq. (45).

c. **Matching conditions for adjacent regions.** Since the general formulation of the FLIDE method can consider numerous flow regions with arbitrarily assigned size and number distributions for the discrete elements, it is necessary to consider special matching conditions for the boundary elements of two adjacent regions. As shown in Fig. 8a, the common dividing boundary between two adjacent regions r and p can correspond to the enclosure surfaces of different numbers of discrete elements in different regions with unknown values on the boundary half-points of their enclosure surfaces. To match the conditions, the

regions are extended with additional, imaginary discrete elements which have the approximate overall conditions of the real elements of the adjacent regions, as shown in Fig. 8b. Considering the two regions r and p in Fig. 8, the sizes of the discrete elements in the regions are related to each other as

$$\Delta y_{l,j}^{(r)} = \sum_{n=0}^N \Delta y_{K,\lambda+n}^{(p)} \quad (81)$$

Hence, the size and the conditions of the additional imaginary discrete element $l+1, j$ in region r can be readily determined as the weighted averages of the size and conditions of the real elements $K, \lambda+n$ in region p :

For element $l+1, j$ in region r :

$$\begin{aligned} \Delta x_{l+1,j}^{(r)} &= \frac{1}{\Delta y_{l,j}^{(r)}} \sum_{n=0}^N \Delta y_{K,\lambda+n}^{(p)} \Delta x_{K,\lambda+n}^{(p)}, \\ \bar{U}_{l+1,j}^{(r)} &= \frac{1}{\Delta y_{l,j}^{(r)}} \sum_{n=0}^N \Delta y_{K,\lambda+n}^{(p)} \bar{U}_{K,\lambda+n}^{(p)}, \\ \bar{V}_{l+1,j}^{(r)} &= \frac{1}{\Delta y_{l,j}^{(r)}} \sum_{n=0}^N \Delta y_{K,\lambda+n}^{(p)} \bar{V}_{K,\lambda+n}^{(p)}, \\ \bar{T}_{l+1,j}^{(r)} &= \frac{1}{\Delta y_{l,j}^{(r)}} \sum_{n=0}^N \Delta y_{K,\lambda+n}^{(p)} \bar{T}_{K,\lambda+n}^{(p)}, \end{aligned} \quad (82)$$

where N represents the number of discrete elements in region p which have common enclosure surfaces with the single element in region r . The conditions for the additional imaginary elements in region p are identically the conditions of the boundary element in region r :

For elements $K+1, \lambda+n$ in region p , $0 \leq n \leq N$:

$$\begin{aligned} \Delta x_{K+1,\lambda+n}^{(p)} &= \Delta x_{l,j}^{(r)}, \quad \bar{U}_{K+1,\lambda+n}^{(p)} = \bar{U}_{l,j}^{(r)}, \\ \bar{V}_{K+1,\lambda+n}^{(p)} &= \bar{V}_{l,j}^{(r)}, \quad \bar{T}_{K+1,\lambda+n}^{(p)} = \bar{T}_{l,j}^{(r)}. \end{aligned} \quad (83)$$

Equations (82) and (83) represent the necessary matching conditions for the consistency of the mathematical systems, Eq. (40), in regions r and p for the common dividing boundary normal to the axes $x^{(r)}$ and $x^{(p)}$.

The necessary matching conditions for two regions r and s with a common dividing boundary normal to the axes $y^{(r)}$ and $y^{(s)}$ can be readily formulated similar to Eqs. (82) and (83) as:

For element $i, J+1$ in region r :

$$\Delta y_{i,J+1}^{(r)} = \frac{1}{\Delta x_{i,J}^{(r)}} \sum_{m=0}^M \Delta x_{k,m,I}^{(s)} \Delta y_{k,m,I}^{(s)},$$

$$\bar{u}_{i,j+1}^{(r)} = \frac{1}{\Delta x_{i,j}^{(r)}} \sum_{m=0}^M \Delta x_{k+m,L}^{(s)} \bar{u}_{k+m,L}^{(s)} \quad (84)$$

$$\bar{v}_{i,j+1}^{(r)} = \frac{1}{\Delta x_{i,j}^{(r)}} \sum_{m=0}^M \Delta x_{k+m,L}^{(s)} \bar{v}_{k+m,L}^{(s)}$$

$$\bar{T}_{i,j+1}^{(r)} = \frac{1}{\Delta x_{i,j}^{(r)}} \sum_{m=0}^M \Delta x_{k+m,L}^{(s)} \bar{T}_{k+m,L}^{(s)}$$

For elements $k+m, L+1$ in region s , $0 \leq m \leq M$:

$$\Delta y_{k+m,L+1}^{(s)} = \Delta y_{i,j}^{(r)}, \quad \bar{u}_{k+m,L+1}^{(s)} = \bar{u}_{i,j}^{(r)} \quad (85)$$

$$\bar{v}_{k+m,L+1}^{(s)} = \bar{v}_{i,j}^{(r)}, \quad \bar{T}_{k+m,L+1}^{(s)} = \bar{T}_{i,j}^{(r)}$$

where M represents the number of discrete elements in region s which have common enclosure surfaces with the single element in region r .

It should be noted that the matching of the velocity conditions considers the necessary direction changes from real to imaginary elements.

The matching formulation for two adjacent regions, Eqs. (82-85), determine the necessary and sufficient conditions for the solution of the FLIDE mathematical system in all flow regions. Although the conditions do not directly determine the half-point values on the boundary enclosure surfaces, the additional imaginary elements enable the application of the standard half-point, second upwind differencing forms for the interior elements, Eqs. (46) and (47), directly for the evaluation of the half-point values on the enclosure surfaces of the boundary elements. Hence, the matching procedure based on additional imaginary elements is indeed equivalent to specifying the half-point values on all boundary element surfaces in all regions.

d. Open region boundary half-point conditions. A mathematical far-field thermal model for coastal and offshore regions must be capable of considering hypothetical boundaries which separate the regions of interest from the rest of the oceans. Although this is not essential from mathematical considerations, since all world oceans can be treated as a flow region with closed or matching boundaries, from practical computational considerations it is essential in the assessment of the thermal impact of power plants. Hence, in the general formulation of the FLIDE model it is necessary to develop the appropriate boundary half-point conditions for open regions (see Fig. 2).

In general, an imaginary boundary which divides a flow region into two parts does not possess any special physical property that can be formulated as a mathematical boundary condition. Hence, for open regions, well-posed boundary value problems, in the sense of Dirichlet, Neumann, or mixed conditions, cannot be formulated without additional plausible assumptions about the behavior of the solutions on the hypothetical boundaries.

In the case of analytical methods, the open boundary conditions are usually specified by the behavior of the solutions at infinity. Since the FLIDE method, or any other finite difference type of method, cannot treat infinity conditions without artificial straining of the coordinates, it is necessary to consider other conditions which can be employed on the boundaries of finite open regions.

One of the seemingly natural assumptions is to specify the temperature on the hypothetical boundary as the local ambient ocean temperature. This condition is definitely unacceptable, since it may cause extremely high temperature gradients near the hypothetical boundaries in order to drop the temperature distributions to ambient conditions. Consequently, it could produce unrealistic results with excessive temperature variations in short distances. Furthermore, the solutions become extremely sensitive to the dimensions of the regions bounded by the imaginary boundaries. This consequence of the assumption is also unacceptable, since the location of the imaginary boundaries must be completely arbitrary from physical considerations.

Another seemingly natural condition is to assume that there exists no thermal diffusion across an imaginary boundary. This condition produces exactly the opposite behavior in the solutions with excessively high temperature values which tend to uniform distributions with increasing time. Furthermore, the solutions again become extremely sensitive to the dimensions of the regions bounded by the imaginary boundaries. Hence, from physical considerations of the thermal energy transport phenomenon, this condition is also unacceptable.

It is evident that the imaginary boundary of an open region must be sufficiently far from the immediate range of influence of the intakes and discharges in the region. Hence, in the neighborhood of the boundary, the variations in temperature will be controlled by the convection and diffusion phenomena in the fluid and by the heat transfer conditions on the surface and possibly at the bottom. Since all these effects will have almost the same influence on the thermal energy transport phenomenon on both sides of the imaginary boundary, it is very plausible to assume that the variations in temperature normal to the boundary are approximately the same on both sides of the boundary.

This assumption is schematically represented in Fig. 9 in terms of a boundary element i, j in the flow region and an additional imaginary discrete element $i + 1, j$ outside the hypothetical boundary.

From mathematical considerations, the assumption corresponds to a linear variation of temperature in the real element i, j and the additional imaginary element $i + 1, j$ across the half-point boundary $i + 1/2, j$. Hence, the value of temperature in the imaginary element $\bar{T}_{i+1,j}^{(r)}$ can be expressed in terms of the temperature values in the boundary element as

$$\frac{\bar{T}_{i+1,j}^{(r)} - \bar{T}_{i,j}^{(r)}}{\Delta x_{i,j}^{(r)}} = 2 \frac{\bar{T}_{i,j}^{(r)} - \bar{T}_{i-1/2,j}^{(r)}}{\Delta x_{i,j}^{(r)}} \quad (86)$$

Rearranging Eq. (86),

$$\bar{T}_{i+1,j}^{(r)} = 3\bar{T}_{i,j}^{(r)} - 2\bar{T}_{i-1/2,j}^{(r)} \quad (87)$$

The half-point temperature $\bar{T}_{i-1/2,j}^{(r)}$ can be readily eliminated according to the linear averaging form, Eq. (46), for the internal discrete elements; that is,

$$\bar{T}_{i-1/2,j}^{(r)} = \frac{[\Delta x_{i,j}^{(r)} \bar{T}_{i-1,j}^{(r)} + \Delta x_{i-1,j}^{(r)} \bar{T}_{i,j}^{(r)}]}{[\Delta x_{i,j}^{(r)} + \Delta x_{i-1,j}^{(r)}]} \quad (88)$$

Substituting Eq. (88) in Eq. (87) and rearranging,

Fig. 9. Schematic representation of the open-end boundary conditions.

$$T_{i+1,j}^{(r)} = T_{i,j}^{(r)} + \frac{2 \Delta x_{i,j}^{(r)}}{[\Delta x_{i,j}^{(r)} + \Delta x_{i-1,j}^{(r)}]} [T_{i,j}^{(r)} - T_{i-1/2,j}^{(r)}] \quad (89)$$

which represents the value of temperature in the external imaginary discrete element.

The velocity conditions for the additional imaginary elements need not be formulated, since according to the general assumption of the mathematical model, the flow field is completely known in the region. Hence, by defining the thermal region sufficiently small, the imaginary elements can always be located exterior to the thermal region but within the bounds of the domain of definition of the flow field.

The conditions for the hypothetical open boundaries normal to the $y^{(r)}$ direction can be readily formulated similar to Eq. (89) as

$$T_{i,j+1}^{(r)} = T_{i,j}^{(r)} + \frac{2 \Delta y_{i,j}^{(r)}}{[\Delta y_{i,j}^{(r)} + \Delta y_{i,j-1}^{(r)}]} [T_{i,j}^{(r)} - T_{i,j-1}^{(r)}] \quad (90)$$

The conditions for open region boundaries, Eqs. (89) and (90), represent the necessary and sufficient conditions for the solution of the FLIDE system of differential equations, Eq. (40). Since the general half-point, second upwind differencing forms for the internal elements, Eqs. (46) and (47), can be directly employed to the real and imaginary elements, the formulation of the temperature conditions, Eqs. (89) and (90), is equivalent to specifying the half-point values on all boundary element surfaces corresponding to the hypothetical boundary of the open region.

From mathematical considerations, the assumption of the rates of change of temperature being approximately equal on both sides of an imaginary open region boundary corresponds to direct linear extrapolation of the temperature value on the boundary in terms of the temperature values in the last two interior discrete elements in the region. Hence, the assumption is only valid if the second derivative of temperature along a direction normal to the boundary is negligible. Considering the differential form of the energy equation under flow conditions normal to the imaginary boundary, for approximately constant eddy diffusivity conditions, the vanishing second derivative of temperature corresponds to the elimination of the diffusion transport effect. Hence, for negligible surface heat transfer, the assumption implies that the substantial derivative of temperature vanishes on the imaginary open region boundary. Hence, from physical considerations, the mathematical extrapolation boundary condition implies that the temperature remains approximately constant along the motion of the fluid as it crosses the imaginary boundary. It must be emphasized that this condition is an approximation which is only true in regions where the temperature gradients are not significant; hence, it can only be applied as a realistic open region boundary condition at sufficiently large distances from the neighborhood of the intakes and discharges in the flow region.

From computational considerations, the conditions for the imaginary boundary, Eqs. (89) and (90), are very acceptable. The temperature solutions are extremely insensitive to the location of the boundary beyond a critical distance from the intakes and discharges.

Hence, the half-point values on the enclosure surfaces of the boundary discrete elements can be completely determined, from appropriate physical and matching conditions, to eliminate the associated inconsistency problems in the general mathematical system of the FLIDE method.

2.7 Surface and Bottom Heat Transfer Conditions

To render the general mathematical system of the FLIDE method, Eqs. (40) or (45), consistent, the terms associated with the bottom and surface heat transfer conditions must be determined either as specified quantities or in terms of the principal unknown values of temperature in the discrete elements.

In the application of the model, if the total depth is considered as the thermally affected vertical range of the thermal plume, then the bottom heat and mass transfer effects become negligible; that is,

$$(T_b^*)_{i,j} = \text{arbitrary}, \quad (\dot{g}_b^*)_{i,j}^{(r)} = 0, \quad (q_b^*)_{i,j}^{(r)} = 0, \quad (91)$$

where the temperature of the water inflow from the bottom $(\bar{T}_b^*)_{i,j}^{(r)}$ need not be specified, since the flow rate $(\dot{g}_b^*)_{i,j}^{(r)}$ is assumed to be zero everywhere in the region.

If the thermal plume is highly stratified, in such a manner that only a portion of the depth near the surface is affected, then the terms associated with the bottom heat transfer conditions can be employed to represent the conditions on the bottom surface of the stratified layer.

It should be emphasized that a stratified thermal plume in the far-field region does not behave like a surface jet in the near-field region. Hence, the bottom heat transfer conditions should not be misinterpreted as entrainment, since no appreciable mass transfer velocity $(\dot{g}_b^*)_{i,j}^{(r)}$, in the sense of dilution, can exist between the bottom and the stratified layer. Indeed, for shallow water regions, the bottom flow rate can be assumed as negligible for both vertically mixed and stratified thermal plumes.

The analysis of the heat transfer conditions at the bottom of a stratified thermal plume from a continuous discharge requires extensive study of the vertical turbulent diffusion phenomenon with density variations and under oceanic flow conditions. Furthermore, since the ocean bottom can always be assumed as impermeable and insulated, such a study will require at least a two-layer thermal model to incorporate

the severe variations in vertical direction. Such an analysis is definitely outside the scope of the present development of the FLIDE model.

However, for shallow water regions and for sufficiently deep stratified surface thermal plumes, the bottom heat transfer effect ($q_b^{*f(r)}$) can be neglected as a conservative assumption. First, near the bottom of the ocean, particularly inside the laminar sublayer, the velocity and turbulent diffusion effects become extremely low, comparable to the molecular diffusion; hence, the vertical diffusion which controls the bottom heat transfer conditions on the plume becomes negligible. Second, if there exists heat transfer between the stratified plume and the bottom layer, this will result in the heating of the bottom layer until thermal quasi-steady state conditions are reached in the flow region after sufficiently many tidal cycles. This effect will ultimately decrease the vertical temperature gradients; hence the bottom heat transfer conditions of the stratified thermal plume, particularly in the neighborhood of the discharges in shallow water regions.

From mathematical considerations of the FLIDE method, the only requirement is the specification of bottom heat transfer effects in terms of the flow conditions, temperatures in the discrete elements, and possibly, a characteristic temperature of the ocean, T_{oc}^* ; that is,

$$\begin{aligned}(T_b^{*f(r)}) &= (T_b^{*f(r)}) \left[\bar{D}_{i,j}^{(r)}, \bar{U}_{i,j}^{(r)}, \bar{V}_{i,j}^{(r)}, \bar{T}_{i,j}^{(r)}, T_{oc}^* \right], \\ (\dot{g}_b^{*f(r)}) &= (\dot{g}_b^{*f(r)}) \left[\bar{D}_{i,j}^{(r)}, \bar{U}_{i,j}^{(r)}, \bar{V}_{i,j}^{(r)} \right], \\ (q_b^{*f(r)}) &= (q_b^{*f(r)}) \left[\bar{D}_{i,j}^{(r)}, \bar{U}_{i,j}^{(r)}, \bar{V}_{i,j}^{(r)}, \bar{T}_{i,j}^{(r)}, T_{oc}^* \right].\end{aligned}\quad (92)$$

Equation (92) represents the general functional forms of bottom heat transfer and mass inflow velocity conditions from mathematical considerations. The determination of the specific forms requires careful study of the physical stratification and turbulent diffusion phenomena associated with actual discharge and oceanic flow conditions in a region.

Hence, in the application of the FLIDE mathematical model to a specific problem for the assessment of the thermal impact of a power plant discharge in a coastal or offshore region, if sufficient information is not available for accurate determination of the bottom heat transfer effects on the stratified plume, the conservative assumption of negligible heat and mass transfer conditions, Eq. (91), is inherently more acceptable and reliable than arbitrarily assigning approximate values to these terms in the thermal analysis.

In the formulation of surface heat transfer conditions, $(\dot{q}_s^{*f(r)})$ represents the velocity of the general surface mass transfer effects, which consist of evaporation and rainfall. From thermal considerations, the evaporation effect can be incorporated in the surface heat flux term ($q_s^{*f(r)}$); hence the surface mass transfer velocity $(\dot{g}_s^{*f(r)})$ only represents the known rainfall conditions in the region; that is,

$$(\dot{g}_s^{*f(r)}) = (\dot{g}_s^{*f(r)})(t) \quad (93)$$

as a specified function of time t . Consequently, the temperature $(T_s^{*f(r)})$ associated with the surface mass transfer conditions represents the temperature of the rainfall; that is,

$$(T_s^{*f(r)}) = T_s^*(t) \quad (94)$$

as a specified function of time t . It should be noted that the velocity, or the flow rate per unit area, of rainfall on the surface $(\dot{g}_s^{*f(r)})$ is retained as a distribution on the horizontal plane to include the effects of

possible variations in rainfall over large regions. However, the rainfall temperature T_s^* is specified as a function of time only, since daily and seasonal changes are approximately uniform over areas which are significantly larger than the flow regions considered in thermal impact analyses.

The formulation of the surface heat flux term $(q_{s,i,j}^*)^{(r)}$ is based on approximate modeling of four fundamental modes of heat transfer between the ocean surface and the atmosphere. Since the present mathematical system considers detailed, short-time transient temperature variations, surface heat transfer models based on the equilibrium temperature concept, see, for example, Edinger and Geyer [51], will not be practical from computational considerations. However, the present surface heat flux model will employ approximate formulations very similar to ones considered in previous studies, for example, Wada [39], Loziak, Anderson, and Belytschko [40], Edinger and Geyer [51], etc., for the different fundamental modes which constitute the overall heat transfer conditions on the ocean surface.

a. **Daytime solar irradiation $(q_{sR,i,j}^*)^{(r)}$.** The maximum solar irradiation Q_{sR}^* on the ocean surface can be readily obtained from available measured data, for example, Table 7-35 [52], based on the latitude of the flow region under consideration. Assuming midnight as the zero hour and sunup and sundown at approximately six and eighteen hours, respectively, the maximum solar irradiation value Q_{sR}^* at noon can be employed to obtain an expression for short-time solar irradiation $(q_{sR,i,j}^*)^{(r)}$ on the ocean surface as

$$(q_{sR,i,j}^*)^{(r)} = \begin{cases} 0; & 0 < t < 6 \text{ hr}, \quad 18 \text{ hr} < t < 24 \text{ hr}, \\ \bar{Q}_{sR} \sin \left[\frac{\pi}{12 \text{ hr}} (t - 6 \text{ hr}) \right]; & 6 \text{ hr} \leq t \leq 18 \text{ hr}. \end{cases} \quad (95)$$

The cloud cover conditions which reduce the hourly solar irradiation $(q_{sR,i,j}^*)^{(r)}$ can be approximately accounted for in the amplitude Q_{sR}^* as time dependent variations; that is,

$$\bar{Q}_{sR} = \bar{Q}_{sR}(t) \leq Q_{sR}^* (\text{maximum}). \quad (96)$$

The expression for hourly solar irradiation $(q_{sR,i,j}^*)^{(r)}$, Eq. (95), is based on a simple, discontinuous sine function form which only approximately represents the actual rate of solar energy gained by the water per unit area of the surface.

b. **Daytime and nighttime black body back radiation $(q_{BR,i,j}^*)^{(r)}$.** This mode of radiation heat transfer depends on the local surface temperature $(T_{s,i,j})^{(r)}$, and it can be approximated by the Stefan-Boltzmann law for radiation to zero temperature space depending on surface area factors; that is,

$$(q_{BR,i,j}^*)^{(r)} = -\sigma \Gamma_e \Gamma_A \left[(T_{s,i,j})^{(r)} + 460^\circ \text{R} \right]^4. \quad (97)$$

where

$$\sigma = 0.222 \times 10^{-8} \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{°R})^{-4}$$

= Stefan-Boltzmann constant.

Γ_e = dimensionless factor based on the emissivities (absorptivities) of the water surface and the sky cloud cover.

Γ_A = geometric shape factors related to the orientation of radiating surfaces of water and cloud cover.

The maximum value for both factors Γ_e and Γ_A is unity, which represents the case of a small black body directly radiating to semispherical form of infinite space at zero temperature. For the ocean surface and the sky, this condition can never be realized; hence the values for the dimensionless factors are

appropriately reduced to represent the water surface and sky cloud conditions. For coastal and offshore regions, the values for the radiation factors Γ_c and Γ_A can be considered as constants within certain bounds; that is,

$$0.6 \leq \Gamma_c \leq 0.9, \quad 0.1 \leq \Gamma_A \leq 0.8. \quad (98)$$

Accurate determination of the radiation shape factors Γ_c and Γ_A requires extensive analyses in radiation heat transfer, which is not within the scope of the present study.

c. Surface evaporation heat transfer $(q_{EV}^*)_{i,j}^{(n)}$. Although this particular mode of surface heat transfer is significant in cooling the water, unfortunately, for coastal and offshore regions the applicability of the presently available mathematical models is not established. The elaborate forms of these models, for example, [51], contain numerous parameters which were determined from field data measured over lake surfaces.

Considering the rate of evaporation from a water surface to air as a simple relation of the form

$$\dot{m}_{EV} = \gamma_{EV}(1 - R_H) \left[1 + \frac{V_w}{(V_w)_{ref}} \right] (T_s - T_a), \quad (99)$$

where

$\dot{m}_{EV}$ = mass rate of evaporation per unit area,

γ_{EV} = coefficient of evaporation to be determined,

R_H = relative humidity of air,

V_w = wind speed,

$(V_w)_{ref}$ = reference value of wind speed associated with coefficient of evaporation,

T_s = water surface temperature,

T_a = air temperature.

The values for the coefficient of evaporation γ_{EV} and reference value of wind speed $(V_w)_{ref}$ can be approximately determined from available experimental data in engineering, for example, extrapolating the data in Table 5-44 [52] to the range of environmental water surface and atmospheric air conditions.

$$\gamma_{EV} \cong 0.0028 \text{ lb hr}^{-1} \text{ ft}^{-2} (\text{°F})^{-1}, \quad (V_w)_{ref} \cong 5 \text{ ft/sec}. \quad (100)$$

Assuming that all of the latent heat of evaporation h_{fg} is supplied by the water, the rate of heat transfer from the surface due to evaporation becomes

$$(q_{EV}^*)_{i,j}^{(n)} = \begin{cases} -h_{EV}^* (1 - R_H) \left[1 + \frac{V_w}{(V_w)_{ref}} \right] |(\bar{T}_s)_{i,j}^{(n)} - \bar{T}_a(t)|, & \text{for } (\bar{T}_s)_{i,j}^{(n)} > \bar{T}_a(t), \\ 0, & \text{for } (\bar{T}_s)_{i,j}^{(n)} \leq \bar{T}_a(t). \end{cases} \quad (101)$$

where

$$h_{EV}^* = h_{fg} \gamma_{EV} \cong 3.0 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{°F})^{-1}. \quad (102)$$

The nonlinear cutoff condition in Eq. (101) is imposed to prevent the possible heating result from the form of the equation when the air temperature is higher than the water temperature. This particular heat transfer effect should not be considered in the evaporation mode of surface heat transfer.

d. Surface convection heat transfer $(q_{CN}^*)_{i,j}^{(r)}$. The convection heat transfer flux is also approximated in a form similar to evaporation heat flux; that is

$$(q_{CN}^*)_{i,j}^{(r)} = -h_{CN}^* \left[1 + \frac{V_w}{(V_w)_{ref}} \right] \left| (\bar{T}_s)_{i,j}^{(r)} - \bar{T}_a(t) \right|, \quad (103)$$

where the convection surface film coefficient h_{CN}^* can be determined from experimental data, for example, Table 5-33 [52], for turbulent flow of air at atmospheric pressure over a flat surface as

$$h_{CN}^* \cong 1.0 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{°F})^{-1}. \quad (104)$$

The expressions for four different modes of surface heat transfer, Eqs. (95), (97), (101), and (103), were employed to compute daily heat transfer rates per unit area from lake surfaces under observed atmospheric and water conditions. The results after conversion to equilibrium temperature forms were verified by the results of other surface heat transfer models and observations [48].

The net rate of heat transfer per unit area across the water surface, $(q_s^*)_{i,j}^{(r)}$, is the sum of the different modes; that is,

$$(q_s^*)_{i,j}^{(r)} = (q_{SR}^*)_{i,j}^{(r)} + (q_{BR}^*)_{i,j}^{(r)} + (q_{EV}^*)_{i,j}^{(r)} + (q_{CN}^*)_{i,j}^{(r)}; \quad (105)$$

hence, the unknown terms for the surface heat transfer conditions, $(\bar{g}_s^*)_{i,j}^{(r)}$, $(\bar{T}_s^*)_{i,j}^{(r)}$ and $(q_s^*)_{i,j}^{(r)}$, are completely eliminated from the general mathematical system of FLIDE equations in terms of known specified time variations and temperature conditions $\bar{T}_{i,j}^{(r)}$ in the discrete elements.

It should be noted that more elaborate surface heat transfer models can be readily incorporated in the general formulation of the FLIDE method as they become more reliable in representing the actual heat transfer conditions on the ocean surface in coastal and offshore regions.

2.8 Formulation of Intake and Discharge Conditions

The volumetric flow rates for intakes $\dot{G}_m$ and discharges $\dot{G}_n$, Eqs. (40) and (45), are determined according to the distribution of the discrete elements, the locations of the intake and discharge structures, and the operational conditions of the power plants in the region. They do not necessarily represent the net condenser flow rates of the power plants.

If the structure of an intake or a discharge spans more than one discrete element, the net flow rate must be divided into different parts which represent different flow rates in different elements. Consequently, a power plant having only one intake and one discharge may be represented by numerous intake and discharge flow rates $\dot{G}_m$ and $\dot{G}_n$ in the formulation of the FLIDE model. The flow rates are specified as time-dependent variations, i.e.,

$$\dot{G}_m = \dot{G}_m(t), \quad \dot{G}_n = \dot{G}_n(t), \quad (106)$$

based on the operational conditions of the power plants.

The intake temperature T_m represents the temperature of the water which enters the intake in the discrete element (i,m) , $j(m)$ which contains the intake m . If severe thermally stratified conditions exist in the element, then, the instantaneous value of the intake temperature T_m may depend on the shape, orientation, depth, and velocity of the intake. Hence, the intake temperature can be expressed as

$$T_m = \bar{T}_{i,j}^{(r)} + (\Delta T_m)_{i,j,m}^{(r)} \quad (107)$$

where $(\Delta T_m)_{i,j,m}^{(r)}$ represents the difference between the intake temperature and the local mean temperature $\bar{T}_{i,j}^{(r)}$ in the discrete element of the intake m . Accurate determination of this difference requires a detailed and sophisticated near-field analysis of the behavior of the intake flow under time-dependent oceanic currents and stratification conditions. Such an analysis is outside the scope of this study and very possibly beyond the limits of the existing near-field models employed in environmental impact studies.

However, as discussed earlier in the development of the general FLIDE equations, it is more plausible and realistic to assume the intake temperature T_m as equal to the local temperature $\bar{T}_{i,j}^{(r)}$; that is,

$$T_m \equiv \bar{T}_{i,j}^{(r)} \quad (\Delta T_m)_{i,j,m}^{(r)} = 0 \quad (108)$$

for a conservative assessment of the thermal impact of a power plant discharge. Hence, the near-field analysis of the intake conditions can be neglected without introducing appreciable error in the solution of the far-field thermal problem.

The discharge temperature T_n depends on the conditions at the inlet and outlet to the condenser where the temperature rise is determined according to the design specifications. For a condenser with a multi-intake structure which spans more than one discrete element, the inlet temperature to the condenser $(T_{in})_{CND}$ must be determined according to the flow rates $\dot{G}_m$ and local intake temperatures T_m in the discrete elements; that is,

$$(\bar{T}_{in})_{CND} = \frac{1}{\dot{G}_{CND}} = \sum_{m=1}^M \dot{G}_m T_m \quad \dot{G}_{CND} = \sum_{m=1}^M \dot{G}_m \quad (109)$$

Then, the discharge temperature T_n is determined according to the designed condenser temperature increase ΔT_{CND} as

$$T_n = (\bar{T}_{in})_{CND} + \Delta T_{CND} \quad (110)$$

provided there exists no in-line cooling system between the condenser and the discharge. In a case when an additional cooling system, for example, cooling tower, etc., is employed before discharging the condenser cooling water into the ocean, the present formulation can be readily modified, by appropriately changing the discharge flow rates $\dot{G}_n$ and the condenser temperature rise ΔT_{CND} , in its application to the particular thermal problem.

It should be emphasized that the present formulation of the intake and discharge conditions does not employ the artificial concept of specifying the percentages of recirculating discharge flows which can reach the intakes. Arbitrarily assigned values for recirculation cannot be justified under periodic, reversing flow conditions. The short-time, fully transient formulation of the FLIDE mathematical model correctly incorporates the complete recirculation effect according to the convection and diffusion phenomena controlled by the oceanic flow conditions in coastal and offshore regions.

2.9 Mathematical Properties, Stability Analysis, and Numerical Solution of FLIDE Equations

The general mathematical formulation of the FLIDE model represents an initial-value problem associated with a nonautonomous system of simultaneous ordinary differential equations; that is,

$$\frac{\partial \bar{T}_{i,j}^{(r)}}{\partial t}(t) = F_{i,j}^{(r)} \left[\bar{T}_{i-1,j}^{(r)}(t), \bar{T}_{i+1,j}^{(r)}(t), \bar{T}_{i,j}^{(r)}(t), \bar{T}_{i,j-1}^{(r)}(t), \bar{T}_{i,j+1}^{(r)}(t) \right] + G_{i,j}^{(r)}(t), \quad (111)$$

$$\bar{T}_{i,j}^{(r)}(t_0) = [t_0]_{i,j}^{(r)}, \quad \text{for all } r, i, j. \quad (112)$$

According to the formulations of integrated conservation equations, Eq. (40), internal discrete-element half-point values, Eqs. (46) and (47), diffusion heat flux terms, Eqs. (75) and (76), boundary half-point conditions, Eqs. (79)–(85) and (88)–(90), surface and bottom heat transfer conditions, Eqs. (91)–(97) and (101)–(105), and intake and discharge conditions, Eqs. (106)–(110), and since the flow field conditions $\bar{U}_{i,j}^{(r)}$, $\bar{V}_{i,j}^{(r)}$, $\bar{W}_{i,j}^{(r)}$ are finite for all time, the right-hand-side functions $F_{i,j}^{(r)}$ and $G_{i,j}^{(r)}$ satisfy the Dirichlet conditions, that is, sectionally continuous with a finite number of finite discontinuities, in the neighborhood of the initial point, t_0 , $[\bar{T}_0]_{i,j}^{(r)}$; hence, a unique set of continuous solutions $\bar{T}_{i,j}^{(r)}$, as extensions of the initial conditions $[\bar{T}_0]_{i,j}^{(r)}$, exists for the initial-value problem, Eqs. (111) and (112), for $t > t_0$.

Furthermore, since $G_{i,j}^{(r)}$ is at least periodic for all time t due to solar irradiation, the mathematical system, Eq. (111), is nonautonomous for all time t ; consequently, there exist no singular points in the solution space $t, \bar{T}_{i,j}^{(r)}$.

Hence, unique and continuous solutions for temperature $\bar{T}_{i,j}^{(r)}(t)$ can be obtained for the initial-value problem, Eqs. (111) and (112), of the FLIDE (fluid-in-discrete-element) mathematical model for all time by either analytical or numerical integration methods.

Considering R number of regions r with minimum $(I_r)_{\min}$, $(J_r)_{\min}$ and maximum $(I_r)_{\max}$, $(J_r)_{\max}$ number of integers i, j , which specify the positions of the discrete elements in the regions, the total number N_{FLIDE} of first-order differential equations associated with the FLIDE mathematical model is bounded by

$$\sum_{r=1}^R (I_r)_{\min} (J_r)_{\min} \leq N_{\text{FLIDE}} \leq \sum_{r=1}^R (I_r)_{\max} (J_r)_{\max}. \quad (113)$$

Since for complex shoreline boundaries a Cartesian system of coordinates cannot divide the flow regions into equal numbers of discrete elements along each line, the minimum and maximum numbers for the integers $(I_r)_{\min}$, $(J_r)_{\min}$ and $(I_r)_{\max}$, $(J_r)_{\max}$, respectively, must be considered as unequal except for rectangular regions. Even for a rather coarse division of the flow region into one region, $R = 1$, with minimum number of integers as $(I_r)_{\min} = 10$ and $(J_r)_{\min} = 10$, the total number of equations is $N_{\text{FLIDE}} > 100$; hence, analytical solutions to the system, although theoretically possible, are extremely cumbersome both from formulation and computational considerations.

However, considering the general form of the mathematical system, Eqs. (111) and (112), it is evident that the simultaneous differential equations are very weakly coupled since regardless of the total number of equations N_{FLIDE} , each differential equation for $\bar{T}_{i,j}^{(r)}$ depends only on five discrete-element temperatures, $\bar{T}_{i-1,j}^{(r)}$, $\bar{T}_{i+1,j}^{(r)}$, $\bar{T}_{i,j}^{(r)}$, $\bar{T}_{i,j-1}^{(r)}$, and $\bar{T}_{i,j+1}^{(r)}$. Hence, the simultaneous system of ordinary, first-order differential equations, Eq. (111), is extremely suitable for numerical integration from the known set of initial conditions, Eq. (112), by well-established marching computational schemes. Since any marching numerical

method can only approximate the actual exact solution with some truncation and/or roundoff error at each time step, it is essential to guarantee the stability of the differential equations of the FLIDE method.

Before presenting a detailed stability analysis of the mathematical system, it would be appropriate to consider the relation between the semidiscretized, finite form of the FLIDE (fluid-in-discrete-element) model and the classical infinitesimal, partial differential equation for the conservation of energy principle. As discussed earlier, the governing equations of the simulation-type FLIDE and FLIC mathematical models cannot be obtained directly from the partial differential form of the energy equation. Since the second upwind differencing or donor cell forms for the convective terms are based on physical considerations rather than rigorous mathematical formulations, classical finite-difference approximation based on Taylor's series expansion of the energy partial differential equation does not result in the semidiscrete form of the FLIDE equations. However, the converse is definitely true; that is, the infinitesimal limit, $\Delta x_{i,j}^{(n)} \rightarrow 0$, $\Delta y_{i,j}^{(n)} \rightarrow 0$, of the FLIDE mathematical system, Eq. (40), indeed leads to the conservative form of the partial differential equation for the energy conservation principle; that is,

$$\begin{aligned} \frac{\partial T^{(n)}}{\partial t} = \frac{1}{D^{(n)}} & \left\{ -\frac{\partial}{\partial x} [D^{(n)} U^{(n)} T^{(n)}] - \frac{\partial}{\partial y} [D^{(n)} V^{(n)} T^{(n)}] - \frac{\partial}{\partial x} [D^{(n)} q_x^{(n)}] \right. \\ & - \frac{\partial}{\partial y} [D^{(n)} q_y^{(n)}] + \dot{E}_m T_m + \dot{E}_n T_n + \frac{1}{\rho C^0} [(q_b^0 f^{(n)} + (q_s^0 f^{(n)} \\ & + \frac{D^{(n)} q_x^{(n)}}{\rho C^0} + (q_b^0 f^{(n)} (T_b^0 f^{(n)} + (q_s^0 f^{(n)} (T_s^0 f^{(n)} \left. \right\} + \frac{T^{(n)}}{D^{(n)}} \left[\frac{\partial}{\partial x} [D^{(n)} E^{(n)}] \right. \\ & \left. \left. + \frac{\partial}{\partial y} [D^{(n)} V^{(n)}] \left[\dot{E}_m + \dot{E}_n + (q_b^0 f^{(n)} + (q_s^0 f^{(n)} \right] \right] \right\}. \quad (114) \end{aligned}$$

Similarly, taking the limit, $\Delta x_{i,j}^{(n)} \rightarrow 0$, $\Delta y_{i,j}^{(n)} \rightarrow 0$, of the mass conservation equation, Eq. (22), of the FLIDE model,

$$\frac{\partial D^{(n)}}{\partial t} = \left\{ \frac{\partial}{\partial x} [D^{(n)} U^{(n)}] + \frac{\partial}{\partial y} [D^{(n)} V^{(n)}] - [\dot{E}_m + \dot{E}_n + (q_b^0 f^{(n)} + (q_s^0 f^{(n)}] \right\}, \quad (115)$$

where $\dot{E}_m$ and $\dot{E}_n$ represent the intake and discharge flow rates per unit area.

Substituting Eq. (115) in Eq. (114) and rearranging,

$$\begin{aligned} \frac{\partial}{\partial t} [D^{(n)} T^{(n)}] + \frac{\partial}{\partial x} [D^{(n)} U^{(n)} T^{(n)}] + \frac{\partial}{\partial y} [D^{(n)} V^{(n)} T^{(n)}] \\ = \frac{\partial}{\partial x} [D^{(n)} q_x^{(n)}] - \frac{\partial}{\partial y} [D^{(n)} q_y^{(n)}] + \frac{1}{\rho C^0} [D^{(n)} q_y^{(n)} + (q_b^0 f^{(n)} + (q_s^0 f^{(n)} \\ + [\dot{E}_m T_m + \dot{E}_n T_n + (q_b^0 f^{(n)} (T_b^0 f^{(n)} + (q_s^0 f^{(n)} (T_s^0 f^{(n)}] \quad (116) \end{aligned}$$

which is the familiar conservative form of the energy equation for variable depth $D^{(n)}$ with the arbitrary heat flux components $q_x^{(n)}$ and $q_y^{(n)}$ to be determined according to the molecular and turbulent diffusion transport phenomena in the flow region. Hence, the solution of the FLIDE mathematical model Eq. (40) in a sense is equivalent to the approximate solution of the partial differential form of the energy equation under specified initial and boundary conditions.

From truly mathematical considerations, since the FLIDE (fluid-in-discrete-element) model represents a vector system of ordinary differential equations, which is not the finite-difference approximation of a partial differential equation, the stability of the system, in the sense of Liapunov [53], depends only on the properties of the system; and it is independent of the particular mathematical method employed in its solution. However, from practical computational considerations, the establishment of the Liapunov stability, which is extremely difficult for most systems, does not contribute significant information to the numerical solution procedure of the initial-value problem. Hence, the Liapunov stability concept will be discarded and a more familiar stability analysis method associated with finite-difference equations will be employed in the study of the stability of the FLIDE mathematical system.

The general mathematical system, Eq. (40), and the definitions of heat flux terms, Eqs. (75) and (76), surface and bottom heat transfer conditions, Eqs. (91)–(105), and intake and discharge conditions, Eqs. (100)–(109), can be simplified by defining appropriate coefficients and retaining only the necessary terms in the stability analysis. Furthermore, since in general the stability of the equations will be independent of the regions, the superscript (1) can be omitted for notational simplicity. Hence, by defining the following,

$$\begin{aligned}
 C_{i-1/2,j} &= \left| \frac{\bar{D}_{i-1/2,j}^{(n)}}{\bar{D}_{i,j}^{(n)}} \right| \left| \frac{\Delta y_{i-1/2,j}^{(n)}}{\Delta y_{i,j}^{(n)}} \right| > 0, \\
 C_{i,j-1/2} &= \left| \frac{\bar{D}_{i,j-1/2}^{(n)}}{\bar{D}_{i,j}^{(n)}} \right| \left| \frac{\Delta x_{i,j-1/2}^{(n)}}{\Delta x_{i,j}^{(n)}} \right| > 0, \\
 C_{i-1,j} &= \left| \frac{\bar{D}_{i-1,j}^{(n)}}{\bar{D}_{i,j}^{(n)}} \right| \left| \frac{\Delta y_{i-1,j}^{(n)}}{\Delta y_{i,j}^{(n)}} \right| \left| \frac{\Delta x_{i,j}^{(n)}}{\Delta x_{i,j}^{(n)} + \Delta x_{i-1,j}^{(n)}} \right| > 0, \\
 C_{i,j-1} &= \left| \frac{\bar{D}_{i,j-1}^{(n)}}{\bar{D}_{i,j}^{(n)}} \right| \left| \frac{\Delta x_{i,j-1}^{(n)}}{\Delta x_{i,j}^{(n)}} \right| \left| \frac{\Delta y_{i,j}^{(n)}}{\Delta y_{i,j}^{(n)} + \Delta y_{i,j-1}^{(n)}} \right| > 0, \\
 c_{i-1/2,j} &= \frac{\kappa}{\rho C_p} + \frac{k_0 \sqrt{\lambda}}{6} \left| \bar{D}_{i-1/2,j}^{(n)} \right| \left\{ \left| U_{i-1/2,j}^{(n)} \right|^2 + \left| V_{i-1/2,j}^{(n)} \right|^2 \right\}^{1/2} > 0, \\
 \bar{c}_{i,j-1/2} &= \frac{\kappa}{\rho C_p} + \frac{k_0 \sqrt{\lambda}}{6} \left| \bar{D}_{i,j-1/2}^{(n)} \right| \left\{ \left| U_{i,j-1/2}^{(n)} \right|^2 + \left| V_{i,j-1/2}^{(n)} \right|^2 \right\}^{1/2} > 0, \\
 R_n &= \frac{G_n}{\Delta x_{i,j}^{(n)} \Delta y_{i,j}^{(n)} \bar{D}_{i,j}^{(n)}} > 0, \\
 h_i^* &= \left| h_{iV}^* (1 - R_n) + h_{iN}^* \right| \left| 1 + \frac{V_w}{(V_w)_{ref}} \right| > 0,
 \end{aligned}
 \tag{117}$$

$$(T_{1,j}^{(0)} - \bar{T}_{1,j}^{(0)}) > 0, \quad T_{2,j} = \bar{T}_{2,j}^{(0)} > 0,$$

$$\sigma^* = \sigma(T_1, T_2) > 0,$$

$$(q_A^*)^{(0)} = 0, \quad (q_B^*)^{(0)} = 0, \quad (q_C^*)^{(0)} = 0, \quad (q_N^*)^{(0)} = 0,$$

where all nonzero terms are positive, the governing system of equations for the FLDE method can be expressed in a simple form as

$$\begin{aligned} \frac{\partial \bar{T}_{1,j}}{\partial t} = & C_{1,j+1/2,j} \frac{U_{1,j+1/2,j}}{\Delta x_{1,j}} (\bar{T}_{1,j+1/2,j} - \bar{T}_{1,j}) + C_{1,j-1/2,j} \frac{\bar{U}_{1,j-1/2,j}}{\Delta x_{1,j}} (\bar{T}_{1,j-1/2,j} - \bar{T}_{1,j}) \\ & + C_{1,j+1/2} \frac{V_{1,j+1/2}}{\Delta y_{1,j}} (\bar{T}_{1,j+1/2} - \bar{T}_{1,j}) + C_{1,j-1/2} \frac{V_{1,j-1/2}}{\Delta y_{1,j}} (\bar{T}_{1,j-1/2} - \bar{T}_{1,j}) \\ & + C_{1,j+1,j} \frac{C_{1,j+1/2,j}}{\Delta x_{1,j}^2} (\bar{T}_{1,j} - \bar{T}_{1,j+1,j}) + C_{1,j-1,j} \frac{C_{1,j-1/2,j}}{\Delta x_{1,j}^2} (\bar{T}_{1,j-1,j} - \bar{T}_{1,j}) \\ & + C_{1,j+1} \frac{\bar{C}_{1,j+1/2}}{\Delta y_{1,j}^2} (\bar{T}_{1,j} - \bar{T}_{1,j+1}) + C_{1,j-1} \frac{\bar{C}_{1,j-1/2}}{\Delta y_{1,j}^2} (\bar{T}_{1,j-1} - \bar{T}_{1,j}) \\ & + \bar{h}_n (\bar{T}_n - \bar{T}_{1,j}) + \frac{1}{\rho(C^* D_{1,j})} \left[q_{SR}^* - h_A^* (\bar{T}_{1,j} - T_2) - \sigma^* \bar{T}_{1,j}^4 \right], \end{aligned} \quad (118)$$

$$\bar{T}_{1,j}(0) = (T_n)_{1,j}, \quad \text{for all } 1,j. \quad (119)$$

The initial-value problem, Eqs. (118) and (119), can be numerically integrated by numerous marching techniques. For the simplest form of Euler-Cauchy method, that is, approximating the time derivatives by the forward time differences, the resulting mathematical system becomes very similar to the donor cell type explicit methods which are extensively employed in computational fluid mechanics (see Roache [48]). It is also possible to employ higher-order, explicit methods with multistep and multilevel time formulations, for example, midpoint leapfrog [54, 55] and Du Fort - Frankel [53], etc., by properly approximating the time derivatives and evaluating the right-hand-side functions at various time steps. Furthermore, by considering the right-hand-side functions at the forward time step $t + \Delta t$, the initial-value problem, Eqs. (118) and (119), can also be formed into an implicit mathematical system which can be solved by either relaxation or alternating direction, modified, tridiagonal matrix reduction methods. The stability criteria for different methods can be quite different, particularly for the implicit method, and the decision on the choice of a particular method depends on the actual computer time requirements for the overall solution during a specified time period.

In the solution of the present FLDE mathematical model for transient temperature distributions, the implicit formulations will not be considered since the two-dimensional formulation of the model will require excessive computational time in the relaxation-type solution of the resulting pentadiagonal matrices or modified alternating direction Gaussian reduction of tridiagonal matrices.

The initial-value problem, Eqs. (118) and (119), will be directly integrated either by the simple Euler-Cauchy method or by the standard fourth-order Runge-Kutta method with Gill coefficients which were successfully employed in numerous studies, for example, Lomax, Kutlar, and Fuller [57], Dawson and Marcus [58], etc., for the numerical solution of the semidiscretized finite-difference equations associated with external flow problems. Since both methods are explicit formulations, it is essential to consider the necessary stability criteria for the application of the methods to the solution of the mathematical system. The stability analysis of the Euler-Cauchy method is quite straightforward, although by no means trivial, since it can employ the stability methods which were developed for explicit finite-difference equations. On the other hand, since the Runge-Kutta method employs four time levels for one time step integration, its stability analysis would clearly require considerably more effort in the derivation of the stability criteria. However, considering the fact that the Runge-Kutta method is, in a sense, a higher-order-accuracy, modified form of the basic Euler-Cauchy method with more stable properties for approximate integration, the stability criteria for the Euler-Cauchy method are definitely sufficient to guarantee the stability conditions for the Runge-Kutta method. Hence, the stability analysis of the FLIDE mathematical model will be based on the application of the Euler-Cauchy method to the governing differential equations of the initial-value problem, Eqs. (118) and (119).

The formulations of the necessary and sufficient stability criteria according to well-established, mathematically systematic methods, for example, von Neumann [59], Hirt [60], and others (see Roache [48]), become extremely difficult, cumbersome, and indeed questionable for finite-difference equations with variable coefficients and individually different cell sizes. Since the general mathematical system contains convective terms with velocity components as coefficients, which are variable in time and space, and the dimensions of the discrete elements are completely arbitrary, the classical, rigorous stability analysis methods cannot be directly applied to the formulation of the stability criteria for the solution of the FLIDE differential equations.

The method which will be employed in the present study is called the "discrete perturbation stability analysis." It was first presented by Thom and Apelt [61] and was further developed by Thoman and Szewczyk [62] and by Roache [48] for the stability analysis of finite-difference equations associated with the computational fluid mechanics problems. The method is conceptually simple in its basic formulation; and it is extremely direct in the derivation of the stability conditions. Although it is not as systematic and formalized as the von Neumann [59] or Hirt [60] analyses, in its numerous applications to simple finite-difference equations it has successfully predicted the same stability conditions as the more rigorous methods.

Considering the initial-value problem, Eqs. (118) and (119), the system of differential equations can be represented in a general form as

$$\begin{aligned} \frac{\partial \bar{T}_{i,j}}{\partial t}(t) = \bar{F}_{i,j}(t) = \bar{F}_{i,j} \left[t, \bar{T}_{i-1/2,j}(t), \bar{T}_{i,j}(t), \bar{T}_{i,j+1/2}(t) \right] \\ = (\bar{F}_C)_{i,j}(t) + (\bar{F}_D)_{i,j}(t) + (\bar{F}_P)_{i,j}(t) + (\bar{F}_S)_{i,j}(t), \quad (120) \end{aligned}$$

where the right-hand-side function $\bar{F}_{i,j}(t)$ is separated into four parts as the convective term $(\bar{F}_C)_{i,j}(t)$, diffusion term $(\bar{F}_D)_{i,j}(t)$, plant discharge term $(\bar{F}_P)_{i,j}(t)$, and surface heat transfer term $(\bar{F}_S)_{i,j}(t)$ as

$$(\bar{F}_C)_{i,j}(t) = C_{i-1/2,j} \frac{\bar{U}_{i-1/2,j}}{\Delta x_{i,j}} (\bar{T}_{i-1/2,j} - \bar{T}_{i,j})$$

$$\begin{aligned}
& -C_{i+1/2,j} \frac{\bar{U}_{i+1/2,j}}{\Delta x_{i,j}} (\bar{T}_{i+1/2,j} - \bar{T}_{i,j}) + C_{i,j-1/2} \frac{\bar{V}_{i,j-1/2}}{\Delta y_{i,j}} (\bar{T}_{i,j-1/2} - \bar{T}_{i,j}) \\
& - C_{i,j+1/2} \frac{\bar{V}_{i,j+1/2}}{\Delta y_{i,j}} (\bar{T}_{i,j+1/2} - \bar{T}_{i,j}), \quad (121)
\end{aligned}$$

$$\begin{aligned}
(\bar{F}_D)_{i,j}(t) = & -C_{i-1,j} \frac{\bar{\epsilon}_{i-1/2,j}}{\Delta x_{i,j}^2} (\bar{T}_{i,j} - \bar{T}_{i-1,j}) + C_{i+1,j} \frac{\bar{\epsilon}_{i+1/2,j}}{\Delta x_{i,j}^2} (\bar{T}_{i+1,j} - \bar{T}_{i,j}) \\
& - C_{i,j-1} \frac{\bar{\epsilon}_{i,j-1/2}}{\Delta y_{i,j}^2} (\bar{T}_{i,j} - \bar{T}_{i,j-1}) + C_{i,j+1} \frac{\bar{\epsilon}_{i,j+1/2}}{\Delta y_{i,j}^2} (\bar{T}_{i,j+1} - \bar{T}_{i,j}), \quad (122)
\end{aligned}$$

$$(\bar{F}_p)_{i,j}(t) = \dot{g}_n (T_n - \bar{T}_{i,j}), \quad (123)$$

$$(\bar{F}_s)_{i,j}(t) = \frac{1}{\rho C^* D_{i,j}} \left[q_{SR}^* - h_S^* (\bar{T}_{i,j} - T_a) - \sigma^* T_{i,j}^4 \right]. \quad (124)$$

During the integration of the initial-value problem, Eqs. (118) and (119), the actual computed temperature $\tilde{T}_{i,j}(t)$ may not be equal to the exact solution $\bar{T}_{i,j}(t)$ due to truncation and machine errors which may be introduced along the computation. In the formulation of the discrete perturbation stability analysis method it is assumed that up to any arbitrary time t the actual computed temperatures $\tilde{T}_{i,j}(t)$ are equal to the exact solution $\bar{T}_{i,j}(t)$ for all i, j ; that is, there exists no error in the solution. At the arbitrary time step t , a discrete perturbation error $\delta_{i,j}(t)$ is introduced only at one arbitrary discrete element i, j , such that the actual computed solution $\tilde{T}_{i,j}(t)$ becomes different than the exact solution $\bar{T}_{i,j}(t)$; that is,

$$\tilde{T}_{i,j}(t) = \bar{T}_{i,j}(t) + \delta_{i,j}(t), \quad \tilde{T}_{i \pm 1,j}(t) = \bar{T}_{i \pm 1,j}(t), \quad \tilde{T}_{i,j \pm 1}(t) = \bar{T}_{i,j \pm 1}(t). \quad (125)$$

Consequently, the differential equation for $\tilde{T}_{i,j}(t)$, Eq. (120), ceases to be exact; and it represents the form for the approximate solution $\tilde{T}_{i,j}(t)$ with the error $\delta_{i,j}(t)$, that is,

$$\frac{\partial \tilde{T}_{i,j}}{\partial t}(t) = \tilde{F}_{i,j}(t) = \bar{F}_{i,j} \left[t, \tilde{T}_{i \pm 1,j}(t), \tilde{T}_{i,j}(t), \tilde{T}_{i,j \pm 1}(t) \right]. \quad (126)$$

Substituting Eq. (125) in Eq. (126),

$$\frac{\partial \tilde{T}_{i,j}}{\partial t}(t) + \frac{\partial \delta_{i,j}}{\partial t}(t) = \bar{F}_{i,j}(t) + \delta F_{i,j}(t), \quad (127)$$

where

$$\delta F_{i,j}(t) = \tilde{F}_{i,j}(t) - \bar{F}_{i,j}(t) \quad (128)$$

represents the error introduced in the right-hand-side function $\bar{F}_{i,j}(t)$ due to the discrete perturbation error $\delta_{i,j}(t)$. Hence, noting that $\bar{T}_{i,j}(t)$ and $\bar{F}_{i,j}(t)$ satisfy the differential equation for the exact solution, Eq. (120), Eq. (127) reduces to

$$\frac{\partial \delta_{i,j}}{\partial t}(t) = \delta F_{i,j}(t), \quad (129)$$

which represents the differential equation for the discrete perturbation error at element i, j . To obtain the approximate error $\delta_{i,j}(t + \Delta t)$ at the next time step $t + \Delta t$, the time derivative can be approximated by its difference form, that is,

$$\frac{\delta_{i,j}(t + \Delta t) - \delta_{i,j}(t)}{\Delta t} = \delta F_{i,j}(t). \quad (130)$$

Hence, solving for the ratio of the errors at two time levels t and $t + \Delta t$,

$$\frac{\delta_{i,j}(t + \Delta t)}{\delta_{i,j}(t)} = 1 + \Delta t \frac{\delta F_{i,j}(t)}{\delta_{i,j}(t)}. \quad (131)$$

For the stability of the solution, the perturbation error $\delta_{i,j}$ must not increase as time increases, which implies that at the first computational time step $t + \Delta t$, after the introduction of the error at time step t , the magnitude of error must decrease or remain the same; that is,

$$\left| \frac{\delta_{i,j}(t + \Delta t)}{\delta_{i,j}(t)} \right| = \left| 1 + \Delta t \frac{\delta F_{i,j}(t)}{\delta_{i,j}(t)} \right| \leq 1. \quad (132)$$

The standard stability condition, Eq. (132), is the necessary and sufficient condition for the discrete element i, j only at the first computational time step; and it can also be expressed as

$$1 \leq \frac{\delta_{i,j}(t + \Delta t)}{\delta_{i,j}(t)} = 1 + \Delta t \frac{\delta F_{i,j}(t)}{\delta_{i,j}(t)} \leq 1. \quad (133)$$

The right-hand inequality is referred to as the "static stability" requirement, which guarantees that the error $\delta_{i,j}$ does not grow along its own direction; and the left-hand inequality is referred to as the "dynamic stability" requirement, which guarantees that the error does not overshoot more than its magnitude in the opposite direction between the two time steps t and $t + \Delta t$.

In its rigorous application, the discrete perturbation method requires a complete time history of error propagation in all the discrete elements for successive later time levels. Consequently, at time level $t + \Delta t$, it becomes necessary to evaluate the errors $\delta_{i+1,j}(t + \Delta t)$, $\delta_{i,j+1}(t + \Delta t)$ in the discrete elements adjacent to i, j where the error $\delta_{i,j}(t)$ was introduced in the analysis. The error propagation process needs to be followed until the imposed stability conditions, similar to Eq. (133), do not continue in getting more restrictive at successive time levels. This procedure becomes extremely cumbersome for two-dimensional systems, particularly at later time levels in the stability analysis. Thoman and Szewczyk [62] have formally proposed a more restrictive dynamic stability requirement which does not allow any overshoot of error between time steps t and $t + \Delta t$; that is,

$$0 \leq \frac{\delta_{i,j}(1 + \Delta t)}{\delta_{i,j}(t)} = 1 + \Delta t \frac{\delta F_{i,j}(t)}{\delta_{i,j}(t)} \leq 1. \quad (134)$$

Although the left-hand inequality is not a "stability requirement," in the sense of a disturbance decreasing in magnitude, it was formally shown in numerous applications (see Roache [48]) that this condition indeed corresponds to the necessary and possibly sufficient stability requirement which can be obtained by other stability analysis methods, for example, von Neumann [59].

Hence, considering the inequalities only, the combined stability requirement, Eq. (134), can be represented in two parts as:

static stability condition:

$$\delta F_{i,j}(t) < 0; \quad (135)$$

dynamic stability condition:

$$\Delta t < \left| \frac{\delta_{i,j}(t)}{\delta F_{i,j}(t)} \right| \quad (136)$$

The static stability condition, Eq. (135), is a necessary requirement which must be satisfied to guarantee the validity of the computed solution. If this condition is violated, the approximate solution $\tilde{T}_{i,j}(t)$ diverges in an unbounded manner from the exact solution $T_{i,j}(t)$, regardless of the time step size Δt ; hence, it violates the stability criterion, in the sense of Liapunov, and becomes meaningless.

The dynamic stability condition, Eq. (136), is also a necessary requirement which specifies the maximum allowable time step size Δt for stable solutions. From computational considerations, this condition is the desired practical criterion for the numerical solution of the initial value problem, Eqs. (118) and (119).

It should be emphasized that both the static stability, Eq. (135), and the modified dynamic stability, Eq. (136), requirements are necessary but not sufficient conditions; that is, their satisfaction does not totally guarantee the stability of the solutions. However, since this undesirable weakness is common to all stability analysis methods, for example, von Neumann [59], it is always considered only as a cautionary point in the application of the discrete perturbation stability analysis.

The disturbance $\delta F_{i,j}(t)$, Eq. (128), which results from the introduction of error $\delta_{i,j}(t)$ in the element i,j , can be evaluated from the definitions of the parts of the right-hand-side function $F_{i,j}(t)$, Eqs. (121)–(124), of the general differential equation, Eq. (120).

Since the second upwind differencing or the donor cell method determines the half-point values on the enclosure surfaces of the discrete elements according to the directions of the velocity components normal to the surfaces, it is necessary to consider the flow conditions which will cause the maximum error in the contribution of convective terms to the right-hand-side function $(F_c)_{i,j}(t)$; that is,

$$\begin{aligned} U_{i-1/2,j} = \bar{U}_{i-1/2,j}^* > 0 &\Rightarrow \tilde{T}_{i-1/2,j}(t) = \tilde{T}_{i-1,j}(t) = \bar{T}_{i-1,j}(t), \\ \bar{U}_{i+1/2,j} = -\bar{U}_{i+1/2,j}^* < 0 &\Rightarrow \tilde{T}_{i+1/2,j}(t) = \tilde{T}_{i+1,j}(t) = \bar{T}_{i+1,j}(t), \\ \bar{V}_{i,j-1/2} = \bar{V}_{i,j-1/2}^* > 0 &\Rightarrow \tilde{T}_{i,j-1/2}(t) = \tilde{T}_{i,j-1}(t) = \bar{T}_{i,j-1}(t), \\ \bar{V}_{i,j+1/2} = -\bar{V}_{i,j+1/2}^* < 0 &\Rightarrow \tilde{T}_{i,j+1/2}(t) = \tilde{T}_{i,j+1}(t) = \bar{T}_{i,j+1}(t) \end{aligned} \quad (137)$$

Substituting Eqs. (125) and (137) in Eq. (121) and rearranging, the error $(\delta F_C)_{i,j}(t)$ which is introduced in the convective term $(F_C)_{i,j}(t)$ due to the discrete perturbation error $\delta_{i,j}(t)$ becomes

$$\begin{aligned}
 (\delta F_C)_{i,j}(t) &= (\tilde{F}_C)_{i,j}(t) - (F_C)_{i,j}(t) \\
 &= -\delta_{i,j}(t) \left[C_{i-1/2,j} \frac{\bar{U}_{i-1/2,j}^*}{\Delta x_{i,j}} + C_{i+1/2,j} \frac{\bar{U}_{i+1/2,j}^*}{\Delta x_{i,j}} \right. \\
 &\quad \left. + C_{i,j-1/2} \frac{\bar{V}_{i,j-1/2}^*}{\Delta y_{i,j}} + C_{i,j+1/2} \frac{\bar{V}_{i,j+1/2}^*}{\Delta y_{i,j}} \right] < 0.
 \end{aligned} \tag{138}$$

It can easily be shown that any other combination for the velocity components $\bar{U}_{i\pm 1/2,j}$, $\bar{V}_{i,j\pm 1/2}$ results in error $(\delta F_C)_{i,j}$ consisting of various terms which are already included in the maximum error, Eq. (138).

Substituting Eq. (125) in Eq. (122) and rearranging, the error $(\delta F_D)_{i,j}(t)$ which is introduced in the diffusion term $(F_D)_{i,j}(t)$ due to the discrete perturbation error $\delta_{i,j}(t)$ becomes

$$\begin{aligned}
 (\delta F_D)_{i,j}(t) &= (\tilde{F}_D)_{i,j}(t) - (F_D)_{i,j}(t) \\
 &= -\delta_{i,j}(t) \left[C_{i-1,j} \frac{\bar{\epsilon}_{i-1/2,j}}{\Delta x_{i,j}^2} + C_{i+1,j} \frac{\bar{\epsilon}_{i+1/2,j}}{\Delta x_{i,j}^2} \right. \\
 &\quad \left. + C_{i,j-1} \frac{\bar{\epsilon}_{i,j-1}}{\Delta y_{i,j}^2} + C_{i,j+1} \frac{\bar{\epsilon}_{i,j+1}}{\Delta y_{i,j}^2} \right] < 0.
 \end{aligned} \tag{139}$$

Similarly, substituting Eq. (125) in Eqs. (123) and (124) and rearranging, the errors $(\delta F_P)_{i,j}(t)$ and $(\delta F_S)_{i,j}(t)$ which are introduced in the plant discharge and surface heat transfer terms $(F_P)_{i,j}(t)$ and $(F_S)_{i,j}(t)$, respectively, become

$$\begin{aligned}
 (\delta F_P)_{i,j}(t) &= (\tilde{F}_P)_{i,j}(t) - (F_P)_{i,j}(t) \\
 &= -\delta_{i,j}(t) [\dot{\epsilon}_n] < 0.
 \end{aligned} \tag{140}$$

$$\begin{aligned}
 (\delta F_S)_{i,j}(t) &= (\tilde{F}_S)_{i,j}(t) - (F_S)_{i,j}(t) \\
 &= -\frac{\delta_{i,j}(t)}{\rho C^* \bar{D}_{i,j}} \left[h_s^* + 4\sigma^* \bar{T}_{i,j}^3 \right] < 0.
 \end{aligned} \tag{141}$$

where in Eq. (141) it is assumed that the error $\delta_{i,j}(t)$ is smaller than the local temperature $T_{i,j}(t)$. According to Eqs. (138) (141), it is immediately evident that the static stability condition, Eq. (135), is unconditionally satisfied; that is,

$$\delta F_{i,j}(t) = (\delta F_C)_{i,j}(t) + (\delta F_D)_{i,j}(t) + (\delta F_P)_{i,j}(t) + (\delta F_S)_{i,j}(t) < 0 \tag{142}$$

for the positive discrete perturbation error $\delta_{i,j}(t)$.

Substituting Eqs. (138)–(141) in Eq. (142), the dynamic stability condition, Eq. (136), establishes the necessary computational criterion for the selection of the time step Δt for the solution $\bar{T}_{i,j}(t)$ in the discrete element i, j , as

$$\Delta t < 1 / \left[C_{i-1/2,j} \frac{\bar{U}_{i-1/2,j}^*}{\Delta x_{i,j}} + C_{i+1/2,j} \frac{\bar{U}_{i+1/2,j}^*}{\Delta x_{i,j}} + C_{i,j-1/2} \frac{\bar{V}_{i,j-1/2}^*}{\Delta y_{i,j}} + C_{i,j+1/2} \frac{\bar{V}_{i,j+1/2}^*}{\Delta y_{i,j}} + C_{i-1,j} \frac{\bar{\epsilon}_{i-1,j}}{\Delta x_{i,j}^2} + C_{i+1,j} \frac{\bar{\epsilon}_{i+1,j}}{\Delta y_{i,j}^2} + C_{i,j-1} \frac{\bar{\epsilon}_{i,j-1}}{\Delta y_{i,j}^2} + C_{i,j+1} \frac{\bar{\epsilon}_{i,j+1}}{\Delta y_{i,j}^2} + \bar{\epsilon}_n + \frac{1}{\rho C^* \bar{D}_{i,j}} (h_i^* + 4\sigma^* \bar{T}_{i,j}^3) \right] \quad (143)$$

To make sure that the selected time step Δt satisfies the dynamic stability criterion, Eq. (136), for all the discrete elements i, j , it is necessary to consider the most critical conditions. Hence, defining the appropriate minimum and maximum values for the terms in Eq. (143) as

$$\begin{aligned} (C_{i\pm 1/2,j\pm 1/2})_{\max} &= \text{Maximum}(C_{i\pm 1/2,j}, C_{i,j\pm 1/2}), \\ (C_{i\pm 1,j\pm 1})_{\max} &= \text{Maximum}(C_{i\pm 1,j}, C_{i,j\pm 1}), \\ (\bar{W}_{i\pm 1/2,j\pm 1/2}^*)_{\max} &= \text{Maximum}(\bar{U}_{i\pm 1/2,j}^*, \bar{V}_{i,j\pm 1/2}^*), \\ (\Delta s_{i,j})_{\min} &= \text{Minimum}(\Delta x_{i,j}, \Delta y_{i,j}), \\ (\bar{\epsilon}_{i\pm 1/2,j\pm 1/2})_{\max} &= \text{Maximum}(\bar{\epsilon}_{i\pm 1/2,j}, \bar{\epsilon}_{i,j\pm 1/2}), \end{aligned} \quad (144)$$

the necessary criterion for the time step Δt which will guarantee the dynamic stability requirement, Eq. (136), for the solution $\bar{T}_{i,j}(t)$ in all discrete elements i, j becomes

$$\begin{aligned} \Delta t &< \frac{(\Delta s_{i,j})_{\min}}{4(C_{i\pm 1/2,j\pm 1/2})_{\max} (\bar{W}_{i\pm 1/2,j\pm 1/2}^*)_{\max}} \\ &\times \left\{ 1 + \frac{(C_{i\pm 1,j\pm 1})_{\max}}{(C_{i\pm 1/2,j\pm 1/2})_{\max}} \frac{(\bar{\epsilon}_{i\pm 1/2,j\pm 1/2})_{\max}}{(\bar{W}_{i\pm 1/2,j\pm 1/2}^*)_{\max} (\Delta s_{i,j})_{\min}} \right. \\ &+ \frac{(\Delta s_{i,j})_{\min}}{4(C_{i\pm 1/2,j\pm 1/2})_{\max} (\bar{W}_{i\pm 1/2,j\pm 1/2}^*)_{\max}} \left[(\bar{\epsilon}_n)_{\max} \right. \\ &\left. \left. + \frac{(h_i^*)_{\max}}{\rho C^* \bar{D}_{\min}} + \frac{4(\sigma^*)_{\max} (\bar{T}_{i,j})_{\max}^3}{\rho C^* \bar{D}_{\min}} \right] \right\}^{-1} \quad (145) \end{aligned}$$

The first term on the right-hand side of Eq. (145) can be considered as the critical time step Δt_{CR} based on a generalized form of "Courant number" [48]; that is,

$$\Delta t_{CR} = \frac{(\Delta s_{i,j})_{\min}}{(C_{i-1/2,j-1/2})_{\max} (\bar{W}_{i-1/2,j-1/2}^*)_{\max}} \quad (146)$$

Furthermore, considering a generalized form of discrete element Reynolds number Re , similar to the conventional form of "cell Reynolds number" [45], and defining the overall combined effect k_{ns} for the plant discharges and surface heat transfer conditions as

$$Re = \frac{(C_{i-1/2,j-1/2})_{\max} (\bar{W}_{i-1/2,j-1/2}^*)_{\max} (\Delta s_{i,j})_{\min}}{(C_{i-1,j-1})_{\max} (\epsilon_{i-1/2,j-1/2})_{\max}} \quad (147)$$

$$k_{ns} = (k_n)_{\max} + \frac{1}{\rho C^*(D_{i,j})_{\min}} \left\{ (h_s^*)_{\max} + 4(\sigma^*)_{\max} (\bar{T}_{i,j})_{\max} \right\}.$$

the dynamic stability requirement, Eq. (136), which establishes the necessary criterion for the selection of the computational time step Δt , becomes

$$\Delta t < \frac{\Delta t_{CR}}{4} \frac{1}{[1 + (1/Re) + \Delta t_{CR} k_{ns}]} \quad (148)$$

It should be emphasized that this requirement was obtained by considering one-step, two-time-level integration by approximating the time derivative for the error in its difference form. Hence, the dynamic stability criterion, Eq. (148), is for the integration of the FLIDE differential equations, Eq. (118), by the simple Euler-Cauchy method, which in a sense is equivalent to the solution of the system by an explicit, forward-time-step finite-difference method. For the numerical integration of the FLIDE initial-value problem, Eqs. (118) and (119), by more accurate methods, fourth-order Runge-Kutta, etc., the criterion for the size of the time step Δt , Eq. (148), is extremely on the conservative side. Indeed, during actual computation, ten times larger step sizes were employed without any detectable error in the solutions. However, considering that the Runge-Kutta method requires four times as much computation as the Euler-Cauchy method at each time step, the actual computer times for these solutions were only two and a half times less than the ones required for the Euler-Cauchy method with the time step specified by the dynamic stability requirement, Eq. (148).

Hence, according to discrete perturbation stability analysis, it is clearly established that the governing mathematical system of the FLIDE model, Eq. (40) or Eq. (118), with the second upwind differencing (donor cell) forms for the convective half-point values, Eq. (47), can be numerically integrated from specified initial conditions, Eq. (118), to obtain valid, stable approximate solutions for all time.

3. APPLICATION TO THERMAL IMPACT ANALYSIS OF A HYPOTHETICAL NUCLEAR GENERATING STATION IN A COASTAL REGION

The general FLIDE (fluid-in-discrete-element) mathematical model was applied to the assessment of the thermal impact of a hypothetical nuclear generating station in a coastal region.

It is assumed that the generating station consists of a presently operational 500-MW single nuclear unit with a once-through condenser cooling system using submerged, single-port intake and discharge structures. It is proposed to increase the electric power output of the station by two additional 1150-MW nuclear units with once-through condenser cooling systems designed as submerged, single-port intakes and 2000-ft multiport, vertical discharge diffusers. The condensers of all three units are designed for 20°F temperature rise in the cooling water based on the condenser flow rates at the rated power outputs of the individual units.

Considering the proposed layout for the locations of the intake and discharge structures (see Fig. 10), it is evident that under adverse oceanic flow conditions, the recirculation and re-entrainment of the heated water could result in the substantial thermal impact of the generating station on the coastal region. To simulate a typical adverse case, from thermal impact considerations, in the application of the FLIDE far-field model, it is assumed that the flow conditions are strictly tidal periodic, without any net drift in the

Fig. 10. Schematic layout of intake and discharge structures of the three units of the hypothetical nuclear generating station.

form of geostrophic or wind-induced currents. Under the assumed periodic flow conditions, the thermal energy input to the coastal region by the condenser cooling water of the nuclear units cannot be convected out of the far-field zone. Hence, the ultimate cooling of the thermal effluent can only occur due to the atmospheric cooling from the ocean surface and by the meso-scale, tidal-averaged, turbulent transport from the open boundaries of the coastal region.

In the assessment of the environmental impact of large nuclear power plants, it is generally required to consider the size of the areas in the discharge region which contain heated water at various specified excess temperatures from the ambient conditions. Furthermore, it is usually necessary to estimate the discharge temperatures and the distances certain specified isotherms extend from the location of the discharge structures. Hence, in the numerical simulation of the short-time transient (within tidal cycle) temperature distributions in the coastal region, special emphasis will be placed on the discussions of these particular aspects of environmental impact assessment to identify the possible adverse thermal effects of recirculation and re-entrainment of the heated water between the intakes and the discharges under the assumed, strictly periodic tidal flow conditions.

3.1. Construction of the FLIDE Model for the Problem

The coastal region for the application of the FLIDE model was taken as approximately 63,000 ft along and 32,000 ft normal to the shoreline with four subregions, according to the location of the diffusers of the proposed additional units, and with approximately 480 discrete elements (see Fig. 11). Since the far-field model considers the tidal flows as directly contributing to the convective transfer of thermal energy, rather than as meso-scale eddies contributing to the turbulent oceanic transport, the dimensions of the region were specifically taken as more than twice the distances of the assumed tidal excursions along and normal to the

Fig. 11. Discrete-element representation of the coastal region for the FLIDE model.

shoreline to guarantee the inclusion of the periodic tidal recirculation in the analysis. Although a single region would be sufficient for the particular application, the coastal region was divided into four subregions to check the general formulation of the FLIDE model and to reduce the computer printout for considering the thermal conditions only in certain specified small areas near the discharges and the shoreline.

It is assumed that the submerged intake and discharge structures are installed on a relatively flat ocean bottom region at approximately 30 ft average depth at mean water level, as shown in Fig. 12, which slopes rapidly down to 1200 ft depth between 8000 ft and 40,000 ft from the shoreline. The structures for the single-port intake and discharge of Unit 1 and the single-port intakes and multiport diffusers of Units 2 and 3 are assumed to be constructed with vertical openings at approximately 10 ft from the bottom, at approximately 20 ft average depth below the surface at mean water level (see Fig. 12).

The minimum dimensions of the discrete elements were determined according to the geometry of the shoreline and the distances between the intake and discharge structures of the three units (see Figs. 10, 11, and 12); and the maximum dimensions were arbitrarily specified based on the overall dimensions of the assumed coastal region, as:

$$\Delta x_{\min} = 458 \text{ ft.} \quad \Delta y_{\min} = 500 \text{ ft} \quad (149)$$

$$\Delta x_{\max} = 8000 \text{ ft.} \quad \Delta y_{\max} = 16,000 \text{ ft}$$

Since the FLIDE simulation model is intended for far-field analysis in large coastal regions, the temperature variations within the discrete elements cannot be accurately determined from the solutions.

Fig. 12. Schematic representation of intake and discharge structures for units 1, 2, and 3 and bottom profile for the coastal region.

Consequently, it is necessary to assume that within the discrete elements which contain the discharges the near-field flow conditions result in rapid mixing of the heated discharge water with the local ocean water to result in a mixture in thermal equilibrium which can be represented by its mean temperature in the discrete element. Considering the vertical discharge conditions of all intakes in relatively shallow water, this assumption is indeed valid for the far-field analysis.

Hence, the actual intake and discharge conditions, Fig. 10, are represented by specified flow conditions in the discrete elements which contain them, Fig. 11, based on the operational conditions of the three units of the hypothetical nuclear generating station. It is assumed that during the period of this study, the intake and discharge conditions are maintained at approximately maximum load conditions of the generating station as shown in Table 1.

The assessment of the environmental impact of a nuclear generating station generally requires the consideration of various meteorological conditions in the coastal region and the operational loads of the power plant during various seasons in the year to determine the possibility of critical combinations which could lead to adverse ecological impact. In the present application of the FLIDE model to the assessment of the thermal impact of the hypothetical generating station, the meteorological and ocean temperature conditions will be assumed as approximately representative of the coastal regions in the continental United States for the summer season when the atmospheric surface heating during the day is substantial, particularly in shallow water regions near the shoreline.

The formulation of the FLIDE model requires the specification of transient depth and velocity components as input data in all discrete elements in the coastal region. Since for short-time (within tidal cycle) simulations of the thermal conditions the dominant convective transfer is based on the velocity components, the accuracy of the temperature solutions strongly depends on the accuracy and detailed representation of the oceanic flow conditions. It is evident that the transient variations of local depth and velocity components must necessarily satisfy the conservation of mass principle at every point in the coastal region. However, for two-dimensional representations, the satisfaction of the two-dimensional form of the principle by no means guarantees the realistic description of the actual flow field. Since the velocity measurement at different depths can be substantially different, particularly for two-layer flow situations caused by the combination of tidal and wind-induced currents, it is extremely important to carefully analyze the velocity data, by noting the depth of the measurements, before considering the satisfaction of the mass conservation principle to check the validity of the observed flow conditions. From mathematical

Table 1. Intake and discharge conditions in the discrete elements for units 1, 2, and 3

Unit	Intake				Discharge				Condenser temperature rise (°F)
	Region	i	j	Flow rate (cfh)	Region	i	j	Flow rate (cfh)	
1	4	2	7	3.20×10^6	4	2	8	3.20×10^6	20.0
2	4	1	5	6.72×10^6	1	1	1	1.68×10^6	20.0
					1	1	2	1.68×10^6	20.0
					1	1	3	1.68×10^6	20.0
					1	1	4	1.68×10^6	20.0
3	3	1	5	6.72×10^6	3	1	1	1.68×10^6	20.0
					3	1	2	1.68×10^6	20.0
					3	1	3	1.68×10^6	20.0
					3	1	4	1.68×10^6	20.0

considerations of the development of FLIDE: numerical simulation, far-field thermal model, further elaboration on the methods of analysis of the necessary input data for the flow conditions is definitely outside the scope of this study.

Hence, for the present application of the model to the assessment of the thermal impact of the hypothetical generating station under hypothetical adverse flow conditions, it will be assumed that the available field data for depth and the velocity components can be schematically represented by the periodic variations as shown in Fig. 13.

To obtain an approximate representation for the depth variations, it is assumed that the tidal height is approximately the same and the time lag is negligible for all the discrete elements. Furthermore, it is assumed that the depth variation parallel to the shoreline is not appreciable; that is, the bottom topography varies only normal to the shore as in Fig. 12. Hence, under these assumptions, the transient depth variations in the discrete elements can be approximated as simple sine functions.

Fig. 13. Schematic representation of the assumed flow conditions in the coastal region.

$$\bar{D}_{i,j}^{(r)}(t) = (\bar{D}_{MWL}^*)_i^{(r)} + \bar{D}_T^* \sin \left[\frac{2\pi}{\tau_T} t \right] \quad (150)$$

where

$$\begin{aligned} (\bar{D}_{MWL}^*)_i^{(r)} &= \text{mean water level in discrete element } i, j \text{ in region } r \text{ according to Fig. 12,} \\ \bar{D}_T^* &= \text{half tidal height} = 4.0 \text{ ft,} \\ \tau_T &= \text{tidal period} = 12 \text{ hr } 24 \text{ min.} \end{aligned} \quad (151)$$

The velocity components are also assumed the same in all discrete elements according to the data of Fig. 13; hence, the transient velocity variations in the discrete elements can also be approximated as simple sine functions.

$$\begin{aligned} \bar{U}_{i,j}^{(r)}(t) &= \bar{U}_{MAX}^* \sin \left[\frac{2\pi}{\tau_T} (t - t_U^*) \right], \\ \bar{V}_{i,j}^{(r)}(t) &= \bar{V}_{MAX}^* \sin \left[\frac{2\pi}{\tau_T} (t - t_V^*) \right], \end{aligned} \quad (152)$$

where

$$\begin{aligned} \bar{U}_{MAX}^* &= \text{maximum tangential velocity} = 0.68 \text{ ft/sec,} \\ \bar{V}_{MAX}^* &= \text{maximum normal velocity} = 0.34 \text{ ft/sec,} \\ t_U^* &= \text{time lag for velocity component parallel to the shoreline} = 0.0 \text{ hr,} \\ t_V^* &= \text{time lag for velocity component normal to the shoreline} = 6.2 \text{ hr.} \end{aligned} \quad (153)$$

It is again emphasized that the extremely simplistic, periodic forms for the transient variations of depth and velocity components, Eqs. (150) and (152), are necessitated by the assumed adverse tidal conditions and not by the limitations of the general formulation of the FLIDE model.

The necessary constants for the definitions of equivalent diffusion coefficients, Eq. (76), were specified as:

$$\begin{aligned} \kappa &= 0.35 \text{ Btu hr}^{-1} \text{ ft}^{-2} (^\circ\text{F})^{-1}, \\ \rho &= 62.4 \text{ lb ft}^{-3} \\ C^* &= 1.0 \text{ Btu lb}^{-1} (^\circ\text{F})^{-1} \\ \lambda &\cong 0.002 \text{ (Bowden [18])}. \end{aligned} \quad (154)$$

The necessary meteorological data was assumed as representative conditions which can exist in a coastal region during the summer; and it was incorporated in the formulation of the various terms of the different modes of the surface heat flux, Eqs. (95)–(103), as:

$$\begin{aligned} Q_{SK}^* &\cong 240 \text{ Btu hr}^{-1} \text{ ft}^{-2} \text{ (Table 7-35 [49])}, \\ \Gamma_e &\cong 0.9, \quad \Gamma_A \cong 0.4, \\ \bar{T}_a(t) &= 75.0^\circ\text{F} + 5.0^\circ\text{F} \sin \left[\frac{\pi}{12 \text{ hr}} (t - 6 \text{ hr}) \right], \end{aligned} \quad (155)$$

$$R_H = R_H(t) = 0.4 + 0.2 \sin \left[\frac{\pi}{12 \text{ hr}} (t - 6 \text{ hr}) \right] .$$

$$V_w = V_w(t) = 5.0 \text{ ft/sec} \left[1 + \left| \sin \frac{\pi}{12 \text{ hr}} (t - 6 \text{ hr}) \right| \right] .$$

The expressions for the assumed atmospheric conditions, Eq. (155), consider day and night variations for air temperature $\bar{T}_a$, relative humidity R_H and wind speed V_w in the coastal region, consistent with the short-time transient (within tidal cycle) application of the FLIDE model.

Considering the possibility of stratification of the thermal effluent, two limiting cases were considered in the assessment of the thermal impact of the generating station.

Case 1: It is assumed that stratification definitely exists throughout the coastal region, with the stratified heated water layer restricted to approximately constant 20 ft thickness below the surface.

Considering that at mean water level the average distance between the surface and the openings of the single-port discharge of Unit 1 and the diffusers of Units 2 and 3 is approximately 20 ft (see Fig. 12), the assumption of 20 ft thickness for the stratified layer of heated water is definitely realistic in the vicinity of the intake and discharge structures; but it is obviously not acceptable in shallow water regions near the shoreline where approximately 8 ft of tidal height results in the reduction of the actual depth below 20 ft during the tidal cycle. However, although this discrepancy can cause an unconservative effect in the temperature solutions near the shoreline, it cannot significantly influence the thermal conditions in the rest of the coastal region under the stratified assumptions of the limiting case.

Since it is assumed that 20 ft deep stratification exists everywhere in the region, the effective depth of the thermal plume is also limited by 20 ft; hence, regardless of the actual depth, the effective depth of 20 ft is employed in the formulation of the differential equations of the FLIDE model for the temperature solutions in the discrete elements. However, since the turbulent diffusion transport depends on the hydrodynamic conditions which determine the turbulent velocity and shear stress distributions, independent of thermal stratification, the actual depth is necessarily employed in the evaluation of the equivalent diffusion coefficients $\bar{\epsilon}_{i-1/2,i}^{(r)}$ and $\bar{\epsilon}_{i,i+1/2}^{(r)}$, Eq. (76), in the solution.

Case 2: It is assumed that due to strong tidal flow conditions full vertical mixing occurs during most of the tidal cycle; hence, no appreciable stratification exists anywhere in the coastal region.

Although this assumption is realistic for shallow regions near the shoreline, its validity becomes questionable in deep water, particularly for discrete elements far from the shore at approximately 1200 ft depth. Indeed, it can be considered as an unconservative assumption for short-time thermal analyses, since large, thermally effective depth of the plume tends to delay the increase of temperature in the coastal region. However, since it is a plausible limiting condition, it is included as a case in the application of the FLIDE mathematical model for the assessment of the thermal impact of the hypothetical nuclear generating station.

Considering the shallow depth at the locations of the intakes of all three units, approximately 30 ft at mean water level conditions, the elevation of the intake openings from the bottom, 10 ft (see Fig. 12), and the maximum tidal height, 8 ft, it is evident that the cooling water to the condensers will enter the intakes at the existing local temperatures of their discrete elements; hence, special modification due to stratification is not necessary in the general formulation of the intake conditions for either of the two limiting cases.

The appropriate time step size Δt for the numerical integration of the governing system of equations of the FLIDE model, Eq. (40), was determined according to the already established stability criterion, Eq. (148), and numerical experimentation. The necessary terms for the stability criterion were approximately

determined from the minimum size of the discrete elements in the coastal region, oceanic flow conditions, maximum discharge flow rate, and surface heat transfer conditions according to Eqs. (117), (146), and (147) as

$$\begin{aligned}
 (C_{i+1/2,j+1/2})_{\max} &\cong 1, \\
 (C_{i+1,j+1})_{\max} &\cong 1.5, \\
 (\bar{W}_{i+1/2,j+1/2})_{\max} &\cong 0.5 \text{ ft/sec}, \\
 (\Delta s_{i,j})_{\min} &\cong 500 \text{ ft}, \\
 (\epsilon_{i+1/2,j+1/2})_{\max} &\cong 2.0 \text{ ft}^2/\text{sec}^2, \\
 Re &\cong 85.0, \\
 (\dot{g}_n)_{\max} &\cong 0.0001/\text{sec}, \\
 (h_s^*)_{\max} &\cong 8.0 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{°F})^{-1}, \\
 4(\sigma_{\max}^*)(\bar{T}_{i,j})_{\max}^3 &\cong 0.4 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{°F})^{-1}, \\
 k_{ns} &\cong 0.0001/\text{sec}.
 \end{aligned} \tag{156}$$

The time step Δt_{CR} based on the Courant number becomes

$$\Delta t_{CR} \cong \frac{500 \text{ ft}}{1 \times 0.5 \text{ ft/sec}} = 1000 \text{ sec}. \tag{157}$$

Hence, substituting Eqs. (156) and (157) in Eq. (148), the critical computational time step Δt for stable numerical integration becomes

$$\Delta t \leq \frac{1000 \text{ sec}}{4} \frac{1}{[1 + (1/85.0) + 1000 \text{ sec} (0.0001/\text{sec})]} \cong 250 \text{ sec}. \tag{158}$$

which represents the maximum stability limit for the Euler-Cauchy method.

The governing system of equations, Eq. (40), for the particular application was numerically integrated according to the initial-value problem, Eqs. (118) and (119), by the fourth-order Runge-Kutta-Gill method with a time step size Δt as 0.155 hr (558 sec), which corresponds to $1/80$ th of the tidal period of 12.4 hr. Since this time step is larger than the critical one, Eq. (158), dictated by the stability criterion, Eq. (148), for numerical integration by the Euler-Cauchy method, at the beginning of each computational run for different cases, the computational solutions were obtained for both time step sizes, 250 sec and 558 sec, according to Runge-Kutta-Gill integration for sufficiently many time steps. The results were compared to guarantee the stability of the larger time step size of 558 sec for further integration of the solutions by the Runge-Kutta-Gill method. In all test cases it was found that the differences in solutions with the two different time step sizes, 250 sec and 558 sec, were less than the truncated error in single-precision accuracy of the computer results.

The numerical solutions for all cases were arbitrarily started at midnight (00:00 hr) on August 18 according to the assumed tidal flow data as shown in Fig. 13. It was assumed that at the initial conditions

the natural ocean temperature was 70°F throughout the coastal region without the generating station. Immediately after midnight, it was assumed that the generating station starts operating according to the conditions given in Table 1, either as the existing hypothetical Unit 1 or as all three proposed Units 1, 2, 3; and it continuously operates at full load throughout the period of computations for the assessment of its thermal impact on the coastal region.

The temperature distributions for the two cases and the existing and proposed operational conditions were obtained from the associated computer code FAROUT of the general, transient two-dimensional FLIDE mathematical far-field thermal model for coastal and offshore regions. The general computer code was programmed for the IBM 360-91 computer. The numerical results for temperature in approximately 480 discrete elements of the region during a complete tidal period, 12 hr 24 min, required approximately 1 min computational time, which was very reasonable considering the complexity of the problem.

Although the temperature solutions were obtained for all the discrete elements in the large coastal region, approximately 63,000 ft by 32,000 ft, the results are presented only in the small region, approximately 15,000 ft by 11,000 ft, as shown in Fig. 11, which contains the intake and discharge systems of the generating station.

The numerical results for the transient temperature development are presented in Figs. 14- 77 both as isotherms and also as perspective plots of temperature distributions as observed from a point on the left bottom corner of the coastal region as shown in Fig. 11.

Each figure presents the results for a full tidal cycle, 12 hr 24 min, consisting of eight time frames at intervals of 1 hr 33 min corresponding to one-eighth of the tidal period, starting at the top left corner and completing the cycle at the top right corner in the counterclockwise direction. In all figures, frames 4 and 8 correspond to low slack water and high slack water conditions, respectively, when the oceanic flow conditions are negligible in the coastal region.

In figures for the isotherms, the minimum temperature at each time step is indicated at the top right-hand corner of the frame; and it corresponds to the outermost isotherm in the region within the frame. The difference between two isotherms represents 0.5°F, with the higher temperature always corresponding to the inner isotherm.

The perspective plots for temperature distributions are presented to complement the results represented as isotherms. Although these figures do not represent quantitatively accurate temperature results, from qualitative considerations, they clearly illustrate the periodic motions and the spreading of the heated water during the tidal cycles in the coastal region.

The subsequent discussions of the results for the assessment of the thermal impact of the hypothetical generating station will be based on both isotherm and perspective representations of the temperature distributions in the coastal region.

3.2 Discussion of Results for Existing Unit 1 Operation

The FLIDE mathematical model was first employed to determine the transient temperature development in the coastal region after the start of the existing Unit 1 of the hypothetical generating station.

The numerical solutions for case 1, which assumes 20 ft thermally effective depth for the stratified heated water layer, are presented as isotherms in Figs. 14, 16, 18, 20, 22, and 24 and as perspective plots of temperature in Figs. 15, 17, 19, 21, 23, and 25 for tidal cycles 1, 2, 5, 6, 11, and 12 during an approximately six-day period after the start of the operation of Unit 1 at midnight, August 18.

It is clearly evident, from Figs. 14- 25, that the convection is the dominant phenomenon which controls the short-time variations of thermal conditions in the coastal region. The overall size, shape and

direction of the thermal plume from the Unit 1 discharge are almost completely determined by the oceanic flow conditions (Fig. 13) within the period of the tidal cycle. The turbulent diffusion transport and surface heat transfer effects are important only in the immediate vicinity of the discharge where there exists sufficiently high local temperatures and temperature gradients. Consequently, as expected, the maximum temperature in the coastal region always occurs at the point of discharge, during slack water conditions (frames 4 and 8 in all Figs. 14-25), when the flow conditions are minimized. Although the mean temperature in the discrete element of the discharge reaches 80°F , 10°F above the assumed ambient temperature 70°F , during the conditions, it rapidly drops to less than 74°F within 1000 ft from the location of the discharge.

The overall, long-time spreading of the heated water, in the part of the coastal region which is considered in the discussions (see Fig. 11), requires approximately six days after the start of the operation of Unit 1 to reach pseudo quasi-steady-state conditions (Figs. 22-25), when the minimum temperature in the region ceases to increase between cycles and starts to fluctuate between 72°F and 72.5°F . During this long time period, consisting of 12 tidal cycles, the turbulent diffusion transport and surface heat transfer phenomena become as important as the convective transport in adjusting the temperature distributions. Indeed, since, according to the specified oceanic flow conditions (Fig. 13), the heated water periodically returns to its initial location, the overall, long-time, tidal-averaged effect of the convection cannot be considered as the dominant phenomenon. For the long-time behavior of the thermal plume, the periodic flow conditions, in a sense, create the meso-scale oceanic eddies which, combined with the micro-scale turbulent diffusion phenomenon, result in the oceanic meso-scale turbulent transport without any net, long-time averaged convective transfer. This can be clearly observed in Figs. 14, 16, 18, 20, 22, and 24 by noting the increase in the number of isotherms and in Figs. 15, 17, 19, 21, 23, and 25 by considering the gradual increase of the mean temperature surface of the perspective plots, in the region with increasing number of tidal cycles.

Fig. 14. Isotherms (0.5°F gradation) during cycle I for case I with only unit I operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

4-1-1

4-1-8

4-1-2

4-1-7

4-1-3

4-1-6

4-1-4

4-1-5

Fig. 15. Perspective plots of temperature distribution (datum 69°F) during cycle 1 for case 1 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

Fig. 16. Isotherms (0.5°F gradation) during cycle 2 for case 1 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:47 hr.

4-2-1

4-2-8

4-2-2

4-2-7

4-2-3

4-2-6

4-2-4

4-2-5

Fig. 17. Perspective plots of temperature distribution (datum 69°F) during cycle 2 for case 1 (with only unit 1 operational). Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

Fig. 18. Isotherms (0.5°F gradation) during cycle 5 for case I with only unit I operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

4-5-1

4-5-8

4-5-2

4-5-7

4-5-3

4-5-6

4-5-4

4-5-5

Fig. 19. Perspective plots of temperature distribution (datum 69°F) during cycle 5 for case 1 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

Fig. 20. Isotherms (0.5 F gradation) during cycle 6 for case 1 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr: 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

4-6-1

4-6-8

4-6-2

4-6-7

4-6-3

4-6-6

4-6-4

4-6-5

Fig. 21. Perspective plots of temperature distribution (datum 69°F) during cycle 6 for case 1 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

Fig. 22. Isotherms (0.5°F gradation) during cycle 11 for case 1 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 05:33 hr to day 5, 16:24 hr.

4-11-1

4-11-8

4-11-2

4-11-7

4-11-3

4-11-6

4-11-4

4-11-5

Fig. 23. Perspective plots of temperature distribution (datum 69°F) during cycle 11 for case 1 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 05:33 hr to day 5, 16:24 hr.

Fig. 24. Isotherms (0.5 F gradation) during cycle 12 for case 1 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (11 hr 33 min), counterclockwise, starting at top left corner, from day 5, 17:57 hr to day 6, 04:48 hr.

4-12-1

4-12-8

4-12-2

4-12-7

4-12-3

4-12-5

4-12-4

4-12-6

Fig. 25. Perspective plots of temperature distribution (datum 69 F) during cycle 12 for case 1 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 17:57 hr to day 6, 04:48 hr.

The computed results for the thermal impact of Unit 1 operation also indicate the strong influence of the surface heat transfer conditions on the short-time variations in temperature. Considering the corresponding first time frames in Figs. 22 and 24, there exists 1.0°F temperature difference between the minimum temperatures as 71.5°F and 72.5°F, respectively, in the region. Since the initial temperature development is already completed by cycle 11 and both frames correspond to the same time level in two successive tidal cycles, it is clearly evident that this difference cannot be attributed to convection or turbulent diffusion transport phenomena. If one considers the actual time of day for each frame, the importance of the surface heat transfer effect immediately becomes evident. Noting that the solution assumes midnight as the start of operation of Unit 1, in Fig. 22, frame 1 corresponds to 05:33 hr in the morning; and in Fig. 24, frame 1 corresponds to 17:57 hr in the evening. Consequently, frame 1 in Fig. 22 represents the thermal conditions after approximately 12 hr of night cooling as combined blackbody back radiation, evaporation, and convection; whereas frame 1 in Fig. 24 represents the thermal conditions after approximately 12 hr of partial heating due to solar irradiation and severely reduced evaporative and convective surface cooling due to higher daytime atmospheric temperature and humidity conditions. Considering that the present results for case 1 are based on the assumption that the thermally effective depth of the stratified plume is only 20 ft, regardless of the actual depth in the region, it is indeed realistic to expect 1.0°F variation between the morning and evening temperatures due to significantly different night and day surface heat transfer conditions during August in the coastal region which is influenced by the discharge of Unit 1.

The appreciable influence, 1.0°F, of the surface heat transfer phenomenon on the temperature variations has a very important consequence in relation to the possibility of existence of quasi-steady-state thermal conditions in the coastal region. Since the surface conditions are approximately 24 hr periodic, whereas the tidal flow conditions are 12 hr 24 min periodic, a truly quasi-steady-state condition with variations of short-time periodicity cannot exist for thermal equilibrium conditions. Furthermore, since the tidal cycles have other variations and the atmospheric conditions have definitely weather and seasonal variations which are out of phase with the tidal flow, there can be no truly quasi-steady-state thermal equilibrium conditions in the coastal region. Consequently, the assessment of the thermal impact of the Unit 1 operation based on simple tidal-averaged meso-scale turbulent-diffusion-type models cannot be more accurate than 1.0°F error range in their predictions.

It is also interesting to note that there exists approximately 6 hr lag between the conditions of maximum solar irradiation at noon and the maximum thermal conditions of the water in the coastal region in the evening. This type of time lag was also observed and computed by Loziuk, Anderson, and Belytschko [40] for small cooling lakes with steady flow conditions resulting from power plant operations. This result is to be expected since the solar radiative power results in time rate of change of temperature as inversely proportional to the overall heat capacity of the water in the coastal region. However, the lag time of 6 hr should not be generalized as true for all coastal regions since it depends very strongly on the thermally effective depth that is considered in the analysis.

In the assessment of the environmental impact of any power plant, in general, it is usually required to consider certain specific characteristics of the thermal conditions associated with the behavior of the thermal effluent as it cools down to ambient conditions in the discharge region. These specific thermal conditions usually consist of a limit on the discharge temperature, the size of the mixing zone, and a limit on the excess temperature above ambient conditions for the portion of the thermal plume outside of the mixing zone. From environmental impact considerations, these criteria are considered as the tolerable limits for the operation of the power plant; and thermal conditions created by condenser cooling water discharges in excess of these specified limits are considered as potentially detrimental to the ecology of the region.

Hence, in the assessment of the environmental impact of an actual power plant, it is usually desired to have simulated temperature distributions such that the predicted conditions can be compared to the specified environmental limits on the thermal impact.

Similar to assessments for actual power plants, the advantages of the FLIDE far-field numerical simulation model can best be exemplified by considering the results of its application to the hypothetical generating station based on discussions as comparisons to some specified thermal impact criteria for the coastal region. Hence, for the present application, a set of limiting criteria, realistically representative of actual environmental thermal impact limits, was arbitrarily selected as: (a) 95°F limit, 25°F above assumed ambient 70°F, for discharges; (b) distance limit of 1000 ft from the discharges for the mixing zone; and (c) 74°F limit, 4°F above assumed ambient 70°F, for the temperatures outside of the mixing zone, for durations not more than half of the tidal period.

The subsequent, detailed discussions of the numerically simulated temperature distributions, resulting from various operational conditions of the hypothetical generating station, will be presented as comparisons with the above-stated, arbitrarily specified thermal criteria for the coastal region.

The results for only Unit 1 operation indicate that 74°F temperature, 4°F above the assumed ambient 70°F, does exist beyond 1000 ft from the discharge at various times during the tidal cycles depending on the transient oceanic flow and atmospheric conditions in the region. The times when the reach of the 74°F isotherm extends beyond the assumed 1000-ft limit are listed according to their order as they appear in the tidal cycles beginning with the assumed starting time of Unit 1 operation at midnight, August 18.

1. Third day at 15:33 hr (frame 1, Figs. 20, 21), maximum reach 1800 ft along the direction of the tidal flow toward the shore.

This is the first time after the start of the operation of Unit 1 when the 74°F isotherm extends beyond the 1000-ft limit from the discharge. It occurs 1 hr 33 min after slack water conditions (frame 8, Figs. 18, 19) when the maximum temperature was attained at the location of the discharge during the previous tidal cycle (frame 8, Figs. 18, 19).

During subsequent tidal cycles the frequency of occurrence of the 74°F isotherm reaching beyond the 1000-ft limit increases until the 11th and 12th cycles, after which the frequency remains approximately uniform depending on the periodic behavior of the tidal flow and night and day surface heat transfer conditions.

2. Sixth day at 05:33 hr (frame 1, Figs. 22, 23), maximum reach 1600 ft along the direction of the tidal flow toward the shore.

3. Sixth day at 11:45 hr (frame 5, Figs. 22, 23), maximum reach 1400 ft along the direction of the tidal flow toward the ocean.

4. Sixth day at 13:18 hr (frame 6, Figs. 22, 23), maximum reach 4000 ft along the direction of the tidal flow toward the ocean.

5. Sixth day at 14:51 hr (frame 7, Figs. 22, 23), maximum reach 5000 ft along the direction of the tidal flow toward the ocean.

6. Sixth day at 16:24 hr (frame 8, Figs. 22, 23), maximum reach 7000 ft along the direction of the tidal flow toward the ocean.

In cycle 11, the results indicate that more than half of the duration of the tidal period, the 74°F isotherm reaches distances beyond the 1000-ft limit from the discharge.

7. Sixth day at 17:57 hr (frame 1, Figs. 24, 25), maximum reach 4000 ft opposing the direction of the tidal flow toward the ocean, 3000 ft along the direction of the tidal flow toward the shore, 2800 ft parallel to the shoreline, and 400 ft on the shore.

This frame represents one of the most critical times from thermal impact considerations. The 74°F isotherm covers approximately 20,000 sq ft area around the Unit 1 discharge.

8. Sixth day at 19:30 hr (frame 2, Figs. 24, 25), maximum reach 2500 ft parallel to the shoreline and 6000 ft on the shore.

This frame represents the most critical case of excessive temperatures on the shore for the operation of the existing Unit 1.

9. Sixth day at 21:03 hr (frame 3, Figs. 24, 25), maximum reach 2800 ft along the direction of the tidal flow toward the shore.

10. Sixth day at 22:36 hr (frame 4, Figs. 24, 25), maximum reach 1800 ft along the direction of the tidal flow toward the ocean.

Again, in cycle 12, the results indicate that more than half of the duration of the tidal period, the 74°F isotherm reaches distances beyond the 1000-ft limit from the discharge.

From realistic considerations, the numerically simulated temperature distributions from the FLIDE model do not necessarily indicate that the assumed operational conditions of the existing Unit 1 of the hypothetical generating station result in thermal conditions which exceed the assumed limits of tolerable thermal impact in the coastal region.

First, the solutions are based on case 1 assumption that the thermally effective depth of the discharge plume is 20 ft everywhere in the coastal region regardless of the actual depth conditions. Since this condition represents only an assumed limiting case, the thermal impact of the Unit 1 operation may be less than the predictions of case 1 if strong vertical mixing exists in the region. Hence, it is also necessary to consider the results based on the case 2 limiting assumption before deciding on the compliance of the operation with the assumed limits of the environmental impact.

Second, the 74°F isotherm cannot be truly defined as the 4°F excess temperature isotherm above the assumed ambient condition, 70°F. Since the results clearly indicate that the variations in night and day surface heat transfer conditions can cause approximately 1.0°F difference in the minimum temperature in the region under consideration, the actual short-time transient ambient temperature can vary between 69.5°F and 70.5°F. Indeed, this effect is clearly illustrated in Fig. 14, frames 1-5, during a period approximately 10 hr following the start of Unit 1 operation when the minimum temperature drops down to 69.5°F, from the initially assumed value of 70°F, due to severe nighttime cooling which occurs before the spreading of the thermal plume in the coastal region. Consequently it may be more realistic to consider the 74.5°F isotherm in checking the compliance of the Unit 1 operation with the assumed standards. This isotherm extends beyond the 1000-ft limit only in two instances during tidal cycle 12 as:

1. Sixth day at 17:57 hr (frame 1, Figs. 24, 25), maximum reach 2200 ft along the direction of the tidal flow toward the shore.

2. Sixth day at 19:30 hr (frame 2, Figs. 24, 25), 800 ft on the shoreline.

Hence, the 74.5°F isotherm remains within the 1000-ft limit distance from the discharge more than half the duration of the tidal cycle. Consequently, it can be concluded that the temperature solutions for case 1 indicate that the Unit 1 operation of the hypothetical nuclear generating station complies with the assumed thermal impact limits for the coastal region.

The numerical results for case 2, which assumes the actual transient local depth as the thermally effective depth of the heated water layer under strong vertical mixing conditions, are presented as isotherms in Figs. 26, 28, 30, 32, 34, 36, 38, 40, 42, and 44 and as perspective plots of temperature in Figs. 27, 29, 31, 33, 35, 37, 39, 41, 43, and 45 for tidal cycles 1, 2, 5, 6, 9, 10, 15, 16, 19, and 20 during approximately ten days period after the start of the operation of Unit 1 at midnight, August 18.

The overall behavior of the transient temperature distributions for case 2 is considerably more complex than for case 1 since the thermally effective depth of the heated water layer as the actual depth

significantly varies in the coastal region. The mean water level depth changes from approximately 10 ft near the shoreline to 100 ft at the boundary of the region where the results are presented and to 1200 ft at the boundary of the region employed in the computations (see Fig. 12). Furthermore, in contrast to the constant depth of 20 ft for case 1, the thermally effective depth of the discharge plume for case 2 varies periodically with the tidal elevations (see Fig. 13), particularly near the shoreline where the water level varies more than 100%, approximately from 6 ft to 14 ft, during a tidal cycle.

The pseudo quasi-steady-state condition for case 2, when the minimum temperature in the region periodically varies within steady bounds of 70.5°F and 71.0°F , is reached approximately 20 cycles, in comparison to 12 cycles for case 1, after the start of the Unit 1 operation. This result is to be expected from physical considerations since the overall heat capacity of the coastal region for case 2 is significantly larger than for case 1 due to the actual local depth being employed in the mathematical model. Since the time rate of thermal energy discharge into the region is the same for both cases, the overall time rate of increase of temperature in the coastal region is considerably slower for case 2 than for case 1 due to the inverse proportionality effect of the overall heat capacity.

The transient variation of the depth of the heated water layer is physically equivalent to the transient variation of the thermally effective fluid height in the discrete elements of the mathematical model. Consequently, for case 2, there exists an additional periodic effect of time rate of accumulation or reduction of fluid in each discrete element. Since the water temperature depends on the direction of the oceanic flow field, the accumulated water in an element may either be the heated water from the discharge or the cooler water from the deep ocean regions. This phenomenon in a sense produces an additional effect of periodic variations of the heat capacities of the elements which significantly alters the transient conditions of the temperature distributions corresponding to the constant depth of 20 ft assumption for case 1. Hence, the periodicity of the transient temperature solutions for case 2 become considerably out of phase with the periodic conditions of the tidal cycle. This behavior is clearly illustrated in all Figs. 26-45 both as isotherms and as perspective plots of the temperature solutions. The same time frames in successive cycles are both qualitatively and quantitatively different from each other particularly for cycles immediately after the start of the Unit 1 operation before the spreading of the thermal plume in the coastal region and for non-slack-water conditions when the convective effects are dominant.

The assumption of local and transient variations of the depth of the heated water layer for case 2 also introduces significant differences from the results of case 1 for the evaluation of the convective terms in the analysis. Since the local depth in the coastal region varies considerably, approximately from 6 ft to 1200 ft, depending on the ocean bottom topography and the transient tidal elevations, in contrast to case 1, the convection phenomenon for case 2 very strongly depends on the actual depth conditions. Consequently, near the shoreline where the transient variations in depth are most important, approximately 220% change between 6 ft and 14 ft, the temperature changes during the tidal period are considerably more pronounced for case 2 than for case 1, particularly during the cycles immediately after the start of the Unit 1 operation and before the long-time spreading of the thermal plume in the coastal region.

Fig. 26. Isotherms (0.5°F gradation) during cycle 1 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

14-1-1

14-1-2

14-1-3

14-1-4

14-1-5

14-1-6

14-1-7

14-1-8

Fig. 27. Perspective plots of temperature distribution (datum 69°F) during cycle 1 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

Fig. 28. Isotherms (0.5° F gradient) during cycle 2 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

14-5-1

14-5-2

14-5-3

14-5-4

14-5-5

14-5-6

14-5-7

14-5-8

Fig. 29. Perspective plots of temperature distribution (datum 69°F) during cycle 2 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

Fig. 30. Isotherms (0.5°F gradation) during cycle 5 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

Fig. 31-1

Fig. 31-2

Fig. 31-3

Fig. 31-4

Fig. 31-5

Fig. 31-6

Fig. 31-7

Fig. 31-8

Fig. 31. Perspective plots of temperature distribution (datum 69°F) during cycle 5 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

Fig. 32. Isotherms (0.5°F gradation) during cycle 6 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

Fig. 33. Perspective plots of temperature distribution (datum 69°F) during cycle 6 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

Fig. 34. Isotherms (0.5°F gradation) during cycle 9 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 04:45 hr to day 4, 15:36 hr.

14-10-1

14-10-2

14-10-3

14-10-4

14-10-5

14-10-6

14-10-7

14-10-8

Fig. 35. Perspective plots of temperature distribution (datum 69 F) during cycle 9 for case 2 with only unit 1 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 04.45 hr to day 4, 15.36 hr.

Fig. 36. Isotherms (0.5° F gradation) during cycle 10 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 17:09 hr to day 5, 04:00 hr.

Fig. 37. Perspective plots of temperature distribution (datum 60°F) during cycle 10 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 17:09 hr to day 5, 04:00 hr.

Fig. 36. Isotherms (0.5°F gradient) during cycle 15 for case 2 with only unit i operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 07:09 hr to day 7, 18:00 hr.

14-15-1

14-15-8

14-15-2

14-15-7

14-15-3

14-15-6

14-15-4

14-15-5

Fig. 39. Perspective plots of temperature distribution (datum 69°F) during cycle 15 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 07:09 hr to day 7, 18:00 hr.

Fig. 48. Isotherms (0.5°F graduation) during cycle 16 for case 2 with only unit 1 operational. Frames at $\frac{1}{2}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 19:33 hr to day 8, 06:24 hr.

14-16-1

14-16-2

14-16-3

14-16-4

14-16-5

14-16-6

14-16-7

14-16-8

Fig. 41. Perspective plots of temperature distribution (bottom 69°F) during cycle 16 for case 2 with only unit 1 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 19:33 hr to day 8, 06:24 hr.

Fig. 42. Isotherms (0.5° F gradient) during cycle 19 for case 2 with only unit 1 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 08:45 hr to day 9, 19:36 hr.

14-19-1

14-19-2

14-19-3

14-19-4

14-19-5

14-19-6

14-19-7

14-19-8

Fig. 43. Perspective plots of temperature distribution (datum 69°F) during cycle 19 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 08:45 hr to day 9, 19:36 hr.

Fig. 44. Isotherms (0.5°F gradation) during cycle 20 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (11 hr 33 min), counterclockwise, starting at top left corner, from day 9, 21:09 hr to day 10, 08:00 hr.

14-20-1

14-20-6

14-20-2

14-20-7

14-20-3

14-20-8

14-20-4

14-20-9

Fig. 45. Perspective plots of temperature distribution (datum 69°F) during cycle 20 for case 2 with only unit 1 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 21:09 hr to day 10, 08:00 hr.

The computed results for case 2 also indicate that, near the shoreline, the effect of the surface heat transfer phenomenon on the temperature distributions is more important than for case 1. In Fig. 28, in the time frames 7 and 8 corresponding to 23:15 hr and 00:48 hr, respectively, during cycle 2, after the start of operation of Unit 1, the minimum temperature isotherm 69°F is immediately adjacent to the shoreline of the coastal region. This value for case 2 is lower than the minimum temperature 69.5°F computed for case 1. This interesting result can be readily explained if one considers the combined convection and surface heat transfer phenomena during the tidal cycle prior to the instant when the minimum temperature occurs. Since during cycle 2, the ocean water in deep and sufficiently far regions from the discharge is not yet heated by the spreading of the thermal plume, when the tidal flow starts moving the water away from the shore, along the direction determined by the components of the velocity field (Fig. 13), frames 5 and 6 in Figs. 28 and 29, corresponding to 20:09 hr and 21:42 hr, respectively, the region near the shoreline starts receiving some of this cool ocean water at temperatures 70.5°F and 69.5°F , respectively, for frames 5 and 6. Considering the tidal conditions during the remainder of the tidal cycle (see Fig. 13), the depth of the region near the shoreline remains in the neighborhood of the minimum water level conditions of approximately 6 ft. Noting that it is almost midnight, there exist severe surface cooling conditions in the region as blackbody back radiation. Since the thermally effective depth of the water is assumed as the local depth for case 2, which is approximately 6 ft during this period, the overall heat capacity of the water in the shallow region is considerably less than for case 1 which assumes constant 20 ft depth for the thermally effective layer. Consequently, for case 2, the water near the shoreline cools down more rapidly to the minimum temperature value of 69°F , which is lower than 69.5°F for case 1.

During later cycles 5-12, after the spreading of the thermal plume in the coastal region, the transient variations of depth for case 2 also cause higher temperature conditions near the shoreline in the afternoons and in the evenings. During the afternoon of the third day after the start of Unit 1 operation (Figs. 30, 31, frame 8, and Figs. 32, 33, frame 1, corresponding to 14:00 hr and 15:33 hr, respectively), the temperature reaches approximately 74°F in a region parallel to the shoreline and more than 8000 ft in both directions from the discharge. Similar high water temperature conditions near the shoreline and on the shore also occur in late afternoons and evenings during later tidal cycles due to the solar radiative heating of the thermal plume in the coastal region.

A very interesting consequence of the actual transient local depth assumption for the thermally effective thickness of the heated water layer is shown in Figs. 30 and 31, frame 1. A distinct warmer temperature spot of substantial dimensions, approximately 1000 ft by 4000 ft, appears in the coastal region with its center located approximately 6000 ft from the discharge of Unit 1. Although the temperature difference between the area of the spot (70°F) and the surrounding water (69.5°F) is quite small, it is very important to realize that the mathematical model is indeed capable of predicting the possible existence of such warmer areas in a coastal region resulting from the thermal impact of only a single port discharge structure. The existence of warmer areas of this type in the middle of cooler regions away from the location of a single port discharge (frame 1, Figs. 30, 31, and frame 8, Figs. 32, 33) can only be explained by considering all transient oceanic flow and atmospheric conditions; and it cannot be predicted by any simpler Eulerian mathematical model than the present FLIDE model.

The numerical results for case 2 indicate that the thermal impact conditions of Unit 1 operation cause higher temperatures in larger areas but for shorter durations than the results for case 1 in the coastal region. The times when the temperature exceeds the 74°F limit beyond 1000 ft from the discharge are listed according to their order as they appear in tidal cycles beginning with the start of Unit 1 operation at midnight, August 18.

1. Third day at 14:00 hr (frame 8, Figs. 30, 31), 74°F isotherm is 2800 ft away and parallel to the shoreline extending more than 8000 ft in both directions from the discharge.

This is the first time for case 2, after the start of operation of Unit 1, when the 74°F isotherm extends beyond the 1000-ft limit from the discharge. It occurs at slack water conditions at the end of cycle 5 when the maximum temperature in the region is at the point of discharge.

2. Third day at 15:33 hr (frame 1, Figs. 32, 33), 74°F isotherm starts on the shore at 6500 ft down coast from the line of the Unit 1 discharge and extends more than 8000 ft up coast at 2200 ft from the shoreline. The 75°F isotherm starts on the shore at 3000 ft down coast and extends to 6000 ft up coast on the shore from the line of the Unit 1 discharge.

This rather severe high-temperature condition on the shore occurs due to the motion of the high-temperature water, which was over the Unit 1 discharge at the previous time frame at 14:00 hr (frame 8, Figs. 30, 31), along the tidal flow toward the shore during a period of 1 hr 33 min. Since the depth of water near the shoreline is very shallow, approximately 8 ft, and the solar radiative heating on the surface is almost maximum in the early afternoon, the already heated layer of water, which was over the discharge, is further heated as it spreads and accumulates in the region near the shoreline. However, this high-temperature condition does not last long since the colder water from the deeper parts of the region follows the thermal plume to the shore with the tidal flow to reduce the temperature of the already existing heated water.

3. Fifth day at 14:03 hr (frame 7, Figs. 34, 35), 74°F along the shore more than 8000 ft in both directions, 75°F isotherm at 1200 ft away and parallel to the shoreline extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

4. Fifth day at 15:36 hr (frame 8, Figs. 34, 35), 75°F along the shore more than 8000 ft in both directions, 75.5°F isotherm at 1000 ft away and parallel to the shoreline extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

5. Fifth day at 17:09 hr (frame 1, Figs. 36, 37), 76°F on the shore from 6000 ft down coast to 8000 ft up coast, 76.5°F isotherm reaching the shore at a point 1000 ft up coast of the line of Unit 1 discharge. The 75°F isotherm starts on the shore at 7000 ft down coast from the line of Unit 1 discharge and extends more than 8000 ft up coast at 2000 ft from the shoreline. The 74°F isotherm extends more than 8000 ft in both directions from the line of Unit 1 discharge.

6. Fifth day at 18:42 hr (frame 2, Figs. 36, 37), 74°F on the shore from 500 ft down coast to 1000 ft up coast from the line of Unit 1 discharge.

The results clearly indicate that during an approximately 5-hr period, between 14:00 hr and 19:00 hr, the thermal impact of Unit 1 operation severely exceeds the limitations based on the 74°F isotherm in the coastal region. However, after 19:00 hr, the temperature rapidly drops due to the tidal flow conditions and atmospheric nighttime cooling in the region; and it remains lower than the 74°F temperature limit throughout the remainder of the tidal cycle (Figs. 36, 37).

7. Eighth day at 14:54 hr (frame 6, Figs. 38, 39), 74°F isotherm on the shore from 6500 ft down coast to 9000 ft up coast from the line of Unit 1 discharge, 74.5°F isotherm at 2500 ft away and parallel to the shoreline extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

8. Eighth day at 16:27 hr (frame 7, Figs. 38, 39), 75°F isotherm on the shore from 5000 ft down coast to 2000 ft up coast from the line of Unit 1 discharge, 75.5°F isotherm at 2500 ft away and parallel to the shoreline extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

9. Eighth day at 18:00 hr (frame 8, Figs. 38, 39), 75.5°F isotherm in a region from the shoreline, including the shore, to 3000 ft, extending more than 8000 ft in both directions from the line of Unit 1 discharge; 75°F and 74°F isotherms 4000 ft and 5500 ft, respectively, away and parallel to the shoreline and extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

10. Eighth day at 19:33 hr (frame 1, Figs. 40, 41), 76°F isotherm reaching the shore at a point 3000 ft from the line of Unit 1 discharge, 75.5°F isotherm on the shore from 3500 ft down coast to 10,000 ft up coast, 75°F isotherm on the shore from 7000 ft to 3500 ft down coast from the line of Unit 1 discharge, 74°F isotherm 3800 ft away and parallel to the shoreline extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

11. Eighth day at 21:06 hr (frame 2, Figs. 40, 41), 74.5°F isotherm on the shore from 3500 ft down coast to 10,000 ft up coast of the line of Unit 1 discharge, 74°F isotherm starting on the shore at 6500 ft down coast and extending beyond 8000 ft up coast from the line of Unit 1 discharge, 4000 ft away and parallel to the shoreline.

Again, during tidal cycles 15 and 16 after the start of Unit 1 operation, in the afternoon and the evening, for approximately 6 hr, from 15:00 hr to 21:00 hr, there exist severe excess temperature conditions in the coastal region. During most of this period, the temperature on the shore exceeds 75°F and the 74°F isotherm extends well beyond the 1000-ft limit from the discharge reaching more than 10,000 ft in both down-coast and up-coast directions parallel to the shoreline.

12. Tenth day at 18:03 hr (frame 7, Figs. 42, 43), 74.5°F isotherm on the shore from 1000 ft down coast to 7000 ft up coast from the line of Unit 1 discharge; 75°F isotherm, maximum reach 3200 ft from the discharge along the tidal motion toward the ocean; 74°F isotherm 4000 ft away and parallel to the shoreline, extending beyond 8000 ft in both directions from the line of Unit 1 discharge.

13. Tenth day at 19:36 hr (frame 8, Figs. 42, 43), 74°F isotherm in a region from the shoreline to 5000 ft, extending more than 8000 ft in both directions from the line of Unit 1 discharge; 74.5°F isotherm, maximum reach 6000 ft, and 75°F isotherm, maximum reach 3000 ft from the discharge, parallel to the shoreline along the down-coast direction from the Unit 1 discharge.

14. Tenth day at 21:09 hr (frame 1, Figs. 44, 45), 74.5°F isotherm reaching the shore at a point 1000 ft from the line of Unit 1 discharge and covering a region approximately 8000 ft by 2500 ft around the discharge; 75°F isotherm, maximum reach 3500 ft from the discharge, parallel to the shoreline along the up-coast direction from Unit 1 discharge; 74°F isotherm 3800 ft away and parallel to the shoreline, extending 7500 ft down coast and beyond 8000 ft up coast from the line of Unit 1 discharge.

Again, during tidal cycles 19 and 20, after the start of Unit 1 operation, in the evening, for approximately 3 hr, from 18:00 hr to 21:00 hr, there exist severe excess temperature conditions in the coastal region. During most of this period the temperature on the shore exceeds 74°F and the isotherm reaches 5000 ft from the shoreline and more than 8000 ft in both down-coast and up-coast directions from the Unit 1 discharge.

Although the results for case 2, with the assumption that the thermally effective layer is the actual transient local depth under strong vertical mixing conditions, indicate that the temperatures in the coastal region reach values higher than 74°F, 4°F above the assumed ambient 70°F, at distances well beyond the 1000-ft limit from the discharge, during the afternoons and the evenings, the operational conditions of Unit 1 do not cause thermal conditions that exceed the assumed thermal impact criteria for the coastal region, since these critical conditions do not last more than half of the tidal period.

The simulated temperature distributions, Figs. 14-45, from the solution of the FLIDE model, for the assessment of the thermal impact of Unit 1 operation were discussed in extensive detail because of two fundamental reasons.

First, it was necessary to identify the importance of various natural phenomena which control the thermal conditions in the coastal region as they appeared in the numerical results of the mathematical model. The short-time transient (within tidal cycle) temperature distributions indicated that the tidal flow conditions, both the velocity field and the depth variations, result in the dominant convective phenomenon

which determines the behavior of the thermal plume. It was evident that heated water can travel significant distances from the discharge along the tidal flow direction, without appreciable cooling by the surface heat transfer and turbulent diffusion transport, in the coastal region. Hence, according to the assumed, strictly periodic tidal flow conditions, the heated water definitely returned to the vicinity of the intake and discharge structures during the tidal cycle, resulting in substantial recirculation and re-entrainment. The results also indicated that the surface heat transfer, particularly the daytime solar irradiation, could interact with the tidal motion of the thermal effluent in shallow water regions to cause unusually high temperature conditions near the shoreline during certain afternoons and evenings depending on the daily tidal conditions in the coastal region.

Second, since it was assumed that the hypothetical nuclear generating station already has an operational unit, theoretically, the results of the mathematical model can be verified by appropriate measurements of the temperature conditions in the coastal region. The results clearly indicated that since the temperature variations are extremely transient and complex, depending on the tidal flow and surface heat transfer conditions, such a monitoring program must be carefully planned to incorporate the simultaneous measurements of all the pertinent thermal and hydrodynamic flow conditions in the coastal region during sufficiently long periods of observation. That is, the verification of the mathematical model can only be accomplished by obtaining all the necessary field data according to the requirements of the model.

3.3 Discussion of Results for Proposed Units 1, 2, 3 Operation

The FLIDE mathematical model was also employed to estimate the thermal impact of the hypothetical generating station after the construction of the proposed two additional 1150-MW nuclear units with discharge locations as shown in Figs. 10 and 12. To directly compare the results, the numerical solutions for the temperature distributions in the coastal region, with all three Units 1, 2, 3 operating, were obtained under the same tidal flow, surface heat transfer, and initial conditions which were employed in the solution for Unit 1 operation.

The numerical results for case 1, which assumes 20 ft thermally effective depth for the stratified heated water layer, are presented as isotherms in Figs. 46, 48, 50, 52, 54, and 56 and as perspective plots of temperature in Figs. 47, 49, 51, 53, 55, and 57 for tidal cycles 1, 2, 5, 6, 11, and 12, during approximately 6 days after the start of the operation of all three Units 1, 2, 3 simultaneously at midnight, August 18.

The results clearly indicate that the dominant convective phenomenon causes severe thermal interaction between the single port discharge of Unit 1 and the diffusers of Units 2 and 3 during all tidal cycles (Figs. 46-57), due to the assumed periodic flow conditions in the coastal region. The localized high temperature distributions around the discharges exist only during high and low slack water conditions (frames 4 and 8, respectively, in Figs. 46-57), when the tidal flow velocities are minimum.

The heated water from the discharges never develops into three well-defined, distinct thermal plumes as may be expected. During most of the tidal cycle, with the exception of slack water conditions, the overall velocity field forces the heated water into a single irregular plume which moves according to the tidal flow conditions. Since the rate of rejected heat for the proposed three-unit operation is approximately five times more than for the existing single-unit operation, the combined thermal plume of the discharges of Units 1, 2, 3 covers significantly large areas in the coastal region. This effect becomes evident immediately after the start of the operation of Units 1, 2, 3, during cycle 1 (Figs. 46, 47, particularly frames 2, 5, 7), without requiring the long-time spreading of the heated water in the coastal region.

The 74°F isotherm, corresponding to 4°F excess over assumed ambient 70°F, reaches beyond the 1000-ft limit from the discharge for the first time during cycle 2 after the start of the operation of Units 1,

2, 3, at 13:57 hr (Fig. 48, frame 1), at 1600 ft up coast of Unit 2 discharge and 1200 ft from Unit 1 discharge along the tidal flow toward the shore. At various other times during the cycle, 15:30 hr, 20:09 hr, 21:42 hr (frames 2, 6, 7, respectively, Fig. 48), hot spot areas at 74°F appear in the region resulting from the thermal interaction of the discharges according to the tidal flow conditions.

The thermal impact conditions of the hypothetical nuclear generating station, with the proposed three units, start exceeding the assumed thermal impact criteria for the coastal region during cycle 5, after the start of the Units 1, 2, 3 operation. During the complete 12.4-hr cycle from 03:09 hr through 14:00 hr (Figs. 50, 51, frames 1-8), the 74°F isotherm extends beyond the 1000-ft limit from the discharges under all tidal flow conditions. Indeed, for more than 50% duration of the tidal cycle there exist hot spot areas equal to or higher than 75°F in the region (Fig. 50, frames 2, 3, 6, 7, 8), with temperatures in excess of 74°F at different parts of the shore. Since most of this cycle occurs during the night and the morning, under high atmospheric cooling and low solar irradiation conditions, the existence of high-temperature areas is strictly caused by the thermal discharge conditions of the generating station. During cycle 6, which corresponds mostly to the afternoon and evening conditions, between 15:33 hr and 02:24 hr (Figs. 52, 53, frames 1-8), there exist areas in the coastal region in excess of 76°F, more than 1000 ft away from the discharges; and throughout the tidal cycle, the temperature remains in excess of 75°F at different parts of the shoreline.

After pseudo quasi-steady-state thermal conditions are reached, during tidal cycles 11 and 12 (Figs. 54-57), the minimum temperature in the coastal region varies between 72.5°F and 73.5°F which is only 1°F above the conditions of Unit 1 operation. However, in contrast to the temperature distributions for Unit 1, throughout the complete tidal cycle, the 74°F isotherm extends well beyond the 1000-ft limit from the discharges for the operation of all three units. Indeed, during the complete cycle (Figs. 54-57), the 76°F isotherm reaches distances in excess of 8000 ft up coast of the discharge structures, with hot-spot areas at 79.5°F appearing in the coastal region (Figs. 56, 57, frame 2), due to the thermal interaction of the intakes and discharges under the influence of the oceanic flow conditions.

The maximum temperature on the shore never drops below 77°F during the tidal cycle; and at various instances, it reaches 78°F depending on the tidal and surface heat transfer conditions in the coastal region.

Hence, the simulated temperature distributions for case 1, which assumes 20 ft for the thermally effective depth of the stratified heated water layer, under the assumed, strictly periodic, tidal flow conditions, indicate that the thermal conditions resulting from simultaneous operation of all three units of the hypothetical generating station will exceed the assumed thermal impact criteria for the coastal region.

Fig. 46. isotherms (0.5 F gradation) during cycle 1 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

8- 1-1

8- 1-8

8- 1-2

8- 1-7

8- 1-3

8- 1-6

8- 1-4

8- 1-5

Fig. 47. Perspective plots of temperature distribution (datum 69°F) during cycle I for case I with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

Fig. 48. Isotherms (0.5°F gradient) during cycle 2 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counter-clockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

0-2-1

0-2-8

0-2-2

0-2-7

0-2-3

0-2-6

0-2-4

0-2-5

Fig. 49. Perspective plots of temperature distribution (surface 00°P) during cycle 2 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{6}$ cycle (1 hr 53 min), counter-clockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:40 hr.

Fig. 38. Isotherms (0.5°F gradient) during cycle 5 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), commencing at top left corner, for: day 2, 03:00 hr to day 2, 14:00 hr.

0- 5-1

0- 5-0

0- 5-2

0- 5-7

0- 5-3

0- 5-6

0- 5-4

0- 5-5

Fig. 51. Perspective plots of temperature distribution (datum 69°F) during cycle 5 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:00 hr to day 2, 14:00 hr.

Fig. 52. Isotherms (0.5°F gradation) during cycle 6 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

8-6-1

8-6-8

8-6-2

8-6-7

8-6-3

8-6-6

8-6-4

8-6-5

Fig. 53. Perspective plots of temperature distribution (datum 69°F) during cycle 6 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

Fig. 54. Isotherms (0.5°F gradation) during cycle 11 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 05:33 hr to day 5, 16:24 hr.

Fig. 55. Perspective plots of temperature distribution (datum 69°F) during cycle 11 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 05:33 hr to day 5, 16:24 hr.

Fig. 56. Isotherms (0.5° F gradation) during cycle 12 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 17:57 hr to day 6, 04:48 hr.

Fig. 57. Perspective plots of temperature distribution (datum 69°F) during cycle 12 for case 1 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 5, 17:57 hr to day 6, 04:48 hr.

The numerical results for case 2, which assumes the actual transient local depth as the thermally effective depth of the heated water layer under strong vertical mixing conditions, are presented as isotherms in Figs. 58, 60, 62, 64, 66, 68, 70, 72, 74, and 76 and as perspective plots of temperature in Figs. 59, 61, 63, 65, 67, 69, 71, 73, 75, and 77 during approximately ten days period after the start of the operation of all three Units 1, 2, 3 simultaneously at midnight, August 18.

Similar to case 1, the results for case 2 indicate that the dominant convective phenomenon causes severe thermal interaction between the discharges of the three units during all tidal cycles (Figs. 58–77), due to the assumed, periodic tidal flow conditions in the coastal region. Again, the localized high temperature distributions around the immediate vicinity of the discharges occur only during high and low slack water conditions (frames 4 and 8, respectively, Figs. 58–77), when the tidal flow velocities are minimum. However, since for case 2 it is assumed that the thermally effective depth of the heated water layer is the actual transient local depth in the region, during early cycles (Figs. 58–61) after the start of Units 1, 2, 3 operation, the temperatures increase slower than for case 1 due to the higher overall heat capacity of water under the assumption of case 2.

The heated water from the single port discharge of Unit 1 and the diffusers of Units 2 and 3 again combines into a single irregular-shaped thermal plume which moves according to the tidal flow conditions in the coastal region. Since, according to the assumption of case 2, the thermally effective depth of the heated water varies timewise during the tidal cycles and locally in the region, the temperature distributions exhibit more complex behavior than for case 1, particularly in shallow water regions near the shoreline where the surface heat transfer effect is appreciable during the afternoons and the evenings at low slack water conditions. The thermal plume spreads considerably slower for case 2 than for case 1 since the overall heat capacity of the water in the coastal region is assumed to be larger for case 2 than for case 1.

The 74°F isotherm, corresponding to 4°F excess over assumed ambient 70°F, reaches beyond the 1000-ft limit from the discharge for the first time during cycle 5, in comparison to cycle 2 for case 1, after the start of operation of Units 1, 2, 3, at 12:27 hr (Fig. 62, frame 7), at more than 8000 ft up coast from the Unit 1 discharge parallel to the shoreline. At various other instances during cycle 5 (Fig. 62, frame 8) and the subsequent cycle 6 (Fig. 64, frames 1–4), during the afternoon and the evening, between 14:00 hr and 20:12 hr, the 74°F isotherm extends beyond 8000 ft up coast parallel to the shoreline when temperatures in excess of 75°F occur on the shore. The most critical conditions from thermal impact considerations occur at 15:33 hr during tidal cycle 6 when the temperature on the shore reaches 77°F along an 800-ft section, approximately 1000 ft up coast from the line of Unit 1 discharge. At this time the temperature on the shore exceeds 75°F for approximately 8000 ft in both directions from the location of the generating station; and the 76°F isotherm extends more than 8000 ft up coast parallel to the shoreline.

The thermal impact conditions of the hypothetical nuclear generating station, with the proposed three units, start exceeding the assumed thermal impact criteria for the coastal region on the fifth day, during cycle 9, after the start of Units 1, 2, 3 operation. During this cycle (Figs. 66, 67) and the subsequent cycle 6 (Figs. 68, 69), throughout the tidal period (frames 1–8), the 74°F isotherm reaches distances beyond the 1000-ft limit from the discharges. At 17:09 hr during cycle 10 (Figs. 68, 69, frame 1), the temperatures beyond the 1000-ft limit reach 78.5°F from the Unit 1 discharge, along the tidal flow, toward the shore; and the 78°F isotherm extends approximately 8000 ft up coast on the shoreline.

During the subsequent cycles 10–16 (Figs. 68–73), the thermal conditions continuously exceed the assumed criteria by creating temperatures in excess of 74°F at distances beyond the 1000-ft limit from the discharges for periods more than half of the duration of the tidal period.

After pseudo quasi-steady-state thermal conditions are reached, during cycles 19 and 20 (Figs. 74–77) on the tenth day after the start of the Units 1, 2, 3 operation, the minimum temperature in the coastal

region varies between 70.5°F and 71.5°F which is identical to the result obtained for case 2 with only Unit 1 operating. Since the rate of rejected heat for the proposed three-unit operation is approximately five times more than for the existing single Unit 1 operation, this result may seem peculiar from thermal discharge considerations. However, noting that based on the assumption of case 2, the thermally effective depth of the heated water is the actual depth in the coastal region, the formulation of the problem considers the depth of the thermal plume as much as 1200 ft at the boundary of the computational region (Fig. 11) according to the topography of the ocean bottom (Fig. 12). Consequently, the evaluation of the dominant convective thermal energy transfer terms in deep regions necessarily employs the total depth in the general formulation. Hence, the convective energy transport out of the computational region becomes unrealistically large, resulting in excessive cooling in the deep ocean parts of the coastal region. Furthermore, since the depth near the boundary of the region where the results are presented (Fig. 11) is approximately 90 ft, according to the assumption of case 2, the rate of heating of the thermally effective layer of 90 ft is considerably slower than for case 1, which considers constant 20 ft depth for the thermal plume. That is, the deepwater regions, near the boundary where the minimum temperatures occur, do not respond as rapidly to the heated water from the discharges, under the assumption of full vertical mixing conditions of case 2, as for the assumption of stratified thermal plume conditions for case 1. Hence, the significant increase in the rate of thermal energy input in the shallow water regions for the proposed Units 1, 2, 3 operation does not appreciably change the minimum temperature conditions from the single Unit 1 operation in deepwater regions under the assumption of case 2 for the actual local depth as being fully thermally effective in the analysis.

However, although the minimum temperature bounds remain approximately unchanged for the proposed Units 1, 2, 3 operation, for case 2, under pseudo quasi-steady-state thermal conditions, the maximum temperatures continuously remain in excess of 74°F at distances more than 1000 ft from the discharges, in contrast to the conditions of the existing Unit 1 operation. During cycles 19 and 20, 10 days after the start of the Units 1, 2, 3 operation, the 75°F isotherm extends beyond 8000 ft up coast from the location of the discharges, more than half of the duration of the tidal period (Figs. 74-77). Furthermore, the temperatures on the shoreline remain in excess of 75°F longer than half of the duration of the tidal cycle; and they reach maximum values of approximately 76°F in the afternoons and the evenings depending on the tidal flow conditions (Figs. 74, 75, frames 6-8; Figs. 76, 77, frames 1-3).

Hence, the simulated temperature distributions for case 2, which assumes the thermally effective depth of the heated water layer as the local transient depth, for strong vertical mixing conditions, under the assumed, strictly periodic, tidal flow conditions, indicate that the thermal conditions resulting from simultaneous operation of all three units of the hypothetical generating station will exceed the assumed thermal impact criteria for the coastal region.

Fig. 58. Isotherms (0.5°F gradient) during cycle 1 for case 2 with all units 1, 2, 3 operational. Frames at V_0 cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

10-1-1

10-1-6

10-1-2

10-1-7

10-1-3

10-1-8

10-1-4

10-1-5

Fig. 59. Perspective plots of temperature distribution (datum 69°F) during cycle 1 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 01:33 hr to day 0, 12:24 hr.

Fig. 68. Isotherms (0.5°F gradation) during cycle 2 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

10- 2- 1

10- 2- 8

10- 2- 2

10- 2- 7

10- 2- 3

10- 2- 6

10- 2- 4

10- 2- 5

Fig. 61. Perspective plots of temperature distribution (datum 69° F) during cycle 2 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 0, 13:57 hr to day 1, 00:48 hr.

Fig. 62. Isotherms (0.5°F gradation) during cycle 5 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

10-5-1

10-5-6

10-5-2

10-5-7

10-5-3

10-5-6

10-5-4

10-5-5

Fig. 63. Perspective plots of temperature distribution (datum 69° F) during cycle 5 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 03:09 hr to day 2, 14:00 hr.

Fig. 64. Isotherms (0.5°F gradient) during cycle 6 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 2, 15:33 hr to day 3, 02:24 hr.

18-6-1

18-6-2

18-6-3

18-6-4

18-6-5

18-6-6

18-6-7

18-6-8

Fig. 65. Perspective plots of temperature distribution (datum 69°F) during cycle 6 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner from day 2, 15:33 hr to day 3, 02:24 hr.

Fig. 66. Isotherms (0.5°F gradient) during cycle 9 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 04:45 hr to day 4, 15:36 hr.

10-9-1

10-9-8

10-9-2

10-9-7

10-9-3

10-9-6

10-9-4

10-9-5

Fig. 67. Perspective plots of temperature distribution (datum 69°F) during cycle 9 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 04:45 hr to day 4, 15:36 hr.

Fig. 68. Isotherms (0.5°F gradation) during cycle 10 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 17:09 hr to day 5, 04:06 hr.

18-10-1

18-10-8

18-10-2

18-10-7

18-10-3

18-10-6

18-10-4

18-10-5

Fig. 69. Perspective plots of temperature distribution (datum 69°F) during cycle 10 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{5}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 4, 17:09 hr to day 5, 04:00 hr.

Fig. 70. Isotherms (0.5°F graduation) during cycle 15 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 07:09 hr to day 7, 18:00 hr.

18-15-1

18-15-8

18-15-2

18-15-7

18-15-3

18-15-6

18-15-4

18-15-5

Fig. 71. Perspective plots of temperature distribution (datum 69°F) during cycle 15 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 07:09 hr to day 7, 18:00 hr.

Fig. 72. Isotherms (0.5°F gradient) during cycle 16 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{4}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 19:33 hr to day 8, 06:24 hr.

18-16-1

18-16-6

18-16-2

18-16-7

18-16-3

18-16-6

18-16-4

18-16-5

Fig. 73. Perspective plots of temperature distribution (datum 69°F) during cycle 16 for case 2 with all units 1, 2, 5 operational. Frames at $\frac{1}{6}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 7, 19:33 hr to day 8, 06:24 hr.

Fig. 74. Isotherms (0.5°F graduation) during cycle 19 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 08:45 hr to day 9, 19:36 hr.

18-19-1

18-19-2

18-19-3

18-19-4

18-19-5

18-19-6

18-19-7

18-19-8

Fig. 75. Perspective plots of temperature distribution (datum 69°F) during cycle 19 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 08:45 hr to day 9, 19:36 hr.

Fig. 76. Isotherms (0.5°F gradient) during cycle 20 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{8}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 21:09 hr to day 10, 08:00 hr.

18-20-1

18-20-8

18-20-2

18-20-7

18-20-3

18-20-6

18-20-4

18-20-5

Fig. 77. Perspective plots of temperature distribution (datum 69°F) during cycle 20 for case 2 with all units 1, 2, 3 operational. Frames at $\frac{1}{3}$ cycle (1 hr 33 min), counterclockwise, starting at top left corner, from day 9, 21:09 hr to day 10, 08:09 hr.

3.4 Comparison of the Results for Existing Unit 1 Operation and for Proposed Units 1, 2, 3 Operation

In comparing the thermal impact conditions in the coastal region for the existing Unit 1 operation and the proposed Units 1, 2, 3 operation of the hypothetical generating station, based on the numerically simulated temperature results of the FLIDE far-field model under the assumed, strictly periodic, tidal flow conditions, it is important to consider the physical consequences of the assumptions associated with case 1 and case 2 in relation to the intake and discharge conditions of the three nuclear units.

According to case 1, it is assumed that the thermally effective depth of the heated water layer from all discharge structures remains approximately constant as 20 ft. This value is not arbitrarily chosen, but it is based on the average distance between the discharge ports and the ocean surface (Fig. 12) during the tidal cycle when the depth varies approximately 8 ft in the coastal region. Since the discharge water will definitely tend to stratify into a layer near the surface, the increase of the depth of the thermal plume will be caused predominantly by the vertical diffusion according to the turbulent velocity profiles of the tidal flow conditions. The correct modeling of the transient local depth of the heated water layer in the region will require actual field data, and it will definitely depend on the transient local Froude number. However, considering the relatively small distances, approximately 500 ft, between the intakes and the discharges of the existing and proposed units, it is realistic to assume that the depth of the thermal plume does not change appreciably as it moves from the discharges toward the intakes according to the oceanic flow conditions in the coastal region. Hence, since the single port intakes of all three units are also located approximately 20 ft below the ocean surface (Fig. 12), it is also realistic to assume that the condenser cooling water enters the intakes at the local temperature of the water at the location of the intakes. It should be realized that this definitely does not imply that the heated water from the discharges directly enters the intakes at the temperature of the discharge conditions; but indeed, the assumption automatically incorporates the cooling effects due to the diffusion by mixing, convection, turbulent transport, and surface heat transfer into the recirculation phenomenon as it occurs according to the assumed flow conditions in the coastal region.

Considering that near the shoreline, the actual depth is less than 20 ft and the vertical mixing due to variations in depth and turbulent diffusion may be dominant, the constant stratified 20-ft depth assumptions of case 1 may not be realistic in the shallow water regions between the discharges and the shore. In contrast, in deep regions toward the ocean, where the local depth rapidly drops to more than 50 ft at the end of the Unit 2 diffuser (Fig. 12), the assumption of case 1 may indeed be very realistic since the heated water layer definitely tends to remain stratified near the surface under oceanic flow conditions.

Hence, the results for case 1 can be considered as one limit of the actual thermal conditions in the coastal region, accurately representing the temperature distributions in deep regions but possibly underestimating the conditions in shallow regions near the shoreline.

According to case 2, it is assumed that the thermally effective depth of the heated water layer is the actual transient local depth in the region. Hence, regardless of the distances between the intake and discharge structures and the ocean surface, formulations for case 2 require that the heated water from the discharges rapidly mix in the vertical direction throughout the total depth in the coastal region. If the assumption is considered as valid, then, the single port intakes of all three units again receive condenser cooling water at the local temperature of the ocean water at the location of the intakes.

In shallow water regions near the shoreline, where the depth varies approximately 8 ft during the tidal cycle, severe vertical mixing definitely exists due to the tidal motions and the vertical turbulent diffusion effects. Hence, the assumption of case 2 can indeed be considered as a realistic representation of the actual conditions. However, in deep regions toward the ocean, where the local depth rapidly drops to more than

50 ft at the end of the Unit 2 diffuser and to 1300 ft on the boundary of the computational region (Fig. 12), the assumption of vertical mixing throughout the total actual depth is definitely not valid since the heated water will definitely stratify into a layer near the surface.

Hence, the results for case 2 can be considered as the second limit of the actual thermal conditions in the coastal region, accurately representing the temperature distributions in shallow regions near the shoreline but definitely underestimating the conditions in deep regions toward the ocean.

Since the specified locations of the intakes are in shallow water and, according to the design of the structures, the distances between the intake ports and the ocean surface are approximately 20 ft, the assumptions of both case 1 and case 2 lead to approximately the same local thermal conditions in the neighborhood of the intakes for the recirculation effect of the heated discharge water.

The discussions of the results of the previous sections clearly indicate that the proposed Units 1, 2, 3 operation will create significant thermal interaction between the intakes and the discharges of the three units. The results for minimum temperatures at the intakes and the discharges are presented in Table 2, for the existing Unit 1 operation and the proposed Units 1, 2, 3 operation according to the solutions of the mathematical model under case 1 and case 2 assumptions.

The first line in Table 2 represents the conditions for the existing Unit 1 operation. The overall recirculation effect, due to the oceanic flow conditions in the coastal region, increases the intake temperature of the condenser cooling water approximately 5°F above the natural ambient temperature, 70°F, for both case 1 and case 2. Hence, since the condenser temperature rise is assumed as 20°F, the temperature of the condenser cooling water discharged into the ocean becomes approximately 95°F for both case 1 and case 2.

In terms of the simple concept of percentage recirculation, employed in the elementary thermal analysis models based on excess temperature formulations, the present results indicate approximately 20% maximum recirculation effect between the intake and discharge structures. Noting that the intake and the discharge are only 500 ft apart, in approximately 20 ft of water (Fig. 12), with approximately 0.8 ft/sec minimum periodic velocity conditions, this maximum value for percentage recirculation is very realistic from physical considerations of the oceanic flow conditions in the coastal region.

Table 2. Minimum intake and discharge temperatures for units 1, 2, and 3

Unit	Case 1			Case 2		
	Intake element	Discharge element	Maximum discharge	Intake element	Discharge element	Minimum discharge
1 (existing)	74.98	88.28	94.98	75.52	88.70	95.52
1 (proposed)	79.14	85.19	99.14	77.69	82.98	97.69
2 (proposed)	88.03	88.70	100.03	77.68	76.74	97.68
		88.34			76.69	
		88.27			76.44	
		88.12			75.99	
3 (proposed)	79.74	81.18	99.74	77.48	77.73	97.48
		81.22			78.13	
		81.24			78.48	
		80.88			78.46	

The difference between the maximum discharge temperature and the mean temperature in the discharge discrete element is approximately 15°F for both cases 1 and 2 for only Unit 1 operation. Considering that the temperature in the discharge element represents, in a sense, the ultimate result of the near-field entrainment dilution and mixing of the discharge jet initially at a discharge temperature of 95°F, a 15°F temperature drop represents a very realistic value, from strictly near-field study considerations. This result indicates that the choice of the dimensions of the discharge discrete element, approximately 500 ft by 500 ft, in the application of the FLIDE far-field thermal model was indeed reasonable, based on near-field dilution considerations, for the specified discharge conditions of Unit 1.

The remaining part of Table 2 presents the temperature results, according to the assumptions of case 1 and case 2, for the intake and discharge conditions of the proposed Units 1, 2, 3 operation of the hypothetical nuclear generating station. For the proposed Units 2 and 3, the discharge temperatures are presented at four equidistant locations along the 2000-ft diffuser.

The results clearly indicate that the operation of the additional Units 2 and 3 in the coastal region will definitely alter the thermal conditions of the intake and the discharge of the existing Unit 1. The overall effect of recirculation and re-entrainment of heated water between their intakes and their discharges, according to the assumed flow conditions, will increase the intake temperature of Unit 1 to approximately 79°F for case 1 and to approximately 76°F for case 2. Consequently, the discharge temperature of the condenser cooling water will increase to approximately 99°F for case 1 and to approximately 95°F for case 2.

The maximum intake temperatures of the proposed Units 2 and 3 will be approximately 80°F for case 1 and approximately 76°F for case 2. Consequently, the discharge temperatures of the condenser cooling water for Units 2 and 3 will reach approximately 100°F for case 1 and approximately 95°F for case 2.

It is also interesting to note that the difference between the maximum discharge temperature and the temperature in the discrete element of the discharge remains approximately the same, 15°F, for only Unit 1 operation and also for Units 1, 2, 3 operation, for both cases 1 and 2. Since the near-field dilution by entrainment within the discharge discrete element is determined by the size of the element, by oceanic flow conditions representing the ambient flow for the near-field, and by the specified discharge flow rate, and since it is independent of the particular values of the discharge and ambient temperatures, this result is to be expected. Furthermore, the same difference for the submerged diffusers of the additional Units 2 and 3 indicates 20°F, thus illustrating the overall advantage of multiport diffusers for the near-field dilution of the heated water by spreading the discharge over a larger volume of colder, ambient water. However, the near-field dilution efficiency of the diffusers, in a sense, becomes insignificant due to the recirculation and re-entrainment effects which raise the discharge temperature to approximately 100°F, 10°F higher than the 90°F value based on ambient 70°F temperature without recirculation and reentrainment.

It is also interesting to note that, although case 1 represents highly stratified conditions and case 2 represents vertically mixed conditions as the two limiting assumptions for the behavior of the heated water layer, the results for the maximum temperature conditions at the intakes and discharges only differ approximately 2°F for Unit 1 and approximately 3°F for Units 2 and 3 between the two solutions. Hence, since the actual thermal conditions in the coastal region will be somewhere in between the two extreme conditions of case 1 and case 2, the maximum temperature results for the intake and discharge conditions in Table 2 must be representative of the actual conditions that could exist under the assumed strictly periodic tidal flow condition.

The results for the temperature distributions indicate that the most severe thermal conditions in the coastal region always occur during the late afternoons and evenings after the day-long surface heating conditions due to solar irradiation. The exact hours and the distributions depend on the different

combinations of the tidal conditions of 12 hr 24 min periodicity and the solar irradiation conditions of 24 hr periodicity, the operational conditions of the hypothetical nuclear generating station (as the existing Unit 1 or the proposed Units 1, 2, 3), and the assumptions about the thermal plume (as stratified for case 1 and vertically mixed for case 2), which are employed in the formulation of the mathematical model.

The comparison of the temperature distributions for the existing Unit 1 operation and the proposed Units 1, 2, 3 operation for the most critical thermal conditions in the coastal region, at 19:30 hr during the sixth day and the 12th cycle after the start of the operation of the generating station at midnight, August 18, is presented both as isotherms and as perspective plots in Fig. 78 for case 1, which considers constant 20 ft for the thermally effective depth of the stratified heated water layer. The results clearly indicate that the additional Units 2 and 3 will significantly increase the thermal impact of the hypothetical nuclear generating station in the coastal region during the thermally critical hours in the evening.

For the existing Unit 1 operation, the maximum temperature on the shore reaches to only 74°F along a relatively short section of approximately 1000 ft, at a location approximately 3000 ft up coast of the generating station. However, for the proposed Units 1, 2, 3 operation, the maximum temperature on the shore increases to 78°F along a section of approximately 1600 ft, at a location of approximately 3000 ft up coast of the generating station, with temperatures in excess of 75°F to distances 4000 ft down coast and in excess of 77°F to distances 5000 ft up coast from the location of the generating station on the

Fig. 78. Comparison of temperature distributions for critical thermal conditions at 19:30 hr for the existing unit 1 operation and the proposed units 1, 2, 3 operation for case 1.

Fig. 79. Comparison of temperature distributions for critical thermal conditions at 21:09 hr for the existing unit 1 operation and the proposed units 1, 2, 3 operation for case 2.

shoreline. Hence, the additional two Units create approximately 4°F higher temperature conditions on the shore during the thermally critical conditions in the evening.

Furthermore, although the 74°F isotherm extends to approximately 6000 ft from the discharge, along the direction of the tidal motion toward the shore, for the existing Unit 1 operation, the 75°F isotherm extends beyond 8000 ft in both directions at approximately 6000 ft away and parallel to the shoreline for the proposed Units 1, 2, 3 operation.

The comparison of the temperature distributions for the existing Unit 1 operation and the proposed Units 1, 2, 3 operation for the most critical thermal conditions in the coastal region, at 21:09 hr during the tenth day and the 20th cycle after the start of the operation of the generating station at midnight August 18, is presented both as isotherms and as perspective plots in Fig. 79 for case 2, which considers the actual transient local depth as the thermally effective depth of the heated water layer under fully mixed conditions in the vertical direction. The results again indicate that the additional Units 2 and 3 will appreciably increase the thermal impact of the generating station in the coastal region during the thermally critical hours in the evening.

According to the results for case 2, for the existing Unit 1 operation, the maximum temperature on the shore reaches to 74.5°F only at a point approximately 1000 ft up coast of the generating station, and the up-coast section of the shore attains 74°F , extending beyond 8000 ft. For the proposed Units 1, 2, 3 operation the maximum temperature on the shore reaches 76°F at the same point as only Unit 1 operation. The up-coast section of the shore, extending beyond 8000 ft, reaches 75.5°F , and the down-coast section exceeds 74.5°F to approximately 6000 ft from the location of the generating station.

For the existing Unit 1 operation, the 74°F isotherm extends to approximately 6000 ft down coast and beyond 8000 ft up coast at approximately 2500 ft away and parallel to the shoreline. However, for the proposed Units 1, 2, 3 operation, the 75°F isotherm extends to approximately 6000 ft down coast and beyond 8000 ft up coast, reaching to distances approximately 7000 ft from the shoreline.

It is evident that the solutions from the mathematical model, under the assumption of case 2, indicate considerably less differences in temperature, compared to the results of case 1, during the thermally critical hours in the evening, between the existing Unit 1 operation and the proposed Units 1, 2, 3 operation. Since case 2 assumes that the thermally effective depth of the heated water layer is the actual transient local depth in the coastal region, the additional thermal energy discharged from the diffusers of the proposed Units 2 and 3 is assumed to immediately mix with the ocean water at approximately 40 ft depth (Fig. 12) in the vicinity of the diffusers. Furthermore, since the convective transport terms for case 2 are also based on the actual depth, rather than the 20 ft depth as for case 1, the mathematical model assumes that the heated water is being convected out of the computational region with an effective depth of 1200 ft (Fig. 12). Hence, the predicted temperature increases according to case 2, resulting from the additional thermal discharges of the proposed Units 2 and 3, are considerably on the unconservative side from physical considerations since the thermal plume definitely tends to stratify in the deep ocean regions. Nevertheless, the results for the temperature distributions, according to the case 2 assumption, still indicate that the additional thermal impact of the proposed Units 2 and 3 creates appreciable temperature differences from the conditions of the existing Unit 1 operation in the coastal region during the thermally critical hours in the evening.

Hence, under the assumed, strictly periodic tidal flow conditions, the numerical simulation results for temperature distributions, from the FLIDE far-field thermal model, indicate that the proposed three unit operation, with two additional 1150-MW nuclear units, could substantially increase the thermal impact of the existing, single, 500-MW nuclear unit operation of the hypothetical generating station in the coastal region. This result could be attributed to various adverse combinations associated with the assumed layout of the submerged intake and discharge structures of the hypothetical generating station and the assumed periodic tidal flows in the coastal region.

First, since all three intakes are assumed to be located between the single port discharge of Unit 1 and 2000-ft-long diffusers of Units 2 and 3, and along the assumed direction of the periodic tidal flow, the heated water from the discharges continuously flows over the intake structures throughout the tidal cycle. Furthermore, since the intakes are assumed to be located not more than 500 ft away from any one of the discharges (see Fig. 11), the heated water does not have sufficient distance to cool, according to the turbulent diffusion and surface heat transfer effects, before reaching the location of the intakes.

Second, the intakes for the additional Units 2 and 3 are assumed to be located between the two diffusers and the shore, in shallow water of approximately 30 ft depth at mean water level conditions (Fig. 12). According to the assumed forms of the structures, the water layer between the intake ports and the surface varies between 16 ft and 24 ft during the tidal cycle. Since the diffusers are assumed to be located in deeper regions than the intakes, at least a part of the stratified heated water could enter the intakes as it moves toward the shoreline under the assumed tidal flow conditions.

The above discussion indicates that for the assumed, typically adverse, strictly periodic tidal flow conditions, the assumed layout and the depths at the locations of the intake and discharge structures of the three units of the hypothetical generating station could cause substantial recirculation and re-entrainment problems during the dissipation of the thermal effluent in the coastal region. Hence, it illustrates the definite necessity to consider the typical oceanic flow conditions in planning the condenser cooling systems of multi-unit power plants in coastal and offshore regions.

4. CONCLUSIONS AND RECOMMENDATIONS

A short-time transient (within tidal cycle), two-dimensional, far-field model is presented for the assessment of the thermal impact of power plants in large coastal and offshore regions. The Eulerian formulation of the FLIDE (fluid-in-discrete-element) model is based on the combination of the fundamental concepts of the FLIC (fluid-in-cell) numerical simulation method, for the solution of hydrodynamic problems, and the "segment" or "box" type models previously employed in the analysis of water quality conditions in bays and estuaries.

The present FLIDE far-field mathematical model applies the integral forms of mass and energy conservation principles to arbitrarily shaped and sized discrete elements; hence, from computational considerations, it is substantially more efficient than the FLIC method, which employs uniform-size cells, for the transient, two-dimensional numerical simulation of temperature distributions in large coastal and offshore regions.

In the development of the mathematical system of the FLIDE model, the conservation principles are rigorously integrated over the enclosure surfaces of the discrete elements by considering the vertical variations in velocity components and temperature. Since its formulation clearly identifies and incorporates the specific forms for depth averaged turbulent transport fluxes and additional nonconvective transport effects due to thermal stratification, the present method is physically more realistic than the "segment" or "box" models which only consider simple and questionable dispersion-type formulations for the turbulent transport.

The general far-field model is capable of incorporating completely arbitrary, transient conditions for depth variations, tidal, geostrophic and wind-induced oceanic currents as locally specified, known inputs in its application to particular problems. Although the arbitrary specification of the flow conditions may not satisfy the conservation of mass principle and may hence introduce errors in the realistic representation of the temperature distributions, the special numerical simulation form of the energy equation, which includes the mass conservation equation, limits these errors to physically acceptable bounds; and it avoids the possibility of certain physically impossible heat transfer conditions that exist in various far-field models which employ heat source type formulations for the discharge conditions.

The model considers surface heat transfer conditions according to short-time (hourly) variations based on solar irradiation, black body back radiation, evaporation and convection modes for realistic representation of daytime and nighttime surface heating and cooling effects which are particularly important in shallow water coastal regions.

The mathematical properties of the governing system of the FLIDE model were extensively discussed in the study. It was shown that the general mathematical model consists of a system of weakly coupled ordinary differential equations in time which can be readily integrated from any specified initial conditions. Based on a well-known theorem, it is rigorously established that the solutions to the initial-value problem exist and are unique extensions of the initial conditions; hence, it was concluded that solutions to the mathematical system can be obtained by marching type numerical integration methods.

The stability analysis of the numerical solutions was based on a discrete perturbation stability analysis method which was developed for the finite-difference formulations of classical partial differential equations. The general mathematical system of the FLIDE model was transformed into the appropriate finite-difference form by employing the Euler-Cauchy integration formulae for the approximation of time derivatives. It was shown that according to the second upwind differencing (donor cell) form of the convective terms, the static stability requirement was unconditionally satisfied by the governing system. Based on the dynamic stability requirement of the discrete perturbation method, the essential criterion for the selection of the time step size for the numerical integration was rigorously established for the most

general conditions including the effects of the variations in discrete-element dimensions, surface heat transfer, and power plant discharge conditions. Hence, it was concluded that stable numerical solutions can always be obtained, by any standard marching-type integration method, as valid approximations of the solution of the governing system of equations of the model.

The general FLIDE mathematical model was applied to a particular far-field problem for the assessment of the thermal impact conditions of a hypothetical generating station in a large coastal region during the summer. It was assumed that the generating station has an existing 500-MW operational nuclear unit with submerged single-port intake and discharge structures for the once-through condenser cooling system. It was further assumed that, to increase the output capacity of the generating station, it was proposed to install two additional 1150-MW nuclear units with two submerged single port intakes and 2000-ft multiport diffusers to be constructed in the vicinity of the existing cooling system.

To consider a typical adverse case for the dissipation of the thermal effluent, it was assumed that the oceanic flow conditions consist of strictly periodic tidal currents, without any geostrophic or wind-induced net drift current in the coastal region.

The results clearly indicated that the convective energy transfer is the dominant phenomenon which controls the thermal conditions in the coastal region, during the tidal cycle, for both limiting cases of stratified and vertically mixed behavior of the thermal plume. For the proposed three unit operation, strictly periodic tidal flow conditions caused severe recirculation and reentrainment between the intakes and the discharges, resulting in excessively high temperature conditions in the vicinity of the discharges of the power plant.

The comparison of temperature distributions during successive tidal cycles indicated that the thermal response of the coastal region to reach quasi-steady-state conditions required considerable time after the start of the operation of the generating station. Starting with the assumed initial condition as the ambient ocean temperature, the quasi-steady-state conditions were reached after approximately six days for stratified and ten days for vertically mixed thermal plume conditions. Hence, it is evident that a realistic study of the thermal impact of a power plant must necessarily consider long time periods, extending through numerous tidal cycles, to be able to predict the actual critical thermal conditions in a coastal region.

The short-time temperature distributions during the tidal cycles also indicated that the daily critical thermal conditions always occurred in the evenings, particularly near the shoreline, indicating the strong influence of the solar heating of the water in the shallow regions. The exact time and temperature of these critical conditions depended on the relative time lag between the 24-hr periodic solar heating and 12.4-hr periodic tidal flow conditions. Hence, the results clearly illustrated the possibility of occurrence of uncommonly severe, thermal conditions, for short durations, in the evenings, near the shoreline, depending on the adverse combinations of solar heating, tidal flow and thermal discharge conditions.

It should be stated that the reliability of the solutions of the FLIDE mathematical model in representing the actual temperature conditions in a specified coastal or offshore region depends very strongly on the input information which is required for the construction of the particular problem from the general formulation of the model. Although the general simulation is complete from physical considerations, since it exactly satisfies the conservation of energy principle in each and every element in the region, and it is mathematically valid, since unique solutions exist and can be approximately obtained according to stable numerical integration methods, nevertheless, the FLIDE mathematical model, in its applications, still possesses certain fundamental limitations associated with the general class of transient two-dimensional thermal energy transport models.

First, the mathematical model requires the transient local flow conditions for the depth and the two velocity components in the horizontal plane at the locations of all the discrete elements in the specified computational region. Although it is indeed possible to solve for the hydrodynamic flow field, by extending the FLIDE model to include the necessary integral forms of the momentum principle, this approach may be considerably difficult to formulate for the open boundary conditions of the coastal and offshore regions and computationally costly for the numerical solution of three additional sets of differential equations for the surface elevation and the two velocity components.

Since the tidal and geostrophic flow conditions in the ocean are definitely independent of the hydrodynamic conditions of the discharges, except in the immediate vicinity of the structures, the necessary transient local flow conditions for the formulation of the FLIDE model can always be obtained in the coastal region before the actual installation of the condenser cooling systems of the proposed power plant. This does not necessarily imply that transient depth and velocity data must be accurately measured at every point in the actual coastal region corresponding to the locations of the discrete elements of the computational region of the FLIDE model. Although such high-resolution field data are most desirable for the construction of the mathematical model, they are admittedly impractical from the cost considerations of the necessary instrumentation equipment. In actual practice, since standard interpolation techniques are readily available, it is only necessary to accurately measure the oceanic flow conditions at sufficiently many and appropriately selected locations in the coastal region, such that reliable high-resolution values for the discrete elements of the FLIDE formulation can be approximately interpolated from the field-measured data.

The significantly important point that must be realized in the design of the field experiments is that, according to the requirements of the mathematical model, the transient local data for depth and the two components of velocity must be simultaneously measured, at all the specified points in the coastal region, at the same time, during sufficiently long periods of observation and at sufficiently many intervals during the tidal cycles. It is also advisable to obtain the necessary atmospheric conditions, particularly the wind speed and the direction, from the available meteorological information, to enable the necessary data reduction for the separation of the tidal and geostrophic flow conditions from the wind-induced effects in the coastal region.

Hence, it is strongly recommended that in the assessment of the environmental thermal impact conditions of power plants in coastal and offshore regions, the development of the particular mathematical models and the planning of the field experiments be undertaken concurrently to guarantee the acquisition of the necessary and reliable oceanic flow data which can be employed in the applications of reliable far-field models for the simulation of the thermal conditions.

Second, the mathematical model is based on the fundamental assumption that the spatial variations are strictly two dimensional. Hence, the variations in depth are incorporated as averaged quantities in the general formulation of the model. Consequently, although the mathematical formulation clearly identifies and establishes the necessary forms for the special thermal energy transport terms resulting from the additional effect of the stratification of the heated water layer, the actual evaluation of these terms requires the specification of the vertical variations of the temperature and the velocity components according to the measured field data in the coastal region. Since these data cannot be obtained without the actual operational conditions of a specific power plant under consideration in a coastal or offshore region, this particular limitation, which is common to all existing two-dimensional models, seems more drastic than the requirements for the accurate representation of the flow conditions. However, the problem can be approximately resolved by either considering the limiting cases for severe stratification and full vertical mixing conditions, similar to the two cases which were considered in the present study, or by incorporating

the available theoretical and empirical information about the vertical variations of temperature and the velocity components under stratified turbulent flow conditions.

The general problem of stratifications is inherently related to the turbulent shear flow phenomenon which was considered in detail in various classical studies associated with the dispersion of an initially localized concentration of matter under the influence of specified turbulent flow conditions. The additional problem about the evaluation of the depth-averaged turbulent diffusion flux terms is also related to the vertical distribution of the turbulent shear stress under the rather questionable simplifying assumption of isotropic turbulence. Furthermore, the problem associated with the realistic modeling of the intake conditions, which in a sense determines the overall recirculation effect between the discharges and the intakes, also depends on the stratification of the heated fluid layer. However, unfortunately, the presently available knowledge about the stratified turbulent flow conditions, particularly in shallow water regions, is extremely limited.

Hence, considering the importance of the nonconvective thermal transport terms, consisting of turbulent diffusion and stratification effects, in the formulation of any far-field energy transport model, it is strongly recommended that further theoretical and experimental research effort be extended for the development of reliable models, based on accurate stratified turbulent velocity distributions, which can be incorporated in the general formulation of the FLIDE mathematical model.

In general, if sufficiently accurate field data for oceanic flow conditions are available and if the vertical stratification of the heated water layer can be approximately estimated, the FLIDE mathematical model can be readily employed for the assessment of the thermal impact conditions of the power plant discharges in coastal and offshore regions. Indeed, considering the results of the hypothetical problem of the present study, it becomes evident that the mathematical model can be applied for long periods of time to investigate the thermal conditions in a coastal region under slowly varying seasonal changes in atmospheric conditions without excessive cost in computer time. According to the presently available exploratory computer code FAROUT of the FLIDE mathematical model, which is not yet computationally optimized as a production program, in a coastal region of approximately 12 miles by 6 miles, with approximately 500 discrete elements, which produces a considerably high resolution, transient temperature distributions at approximately 1.5-hr intervals can be obtained for a complete year with realistically varying oceanic flow and atmospheric conditions, depending on the weather and monthly and seasonal changes, within approximately 12 hr of computation time by an IBM 360-91 computer. Since the computer code has flexible restart capabilities, the required computational time is extremely realistic and acceptable from both machine usage and cost considerations.

Hence, in conclusion, it can be stated that the FLIDE (fluid-in-discrete-element) numerical simulation model of the present study is superior to the existing transient two-dimensional far-field models, both from rigorous mathematical formulation and computational efficiency considerations; and it can be immediately employed as a practical method for the assessment of the environmental thermal impact conditions of power plant discharges in coastal and offshore regions.

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NOMENCLATURE

A	enclosure surface area of the discrete element
C	dimensionless coefficient in the formulation for stability analysis [Eq. (117)]
C^*	constant specific heat of ocean water
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
$\bar{D}(t, x, y)$	mean time-averaged depth
$\bar{D}_{MWL}^*$	mean water level depth
$\bar{D}_T^*$	amplitude of the tidal wave
dA	area element on the enclosure surface
dV	volume element in the discrete element
e	specific internal energy
e_0^*	specific internal energy of formation at reference temperature T_0^*
F	right-hand-side functions of the general system of differential equations [Eq. (45)]
F_C	convection effect of mean time-averaged and depth-averaged flow field [Eq. (41)]
F_D	overall diffusion effect of depth-averaged molecular and turbulent transport phenomena [Eq. (42)]
F_P	thermal effect of power plant intakes and discharges [Eq. (43)]
F_S	overall effect of surface and bottom conditions and heat generation [Eq. (44)]
FLIDE	abbreviation for Fluid-in-Discrete-Element
$\tilde{F}(t)$	actual computed value of the right-hand-side function including the effect of discrete perturbation error [Eq. (125)]
$f(\xi)$	dimensionless function for turbulent velocity distribution along vertical direction [Eq. (53)]
$\bar{f}$	depth mean average of the dimensionless function for turbulent velocity distribution [Eq. (55)]
$f'(\xi)$	derivative of the dimensionless function for turbulent velocity distribution
f_b	body force per unit volume
$G(t)$	time-dependent part of the right-hand-side function of the general nonautonomous system of differential equations [Eq. (111)]
$\dot{G}_{CND}$	condenser flow rate [Eq. (109)]
$\dot{G}_m$	volumetric flow rate at the intake m
$\dot{G}_n$	volumetric flow rate of the discharge n
$g(\xi)$	dimensionless function for the vertical distribution of temperature difference from its depth-averaged value [Eq. (56)]
$\dot{g}_b^*$	volumetric flow rate per unit area (velocity) at the ocean bottom
$\dot{g}_m$	volumetric intake flow rate per unit volume (inverse time) [Eq. (116)]
$\dot{g}_n$	volumetric discharge flow rate per unit volume (inverse time) [Eq. (116)]
$\dot{g}_s^*$	volumetric flow rate per unit area (velocity) on the ocean surface
h	specific enthalpy of ocean water
h_{CN}^*	convection surface heat transfer coefficient

h_{EV}^o	evaporation surface heat transfer coefficient
h_s^o	equivalent surface heat transfer coefficient [Eq. (116)]
h_g^o	specific enthalpy of formation at the reference temperature T_o^o
$(I_r)_{min}$	minimum number of discrete elements along x direction in region r
$(I_r)_{max}$	maximum number of discrete elements along x direction in region r
i	unit vector along x direction
$(J_r)_{min}$	minimum number of discrete elements along y direction in region r
$(J_r)_{max}$	maximum number of discrete elements along y direction in region r
j	unit vector along y direction
k_{ns}	parameter representing the characteristic time associated with plant discharges and surface heat transfer conditions in the stability analysis [Eq. (147)]
k_o	von Kármán's constant
$\dot{m}_{EV}$	mass rate of evaporation from the ocean surface [Eq. (99)]
N_{FLIDE}	total number of discrete elements in the formulation
n	unit normal vector outward positive to the enclosure surface of the discrete element
P	pressure
Q_{SR}^o	maximum solar irradiation on the ocean surface at noon
Q_{sx}	turbulent heat flux along x direction due to stratification [Eq. (71)]
Q_{sy}	turbulent heat flux along y direction due to stratification [Eq. (71)]
$\bar{q}$	combined molecular and turbulent diffusion flux, along x or y direction [Eq. (39)]
q	general heat flux vector
q_b^o	heat transfer flux at the bottom
q_{BR}^o	blackbody back radiation flux on the ocean surface
q_{CN}^o	convection surface heat transfer flux
q_{EV}^o	evaporation surface heat transfer flux [Eq. (101)]
$\bar{q}_m^o$	heat flux due to molecular diffusion transport along x or y direction
q_s^o	heat transfer flux on the ocean surface
Q_{SR}^o	daytime solar irradiation flux on the ocean surface
q_T^o	general nonconvective turbulent transport flux along x or y direction [Eq. (33)]
$\dot{q}_v$	heat generation rate per unit volume
R	total number of regions in the formulation
R_e	discrete element Reynolds number [Eq. (147)]
R_H	relative humidity
$T(t, x, y, z)$	instantaneous local temperature
$\bar{T}(t, x, y)$	mean time-averaged and depth-averaged temperature
$\tilde{T}(t)$	actual discrete-element temperature at time t including the discrete perturbation error [Eq. (125)]
$T'(t, x, y, z)$	random turbulent fluctuation of temperature
$\bar{T}^o(t, x, y, z)$	difference between the actual mean time-averaged vertical distribution and its depth-averaged value for temperature [Eq. (12)]

T_a	atmospheric temperature
T_b^o	temperature of the fluid inflow at the ocean bottom
$(\bar{T}_m)_{CND}$	cooling water temperature from the intake: entering the condenser [Eq. (109)]
T_m	temperature of the fluid at intake m
T_n	temperature of the fluid at discharge n
T_s	temperature of the water surface
T_s^o	temperature of the inflow on the ocean surface
T_o^o	reference temperature of formation
t	time
t_U^o	time lag for velocity component parallel to the shoreline [Eq. (153)]
t_V^o	time lag for velocity component normal to the shoreline [Eq. (153)]
$\bar{U}(t, x, y)$	mean time-averaged and depth-averaged velocity component along x direction
$\bar{U}^o$	magnitude of the velocity component along x direction on the half-point surfaces
$\bar{U}_{MAX}^o$	maximum value of the velocity component parallel to the shoreline [Eq. (153)]
$u(t, x, y, z)$	instantaneous local velocity component along x direction
$u'(t, x, y, z)$	random turbulent fluctuations of the velocity component along x direction
$\bar{u}''(t, x, y, z)$	difference between the actual mean time-averaged vertical distribution and its depth-averaged value for the velocity component along x direction [Eq. (12)]
u_f^o	friction velocity along x direction [Eq. (54)]
$\bar{V}(t, x, y)$	mean time-averaged and depth-averaged velocity component along y direction
V	velocity vector
V^o	magnitude of the horizontal velocity
$\bar{V}^o$	magnitude of the velocity component along y direction on the half-point surfaces
$\bar{V}_{MAX}^o$	maximum value of the velocity component normal to the shoreline [Eq. (153)]
V_w	wind speed
$(V_w)_{ref}$	reference value of wind speed associated with the evaporation surface heat transfer coefficient
V_f^o	magnitude of the friction velocity based on the bottom shear stress [Eq. (66)]
V	volume of the discrete element
$v(t, x, y, z)$	instantaneous local velocity component along y direction
$v'(t, x, y, z)$	random turbulent fluctuations of the velocity component along y direction
$\bar{v}''(t, x, y, z)$	difference between the actual mean time-averaged vertical distribution and its depth-averaged value for the velocity component along y direction [Eq. (12)]
v_f^o	friction velocity along y direction [Eq. (54)]
$\bar{W}^o$	maximum value of either of the two mean time-averaged and depth-averaged velocity components [Eq. (144)]
x	Cartesian coordinate of the formulation
y	Cartesian coordinate of the formulation
β	dimensionless constant [Eq. (58)]
γ_{EV}	coefficient associated with surface evaporation rate [Eq. (100)]

Δs	maximum dimension of the discrete elements [Eq. (144)]
ΔT_{COND}	temperature increase in the condenser [Eq. (110)]
ΔT_{in}	difference between the local and intake temperatures [Eq. (107)]
$\overline{\Delta T}_s^*$	maximum difference due to stratification between the actual mean time-averaged vertical distribution and its depth-averaged value for temperature [Eq. (56)]
Δt	time step size employed in the numerical solution
Δt_{CR}	time step size based on generalized form of the Courant's number for stability [Eq. (146)]
Δx	dimension of the discrete element along x direction
Δy	dimension of the discrete element along y direction
δ	Kronecker delta associated with the location of intakes and discharges in the discrete elements [Eq. (19)]
$\delta(t)$	discrete perturbation error at time t
$\delta F(t)$	error introduced in the right-hand-side function due to the discrete perturbation error [Eq. (128)]
$\bar{\epsilon}$	equivalent thermal diffusion coefficient [Eq. (76)]
ϵ_{Tx}	turbulent thermal eddy diffusivity along x direction [Eq. (62)]
ϵ_{Ty}	turbulent thermal eddy diffusivity along y direction [Eq. (62)]
ϵ_{Tz}	turbulent thermal eddy diffusivity along z direction [Eq. (63)]
ϵ_{Vz}	turbulent momentum eddy diffusivity along z direction [Eq. (65)]
ξ	dimensionless vertical coordinate [Eq. (53)]
ξ_0^*	dimensionless thickness of the laminar sublayer on the ocean bottom
κ	coefficient of molecular thermal conductivity
λ_x	bottom friction coefficient along x direction [Eq. (60)]
λ_y	bottom friction coefficient along y direction [Eq. (60)]
ρ	density of the ocean water
Γ_A	geometric shape factor for blackbody back radiation [Eq. (98)]
Γ_ϵ	dimensionless factor based on emissivities (absorptivities) for blackbody back radiation [Eq. (98)]
σ	Stefan-Boltzmann constant
$\vec{\sigma}$	surface force vector on the enclosure surface
σ^*	equivalent coefficient for blackbody back radiation [Eq. (117)]
τ	local shear stress component in a horizontal plane
τ_T	tidal period
τ_w^*	bottom shear stress due to turbulent flow conditions [Eq. (66)]
τ_{wx}	bottom shear stress along x direction
τ_{wy}	bottom shear stress along y direction

Superscripts

(r)	integer identifying the region in the formulation
(s)	integer identifying the region for matching conditions

Subscripts

I	integer identifying the discrete element on x boundary
i	integer identifying the location of the discrete element along x direction
i$\pm$1/2	integer identifying the location of the half-point enclosure surfaces of the discrete element normal to x direction
J	integer identifying the discrete element on y boundary
j	integer identifying the location of the discrete element along y direction
j$\pm$1/2	integer identifying the location of the half-point enclosure surfaces of the discrete element normal to y direction
K	integer identifying the discrete element for x boundary matching conditions for adjacent regions
k	integer identifying the discrete element for y boundary matching conditions for adjacent regions
L	integer identifying the discrete element for y boundary matching conditions for adjacent regions
l	integer identifying the discrete element for x boundary matching conditions for adjacent regions
m	integer identifying the extra discrete elements for y boundary matching conditions for adjacent regions
n	integer identifying the extra discrete elements for x boundary matching conditions for adjacent regions
x	specified value on the x boundary
y	specified value on the y boundary
0	initial conditions

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