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IN URANIUM-WATER LATTICES

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MIGRATION AREAS OF FISSION NEUTRONS IN URANIUM - WATER LATTICES

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Abstract: Measurements were made of the migration areas of fission neutrons in lattices of .600" diameter uranium rods in ordinary water. A range of water-to-uranium volume ratios from 1:1 to 4:1 was explored. Values were obtained in two different but related ways; the methods, analysis, and results are given and discussed. It is shown that the results are more consistent with a continuous slowing-down critical equation than they are with a one-group critical equation.

Introduction: The major reason for measuring the migration areas of uranium-water assemblies is the desire to obtain information on their reactivity as reactor cores. When both the buckling and the migration area are known, the infinite pile reproduction factor (k_{∞}) is also known, and this is one of the chief numbers in the analysis of the neutron economy.

In addition, the measured values can lead to information on the inelastic and transport cross-sections of uranium, when comparison is made with migration area calculations. This course has to be followed cautiously, however, because there are reasons why the measured values could differ

from the calculated ones, and both still be right. This point will be discussed later in the light of the measured values.

The measurements we shall describe here were done in the course of our program of studies of lattices of .600" diameter uranium rods in ordinary water. The experimental results were found in two different but related ways:

- (1) The moderator water was poisoned with B_2O_3 , and the thermal utilization and buckling were measured at several such concentrations. We shall call this the poison method.
- (2) Values of thermal utilization and buckling were obtained as a function of fuel rod enrichment. This we term the enrichment method.

Theoretical Basis: Both of the methods depend on use of the critical equation:

$$k p f \eta \mathcal{L} = 1 \quad (1)$$

where $\mathcal{L}$ is the fractional neutron non-leakage in a single fission cycle.

According to one-group theory,

$$\mathcal{L} = (1 + M^2 B^2)^{-1} \quad (2)$$

On the other hand, in accordance with age theory (continuous slowing-down),

$$\mathcal{L} = e^{-\tau B^2} (1 + L^2 B^2)^{-1}$$

For the assemblies we have studied, B^2 is at most about 6×10^{-3} , and L^2 at most about 4 cm^2 . Thus at most

$$B^2 L^2 \sim 2.4 \times 10^{-2}$$

and we can take

$$(1 + B^2 L^2)^{-1} \sim e^{-B^2 L^2}$$

$$\mathcal{L} = e^{-B^2 \tau} (1 + L^2 B^2)^{-1} \approx e^{-M^2 B^2} \quad (3)$$

At the outset we are faced with having to make a choice between describing the leakage by means of (2) or (3). There are reasonable a priori arguments for each; (3) is the limiting expression for multigroup diffusion theory, while on the other hand the thermal flux distribution arising from slowing down of neutrons in water can be fitted reasonably well by one-group theory. Use of the one-group kernel, though, leads to (2). Since grounds for a choice are not obvious at the beginning, we shall present here analyses based on both assumptions. It will be seen as a result that the age-theory expression (3) seems to fit the results better.

The poison method consists in adding B_2O_3 to the moderator water, and then measuring B^2 and f . The presence of the boron does not materially change the values of ϵ, p, η , and M^2 , and so from (2)

$$\epsilon p \eta \left[\frac{df}{d(B^2)} \right] = M^2$$

However, also from (2)

$$\epsilon p \eta (f)_{B^2=0} = 1$$

Therefore

$$M^2 = \frac{1}{(f)_{B^2=0}} \cdot \frac{df}{d(B^2)} \quad (4)$$

The one-group theory formulation thus relates the measured values of f and B^2 linearly, and the migration area is the slope of the straight line divided by its intercept.

The analysis of the poison method by the age theory kernel proceeds from recasting (3) in the form

$$\ln(\epsilon p \eta) + \ln f = M^2 B^2$$

In this case it is $\ln f$ which is linearly related to B^2 , and M^2 is

calculated as

$$M^2 = \frac{d(\ln f)}{d(B^2)} \quad (5)$$

To obtain the migration area from enrichment changes, we must first notice that η is also a variable in this case. Having observed this, we similarly derive, assuming (2)

$$M^2 = \left[\frac{1}{(\eta f)_{B^2=0}} \right] \times \left[\frac{d(\eta f)}{d(B^2)} \right] \quad (6)$$

or assuming (3)

$$M^2 = \frac{d[\ln(\eta f)]}{d[B^2]} \quad (7)$$

Experimental Methods: Buckling measurements were made by either of the two methods reported in earlier memoranda. (1)

The first is based on experimental determination of the radial and axial thermal neutron flux distributions throughout the cylindrical array of uranium and water. These flux plots are fitted respectively to the functions

$$A J_0 \left(2.4048 \frac{r}{R} \right)$$

and

$$C \sinh \left(\frac{Z_0 - Z}{L} \right)$$

Then B^2 is found as

$$B^2 = \left(\frac{2.4048}{R} \right)^2 - \frac{1}{L^2}$$

This method is the one commonly used in exponential experiments.

The second method depends on measuring only the axial flux distribution for a number of rod loadings. From these, values of L are obtained by

fitting to a sinh function. Then B^2 and the reflector savings λ are found from a two-constant least squares fit to the expression

$$L = \left\{ \left(\frac{2.4048}{R_f + \lambda} \right)^2 - B^2 \right\}^{-\frac{1}{2}}$$

R_f being the loaded radius, and L being the relaxation length. Inasmuch as we have never seen any systematic difference between values of the buckling found by these two methods, we shall make no distinction here between the numbers obtained one way or the other.

In order to get the desired values of f , we measured the intracell variation of the thermal neutron flux. Using these experimental plots, we calculated the moderator and fuel disadvantage factors, and got the thermal utilizations from these and published cross-sections. A detailed discussion of the methods is given in earlier reports. ⁽²⁾

For the poison measurements, boron was added to the water as boric acid crystals. Care was taken to assure complete solution of the salt, the mixture in each case being agitated for several hours with a mechanical stirrer. The boron concentrations were then found by chemical analysis. ⁽³⁾ In each case at least two quantitative determinations were made, and these generally agreed to better than 0.5%.

The poison method was carried out with 1.15% enriched rods, .600" in diameter, at volume ratios 1:1, 1.5:1, 2:1, 3:1, 4:1. The enrichment method used rods of 1.027%, 1.15%, and 1.3%, of the same diameter and the same volume ratios. In each case the volume ratio reported is that of water to uranium, the aluminum not being included in either fraction.

Results of Poison Measurement: The measured values of buckling for the different volume ratios and poison concentrations are listed in table I, and are plotted in figures 1 and 2. The values of thermal utilization which were found are given in table II, and are plotted in figure 3 as a function of boron concentration. These measurements of f were discussed in detail in an earlier report.⁽⁴⁾ As stated there, the value of f given for the clean 1:1 volume ratio lattice was not measured directly, but was inferred from a plot of f vs. volume ratio.

Figures 4 and 5 show B^2 plotted against f and $\ln f$, respectively. Since the two variables were usually not measured at the same poison concentrations, the values upon which figures 4 and 5 are based are interpolations from figures 1, 2 and 3. Of course, this procedure is a possible source of error, because of the degree of freedom available in selecting smooth curve fits to the measured data.

The interpolated values of f , $\ln f$, and B^2 on which figures 4 and 5 are based are given in table III. In both tables and figures we have used $(1 + \ln f)$ instead of $\ln f$, because the latter number is negative. Of course, the slopes of the plotted curves are not influenced by this.

The best straight-line fits to the curves (actually to the numbers in table III) were obtained by the method of least squares. The migration areas were then evaluated by means of (4) and (5). The results of the analysis are listed in table IV, and are plotted in figures 6 and 7.

The errors assigned to the numbers are not intended to indicate probable departures from the correct values; rather, they give the probable contribution of statistical inaccuracies to the overall error. Thus the

actual errors may be somewhat larger than those indicated.

For neutron economy considerations only the values of k_{∞} implied by these migration areas are important. The corresponding values of k_{∞} are therefore given in table V for the clean, 1.15% enriched rods. The values obtained through use of the two separate critical equations agree substantially, in spite of the fact that the migration areas themselves are considerably different.

The probable errors in table V are obtained by compounding those in tables I and IV.

Results of Enrichment Measurement: The determination of the migration area from the effect of enrichment change on B^2 and f is not so well-founded as that done by the previous method, because of the earlier mentioned uncertainty here involved in choosing good values of η . Consistency in the choices for the different enrichments is easily arrived at. The method we chose to get such consistent values of η is as follows: One begins by choosing a value of η for natural uranium, and writing it as

$$\eta_{\text{nat}} = \frac{\eta_{25}}{1 + N \frac{\sigma_a^{28}}{\sigma_a^{25}}}$$

with N the atomic ratio of 28 to 25 in natural uranium. η_{25} is assumed also known. These choices of η_{nat} and η_{25} then determine a value for the ratio $\sigma_a^{28}/\sigma_a^{25}$, and the enrichment dependence of η is obtained from the variation of N .

The present best values of the cross-sections (5) give

$$\eta_{\text{nat}} = 1.327 \pm .02$$

The analysis of various reactor experiments has not been any more successful in fixing this vital value; in the literature one is presented with a choice anywhere in the range from 1.28 to 1.35. On the basis of what evidence is available, we have assumed the correct quantity to be somewhere in the range 1.310 to 1.340, and have computed the migration areas assuming values at the extremes and at the midpoint of this interval.

The values of η for the three enrichments used, determined by the procedure described above, are listed in table VI.

The experimental values of f and B^2 upon which the analysis is based have been published in earlier memoranda.⁽²⁾⁽⁶⁾ These quantities are listed in tables VII and VIII and are plotted as figures 8 and 9.

The one-group migration areas obtained from use of (6) were found from straight-line least squares fits of (ηf) against B^2 . The age theory values implied by (7) were obtained from straight line least squares fits of $\ln(\eta f)$ against B^2 . The data used are summarized in table IX. It may be noticed that the values of f in this table are not identical with those in table VIII. Some of the latter quantities are known to be wrong, probably because of errors in experimental methods. Others show a normal scatter to be expected from the statistical fluctuations of foil-counting. In order to minimize the effects of such errors we based the analysis for M^2 on the values of f implied by the smooth curves of figure 9; it is these curve-read numbers which are given in table IX.

The one-group values of M^2 deduced from use of (6) are listed in table X, for the three preassumed values of η for natural uranium. The age theory

results found by analyzing according to (7) are given in table XI, also for the three values of η .

As a check on the effect of using in the analysis the smooth curve values of f from figure 9, we also subjected the data to a similar analysis which involved measured values of B^2 , but calculated values of f . Two sets of calculations were done. In the first, we used values of f obtained by Fleck and Chernick from P^3 calculations of the intracell thermal neutron flux traverses in these lattices.⁽⁷⁾ In the second, we used thermal utilizations found from simple diffusion, fudged to give the same flux dip in a fuel rod as was observed with flux traverses with 1.027% enriched rods. The values of M^2 obtained with use of these values of f are given in tables XII and XIII (calculations were done only with the assumption $\eta = 1.325$ for natural uranium).

Error Analysis: The values of M^2 given in tables IV, V, I, and XI have numerical estimates of probable error beside them. The manner in which these estimates were obtained can be best described by an outline of the way they were arrived at in one case. Consider the age-theory analysis of the poison method; here M^2 is found from equation 5. The actual least squares fit which provides M^2 is contained in the expression

$$M^2 = \frac{n \sum B_1^2 \ln f_1 - \sum B_1^2 \sum \ln f_1}{n \sum (B_1^2)^2 - (\sum B_1^2)^2} \quad (8)$$

n is the number of points being fitted to the straight line of equation (3), and B_1^2 and f_1 are the pairs of values of buckling and thermal utilization.

An experimental error δB^2 in a single value of the buckling (say B_k^2) will

produce an error in M^2 of amount

$$\delta_1 M_k^2 = \left(\frac{\partial M_k^2}{\partial B_k^2} \right) \delta B_k^2 \quad (9)$$

and similarly an error in an experimental value of f will produce an error in M^2

$$\delta_2 M_k^2 = \left(\frac{\partial M_k^2}{\partial f_k} \right) \delta f_k \quad (10)$$

Since the measurements of B^2 and f are not interdependent, their errors add randomly, and the overall uncertainty in M^2 is

$$\delta M^2 = \left\{ \frac{\sum_{k=1}^K (\delta_1 M_k^2)^2 + \sum_{k=1}^K (\delta_2 M_k^2)^2}{K} \right\}^{1/2} \quad (11)$$

The values of B^2 used in the analysis already have probable errors associated with them; these errors are derived solely from the internal consistency of data, but experience has shown that they are also good measures of the reliability of the values of the buckling. Hence they may be regarded as true estimates of the probable errors of the quantities they refer to, and are the δB^2 used in (9) and (11).

On the other hand it is more difficult to estimate the accuracy of the values of thermal utilization used. One would hope that their major uncertainties have origin in the statistical fluctuations of foil counts on which the intracell flux distributions are based. To find values of δf to use in (10) and (11), we assumed an uncertainty of 2% in the water disadvantage factor; this amount takes care of statistical inaccuracies in foil counting, and leaves some allowance for other experimental errors.

Thus the probable errors cited are compounded of known uncertainties in the values of B^2 , and a guess at the uncertainties in f .

Discussion of Results: Tables X and XI show how little M^2 is affected by a variation in the choice of η_{nat} . Taking alternately $\eta_{nat} = 1.310$ and 1.340 changes the implied M^2 by an amount roughly equal to the probable error of the measurement. If we suppose $\eta_{nat} = 1.325 \pm .015$, as was done in the analysis, we then arrive at a single set of values of M^2 for the one-group analysis of the enrichment method, and another set of values for the age theory method. These are given in table XIV, along with the errors compounded from those of tables X and XI, and the above assumed probable error in η .

We are now able to compare the results of the poison method with those of the enrichment method; one simply compares tables IV and XIV. It is immediately apparent that the values of M^2 from the one-group theory analysis do not agree, while those from age theory analysis agree quite well. In other words, the age theory critical equation leads to a much better fit to all the data available than does the one-group critical equation.

This can also be seen in geometrical terms. Equations (1) and (2) combined give

$$e p f \eta = 1 + M^2 B^2 \quad (12)$$

and (1) and (3) combined give

$$e p f \eta = e^{M^2 B^2} \quad (13)$$

(12) and (13) may be regarded as one parameter families of surfaces in the space of f , η and B^2 , the parameter being M^2 . The question to be considered then is whether the experimental data for a single rod size and volume ratio falls on one surface of the sets (12) or (13). The conclusion one is led to is that the data falls on one surface of the set (13), but on no surface the set (12). Thus (13) must be considered as describing the experimental data.

Best Values of the Migration Areas: Having arrived at the conclusion that the measurements must be analyzed in terms of the age-theory critical equation, we can now combine all those done by the two methods into a single set of analyses. The results of this final computation are to be found in table XV, and are plotted in figure 11. Figure 10 is a compilation of all measurements made with different fuel enrichments and boron poisonings, and shows the least squares fit straight lines which lead to the best values of M^2 for these lattices.

Conclusions: From figure 11 it can be seen that the migration area varies only slightly over the range of volume ratios studied; as might be expected, it has its largest values at the smaller volume ratios.

As mentioned earlier, one must be careful about identifying these experimental values of M^2 with those calculated by neutron slowing-down theory. The lattices we are studying are strongly heterogeneous, and strictly speaking the migration area we have evaluated is defined only by the non-leakage probability (equation (13)). It is not at all certain that this quantity is related to $\bar{r}^2$ in the same way as the migration area for homogeneous assemblies. In the past, attempts have been made to "homogenize" heterogeneous reactor theory by defining quantities like the mean age, cross-sections, and diffusion length over a lattice cell, but these efforts have all depended on some simplified model of the slowing-down and diffusion processes. The most which can be said in general is that the calculated (from slowing-down theory) and measured values of migration area for the heterogeneous assemblies we are studying should be of the same orders of magnitude.

References

- (1) These methods are discussed in detail and compared in the report on buckling measurements with 1.027% enriched rods, .600" diameter. This report is BNL Log No. C-7592, Kouts, Downes, Price, Sher, Walsh, (November 5, 1953).
- (2) Kouts, Downes, Price, Sher, Walsh. BNL Log No. C-7568 (October 28, 1953), BNL Log No. C-7931 (March 24, 1954), BNL-1987 (August 11, 1954).
- (3) We wish to express our thanks to Dr. R. W. Stoenner and Miss Elinor Norton of the Chemistry Department for having done the boron analyses for this experiment.
- (4) Kouts, Downes, Price, Sher, Walsh. BNL-1987 (August 11, 1954).
- (5) Brookhaven Neutron Cross Section Compilation, BNL-250, (August 1, 1954).
- (6) Kouts, Downes, Price, Sher, Walsh, BNL Log No. C-7592 (November 5, 1953), BNL Log No. C-7915 (March 17, 1954).
- (7) J. Chernick, BNL Log No. C-7979 (April 13, 1954).

Table I

Bucklings of Lattices with 1.15%, .600" Diameter Rods

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	Boron Content $\frac{\text{mg B}_2\text{O}_3}{\text{ml H}_2\text{O}}$	B^2 $\text{cm}^{-2} \times 10^4$	λ Reflector Savings cm
1	0	$21.19 \pm .23$	$8.15 \pm .13$
1	3.327	$-7.05 \pm .58$	$6.29 \pm .14$
1.5	0	$40.10 \pm .32$	$7.53 \pm .10$
1.5	2.022	$13.16 \pm .49$	$6.47 \pm .13$
1.5	3.930	$-8.70 \pm .44$	$5.79 \pm .11$
2	0	$48.22 \pm .31$	$7.10 \pm .08$
2	1.911	$12.47 \pm .61$	$6.48 \pm .17$
2	3.020	$-5.80 \pm .45$	$6.03 \pm .15$
3	0	$47.12 \pm .33$	$6.68 \pm .10$
3	1.308	$6.70 \pm .73$	$6.56 \pm .25$
4	0	$36.03 \pm .16$	$6.43 \pm .06$
4	.344	$22.03 \pm .44$	$6.19 \pm .21$
4	.746	$5.90 \pm .54$	$6.39 \pm .27$
4	1.300	$-13.62 \pm .50$	$6.31 \pm .24$

Table II

Thermal Utilizations of Lattices with 1.15%, .600" Diameter Rods

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	$\frac{\text{mg B}_2\text{O}_3}{\text{ml H}_2\text{O}}$	k
1	0	.945*
1	2.950	.869
1.5	0	.917
1.5	0.933	.880
1.5	2.022	.840
1.5	2.950	.809
2	0	.889
2	1.911	.800
2	2.920	.758
3	0	.842
3	1.300	.750
4	0	.793
4	0.344	.760
4	0.746	.727
4	1.300	.690

* Not a measured value. See text

Table III

B^2 vs. f and $\ln f$ for Boron Poisoned Lattices. 1.15% Enriched Uranium Rods in Ordinary Water. (Except for clean lattices, values are interpolated from figures 1-3)

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	$\frac{\text{mg B}_2\text{O}_3}{\text{ml H}_2\text{O}}$	B^2 $\text{cm}^{-2} \times 10^4$	f	$1 + \ln f$
1	0	21.19	.945	.9434
	1	11.9	.918	.9144
	2	3.1	.892	.8857
	3	- 4.7	.868	.8584
1.5	0	40.10	.917	.9134
	1	26.0	.878	.8699
	2	13.3	.841	.8268
	3	1.5	.807	.7856
2	0	48.22	.889	.8823
	1	28.5	.840	.8256
	2	10.8	.796	.7718
	3	- 6.3	.754	.7176
3	0	47.12	.842	.828
	.4	34.1	.811	.7905
	.8	21.5	.783	.7554
	1.2	9.7	.757	.7216
4	0	36.03	.793	.7680
	.4	19.4	.754	.7176
	.8	3.6	.724	.6770
	1.2	-10.7	.696	.6376

Table IV

Migration Areas of Lattices According to the Poison Method

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	M^2 (cm ²) from One-group Critical Equation	M^2 (cm ²) from Age Theory Critical Equation
1	33.66 $\pm$ 1.08	32.78 $\pm$ 1.08
1.5	35.55 $\pm$ 1.00	33.15 $\pm$.88
2	32.19 $\pm$.77	30.21 $\pm$.69
3	30.91 $\pm$ 1.26	28.38 $\pm$ 1.07
4	28.75 $\pm$ 1.17	27.72 $\pm$ 1.08

Table V

k_{∞} for 1.15% Enriched, .600" Diameter Uranium Rods
in Ordinary Water. (Values based on measured values
of B^2 and on the values of M^2 in table IV)

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	k_{∞} from One-group Analysis	k_{∞} from Age Theory Analysis
1	1.071 $\pm$.003	1.072 $\pm$.003
1.5	1.143 $\pm$.005	1.142 $\pm$.005
2	1.155 $\pm$.005	1.157 $\pm$.004
3	1.146 $\pm$.007	1.143 $\pm$.006
4	1.104 $\pm$.005	1.105 $\pm$.005

Table VI

Values of η for Slightly Enriched Uranium, Based on
Different Assumed Values of η for Natural Uranium

Enrichment	$\eta_{\text{nat}} =$ 1.310	$\eta_{\text{nat}} =$ 1.325	$\eta_{\text{nat}} =$ 1.340
1.3 %	1.5871	1.5989	1.6107
1.15 %	1.5359	1.5485	1.5611
1.027 %	1.4861	1.4993	1.5126

Table VII

Buckling of Light Water Moderated, Slightly Enriched Uranium
Rod Lattices (.600" diameter rods) for Varying Rod Enrichment

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	$B^2 \text{ (cm}^{-2} \times 10^4)$		
	1.027% Enrichment	1.15% Enrichment	1.3% Enrichment
1	32.11 $\pm$.54	21.19 $\pm$.23	9.66 $\pm$.52
1.5	51.87 $\pm$.50	41.10 $\pm$.30	29.60 $\pm$.70
2	61.08 $\pm$.32	48.22 $\pm$.31	35.83 $\pm$.47
3	60.99 $\pm$.26	47.12 $\pm$.33	32.93 $\pm$.34
4	50.28 $\pm$.27	36.03 $\pm$.16	21.01 $\pm$.29

Table VIII

Thermal Utilization of Light Water Moderated, Slightly Enriched
Uranium Rod Lattices (.600" diameter rods) for Varying Rod Enrichment

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	f		
	1.027% Enrichment	1.15% Enrichment	1.3% Enrichment
1	.941	.945	.948
1.5	.913	.917	.925
2	.886	.888	.900
3	.833	.842	.846
4	.781	.793	.784

Table IX

B^2 , f , ηf , and $\ln \eta f$ for Slightly Enriched, .600" Diameter Uranium Rods in Ordinary Water
 Values of f are taken from the smooth curves of figure 9.
 η_{nat} = assumed value of η for natural uranium.

Volume Ratio	Enrichment	B^2 $\text{cm}^{-2} \times 10^4$	f	$\eta_{nat} = 1.310$		$\eta_{nat} = 1.325$		$\eta_{nat} = 1.340$	
				ηf	$\ln \eta f$	ηf	$\ln \eta f$	ηf	$\ln \eta f$
1	1.027%	9.66	.941	1.3984	.33533	1.4108	.34415	1.4234	.35306
	1.15	21.19	.945	1.4514	.37254	1.4633	.38069	1.4752	.38879
	1.3	32.11	.949	1.5062	.40959	1.5174	.41700	1.5286	.42435
1.5	1.027	29.60	.912	1.3553	.30404	1.3674	.31292	1.3795	.32172
	1.15	41.10	.918	1.4100	.34359	1.4215	.35172	1.4331	.35985
	1.3	51.87	.924	1.4665	.38288	1.4774	.39029	1.4883	.39764
2	1.027	35.83	.885	1.3152	.27399	1.3269	.28285	1.3387	.29169
	1.15	48.22	.893	1.3716	.31599	1.3828	.32411	1.3941	.33224
	1.3	61.08	.899	1.4268	.35543	1.4374	.36284	1.4480	.37019
3	1.027	32.93	.833	1.2379	.21343	1.2489	.22227	1.2600	.23111
	1.15	47.12	.842	1.2932	.25713	1.3038	.26528	1.3144	.27339
	1.3	60.99	.851	1.3506	.30056	1.3607	.30800	1.3707	.31532
4	1.027	21.01	.781	1.1606	.14893	1.1710	.15787	1.1813	.16662
	1.15	36.03	.793	1.2180	.19722	1.2280	.20539	1.2380	.21350
	1.3	50.28	.804	1.2760	.24373	1.2855	.25114	1.2950	.25851

Table X

One-Group Theory Values of M^2 , from Enrichment Method

<u>Volume Water</u> <u>Volume Uranium</u>	$\eta_{\text{nat}} = 1.310$	$\eta_{\text{nat}} = 1.325$	$\eta_{\text{nat}} = 1.340$
1	$35.06 \pm 1.43 \text{ cm}^2$	$34.34 \pm 1.43 \text{ cm}^2$	$33.55 \pm 1.43 \text{ cm}^2$
1.5	41.03 ± 1.89	39.91 ± 1.89	39.02 ± 1.89
2	37.74 ± 1.17	36.94 ± 1.17	36.11 ± 1.17
3	36.69 ± 1.06	36.08 ± 1.06	35.18 ± 1.06
4	36.89 ± 1.10	36.23 ± 1.10	35.63 ± 1.10

Table XI

Age Theory Values of M^2 , from Enrichment Method

<u>Volume Water</u> <u>Volume Uranium</u>	$\eta_{\text{nat}} = 1.310$	$\eta_{\text{nat}} = 1.325$	$\eta_{\text{nat}} = 1.340$
1	$32.93 \pm 1.23 \text{ cm}^2$	$32.29 \pm 1.23 \text{ cm}^2$	$31.59 \pm 1.23 \text{ cm}^2$
1.5	35.25 ± 1.51	34.59 ± 1.51	33.94 ± 1.51
2	32.14 ± 0.99	31.57 ± 0.99	30.98 ± 0.99
3	31.12 ± 1.18	30.62 ± 1.18	30.08 ± 1.18
4	32.43 ± 1.24	31.91 ± 1.24	31.44 ± 1.24

Table XII

One-Group Theory Values of M^2 from Enrichment Method
(based on calculated values of f)

<u>Volume Water</u> <u>Volume Uranium</u>	M^2 Based on P^3 Calculations of f	M^2 Based on Diffusion Theory Values of f
1	34.43 cm ²	34.16 cm ²
1.5	39.15	39.72
2	36.38	36.34
3	34.59	34.91
4	33.68	34.03

Table XIII

Age Theory Values of M^2 from Enrichment Method
(based on measured values of f)

<u>Volume Water</u> <u>Volume Uranium</u>	M^2 based on P^3 Calculations of f	M^2 Based on Diffusion Theory Values of f
1	32.13 cm ²	31.89 cm ²
1.5	33.75	33.65
2	30.94	30.90
3	29.75	30.00
4	30.08	30.35

Table XIV

Best Values of M^2 from Enrichment Method

<u>Volume Water</u> <u>Volume Uranium</u>	One-Group Critical Equation	Age Theory Critical Equation
1	$34.34 \pm 1.63 \text{ cm}^2$	$32.29 \pm 1.41 \text{ cm}^2$
1.5	39.91 ± 2.09	34.59 ± 1.64
2	36.94 ± 1.44	31.57 ± 1.15
3	36.08 ± 1.39	30.62 ± 1.30
4	36.23 ± 1.25	31.91 ± 1.33

Table XV

Best Values of Migration Area Fission Neutrons in Lattices of
.600" Diameter Uranium Rods in Ordinary Water

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	$M^2 \text{ (cm}^2\text{)}$
1	$32.70 \pm .77$
1.5	$33.05 \pm .55$
2	$30.78 \pm .44$
3	$29.31 \pm .73$
4	$28.70 \pm .71$

FIG. 1

B^2 VS BORON CONCENTRATION IN MODERATOR
1.15% ENRICHED, .600" DIAMETER URANIUM
RODS IN ORDINARY WATER.

FIG. 2

B^2 VS BORON CONCENTRATION IN MODERATOR
1.15% ENRICHED, .600" DIAMETER URANIUM
RODS IN ORDINARY WATER

FIG. 3

f VS BORON CONCENTRATION IN MODERATOR
1.15% ENRICHED, .600" DIAMETER URANIUM
RODS IN ORDINARY WATER.

FIG. 4

B^2 VS f FOR BORON POISONED LATTICES 1.15% ENRICHED, .600" DIAMETER URANIUM RODS IN ORDINARY WATER. PLOTTED POINTS INTERPOLATED FROM FIGURES 1-3.

FIG. 5

B^2 VS $1 + \ln(f)$ FOR BORON POISONED LATTICES,
1.15% ENRICHED, .600" DIAMETER URANIUM RODS
IN ORDINARY WATER. PLOTTED POINTS BASED ON
INTERPOLATIONS FROM FIGURES 1-3.

FIG. 6

ONE-GROUP THEORY VALUES OF MIGRATION AREA
FOR URANIUM-WATER LATTICES. VALUES OBTAINED
BY POISON MEASUREMENT.

FIG. 7

AGE THEORY VALUES OF MIGRATION AREA FOR
URANIUM-WATER LATTICES. VALUES OBTAINED
FROM POISON MEASUREMENT.

FIG. 8

B^2 VS VOLUME RATIO. .600" DIAMETER URANIUM
RODS IN ORDINARY WATER

ENRICHMENT

O - 1.3 %

+ - 1.15 %

□ - 1.027 %

FIG. 10

B^2 VS. $LN(\eta f)$ FOR .600" DIAMETER, SLIGHTLY ENRICHED URANIUM RODS IN ORDINARY WATER.

○ MEASUREMENTS WITH CLEAN WATER.

+ MEASUREMENTS WITH BORON POISONED WATER.

FIG. II

BEST MEASURED VALUES OF MIGRATION AREA OF
FISSION NEUTRONS IN LATTICES OF .600"
DIAMETER URANIUM RODS IN ORDINARY WATER.