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ANL-4284

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DATE FEB 12 1957 *let*

For The Atomic Energy Commission

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THEORETICAL NUCLEAR PHYSICS DIVISION

Report for Period
ending March 28, 1949

F. C. Hoyt, Director

Photostat Price \$ 7.20

Microfilm Price \$ 3.60

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

May 2, 1949

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THE ATTENUATION OF NEUTRONS IN A CONICAL SHIELD

Written by E. J. Hellund
Work done by E. J. Hellund

1. Introduction. We consider a cone of revolution of half angle β bounded by spherical caps at $R = a$ and $R = b$ ($b > a$).

The neutrons are supposed to be fed into the smaller cap. Two cases are considered. The first is that of constant density over the inner cap, and the second is that of constant diffusion current over the inner cap. In both cases it is assumed that no capture takes place, only elastic scattering. It is further assumed that the remaining bounding surfaces behave as perfect sinks, that is, the density of neutrons falls to zero at these surfaces. The problem is then to find the attenuation of neutron flux outward from the inner cap, at various angles from the axis of the conical shield.

In the first case considered it should be clear as to the difference in flux conditions relative to the second case. The "stream lines" of neutron flow can be seen qualitatively to be radial near the cone axis and near the boundary to curve sharply so as to intersect the surface of revolution at right angles. This curvature is more marked near the inner cap and becomes less pronounced as points are taken closer to the outer cap.

It is clear from the representation by stream lines that the flux must fall off faster than the inverse square law, (geometrical spreading). Further, one may easily conclude that the form of the attenuation of the flux is chiefly dependent on the cone angle. That is, for large cone angle one expects adherence to the inverse square attenuation, but for small cone

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angles the attenuation should approach an exponential form.

Instead of the boundaries previously chosen, one may easily see that a surface of revolution whose generators are all stream lines could be taken to be a boundary formed by a perfect reflector of neutrons.

2. Mathematical Analysis.

Notation:

$$\rho = \rho(R, \theta) = \text{neutron density.} \quad (1)$$

$$\theta = \text{angle from cone axis.} \quad (2)$$

$$R = \text{distance from cone vertex.} \quad (3)$$

$$- \frac{\partial \rho}{\partial R} = \text{radial neutron flux.} \quad (4)$$

$$- \frac{1}{R} \frac{\partial \rho}{\partial \theta} = \text{transverse neutron flux.} \quad (5)$$

Differential equation for ρ :

$$R \frac{\partial^2}{\partial R^2} (R \rho) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \rho}{\partial \theta}) = 0. \quad (6)$$

Boundary conditions:

$$\rho(a, \theta) = 1, \quad (7a)$$

$$\rho(b, \theta) = 0, \quad (7b)$$

$$\rho(R, \beta) = 0. \quad (7c)$$

"Eigensolutions" (not satisfying boundary conditions):

$$\frac{1}{R^{m+1}} P_m(\cos \theta), \quad (8a)$$

$$R^m P_m(\cos \theta). \quad (8b)$$

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where the $P_m(\cos \theta)$ are the Legendre functions. Superposition of eigensolutions gives the solution:

$$\rho(R, \theta) = \sum_m \left(\frac{A_m}{R^{m+1}} P_m + B_m R^m P_m \right). \quad (9)$$

Insertion of conditions (7a) and (7b) leads to the following pair of equations:

$$\frac{A_m}{a^{m+1}} + B_m a^m = 0, \quad m \neq 0, \quad (10a)$$

$$\frac{A_m}{b^{m+1}} + B_m b^m = 0. \quad (10b)$$

For the special case of $m = 0$,

$$\frac{A_0}{a} + B_0 = 1, \quad (10c)$$

$$\frac{A_0}{b} + B_0 = 0. \quad (10d)$$

In order that non-trivial solutions of (10a) and (10b) exist, one must have:

$$\begin{vmatrix} \frac{1}{a^{m+1}} & a^m \\ \frac{1}{b^{m+1}} & b^m \end{vmatrix} = 0, \quad (11)$$

giving

$$\frac{b^m}{a^{m+1}} - \frac{a^m}{b^{m+1}} = 0. \quad (12)$$

Equation (12) provides the values of m which will permit a solution of (6) subject to the proper boundary conditions, i.e.

$$\left(\frac{b}{a}\right)^{2m+1} = 1 \quad (13)$$

or

$$e^{(2m+1) \log (b/a)} = e^{2\pi N i}.$$

Thus

$$m = -\frac{1}{2} + \frac{(N\pi) i}{\log (b/a)} \quad |N| = 0, 1, 2, 3, \dots \quad (14)$$

It is apparent that Legendre functions of complex order are forced into the solution just by the boundary conditions.

These functions, called conal harmonics, have the following expansion (Beverly-Fourier Series and Spherical Harmonics - pages 192-3):

$$\begin{aligned} K_{-\frac{1}{2} + ki}(\cos \theta) &= 1 + \frac{(k^2 + \frac{1}{2})}{2} (1 - \cos \theta) \\ &+ \frac{(k^2 + \frac{1}{2})(k^2 + \frac{3}{2})}{2^2 (2i)^2} \\ &+ \dots \end{aligned} \quad (15)$$

the series converges for all values of θ except $\theta = \pi$.

Rewriting (9),

$$\rho(R, \theta) = \frac{a}{b-a} \left(\frac{b}{R} - 1\right) = \sum_m \frac{K_m^*(\cos \theta)}{K_m^*(\cos \beta)} \sin\left(m\pi \frac{\log \frac{R}{a}}{\log \frac{b}{a}}\right), \quad (16a)$$

with

$$m' = \frac{m \pi}{\log(b/a)}$$

and

$$K m' = P_{-\frac{1}{2}} + m' i$$

and from (7c)

$$\begin{aligned} A_m &= \frac{2}{\log(b/a)} \left(\sqrt{\frac{a}{R}} \right) \int_0^{\log(b/a)} e^{x/2} \left\{ \left(\frac{a}{b-a} \right) \left(\frac{b}{a} e^{-x} - 1 \right) \sin \left(\frac{m \pi x}{\log b/a} \right) dx \right\} \quad (16b) \\ &= \frac{8m \pi}{\log(b/a) \left(2 + 4m^2 \pi^2 \right)} \left(\sqrt{\frac{a}{R}} \right). \end{aligned}$$

This completes the solution.

While the problem of constant flux is apparently not discussed in the literature, it is in principle just as easy to handle.

In place of (10a) and (10b), one has

$$\frac{-A_m(m+1)}{a^{m+2}} + (m) B_m a^{m-1} = 0, \quad m \neq 0 \quad (17a)$$

$$\frac{A_m}{b^{m+1}} + B_m b^m = 0. \quad (17b)$$

In place of (10c) and (10d),

$$-\frac{A_0}{a^2} = -1; \quad -\left(\frac{\partial \rho}{R} = \text{current} = 1 \right), \quad (17c)$$

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$$\frac{A_0}{b} + B_0 = 0, \quad (17d)$$

and (11) is replaced by

$$\left| \begin{array}{cc} \frac{-(m+1)}{a^{m+2}} & , \quad (m) a^{m-1} \\ \frac{1}{b^{m+1}} & , \quad b^m \end{array} \right| = 0. \quad (18)$$

Consequently,

$$\frac{m+1}{m} = - \left(\frac{a}{b} \right)^{2m+1} = e^{(2m+1)\pi i} e^{(2m+1) \log \left(\frac{a}{b} \right)} \quad (19)$$

or

$$m = -\frac{1}{2} + \chi i \quad (20)$$

where χ satisfies

$$\chi \log \frac{b}{a} + \tan^{-1} \left(\frac{1}{2\chi} \right) = (N + \frac{1}{2}) \pi, \quad |N| = 0, 1, 2, \dots \quad (21)$$

When χ is moderately large,

$$2\chi \simeq \frac{(N' + \frac{1}{2})\pi + \sqrt{(N' + \frac{1}{2})^2 \pi^2 + 2 \log(b/a)}}{\log(b/a)}, \quad (b > a) \quad (22)$$

When χ is small,

$$\chi \simeq \frac{(N' + 1)\pi}{2 + \log(b/a)}, \quad (b > a). \quad (23)$$

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Approximations (22) and (23) are valid for the following cases, respectively,

$$\tan^{-1} \frac{1}{2\chi} \approx \frac{1}{2\chi}, \quad (24a)$$

$$\tan^{-1} (2\chi) \approx 2\chi. \quad (24b)$$

Both (22) and (23) are repeated for negative values. The Legendre functions, however, yield no new solutions for these values.

It may be seen that (23) admits the value $\chi = 0$. Ultimately, series (16) is replaced by

$$\rho(R, \theta) = + a \left(\frac{a}{R} - \frac{a}{b} \right) - \sum \frac{C_n}{R^{\frac{1}{2}}} \sin \left(\chi_n \log \frac{R}{b} \right) \frac{K_{\chi_n}(\cos \theta)}{K_{\chi_n}(\cos \beta)}, \quad (25)$$

where C_n is so chosen that,

$$a \left(\frac{a}{R} - \frac{a}{b} \right) = \sum \frac{C_n}{R^{\frac{1}{2}}} \sin \left(\chi_n \log \frac{R}{b} \right). \quad (26)$$

Since the $\sin \left(\chi_n \log \frac{R}{b} \right)$ functions form an orthogonal set over a to b , one has,

$$C_N = \frac{a^{\frac{3}{2}} b^{-\frac{1}{2}}}{\int_0^{\log(b/a)} \sin^2(\chi_N x) dx} \int_0^{\log(b/a)} (e^{x/2} - e^{-x/2}) \sin(\chi_N x) dx \quad (27)$$

$$= \frac{\chi_N a^{3/2} (-1)^{N+1}}{\left[\left(\frac{1}{2} \right)^2 + \chi_N^2 \right]^{3/2} \left[\frac{1}{2} \log(b/a) + \frac{1}{2\chi_N^2 + 1} \right]}$$

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Finally,

$$\rho(R, \theta) = a \left(\frac{a}{R} - \frac{a}{b} \right) + \frac{a^{3/2}}{R^2} \sum \frac{(-1)^N \chi_N \sin \left(\chi_N \log \frac{R}{b} \right)}{\left[\left(\frac{1}{2} \right)^2 + \chi_N^2 \right]^{3/2} \left[\frac{1}{2} \log b/a + \frac{1}{2\chi_N^2 + 1} \right]} \quad (28)$$

$$\cdot \frac{K \chi_N (\cos \theta)}{K \chi_N (\cos \theta)}$$

with χ_N given by (21).

3. Approximate Solutions. Provided points are taken far from the inner cap, one may derive a simple expression for the flux that represents minimum attenuation, namely,

$$\text{Radial flux at } R, \theta < \frac{(\text{flux at inner cap}) \cos \left(\frac{\pi}{2} \frac{\theta}{\beta} \right)}{(R/a)^{3/2 + \frac{1}{2}} \left(1 + \frac{2\pi^2}{\beta^2} \right)^{\frac{1}{2}}} \quad (29)$$

$$\beta < \frac{\pi}{2}$$

The added attenuation apart from the inverse square spreading is,

$$\frac{\cos \frac{\pi}{2} \frac{\theta}{\beta}}{(R/a)^{\frac{1}{2}} \left[\sqrt{1 + \frac{2\pi^2}{\beta^2}} - 1 \right]}, \quad \beta < \frac{\pi}{2} \quad (30)$$

It is to be noted that the attenuation increases with the smallness of β and decreases with increasing β .

Expression (30) is actually derived by "perturbing" the continuity equation to the following form,

$$R \frac{\partial^2}{\partial R^2} (R \rho) + \frac{k^2}{\sin(k\theta)} - \frac{\partial}{\partial(k\theta)} \left[\sin(k\theta) \frac{\partial \rho}{\partial(k\theta)} \right] \quad (31)$$

The perturbation is directly a modification of the transverse, (θ) , current, which leads to a change in radial current.

The perturbation in transverse current is,

$$\left[\cot \theta - k \cot(k\theta) \right] \frac{\partial \rho}{\partial \theta} \quad \text{subtracted}$$

$$\sim \frac{\theta}{3} (1 - k^2) \frac{\partial \rho}{\partial \theta} \quad \text{for small } \theta.$$

Thus the perturbation is small if θ , or $k-1$, or $\frac{\partial \rho}{\partial \theta}$ is small.

Actually in application to large cones k must be taken near 1, and for small angle cones θ is necessarily small so the perturbation does in general behave as a physically small quantity.

Proceeding from (31), we take as solution $\cos(k\theta) R^m$. Inserting in (31) we find that one must have,

$$m(m+1) = 2k^2 \quad (33)$$

or

$$m = \frac{-1 \pm \sqrt{1 + 8k^2}}{2} \quad (34)$$

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Now if $\cos(k\theta) = 0$, $\theta = \beta$ and we have

$$k\beta = \frac{\pi}{2} \quad (35)$$

or

$$k = \frac{\pi}{2} \left(\frac{1}{\beta} \right),$$

from which (29) and (30) follow.

The reason for excluding regions near the inner cap is that boundary perturbations will distort the solutions near the inner cap.

For $\beta = \frac{\pi}{2}$, (maximum angle), the attenuation, (extra), is

$$\frac{\cos(\theta)}{R/a}$$

4. Calculations. Computations have been carried on (16), but none on (28). However, in the region $R \rightarrow b$ it was far more accurate to use (29) and (30), as the required accuracy in the series for the asymptotic region becomes prohibitive.

References

R. W. Hobson - A Class of Spherical Harmonics of Complex Degree p. 211 - Trans Cambridge Phil. Soc., Vol. 14, (1889).

Byerly - Fourier Series and Spherical Harmonics

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COLLECTION EFFICIENCY FOR BROMINE IONS

Written by E. J. Hellund
Work done by E. J. Hellund

1. Introduction. The problem considered here arose in the experiment performed by Wexler and Davies in which a study was made of the charge and energy of gamma recoil bromine nuclei. The bromine was present as neutron irradiated ethyl bromide. The recoil on gamma emission breaks the chemical bond and frees the bromine nucleus which may then be collected by means of an electrical field, upon a rod or wire, if the bromine is charged. The collecting chamber actually consisted of two concentric cylinders with a potential difference such that collection was possible due to subsequent β emission.

The present report is concerned with the analysis of the efficiency of collection as a function of the charge, mass, and energy of the bromine nuclei, the radii of the outer and inner cylinders, and potential difference between the cylinders. No consideration of end effects was attempted due to the fact that the ratio of the length of the tube to the diameter of the outer cylinder was about 30 to 1.

A later report will treat pressure effects.

2. Mathematical Analysis. It can easily be shown that conservation of energy and angular momentum lead to the following expression for the radial velocity of the ion.

$$\frac{mR^2}{2} = \frac{-E_0}{\left(\frac{R}{R_0}\right)^2} - \frac{q V_0}{\log(b/a)} \log \frac{R}{R_0} + E_R + E_\theta \quad (1)$$

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where R is the radial distance R_0 the point at which the recoil occurs, q is the charge of the bromine nucleus, V_0 the voltage across the tube, a and b the inner and outer radii respectively, and E_R and E_θ are the energies of recoil along the radius vector and perpendicular, (θ direction), to the radius vector respectively.

For minimum or maximum radius, $\frac{dR}{dt} = 0$. Therefore, collection of the ion is determined by the roots of the following equation,

$$y = \Gamma \log(y + H), \quad (2)$$

where

$$\Gamma = \frac{q V_0}{2E_\theta \log(b/a)}, \quad (3)$$

and

$$H = \frac{E_R}{E_\theta} + \frac{E_\theta}{E_\theta}, \quad (4)$$

and

$$y = \left(\frac{R_0}{R} \right)^2 - \frac{E_R}{E_\theta} + \frac{E_\theta}{E_\theta}. \quad (5)$$

All energies including $q V_0$ may be taken in electron volts.

While it is a simple matter to obtain approximations for the roots of (2), it is far more practical to obtain these values graphically for particular values of Γ and H . Then one may fit the data so obtained by suitable analytical expressions and obtain the efficiency from the latter.

If one denotes by R_0^a and R_0^b the points of recoil, (with particular values of E_θ and E_R), such that grazing collisions occur with the inner and outer cylinders respectively, the collection to the center is,

$$\frac{(R_o^a)^2 - a^2}{b^2 - a^2}, \quad R_o^a < R_o^b \quad (6a)$$

$$\frac{(R_o^a)^2 - a^2}{b^2 - a^2}, \quad \begin{matrix} R_o^a > R_o^b \\ \dot{R}_o < 0 \end{matrix} \quad (6b)$$

$$\frac{(R_o^b)^2 - a^2}{b^2 - a^2}, \quad \begin{matrix} R_o^a > R_o^b \\ \dot{R}_o > 0 \end{matrix} \quad (6c)$$

Correspondingly, for the outer wall collection is given by,

$$\frac{b^2 - (R_o^b)^2}{b^2 - a^2}, \quad R_o^a < R_o^b \quad (7a)$$

$$\frac{b^2 - (R_o^a)^2}{b^2 - a^2}, \quad \begin{matrix} R_o^a > R_o^b \\ \dot{R}_o < 0 \end{matrix} \quad (7b)$$

$$\frac{b^2 - (R_o^b)^2}{b^2 - a^2}, \quad \begin{matrix} R_o^a > R_o^b \\ \dot{R}_o < 0 \end{matrix} \quad (7c)$$

In equations (6) and (7) it is assumed that the sticking probability on collision is unity. Experiments performed to test this assumption indicate that for copper amalgam surfaces this is very nearly correct.

3. Application. For the experiment considered, the following data are fixed.

$$\begin{aligned} V_0 &= 4000 \text{ volts} \\ a &= 0.0795 \\ b &= 1.10 \text{ cm} \\ q &= 1 \text{ electron unit (assumed)} \end{aligned}$$

The elementary analysis considered yielded the following approximate analytical expression for the collection efficiency for a given E_g :

$$\left(1 - \frac{E_g}{E'}\right)^4 \text{ to center,} \quad (8)$$

where $E' \sim 350 \text{ ev}$, and $E_g < 300 \text{ ev}$.

Assuming that the recoil is isotropic in direction, one has, for a given total energy of recoil,

$$\begin{aligned} \text{efficiency for} \\ \text{collection to} \\ \text{the center} \end{aligned} = 1 - \frac{4}{3} \left(\frac{E_t}{E'}\right) + \frac{6}{5} \left(\frac{E_t}{E'}\right)^2 - \frac{4}{7} \left(\frac{E_t}{E'}\right)^3 + \frac{1}{9} \left(\frac{E_t}{E'}\right)^4, (E_t < E') \quad (9)$$

where E_t is the total energy of recoil in electron volts.

The inequality is necessary because of the fact that the approximate expression (8) fails for high energies. Actually the available gamma recoil energy does lie in the region governed by (9).

A similar analysis is required to get the efficiency of collection to the outer cylinder. It is, of course, obvious that this efficiency added to the former does not give 100%.

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TABLE OF ROOTS OF EQUATION (2)

For various distributions E_R , E_θ , and for

$$q V_0 = 4000 \text{ ev}$$

$$b = 1.10 \text{ cm}$$

$$a = 0.0795 \text{ cm}$$

E_θ (electron volts)	E_R	(R/R_0) minimum	(R/R_0) maximum
340	0	0.4863	1.0000
250	90	0.3653	1.0884
150	190	0.2462	1.1623
50	290	0.1179	1.2231
25	315	0.0767	1.2405
250	0	0.3816	1.0000
150	100	0.2530	1.0835
50	200	0.1199	1.1495
150	0	0.2616	1.0000
100	50	0.1945	1.0383
50	100	0.1221	1.0725

Note dependence of $(R/R_0)_{\min.}$ on E_θ and $(R/R_0)_{\max.}$ on E_R .

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ON CLOSED SHELLS IN NUCLEI. II

Maria G. Mayer

Feenberg^{(1),(2)} and Nordheim⁽³⁾ have used the spins and magnetic moments of the even-odd nuclei to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin orbit coupling, increasing with angular momentum is assumed, a level assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

Table 1 contains in column two in order of decreasing binding energy the levels of the square well potential. The quantum number gives the number of radial nodes. Two levels of the same quantum number gives the number of radial nodes. Two levels of the same quantum number cannot cross for any type of potential well, except due to spin-orbit splitting. No evidence of any crossing is found. Column three contains the usual spectroscopic designation of the levels, as used by Nordheim and Feenberg. Column one groups together those levels which are degenerate for a three-dimensional isotropic oscillator potential. A well with rounded corners

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(1) Eugene Feenberg, PHYS. REV. 75, 320, (1949)

(2) Eugene Feenberg, PHYS. REV. (1949)

(3) Lothar Nordheim, PHYS. REV. (1949)

The author is indebted to these authors for having obtained copies of both (2) and (3) before publication.

will have a behavior in between these two potentials. The shell grouping is given in column five, with the numbers of particles per shell and the total number of particles up to and including each shell in column six and seven respectively.

Within each shell the levels may be expected to be close in energy, and not necessarily in the order of the table, although the order of levels of the same orbital angular momentum and different spin should be maintained. Two exceptions, $^{23}_{11}\text{Na}$ with spin $3/2$ instead of the expected $d_{5/2}$, and $^{55}_{25}\text{Mn}$ with $5/2$ instead of the expected $f_{7/2}$, are the only violations.

Table 2 lists the known spins and orbital assignments from magnetic moments⁽⁴⁾ when these are known and unambiguous, for the even-odd nuclei up to 83. Beyond 83 the data is limited and no exceptions to the assignment appear.

Up to Z or $N = 20$ the assignment is the same as that of Feenberg and Nordheim. At the beginning of the next shell, $f_{7/2}$ levels occur at 21 and 23, as they should. At 28 the the $f_{7/2}$ levels should be filled, and no spins of $7/2$ are encountered any more in this shell. This subshell may contribute to the stability of Ca^{48} . If the $g_{9/2}$ level did not cross the $p_{1/2}$ or $f_{5/2}$ levels, the first spin of $9/2$ should occur at 41, which is indeed the case. Three nuclei with N or $Z = 49$ have $g_{9/2}$ orbits. No s or d levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin $5/2$ instead of $7/2$ for Mn^{55} and the fact that the magnetic moment of

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(4) H. H. Goldsmith and D. R. Inglis, the Properties of Atomic Nuclei. I., Information and Publications Division, Brookhaven National Laboratory.

$^{59}\text{Co}_{27}$ indicates a $g_{7/2}$ orbit instead of the expected $f_{7/2}$.

In the next shell two exceptions to the assignment occur. The spin of $1/2$ for Mo^{95} with 53 would be a violation, but is experimentally doubtful. The magnetic moment of Eu^{153} indicates $f_{5/2}$ instead of the predicted $d_{5/2}$. No $h_{11/2}$ levels appear. It seems that these levels are filled in pairs only which does not seem a serious drawback of the theory as this tendency already shows up at the filling of the $g_{9/2}$ levels. Otherwise, the agreement is satisfactory. The shell begins with 51Sb , which has two isotopes with $d_{5/2}$ and $g_{7/2}$ levels respectively, as it should. The Thallium isotopes with 81 neutrons and a spin of $1/2$ indicate a crossing of the $h_{11/2}$ and $3s$ levels. This is not surprising, since the energies of these levels are close together in the square well. The assignment demands that there be no spins of $9/2$ in this shell, and none have been found. No f or p levels should occur and, except for Eu^{153} , there is no indication of any.

The spin and magnetic moment of 83Bi , indicating an $h_{9/2}$ state, is a beautiful confirmation of the correct beginning of the next shell. Here information begins to be scarce. The spin and magnetic moment of Pb^{207} with 125 neutrons interpret as $p_{1/2}$. This is the expected end of the shell since $7i$ and $4p$ have practically the same energy in the square well model. No spins of $11/2$ and no s , d or g orbits should occur in this shell and the data indicates none.

The prevalence of isomerism in certain regions of the isotope chart, noticed by both Feenberg and Nordheim, is readily understood by this assignment. Long-lived isomeric states will occur in regions where levels of very different spin lie close together. These regions lie toward the

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end of the shells of 50, 82, and 126, where the levels of lowest angular momentum of one oscillator level almost coincide in energy with those of highest angular momentum from the next oscillator level. One is lead to the prediction that for nuclei of odd A isomerism should occur between $39 \leq Z$ or $N \leq 49$. This is the region where the $g_{9/2}$ and $p_{1/2}$ levels have closely the same energy and compete for the ground state. From 51 on there is a competition between $g_{7/2}$ and $d_{5/2}$, which would not lead to long-lived isomers. Later in the shell, from about 65 on, competition should occur between the $h_{11/2}$ and the $s_{1/2}$ and $d_{3/2}$ levels, and the occurrence of isomerism is predicted between $65 \leq Z$ or $N \leq 81$. The beginning of the new shell should again be free of isomerism. The experimental facts bear out the conclusions exceedingly well. Below are listed the number of long-lived isomeric states known and listed as A in the table by Seaborg and Perlman. Only isomers of odd A are used, and these are attributed to the odd one of the numbers N or Z .

N or Z =	29	<u>31 - 37</u>	39	41	43	45	47	49	<u>51 - 61</u>
No of isomers =	1	0	3	3	3	2	5	4	0

N or Z =	63	65	67	69	71	73	75	77	79	81	<u>83 - 93</u>
No of isomers =	1	1	1	4	1	2	2	2	2	2	0

In both regions, the level of high spin has opposite parity to the one of low spin. Consequently, one would expect electric 5^{th} pole and magnetic 4^{th} pole radiation to occur, but not electric 4^{th} pole.

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The assignment of orbits makes possible the prediction of spin and parity in cases in which the spin has not been observed. Since spin change and change of parity determine the selection rules for β - decay, it should be possible to test the theory against experiment. Work on this is in progress.

TABLE 1.

Osc. No.	Square Well	Spect. term	Spin term	No. of States	Shells	Total No.
0	1s	1s	1s _{1/2}	2	2	2
1	1p	2p	1p _{1/2} 1p _{3/2}	4 2	6	8
2	1d 2s	3d 2s	1d _{5/2} 1d _{3/2} 2s _{1/2}	6 4 2	12	20
3	1f 2p	4f 3p	1f _{7/2} 1f _{5/2} 2p _{3/2} 2p _{1/2} 1g _{9/2}	8 6 4 2 10	8? 22	28? 50
4	1g 2d 3s	5g 4d 3s	1g _{7/2} 2d _{5/2} 2d _{3/2} 3s _{1/2} 1h _{11/2}	8 6 4 2 12	32	82
5	1h 2f 3p	6h 5f 4p	1h _{9/2} 2f _{7/2} 2f _{5/2} 3p _{3/2} 3p _{1/2} 1i _{13/2}	10 8 6 4 2 14	44	126
6	1i 2g 3d 4s	7i 6g 5d 4s	1i _{11/2}			

TABLE 1
SPINS OF EVEN-ODD NUCLEI

No. of neutrons or protons	Odd Protons				Odd Neutrons				O = Neutron X = Proton					No. odd p or n	Levels	
	Element	Mass No.	Orbit	Mass No.	Orbit	Element	Mass No.	Orbit	SPIN							
									p	d	f	g	h			
									1	2	3	4				
									1/2	3/2	5/2	7/2	9/2			
1	H	1	s	2	s	He	3	s	OX						1	1s
3	Li	7	p							X					3	$p_{3/2}$
5	B	11	s			Be	9	s		OX					5	
7	N	15	p			C	13	p	OX						7	$p_{3/2}$
9	F	19	s							X					9	
11	Na	23									X				11	$d_{5/2}$
13	Al	27	d									X			13	
15	P	31								X					15	$p_{3/2}$
17	Cl	35	d	37	d	S	33			OX					17	$d_{3/2}$
19	K	39	d	41	d						X				19	
21	Sc	45	f										X		21	$f_{7/2}$
23	V	51											X		23	
25	Mn	55										X			25	
27	Co	59	s										X		27	
29	Cu	63		65							X				29	$p_{3/2}$
31	Ga	69									X				31	
33	As	75									X				33	
35	Br	79		81							X				35	$f_{5/2}$
37	Rb	85 $f(5/2)$		87 $p(3/2)$		Zn	67	f			X	OX			37	
39	Y	89								X					39	$p_{1/2}$
41	Cb	93	s											X	41	$f_{9/2}$
43	Tc					Se	77			O					43	
45	Rh														45	
47	Ag	107	p	109	p	Er	63	s		X				O	47	
49	In	113	s	115	s	Br	67	s						OX	49	
51	Sb	121 $d(5/2)$		123 $g(7/2)$								X	X		51	$f_{7/2}$
53	I	127				Mo	95			O		X			53	$d_{5/2}$
55	Cs	133	s	137	s								X		55	
57	La	139	s										X		57	
59	Pr	143										X			59	
61	Pm														61	
63	Eu	151		153	f	Cd	111	s		O		X			63	
65	Tb	159				Cd	113	$g_{9/2}$		O	X				65	$h_{11/2}$
67	Ho	165				Gd	117	s		O			X		67	
69	Tm					Er	119	s		OX					69	
71	Lu	175	s										X		71	
73	Ta	181	s										X		73	
75	Re	185		187		Xe	129	s		O		X			75	
77	Ir	191 $(1/2)$		193 $(3/2)$		Xe	131	d		X	OX				77	$d_{3/2}$
79	Au	197	d			Ba	135	d			OX				79	
81	Tl	203		205		Ba	137	d		X	O				81	$p_{1/2}$
83	Bi	209	s											X	83	$h_{9/2}$

MONTE CARLO CALCULATIONS ON FAST REACTORS I.

Written by E. Eisner and M. Mayer
Work done by J. Alexander, E. Eisner, M. Mayer

1. Introduction. Calculations by the Monte Carlo method on the neutron distributions in two fast reactors were carried out during a three-week period in December when the Eniac was made available through the courtesy of the Aberdeen Proving Ground. The reactors investigated were the Experimental Breeder (CP-4) [now in an advanced stage of design at the Argonne National Laboratory] and the Los Alamos Fast Reactor (Clementine), [now in operation.] Analysis of the data obtained, consisting of about 100,000 IBM cards containing information on as many neutrons, is nearly complete and the results to date, together with tentative conclusions [that can be drawn from them] are [here] presented in this first report.

Acknowledgement is due to L. C. Merrill and J. W. Webster of Nepa for participation in part of the work and to R. Richtmyer and E. Teller for valuable suggestions and advice.

2. The Monte Carlo Method. In applying this method, a neutron is given an initial location and velocity, and a random number is used to assign the multiple of a mean free path it travels before making a collision. The total cross section along the path then determines the location of the collision, and another random number selects the fate that befalls it on the basis of the differential cross sections. If it is absorbed, a fresh neutron is started. If scattering or fission occur, random numbers further select from the given distribution functions, the velocity,

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and in the case of fission, the number of emerging neutrons. These emerging neutrons are similarly followed until all have been absorbed, escaped across the boundaries, or else have fulfilled some other condition. By following a sufficiently large and varied initial sample in this fashion, all questions concerning the behavior of neutrons in the particular reactor studied find their answers.

With the Eniac it is feasible to carry out this procedure for a usefully large sample of neutrons. It was, however, not feasible to keep a complete record of all neutron paths. Instead, a censusing procedure was used: The position and velocity of all neutrons were recorded at the instant they had covered a specified path distance. In previous Los Alamos work a census based on time was used. The large range of neutron velocities involved in our work made it advisable to take advantage of the relative constancy of free-paths as compared with free-times by the use of the distance-census. (The interpretation of the results is slightly modified and is discussed in detail in a forthcoming report. As shown by Fermi and Richtmyer,⁽¹⁾ a distance census gives directly the flux distribution rather than the density distribution for a critical system.

The location and incident velocity of all absorptions, fissions, and escapes across the outer boundary were recorded. A complete genealogical record of the neutrons was also kept.

3. Geometrical Configurations. Although no difficulties in principle arise in treating arbitrarily complex configurations, consideration of

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(1) LAMS-805

the capabilities of the Eniac dictated limitation on this score. One problem had spherical symmetry, but most of the problems treated were in cylindrical geometry. The space variation of cross sections was represented by means of a number of zones, each of which was considered to be homogeneous in composition. The configuration of the zones used in the cylindrical cases is illustrated schematically in Fig. 1.

Fig. 1

For convenience of identification, these zones have been labeled by an index i , referring to the radius of the zone boundary closest to the axis of the cylinder, and by an index j , referring to the height of the lower zone boundary plane. Only the central ($i = 0$) zone was divided into layers.

4. The Differential Cross Sections. The calculation can be no better than the physical data on which it is based. There are many gaps in our knowledge

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of cross sections, and inspired guessing had to be resorted to occasionally. The results indicate that the set of numbers used may contain significant errors. As an aid in evaluating the results, the assumed detailed behavior with energy of all cross sections used is appended in Figs. 2-9. The total cross section σ_t is the sum of the cross sections σ_e , σ_i , σ_f , σ_a for elastic scattering, inelastic scattering, fission and absorption, respectively. The elastic scattering σ_e is not plotted, but may be obtained by subtraction. The Eniac does not have sufficient memory to hold the detailed energy variation, and so it was necessary to approximate the curves in some simple fashion. For this purpose the energy range was divided into five regions, labeled by the index k. In the lowest (k = 0) region $1/v$ fission and absorption cross sections, and energy independent elastic scattering and absorption cross sections were included. In the k = 1, 2, 3, 4, regions, constant elastic scattering, absorption, and fission cross sections were used; a constant inelastic scattering cross section was included in the k = 3, 4 regions; and fast-fission was included in region k = 4. The boundaries between the various regions used in the calculations on CP-4 are listed in Table 1 below.

	k = 0	k = 1	k = 2	k = 3	k = 4
v in(m/sec) $\times 10^{-8}$	<.001383	.001383 to .49061	.49062 to .09772	.09772 to .1749	>.1749
E	< 100 ev	100 ev to 125 kev	125 kev to 500 kev	500 kev to 1.6 mev	>1.6 mev

TABLE 1.

Energy Zones Used for CP-4.

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It is necessary to average the cross sections over each energy range in some fashion. While this question will not be discussed here in detail, it is worth pointing out that the major weighting of the $k = 1$ region should be given to the high velocity end. The atomic composition of the zones for CP-4 is given in Table 2.

Zone	00	01	02	03	04	10	20
28	350.22		230.61	23.79	232.27	251.85	356.64
25	2.55		1.68	216.84	1.69	1.84	2.60
Fe	47.93	112.90	10.15	2.09	5.08	52.75	43.26
Ni	13.00	149.47	128.04	142.74	144.20	55.08	4.55
Cr	12.69	29.90	2.69	.55	1.35	13.97	11.45
Cb	.39	.93	.08	.02	.04	.43	.36
C	.31	.72	.06	.01	.03	.35	.29
Ta			.66	.41	.64		
Na		30.17	16.24	15.42	15.34	13.22	
K		65.17	35.08	33.29	33.11	28.55	

Zone 30 is an air gap. Zone 40 is the graphite, with 827.54×10^{20} atoms/cm³.

TABLE 2.

Composition of CP-4 by Zones.

The units are (atoms/cm³) $\times 10^{-20}$

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The macroscopic cross sections actually used are presented for reference in Table 3.

The direction of a neutron emerging from a collision was always taken as uncorrelated with its original direction and selected from 100 isotropically distributed possibilities.

The energy loss on elastic scattering was taken as zero.

Following inelastic collision the energy of the emerging neutron had 10 equally probable choices, ranging from 2.5% to 47.5% of its original energy in equal steps.

Following fission the velocity of the emerging neutron had 100 equally probable choices, given by $V = V(M) + 0.1 N [V(M+1) - V(M)]$ where M and N run from 0 to 9 independently. The choice of $V(M)$ which gave the best fit with the percentile points according to $F(E) = .7476 \sqrt{E} \exp(-0.76E)$, the assumed form of the fission spectrum, is listed in Table 4.

<u>M</u>	<u>V(M) in (m/sec) $\times 10^{-8}$</u>
0	0.039733
1	0.090291
2	0.113451
3	0.135047
4	0.154247
5	0.173471
6	0.193550
7	0.215083
8	0.243697
9	0.275232
10	0.384888

TABLE 4.

Fission Spectrum

$$\bar{\nu} = 2.5$$

Zone 40 contained graphite. In order to conserve machine time, the procedure used in the other zones was modified here. Only elastic scattering was allowed, and free path and energy loss were always assigned.

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TABLE 3.

Macroscopic Cross Sections as Used.

Σ_e , Σ_i , Σ_f , Σ_a , are cross sections for elastic and inelastic scattering, fission and absorption respectively, in m^{-1} . In zone 40 (graphite) λ is the mean free path for scattering in cm.

$k = 0$

Cross sections are assumed proportional to $1/v$, the values given being for $v = 10^5$ cm/sec.

ij	$v \Sigma_f$	$\Sigma_a + \Sigma_f$	Σ_e
00	77	95	.43
01	0	28	.42
02	51	75	.47
03	6559	3115	.47
04	51	77	.49
10	56	76	.42
20	79	95	.42
30	0	0	0
40	$\lambda = 2.57$ cm		

TABLE 3. (continued)

k = 1

lj	Σ_e	Σ_i	Σ_f	Σ_a
00	51.90	0	.06	1.08
01	22.37	0	0	0.13
02	44.93	0	.04	0.74
03	25.2	0	4.77	.40
04	41.90	0	.04	.76
10	43.55	0	.04	.79
20	52.00	0	.06	1.09
30	0	0	0	0
40	$\lambda = 2.650 \text{ cm}$			

k = 2

lj	Σ_e	Σ_i	Σ_f	Σ_a
00	39.76	1.21	.04	.62
01	13.63	0	0	.03
02	31.46	0.79	.03	.42
03	18.3	0.90	3.26	.24
04	31.98	0.80	.03	.43
10	32.38	0.87	.03	.46
20	39.94	1.23	.04	.63
30	0	0	0	0
40	$\lambda = 2.232 \text{ cm}$			

λ = 2.232 cm

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TABLE 3. (continued)

k = 3

lj	Σ_e	Σ_i	Σ_f	Σ_a
00	23.59	4.55	.04	.40
01	10.80	0	0	.02
02	19.23	3.56	.02	.27
03	11.91	3.51	2.97	.12
04	19.55	3.59	.02	.28
10	19.34	3.90	.03	.30
20	22.57	5.48	.04	.40
30	0	0	0	0
40	$\lambda = 4.952 \text{ cm}$			

k = 4

lj	Σ_e	Σ_i	Σ_f	Σ_a
00	11.71	9.40	1.83	0.15
01	8.77	2.41	0	.02
02	10.42	7.08	1.20	.10
03	7.45	5.6	2.95	.09
04	10.45	7.18	1.22	.10
10	10.83	7.62	1.32	.11
20	11.28	9.91	1.86	.15
30	0	0	0	0
40	$\lambda = 7.150 \text{ cm}$			

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Neutron Population in CP-4 I.

ij	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$
00	0	11	17	5	1
01	0	28	33	14	3
02	0	74	136	90	45
03	0	237	457	458	371
04	0	142	173	107	64
10	0	1496	1823	819	353
20	2	1153	924	248	75
30	22	176	74	18	7

Neutron Population in CP-4 II.

ij	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$
00	0	4	1	0	0
01	0	1	10	0	0
02	0	24	15	16	7
03	0	49	76	106	69
04	0	29	42	15	5
10	0	469	486	181	85
20	2	422	273	42	15
30	15	84	30	8	1

TABLE 6.

Neutron Populations

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their average values. Isotropic scattering was assumed and represented by the choice of 8 octant diagonal directions.

A neutron reflected back into the blanket was considered absorbed in a uranium resonance if its energy lay between 5.8 and 9.1 e.v.

5. Description of Problems. The principal problems treated are listed below.

CP-4 I. This was a run on CP-4 with design dimensions as given in Table 5. The cross sections were

R_1	= 8.55 cm	h_1	= 18.10 cm
R_2	= 20.46	h_2	= 26.67
R_3	= 38.57	h_3	= 38.74
R_4	= 46.196	h_4	= 57.79
R_5	= 100.00	h_5	= 78.11

TABLE 5.

Design Dimensions for CP-4

The dimensions are the boundaries of the zones as shown in Fig. 1

those listed in Table 3. The effect of removing the plug (i.e., replacing zone 00 by a vacuum) was observed. The reactor proved to be supercritical with these cross sections and dimensions.

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CP-4 II. For this run (because of the supercriticality of the first run) the core radius $R_1 = 8.55$ cm was reduced to 7.55 cm, giving a closer approximation to criticality.

CP-4 III. For comparison with analytical calculations and to investigate the effect of geometry a run was made on CP-4 in spherical form.

Clementine I. This was a run on the plutonium fast reactor with a set of cross sections supplied by Los Alamos. The reactor under these conditions proved highly supercritical.

Clementine II. A second run on Clementine was made with modified cross sections, resulting in a still higher degree of supercriticality.

6. Present Status of Results. In none of the problems has a sufficiently careful study of the neutron data yet been made. However, it seems desirable to record here some basic results on flux distributions and calculations based on them.

Flux Distributions in CP-4 I and II. Statistical fluctuations make it impossible to observe too detailed a distribution of the neutrons in position and energy. It is accordingly necessary to compromise in the choice of intervals, and this may be done in various ways according to character of the information to be obtained from the distributions. The graphs in Figs. 10-12 thus represent only some of the many ways in which the card data may be interpreted. The procedure has always been to combine neutron data for a large number of census intervals on the assumption that they are statistically independent.

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Fluxes have been obtained directly by counting neutron populations on a distance census. The ordinates give the actual numbers of neutron observations.

Fig. 10 gives radial distributions $R(r)$ integrated in height for the supercritical CP-4 and the nearly critical CP-4 II.

Fig. 11 gives vertical distributions integrated over the indicated radial intervals. The notation $Z_i(z)$ means the flux at a distance z from the base of the reactor for the i^{th} column in Fig. 1.

Fig. 12 gives the energy distributions in the various zones, the ordinates being actual numbers of neutron observations.

Table 6 gives a listing of the neutron observations by zones. It should be recalled that the numbers N_{ijk} for the ij zone and the k^{th} energy region are proportional to flux and not to neutron density.

Flux Distributions in Clementine. Both runs on Clementine were supercritical and only the first, which was less supercritical, has so far been analyzed. The spatial distribution of the flux is given in Figs. 13 and 14 and the energy distribution by zones in Figs. 15 and 16.

Some experimental information is available on the energy distribution in Clementine, consisting in counts of the total number of fissions produced in different positions in the reactor.⁽¹⁾ The sharpest comparison between these experiments and the present calculations is obtained

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(1) D. and J. Hall LAMS-727 and report at the Oak Ridge Symposium on shielding.

by computing the ratios of the average fission cross sections for these materials. This has been

Distance from center in cm	25/28		37/28		25/37	
	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
0	6.07	22	4.58	5.34	1.32	4.11
10	28	23.6	9.65	6.04	2.90	3.90
23	123	34.5	15.7	5.18	7.83	6.66

TABLE 7.

Observed and Computed Ratios of Average Fission Cross Sections.
in 25, 28, and 37 Fission Chambers

done from the neutron populations obtained and energy dependent cross sections used. The results are shown in Table 7. The 25/28 ratio is, of course, quite sensitive to the ratio of slow to fast neutrons, and the lack of agreement seems to indicate that the calculation gives too many slow neutrons in the core and too many fast neutrons far out in the reflector. This discrepancy, like the calculated supercriticality, may well be due to errors in the choice of cross sections, most probably in the elastic and inelastic cross section for U and Pu.

Multiplication Rates. The present method only gives directly the multiplication constant in distance, β , defined by the equation

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$$N = N_0 e^{\beta D},$$

where N is the total neutron population at census distance D . A logarithmic plot of $N(D)$ is given in Fig. 17 for the three principal runs. The estimated value of β for CP-4 I is 1.3 reciprocal meters, which corresponds to a time constant α of about 10^7 sec^{-1} . The interpretation of these results for the purpose of making adjustments to critical conditions is being given further study.

Breeding Ratio. Calculations of the breeding ratio (defined here as the number of captures in 28 outside of the core per 25 atom consumed in the core) have been made for CP-4 by summing the cross sections used over the neutron populations N_{ijk} . For the approximately critical CP-4 II the calculated value is 1.30. The supercritical CP-4 I gave directly the much lower value .86, to which an approximate correction to critical conditions can be applied. The result of such a correction (addition of an appropriate amount of absorption uniformly over the whole reactor) is the value 1.46 which is not in good agreement with the value 1.30 for the nearly critical case, the discrepancy probably being due to inaccuracy of the method of approximation.

The above values are not to be regarded as predictions of the actual breeding ratio, as in addition to uncertainties inherent in the calculational procedure, they are based on uncertain cross sections, particularly the low value (.03) of capture to fission cross section in 25 at high energies.

The Adjoint Flux. For use in perturbation calculations it is important to have the adjoint as well as the actual flux. An attempt

has been made to obtain this adjoint for CP-4 from the neutron data by a procedure that amounts to having each neutron back to its point of origin. The number originating at a given point is then the adjoint at that point. Because of statistical fluctuations it proved impossible to obtain the adjoints in more detail than as averages over the regions ij . The results are shown in Table 8 for the sake of the record, although they are highly inaccurate because of fluctuations.

Region ij	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$
00	0	0	0	0	0
01	0	0	0	0	.133
02	0	1.767	1.200	.024	1.856
03	0	2.309	3.952	1.919	2.234
04	0	.025	.511	1.833	0
10	0	.950	.608	1.516	.938
20	0	.246	.223	0	0
30	0	.012	.0	0	0

TABLE 8.

Relative Values of Adjoint Fluxes by Regions in CP-4

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RADIAL DISTRIBUTIONS IN CP-4 I & II FIG. 10

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VERTICAL FLUX DISTRIBUTIONS IN CP-4

FIG. 11

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ENERGY DISTRIBUTION IN GP-4 I

FIG. 12

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FIG. 13

VERTICAL NEUTRON DISTRIBUTIONS IN CLEMENTINE I

FIG. 14

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FIG. 15
ENERGY DISTRIBUTIONS IN CLEMENTINE

ENERGY DISTRIBUTION IN CLEMENTINE

FIG. 16

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GROWTH OF NEUTRON POPULATION WITH CENSUS DISTANCE

CLEMENTINE I

CALCULATIONS FOR A 25 - D₂O REACTOR I.

Written by F. C. Hoyt

Work done by M. Felix, D. A. Flanders, F. C. Hoyt, H. Hummel

1. Introduction. The calculations reported here were carried out for use in connection with the design of a proposed research reactor⁽¹⁾ for the Argonne National Laboratory. The actual reactor consists of an active lattice of 16 fuel elements occupying a volume of about 270 liters in the center of an aluminum tank containing 4,000 liter of D₂O, the whole surrounded by 60 cm of graphite. The present preliminary calculations are based on an approximation by spherical geometry. Standard two-group methods have been used throughout with a first rough approximation to the lattice effect in the core. ^{ANL-4221} _{is not}

2. Dimensions and Two-Group Constants. The following dimensions, corresponding closely to preservation of volumes in spherical geometry, have been kept fixed in the present calculations:

Radius of core	40 cm
Outer radius of D ₂ O region	100 cm
Thickness of Al tank wall	1.27 cm
Outer radius of graphite reflector	160 cm

The amount of structural aluminum in the core has been kept constant at 25 kg, and its absorption cross section has been taken as .21 b.

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(1) For an account of the tentative design see S. Untermyer ANL-4221

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The two-group constants were based on the following values:

Transport mean free path in D_2O (thermal)	2.5 cm ⁽²⁾
Diffusion length in D_2O	123 cm ⁽²⁾
Absorption in D_2O	8.3×10^{-4} (2) barns
Age to thermal in D_2O	127 cm ²

As a roughly estimated allowance for fast fission, 3% was subtracted from the term in the two-group equations which corresponds to loss of fast neutrons to produce slow ones.

The lattice effect was approximated by the usual cell procedure assuming uniform production of thermal neutrons. This cannot be expected to give reliable results for the present lattice, and a more accurate calculation is underway. Using a one-group procedure with infinite cylinders gives .85 for the ratio of the average thermal flux inside a fuel element to the total average thermal flux in a cell.

3. Critical Mass and Flux Distributions. The slow and fast flux distributions shown in Fig. 1 have been computed with an allowance of .19 mb per cm³ of absorption uniformly distributed in the core for poison and a reduction of 9% in k as an allowance for experiments.

An effective absorption cross section for slow neutrons in the core has been introduced and taken equal to 85% of the actual cross section, corresponding to the above rough estimate of the lattice effect.

The fluxes are normalized to one megawatt power.

The effect of the $\frac{1}{2}$ " aluminum tank has been estimated as a discontinuity in the diffusion current at the tank wall of magnitude $(t/\lambda_a)\phi_s$, where t is the thickness of the tank wall, λ_a the absorption mean free path in Al and ϕ_s the slow neutron flux.

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(2) Data of Wattenberg et al for D_2O of isotopic purity 99.84% CP-3406

The dotted curve in Fig. 1 shows the effect on the slow flux distribution.

The critical mass with the tank wall estimate included is 649 gr of 25, the contribution of the tank wall as here calculated amounting to 3.7%. At criticality $k = 1.706$ and the concentration of 25 is 2.42 gr per liter.

4. Adjoint Fluxes. Fig. 2 shows the actual (ϕ_s, ϕ_f) and adjoint ($\tilde{\phi}_s, \tilde{\phi}_f$) fluxes, normalized so that

$$\int_0^{160} (\tilde{\phi}_f \phi_f + \tilde{\phi}_s \phi_s) r^2 dr = 10^3$$

5. Prompt Period. A perturbation calculation gives

$$\alpha = 853 (k - k_{crit}) \text{ sec}^{-1},$$

where α is the reciprocal period. The reactor will thus be decidedly slower than CP-3.

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FIG. 2