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SACRAMENTO PLANT

REPORT NO. RN-DR-0128

TURBINE POWER CONTROL VALVE ACTUATOR
PERFORMANCE EVALUATION
VIBRATION TEST

NERVA PROGRAM

CONTRACT SNP-1

DECEMBER 1967

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NUCLEAR PRODUCTS AND SERVICES GROUP

AEROJET-GENERAL CORPORATION

SACRAMENTO, CALIFORNIA

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G. G. Henderson

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TURBINE POWER CONTROL VALVE ACTUATOR
PERFORMANCE EVALUATION
VIBRATION TEST

H. O. Suttkus

C. M. Rice
Program Manager
Nuclear Rocket Operations

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I. INTRODUCTION

This report describes the vibration test of the Bendix Model NT-D1 Turbine Power Control Valve Actuator (TPCVA), Serial No. 17. The vibration test is part of the overall qualification test and was performed in accordance with the requirements of RN-S-0425, Qualification Test Plan, Bendix Model NT-D1 Turbine Power Control Valve Actuator.

II. SUMMARY

Prior to performing the vibration test described in this report, a vibration test was started but not completed due to faulty instrumentation and consequent minor damage to the actuator. After the test equipment and the actuator were repaired, the entire test was conducted through completion using both sinusoidal and random vibrations.

Performance tests which were conducted before and after each vibration test indicated no appreciable changes in performance.

No deterioration in performance was noted during the vibration testing (either sinusoidal or random) when conducted in accordance with the applicable specification. In the Y axis only, performance deterioration was noted during dwell at a 573 cps resonant point. However, the maximum input vibration frequency for the Y axis, specified in NDC-51A, Actuator, Electro-Pneumatic, Rotary, for Turbopump Power Control Valve, is 500 cps at 1 g.

III. TECHNICAL DISCUSSION

A. OBJECTIVE

The purposes of the sinusoidal vibration test were: (1) to obtain data which would show the frequencies at which the actuator had resonant peaks; (2) to determine if continued excitation at the resonant

frequencies would cause damage to, or permanently degrade the performance of the actuator; and (3) to determine the nature and magnitude of any performance degradation which might occur during vibration at the resonant frequencies.

The random vibration test was made to determine (1) if random-input frequencies were of sufficient magnitudes to cause the development of high accelerations at the resonant frequencies determined during the sinusoidal vibration test noted above; and (2) the nature and magnitude of any performance degradation which might occur during the test.

B. TEST SETUP

The electrical power and control for the actuator is supplied from a console which is shown schematically in Figure 1 and pictorially in Figure 2. The actuator was installed in a special vibration test fixture for adaptation to the vibration table as shown in Figure 3.

The vibration table and its associated instrumentation and control equipment is noted in Table 1.

C. PROCEDURE

The Bendix TPCVA was examined and then mounted on its test fixture, using prescribed torque values and procedures. A supply of helium gas was provided and the actuator was connected to the operating console (see Figure 4). A performance reference test was conducted both prior to and following the vibration tests. The performance reference test consisted of (1) threshold test, (2) frequency response test, and (3) step response test. Detailed procedures for the 3 noted tests which comprise the performance reference test may be found in Report No. RN-DR-0084, entitled Electro-Pneumatic Servo Valve Actuator Evaluation Procedure, March 1967

Figure 2 - Test Console and Recorder

Figure 3 - Actuator Installed on Test Fixture

TABLE 1

TEST EQUIPMENT LIST

<u>Item</u>	<u>Manufacturer</u>	<u>Part No./ Model No.</u>	<u>Serial No.</u>	<u>Calibration Plates</u>
Shaker	Ling	B 290	108	4/28/66 10/15/66
Vibration Console	Ling	SCO-3	Ling #1	3/30/66 9/18/66
Oscilloscope	Tecktronix	502A	22889	5/20/66 8/25/66
Oscillograph	Consolidated Electrodynamics	1588-51	7286	7/10/66 8/28/66
Tape Recorder	Ampex	FR1300	505	2/25/66 8/28/66
Differential Voltmeter	John Fluke	825A	835	6/5/66 9/1/66
Function Generator	Hewlett Packard	202A	325-13248	3/29/66 9/6/66
Brush Recorder	Brush	MARK 20	634	5/10/66 8/15/66
Preamp	Brush	13-4485-10773	115	5/20/66 9/25/66
15 Volt Power Supply	Power Design	TW4005	T409062	3/15/66 9/8/66
10 Volt Power Supply	Power Design	2015R	V406021	6/15/66 11/7/66
Frequency Response Analyzer	Weston, Boonshaft & Fuchs	DA410A	6001	7/23/66 10/20/66
Servo Amplifier	Burr-Brown	Dwg. No. 2775842 (Modified)		
Test Console	AGC	Dwg No. 1118758		
Torque Wrench	Snap On Tools Corp.	50-14	4900457	6/15/66 9/25/66

Figure 4 - Helium Supply Line to Actuator

The accelerometers were calibrated and ranged on the recorder. The actuator was installed on the vibration stand as shown in Figure 5. The accelerometers were placed on the actuator as shown in Figure 6. The servo-valve spool central axis and the output shaft central axis were chosen as the basis of the coordinate system for the definition of the three mutually perpendicular vibration input axes. The cross-spool axis was designated as the X axis, the spool central as the Y axis, and the output-shaft central axis as the Z axis (see Figure 6). Electrical and pneumatic power was applied to the actuator, and the sweep portion of the vibration test in the X axis was conducted using the following values: From 5 to 17 cps the double amplitude (D.A.) was maintained at 0.135 inch, and between 17 and 2000 cps, the 2 g input was kept constant. During the entire frequency sweep from 5 to 2000 to 5 cps, the actuator command signal input consisted of a triangular wave function, at 0.1 cps and approximately 60° peak-to-peak amplitude, while resonant peaks, as indicated by the various accelerometers, were noted. At the end of the sweep phase, the frequencies producing major (high-amplitude) resonances were determined by examination of the data. The vibration table was then operated at the major resonance point, with an input sufficient to impart a 1 g acceleration to the actuator. The actuator was subjected to this input acceleration for 10 minutes, during which time the performance of the actuator was checked by running a performance reference test as described above.

Following the X-axis vibration test the entire procedure noted above was repeated using the Y axis of the actuator as the input axis.

Following the Y-axis vibration, a similar procedure was followed for the Z axis of the actuator except that the vibration was from 16 to 2000 to 16 cps at the following values: From 16 to 45 cps at an amplitude sufficient to produce 1.33 g acceleration; from 45 to 87 cps, at a constant amplitude of 0.013 inch (D.A.); and from 87 to 2000 cps at sufficient amplitude to produce 5 g input acceleration. After the resonant peaks had been established, the 10-minute dwell test was evenly divided between the two most prominent resonant points, using a 5 g input acceleration.

Figure 5 - Actuator Mounted on Vibration Table in X Axis Direction

Figure 6 - Actuator Mounted on Vibration Table in Z Axis Direction

D. RESULTS

Midway through the Z-axis dwell of the first vibration test, the performance of the actuator deteriorated and a fractured gear-motor pressurization-line bellows was discovered (see Figures 7 and 8), at which point the test was discontinued pending investigation. The ensuing investigation revealed that the accelerometer controlling the feedback to the vibration-table amplifier had been damaged during installation and its output was thus materially reduced. This output error caused the input excitation to the actuator to be correspondingly higher than specified (by an estimated factor of 4). The higher than normal input excitation due to the damaged accelerometer and misalignment of the brazed joint between the tubular elbow and bellows, which was introduced during its manufacture, is believed to have contributed to the failure. It should be noted that the bellows on the other gear-motor pressurization line was subjected to the same vibration and had not failed and did not fail in the testing that followed the repair.

Vibration at the major resonant points in the X and Z axis produced no appreciable effect on actuator performance, as evidenced by the performance reference tests conducted prior to and during the vibration (see Figures 9 through 12 for Bode Plots and Figure 13 for resonance data).

In the Y axis, during vibration at the 573 cps major resonant point actuator performance was deleteriously affected causing noise on the torque motor current and position feedback voltage. Thus, preventing detailed data interpretation for this dwell condition.

The random vibration test was conducted using the same test configuration as in the Z-axis sinusoidal vibration test. The actuator was subjected to a random frequency excitation with a power-spectral density

of 0.06 g^2 per cps, ± 3 decibels. The spectrum was from 100 through 2000 cps. The data collected during the random-vibration input in the Z axis indicated no resonant peaks or operational problems. (See Figures 14 through 16 for Bode plots).

E. CONCLUSIONS

The TPCVA suffers no permanent ill effects resulting from subjecting the actuator to either the sinusoidal or random vibration testing in conformance with parameters specified by NDC-51A.

The TPCVA exhibits no appreciable performance degradation during the noted vibration testing.

In the Y axis only, performance deterioration was noted at a 573 cps major resonant; however, this is above the specified vibration spectrum maximum of 500 cps. In addition, an analysis of the TPCVA vibration environment during engine operation indicates that input frequencies as well as amplitudes from the XE turbine inlet line will be well below the maximum specified frequencies and amplitudes. A complete report of the noted analysis is given in 7730:002R entitled "Environmental Dynamic Analysis of the Turbine Power Control Valve and the Turbine Inlet Line". A facsimile of this report is included herein as Appendix I.

With regard to the aborted test, even though the vibration was considerably in excess of the specified values, it served to define the misaligned bellows as a weak point of the actuator. Therefore, action was taken by way of appropriate drawing changes to prevent future reoccurrence of the misalignment.

Figure 7 - Actuator Mounted on Vibration Table Showing
Fractured Bellows (Mounted in X Axis)

Figure 8 - Close-Up Showing Fractured Bellows

Figure 9 - Bode Plot; Pre-Vibration Performance Reference Test

Figure 10 - Bode Plot; During X Axis Input Vibration Test

Figure 11 - Bode Plot; During Z Axis Input Vibration Test
at 658 cps

Figure 12 - Bode Plot; During Z Axis Input Vibration Test
at 1505 cps

Figure 13 - TPCV Actuator S/N 17 Vibration Test Data Gain vs Frequency

Figure 14 - Bode Plot; Performance Reference Test, Before
Random Vibration Test

Figure 15 - Bode Plot; Performance Reference Test, During Random Vibration Test

Figure 16 - Bode Plot; Performance Reference Test, After Random Vibration Test

APPENDIX I

ENVIRONMENTAL DYNAMIC ANALYSIS OF THE TURBINE POWER
CONTROL VALVE AND THE TURBINE INLET LINE

(P. Soo)

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I. SUMMARY

This report summarizes the preliminary study on the environmental structural dynamic analysis of the X-E Turbine Power Control Valve (TPCV) and the turbine inlet line. The critical speed of the turbopump, the natural frequencies of bell-mode vibration of the nozzle, the resonant frequencies of the TPCV torque motor component, the natural frequencies of the turbine inlet line, and the estimated dynamic response of the TPCV due to external excitations from both the turbopump and nozzle are determined.

II. INTRODUCTION

The X-E turbine control valve is located immediately below the upper elbow of the turbine inlet line as shown in Figure 1. The function of this unit is to control the flow rate of hot gas to the turbine. The valve is opened or closed to a certain extent depending upon what the "error sign" input feeds into the torque motor. The electric-pneumatic actuator functions the transmission shaft which in turn operates the "butterfly" valve.

The performance of the TPCV servo control unit can be affected seriously by undesirable vibration inputs, resulting from the valve supporting structure or the adjacent structures. Since the TPCV is closely related to the structures such as the turbopump, the turbine inlet line, the diluent line as well as the X-E nozzle, an attempt has been made to study the basic vibration characteristics of the TPCV in this environment. Of specific interest are the resonant frequencies of each structural component and the dynamic response of the valve. A summary of the analytical results is given in the next section.

Figure 1 - Propellant Feed System Components X-Engine

III. SUMMARY OF RESULTS

A. NATURAL FREQUENCIES OF THE X-E TURBINE INLET LINE, SYMMETRICAL MODE

<u>Mode</u>	<u>Frequency (cps)</u>
1st	10.1
2nd	29.9
3rd	38.4
4th	49.0
5th	61.0

B. NATURAL FREQUENCIES OF THE FLAPPER ARM TPCV TORQUE MOTOR COMPONENTS

<u>Mode</u>	<u>Frequency (cps)</u>
1	6,700
2	40,190
3	81,393
4	132,440
5	253,000

C. NATURAL FREQUENCIES OF THE ARMATURE CYLINDER, TPCV FLAPPER ARM

<u>Mode</u>	<u>Frequency (cps)</u>
1st	7
2nd	10
3rd	13
4th	24
5th	36

D. NATURAL FREQUENCIES OF THE X-E NOZZLE, BELL-MODE TYPE

<u>Mode</u>	<u>Anal Frequency (cps)</u>
1st	800
2nd	650
3rd	300-240 Test
4th	570
5th	910

E. UNCOUPLED CRITICAL SPEEDS, X-E TURBOPUMP

<u>Mode</u>	<u>Critical Speed (RPM)</u>
1st	38,000
2nd	48,000
3rd	146,000

F. NATURAL FREQUENCIES OF THE X-E TURBOPUMP
SUPPORTING STRUCTURE

<u>Mode</u>	<u>Natural Frequencies (cps)</u>
1st	111
2nd	180
3rd	315

G. NATURAL FREQUENCIES OF THE X-E TURBOPUMP TURBINE DISK AT
22,000 RPM AND NORMAL TEMPERATURE GRADIENT - 1ST STAGE ROTOR

<u>Mode**</u>	<u>Natural Disk Frequencies (cps)</u>	<u>Critical Shaft Speed RPM</u>
N=0, S=0	911	54,660
N=1, S=0	624	37,440
N=2, S=0	936	28,080
N=3, S=0	1,328	26,520
N=4, S=0	1,828	27,420

H. COUPLED CRITICAL SPEEDS, X-E TURBOPUMP

<u>Mode</u>	<u>Coupled Critical Speeds</u>
1st	6,000
2nd	9,500
3rd	16,500

I. NATURAL FREQUENCIES OF THE ENTIRE X-E ENGINE

<u>Mode</u>	<u>Natural Frequencies (cps)</u>
1st	6
2nd	34
3rd	115
4th	246
5th	450

IV. METHOD OF STRUCTURAL, ENVIRONMENTAL DYNAMIC ANALYSIS

A. CRITICAL SPEED OF THE X-E TURBOPUMP

The turbopump, like all rotational machinery, will vibrate violently at speeds known as the critical speeds. This phenomenon occurs when the pump angular speed coincides with one of the lateral natural frequencies of the rotating shaft together with its rotating elements. Attempt has been made to classify the critical speed analysis into two categories: (1) uncoupled critical speed of the turbopump, and (2) coupled critical speed of the turbopump.

The uncoupled critical speeds refer to those not associated with any pump supporting structure, whereas the coupled critical speeds refer to those associated with pump supporting structure.

The uncoupled critical speed analysis was based on the partial differential equations of motion of a laterally vibrated shaft. This equation includes the internal shear, rotary inertia, and gyroscopic effects. The general solutions of this equation for deflection, slope, moment, and shear at one end of a typical beam segment can be written in terms of those at the other end in matrix form. The static and dynamic properties of the beam can be completely represented by a number of these transfer matrices, which when fitted together provide the static and dynamic properties of the entire system. Setting the final matrix equation to zero yields the natural frequencies of the system.

The coupled critical speed analysis for the NERVA turbopump was based on the equations of motion for the structural interaction of the pump and its supporting structure. In the case of the X-E pump, the structural rigidity of the six rib-panels was determined and the coupling critical speeds were executed by computer program 14071.

B. TURBINE INLET LINE RESONANT FREQUENCIES

The turbine inlet line of the X-E engine is a three dimensional space frame. As a result the frequency analysis for this system is relatively complicated compared with a continuous beam of multi-elastic supports.

The initial step for analyzing the X-E turbine inlet line is to reduce this system from three dimensional space structure into two dimensional plane structure. Notice that there is a lower elbow near the nozzle section and an upper elbow is located near the turbopump section. For symmetrical modes this plane structure is equivalent to a one dimensional continuous beam with additional elastic supports near the elbow.

C. TURBINE DISK VIBRATION ANALYSIS

In addition to the lateral vibration of the shaft of the turbopump, turbine disk vibration is another problem for the NERVA turbopump. When one of the natural frequencies of the turbine disk coincides with the angular speed of its shaft, resonant condition occurs. This resonant phenomenon of a disk has been studied for some time, and a generalized analytical model for computing turbine disk frequencies has been written in a digital computer program 10062. Raleigh-Ritz method was utilized for the analysis.

D. NOZZLE VIBRATION

Since the lower end of the turbine inlet line is attached to the conical nozzle of the X-E engine, some forcing vibrational input to the TIL is expected and it is desirable to study the resonant frequencies of the conical nozzle.

A considerable amount of time has been devoted to investigating the resonant frequencies of the nozzle. Method of nozzle vibration analysis is given in Paragraph VI, E.

V. DISCUSSION

Two types of vibration have been considered for the Turbine Power Control Valve (TPCV). The first is due to the resonance of the valve itself, and the second is due to its structural environment, i.e., vibration arising from external sources other than the valve itself. As an example of environmental vibration, if the turbopump were operated near or at one of its critical speeds, the turbine inlet line could then be expected to vibrate which could induce vibration into the TPCV, since it is mounted in the line. Also, vibration at the nozzle end of the turbine inlet line would be transmitted to the valve.

In either case, the natural frequencies of a subsystem are closely related to the external forcing function. As a result, an attempt has been made to look into the natural frequencies of the nozzle, the inlet line and the critical speeds of the turbopump, as well as the resonant frequencies of the various structural components within the TPCV.

The preliminary investigation shows it is unlikely that the turbine inlet line will encounter a resonant condition, because its first natural frequencies are under 65 Hz, whereas the operational speed of the turbopump is above 20,000 rpm or 330 Hz (the uncoupled critical speed of the turbopump is 32,000 rpm or 630 Hz). The first coupled critical speed of the X-E turbopump with its supporting structure has also been studied. In this case, the first critical coupled speed is expected to be near 6,000 rpm or 100 Hz, which is still far above the first five turbine-inlet-line resonant frequencies. Therefore no danger of resonant vibration is expected from the turbopump. The natural frequencies of the X-E nozzle are also high (varies from the 240 cps to 910 cps for the first five bell modes), comparing with 3 g at 240 cps as the nozzle forcing function. Therefore, no extremely severe vibration is expected from the nozzle which would transmit to the TPCV. Furthermore, there

are flexible bellows on the turbine inlet which will all absorb a relatively large amount of vibration during operation. As a result, the TPCV is somewhat protected to a certain extent from environmental vibration standpoint.

The natural frequencies of the TPCV armature cylinder is extremely low (under 40 Hz for the first five bending modes). The flapper arm is very rigid (first bending mode frequency is above 6500 cps). These resonant frequencies of the flapper are all remote from the forcing functions of the TPA or the nozzle.

Vibration shake tests on the TPCV have been performed in the Aerojet Vibration Test Lab. Test results indicate that the TPCV has a relatively large dynamic response at 545 Hz and 575 Hz. The TPCV transmission shaft's lateral vibration could have been responsible for the resonant condition. If the shaft is non-uniform in cross section and hence in area moment of inertia with respect to two principal mutual axes, two lateral resonant frequencies will be expected, although the possible source of vibration of the oscillating spool in the TPCV unit is not yet discarded.

VI. CALCULATIONS

A. UNCOUPLED CRITICAL SPEED AND SPACE FRAME ANALYSIS

Equation of Motion

$$EI \frac{\partial^3 Y_b}{\partial X^2} + \mu AG \frac{\partial Y_s}{\partial X} - \rho I \frac{\partial^3 Y}{\partial X \partial t^2} = 0$$

$$M \frac{\partial^2 Y_b}{\partial t^2} + M \frac{\partial^2 Y_s}{\partial t^2} - \mu AG \frac{\partial^2 Y_s}{\partial X^2} = 0$$

Transfer Matrix

$$\begin{aligned}
 W & \quad C_0 - \sigma C_2 \quad \ell [C_1 - (\sigma + \tau) C_3] \quad a C_2 \quad \frac{a \ell}{\beta^4} \quad [\beta_{14}] \\
 O & \quad C_3 \beta^4 / \ell \quad C_0 - \tau C_2 \quad \frac{a}{\ell} (C_1 - \tau C_3) \quad a C_2 \\
 = & \\
 M & \quad C_2 \beta^4 / a \quad \frac{\ell}{a} [-\tau C_1 + (\beta^4 + \tau^2) C_3] \quad C_0 - \tau C_2 \quad \ell [\beta_{34}] \\
 V & \quad \beta^4 (C_\ell - \sigma C_3) / a \ell \quad \beta^4 C_2 / a \quad \beta^4 C_3 / \ell \quad C_0 - \sigma C_2
 \end{aligned}$$

B. BASIC EQUATION FOR ANALYZING FORCED VIBRATION OF CONTINUOUS BEAM

Equation of Motion

$$\frac{\partial^2}{\partial X^2} \left(EI \frac{\partial^2 Y}{\partial X^2} \right) + M \frac{\partial^2 Y}{\partial t^2} = \sum \int_{x_0} F_i m \bar{X}_n (X) \bar{X}_n (X_i)$$

LaGrange Equation

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

C. BASIC EQUATION FOR ANALYZING COUPLED CRITICAL SPEEDS OF THE TURBOPUMP

$$m_1 x_1 = -k_1 (x - \xi - e \cos wt)$$

$$m_1 y_1 = -k_1 (y - \zeta - e \cos wt)$$

$$m_2 \xi = -k_2 \xi + k_1 x = -e k \cos wt$$

$$m_2 \ddot{\zeta} = -k_2 \zeta + k_1 y = -ek \cos \omega t$$

$$[(\omega_{CR}^2 - \omega_n^2) \frac{m_2}{m_1} - \omega^2 \omega_{CR}^2] = 0$$

D. BASIC EQUATIONS FOR ANALYZING NON-UNIFORM TURBINE DISK

$$V = \int_0^{2\pi} \int_a^b \frac{D(r)}{2} \left[\nabla^4 - 2(1-\mu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{2} \frac{\partial^2 w}{\partial \theta^2} \right) + \right. \\ \left. + 2(1-\mu) \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right) \right]^2 \right] r dr d\theta$$

$$T = \int_0^{2\pi} \int_a^b \frac{\pi \gamma \Omega}{2} \rho(r) h(r) \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta$$

Displacement

$$w = \sum_{s=1}^k A_s (r-r_o)^{s+1} \sin n\theta \cos \omega t$$

Rotation Stresses

$$\sigma_n = \frac{\phi}{h(r) r}, \quad \sigma_\theta = \frac{d\phi}{h(r) dr} + \rho \omega^2 r^2$$

$$\phi = m r^{n+3} + C_1 r^\alpha + C_2 r^\beta$$

$$m = (3+\mu) \rho \omega^2 c / (\mu n + 3n + 3)$$

$$X^2 = n x + \mu n - 1 = 0 \quad \text{for } \alpha, \beta$$

Thermal Stress

$$\frac{\partial^2 w}{\partial r^2} + (1+n) \frac{1}{r} \frac{\partial u}{\partial r} - (1-n\mu) \frac{u}{r^2} = (1+\mu) \alpha \cdot$$

$$\cdot \left[\frac{nT(r)}{r} + \frac{\partial T(r)}{\partial r} \right]$$

E. BASIC EQUATION FOR ANALYZING FREE VIBRATION OF CONICAL SHELL OF REVOLUTION - NOZZLE.

Strain-displacement Equations

$$\epsilon_1 = \frac{1}{A_1} \frac{\partial u}{\partial \alpha_1} + \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} v + \frac{w}{R_1}$$

$$\epsilon_2 = \frac{1}{A_2} \frac{\partial v}{\partial \alpha_2} + \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} u + \frac{w}{R_2}$$

$$\omega = \frac{A_2}{A_1} \frac{\partial}{\partial \alpha_1} \left(\frac{v}{A_2} \right) + \frac{A_1}{A_2} \frac{\partial}{\partial \alpha_2} \left(\frac{u}{A_1} \right)$$

Curvature Equation

$$K_1 = -\frac{1}{A_1} \frac{\partial}{\partial \alpha_1} \left(\frac{1}{A_1} \frac{\partial w}{\partial \alpha_1} + \frac{u}{R_1} \right) - \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} \left(\frac{1}{A_2} \frac{\partial w}{\partial \alpha_2} - \frac{v}{R_2} \right)$$

$$K_2 = -\frac{1}{A_2} \frac{\partial}{\partial \alpha_2} \left(\frac{1}{A_2} \frac{\partial w}{\partial \alpha_2} + \frac{v}{R_2} \right) - \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} \left(\frac{1}{A_1} \frac{\partial w}{\partial \alpha_1} - \frac{u}{R_1} \right)$$

$$\tau + \left(-\frac{1}{A_1 A_2} \left(\frac{\partial^2 w}{\partial \alpha_1 \partial \alpha_2} - \frac{1}{A_1} \frac{\partial A_1}{\partial \alpha_2} \frac{\partial w}{\partial \alpha_1} - \frac{1}{A_2} \frac{\partial A_2}{\partial \alpha_1} \frac{\partial w}{\partial \alpha_2} \right) + \right. \\ \left. + \frac{1}{R_1} \left(\frac{1}{A_2} \frac{\partial u}{\partial \alpha_2} - \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} u \right) + \frac{1}{R_2} \left(\frac{1}{A_1} \frac{\partial v}{\partial \alpha_1} - \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} v \right) \right)$$

Potential Energy

$$V = \frac{Eh}{\alpha(1-\mu^2)} \int \int [(\epsilon_1 + \epsilon_2)^2 - 2(1-\mu)(\epsilon_1 \epsilon_2 - \frac{\omega^2}{4})] \\ A_1 A_2 d\alpha_1 d\alpha_2 + \frac{Eh^3}{\alpha 4(1-\mu^2)} \int \int [(k_1 + k_2)^2 - 2 \cdot \\ \cdot (1-\mu)(k_1 k_2 - \tau^2)] A_1 A_2 d\alpha_1 d\alpha_2$$

Kinetic Energy

$$T = \frac{h}{\alpha} \int \int (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) A_1 A_2 d\alpha_1 d\alpha_2$$

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