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DYNAMICS OF A PLASMA DISCHARGE IN A MIRROR FIELD

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January 24, 1974

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DYNAMICS OF A PLASMA DISCHARGE IN A MIRROR FIELD*

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ABSTRACT

A cool plasma discharge in mirror geometry is modeled with ions flowing along field lines into electrodes beyond the throats. The velocity of the flow is determined by the electrode sheath properties and nozzle-like phenomena in the mirror and ion source. This model is applicable to the "wet-wood burner" reactor and mirror target schemes.

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I. INTRODUCTION AND SUMMARY

The "wet-wood burner" reactor concept¹ and Baseball target discharge² are two-component systems in which a cool flowing plasma serves as a target for hot confined ions and an energetic neutral beam. The cool component is generated from a flux of neutrals ionized by impact with warm electrons. These ions are heated by further encounters with electrons and hot ions, but their temperature is low $T_i \sim T_e$ because they are not confined by the mirror field. Instead, they pass through the system at approximately the ion sound speed $c_s \approx \sqrt{(kT_e + kT_i)/m_i}$ and there is insufficient time for significant thermal transfer to occur. Electrons are heated by drag on the hot ion component and/or acceleration in the cathode drop if the system is operated as a discharge. The plasma is assumed to be terminated with absorbing end plates which also serve as electrodes. The neutral flux may be introduced through holes in the end plates or radially across field lines. In addition, ions flowing into the end plates knock neutrals back into the plasma. Typical anticipated parameters are

$$\begin{array}{ll} \text{Wet-Wood Burner:} & \begin{cases} n_{\text{hot}} \approx 10^{15} \text{ cm}^{-3} \\ n_{\text{cool}} \approx 10^{16} \text{ cm}^{-3} \\ kT_e \approx 5 \text{ kV} \end{cases} \\ \text{Baseball Target:} & \begin{cases} n_{\text{cool}} \approx 10^{14} \text{ cm}^{-3} \\ kT_e \approx 100 \text{ eV} \end{cases} \end{array}$$

Here we develop a simple model for the gross features of the cool plasma flow. It is shown that the plasma discharge is essentially a constant current device if the electrodes are held at constant temperature, net current being controlled by the gas feed; increased voltage $\Delta\phi$ acts mainly to increase the electron temperature $kT_e \approx \frac{1}{4} e\Delta\phi$. The plasma

flow into the electrodes is always supersonic with the mirror throats acting as virtual electrodes (plasma velocity tends to equal the speed of sound at peak $|B|$). The plasma flow inside the mirror region is supersonic if all the ion source is located outside the throats and subsonic otherwise. We examine a number of simple discharge configurations and calculate the power flow, temperature, and density profile. The fundamental solution for a target arc in a mirror field fed by a hollow anode is sketched in Fig. 1.

In Sec. II we derive the basic equations of the plasma flow. This is done in greater generality than required in order to supply a basis for further numerical study. The end conditions and power balance (at the electrodes) are discussed in Sec. III. Sec. IV is devoted to the study of several simple models of the plasma flow, determination of T_e and ion source strength.

II. BASIC EQUATIONS

We assume the cool ion component behaves as a collisional fluid tied to the field lines with flow velocity large compared with its thermal velocity. The electrons interpenetrate this fluid with equal density to satisfy the quasineutrality condition. The electron mean free path is large compared with the length of the system, so they must be confined by means of an ambipolar potential. This drop appears largely in the electrode sheaths and there is also an electric field associated with density gradients within the plasma body. Since the electron thermal conductivity is effectively infinite, we take the distribution to be isothermal for energies below the escape energy. The electrons

interact with the cool ions mainly through the electron pressure— which couples by means of the ambipolar field. The density of the hot ion component is assumed negligible.

The cool ions (mass m , charge e) are characterized by number density n , flow velocity $\underline{v}$, and temperature T_i . Due to interactions among species there are particle, and energy sources (per unit volume) denoted respectively by S_p , and S_E . The flow satisfies the fundamental conservation laws, which are:

Particles:

$$\frac{\partial n}{\partial t} + \nabla \cdot n \underline{v} = S_p, \quad (1)$$

Momentum

$$\frac{\partial}{\partial t} m n \underline{v} + \nabla \cdot m n \underline{v} \underline{v} = - \nabla n k T_i + e n (-\nabla \phi + \frac{\underline{v} \times \underline{B}}{c}), \quad (2)$$

Energy:

$$\begin{aligned} \frac{\partial}{\partial t} n \left(\frac{m v^2}{2} + \frac{3}{2} k T_i \right) + \nabla \cdot n \underline{v} \left(\frac{m v^2}{2} + \frac{5}{2} k T_i \right) \\ = \nabla \cdot \underline{k} \nabla k T_i - e n \underline{v} \cdot \nabla \phi + S_E. \end{aligned} \quad (3)$$

Here $\underline{k}$ is the tensor of ion thermal conductivity. The potential ϕ is derived from the condition of electron pressure balance:

$$0 = -k T_e \nabla n + e n \nabla \phi. \quad (4)$$

Under our assumptions the velocity $\underline{v}$ must be nearly parallel to $\underline{B}$, otherwise the Lorentz force could not be balanced in Eq. (2). Thus we may set

$$\underline{v} = v \frac{\underline{B}}{B} \quad (5)$$

in Eqs. (1)–(3) provided we consider only the component of the momentum equation parallel to B. It may also be assumed that ion heat conduction is parallel to B. Eqs. (1)–(4) then determine quantities along each field line separately:

$$\frac{\partial n}{\partial t} + B \frac{\partial}{\partial s} \left(\frac{nv}{B} \right) = S_P, \quad (6)$$

$$mn \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} \right) = - \frac{\partial}{\partial s} nkT_i - en \frac{\partial \phi}{\partial s} - S_P mv, \quad (7)$$

$$\begin{aligned} \frac{\partial}{\partial t} n \left(\frac{mv^2}{2} + \frac{3}{2} kT_i \right) + B \frac{\partial}{\partial s} \frac{nv}{B} \left(\frac{mv^2}{2} + \frac{5}{2} kT_i \right) \\ = B \frac{\partial}{\partial s} \frac{K_{ii}}{B} \frac{\partial}{\partial s} kT_i - env \frac{\partial \phi}{\partial s} + S_E, \end{aligned} \quad (8)$$

$$0 = -kT_e \frac{\partial n}{\partial s} + en \frac{\partial \phi}{\partial s}. \quad (9)$$

It is convenient to use Eqs. (6) and (7) to eliminate as many flow terms as possible from Eq. (8). This procedure yields

$$\begin{aligned} \frac{3}{2} n \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial s} \right) kT_i = kT_i \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial s} \right) n \\ + S_P \left(\frac{mv^2}{2} + \frac{5}{2} kT_i \right) + B \frac{\partial}{\partial s} \frac{K_{ii}}{B} \frac{\partial}{\partial s} kT_i + S_E \end{aligned} \quad (10)$$

as a source equation for T_i . Note that in a region where $S_P \neq 0$ there is a source of ion temperature; this is a result of the (implicit) ion-ion collisions required to bring newly created plasma into the mean flow.

If S_p, k_{ii} and S_E vanish, then Eq. (10) may be written

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial s}\right) \left[\frac{(kT_i)^{3/2}}{n} \right] = 0, \quad (11)$$

i.e., entropy per unit mass is conserved following the flow as expected.

The assumption that the ion flow speed is of order $\sqrt{kT_e/m}$ is justified by the end conditions discussed in Sec. III. The main reason the ions are assumed to be collisional is convenience and this may not be always justified on physical grounds. However, the ion temperature plays very little role in this work and it appears that a collisionless treatment gives similar results for T_i . For example, instead of ion temperature generated in a particle source by collisions as described above, the collisionless model gives an equivalent spread of ion velocities due to their creation in the potential gradient which accompanies the source.

In the absence of a hot component the ions are very cool ($T_i \sim .1 T_e$). However, the mechanism of ion-ion drag gives $T_i \approx T_e \approx .1 T_{hot}$ in a two component reactor.

III. END CONDITIONS

At end plates most electrons are reflected by the Debye sheath while ions are simply lost from the plasma. An important role is played by a boundary condition on the ion flow velocity into the plate. On first sight it appears that there is no such boundary condition except that the flow velocity be at least equal to 1/4 the mean ion thermal velocity.⁽⁶⁾ However, it is easily shown⁽³⁾ that for a stable sheath to exist the ion flow velocity must be greater than or equal to the speed of sound:

$$|v|_{end} \geq c_s = \left(\frac{kT_e}{m} + \frac{5}{3} \frac{kT_i}{m} \right)^{1/2}. \quad (12)$$

If the ion flow is slower than C_s then an excess of positive charge appears at the sheath edge, which attracts rather than repels the electrons. The result is a standing charge wave rather than a sheath—which is unsatisfactory since it does not reflect electrons. Although (12) is an inequality it is sufficient to determine a unique solution of the steady flow equations in every case we have studied. Calculations of the structure of sheaths^(4,5) show the existence of a pre-sheath region of scale C_s/v_I (v_I = ionization frequency) extending into the plasma body, in which ions are accelerated by a potential drop of order kT_e . In our model this drop appears in the particle source region ($S_p \neq 0$). We find that flow away from a plasma source satisfies a condition $|v| \leq C_s$. A further acceleration occurs if the plasma flows along a spatially decreasing magnetic field.

The magnitude of the anode drop is determined by the requirement of quasi-neutrality in the plasma; the potential adjusts to a value such that exactly enough electrons leave the system as required to balance the ion flow. This anode drop is generally found to be about $3 kT_e$ nearly independent of all other parameters, and the cathode drop is determined from the net potential applied to the system; if there is no applied potential electrons leave from both ends. The actual sheath structure is a separate problem which does not affect any of our considerations aside from the boundary condition (12). However, we assume that thermionic and field emission from the electrodes is not strong enough to destroy the sheaths or reverse their drops, i.e., the plasma is electron deficient. This is essential for the proper operation of the system since electrons must be thermally isolated to keep T_e high. In a reactor the emerging field lines must be fanned before meeting a wall in order to reduce the power incident on the wall to an acceptable level.

Current and voltage are related by consideration of charge balance as follows. The net drop from anode to cathode is

$$\Delta\phi = \Delta\phi_A - \Delta\phi_C + \Delta\phi_p \quad (13)$$

where, $\Delta\phi_A$ and $\Delta\phi_C$ are (negative) sheath drops and $\Delta\phi_p$ is the drop across the plasma taken between the inside edges of the sheaths—it accompanies the ion flows (see Fig.1). The cathode current density is

$$J_c = J'_c + e n_c |v_c| (1 + \eta_c) \quad (14)$$

with $J'_c \geq 0$ the thermionic current and η_c the coefficient for secondary emission of electrons due to ion impact. If there is no applied voltage an additional contribution is collected thermal electron current:

$$- e n_c \exp\left(\frac{e\Delta\phi_C}{kT_e}\right) \left(\frac{kT_e}{2\pi m_e}\right)^{1/2}$$

The anode current is

$$J_A = - e n_A |v_A| (1 + \eta_A) + e n_A \exp\left(\frac{e\Delta\phi_A}{kT_e}\right) \left(\frac{kT_e}{2\pi m_e}\right)^{1/2} \quad (15)$$

Here we have assumed that all electrons leaving the cathode are thermalized—this depends on an anomalous mechanism such as the excitation of Langmuir waves by streaming electrons. Quasi-neutrality requires

$$J = J_A = J_c \quad (16)$$

Assuming no electrons are gathered at the cathode Eqs.(13) and (14) determine $\Delta\phi_C$ and J , provided the plasma flow has been solved to obtain n , v and $\Delta\phi_p$. Equation (15) yields

$$\frac{e\Delta\phi_A}{kT_e} = \log \left\{ \frac{J + e n_A |v_A| (1 + \eta_A)}{(kT_e / 2\pi m_e)^{1/2} e n_A} \right\} . \quad (17)$$

If we scale v_A with C_s and thermionic current is not too large compared with ion flow, then

$$\frac{e\Delta\phi_A}{kT_e} \approx \log \sqrt{\frac{2\pi m_e}{m}} = - 2.84 . \quad (18)$$

The electron temperature is fixed by power balance of the system as a whole on a flux tube. In steady state

$$(\text{Power in}) = (\text{Power out}) , \quad (19)$$

with

$$(\text{Power in}) = I\Delta\phi + (\text{power into flux tube from neutral beam}). \quad (20)$$

To calculate power out we add up all energy delivered to the end plates as energetic particles. This is

$$\begin{aligned}
 (\text{Power out}) &= \left(\text{ions/sec into anode} \right) \times \left(\text{ion flow energy} + \text{anode drop} + \text{ion thermal energy} + \text{other energy} \right) \\
 &+ \left(\text{cathode ion terms} \right) + \left(\text{transverse electron energy lost at anode} \right) + \left(\text{ion thermal diffusion loss} \right) \\
 &= (n_A |v_A| A_A) \left(\frac{mv_A^2}{2} - e\Delta\phi_A + \frac{5}{2} kT_{iA} + \epsilon \right) \\
 &+ (n_C |v_C| A_C) \left(\frac{mv_C^2}{2} - e\Delta\phi_C + \frac{5}{2} kT_{iC} + \epsilon \right) \\
 &+ n_A A_A \exp\left(\frac{e\Delta\phi_A}{kT_e}\right) \sqrt{\frac{kT_e}{2\pi m_e}} kT_e - A_A \left[K_{||} \frac{\partial T_i}{\partial s} \right]_A + A_C \left[K_{||} \frac{\partial T_i}{\partial s} \right]_C. \quad (21)
 \end{aligned}$$

Here A_A and A_C are the areas of the flux tube at anode and cathode. $\epsilon \approx 32$ eV represents ionization energy and radiation loss per ion. The electrons only carry off kT_e energy per particle at the anode because an escaping electron has just sufficient parallel energy to cross the sheath. (Power out) scales roughly as $T_e^{3/2}$, so T_e rises or falls until the balance (19) is achieved.

The ion temperature satisfies no simple boundary condition except that no ion heat be conducted through the sheaths opposite to the direction of the flow. Thus T_i "floats" at the ends with

$$\left(\frac{\partial T_i}{\partial s} v \right)_{\text{end}} \leq 0. \quad (22)$$

IV. SIMPLE MODELS OF FLOWING PLASMA

The steady flow of cool ions along a field line is described by the laws of conservation of mass and momentum (see Sec. II):

$$B \frac{d}{ds} \frac{nv}{B} = S_p, \quad (23)$$

$$mnv \frac{dv}{ds} = -en \frac{d\phi}{ds} - mvS_p. \quad (24)$$

The ions are created cold and stationary but they are immediately caught up in the mean flow and carried to the end plates. Since T_i is expected to be small, it is neglected in writing Eqs. (22) and (23). Furthermore, the ion energy Eq. (10) is not used since it would only serve to determine T_i . Potential ϕ satisfies

$$0 = en \frac{d\phi}{ds} - kT_e \frac{dn}{ds} , \quad (25)$$

which closes the set (22)–(24) provided $T_e, S_p(s)$ and $B(s)$ are specified. We return to the determination of T_e and S_p later (see cases 5 and 6). First, the general features of the flow are examined in several simple geometries.

Case 1: $S_p = 0$, $B(s)$ arbitrary

Equations (23)–(25) may be integrated immediately to obtain

$$\frac{nv}{B} = \text{const.} , \quad (26)$$

$$\frac{Mv^2}{2} + e\phi = \text{const.}, \quad (27)$$

$$e\phi - kT_e \log n = \text{const.} \quad (28)$$

Eliminating n and ϕ gives

$$\left(\frac{B}{B_0}\right)^2 = \left(\frac{v}{v_0}\right)^2 \exp[-(v/c_s)^2 + (v_0/c_s)^2] \quad (29)$$

with B_0 and v_0 specified at an arbitrary position s_0 . A plot of B vs v is given in Fig. 2 for several values of v_0 . Note that B is a maximum for $v = c_s$ while the normalization factor

$$W = \left(\frac{c_s}{v_0}\right)^2 e^{(v_0/c_s)^2} \quad (30)$$

is a minimum for $v_0 = c_s$. Flow along a decreasing field is either accelerated or decelerated depending on whether it is supersonic or subsonic; the appropriate branch will be determined by the location of the ion sources with respect to the end plates.

Case 2: $B = \text{const.}$, $S_p(s)$ arbitrary

Defining the flux

$$F = nv \quad (31)$$

with

$$S_p = dF/ds, \quad (32)$$

Equation (24) may be written

$$mF \frac{dv}{ds} = -en \frac{d\phi}{ds} - mv \frac{dF}{ds} \quad (33)$$

Substituting from Eq. (25) for ϕ and dividing by m yields

$$F \frac{dv}{ds} = - \frac{kT_e}{m} \frac{dn}{ds} - v \frac{dF}{ds} \quad (34)$$

which may be integrated to give

$$\begin{aligned} Fv + c_s^2 n &= \text{const} \\ &= c_s^2 \tilde{n} \end{aligned} \quad (35)$$

$\tilde{n}$ is the ion density at the stagnation point (which must exist since the flow velocity is directed into both end plates.) Eliminating n with (31) gives a relation between F and v . Although v is unknown, important information is obtained by solving for $F(v)$. This is

$$F(v) = \frac{c_s^2 \tilde{n}}{v + c_s^2/v}, \quad (36)$$

which is plotted in Fig. 3. Note that the maximum value of F occurs for $|v| = c_s$. Since F and v vanish at the stagnation point and $|F|$ is expected to increase following the flow it is seen that $|v|$ does not exceed c_s .

Hence the unique solution of the flow has $\tilde{n}$ adjusted such that $|v_{\text{end}}| = c_s$ to satisfy this requirement and the Bohm boundary condition (12). Denoting cathode and anode quantities by subscripts C and A we have

$$v_C = -v_A = c_s, \quad (37)$$

$$n_C = \frac{F_C}{v_C} = \frac{c_s^2 \tilde{n}}{v_C^2 + c_s^2} = \frac{\tilde{n}}{2}, \quad (38)$$

$$n_A = n_C = \frac{\tilde{n}}{2}. \quad (39)$$

Let

$$g = \int_A^C ds' S_p(s') \quad (40)$$

be the net ion source strength per cm^2 along the flux tube. Then by (37) and (39)

$$F_C = -F_A = \frac{g}{2} \quad (41)$$

and

$$\tilde{n} = 2n_C = \frac{2F_C}{c_s} = \frac{g}{c_s} = g \left(\frac{m}{kT_e} \right)^{1/2}. \quad (42)$$

To locate the stagnation point $\tilde{s}$ we must solve

$$0 = F(\tilde{s}) = F_A + \int_A^{\tilde{s}} ds' S(s') \quad (43)$$

which requires detailed information about

$$S_p(s').$$

Note that if $B(s) \neq \text{const.}$ but S_p is concentrated in a small region along a field line, Eq. (35) is still valid. For example, an incident flux upon a thin positive source is speeded up if the flow is subsonic and slowed if it is supersonic and vice versa for a negative source (a sink). Across a thin source we have the jump conditions

$$F_1 \left(v_1 + \frac{c_s^2}{v_1} \right) = F_2 \left(v_2 + \frac{c_s^2}{v_2} \right) , \quad (44)$$

$$F_1 = F_2 - g . \quad (45)$$

Case 3: Equal Thin Sources Outside the Mirror Throats. (Fig. 4)

By symmetry $v = 0$ inside the mirror. At the outside edges of the sources $|v| = c_s$ since a higher velocity is impossible (see Case 2) — a lower velocity would result in deceleration in the decreasing field and the plate boundary conditions would not be met (see Case 1). If the density just outside the sources is denoted n_s then the by jump condition (44) the stagnet density $\tilde{n}$ in the mirror is

$$\tilde{n} = 2n_s = 2 \frac{F_s}{c_s} = 2 \frac{g}{2c_s} = \frac{g}{c_s} \quad (46)$$

as in the case of $B = \text{const.}$ The velocity at the end plates is the supersonic solution of

$$\left(\frac{B_E}{B_s} \right)^2 = \left(\frac{v_E}{c_s} \right)^2 e^{-(v_E/c_s)^2} + 1 \quad (47)$$

with subscripts E and s referring to ends and sources. An important feature of this example is the fact that $\tilde{n}$ is proportional to the flux near the throat and thus increases with the mirror ratio for fixed power loss. On the other hand, v_{end} is several times c_s due to the fanning of field lines and there is, thus, a somewhat greater energy

loss per exiting ion. Generally, v_{end}/c_s increases only as $\log(B_s/B_E)$ so this not a severe penalty. Furthermore, the ion energy is highly directed so there is a possibility of its direct recovery.

Case 4: Single Thin Source Near Anode Located Outside Throat. (Fig. 1)

The stagnation point lies within the thin source with plasma flowing towards the mirror and a reverse flow into the anode. Regard the flux F_s towards the mirror as given but its velocity $v_s < c_s$ to be found; the reverse flux F'_s will also be determined. It is evident from Fig. 2 that $v = c_s$ at the throats; if on the one hand $v = c_s$ is reached before the first throat then $B = B_{\text{throat}}$ can't be reached; on the other hand, if $v < c_s$ at the throats then $v < c_s$ everywhere and the boundary condition at the cathode is not satisfied. Hence v_s is the slow solution of

$$\left(\frac{B_s}{B_T}\right)^2 = \left(\frac{v_s}{c_s}\right)^2 e^{-(v_s/c_s)^2} + 1, \quad (48)$$

and

$$n_s = F_s / v_s \quad (49)$$

The reverse velocity must equal $-c_s$ so application of the jump conditions (44), (45) gives

$$F'_s = -\frac{1}{2} F_s \left(\frac{v_s}{c_s} + \frac{c_s}{v_s} \right), \quad (50)$$

$$g = F_s - F'_s = \frac{F_s}{2} \left[2 + \frac{v_s}{c_s} + \frac{c_s}{v_s} \right]. \quad (51)$$

An interesting question concerns the branch of the B vs. v curve which applies between the throats. The apparent degeneracy is removed by the

presence of a small sink or source of ions in this region. If there is a sink and the flow is subsonic then the ions are decelerated upon passing it. This results in a renormalization of the B vs v curve such that $B_{\max}$ is greater than B_{throat} and the supersonic branch is inaccessible — the cathode boundary condition can't be satisfied. Thus, the flow must be supersonic in this case. Similarly, if there is a net ion source the flow is subsonic in the mirror region. This consideration must be of importance because the density in the mirror well is lower for the supersonic solution. However, there is an electrostatic potential created which may be useful in confining the hot ion plasma.

Case 5: Energetics with $B = \text{const.}$

Returning to Case 2 the relations between current, voltage and T_e are outlined.

The potential at $\tilde{s}$ is related to the density by

$$\phi(\tilde{s}) - \phi(c) = \frac{kT_e}{e} \log \frac{\tilde{n}}{n_c} = \frac{kT_e}{e} \log 2 \quad (52)$$

and

$$\Delta\phi_p = 0 \quad (53)$$

Equations (14) and (37) give

$$J = J_c = J'_c + e \frac{g}{2} (1 + \eta_c) \quad (54)$$

and Eq. (17) yields

$$\frac{e\Delta\phi_A}{kT_e} = \log \left(\frac{2J/eg + 1 + \eta_A}{\sqrt{m/2m_e}} \right) \quad (55)$$

The cathode drop for given net $\Delta\phi$ is

$$\begin{aligned}\Delta\phi_c &= \Delta\phi_A - \Delta\phi \\ &= \frac{kT_e}{e} \log \left(\frac{2J/eg + 1 + \eta_A}{\sqrt{m/2\pi m_e}} \right) - \Delta\phi.\end{aligned}\quad (56)$$

To find T_e note

$$\begin{aligned}\left(\frac{\text{Power out}}{\text{cm}^2} \right) &= \frac{g}{2} \left(\frac{kT_e}{2} - e\Delta\phi_A + \epsilon \right) + \frac{g}{2} \left(\frac{kT_e}{2} - e\Delta\phi_c + \epsilon \right) + \left(\frac{J}{e} - \frac{g}{2} (1+\eta_A) \right) kT_e \\ &= \left(\frac{\text{Power in}}{\text{cm}^2} \right) = I\Delta\phi + \left(\frac{P_{\text{in other}}}{\text{cm}^2} \right).\end{aligned}\quad (57)$$

Solving for T_e gives

$$kT_e = \frac{I\Delta\phi - g\epsilon - (g/2)e\Delta\phi + P_{\text{in other}}}{g(1 + \eta_A/2 - e\Delta\phi_A/kT_e) + J/e}.\quad (58)$$

In an experiment the quantities J , $\Delta\phi$, $\tilde{n}$ and T_e could be measured. The source strength g is derived from (42) and the thermionic current J' from (54). The temperature T_e derived from (58) serves as a check on our assumptions. If this temperature exceeds the experimental value a likely cause is incomplete thermalization of the cathode electrons.

Case 6: Model of S_p

Gas introduced at the anode only determines a fundamental source g_0 . The actual source strength must take into account neutrals knocked into the plasma by ions striking the anode. If each ion causes the injection of γ neutrals the actual source is

$$g = g_O - \gamma F'_S \quad (59)$$

Recalling the jump conditions

$$\frac{F'_S}{F_S} = -\frac{1}{2} \left(\frac{v_S}{c_S} + \frac{c_S}{v_S} \right),$$

$$g = F_S - F'_S,$$

we find

$$g_O = F_S \left[1 + (1-\gamma) \frac{1}{2} \left(\frac{v_S}{c_S} + \frac{c_S}{v_S} \right) \right] \quad (60)$$

If

$$\gamma = \gamma_{\text{critical}} = 1 + 2 \left(\frac{v_S}{c_S} + \frac{c_S}{v_S} \right)^{-1} \quad (61)$$

then the discharge is self-sustaining with $g_O = 0$. Higher γ results in a runaway situation.

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10 X 10 TO THE HALF INCH

SQUARE

Fig. 2 B vs v with $S_z = 0$.

Fig 4 Flow and ϕ vs s for equal sources near throats

REFERENCES

1. R. F. Post, T. K. Fowler, J. Killeen, A. A. Mirin, Phys.Rev. Letters, 31, 280 (1973).
2. C. C. Damm, et al., Baseball II Program Plan, June 12, 1973, Prop. 105.
3. P. Bohm, Characteristics of Electrical Discharges in Magnetic Fields, (McGraw-Hill, N. Y., N. Y.) Chapt. 3, 1449 (1949).
4. S. A. Self, Phys. Fluids 6, 1726 (1963).
5. S. A. Self and H. N. Ewald, Phys. Fluids, 9, 2486 (1966).
6. R. A. Gibbons, Masters Thesis, Univ. Tenn., March, 1963.

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