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ARGONNE NATIONAL LABORATORY
P. O. Box 5207
Chicago 80, Illinois

NUCLEAR REACTOR SIMULATOR

By
J. M. Harrer

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Harold Etherington, Director

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DISCLAIMER

NUCLEAR REACTOR SIMULATOR

Preface

The purpose of this report is to describe the work done at Oak Ridge National Laboratory in the development of a nuclear reactor simulator. This work is part of the total program of the Power Pile Division and was carried on by the Control Section. No one person in this section spent full time in this development work. Limited personnel made it necessary to stop this work from time to time to design electrical control circuits for the particular reactors being studied by the division. The advantages to be gained from having available a simulator were sufficient to warrant directing the persons involved to put as much effort as possible into its development.

A great deal has been learned about the possibilities of such a simulator and the difficulties which are encountered in its design. In this report no attempt is made to draw any definite conclusions. It is felt that, because of changes in personnel and the fact that the reactor development program has been relocated, all the significant information pertaining to the simulator should be gathered together.

This report includes the following:

Introduction, Page 7, is a general picture of the ideas behind the simulator and its uses.

Section A, Page 14, entitled Network Development, covers the derivation of formulae for the relations between reactor constants and electrical network constants.

Section B, Page 28, entitled Construction of Experimental Network, describes the networks used to determine the feasibility of the simulator. Results of test runs are given and compared to calculated values.

Section C, Page 45, entitled Current Feeding Equipment, gives a description of the methods used to feed electrical current, which simulates the birth of neutrons in the reactor, into the electrical network.

Section D, Page 65, is entitled Experiment and Development Work. As the tests described above were conducted several ideas developed which offered solutions to the various difficulties encountered. Some of these were tested and are described in this section.

Appendix I, Page 90, is entitled Transmission Line Analogy. In this section an attempt is made to present corrective formulae which would make the results of measurements on a lumped network exactly equal to the equations for a distributed network and therefore equivalent to a homogeneous reactor.

SYMBOLS AND DEFINITIONS OF TERMS

General symbols used for reactor constants and quantities throughout this report are:

k = number of thermal energy neutrons produced per thermal neutron absorbed in an infinite core.

Σ_{tth} = transport cross section for thermal energy neutrons, (cm^{-1}).

Σ_{tf} = transport cross section for high energy neutrons, (cm^{-1}).

Σ_{ath} = absorption cross section for thermal energy neutrons, (cm^{-1}).

Σ_{af} = absorption cross section for high energy neutrons, (cm^{-1}).

Φ_f = high energy neutrons per sq cm per second.

Φ_s = thermal energy neutrons per sq cm per second.

$L^2 = \frac{1}{3\Sigma_{tth}\Sigma_{ath}} = \frac{1}{6}$ of the mean square distance a neutron travels from point at which it becomes thermal to point at which it is absorbed.

$T = \frac{1}{3\Sigma_{tf}\,\Sigma_{af}}$ = "Fermi age" of neutrons, $\frac{1}{6}$ of the mean square distance a neutron travels between point of fission and point at which it becomes a thermal energy neutron.

The terms used to describe the electrical system are those generally accepted in electrical engineering. However, the terms are somewhat more specific and are set out here for clarity.

Network is the term applied to that portion of the simulator consisting of resistances interconnected in a mesh configuration.

Simulator is the term applied to the entire system which includes the network and the current feeding equipment.

Node Point is the term describing those points in the resistor network where resistors are joined together.

Lattice Unit is the term used to describe that portion of the network of resistances which contain a node point. In particular refer to fig. A-1.

The lattice unit here consists of the four resistances $R/2$ and the resistor Rg. In the network this point would be surrounded by four similar lattice units a distance h away from x, y, which would contain the other half of R shown here. Each of these four lattice units also contain the value Rg. Fig. A-2 is a typical lattice unit for a three dimensional network. In this figure two lattice units similar to fig. A-1 have been placed on top of one another and a lattice unit refers to this combination. Again, each lattice unit contains the value R and Rg and six of each of the values $R_\theta/2$ and $R_\phi/2$.

Feed Point is the term used to indicate the node points at which current is fed into the network. Feed points occur only in the network which represents reactor region.

NUCLEAR REACTOR SIMULATOR

Introduction

The simulator is an electrical analogue computer. The idea of solving differential equations by the use of electrical models has become a widely accepted technique. If due consideration has been given to keeping the model similar to the physical phenomena described by the differential equations, the practical engineer has at his disposal a means of studying the physical phenomena without continual reference to the differential equations involved. The simulator developed at ORNL, through the combined efforts of E. J. Wade and J. W. Simpson of the Power Pile Division, meets this requirement. It consists of a network of electrical conducting elements and suitable sources of current and current sinks. The design of this network is such that when the proper electrical resistance values are assembled and the current sources and sinks are properly adjusted, the steady state voltage distribution on the network is the same as the steady state flux distribution in the reactor and the current sources are a measure of the critical mass of the reactor.

In elementary diffusion theory, the neutrons are assumed to obey the diffusion equation (a) for any volume.

$$\frac{\lambda_t}{3} \nabla^2 \Phi - A + Q = \frac{dn}{dt}$$

where

$A = \Phi\Sigma_a$ = number of neutrons absorbed per cu cm per sec.

$\frac{\lambda_t}{3}$ = diffusion constant for the reactor material.

$\Phi = (nv)$ = neutron flux = neutrons/cu cm x cm per sec = neutrons per sq cm.

Σ_a = absorption constant for neutrons.

Q = neutrons produced per cu cm per sec which may be a function of their position in the reactor.

n = number of neutrons per cu cm.

t = time.

∇^2 = Laplacian = divergence of the gradient.

(a) AECD-2201
6:76

This equation shows that the rate of change of neutrons for a particular volume of core material equals the net neutrons that diffuse into that volume less those that are absorbed there plus those produced in the same volume.

The net neutron current density is given by equation (b),

$$\underline{J} = -\frac{\lambda_t}{3} \nabla \Phi$$

The diffusion of electrical current in a small volume of electrical conductor obeys the diffusion equation (c),

$$\frac{\nabla^2 V}{R} - C + I = \frac{dQ}{dt}$$

in which

V = electrical potential.

I = electrical current entering the conductor.

Q = electrical charge per unit volume.

t = time.

∇^2 = Laplacian.

R = resistance of the conductor.

C = the sink or loss of current per sec for that volume of conductor. The current flowing in the conductor is given by equation (d)

$$\underline{i} = -\frac{\nabla V}{R} \qquad \text{(d)}$$

The similarity of equations (a) and (c) as well as (b) and (d) can readily be seen.

V is analogous to (nv)

i is analogous to J

Q is analogous to n

A volume of electrical conductor in which there is a sink C and a source I, will serve as an electrical analogue for the differential equation of neutron diffusion. Consideration is restricted to only the steady state

(b) AECD-2201

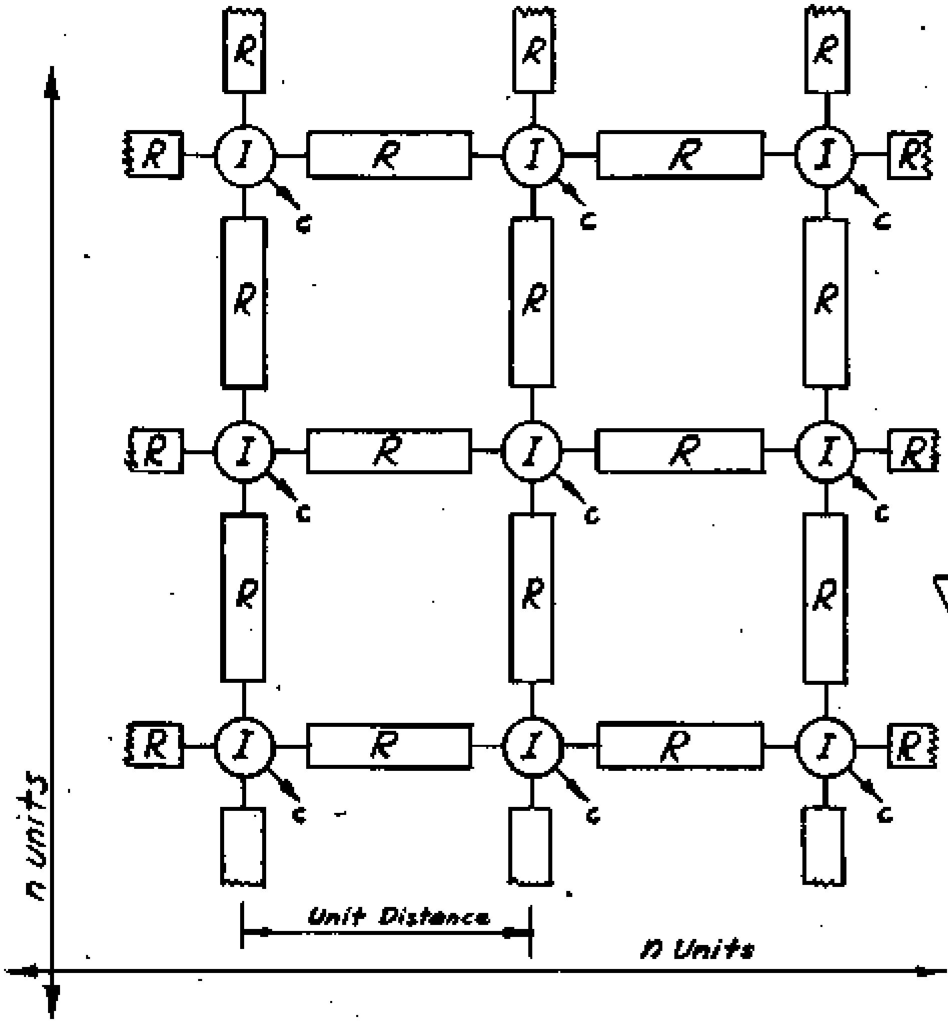
flux condition and therefore in equation (a) and (c) the right hand side is zero.

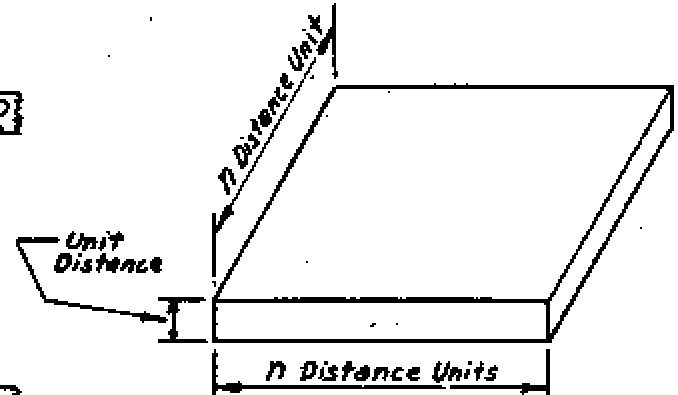
$$\frac{\lambda_t}{3} \nabla^2 \Phi - \Phi\Sigma_a + Q = O \qquad (e)$$

$$\frac{\nabla^2 V}{R} - C + I = O \qquad (f)$$

Practical aspects of building a conductor such as have been described must now be considered. It is not difficult to build a conductor of some material and introduce a source or several sources of current into it through small insulated conductors. However, it is quite impossible to construct an absolutely homogeneous distribution of sources in the conductor. Therefore, a finite spacing of sources must suffice. Since the sources must be spaced at finite intervals the finite volume of conductor, which exists between each source may as well be replaced with a conductor which has the same resistance as that volume of conductor. If, for simplicity, the sources are spaced evenly throughout the volume, the same resistor between each source would be used. So far it has been implied that the current is being removed from all points on the surface of the volume of conductor and that these points are not necessarily at the same potential. The same is true for the network of lumped conductors. The current is removed from all resistors which reach to the boundary of the volume. For the balance of this discussion the distance between the two sources is considered to be a unit distance (assuming evenly spaced sources) and a volume of all unit dimensions is called a unit volume.

For convenience attention is confined to a volume of conductor of unit thickness which contains all the sources in one flat plane of the total conductor volume. Here it is implied that the components of current flow out of this unit thickness volume, perpendicular to the plane of the sources, are known. A network to represent such a volume is shown in fig. 1 where equal spacing of the sources is again used and therefore all the conductors are of equal resistance R ohms. At each of the node points (where resistors join) the current source I is introduced. This current flows through the network eventually reaching ground to complete the electrical circuit. This current I can be any quantity and all the feeds are not necessarily equal. For the analogy this current should be proportional to the voltage which exists at each node point because, in the reactor, the number of neutrons produced by fission is proportional to the thermal flux present at any point in the reactor, where fissions occur. To complete the analogy a sink term C is needed, which for a perfect analogy, requires that the electrical current be destroyed at all points in the volume of conducting material in an amount which is everywhere proportional to the neutron flux present. Here again practical considerations require that the sink C, for the units of volume be lumped at the node points of the network. Since this sink is proportional to the voltage, at each node point, it is possible to take advantage of the linear





Conducting Material
Represented by Network

Network Representing an Electrical Conductor of Unit Thickness

R.P.D.A-1419 Fig. 1

relation of Ohm's law

$$I = E/R'$$

and place a conductor, from each node point to ground, which has a resistance inversely proportional to the absorption constant for neutrons. The current source requirement can be met by a voltage controlled current supply such as a vacuum tube operating in the linear portion of its characteristic curve. With a suitably controlled source in place, the analogy for elementary diffusion theory is complete.

In a thermal-neutron nuclear reactor, neutrons exist at all energies between fission energy and thermal energy, and the "constants" of equation (e) are functions of neutron energy. The "two-group theory"* is used extensively in practical reactor calculations as an approximation to this situation. Here it is assumed that the neutrons exist in two energy groups, fast or high energy and slow or thermal energy. For a steady state flux of either energy level, an equation like e applies. In the case of fast neutrons the A term represents slowing down while for thermal neutrons it continues to be absorption. The electrical analogy requires two similar networks of the type described. Each must have the same lattice construction, that is, equally spaced node points. Thermal neutrons are produced by the slowing down of fast neutrons. The current of the sink term of the network representing fast neutrons must be made to flow into the corresponding node points of the thermal network. This suggests that the conductor which carries the sink current away from the fast network should connect directly between the corresponding node points of the fast and thermal networks instead of to ground. In two-group theory it is assumed that all fissions occur because a thermal neutron flux is present at any point at which fission can occur. For this analogy, then, the current source I, for the fast network is controlled with the voltage of the thermal network. Thus, a single network can be constructed to simulate all the elements of two-group diffusion theory.

A typical lattice unit for this network is shown in fig. A-1 of Section A. In this figure the fast neutron diffusion network resistors are denoted by the subscripts ϕ and the thermal network by θ. R_γ is the slowing down resistor and Rg is the absorption resistor. The source current I is represented by a box which is an electronic amplifier whose output current I is equal to A x V_θ. To operate the simulator a particular value of A must be found which will establish constant voltages on all points in the network. If the voltage V_θ were zero the source I would be zero. In order to start the cycle of production of current, a sourve V_θ must be applied to the network just as in the reactor a source of neutrons must be present in order to start the chain reaction. Once the cycle is started the source can be removed and the cycle simulating the chain reaction will continue.

*Soodak, H., AECD Publication 2201

An ideal analogue would simulate the continuous slowing down process which occurs for the neutrons in the chain reactor. Actually what is wanted is the simulation of many groups of neutron energies. From the discussion of the two-group simulator it is evident that a voltage difference must exist such that the ϕ (fast) network voltage is higher than the θ (thermal) network voltage. The larger this voltage difference, the more nearly the slowing down of neutrons can be simulated in the sink conductor of the fast network. In the simulator of this report (Section B) the ϕ network peak voltage was set at 150 volts and the θ network peak at 3 volts, a ratio of 50:1. Obviously an attempt to simulate three groups with the same ratios results in ratios of 2500:50-1, a peak of 7500 volts for the highest energy for a peak of 3 volts on the lowest energy network. This is not practical but, fortunately, the 50:1 ratio between voltages is not necessary and by using a smaller ratio of, say 10:1, at least three energy groups could be simulated. At ORNL the work was confined to two energy groups.

In Section A of this report, the relations between the network parameters and reactor constants are developed. In Section B the construction of two simulators, each for a different reactor, is described.

A chain reactor consists of a core and a reflector region. So far only a region in which sources of neutrons are present, which is the case in the core, has been considered. The slowing down, absorption and diffusion which occur in the reflector region can be simulated by a network of the same form as the core with the source current (I) zero. The source of thermal neutrons due to slowing down is still present and the absorption of thermal neutrons is much less because of the absence of fissionable material. For all practical purposes, all that is necessary in order to simulate the reflector region is to assign average values of material constants to that region and to adjust the network resistance values to correspond. This is covered more completely in the designs of Section B. The neutron flux is considered to be zero at the outside boundary of the reflector and therefore the network is grounded at that point.

Simulating the insertion of a control rod into the core is a matter of imposing upon the simulator network the known effect of the rod upon the neutrons. If the rod will absorb all the thermal neutrons in a particular reactor region the thermal network can be grounded at those points which correspond to the effective rod dimensions. The practical considerations of this operation are covered more fully in Section B of this report.

With a simulator of this kind available, the determination of critical mass and the flux distribution for complex boundary condition problems is greatly simplified. Analytical solution of the two-group theory differential equations results, for nearly all boundary problems, in a tedious trial and error process. The simulator offers a means of performing this process rapidly through the feed-back action of the electronic amplifier. Examples of these complex problems are: the insertion of control rods along the radius or a chord of a cylindrical core, irregular control rod spacings and the case where the material constants vary as a function of the location of the material in the reactor.

SECTION A

NETWORK DEVELOPMENT

A. Network Theory

Partial differential equations such as those used in two-group theory calculations can be solved by the use of suitable electrical networks.

Consider the point x, y in a network mesh as shown in fig. A-1 (b) (This is one point in the network of fig. A-1 (a).

The electrical current equation at x, y is:

$$\frac{V(x+h,y) - V(x,y)}{R} + \frac{V(x-h,y) - V(x,y)}{R} + \frac{V(x,y+h) - V(x,y)}{R} + \frac{V(x,y-h) - V(x,y)}{R} = \frac{V(x,y)}{R_g}$$

or;

$$[V(x+h,y) + V(x-h,y) - 2V(x,y)] + [V(x,y+h) + V(x,y-h) - 2V(x,y)] = V(x,y)\,\frac{R}{R_g}$$

Using Taylor's expansion*

$$V(x+h,y) = V(x,y) + \frac{h}{1!}\,\frac{\partial V(x,y)}{\partial x} + \frac{h^2}{2!}\,\frac{\partial^2 V(x,y)}{\partial x^2} + \ldots$$

and

$$V(x-h,y) = V(x,y) - \frac{h}{1!}\,\frac{\partial V(x,y)}{\partial x} + \frac{h^2}{2!}\,\frac{\partial^2 V(x,y)}{\partial x^2} - \ldots$$

adding and rearranging terms

$$\frac{\partial^2 V(x,y)}{\partial x^2} = \frac{V(x+h,y) \quad V(x-h,y) \quad 2V(x,y)}{h^2}$$

*In this development a lattice length h = 1 could be conveniently used, and the variables x, y and z could be measured in lattice units. In Section B, however, different values of length for h were used and it was considered more convenient to let h remain as a parameter in the equation.

likewise

$$\frac{\partial^2 V(x,y)}{\partial y^2} = \frac{V(x,y+h) + V(x,y-h) - 2V(x,y)}{h^2}$$

The network equation becomes very nearly:

$$\frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = \frac{V(x,y)}{h^2} \frac{R}{R_g}$$

The solution of this equation V(xy) can be read with a voltmeter from the network point (x, y) to ground.

B. Network Theory Applied to Calculations for Reactor

This fundamental idea can be extended into three directions x, y and z and the resulting network equation will have a $\nabla^2 V$ term where the reactor equation contains $\nabla^2 \Phi$. The network equation would be:

$$\nabla^2 V(x,y,z) = \frac{V(x,y,z)}{h^2} \frac{R}{R_g}$$

In two-group theory the neutrons are assumed to exist only in two energy levels, fast or high energy, and thermal or low energy. In either energy level the general form of the balance equation at steady state conditions of flux is:

- Leakage - absorption + production = 0

$$\frac{\lambda_{tf}}{3} \nabla^2 \Phi_f - \Sigma_{af}\, \Phi_f + k\, \Sigma_{ath}\, \Phi_{th} = 0 \qquad (1p)$$

$$\frac{\lambda_{tth}}{3} \nabla^2 \Phi_{th} - \Sigma_{ath} \Phi_{th} + \Sigma_{af}\, \Phi_f = 0 \qquad (2p)$$

where

$\frac{\lambda_{tf}}{3} \nabla^2 \Phi_f$ = number of fast neutrons diffusing into a cu cm per sec.

$\frac{\lambda_{tth}}{3} \nabla^2 \Phi_{th}$ = number of thermal neutrons diffusing into a cu cm per sec.

$\Sigma_{af}\, \Phi_f$ = number of fast neutrons becoming thermal/cu cm/sec.

$\Sigma_{ath}\Phi_{th}$ = number of thermal neutrons absorbed/cu cm/sec.

$k\Sigma_{ath}\Phi_{th}$ = number of fast neutrons produced/cu cm/sec.

In the electrical network a current I amp can then be introduced at the node point and the balance equation for current will become:

$$\frac{h^2}{R}\nabla^2 V(xyz) - \frac{V(xyz)}{R_g} + I = 0$$

or;

$$\frac{h}{R}\nabla^2 V(xyz) - \frac{V(xyz)}{hR_g} + \frac{I}{h} = 0$$

which is of the same form as (1p) or (2p) above.

For each neutron energy group an electrical resistor network having the above equation can be constructed. For a simultaneous solution of these equations, the current I at each node point in each network must be made proportional to the voltage V of the corresponding node point in the other network.

Rather than construct separate networks to meet these requirements, a single network with suitable resistance elements to represent all of the terms in both energy group equations can be arranged. The network equations can then be written and the coefficients of these equations equated to the coefficients of the reactor equations.

A network of resistances to meet these requirements is shown in fig. A-2.

C. Calculation of Network Constants

The current equation at $V_{(x,y,z)}$ is:

(Equation 1n)

$$\frac{V_\phi(x+a,y,z) - V_\phi(x,y,z)}{R_{\phi x}} + \frac{V_\phi(x-a,y,z) - V_\phi(x,y,z)}{R_{\phi x}} +$$

$$\frac{V_\phi(x,y+b,z) - V_\phi(x,y,z)}{R_{\phi y}} + \frac{V_\phi(x,y-b,z) - V_\phi(x,y,z)}{R_{\phi y}} +$$

$$\frac{V_\phi(x,y,z+c) - V_\phi(x,y,z)}{R_{\phi z}} + \frac{V_\phi(x,y,z-c) - V_\phi(x,y,z)}{R_{\phi z}} + \frac{V_\phi - V_\theta}{R_\gamma} + AV_\theta = 0$$

or collecting terms

$$\frac{V\phi(x+a,y,z) + V\phi(x-a,y,z) - 2V\phi(x,y,z)}{R_{\phi x}} +$$

$$\frac{V\phi(x,y+b,z) + V\phi(x,y-b,z) - 2V\phi(x,y,z)}{R_{\phi y}} +$$

$$\frac{V\phi(x,y,z+c) + V\phi(x,y,z-c) - 2V\phi(x,y,z)}{R_{\phi z}} + \frac{V\phi}{R_{\gamma}} + V\phi\left[A - \frac{1}{R_{\gamma}}\right] = 0$$

Reactor equation for fast flux $(nv)_f \equiv \Phi_f$

$$\frac{\lambda tf}{3}\Delta^2\Phi_f - \Sigma_{af}\Phi_f + k\Sigma_{ath}\Phi_{th} = 0$$

By Taylor Expansion (using a, b, and c to correspond to lattice unit length in x, y, and z directions)

$$\Phi_f(x+a,y,z) = \Phi_f(x,y,z) + \frac{a}{1!}\frac{\partial\Phi_f}{\partial x}(x,y,z) + \frac{a^2}{2!}\frac{\partial^2\Phi_f}{\partial x^2}(x,y,z) + \ldots$$

$$\Phi_f(x-a,y,z) = \Phi_f(x,y,z) - \frac{a}{1!}\frac{\partial\Phi_f}{\partial x}(x,y,z) + \frac{a^2}{2!}\frac{\partial^2\Phi_f}{\partial x^2}(x,y,z) - \ldots$$

Adding

$$\frac{\partial^2\Phi_f(x,y,z)}{\partial x^2} = \frac{\Phi_f(x+a,y,z) + \Phi_f(x-a,y,z) - 2\Phi_f(x,y,z)}{a^2}$$

Similarly, terms for

$$\frac{\partial^2\Phi_f(x,y,z)}{\partial y^2} \quad ; \quad \frac{\partial^2\Phi_f(x,y,z)}{\partial z^2}$$

can be obtained.

Substituting these expressions in 1p we get: (after multiplying through by a, b and c)

Equation 1p

$$\frac{\lambda_{tf}}{3}\cdot\frac{bc}{a}[\Phi_f(x+a,y,z) + \Phi_f(x-a,y,z) - 2\Phi_f(x,y,z)] +$$

$$\frac{\lambda_{tf}}{3}\cdot\frac{ac}{b}[\Phi_f(x,y+b,z) + \Phi_f(x,y-b,z) - 2\Phi_f(x,y,z)] +$$

$$\frac{\lambda_{tf}}{3}\ \frac{ab}{c}\ [\Phi_f(x,y,z+c) + \Phi_f(x,y,z-c) - 2\Phi_f(x,y,z)] -$$

$$abc\ \Sigma_{af}\ \Phi_f(x,y,z) + abck\ \ \Sigma_{ath}\ \Phi_{th}(x,y,z) = 0$$

Equation 1n and 1p′ will be equal if:

$$\Phi_f = V_\phi$$

$$\Phi_{th} = V_\theta$$

$$R_{\phi x} = \frac{3}{\lambda_{tf}} \cdot \frac{a}{bc} \qquad (4\text{-}1)$$

$$R_{\phi y} = \frac{3}{\lambda_{tf}} \cdot \frac{b}{ac} \qquad (4\text{-}2)$$

$$R_{\phi z} = \frac{3}{\lambda_{tf}} \cdot \frac{c}{ab} \qquad (4\text{-}3)$$

$$R\gamma = -\frac{1}{\Sigma_{af}\ abc} \qquad (5)$$

$$-\frac{1}{R\gamma} + A = k\ \Sigma_{ath}\ abc \qquad (6)$$

Likewise, the equation for the θ network is as follows: (Using a, b and c as before).

(Equation 2n)

$$\frac{V_\theta(x+a,y,z) + V_\theta(x-a,y,z) - 2V_\theta(x,y,z)}{R_{\theta x}} +$$

$$\frac{V_\theta(x,y+b,z) + V_\theta(x,y-b,z) - 2V_\theta(x,y,z)}{R_{\theta y}} +$$

$$\frac{V_\theta(x,y,z+c) + V_\theta(x,y,z-c) - 2V_\theta(x,y,z)}{R_{\theta z}} - V_\theta(x,y,z)\left[\frac{1}{R_g} + \frac{1}{R\gamma}\right] + \frac{V_\phi}{R\gamma} = 0$$

The reaction equation for thermal flux $(\Phi)_{th}$ infinite difference form is:

(Equation 2p′)

$$\frac{\lambda_{tth}}{3} \cdot \frac{bc}{a}\ [\Phi_{th}(x+a,y,z) + \Phi_{th}(x-a,y,z) - 2\Phi_{th}(x,y,z)] +$$

$$\frac{\lambda_{tth}}{3} \cdot \frac{ac}{b}\ [\Phi_{th}(x,y+b,z) + \Phi_{th}(x,y-b,z) - 2\Phi_{th}(x,y,z)] +$$

$$\frac{\lambda_{tth}}{3} \cdot \frac{ab}{c} \left[\Phi_{th}(x,y,z+c) + \Phi_{th}(x,y,z-c) - 2\Phi_{th}(x,y,z)\right] -$$

$$abc\, \Sigma_{ath}\, \Phi_{th}(x,y,z) + abc\ \Sigma_{af}\ \Phi_{th}(x,y,z) = 0.$$

Here again 2n will be similar to 2p′ if:

$$V_\theta = \Phi_{th} \tag{7}$$

$$R_{\theta x} = \frac{3}{\lambda_{tth}} \cdot \frac{a}{bc} \tag{8-1}$$

$$R_{\theta y} = \frac{3}{\lambda_{tth}} \cdot \frac{b}{ac} \tag{8-2}$$

$$R_{\theta z} = \frac{3}{\lambda_{tth}} \cdot \frac{c}{ab} \tag{8-3}$$

$$\frac{1}{R_g} + \frac{1}{R_\gamma} = \sum_{ath} abc \tag{9}$$

Having established the above relations the ratios between network resistance values can be set up.

D. Network Resistor Ratios

$$\frac{R_{\phi x}}{R_\gamma} = \frac{3a^2}{\lambda_{tf}} \Sigma_{af} = \frac{a^2}{T} \tag{10}$$

$$\frac{R_{\phi y}}{R_\gamma} = \frac{3b^2}{\lambda_{tf}} \Sigma_{af} = \frac{b^2}{T} \tag{11}$$

$$\frac{R_{\theta z}}{R_\gamma} = \frac{3c^2}{\lambda_{tf}} \Sigma_{af} = \frac{c^2}{T} \tag{12}$$

From 4 - 1, 2, 3 and 8 - 1, 2 and 3

$$\frac{R_{\phi x}}{R_{\theta x}} = \frac{R_{\phi y}}{R_{\theta y}} = \frac{R_{\phi z}}{R_{\theta z}} = \frac{\lambda_{t,th}}{\lambda_{tf}} \tag{13}$$

From 8 - 1 and 7

$$R_{\theta x}\left[\frac{1}{R_g} + \frac{1}{R_\gamma}\right] = \frac{3a^2}{\lambda_{tth}} \Sigma_{ath} = \frac{a^2}{L^2}$$

$$\frac{R_{\theta x}}{R_g} + \frac{R_{\theta x}}{R_\gamma} = \frac{a^2}{L^2} \tag{14}$$

From 13

$$R_{\theta x} = R_{\phi x} \cdot \frac{\lambda_{tf}}{\lambda_{tth}}$$

$$R_{\theta x} = \frac{a^2}{L^2} - \frac{R_{\phi x}}{R_\gamma} \cdot \frac{\lambda_{tf}}{\lambda_{tth}} = \frac{a^2}{L^2} - \frac{a^2}{T} \cdot \frac{\lambda_{tf}}{\lambda_{tth}}$$

$$\frac{R_{\theta x}}{R_g} = \frac{a^2}{L^2}\left[1 - \frac{L^2}{T}\frac{\lambda_{tf}}{\lambda_{tth}}\right] \qquad (14\text{-}1)$$

Likewise:

$$\frac{R_{\theta y}}{R_g} = \frac{b^2}{L^2}\left[1 - \frac{L^2}{T}\frac{\lambda_{tf}}{\lambda_{tth}}\right] \qquad (14\text{-}2)$$

$$\frac{R_{\theta z}}{R_g} = \frac{c^2}{L^2}\left[1 - \frac{L^2}{T}\frac{\lambda_{tf}}{\lambda_{tth}}\right] \qquad (14\text{-}3)$$

From 6

$$A = k\sum_{ath} - \frac{1}{R_\gamma}$$

Substitute 9

$$A = k\left[\frac{1}{R_g} + \frac{1}{R_\gamma}\right] - \frac{1}{R_\gamma}$$

$$= \frac{k}{R_g} + \frac{k-1}{R_\gamma}$$

The current I was defined as AV_θ

or $$A = \frac{I}{V_\theta} \quad \text{(mhos)}$$

In order to supply this current a current generator is required which can be controlled. Its output current must always equal AV_θ. An electronic amplifier will satisfy this requirement. The ideal input-output relation for this amplifier is a straight line, the slope of which is A. The slope A must be adjustable.

E. Kappa Factor

The network under discussion will simulate a reactor described by the two-group theory equation but the current flows i_γ, ig and I of fig. A-2 will not represent exactly any reactor quantities because the ϕ network has been placed on top of the θ network. Consider again the basic network requirement that the absorption resistor (R_γ or R_θ) should go to ground. It is easy to see that this ideal network will be approximated by the combined network if the voltage V_ϕ of this combined network is very large compared to V_θ, for example, in the order of a 50:1 ratio.

This means that there must be a ratio of about 50:1 between the values of R_γ and R_g and between the values of R_ϕ and R_θ.

Formula (13)(with x = y = z) gives the relation

$$\frac{R_\phi}{R_\theta} = \frac{\lambda_{tth}}{\lambda_{tf}}$$

A factor k is now introduced so that

$$\frac{R_\phi}{R_\theta} = k\,\frac{\lambda_{tth}}{\lambda_{tf}} \qquad (13a)$$

and this is called a "kappa factor." This gives the desired arbitrary reduction of the network voltage and if $\lambda_{tth} = \lambda_{tf}$

$$R_\phi = kR_\theta$$

This factor also affects formulas (14) which (with a = b = c) becomes

$$\frac{R_\theta}{R_g} = \frac{a^2}{L^2}\left[1 - \frac{L^2}{T}\,\frac{\lambda_{tf}}{\lambda_{tth}}\right]$$

and before the "kappa factor" was introduced this could have been written (by using 13)

$$\frac{R_\theta}{R_g} = \frac{a^2}{L^2}\left[1 - \frac{L^2}{T}\cdot\frac{R_\theta}{R_\phi}\right]$$

Now inserting the ratio of 13a with the "kappa factor" introduced the result is:

$$\frac{R_\theta}{R_g} = \frac{a^2}{L^2}\left[1 - \frac{L^2}{T}\,k\,\frac{\lambda_{tf}}{\lambda_{tth}}\right] \qquad (14a)$$

In formula 15 the value of R_g which was determined using k is employed. Here, since R_g has been reduced, A for a given k is larger. This means simply that any method of introducing the current $I = AV_\theta$ must supply a larger current per volt than would have been required without the kappa factor."

When the relations of formulas 3 through 15 are built into the electrical network the following direct relations between reactors and network quantities exist.

Network Quantity	Reactor Quantity Represented
$i_{\phi x}$, $i_{\phi y}$, $i_{\phi z}$	Diffusion of fast neutrons/sec/a.b.c. $(cm)^3$ in the directions x, y or z
i_γ	Fast neutrons/sec/a.b.c. $(cm)^3$ which slow down or become thermal
$i_{\theta x}$, $i_{\theta y}$, $i_{\theta z}$	Diffusion of thermal neutrons/sec/a.b.c. $(cm)^3$ in the direction x, y or z
i_g	Thermal neutrons/sec/a.b.c.$(cm)^3$ which cause fissions
I (The current entering the network node point)	Fast neutrons/sec/a.b.c. $(cm)^3$ which result from fissions

A solution to the network equations results when V_ϕ and V_θ are constant and for this condition A for the amplifier must be adjusted to a particular value. The operational procedure is to start current flowing in the network by some external means, and vary A until V_ϕ and V_θ are constant. A is then a measure of k for the reactor by equation 15. The node point voltages V_ϕ and V_θ give the flux distribution in the reactor.

F. Boundary Conditions

So far only the core has been considered and no specific attention has been given to the reflector region. The core is surrounded by a reflector region and it is assumed that no neutrons leave the reflector. Three boundary conditions regularly used for solution of the reactor equations must be met by the simulator; namely, that in the reflector the production of neutrons is zero, that the flux is continuous across the interface between the reflector and the reactor and that the thermal and fast flux are both zero at the outside boundary of the reflector. The first condition is met by not feeding current into the reflector node point. To meet the third of these conditions the perimeter of the network which represents the reflector is grounded. For the second condition reflector section network must be built to contain the constants τ, L^2, λ_{tf} and λ_{tth} for the reflector using the formulas developed for $R\phi$, $R\theta$, $R\gamma$, and R_g and the network for the reactor is joined to this reflector network. Due consideration must be given to the location of the interface with respect to the network node points.

If the interface occurs midway between node points the value of the resistors R_γ and R_g, at the last reactor node point inside of the reactor, and the first node point inside of the reflector need not be adjusted. However, if the interface occurs at a node point the $R\gamma$ and R_g resistors at this point must be adjusted. To do this the respective regions are considered to overlap at that node point and the following value is used.

$$R\gamma\,(\text{interface}) = \frac{2\,R\gamma\,(\text{reactor}) \times R_\gamma\,(\text{reflector})}{R\gamma\,(\text{reactor}) + R_\gamma\,(\text{reflector})} \qquad (16)$$

The method of arriving at this formula is shown in fig. A-3. If the interface occurs some fraction of the way between the last node point in the reactor and the first node point in the reflector, only the diffusion resistor between these node points must be changed if R_ϕ or $R\theta$ for the reactor is not equal to R_ϕ or R_θ for the reflector. In the case where the interface is at one half of the distance between the node points the fast network diffusion resistor are equal to

$$\frac{R_\phi\ (\text{reactor})}{2} \text{ plus } \frac{R_\phi\ (\text{reflector})}{2}$$

and the thermal flux network resistor is computed in the same way.

Another boundary condition which must be simulated is the insertion of a control rod. This condition may require, for example, that the thermal flux be zero at the effective radius of the rod. The thermal flux is made to go to zero by grounding the θ network at points in that network which correspond to the dimensions of the rod. The production of neutrons is zero so again no current feed is needed in that section of the θ network which was grounded. In this way any number of rods of any configuration can be simulated.

G. Conclusions

1. A network can be constructed so as to simulate the reactor described by the two-group theory equations.

2. The network is one of lumped constants and contains a finite distance between the network node points. These finite network lengths have a definite directional relation of 90 degrees to one another.

3. The accuracy of the results obtained from the simulator network depend upon:

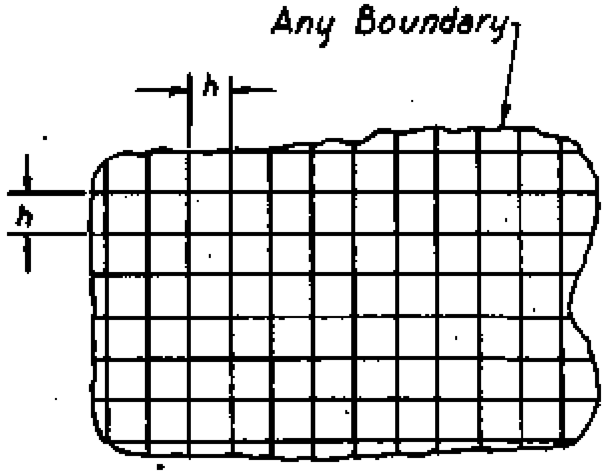
a. Selection of resistors whose values closely approximate the calculated values.

b. The accuracy with which the current feeding equipment fulfills the theoretical requirement: $I = AV_{\theta}$.

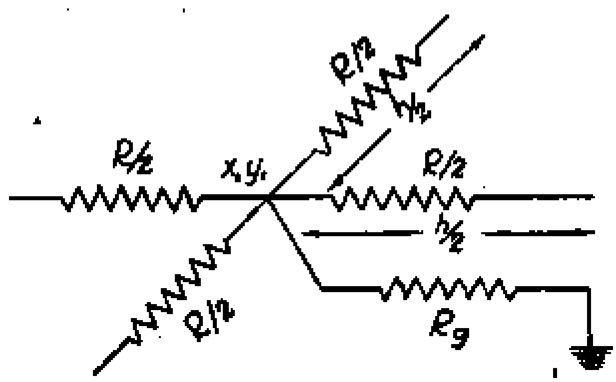
c. The accuracy of instrument reading.

d. The error involved in transforming the differential equation of the reactor into finite difference form. In the Taylor expansion, all odd powered terms cancel out and terms of the fourth, sixth, etc., power constitute the error.

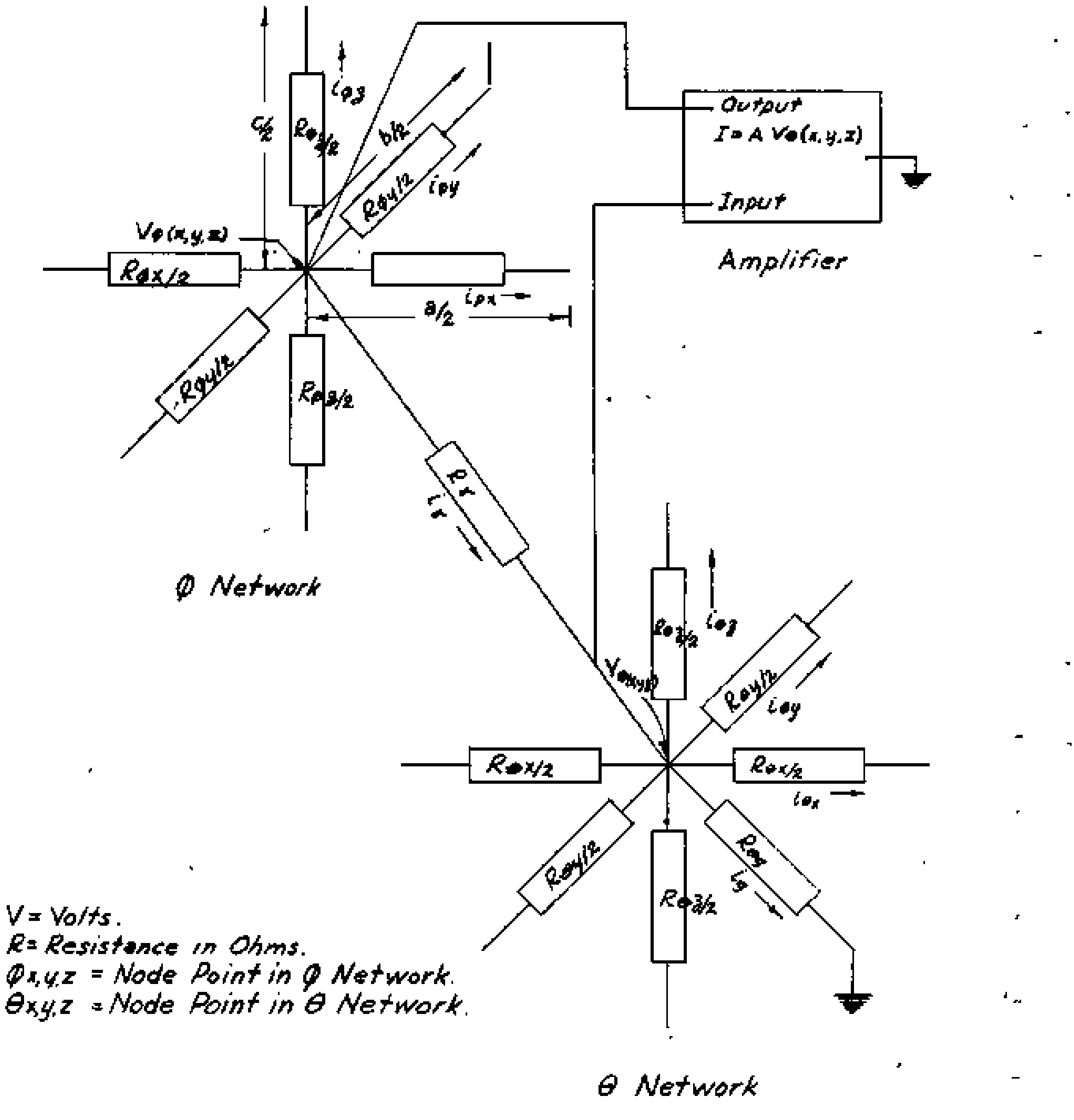
The total error resulting from all of these effects is best determined by constructing a network and comparing the measured results with those obtained by solving the reactor equations analytically.



(a) Square Mesh Network
Fitted To Any Boundary



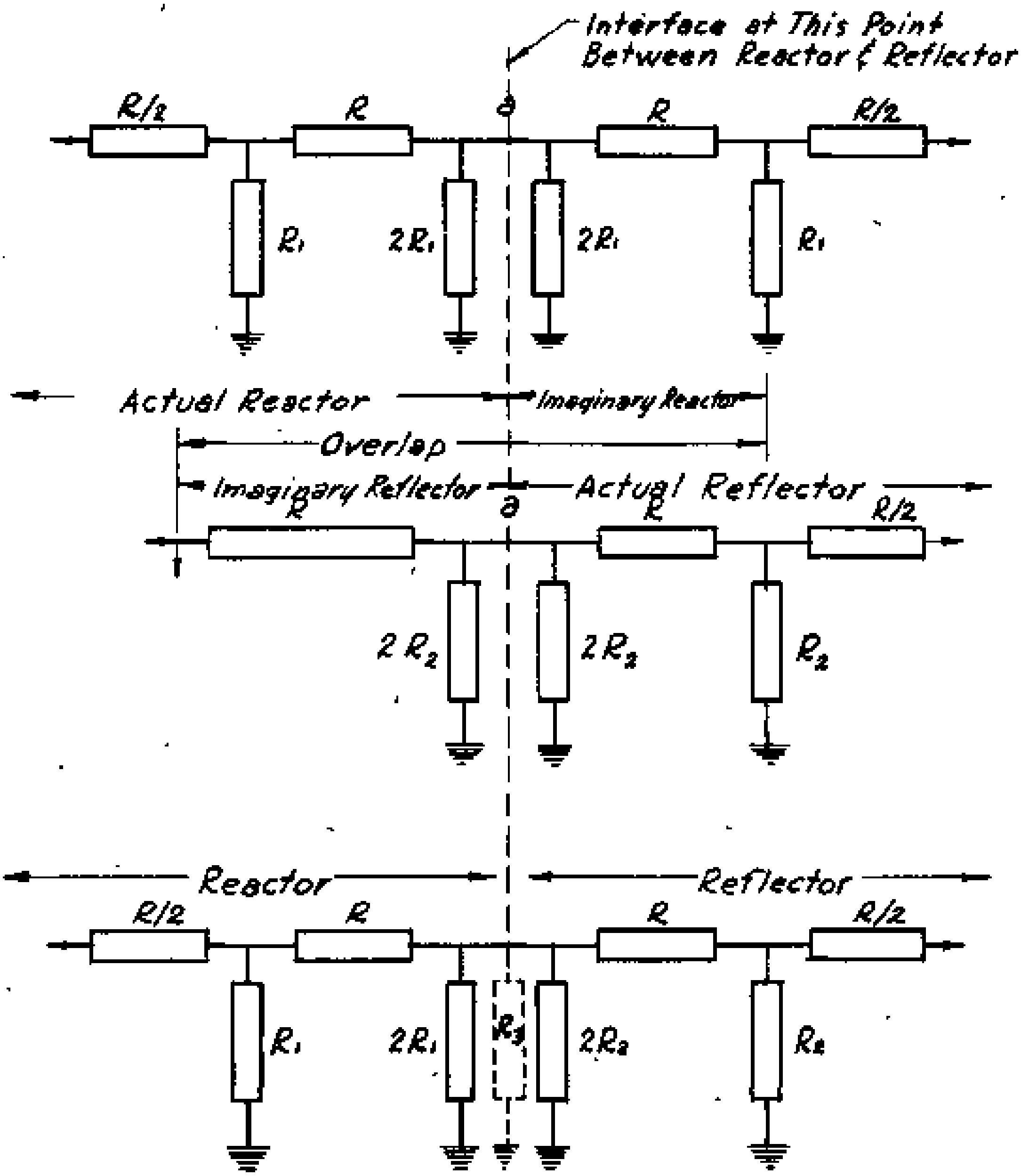
(b) Details of One Lattice Unit from
Network (a)



Lattice Unit of Reactor Simulator Network

R.P.D. A-1423 Fig. A-2

Let: R_T (Reactor) = R_1
R_T (Reflector) = R_2
R_e (Reactor) = R_e (Reflector) = R



In Place of $2R_1$ & $2R_2$ A Single Resistor Can Be Used.

$$\frac{1}{R_3} = \frac{1}{2R_1} + \frac{1}{2R_2} \; ; \qquad R_3 = \frac{2R_1R_2}{R_1 + R_2}$$

Adjustment of R_T Value at Inteface of Reactor & Reflector

R.P.D.A-1404 Fig. A-3

SECTION B

CONSTRUCTION OF EXPERIMENTAL NETWORKS

A. A one dimensional simulator is also called a network system.

1. General Description: The first test network constructed by E. J. Wade is shown on fig. B-5. This drawing contains the complete hook-up including the network and the current feeding amplifier and switch. The network is the equivalent of a slab reactor in two-group theory calculations. A slab reactor is one in which the neutron flux depends on only one coordinate. An $8\,\mu f$ condenser was connected between each point of the ϕ network and ground. The condenser together with the rotating switch and amplifier constitute the current feeding equipment for the network of resistors. As the switch revolves it connects the amplifier input and output successively to corresponding points on the ϕ and θ networks. The amplifier supplies a current directly proportional to the input voltage and independent of the load resistance. When this assembly is in contact with any pair of network points the current charges the condenser. The condenser then continues to feed current into that point of the network after the amplifier and switch assembly moves on to the other points. This process is referred to as "scanning." By using this system the necessity for having an amplifier for each network point is eliminated.

The main purpose of this network was to study the feasibility of this scanning, and to develop a suitable amplifier.

A detailed discussion of the switch and the amplifier will be given in Section C.

2. Network Resistance Values: (In all cases the available standard resistor size closest to the calculated value was used.)

$$a = \frac{\text{reactor length}}{\text{no. of lattice units}} = \frac{240\text{ cm}}{24} = 10\text{ cm}$$

A standard resistor size was selected for $R_\phi = 2.2$ megohms.

By formula 10 Section A, with $T = 250$ sq cm

$$R_\gamma = \frac{R_\phi T}{a^2} = \frac{(2.2)\ (250)}{100} = 5.5 \text{ use } 5.6 \text{ megohms.}$$

A standard resistor was then selected for R_θ = 0.047 megohms which was approximately 1/50 of R_γ then by formula 13a of Section A

$$\frac{R_\phi}{R_\theta} = \frac{2.2}{.047} = 46.8 \doteq k\,\frac{\lambda_{tth}}{\lambda_{tf}}$$

Let $\lambda_{tf} = \lambda_{tth}$. Then k = 46.8 the kappa factor.

With L^2 = 200, by formula 14a of Section A

$$\frac{R}{R_g} = \frac{a^2}{L^2}\left(1 - \frac{L^2}{T}\,\frac{\lambda_{tth}}{k\,\lambda_{tf}}\right)$$

$$\frac{0.047}{R_g} = \frac{100}{200}\left(1 - \frac{200}{(250)\ 46.8}\right)$$

R_g = 0.1 megohms

For the reflector let T = 250 sq cm, the same as in the reactor. Hence the R_γ in the reflector = the R_γ in the reactor when the lattice unit length, a, and R_ϕ are not changed.

Let L^2 for reflector be 950 sq cm

$$\frac{0.047}{R_g} = \frac{100}{950}\left(1 - \frac{950}{(250)\ (46.8)}\right)$$

R_g = .47 megohms

The boundary between reactor and reflector was made by adjusting the values of R_g at point 1 using formula (16). See fig. C.

$$R\ (\text{interface}) = \frac{2\,(100{,}000)\ (470{,}000)}{100{,}000 + 470{,}000} = 164\ 000\ \text{ohms}$$

3. <u>Measurement of k:</u> The X network system is operated by first starting the multicontact switch. The output current of the amplifier for zero signal is not exactly zero and therefore, a small initial current flows causing voltages to be established at the various points of the θ network.

The small initial amplifier current corresponds to the source of neutrons required to start a chain reaction. The value of A for the amplifier, which has been shown to represent k of the reactor in Section A, then feeds a current representing neutrons into the network as dictated by this small initial source producing a voltage V_θ. If A is large enough to represent super critical the voltage of the network rises just as the flux of the reactor increases if more neutrons are produced than are absorbed. When the V_ϕ approaches a convenient value such as 100 volts at the highest point, A is reduced until V_ϕ ceases to rise. This indicates that just enough current is being received by the network to replace the current lost by leakage to ground. This represents the steady state condition in the reactor which calls for leakage plus absorption of neutrons to be equal to neutron production. This A value is measured in amps per volt, which is conductance, and having A, k for the reactor is calculated by the formula, $A = \frac{k}{R_g} + \frac{k-1}{R_\gamma}$

for the close approximation due to "kappa factor" since the large value of R (5.6 megohms) makes the last term small, so that $k = A.R_g$.

4. Boundary Conditions: To operate the network as a core without reflector the ϕ and θ networks were grounded at the network points corresponding to the reactor boundaries.

To simulate the effect of a control rod the θ network was grounded at the point in that network at which the flux was to be zero. For the curve of fig. C-2 the network was grounded at points 11L, 12 and 11R. Each lattice unit represents 10 cm of core material hence the effect of a 1 x 1 x 20 cm rod black to thermal energy neutrons was simulated. In fig. B-1 and B-2 the effective rod dimensions are 1 x 1 x 0 cm because only one network point was grounded.

5. Experimental Results: In the test runs on the simulator k was measured for various boundary conditions. The k was then calculated by solution of the reactor equations so that the accuracy of the measurement could be determined. For the core and no control rods, the measured k was 1.16; the calculated k was 1.078. The balance of the results were taken with the insertion of control rods simulated. Various dimensions of rod were simulated and in each case the rod represents, in a slab reactor, a slab rod of 1 cm x 1 cm x some desired thickness. The thickness was varied in each case.

Figure B-1 describes a core, and a rod of zero effective thickness at the center. The measured k was 1.24; the calculated k was 1.270.

Figure B-2 describes a core with a rod of 20 cm thickness at center. The measured k was 1.32; the calculated k was 1.360.

Figure B-3 describes a core with two, zero effective thickness, rods at points 40 cm out from center. The measured k was 1.695; the calculated k was 1.825.

Figure B-4 describes one half of the core with reflector and no rods. The measured k for this core was 1.151, the calculated k was 1.06.

Each of the above figures shows the plot of the flux distribution, as simulated by the network voltages, as a solid line. For comparison with this the calculated flux distribution from an analytical solution of the reactor equations was plotted as a dotted line for B-1 and B-2.

B. Two Dimensional Simulator

1. General Description: The simulator was built to represent a cylindrical reactor of length 1 cm, and 91.44 cm over-all radius, having 60.96 cm radius reactor. For a two dimensional reactor neutron flow is considered to be in two directions only, that is, the flow under consideration is confined to a plane. The network then represents a sheet of reactor material 1 cm thick having the above dimensions. The flux distribution in such a reactor is symmetrical around the center and therefore the network was built to represent only a half circle of reactor material. The plan view of this network is shown on fig. B-9. To construct the network the circular reactor section was first drawn to scale and then divided into square sections as outlined by the dotted lines. In order to approximate a circular area with a group of small square area units, the size of the units must be small. In this case two sizes of square area units were used, so that the effect of changing network lattice size could be studied. Having laid out the square sections a network node point was placed at the center of each area, and the points were joined by resistors. Each large unit represents a volume of reactor material 15.24 sq cm and 1 cm thick and each small unit represents a volume 7.62 sq cm and 1 cm thick.

The drawing shows a plan view of the network representing one energy group whereas the entire network consists of two such arrangements of resistors with resistors R_γ between corresponding node points of the ϕ and θ netwoskks and resistors Rg to ground from each of the θ network node points.

2. Current Feeding Equipment: There are 60 node points in the section of network which represents the reactor. Each of these ϕ network node points must receive a current feed proportional to the θ network voltage at that point and also proportional to the volume of reactor material represented by the lattice unit containing that point. If the same amplifier is used for both sized volumes, the time the amplifier feeds each point must be proportional to the area represented. This was accomplished by feeding each large lattice unit four times as often as the small units were fed. The large lattice represents four times the volume of reactor material represented by the small lattice.

There are 45 feed points for small volumes and 15 feed points for large volumes so a switch, having (4 x 15) plus 45 or 105 contact pairs was used. It was decided to build these contacts on two separate disks, each disk having 70 pairs of contacts. The construction of this switch is covered in Section C. It is immediately clear that if two switches are used, two amplifiers will be necessary, one for each group of points fed by a switch. These amplifiers must be similar. It should be noted that even though the 105 pairs of contacts could have been built on one disk in this particular case, in the construction of three dimensional simulator it would be necessary to have several switches and amplifiers. Two-disk construction was used so that the problem of matching the characteristic curves of more than one amplifier could be studied.

3. Calculation of Resistor Values: It was pointed out that the lattice size is directly related to the volume of reactor represented by that network section. It is clear, then, that if a value of $R\phi$ is selected and used throughout the network the core volume represented by a network lattice unit is determined by the value of R_γ at the node point of that section of lattice. Let the pile constants for the reactor be $T = 258$, $L^2 = 121$ and $\lambda_{tf} = \lambda_{tth}$. The value of R_ϕ was first selected to be 2.25 megohms and also the value of $R\theta$, 0.047 megohms.

Selecting the values of R_ϕ and R_θ for the small lattice unit fixed the value of R_ϕ and $R\theta$ for the larger lattice units. When changing the lattice unit size the diffusion resistance per unit area must be kept the same. Referring to fig. B-9, for example at point 11, the resistance in either the x or y directions of a small unit is R for the area h^2 (7.62 x 7.62 sq cm) enclosed by the dotted line around point 11. Compare this area with the four points 11, 21, 22, 12 which form a square and are together enclosed by dotted lines of 2h cm on a side. The area contained by these lines is therefore $4h^2$. These dotted lines always bisect a resistor, so in the x direction starting at a left hand dotted area it is found that there is $R/2 + R + R/2$ or 2R ohms along the line of 21 and 11 and the same 2R ohms along the line of 22 and 12. The parallel combination is R, the resistance through which current must pass in going from the left hand boundary of the four units to the right hand boundary in the direction x. The same can be said of the direction y at 90 degrees to x. It follows logically then, that for the area of $4h^2$ a single resistance R could be installed between the two boundaries in either the x or y direction and the current passing across the section would not be changed. This is what was done in the large lattice units. For example, at point 022 $R/2 + R/2 = R$ for the resistance R in either the x or y directions. Only the x and y components of current diffusion which has been demonstrated in Section A in the development of the network formulas are considered. Current which diffuses in any intermediate direction between x and y is broken up into its x and y components. Consider that there exists a voltage at point 022 greater than that at 044. A current cannot flow directly from point 022 to point 044 if a direct conductor connection does not exist between them. However, a current does flow between these points through the balance of the network, because, by the superposition network theorem, the flow of current in any network due to any one voltage is independent of other voltages existing on the network. It is important therefore that in designing a network the X and Y direction resistances per unit square area be maintained the same, no matter what size area is considered. Since the distances in the X and Y directions are equal for square mesh, a = b = c = h = 7.62 cm for the small lattice units, from 10

$$\frac{R\phi}{R\gamma} = \frac{h^2}{T}$$

$$R_\gamma = \frac{(2.25)\ .\ (258)}{(7.62)^2} = 10 \text{ megohms}$$

Here again as in the slab reactor simulator, a " kappa factor" is introduced so that the θ network voltage will be small.

By formula 13a

$$\frac{R\phi}{R\theta} = \frac{2.25}{.047} = 47.87 = \frac{K\,\lambda\,\text{th}}{\lambda\,\text{tf}}$$

Then by formula 14a

$$\frac{R\theta}{R_g} = \frac{h^2}{L^2}\left[1 - \frac{L^2 \cdot \lambda\,\text{tf}}{Tk\ \lambda\ \text{tth}}\right]$$

$$R_g = \frac{121 \times .047}{(7.62)^2 \times \left(1 - \frac{121}{258} \cdot \frac{1}{47.87}\right)} = 0.09795 \text{ megohms}$$

use 0.1 megohms

For the large lattice units h = 15.24 cm

From 10

$$\frac{R\phi}{R\gamma} = \frac{h^2}{T}$$

Since T does not change, and R is the same for both sized lattice units,

$$\frac{R\gamma\ \text{(large mesh)}}{R\gamma\ \text{(small mesh)}} = \frac{h^2\ \text{(small mesh)}}{h^2\ \text{(large mesh)}} = \frac{(7.62)^2}{(15.24)^2} = \frac{1}{4}$$

$$R_\gamma\ \text{(large mesh)}\ \ \frac{R_\gamma\ \text{(small mesh)}}{4} - 2.5 \text{ megohms}$$

Similarly

$$R_g \text{ (large mesh)} = \frac{R_g \text{ (small mesh)}}{4} = 25{,}000 \text{ ohms}$$

Here the nearest available size was 24,000 ohms which is the value used in the network.

For the reflector, letting T = 150 sq cm, L^2 = 100 sq cm and using the diffusion resistor values of R_ϕ and R_θ which are now fixed values the following is obtained.

$$R_\gamma = \frac{(2.25)\ (150)}{(7.62)^2} = 5.813$$ megohms for the small mesh for which the value 5.6 megohms was used.

$$R \text{ (large mesh)} = \frac{5.6}{4} = 1.4$$ megohms for which the standard value 1.5 megohms was used.

$$R_g \text{ (small mesh)} = \frac{(1000)\ (0.047)}{(7.62)^2 \quad (1 - \frac{1000}{150} \cdot \frac{1}{47.87})} = 1 \text{ megohm}$$

$$R_g \text{ (large mesh)} = 0.24 \text{ megohms}$$

4. Boundary Conditions: On fig. B-9 it can be seen that the boundary between the reflector and reactor, represented by the heavy dotted line, passes between the node points of the network. As a result of this choice it is not necessary to adjust the value of R_γ or R_g at this boundary as was necessary in the slab reactor simulator when the boundary passed through a node point. The boundary of the reflector is grounded for both the ϕ and θ networks.

The method of making the connections between the large and small mesh sections must still be justified, (refer to fig. B-9). In the x or y direction the value of R for the small lattice corresponds to the distance h whereas in the large lattice it corresponds to 2h. Hence, the distance h in the large lattice requires the value $\frac{R}{2}$. In the small lattice the distance $\frac{h}{2}$ corresponds to $\frac{R}{2}$. Examining the adjacent points 0, 1 and 01 on the fig. B-9 it can be seen that the distance from 0 or 1 to 01 is the sum of $\frac{R}{2}$ for the large mesh plus $\frac{R}{2}$ for the small lattice. If the resistor $\frac{R}{2}$ for the large lattice is split into two resistors in parallel, each of value R, and these resistors each placed in series with the $\frac{R}{2}$ required for each of the small lattice points, the total resistance between either 0 or 1 and 01 is 1.5 R. This is the value used. This connection corresponds to the physical concept of neutron diffusion inasmuch as the neutrons at 01 which diffuse in the X direction must reach 0 and 1. The reverse is also true because neutrons from points 0 and 1 must flow to 01.

5. Control Rods in Reactor: The network as designed will allow simulating the insertion, into the core, of two sizes of rods, black to thermal neutrons, one 7.62 sq cm and the other 15.24 sq cm by grounding the θ network at the boundaries of a θ network lattice unit.

If, for example, a rod 7.62 sq cm is desired at point 10, the resistors R are disconnected from between points 10 and 20, 10 and 11, 10 and 0. Resistors of value $\frac{R}{2}$ ohms are then connected to ground from each of points 20, 11 and 0. The current feed to point 10 in the ϕ network must then be zero to simulate no production of neutrons in the reactor volume represented by the dotted square around 10. Likewise point 022 can be treated in the same manner. By placing resistors of $\frac{R}{2}$ ohms between 02, 024, 042, 020 and ground, a rod 15.42 sq cm is simulated.

The location of these rods is confined to areas around existing node points of the network. Generally, however, a method is desired of simulating a rod of any given size at any point in the reactor. To do this there must exist at that point a network lattice unit which represents the cross sectional area of rod to be simulated. The method of changing network lattice size described above can be extended through subsequent reductions until the size of area simulated is very nearly the area of control rod desired. The subsequent reductions in area size are, starting with an area h^2; $\frac{h^2}{4}$; $\frac{h^2}{16}$; $\frac{h^2}{64}$ etc. The value of R and Rg for each of these points must be adjusted in accordance with the formulas in Section A for each new section length, equal to $\frac{h}{2}$; $\frac{h}{4}$; $\frac{h}{8}$ etc. At the same time the current feeds to the ϕ network must be changed so that the current fed to each point is proportional to the area represented. This current feed provision is the most difficult to satisfy. There are various methods by which this can be done and they are covered in Section D.

6. Experimental Results: All experimental work to date has been done with the network simulating a reactor without control rods. For this type of reactor the calculation of the flux distribution and k by analytical solution of the reactor equation, is not a very difficult problem.

The process of operating the two dimensional simulator involves inserting a voltage source at some point on the θ network and thereby setting up some distribution of voltage in the network. This simulates the presence of a neutron source. The switch and amplifier are then started and because a current $I = AV_\theta$ is then fed into the ϕ network points, the voltage distribution becomes a function of the network resistor values and the value of amplifier transconductance A. In the simulator used for the experiments two similar amplifiers were used to feed the network through the two separate switches. The voltage distribution is seriously affected by any dissimilarity in the value of A for each of these amplifiers. The characteristic input voltage-output current curves for these amplifiers must coincide throughout the range of current feed. Similarly, the contact time for each switch must be the same because if, for some reason, one switch fed current for somewhat longer intervals than the other, that network section would receive more current. This would result in an improper voltage distribution on the network, because the current feed (I) would not even approximate the desired or design value.

The two amplifiers displayed a tendency to have a shift in their zero signal-current output and the result was a change in the slope A of their characteristic curves. For this reason it was found that adjusting the amplifiers to coincide before a run was not sufficient and that a continual check of this transconductance was needed. In practice this check consisted of installing a milliammeter and a voltmeter at representative network feed points in each section, fed by an amplifier. The current per volt at these points was measured and compared by the operator so that he could adjust the A of the amplifiers to coincide at one point on their characteristic curves. This operation is covered more fully in Section C-5. Inaccuracies due to the switch contact time could not be adjusted in all cases.

The experimental results are therefore somewhat dependent on the operator's observations during a run. It will be necessary in future work to eliminate this accuracy factor by developing a more stable amplifier and a more accurate switch.

The results obtained from the simulator are plotted on fig. B-6 and B-7. The flux distribution obtained by an analytical solution of the reactor equation is plotted on the same graph for comparison. The k measured using the simulator was 1.263 and the analytical value was 1.266.

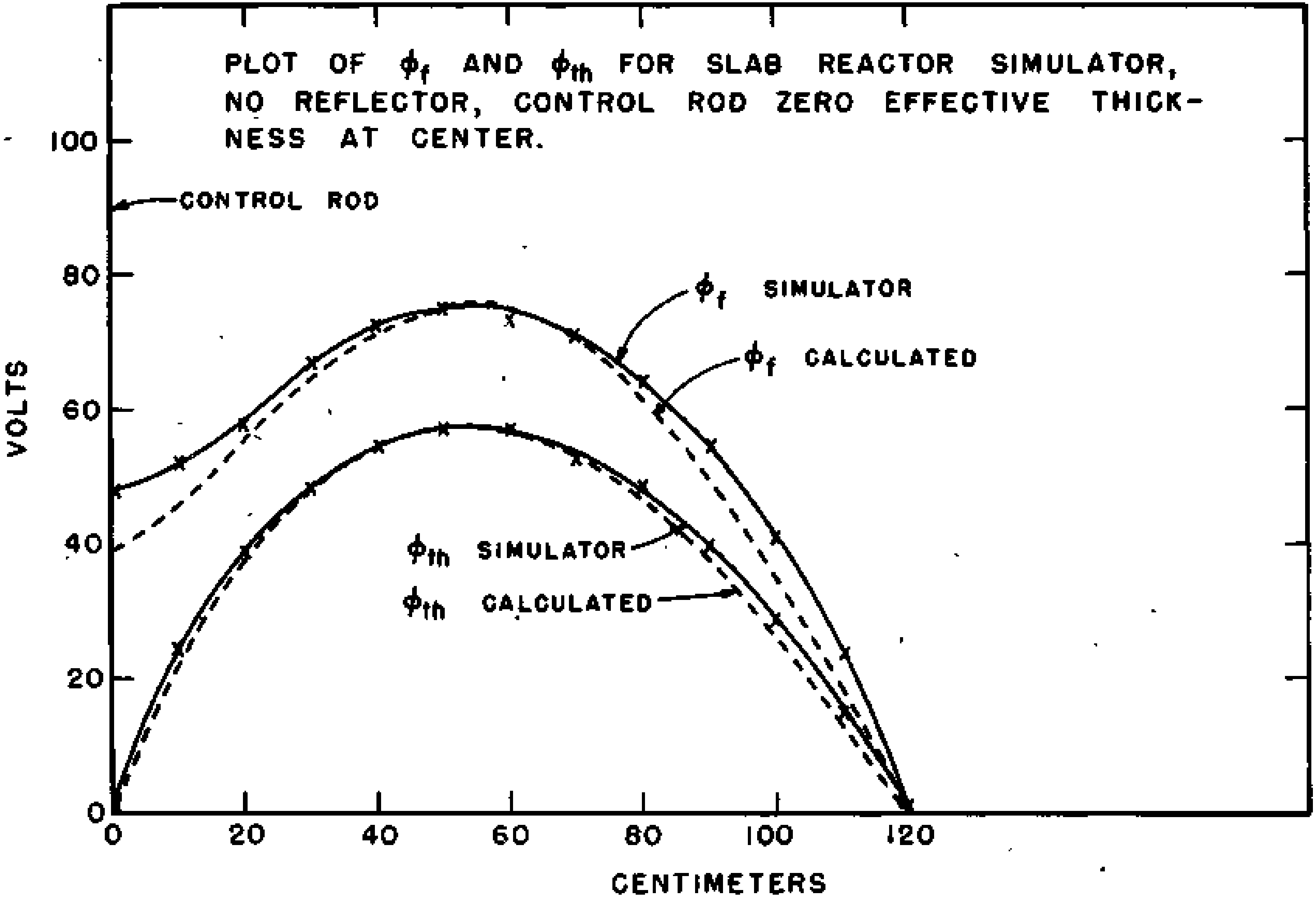


FIG. B-1

PPD-A-1425

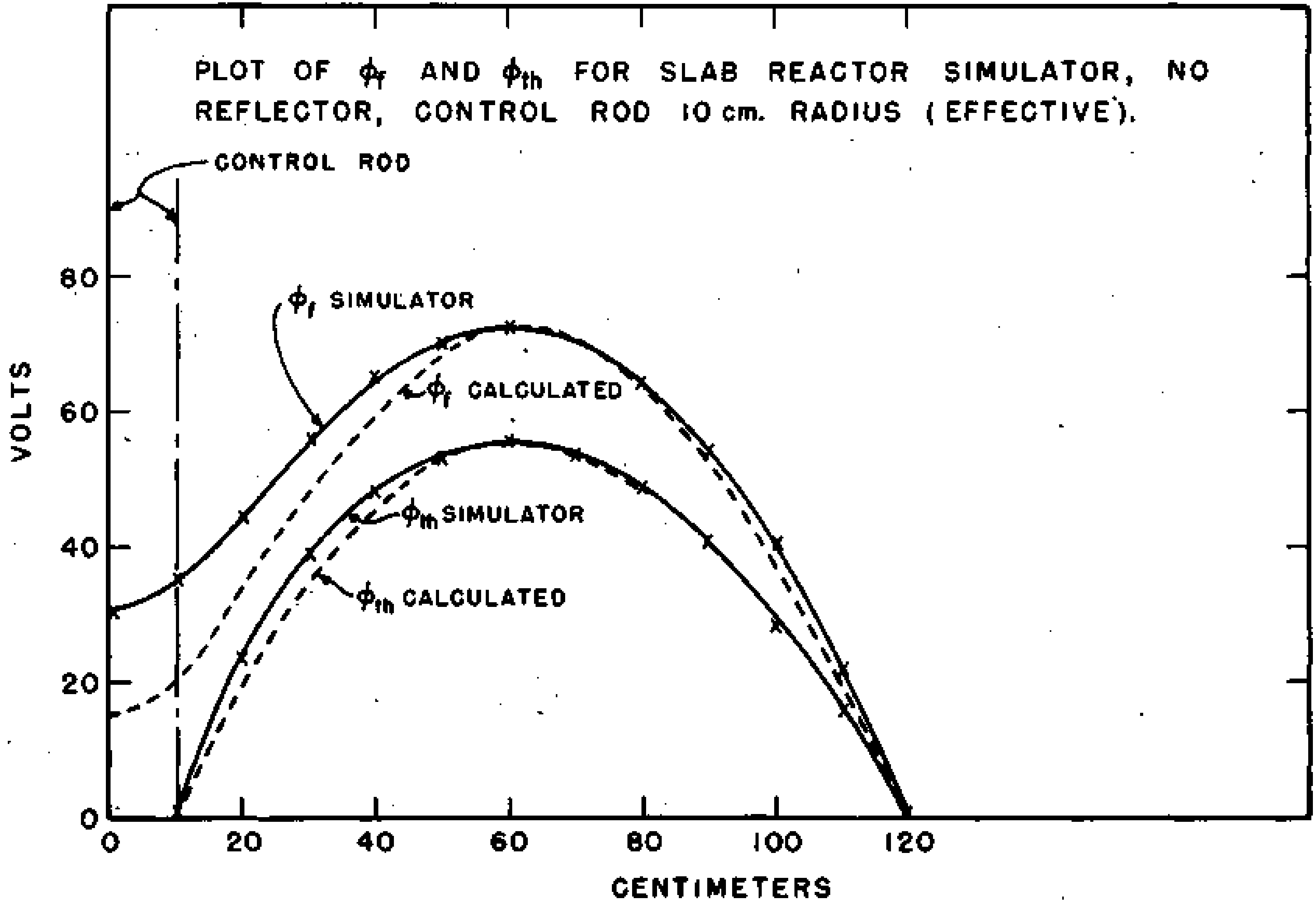


FIG. B-2

PPD-A-1426

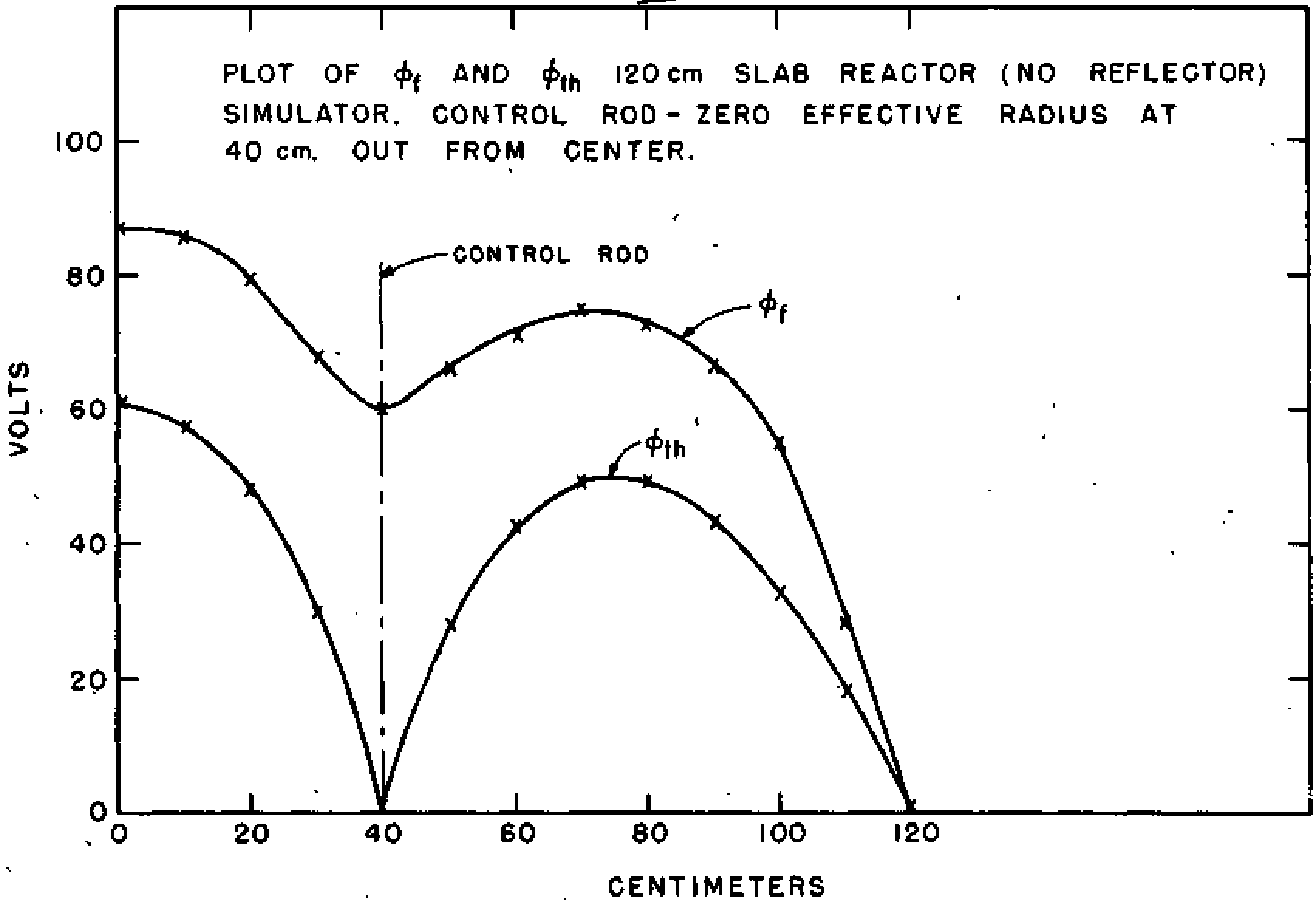


FIG. B-3

PPD-A-1424

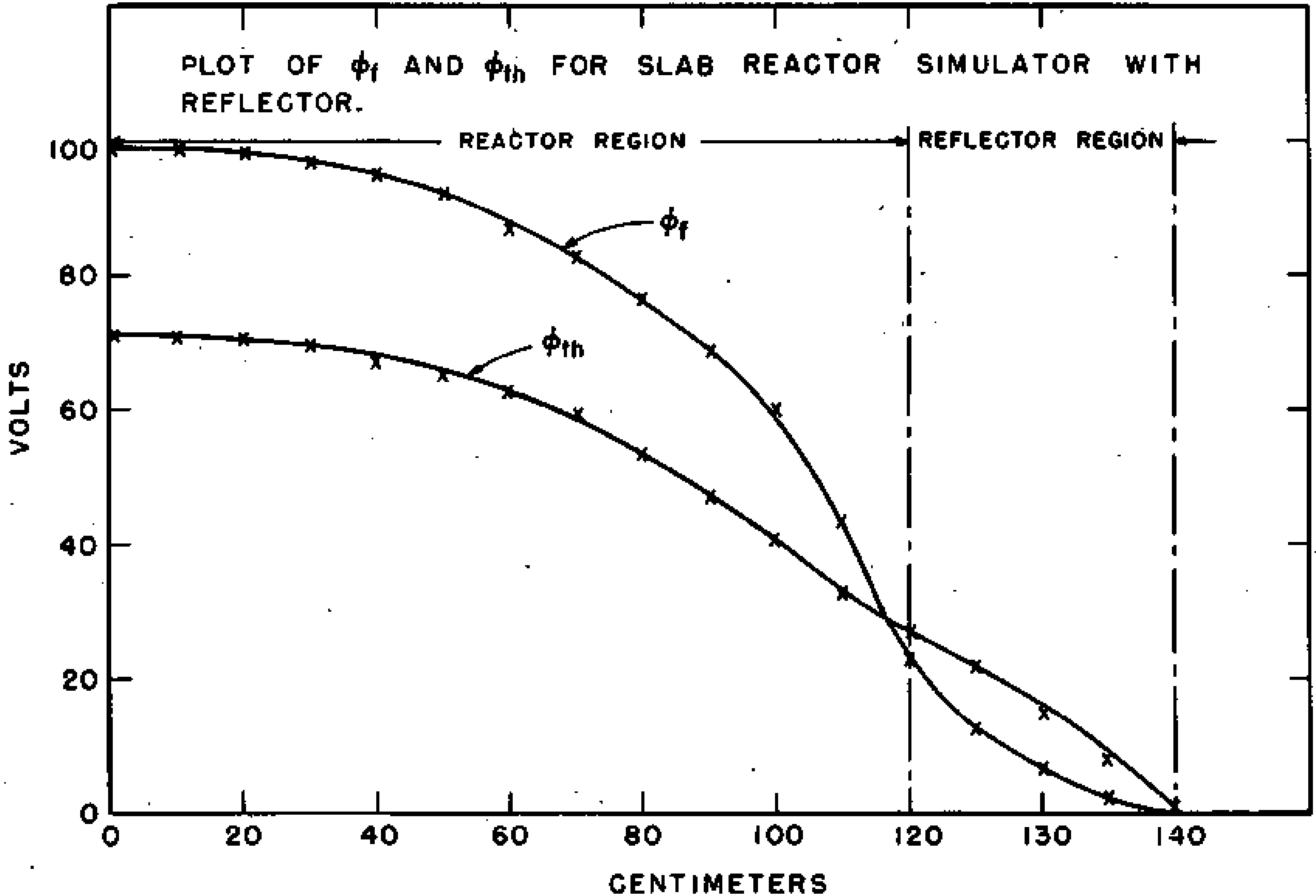
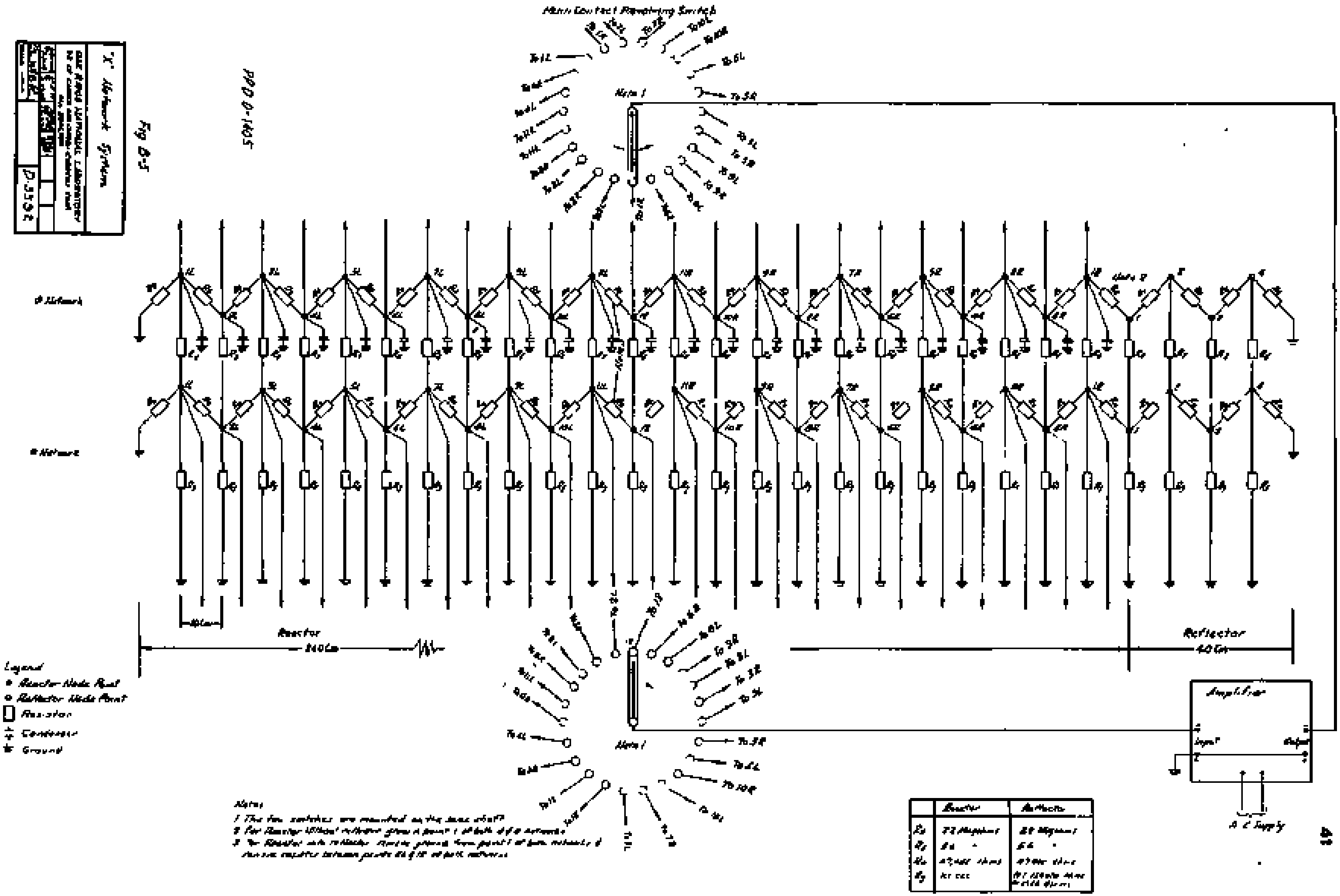


FIG. B-4

PPD-A-1423


PPD-D-1405
Fig B-5
Reactor
Reflector
Amplifier
Input
A C Supply
Legend
Resistor
Condenser
Ground
Notes
Note 1

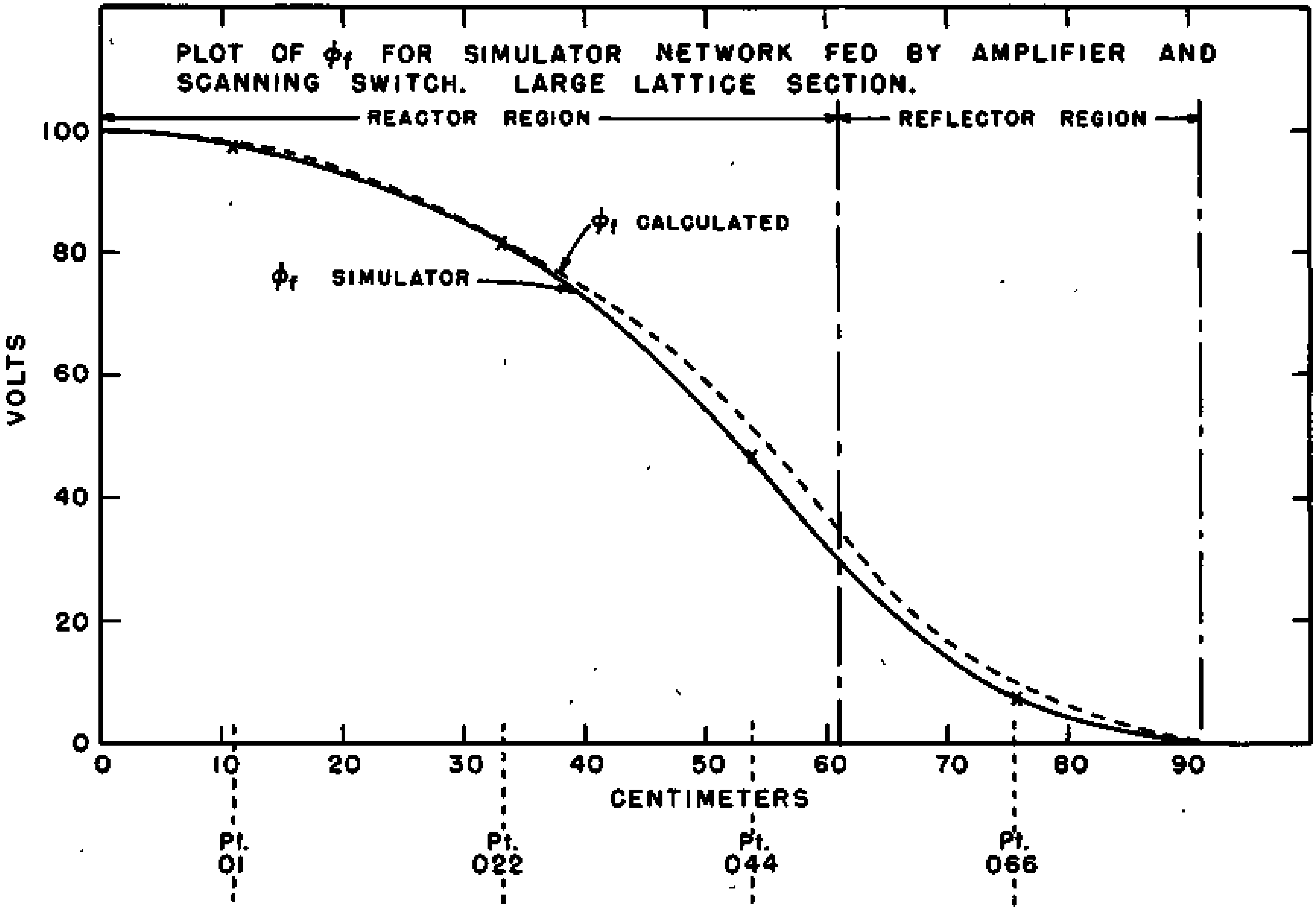


FIG. B-6

PPD-A-1397

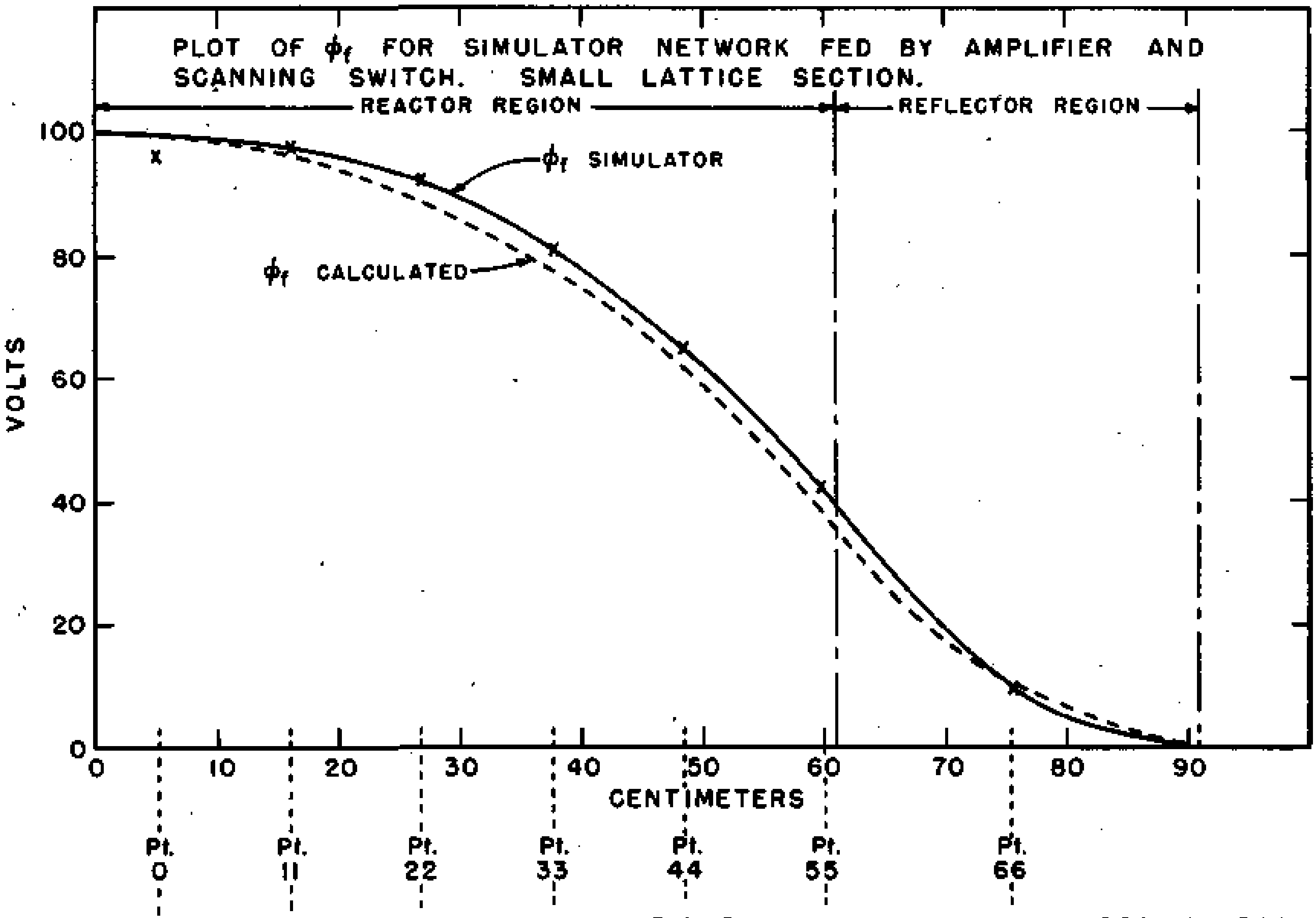


FIG. B-7

PPD-A-1398

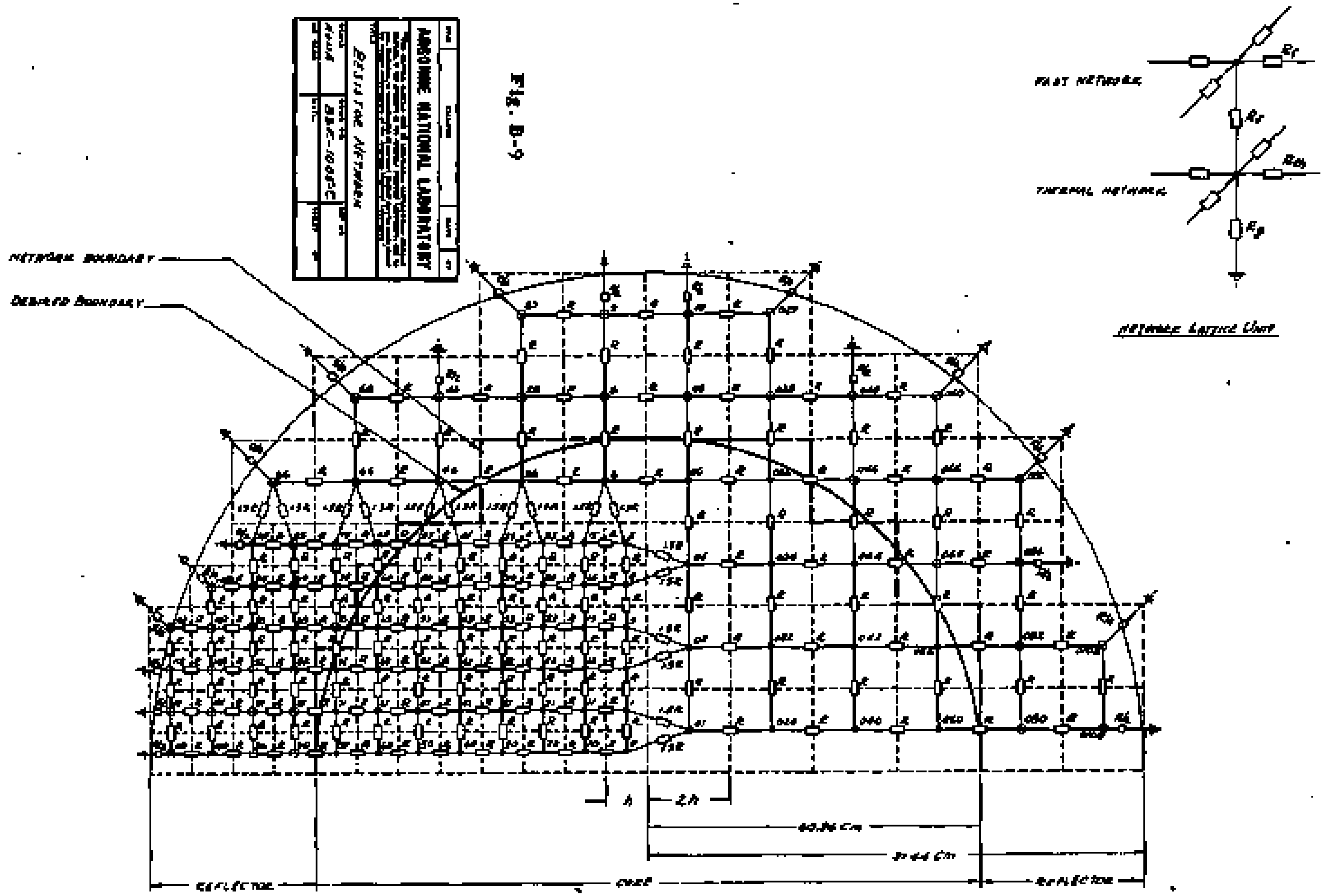


Fig. B-9

	FAST NETWORK R_f	THERMAL NETWORK R_{th}
R	2.2 megohms	47,000 ohms
R/2	1.2 "	24,000 "
R/4	0.56 "	12,000 "
1.5R	3.3 "	68,000 "

RESISTOR BETWEEN NODE POINTS OF FAST & THERMAL NETWORKS - R_R		
	CORE	REFLECTOR
SMALL LATTICE	10 MEGOHMS	8.6 MEGOHMS
LARGE LATTICE	2.3 "	1.5 "

RESISTOR BETWEEN NODE POINTS OF THERMAL NETWORK & GROUND - R_g		
	CORE	REFLECTOR
SMALL LATTICE	0.7 MEGOHMS	1.0 MEGOHMS
LARGE LATTICE	54,000 OHMS	0.24 "

SECTION C

CURRENT FEEDING EQUIPMENT

A. General

The condensers, scanning switch and amplifier constitute the current feeding equipment for the resistor network. The operation of this assembly can be studied by resolving the system into the simple circuit of fig. C-1 (a).

G is a constant current generator which represents the amplifier. The amplifier has virtually a constant current output for any particular network point when the network voltages V_ϕ and V_θ are nearly constant.

C is the 10 μf condenser between the ϕ network node points and ground.

R is the resultant network resistance from any node point to ground for a steady voltage distribution. This resistance changes as the network voltages change since the effective resistance to current flow at any particular point is a function of not only R_γ but also of the voltage difference between the node point being fed and the neighboring node points of the network.

S is one pole of the scanning switch which has an open time of 0.257 sec (to) and a closed time of 0.004 sec (tc) for the small lattice units of the two dimensional network.

I is the current which flows when the switch is closed so that $I = i_1 + i_2$ with $R \sim 7$ megohms $i_1 \gg i_2$.

i is the loop currents which flows when the switch is open. The current flow at point A is plotted in fig. C-1 (b). If I is constant the average value of V

$$\text{Vare} = \frac{R\,I\,tc}{tc + to}$$

The voltage rises to a maximum at t_1 then falls from t_1 to t_2 and rises again from t_2 to t_3 etc.

With RC large the voltage V_ϕ is nearly constant and in practice it was found that the variation in voltage due to scanning was 0.25 volts, at 100 volts on the ϕ network, or a variation of .25 per cent. This is well within the limits of experimental accuracy.

At steady state i is nearly equal to i_2 and

$$i = \frac{V_\phi}{R} = \frac{Q}{C} = \frac{\int_0^t I\,dt}{C}$$

With I constant over the period 0 to t_1

$$i = \frac{I}{c}\int_0^{t_1} dt = \frac{I\,t_c}{c}$$

Thus the current i is seen to be a direct function of the time throughout which the switch is closed. If this time is different on subsequent switch closings i and (as a result) V_ϕ will change. Furthermore, if the time tc were different for the different contacts of a scanning switch the current fed to the respective network feed points would not be a function of the amplifier current alone but would vary with the contact time. It is therefore important that the scanning switch contact time be constant so that the amplifier can perform its function properly.

For proper scanning of the large lattice unit section of the network, the intervals between feeds must be as uniform as possible or an additional voltage variation will result. To understand this, consider the action of scanning a network point four times in one revolution of the switch. In fig. C-1 (c) the time base is one revolution, tc and to are the closed and open times respectively for a contact, and 1, 2, 3, 4 are subscripts denoting the four separate contacts which feed current to the network point.

The voltage rise due to the current feed of contact number 1 is 0.25 volt. For the small lattice units the voltage decay is very slow while for the large lattice units the decay is four times as fast. In fig. C-1 (c) the four feed contacts have been assumed to be spaced evenly, and therefore the peak of the saw tooth wave is always at the same voltage. In the two dimensional simulator the contacts were actually spaced as shown on fig. C-2. Contacts numbers 1 to 2, 2 to 3, 3 to 4 have 13 segments between each, and between numbers 4 and 1 there were 31 segments. The result of this unequal spacing was a variation from maximum to minimum voltage of about 0.35 volts. In future construction more evenly spaced contacts would be desirable.

B. Scanning Sequence

The sequence in which the feed points of the networks are fed by the amplifier has a significant effect on the voltage distribution. It was found in the work with the slab reactor simulator that scanning the points in rotation from one end of the network to the other resulted in an improper voltage distribution whereas, if the network was fed as shown on fig. B-5, the voltage distribution more nearly simulated the calculated flux distribution.

Feeding adjacent network node points consecutively is not desirable because the network voltage of points around the point which has just been fed are at the peak of the saw tooth of fig. C-1. Since the current fed to a point is proportional to the voltage at that point, the current fed will be higher than it would have been had some time elapsed for the voltage to decay before the current feed was applied.

In the two dimensional simulator the scanning sequence was selected so that the points were fed almost at random. The sequence is shown in fig. C-3 for the large lattice section. The sequence for the section containing small mesh is as follows:

Switch segment number 1 was connected to network point number 73:

SW Point	Net. Point	SW Point	Net. Point	SW Point	Net. Point
1	73	25	26	49	61
2	70	26	55	50	45
3	62	27	72	51	71
4	44	28	64	52	-
5	6	29	54	53	-
6	15	30	60	54	-
7	26	31	52	55	-
8	50	32	34	56	-
9	33	33	5	57	-
10	40	34	41	58	6
11	3	35	23	59	-
12	21	36	31	60	26
13	1	37	20	61	-
14	0	38	10	62	-
15	11	39	12	63	-
16	2	40	22	64	-
17	30	41	6	65	-
18	13	42	26	66	-
19	4	43	32	67	-
20	24	44	14	68	-
21	51	45	42	69	-
22	35	46	43	70	-
23	6	47	25		
24	63	48	53		

C. Time Factors

The time required to charge the condensers connected to the network determines the time required to stabilize the network at any desired voltage. This time factor has no connection with the reactor transients.

When the network was completely de-energized and all the capacitors were discharged it was found that the time required to establish steady state network voltage of 100 volts peak value was about 15 minutes. This 15 minutes was not a fixed figure but the average time required by the operators who worked with the simulator. If the amplifier were applied to a completely de-energized network, where there was no V_θ voltage, the theoretical current fed is zero, just as in a reactor, no fissions take place unless a neutron source is present. Actually, the current output of the amplifier was never exactly zero so some current would flow, in the order of 5×10^{-6} amps, and the cycle of production could start. The voltages increase as long as A is above the critical value, in which case a super critical k exists for the reactor simulated. A must, of course, be reduced to the critical value when the desired peak voltage has been established. If this small initial current were depended upon in establishing the network voltages for a run the time consumed in reaching a peak voltage of 100 volts would be very long unless A was set far above the critical value. With A set only slightly super critical a long time is required and the tendency is for the operator to set A very high to speed up the charging process. If this is done the V_ϕ voltages rise rapidly as the V_θ voltages become larger. To prevent the V_ϕ voltages reaching the maximum voltage output of the amplifier A must be reduced to slightly above critical as soon as the V_ϕ voltages begin to rise rapidly.

The amplifier characteristic is not $I = A\, V_\theta$ at load voltages above about 180 volts and the current feed requirement is not being met by the amplifier in this region of saturation so the system does not simulate the reactor. It was found that the best way to start up the simulator was to apply a V_θ voltage at the network node point where the peak voltage would logically occur and hold the value of A just slightly above critical during the buildup time.

The possibility exists that the V_ϕ voltages will rise to the voltage saturation point of the amplifier during a run as well as when starting up. With the network energized and A set at the critical value the system is theoretically stable. Actually this is not the case. A slight change in A due to a shift in amplifier characteristics caused the network voltage to fall or rise. If the value of A was returned to the critical value immediately the voltages fell to zero or rose to the maximum voltage dictated by the amplifier. This rise or fall was slow if only a small shift from critical took place but none the less the voltages changed and the voltage distribution could not be read accurately. To prevent this voltage drift it was imperative therefore to include in the amplifier a means of providing a small step change in A which could be controlled by the network voltage. If the V_ϕ voltage rose above the desired peak value a set of voltage sensitive relay contacts were opened and a resistance was inserted in the A control circuit. With a fall in voltage the reverse took place. The critical A value then was between the high and low value applied through this control. The value of A must be determined with instrument readings taken at the network, namely, average I fed to a point divided by the average V_θ for that same point. The total change in I and V_θ for the network used in the tests was so small that hardly any change was noticed on the instruments. It was therefore not difficult to take readings which represent the actual current per volt fed to the network.

The operator must have the ability to adjust the network to operate at a desired peak voltage, hold it there by adjusting the voltage sensitive control relay, and then read the instruments properly. This ability is not difficult to acquire through a reasonable amount of experience with the simulator system.

D. Scanning Switch

The original scanning switch for the slab reactor simulator was made by inserting pins in a lucite sheet about 1/2 in. thick. Twenty-four pins were spaced evenly around a circle and a contact, carried on a motor-driven arm, was pivoted at the center of the circle. See fig. C-4.

The contact spring was set so that it did not touch two adjacent pins at the same time.

It has been pointed out that uniformity of contact time is very important. Several factors contributed to the poor uniformity of this switch: the contact bounced when the spring hit a pin; the spring tension changed during operation; all the pins were not set absolutely vertical to the mounting sheet, and were not evenly spaced. The ϕ network connected to one ring of pins and the θ network to the other as shown on fig. B-5.

An improved design was developed and, since the two dimensional simulator was in view at the time, it was made with 70 pairs of contacts on each of two contact discs. The complete assembly is shown on fig. C-5, which is a photograph of the switch and network assembly. Figure C-6 shows the detail of the contact assembly. A machine screw held each contact in place on the lucite disc. A wire was connected from each contact to the network. The outer row of contact was connected to the ϕ network points and the inner row was connected to corresponding θ network points. The lucite between each contact was machined flush with the metallic contact surface. A revolving contact arm carried two spring steel contacts which slid over the rings, contacting a corresponding pair simultaneously. Electrical connection was made to these moving contacts through two slip rings which were in turn connected to the amplifier input and output. One side of the network and amplifier was connected to ground to complete the circuit.

The contact time uniformity for this switch was better than for the original switch but was still far from the uniformity required. Figures C-7 and C-8 are photographs of the switch contact operation as shown on a cathode ray oscillograph. In these pictures a slow vertical beam sweep was coupled with a fast horizontal sweep to make successive horizontal lines, which start at the bottom of the scope screen and move upward. The switch contacts when closed caused the blank spaces in these lines. The break in the bottom line represents one contact and the next break above it is the next contact on the disc. The switch was turning at about 230 rpm for fig. C-7 and about 115 rpm for fig. C-8.

The large variation in contact time is easily seen from these two pictures. Although they do not contain all the contacts they show a representative sample of the contact action. The contact time variation was over 100 per cent from the shortest to the longest contact time shown in the photograph. Poor machine shop accuracy caused this variation. It is not likely that this type of switch can be constructed accurately enough to meet the needs of the simulator.

Several ideas were advanced for further improvements and some test work has been done with a view to developing a better switch. This work is described in Section D of this report.

E. Amplifier

It has been pointed out that a current must be introduced at each network feed point in the ϕ network which is proportional to the voltage of the corresponding θ network point. The same amplifier was used for both the slab reactor and the two dimensional simulator.

The ideal input-output relation for this amplifier is a straight line $I\ (\text{out}) = A\ V_\theta\ (\text{signal})$.

The circuit developed by E. J. Wade is shown on fig. C-9. This drawing covers the amplifier circuit alone. The equipment included a standard d-c power supply which is not shown. An additional circuit called a compensator was built onto the same chassis but never used in operation on the simulator. This circuit is not shown in this report. The purpose of this circuit was to compensate for the zero drift of the amplifier by changing the voltage across R5 of the amplifier input circuit. Compensation for zero drift in the amplifier was needed but this particular method did not prove satisfactory.

The power supply was not as ripple free as was desirable and the circuit of fig. C-9 was changed to that shown on fig. C-10, in which the three OB-2 voltage regulator tubes were installed in place of the voltage divider. In addition the 500, 0.5 megohm potentiometer was inserted in place of the fixed resistor in fig. C-9. This gave more range to zero signal adjustment. For operation on the network the amplifier was so adjusted that, at zero voltage input, the output current was approximately 5×10^{-6} amp for all settings of the A control. The desired zero signal current was zero, but, since this point of the characteristic curve was not used in operation, the small zero current was tolerated. In setting the zero current it was important not to suppress the zero point. If this was done a considerable voltage signal was required to produce any current flow. Zero must be set where the output is just 5×10^{-6} amp at zero voltage signal. The test curves of this amplifier have been plotted on fig. C-11. From these curves it can be seen that the effect of changing load is small but that the current flow for a given signal voltage is not entirely independent of the load. For signal voltages above 0.1 volts the curve was almost a straight line. The maximum voltage output for the amplifier was 197 volts and it was necessary, therefore, to operate the network at voltages well below this value. The amplifiers were usually adjusted with the output feeding current to ground. This is nearly the correct loading because the 10 μf capacitors connected to each node point make the effective load on the amplifier almost purely capacitive. The main difficulty with this amplifier was the fact that the zero setting shifted and would not remain constant long enough to complete a run. With the zero set at 5×10^{-6} amp at the start of a run it was found to be 50×10^{-6} after a run. Shifting the zero point caused the characteristic curve to shift. This affected the stability of the network voltages. The net effect of this zero current change on the accuracy of the simulator could not be evaluated during the experimental runs because of the difficulties which existed in the scanning switch. An amplifier is required which minimizes the effect of changes in tube characteristics. Such an amplifier would give more stable simulator operation. From the foregoing discussion of the characteristic of one amplifier it can be easily understood that some difficulty would be encountered in trying to use two amplifiers to feed different sections of the same network. For such an operation the characteristic curves for the two amplifiers should coincide for all values of the A control setting. Here again any change in zero current caused the curves not to coincide and the network section fed by each amplifier did not receive the same current as was specified in the simulator theory, $(I = A\ V_\theta)$.

Suppose the characteristic curves coincide for both amplifiers at start of a run and are shown by line 1 of fig. C-12. Line 2 shows the effect of a change in zero current for one amplifier. The line is almost parallel to the original line but this amplifier then supplies a current per volt which is higher at all points throughout the operating range than its mate. Too high a voltage is then developed on the network section fed by this amplifier and the voltage distribution on the whole network becomes distorted, the peak voltage being shifted toward the section fed by the amplifier having the higher current output per volt. If the A control is then changed for this amplifier, the slope of its curve will change as shown by line 3, causing curves for the two amplifiers to cross or coincide at one point. If this point (X on the figure) is near the center of the operating range the characteristic curves will almost coincide over the operating range for the two amplifiers. In order to make this adjustment during a run an ammeter was installed in the line feeding current to the network node point 022 in the large lattice section and 22 in the small lattice section (see fig. B-9). The current per volt for these two network node points was then adjusted by the operator so that they were the same.

F. Conclusions

In this section the inherent inaccuracies of the current feeding equipment have been pointed out. No evaluation of the effect of each was made since the net effect of all errors introduced by the various elements was the only real concern. A complete investigation of all possible current feeding methods would include:

1. Scanning with an electronic multivibrator circuit in place of the mechanical switch with an amplifier suitable for that type of scanning.

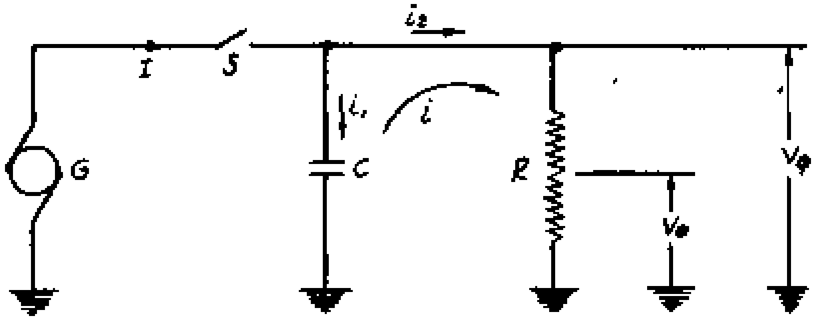
2. Feeding current with manually adjusted variable resistors connected to each feed point from a source of voltage. This is the Iteration Method described in Section D.

3. Development of an automatic system of adjusting the variable resistors (similar to 2) using a servo system.

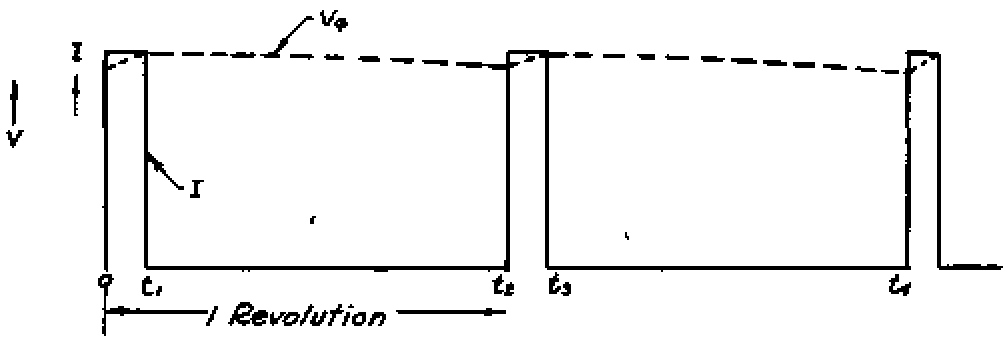
4. A vacuum tube, for each feed point, whose conductance is changed by scanning the grid circuit of the tube with a voltage signal amplifier.

5. Combinations of the above methods. For example the amplifier and scanning switch method can be used to feed the network proper and then a section requiring fractional feed points can be fed by a system of variable resistors.

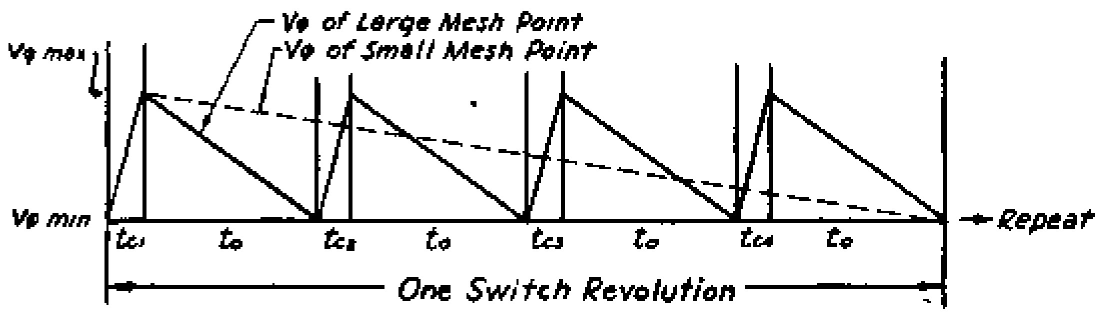
The problem of properly designing a current feeding system for the network could only be solved by constructing the network of resistors, in which no apparent difficulty has been found, and then building up the best possible current feeding system and trying it out. The requirements of this current feeding equipment are now more clearly understood. Almost all the time spent on the simulator was spent in the development of the current feed and it is felt that a suitable system can be devised as a result of this endeavor. Section D which follows shows some of this work in progress, and current thinking on this problem. None of the work can be considered complete and therefore no final design of a current feed system is recommended in this report.



(a) Equivalent Circuit



(b) Node Point Scanning



(c) Node Point Fed Four Times Per Switch Revolution.

Node Point Voltage Variation As A Result of Scanning Switch Operation. P.P.R. A-1407 Fig.C-1

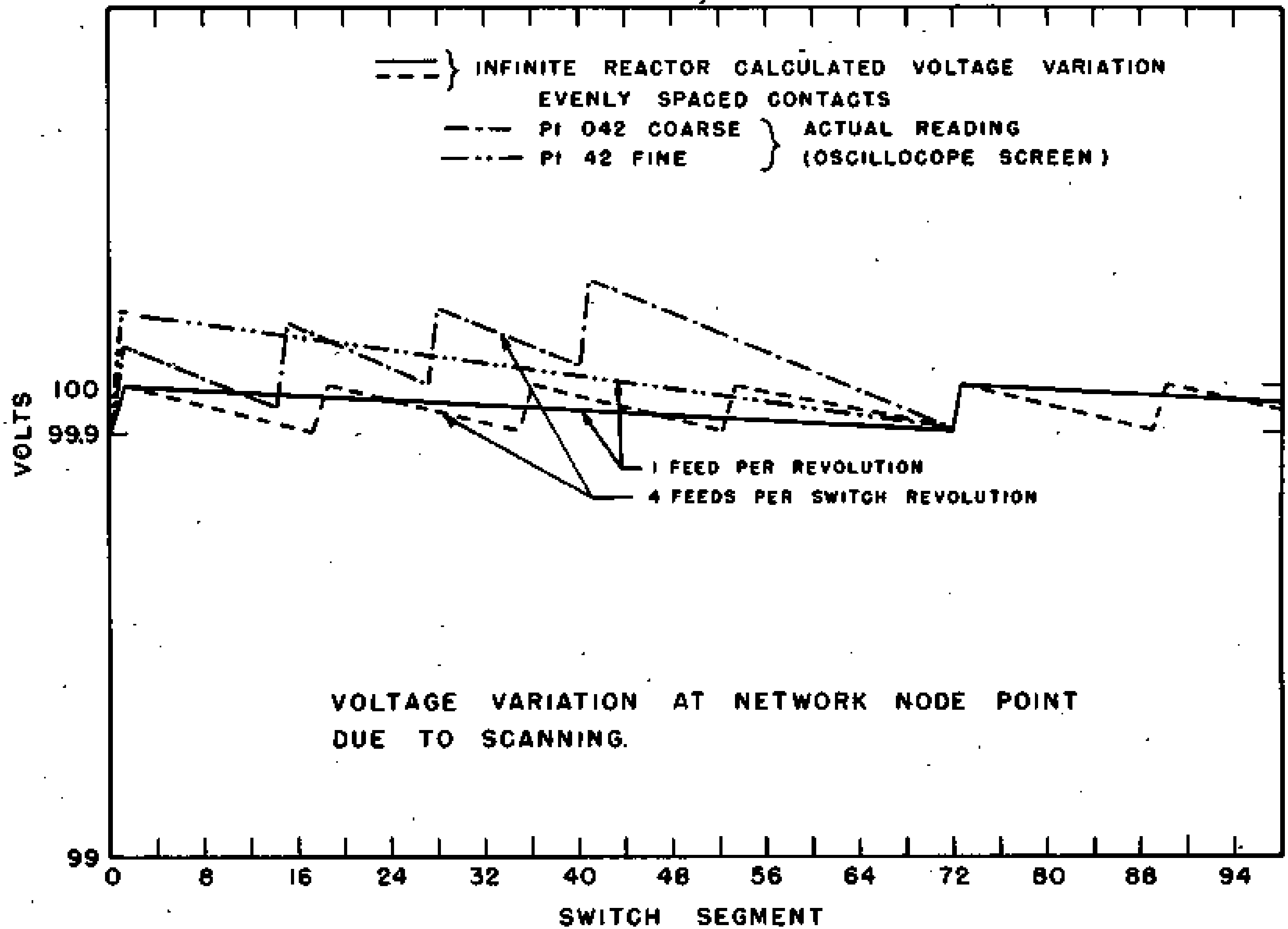


Fig. C-2

PPD-A-1408

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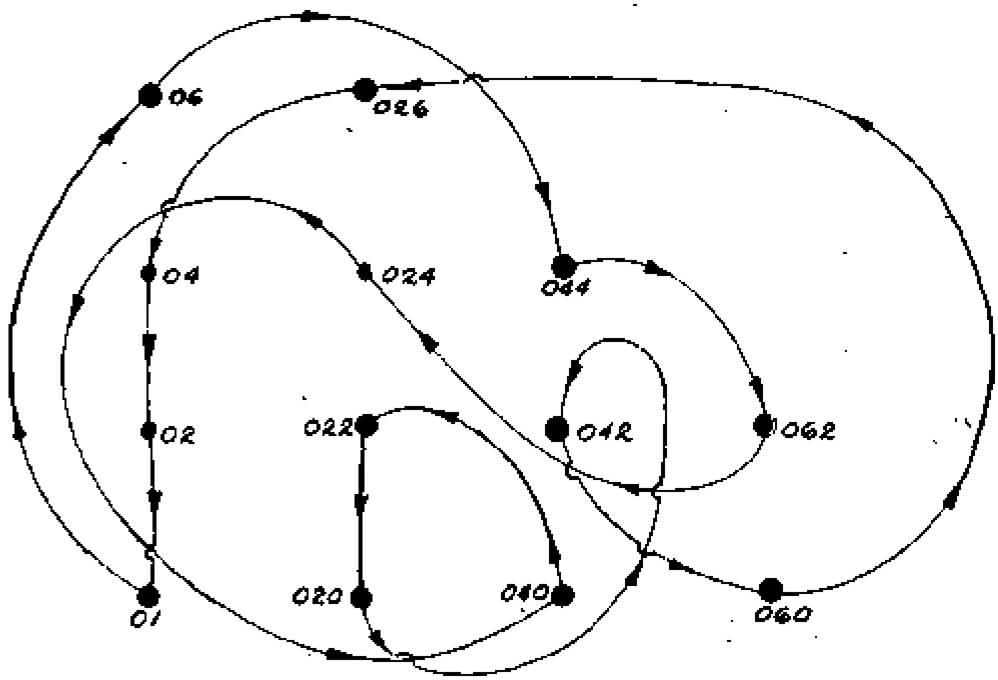


Legend:

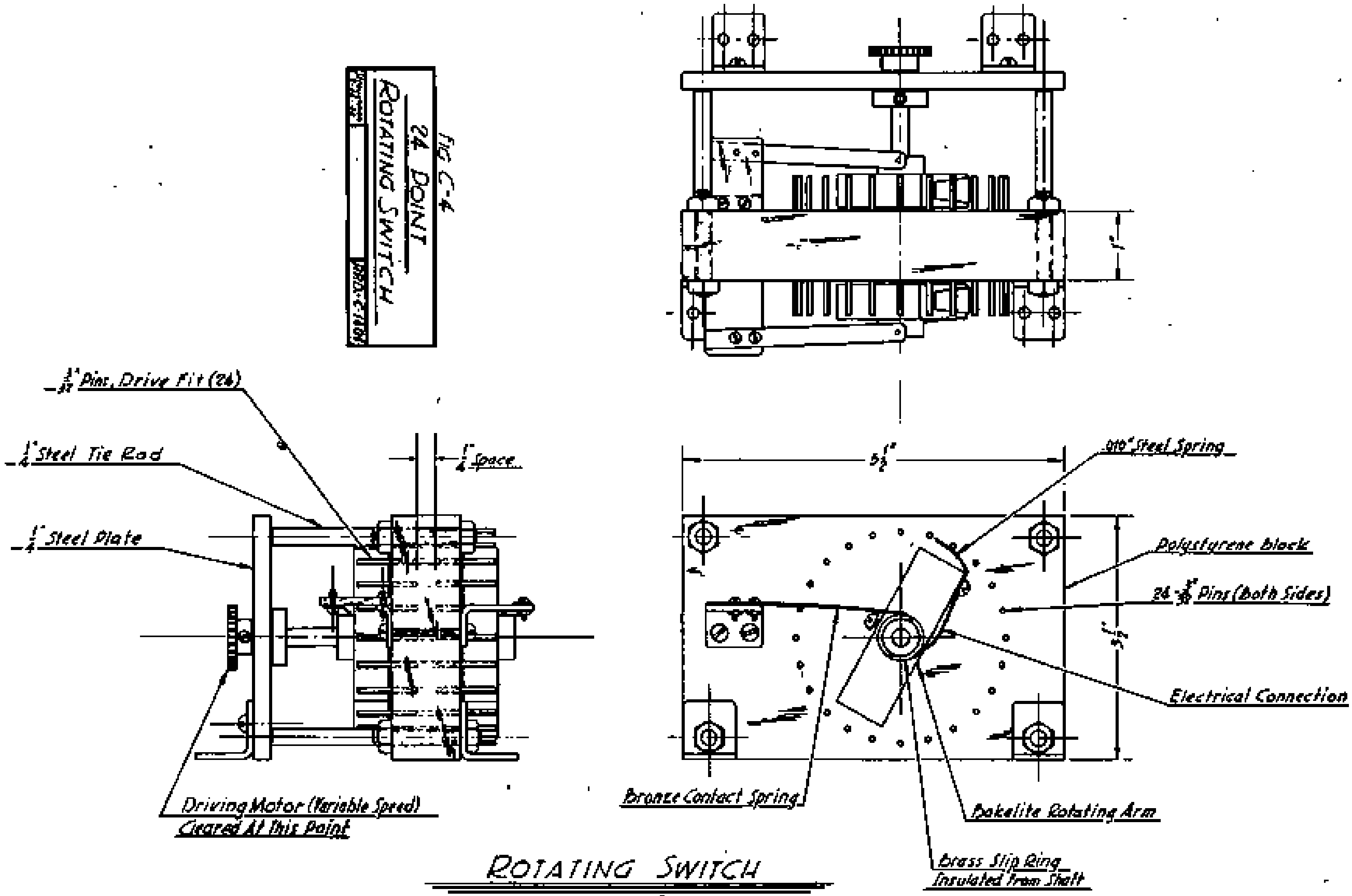
● Node Point in Large Lattice Section of Network
Sequence of Scanning Switch. (See Fig. B-9

Note: Sequence Repeated 4 Times in 1 Switch
Revolution.

Scanning Sequence Large Lattice
Network

P.R.D.A-1409

Fig. C-3



ROTATING SWITCH

SCALE: FULL

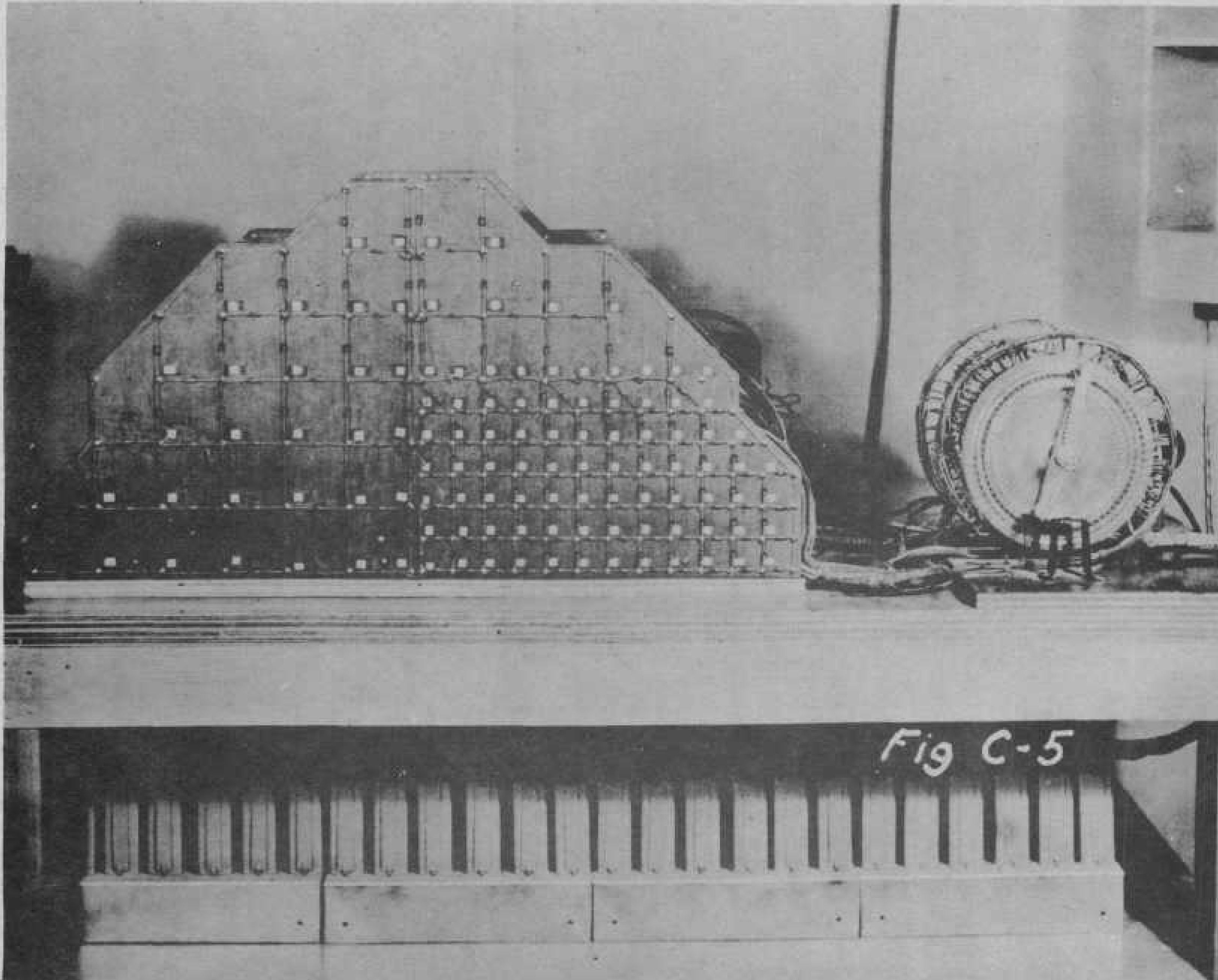
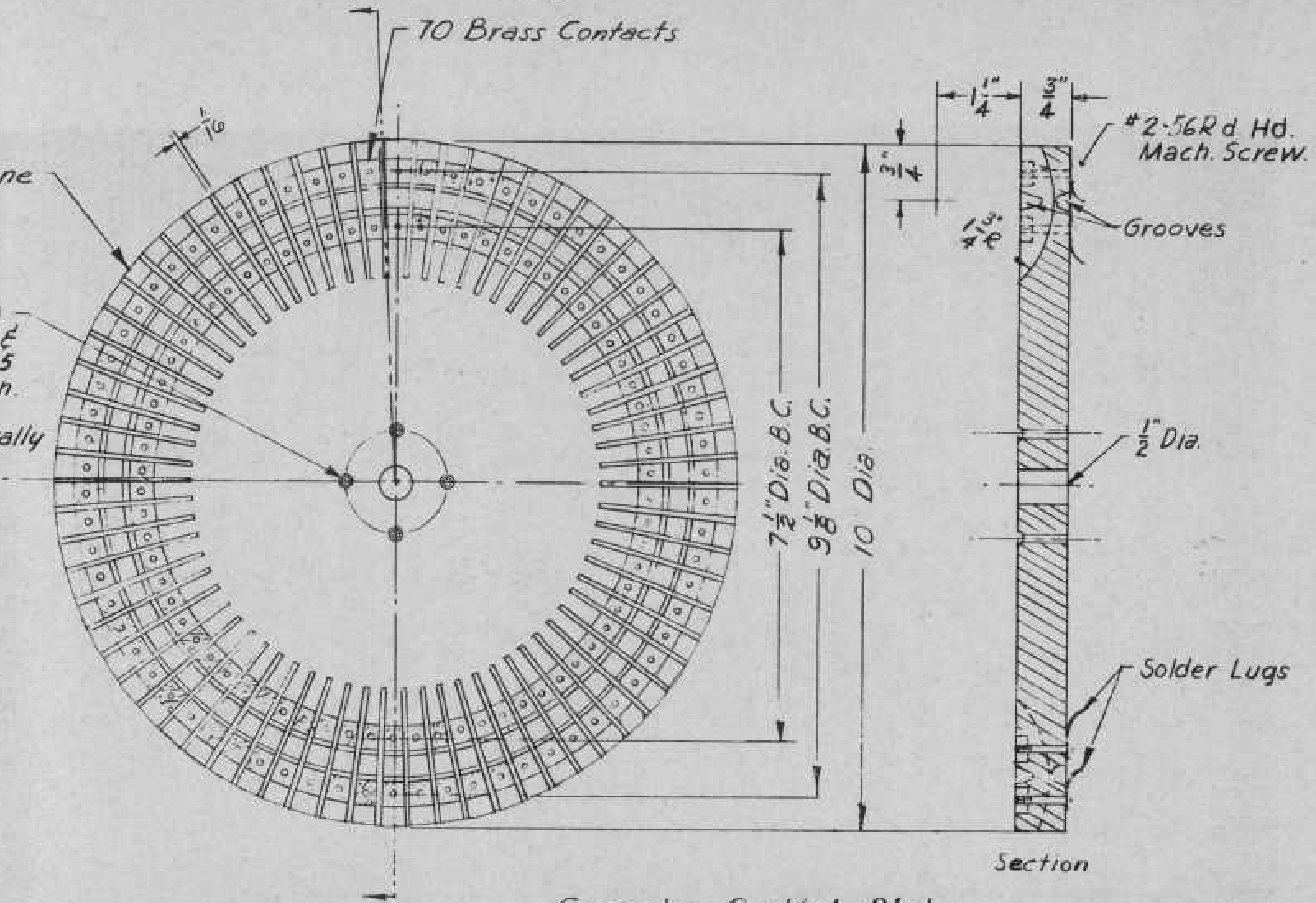


Fig C-5



Scanning Switch Disk

R.P.D. A-1410 Fig. C-6

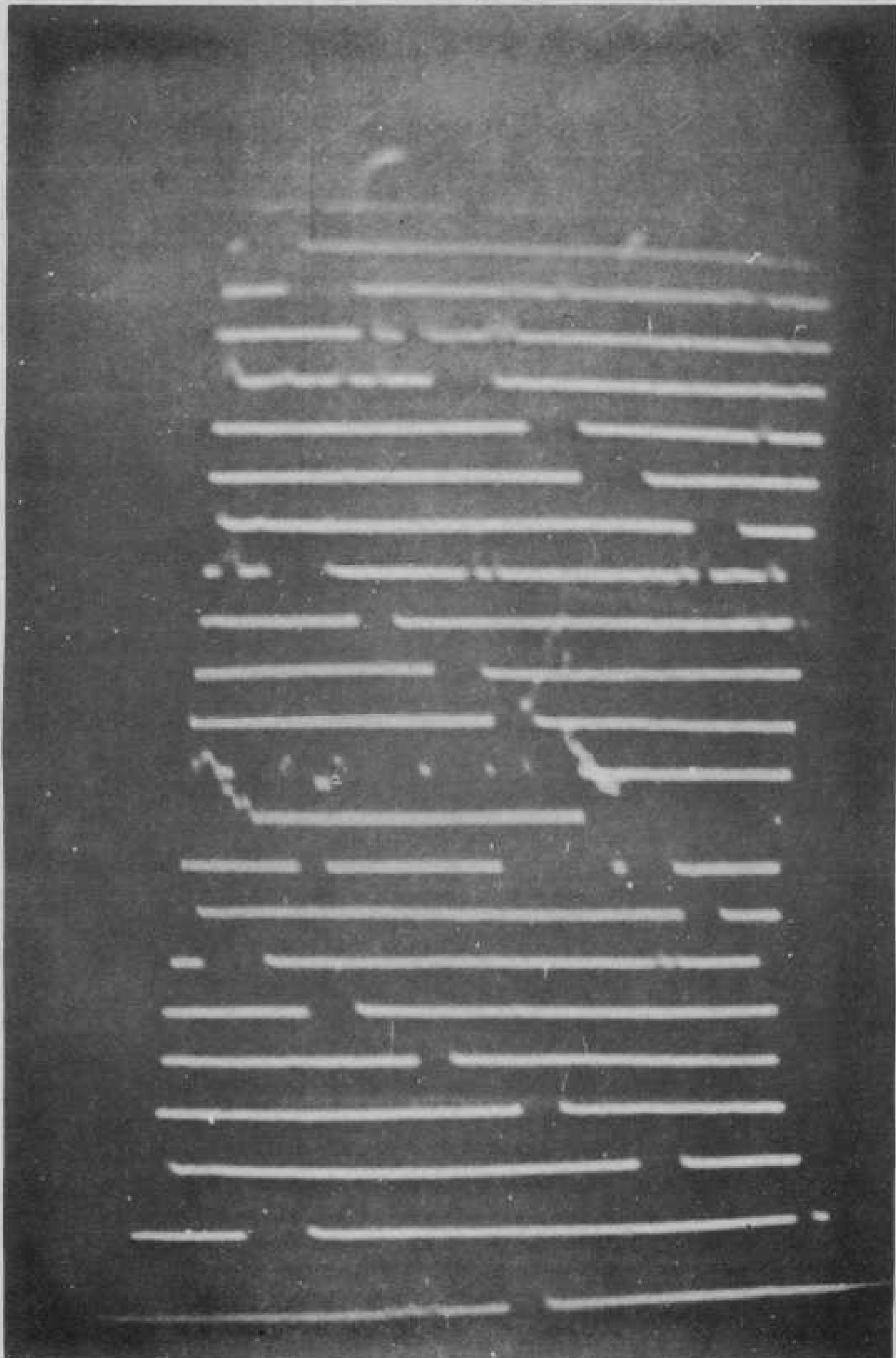


Fig. C-7

60

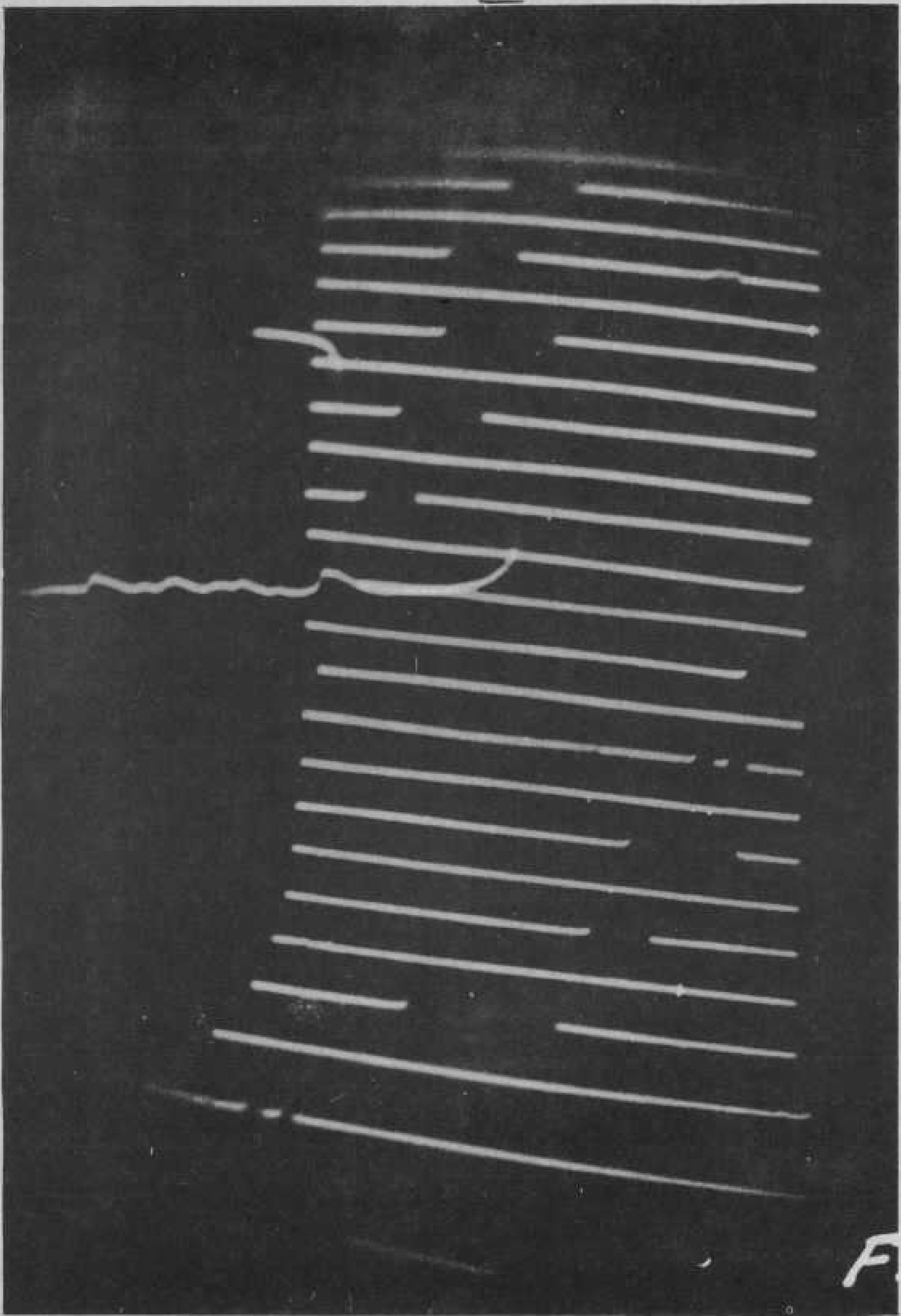
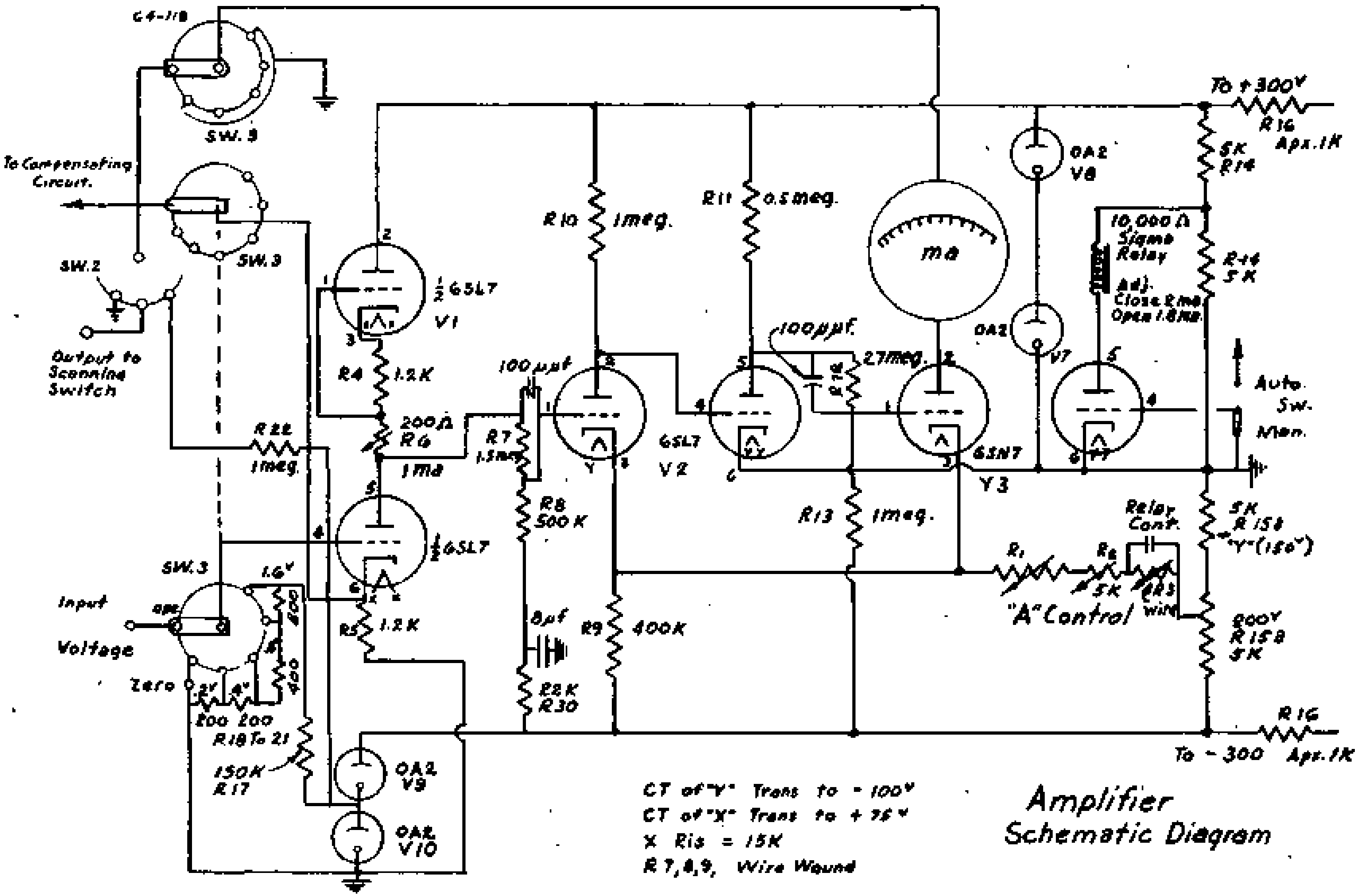


Fig. C-8



Amplifier
Schematic Diagram

P.P.D.A-1411 Fig. C-9

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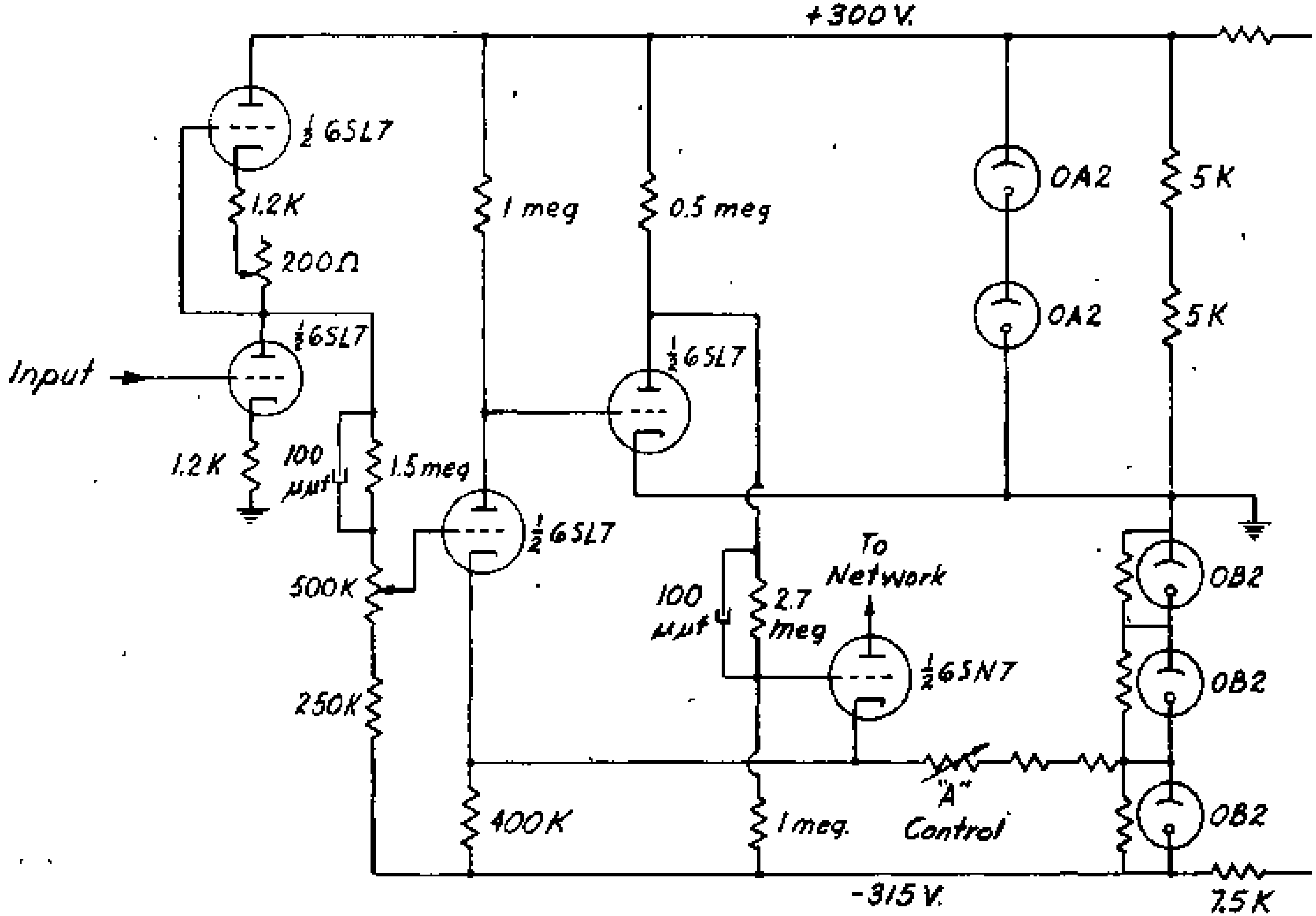
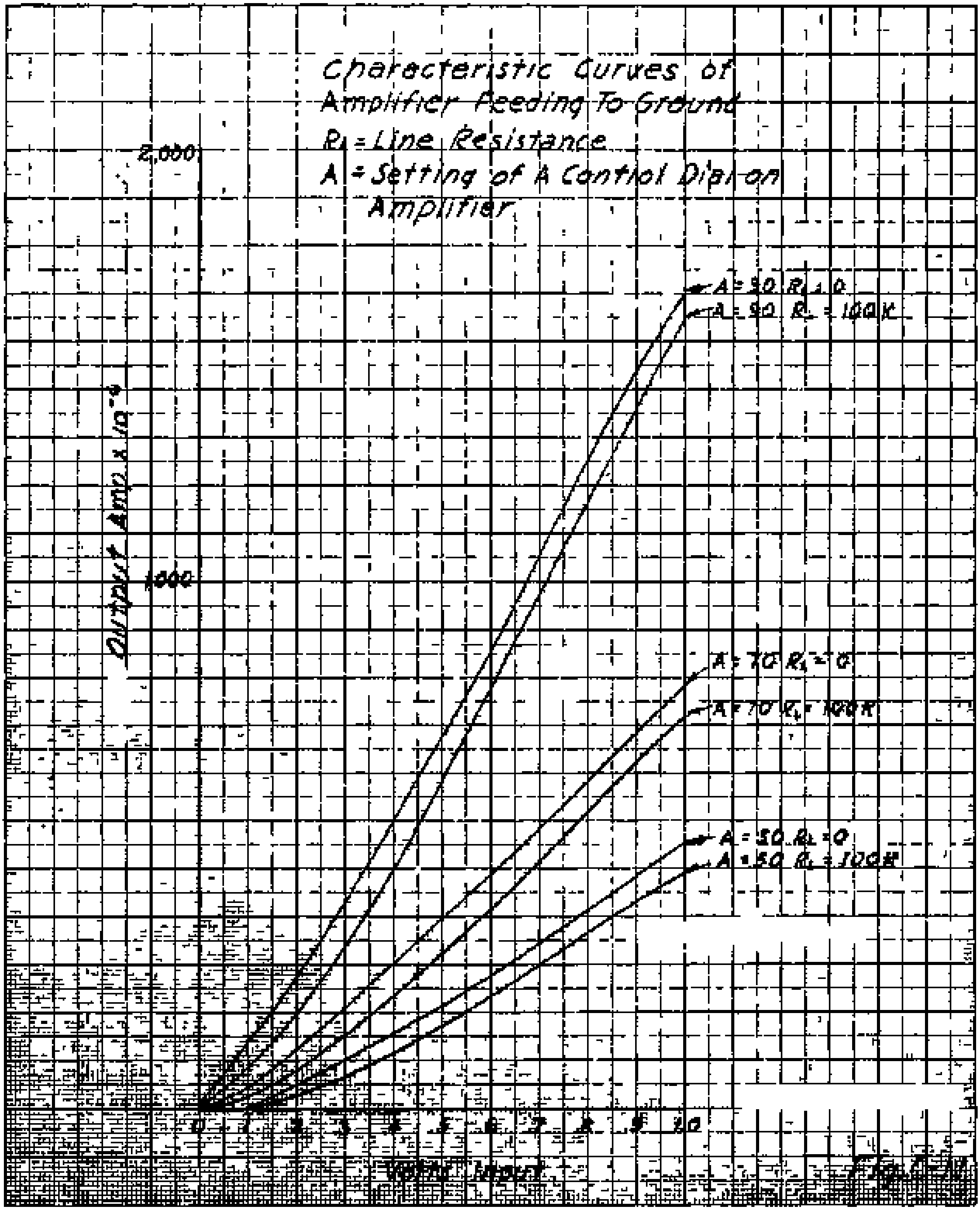


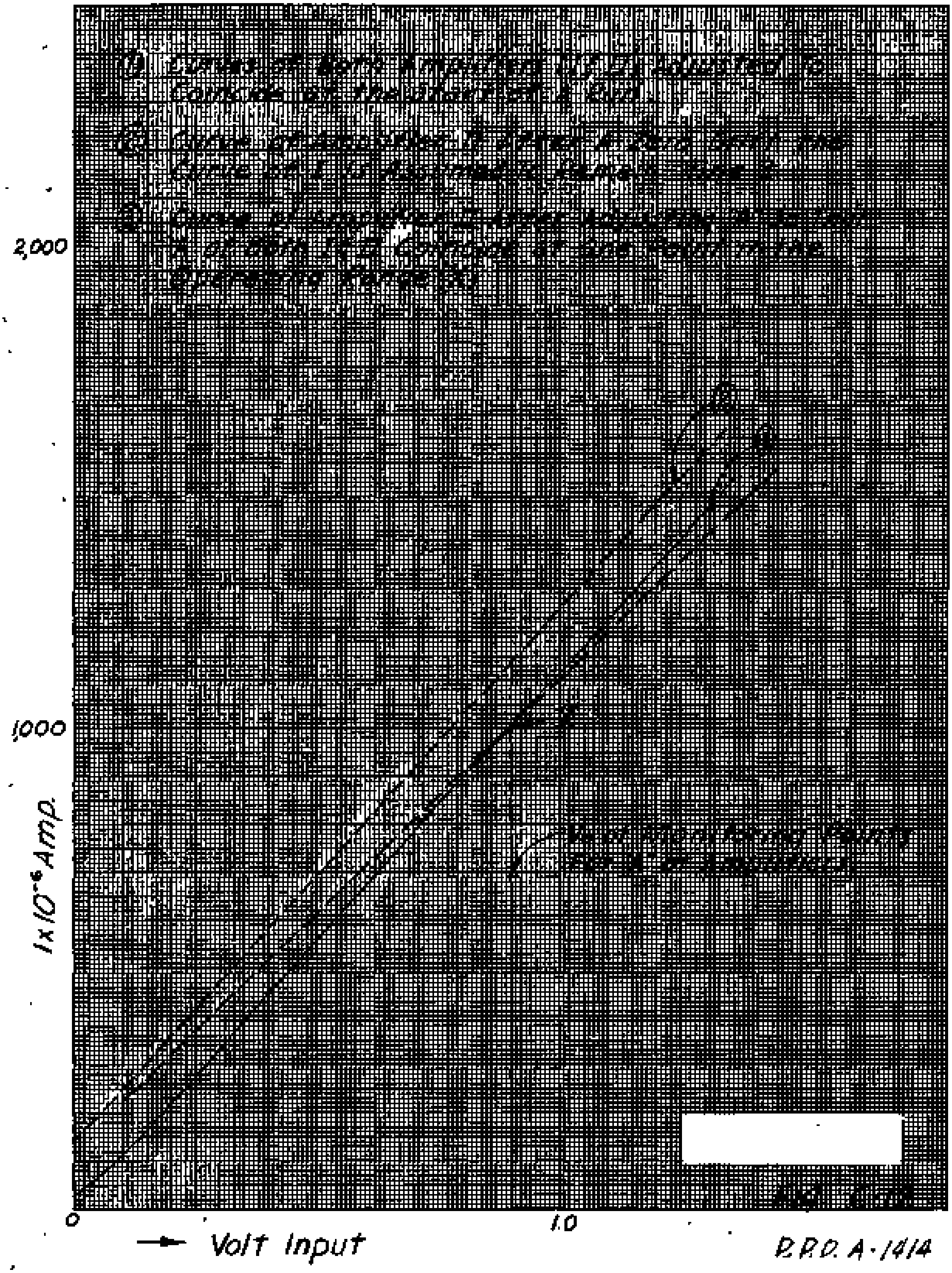
Revised
Amplifier

RRDA-1412

Fig. C-10



Characteristic Curves of
Amplifier Feeding To Ground
R_L = Line Resistance
A = Setting of A Control Dial on
Amplifier
2,000
1000
Output Amp x 10^{-6}
A = 90 R_L = 0
A = 90 R_L = 100K
A = 70 R_L = 0
A = 70 R_L = 100K
A = 50 R_L = 0
A = 50 R_L = 100K
0 1 2 3 4 5 6 7 8 9 10
Volts Input



R.R.D. A-1414

SECTION D

FURTHER EXPERIMENTAL AND DEVELOPMENT WORK

A. Iteration Method

This method of feeding current has been indicated in the other sections of this report. The circuit is shown in fig. D-1 for a two dimensional network. Three node points are shown in fig. D-1 in a network of any number (n) node points.

In Section A it was shown that i_1; i_2; i_3 etc. must be equal to AV_θ (1); AV_θ (2); AV_θ (3) etc. respectively or

$$A = \frac{i_1}{V_\theta(1)} = \frac{i_2}{V_\theta(2)} = \frac{i_3}{V_\theta(3)} = \dots\dots\dots\dots = \frac{i_n}{V_\theta(n)}$$

When the scanning switch and amplifier were used the A was the same for each feed point so it was only necessary to find the value of A which gave a constant V_θ (1), V_θ (2), V_θ (3) etc.

In this method, however, the voltages are always constant no matter what the value of A but this current voltage ratio A, for every point, must be the same and it is this requirement which will be met by only a particular value of A. The process then consists of finding this particular A value by adjusting the variable resistors. As an example, the method of finding A for a particular network is shown in fig. D-2. The Iteration Process for a particular network is shown in fig. D-2. The network is shown at the top of the figure. The total length ℓ has been divided into ten equal sections and the point 5 is the center feed point of the network. The purpose is to find a value A such that i fed into every point will be AV_θ. The shape of the current feed curve (current as a function of ℓ) will then be the same as the shape of the V_θ curve (V_θ as a function of ℓ). Only one particular value of A will satisfy this condition. In this case Step 1 started with a straight line current distribution. This results in the curve F_1 (x) for V_θ which is not a straight line because the end points are at ground or zero voltage. Actually, it will slope approximately as shown in the V_θ curve of Step 1. Then F_1 (x) is a nearer approximation of the required current curve and it is possible to make $i = A_1 F_1(x)$ in Step 2 where A_1 is the current i entering any network lattice unit divided by the corresponding voltage V_θ. It would be best to use the current and voltage at point 5 because it will be affected least by subsequent adjustments, but actually any point may be used. Then in Step 2 a new V_θ curve f_2 (x) is obtained.

Repeating this process in Step 3 f_3 (x) is determined for V_θ which is nearly one loop of a sine curve. (Theoretically the curve should be a sine curve.) Step 4 makes the shape of the curve nearer still to a sine curve. The value A_4, calculated after step 4, would be nearly equal to A_3. This means that more steps will not change the distribution of current materially.

In general when $A(n + 1)$ is nearly equal to $A(n)$ the proper current setting exists. This is the value A which is being sought.

It is evident that had one started with the current distribution according to f_1 (x) or f_2 (x) the iteration process would have been shortened. This can be done since it is known that the ends of the final curve will slope and the peak will occur at the center point and there is no reason for starting with a straight line current distribution. Furthermore, if a point of the θ network is grounded, which is the case when simulating a rod, black to thermal neutrons, inserted at some particular point in the reactor, V_θ goes to zero at that point. The peak voltage occurs at some point between the perimeter ground and the rod and a reasonable distribution of current can easily be estimated. This sort of forethought shortens the iteration process for any problem. The chief advantage offered by this method is that switch and amplifier difficulties are eliminated. This method applied to several hundred feed points could prove tedious, but it is a convergent process and will eventually give an answer to the problem which is set up on the simulator. In order to test this method of feeding current, an assembly of 13 variable resistors was constructed and used to feed the large mesh half of the two dimensional network (see fig. B-9). The assembly is shown on fig. D-3. The plot of voltage distribution obtained with this equipment is shown for two radii on fig. D-4 and fig. D-5. The k as determined from the apparatus was 1.268 which compares well with the analytical value 1.266.

If the switch and amplifier method of feeding current eventually proves impractical this method should be developed more fully. An automatic electromechanical system for adjusting the variable resistors can be developed.

B. Fractional Feed Points

In Section C-5, it was pointed out that the current fed into a network point must be proportional to the volume of reactor material represented by that point. In the following discussion a definite volume of reactor is considered a unit volume. The network representing that area is a unit lattice and the current fed to the node point in the unit lattice is considered a unit of current feed.

Consider again the scanning switch described in Section C. If each segment represents a unit of current feed (amplifier current x time of contact) then a lattice representing twice the unit volume can be fed twice and the current fulfills the requirements of the simulator design. However, in the other direction there is a problem. If a volume of 1/2 is to be fed the segment must be cut in half. Even if this were practical, further reductions of 1/4 and 1/8 are certainly difficult to construct accurately. A better method is to feed all such fractional points with another amplifier and switch. In this case the segments can be of a set standard size but the amplifier will feed a current of say 1/8 or 1/16 of the current being fed to the main network. This establishes a new system of basic units which can be used in multiples of 1, 2, 4 etc. depending upon the volumes to be represented.

Another way to accomplish this is to use the variable resistor feed system, described in Section D-A, to feed the fractional areas still using the amplifier and scanning switch to feed the rest of the network. The mechanics of this combination feed method have not been worked out but should be tried in lieu of adding another amplifier and switch in subsequent work.

C. Re-design of Scanning Switch

Design for an improved scanning switch is shown on fig. D-6. In principle this switch inverts the action of the switch in Section C. The spring contacts are stationary and the contact segment is moved over them. Thus the contact time must be more accurate because the same segment length is used for each contact. In order to test this design a scale model switch, having only six contacts was built and tested. The switch is shown on fig. D-7. The test was set up as shown in fig. D-8.

The test work showed that the contact time was almost absolutely uniform for all contacts at least as nearly as could be measured with an oscilloscope screen picture of the contact operation. Figure D-9 is the photograph of the oscilloscope screen picture of contact operation. To make this picture all six of the contacts were wired together. Each time a contact was made the line on the oscilloscope screen was moved downward. Therefore, the short upper lines represent the contact open time and the longer bottom lines show the contact performance of the switch. Where the contact lower line is clean and solid good contact is indicated. The hash on two of the contact lines indicates the presence of dirt which caused a high contact resistance. When this run was started all the contacts were clean and none of the contact lines on the scope picture showed any hash. The picture was taken after about two hours of operation when some dirt began to accumulate. The uniformity of contact was still very good as can be seen from the picture. The length of the contact line represents time.

The scope beam moved from left to right and completed one screen sweep in 1/30 sec. The contact time was therefore about 0.0032 sec.

The best results (longest satisfactory runs) were made with the rotor very lightly oiled and continuously cleaned with a swab. The design (fig. D-6) provided for running the rotor in an oil bath with a view to reducing surface wear but the oil film established a perfectly insulated surface on the rotor, and this idea had to be discarded.

In addition to the spring contact shown on the design drawing a flat spring was tried, which had no raised wire contact surface. This contact did not work as well as the original design.

A further idea was tried in which a square thread was cut on the rotor so that a groove was formed that wiped across the stationary contact surfaces once in each revolution. This did not offer any marked improvement. The contacts still became dirty after about a six hour run.

No definite conclusions have been reached about this switch design up to the present time and it is intended that several different contact materials will be tried in an effort to find something more satisfactory than steel on brass. These materials were used only because they were readily available from stock. It is felt that a run of 40 hours without cleaning is necessary. In general, the results of test work are encouraging and it is felt that this method of current feeding should not yet be abandoned. Eventually it may become an established fact that a sliding contact is not suitable for this service. Other methods of scanning should be investigated, especially an electronic multivibrator type circuit. The question of transmitting a very low voltage signal through such a circuit may prove to be a problem, since the V_θ signal voltage ranges from 0.2 to 2.0 volt. The V_θ voltage can, however, be increased by changing the "kappa factor" of the network design and thus the voltage can be raised to a more suitable value.

D. Re-design of Current Feeding Amplifier

In Section C-5 some of the difficulties encountered in the amplifier were pointed out. An amplifier which eliminates most of these troubles was designed and tested by F. W. Manning in the electronic shop at ORNL. The following analysis of the problem and amplifier design was done by Manning and is included here for completeness.

It is clear that this amplifier meets the specifications set forth by the author for the simulator amplifier. While this project was in progress it was decided to discontinue work on the simulator and hence no performance tests were made on the network.

The reactor simulator amplifier (fig. D-15) is a direct coupled three terminal amplifier designed to take a negative input signal of up to three volts and deliver a current up to nine milliamperes to the output terminals. The output current is relatively independent of the load circuit impedance. The transconductance of the amplifier may be readily adjusted to any value from 700 to 3000 micromhos. When operated from a regulated a-c line, the transfer characteristic of the amplifier remains essentially a constant negative transconductance at load currents greater than 50 microamperes.

The circuit for the amplifier consists of a cathode follower input stage, a "bootstrap" cathode follower output stage, an attendant amplifier, and the necessary power supplies. The basic circuit is that of a "bootstrap" or, cathode follower used as a constant current device, which may be found on page 94 of Time Bases by O. S. Puckle (John Wiley and Sons). This circuit is shown in fig. D-10.

In this circuit, the current charging the capacitor C remains almost constant after opening switch S until the potential across the condenser approaches a large fraction of the supply voltage. According to the above reference, the linearity of the charging current is the same as if condenser C were being charged through an ohmic resistance from a high potential supply. In the case shown in fig. D-10 with the cathode resistor made equal to the plate resistance, the equivalent supply voltage is:

$$\frac{\mu + 2}{2} E + \frac{1}{2} E_B$$

A modification of the above circuit, shown in fig. D-11 was utilized in the Pile Simulator Amplifier fig. D-15 wherein a 6AG7, pentode connected, V_6, preceded by a 6SL7 difference amplifier, V_8, is used instead of the triode above. A pentode, V_9, is utilized as the cathode resistor of the "bootstrap" which allows operation of the "bootstrap" cathode follower with a reasonable static plate current, thus operating in a more linear portion of the tube characteristics, while retaining the advantage of having a high dynamic impedance in its cathode circuit.

The input signal is applied to V_{11}, a 6AG7 connected as a triode cathode follower. The output of this stage is connected in series with the transconductance control R , and connected thence to the cathode of V_6.

The bias control is so adjusted that the static plate current of V_{11} flows through the bias control and no potential drop occurs across R_C.

The zero control adjusts the static bias on V_6. Under normal operating conditions, this control is used to adjust the output current to zero when both input and output are shorted.

This control also changes the effective gain of the amplifier V_8 and consequently will have a minor effect on the over-all characteristics of the instrument. This is not considered a serious handicap since the zero point will remain fixed for a given set of tubes in the amplifier. Zero drift is minimized when the instrument is operated from a regulated a-c line.

Simplification of fig. D-11 results in fig. D-12 which lends itself more readily to analysis.

V_6 and V_8 of fig. D-11 are replaced by a triode V_2 of fig. D-12 whose plate resistance is identical to that of V_6 and whose amplification factor is greater by the gain of V_8. V_1 of fig. D-12 corresponds to V_{11} of fig. D-11. Since very little current change occurs in V_9 little error is encountered in its omission in fig. D-12. Since current will be considered flowing into the load, little error is introduced by omitting V_{10} from fig. D-12.

It will be observed that the change in grid voltage of V_1 is:

$$de_{g1} = de_s - di_{p1}[R_B+R_C] - di_{p2}R_C \tag{1}$$

and the corresponding change in plate voltage is:

$$de_{p1} = -di_{p1}[R_B+R_C] - di_{p2}R_C \tag{2}$$

By application of the first term of a Taylor series expansion for plate current of a triode vacuum tube, the change in plate current is:

$$di_{p1} = \frac{\partial i_p}{\partial e_g} de_g + \frac{\partial i_p}{\partial e_p} de_p = g_m e_g + \frac{de_p}{r_p}$$

and for V_1

$$di_{p1} = g_{m1}\, de_{g1} + \frac{de_{p1}}{r_{p1}} \tag{3}$$

From 1, 2 and 3:

$$di_p = g_{m1}\left[de_s - di_{p1}(R_B + R_C) - di_{p2}R_C\right] + \frac{1}{r_{p1}}\left[- di_{p1}(R_B+R_C) - di_{p2}R_C\right] \tag{4}$$

Since

$$g_m = \frac{\mu}{r_p}$$

$$di_{p1}\left[r_p+(\mu_1 + 1)(R_B+R_C)\right] = \mu_1 de_s - (\mu_1 + 1) R_C di_{p2} \tag{5}$$

Whence

$$di_{p1} = \frac{\mu_1 de_s - (\mu_1 + 1)R_C\, di_{p2}}{r_{p1} + (\mu + 1)(R_B + R_C)} \tag{6}$$

Likewise for V_2

$$de_{g2} = -di_{p2}(R_A + R_C) \;\; -di_{p2}R_C \tag{7}$$

With a load resistance R_L across the output terminals

$$de_{p2} = -di_{p2}(R_L + R_A + R_C) - di_{p1}R_C \tag{8}$$

$$di_{p2} = g_{m2}\, de_{g2} + \frac{de_{p2}}{r_{p2}} \tag{9}$$

$$di_{p2} = g_{m2}\Big[-di_{p2}(R_A + R_C) - di_{p1}R_C\Big] + \frac{1}{r_{p2}}\Big[-di_{p2}(R_L + R_A + R_C) - di_{p1}R_C\Big] \tag{10}$$

$$di_{p2}\Big[r_{p2} + (\mu + 1)(R_A + R_C) + R_L\Big] = -(\mu_2 + 1)R_C di_{p1} \tag{11}$$

Substituting (6) in (11)

$$di_{p2}\left[\Big[r_{p2} + (\mu_2 + 1)(R_A + R_C) + R_L\Big] - \frac{(\mu_1 + 1)(\mu_2 + 1)R_C^2}{r_{p1} + (\mu_1 + 1)(R_B + R_C)}\right] =$$

$$\frac{R_C\mu_1(\mu_2 + 1)}{r_{p1} + (\mu_1 + 1)(R_B + R_C)}\; de_s \tag{12}$$

Since the transconductance of a network is defined as the ratio of the change in output current to the change in input voltage, from 12 above it is found that:

$$A = \frac{di_p}{de_s} = -\frac{\mu_1(\mu_2 + 1)R_C}{\Big[r_{p1} + (\mu_1 + 1)(R_B + R_C)\Big]\Big[r_{p2} + (\mu_2 + 1)(R_A + R_C) + R_L\Big] - \Big[(\mu_1 + 1)(\mu_2 + 1)R_C^2\Big]} \tag{13}$$

By substituting the following values for the quantities as found in reactor simulator amplifier:

$r_{p1} = 2500$

$\mu_1 = 21$

$R_B = 150$

$R_C = 2200$

$r_{p2} = 130{,}000$

$\mu_2 = Gg_m r_p \cong 30 \times 11000 \times 10^{-6} \times 130{,}000 = 42800$

$R_A = R_A$

Equation (13) then reduces to:

$$A \doteq - \frac{1}{1.172\, R_A + 282}$$

Since a 1000 ohm 10 turn helipot was used for R_A, with the dial calibrated from 0 to 1000, but so oriented that the resistance in the circuit was equal to 1000 minus the dial reading, the above equation reduces to the following in terms of dial reading.

$$A = - \frac{1}{1454 - 1.172D}$$

A plot of this equation and of experimental data taken with a one volt signal applied to the input terminals and a 100,000 ohm resistor across the output terminals is shown in fig. D-13.

It will be noted that close agreement exists between the two curves, substantiating the above circuit analysis.

Typical input voltage - output current curves are shown in fig. D-14 for two settings of the transconductance control. It may be seen that these curves are essentially straight lines. The curve for a dial setting of 975 with a 100,000 ohm load illustrates the saturation of the amplifier at an output voltage of 420 volts corresponding to 4.2 milliamperes through the load resistance.

The action of the amplifier operating into a capacitive load may be analyzed by assuming a capacitor C across the output terminals. The differential equation for the current flowing in the output terminals may be shown to be:

(16)

$$i_{p2}\left[r_{p2}+(\mu_2+1)(R_C+R_A) - \frac{(\mu_1+1)(\mu_2+1)\,R_C{}^2}{r_{p1}+(\mu_1+1)(R_C+R_B)}\right]+\frac{q}{C} = \frac{\mu_1(\mu_2+1)R_C}{r_{p1}+(\mu_1+1)(R_C+R_B)}\,e_s$$

which is of the form

$$IR + \frac{Q}{C} = E \qquad (17)$$

with solution

$$I = \frac{E}{R}\,e^{-\frac{t}{R_C}} \qquad (18)$$

From this it may be seen that the capacitor C is apparently being charged from a voltage source

(19)

$$E \cong -e_s\,\frac{\mu_1(\mu_2+1)\,R_C}{r_{p1}+(\mu_1+1)(R_C+R_B)} \cong -e_s\,\frac{21 \times 42800 \times 2200}{2500 + 22(2200+150)} = -36600\,e_s$$

The apparent series resistance R will be found to be

$$R \cong r_{p2} + (\mu_2+1)\,(R_C+R_A) - \frac{(\mu_1+1)(\mu_2+1)\,R_C{}^2}{r_{p1}+(\mu_1+1)(R_C+R_B)} \qquad (20)$$

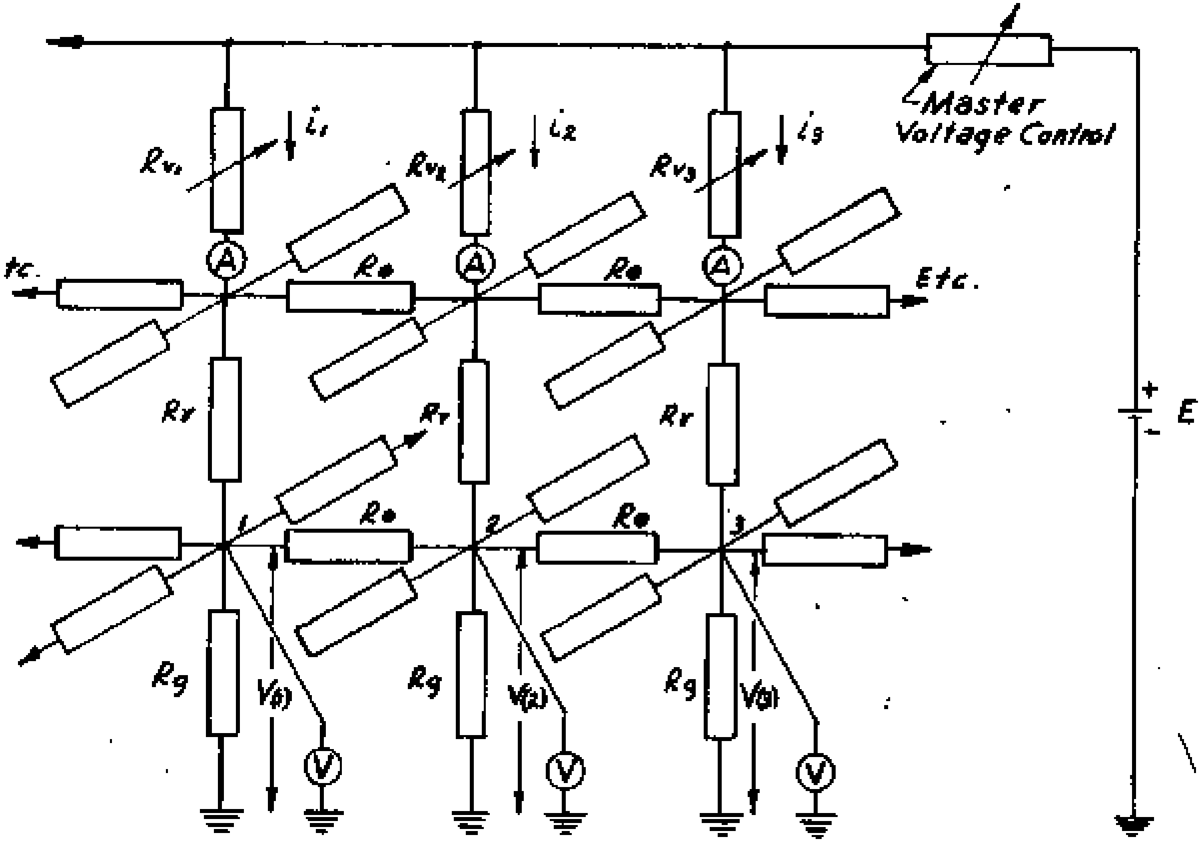
$$\cong 1.03 \times 10^7 + 42800\,R_A$$

Thus the apparent series resistance will vary from about 10 megohms to 53 megohms.

Experimental results showed approximately 10 microamperes change in output current by shorting out a 100,000 ohm load resistor, while the transconductance was adjusted to 2000 micromhos. This implies an apparent voltage of approximately $-40{,}000e_s$ and apparent series resistance of about 20 megohms. This shows a reasonable correlation between calculated and experimental results.

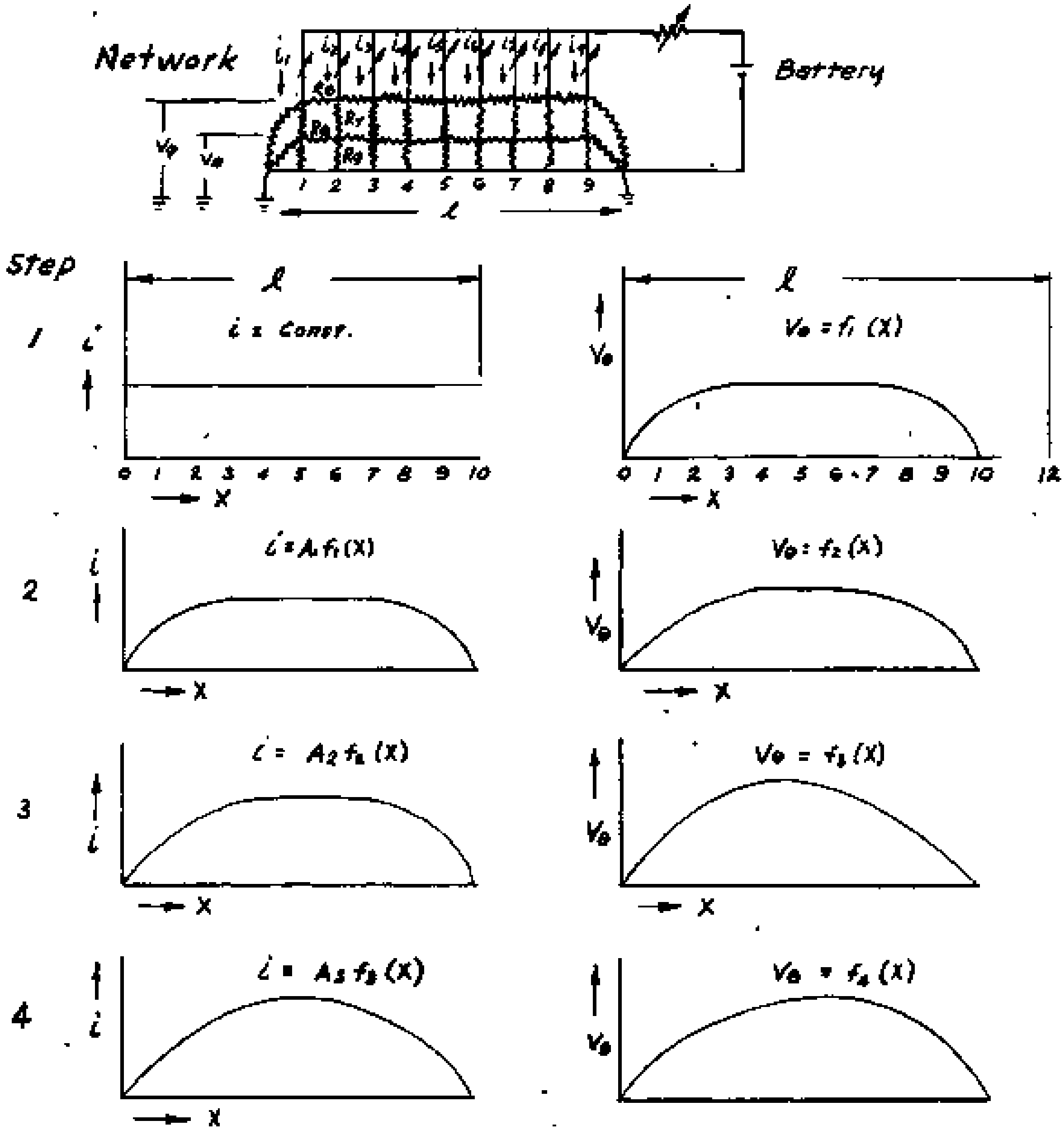
The apparent time constant of the circuit when used with a load capacitance of 10 microfarads would be of the order of 100 to 530 seconds. For periods small with respect to the time constant one would expect to charge the condenser with a reasonably constant current.

It was found necessary to operate the amplifier from a regulated a-c line to prevent excessive drift of the output current. This occurred in spite of the fact that an electronically regulated supply with a stabilization factor of 1:200 was used to furnish the positive and negative 105 volts to the amplifier stage, V_8 of fig. D-11. It is felt that considerable benefit would be obtained by balancing this stage by the addition of a one megohm plate load to the input section and by adding a 15 megohm resistor from its plate to negative 105 volts. This would tend to prevent changes in supply voltage from affecting the bias of the "bootstrap" stage. This would produce an attendant loss in gain and consequently reduces the value of μ_2 by approximately 1/2, thereby modifying all of the above calculated and experimental results. The input impedance of the amplifier is approximately equal to the input grid resistor since the signal is almost completely degenerated in the cathode circuit of V_{11}. Should a higher input impedance be desired, it is suggested that V_{11} be preceded by a high input impedance cathode follower.



Typical Connections for Iteration Method of Determining Flux Distribution And k.

P.P.D. AM15 Fig. D-1



$A_1 = \dfrac{i_s(1)}{V_{os}(1)}$ $\quad A_2 = \dfrac{i_s(2)}{V_{os}(2)}$ $\quad A_3 = \dfrac{i_s(3)}{V_{os}(3)}$

$A_4 = \dfrac{i_s(4)}{V_{os}(4)}$ Etc.

When $A_{n+1} \approx A_n$ Then The Critical A Has Been Found.

Iteration Process

R.P.D.A-1416 Fig. D-2

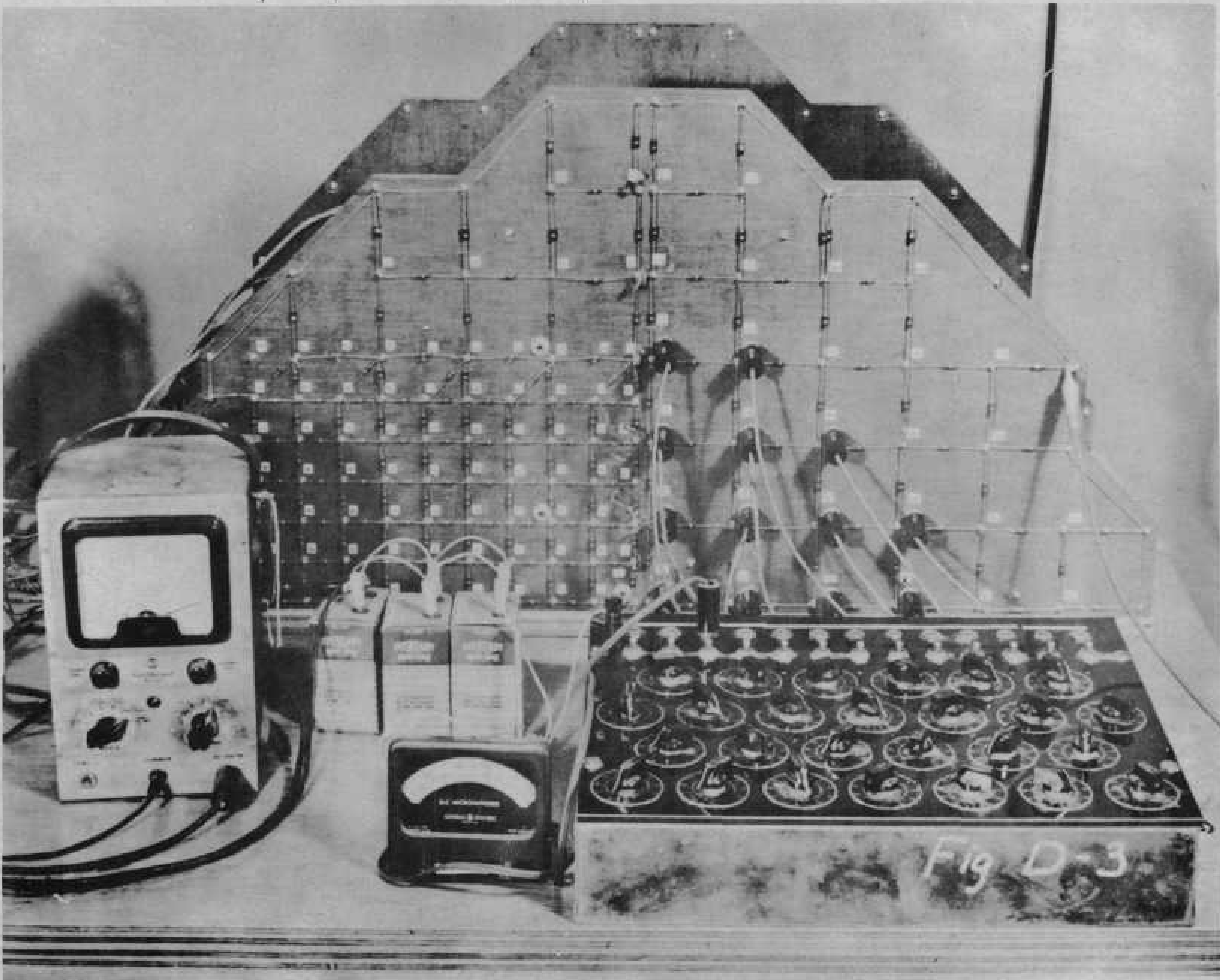


Fig D-3

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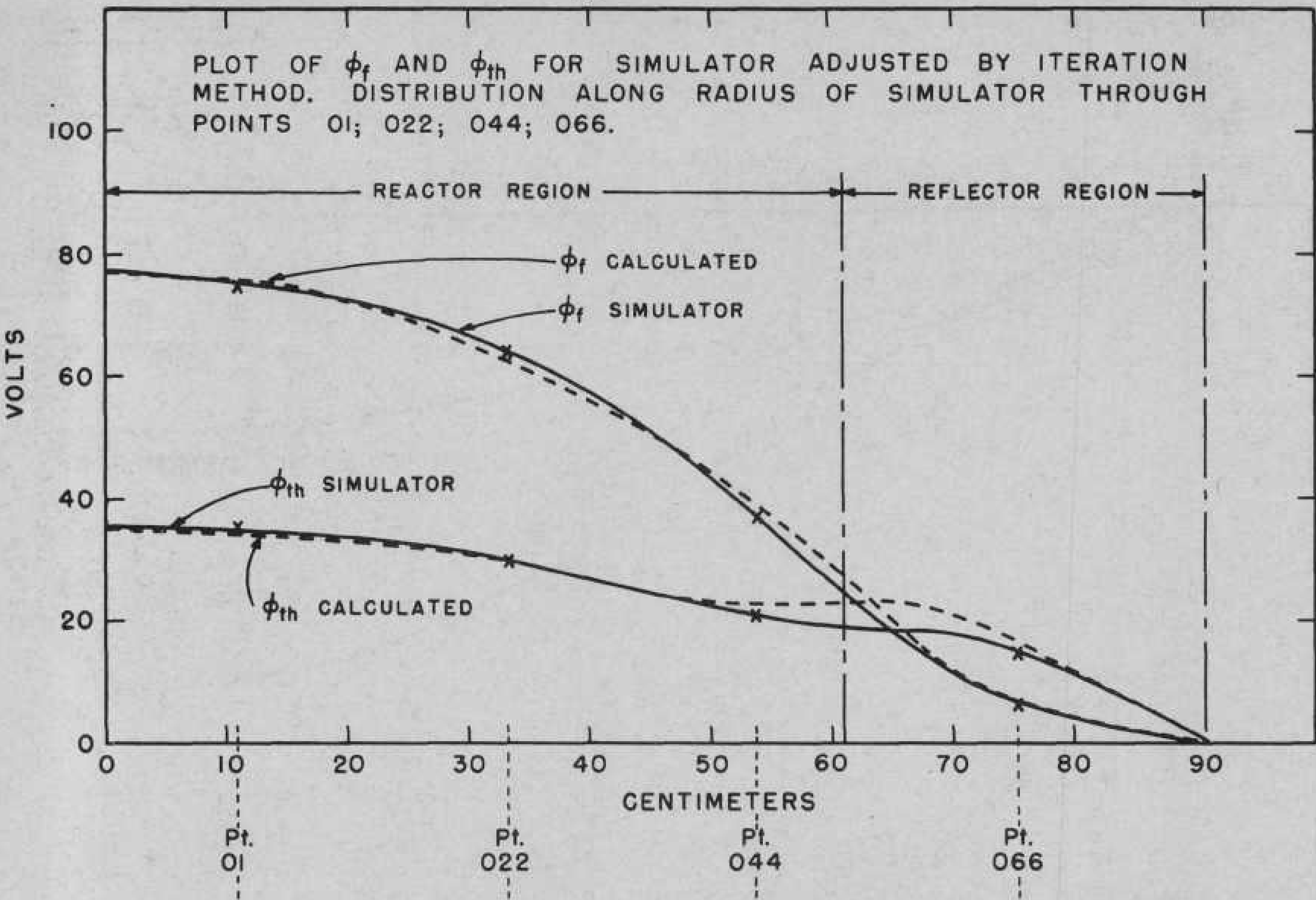


FIG. D-4

PPD-A-1399

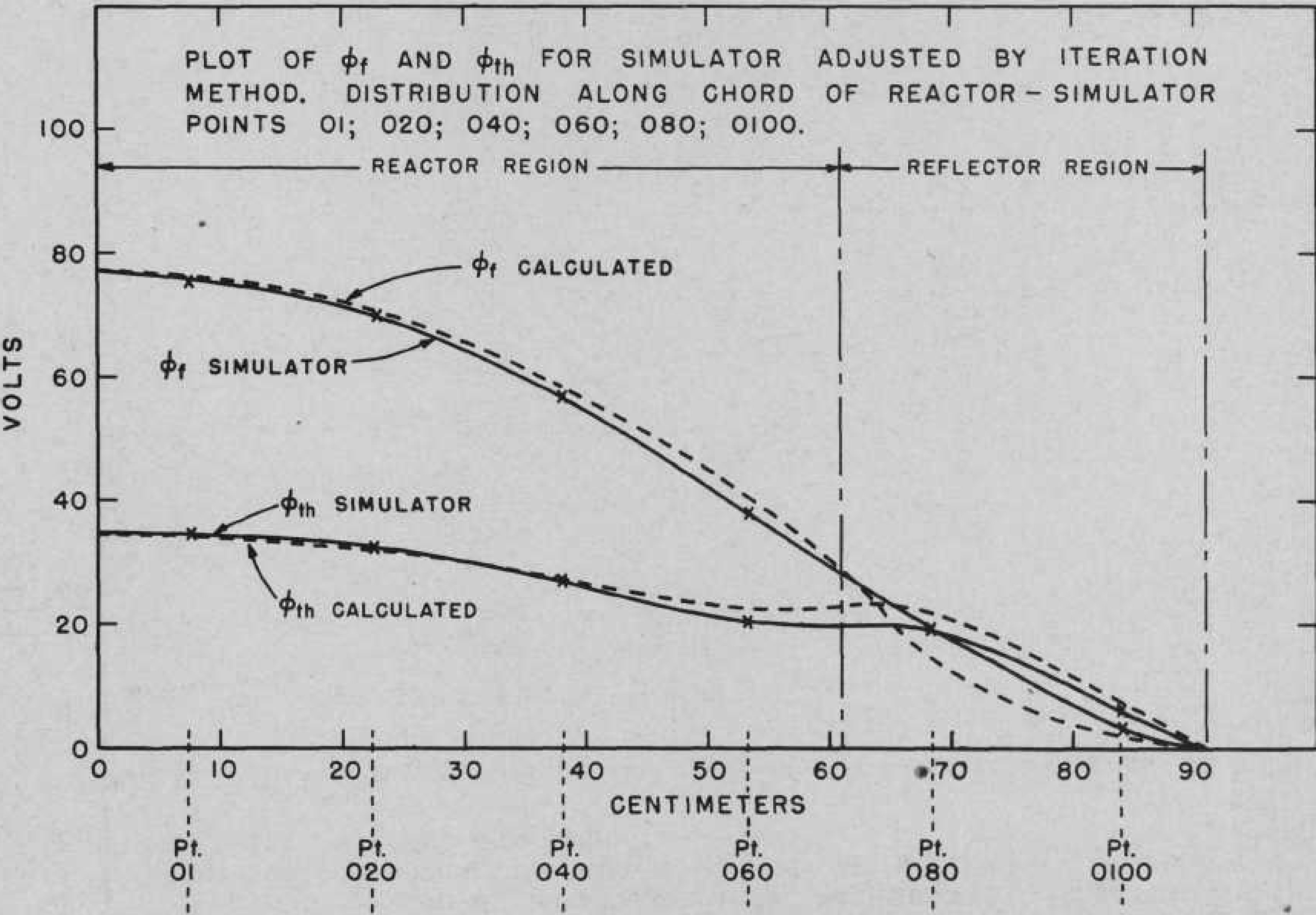
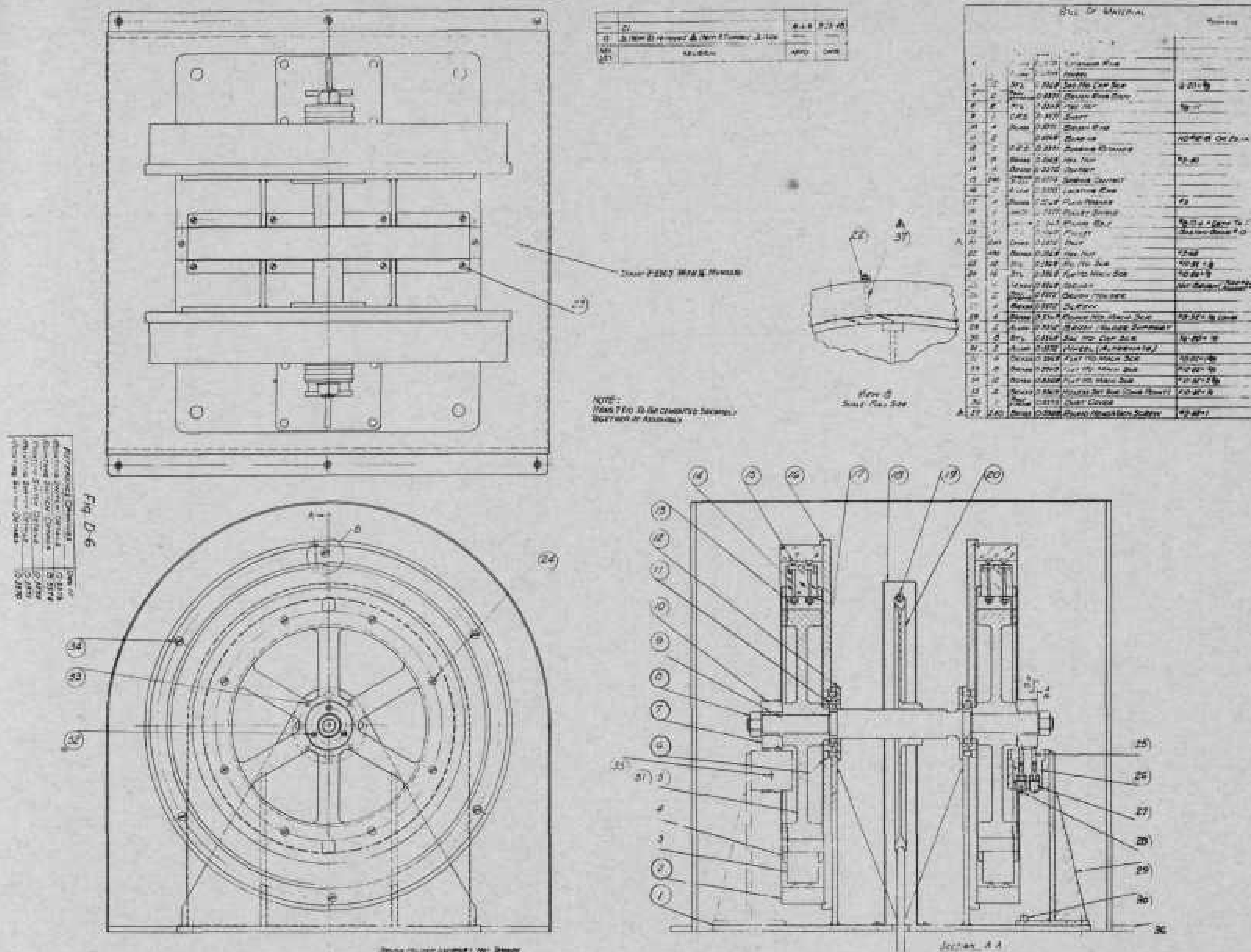
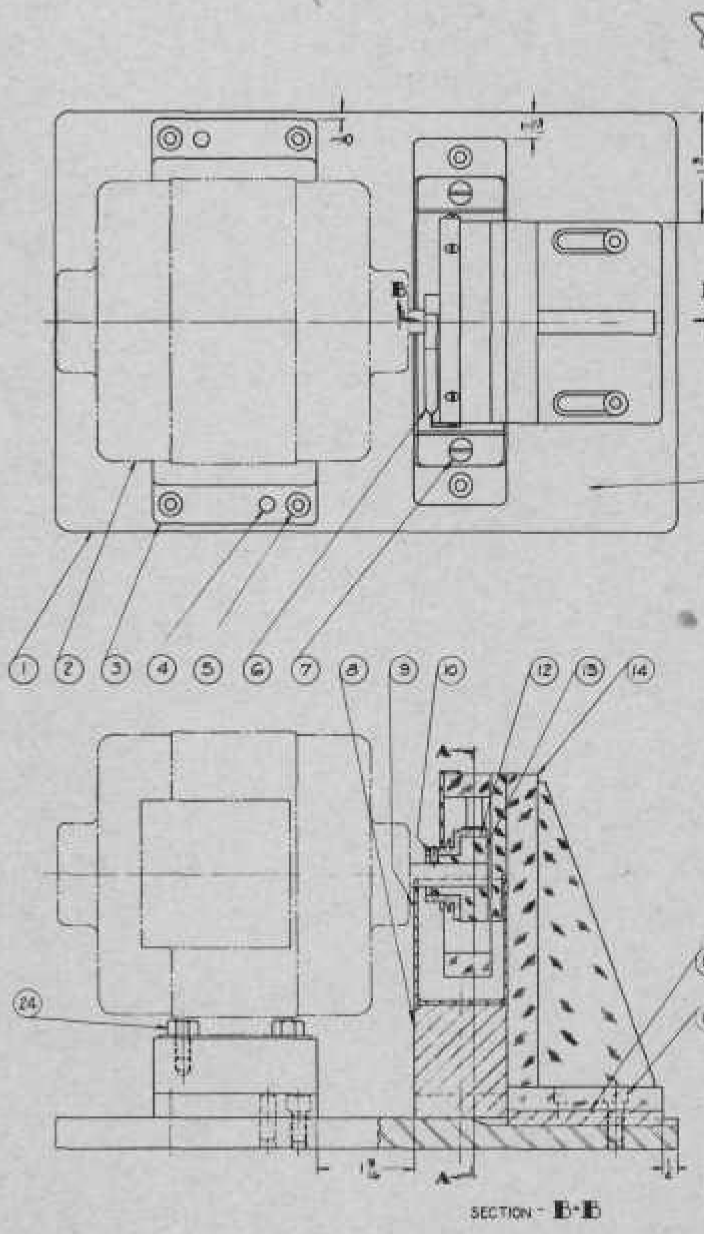
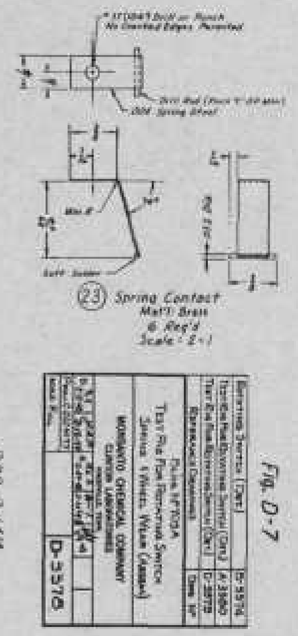


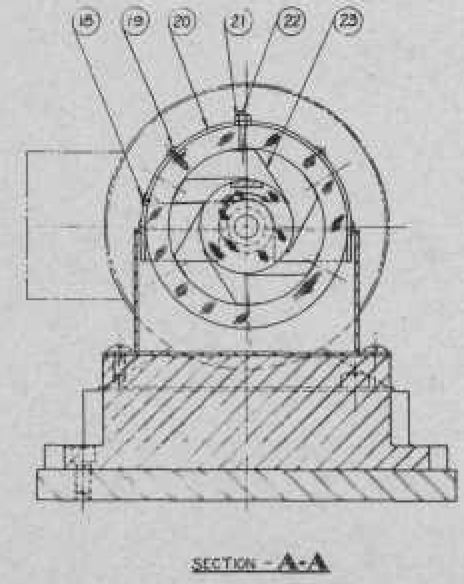
FIG. D-5

PPD-A-1400


Bill of Material
Fig D-6
NOTE:
View B
Section A-A







Item	No. Req.	Mat'l	Dwg. No.	Description	Remarks
1	1	C.R.S.	D-3579	Base Plate	
2	1		D-3378	Electric Motor (See Elec.	5SH25AC1A (On Hand)
3	1	C.R.S.	D-3378	Motor Base	
4	2	D.R.	D-3575	Dowel Pin	1/4 Dia × 5/8
5	2	Stl.	D-3575	Soc. Hd. Cap Scr.	1/4-20 × 1/2
6	1	Spring Brass	A-3380	Spring	
7	2	Stl.	D-3578	Rnd. Hd. Mach. Scr.	#10-32 × 3/8
8	1	C.R.S.	D-3578	Oil Tank Support	
9	1	Sheet Steel	D-3578	Oil Tank	
10	2	Fibre	D-3577	Soc. Set Scr.	#5-40 × 5/16
11	1	Brass	D-3578	Soc. Set Scr.	#5-40 × 1/8
12	1	Brass	D-3578	Contact Wheel	
13	1	Poly-styrene	D-3577	Insulator Wheel	
14	1	Poly-styrene	D-3578	Contact Spring Holder	
15	3	Brass	D-3578	Flat Hd. Mach. Scr.	#2-56 × 3/16
16	1	Steel	D-3578	Key	1/8 × 1/8 × 2 1/2
17	2	Stl.	D-3578	Soc. Hd. Cap Scr.	1/4-20 × 5/8
18	1	Brass	D-3578	Rivet	1/16 Dia × 1/8
19	4	Brass	D-3577	Rnd. Hd. Mach. Scr.	#5-40 × 1/4
20	1	Poly-styrene	D-3578	Oil Shield	#5-45
21	10	Brass	D-3578	Hex. Nut	#5-40
22	6	Brass	D-3578	Rnd. Hd. Mach. Scr.	#5-40 × 5/16
* 23	6	Brass	D-3374	Spring Contact	Increase Angle To 72°
24	4	Stl.	D-3578	Hex. Hd. Mach. Scr.	1/4-20 × 5/8

* Angle May Be Increased By Hand (72°)

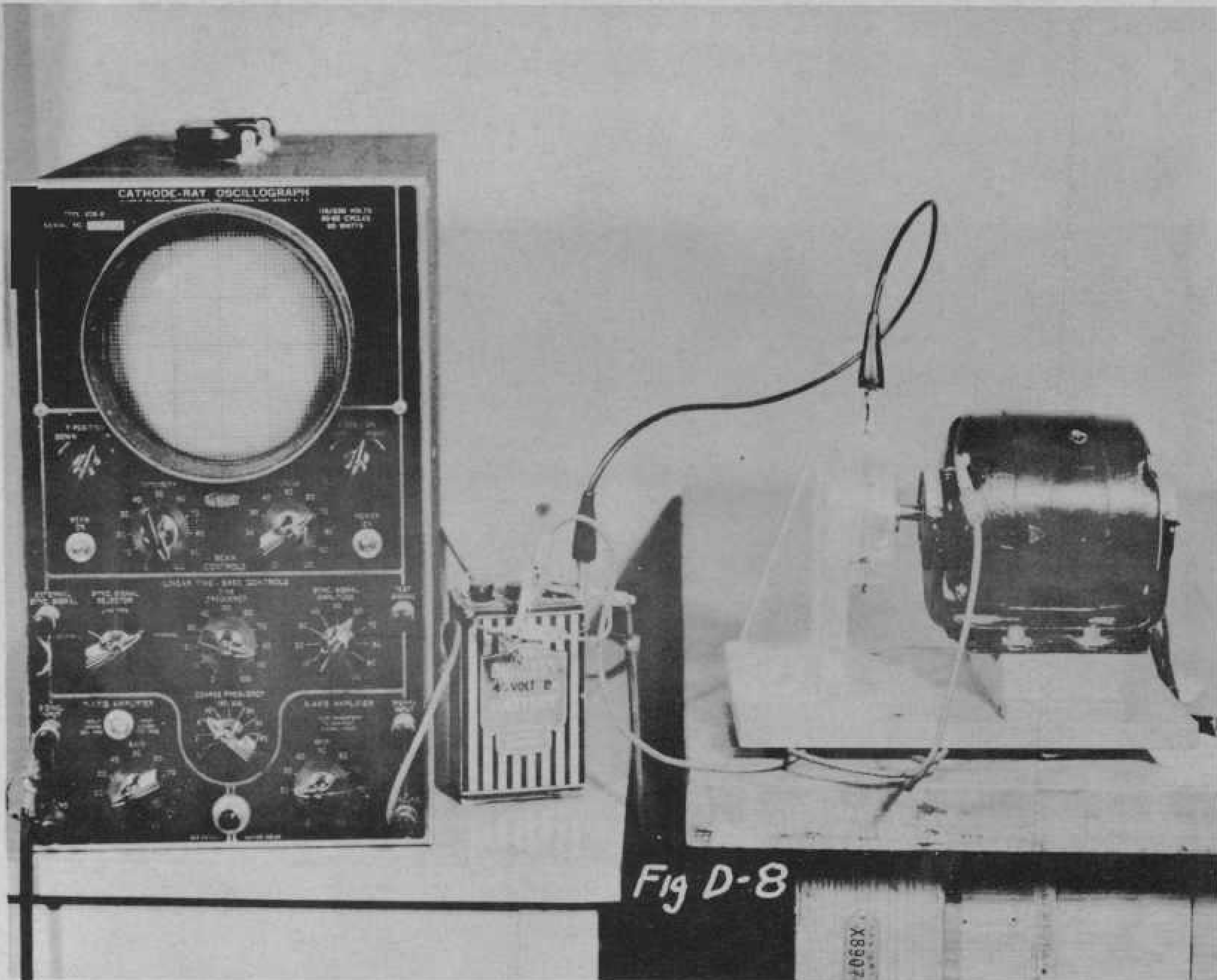


Fig D-8

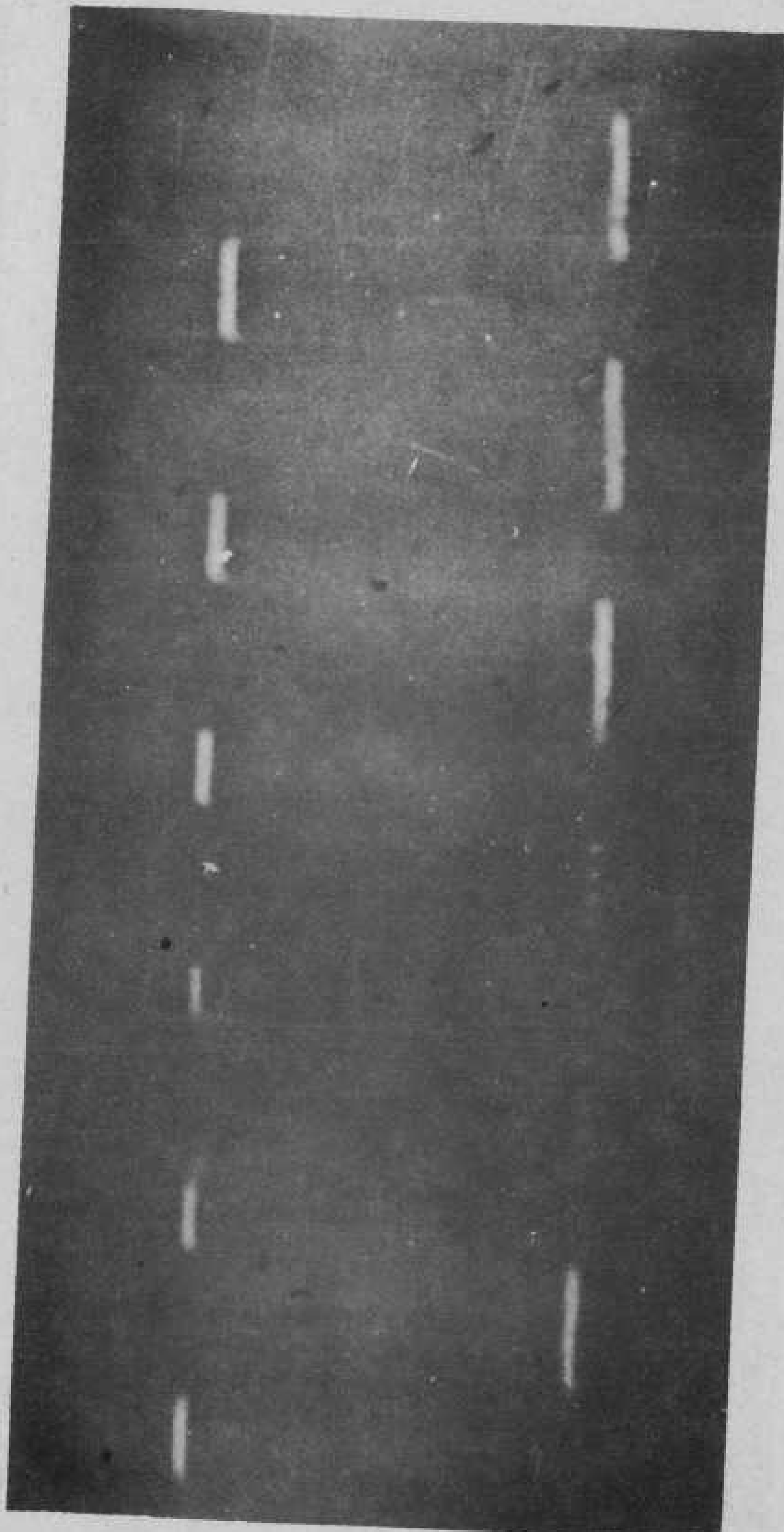


Fig. D-9

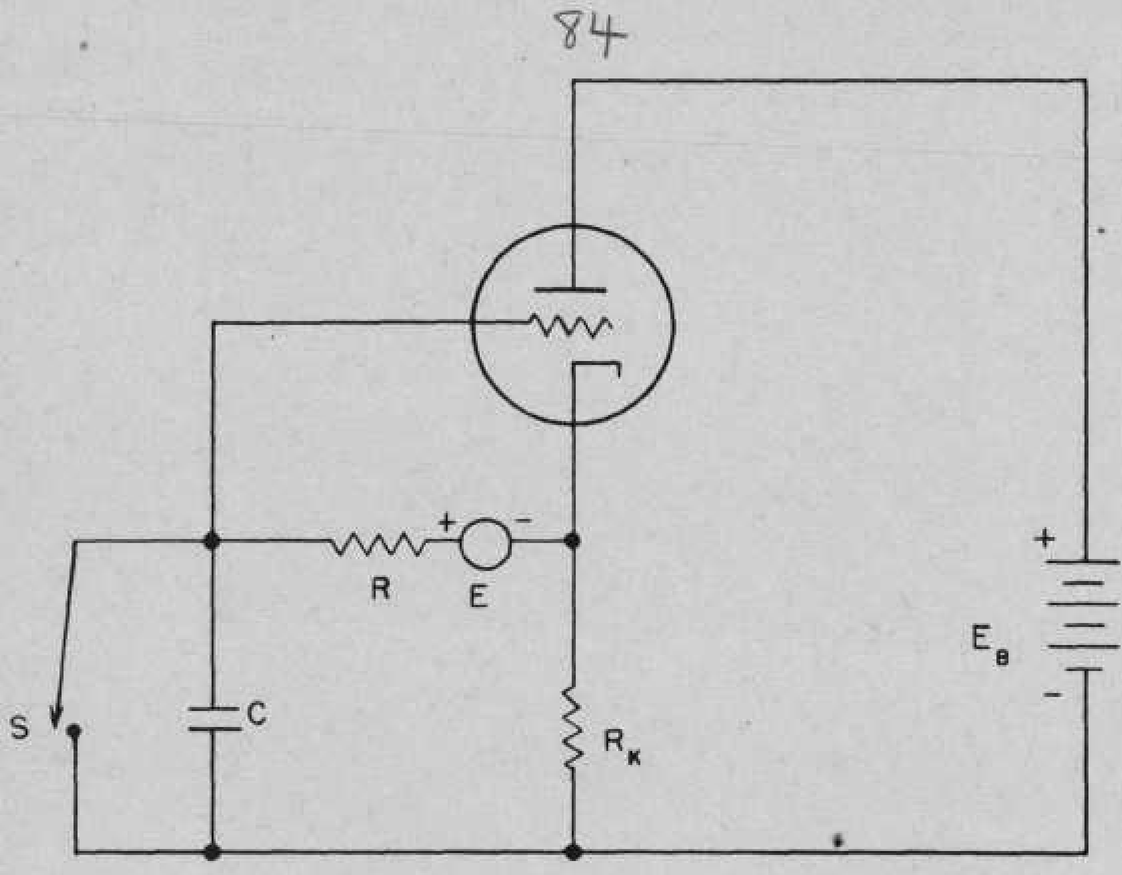


Fig. D-10

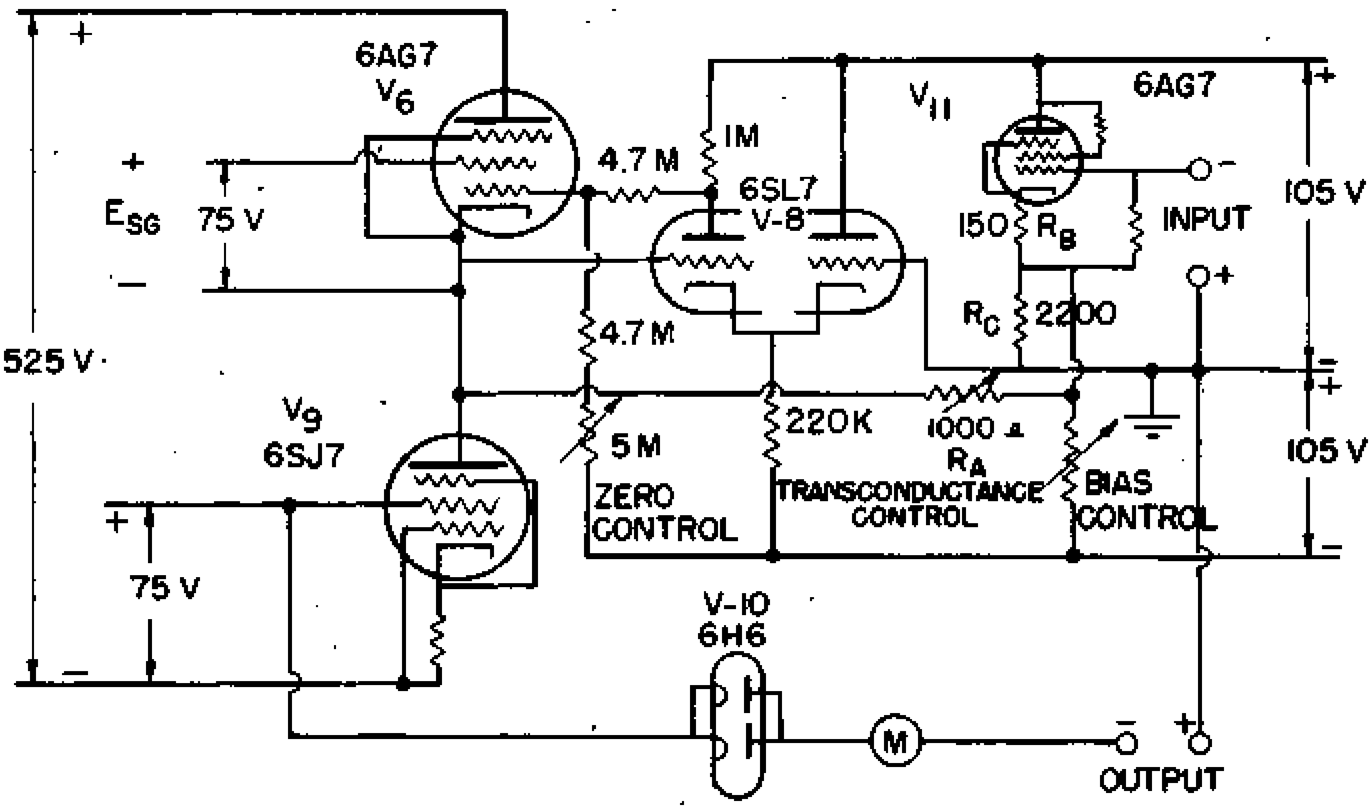

6AG7
V6
+
ESG
75 V
−
525 V
+
−
4.7 M
IM
6SL7
V-8
4.7 M
V11
6AG7
150
RB
RC
2200
INPUT
105 V
105 V
V9
6SJ7
+
75 V
−
5 M
ZERO CONTROL
220K
1000 Ω
RA
TRANSCONDUCTANCE CONTROL
BIAS CONTROL
V-10
6H6
M
OUTPUT

Fig. D-11

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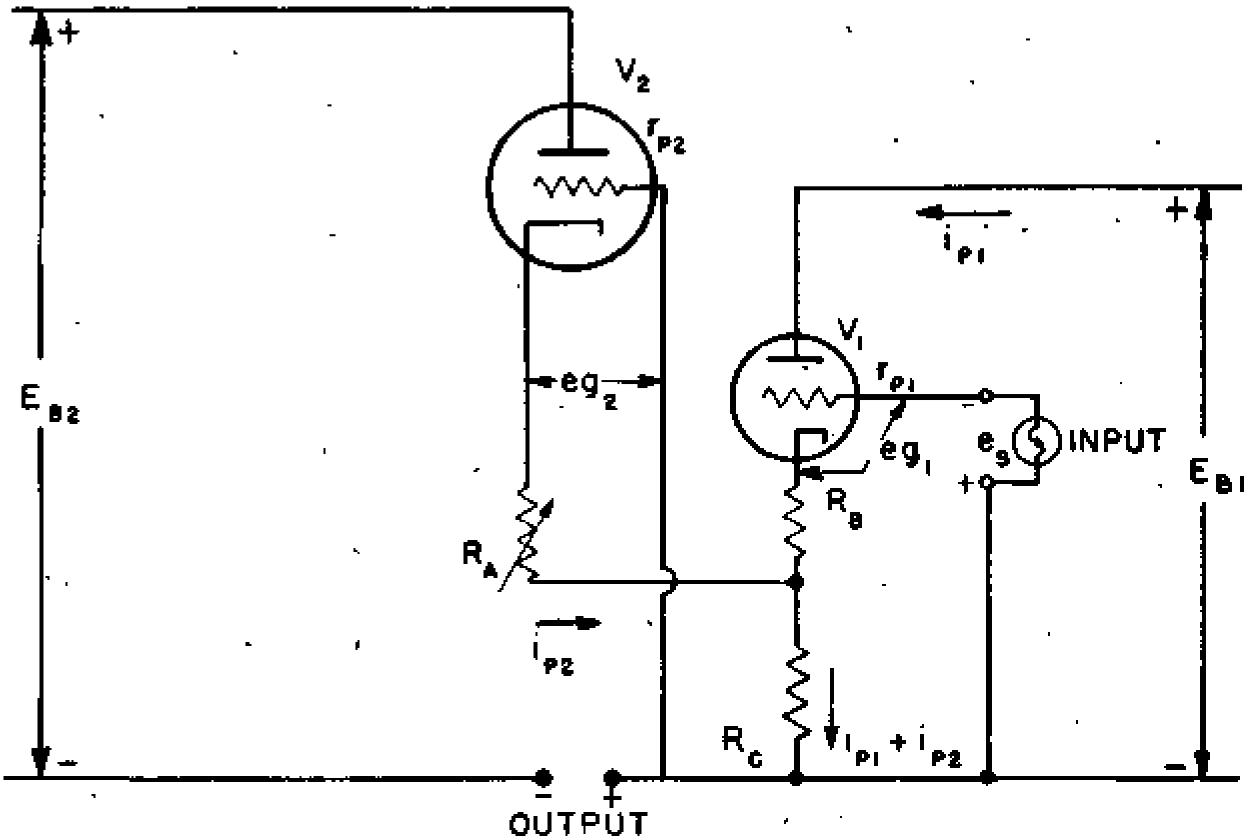

V2
rp2
V1
rp1
ip1
EB2
eg2
eg1
es
INPUT
EB1
RB
RA
RC
ip2
ip1 + ip2
OUTPUT

Fig. D-12

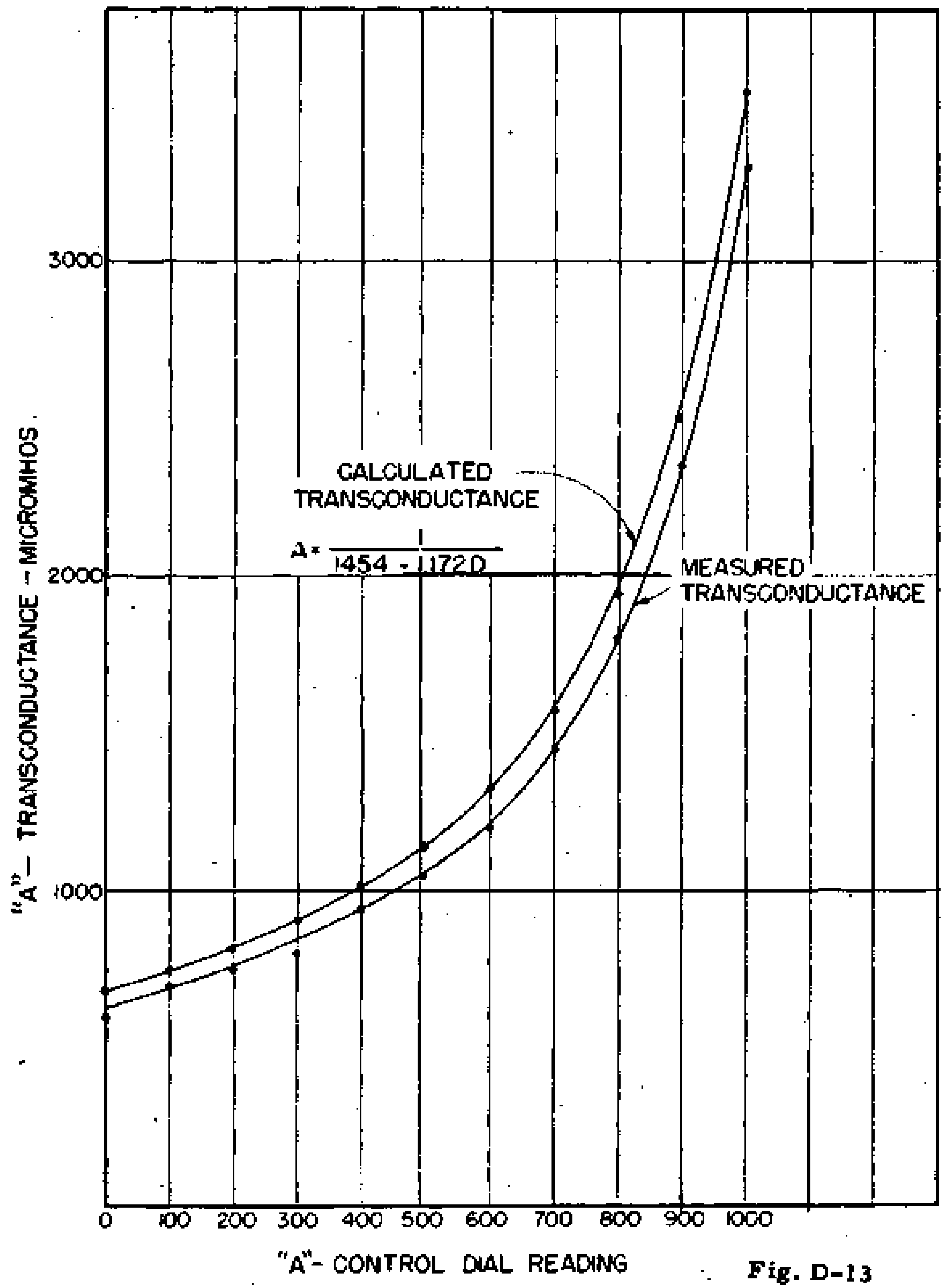

3000
2000
1000
"A" – TRANSCONDUCTANCE – MICROMHOS
0
100
200
300
400
500
600
700
800
900
1000
"A"- CONTROL DIAL READING
CALCULATED
TRANSCONDUCTANCE
A = 1 / (1454 - 1172D)
MEASURED
TRANSCONDUCTANCE

Fig. D-13

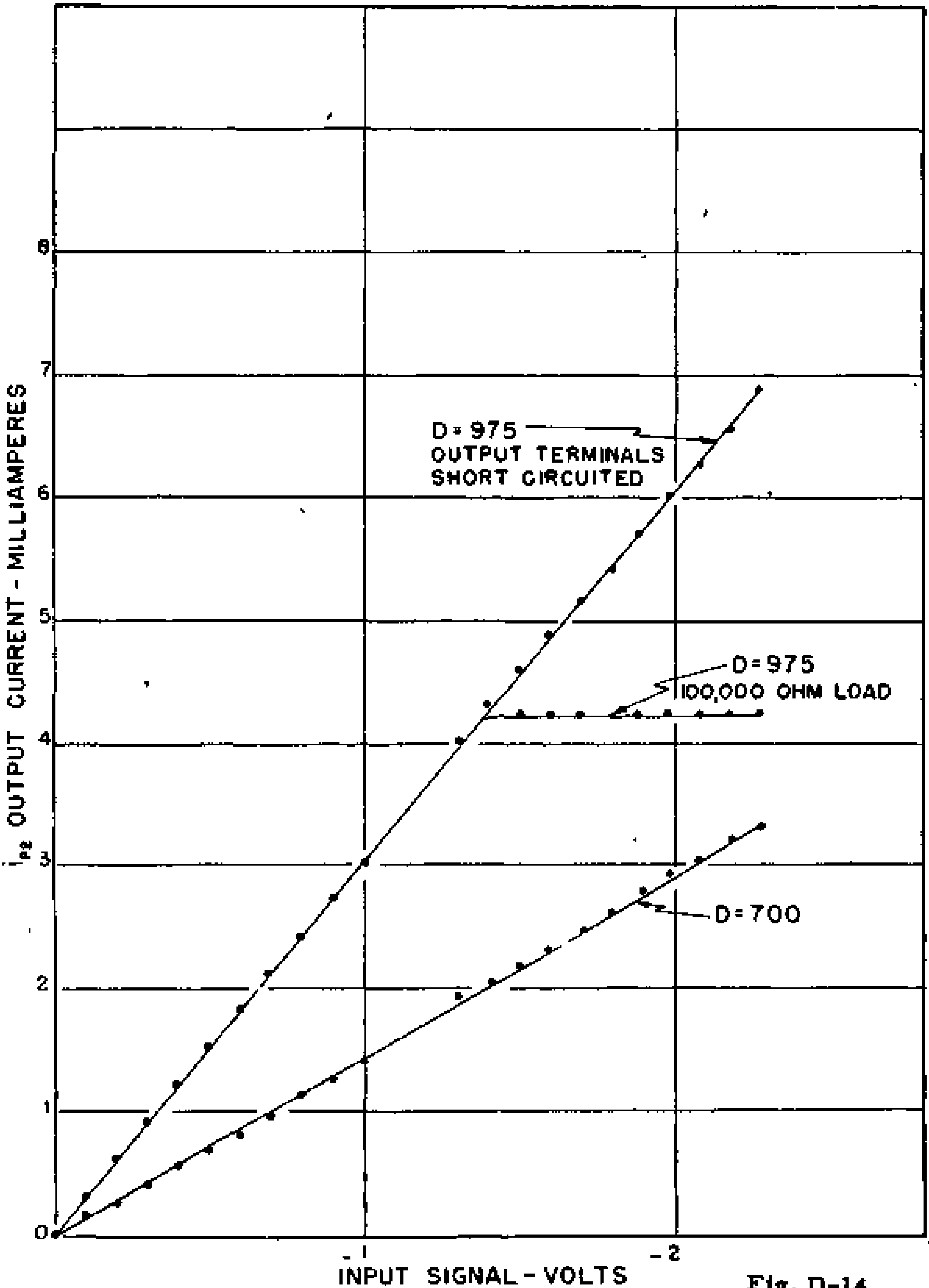

I_{P2} OUTPUT CURRENT - MILLIAMPERES
0
1
2
3
4
5
6
7
8
-1
-2
INPUT SIGNAL - VOLTS
D = 975
OUTPUT TERMINALS
SHORT CIRCUITED
D = 975
100,000 OHM LOAD
D = 700

Fig. D-14

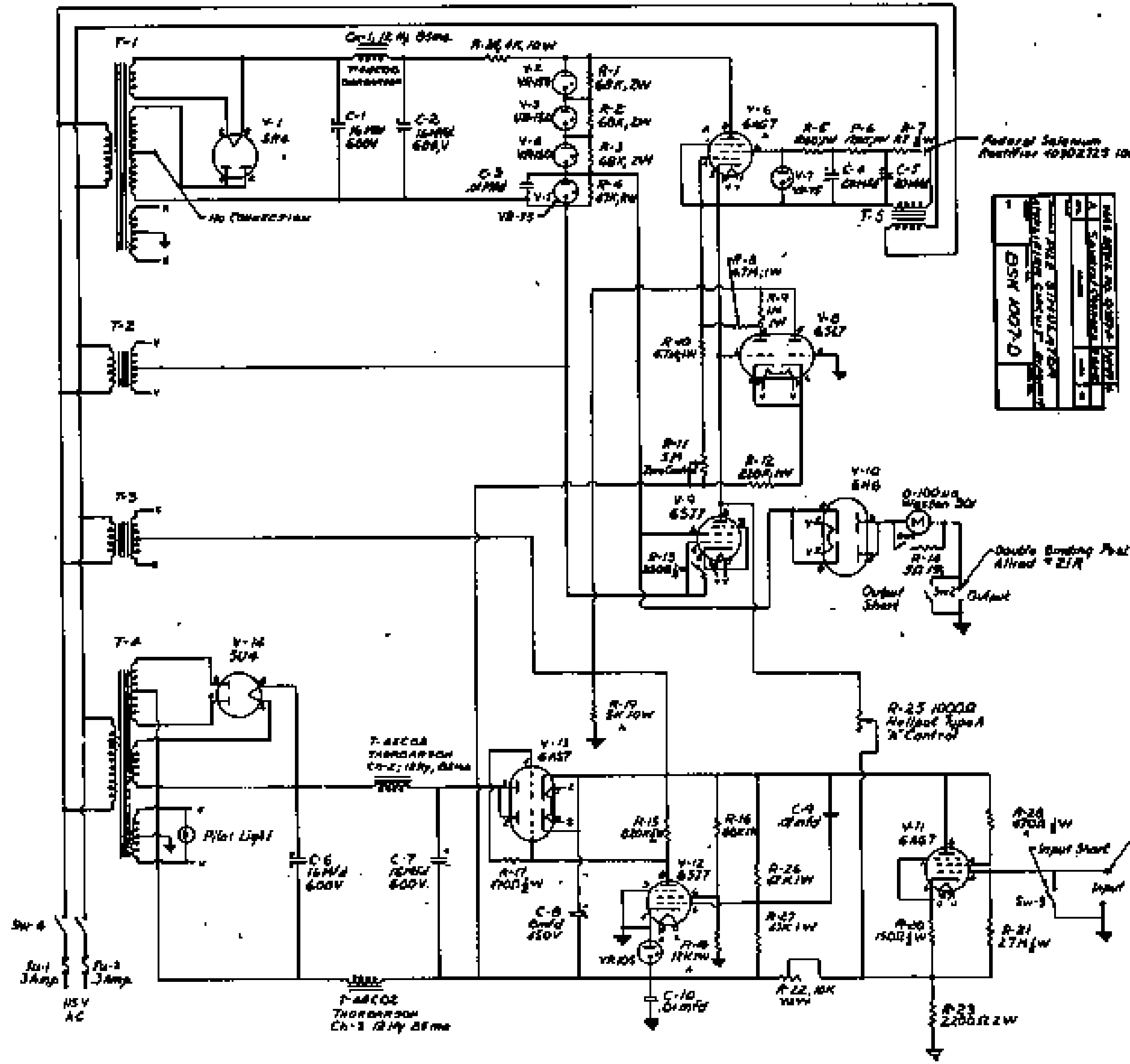


Fig. D-15

Notes

T1,T4 Plate Transformer P-4076, 325-0-325V, @ 40ma, 5.0V @ 2.0Amp, 6.3V @ 2.0Amp

T2,T3, Thordarson Transformer T21F10 6.3V @ 3Amp

T5 Allied 1:1 Isolation Transformer E-117 115V-125V @ 30-40VA.

APPENDIX I

Introduction

In the body of this report it is shown that practical construction considerations necessitate the use of a lumped network for the electrical analogy of a neutron reactor. It was pointed out by J. W. Simpson that the differential equation of an electrical transmission line, with distributed sources and sinks, was exactly similar to the differential equation of a slab reactor in elementary diffusion theory. Having drawn this analogy one then can build a "π" section of resistors which is exactly equivalent, at its terminals, to the transmission line. A lumped network of these "π" sections would then be exactly equivalent to a distributed slab reactor in elementary diffusion theory. The formulas for the equivalent circuit of a transmission line are well known and can be found in any text book on the subject. Here the necessary formulas for such a resistor network have been derived and the analogy extended to include the two group theory of neutron diffusion. The formulas for a three dimensional reactor are also derived.

The formulas for the relation between the various electrical network constants involve the value k of the reactor to be simulated. This means that the analogy would only be exact for one k value. For any value of k other than this particular value, the relations are again approximate. The details of this analogy are included here for completeness and because these more exact formulas may prove useful in the future. No use was made of these formulas in the experimental work to date.

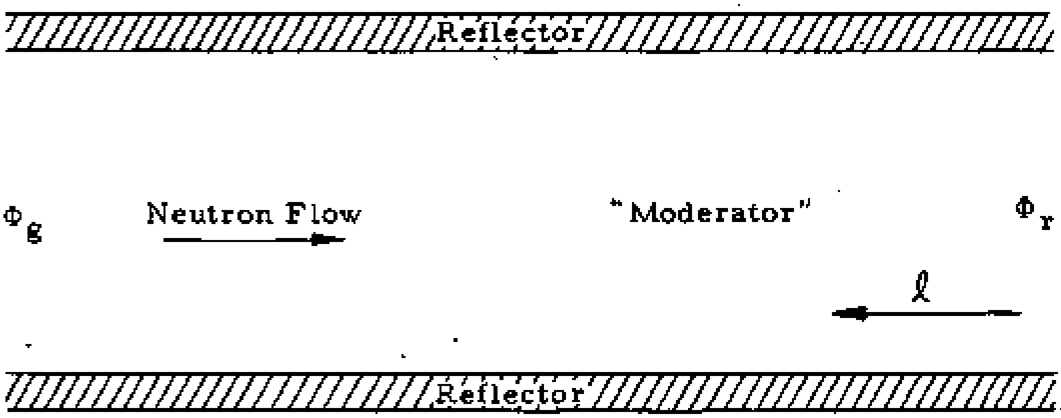
APPENDIX I

Transmission Line Analogy

The results of the finite difference calculation can be reached in a more exact form by making the following analogy between a transmission line and a reactor. It is possible to build lumped network exactly equivalent to a distributed transmission line at the end points of the line. By making the direct analogy, then one can build a network exactly equivalent to a distributed reactor.

A. One-Group Theory

Consider a slab pile of any length ℓ surrounded by a perfect reflector.



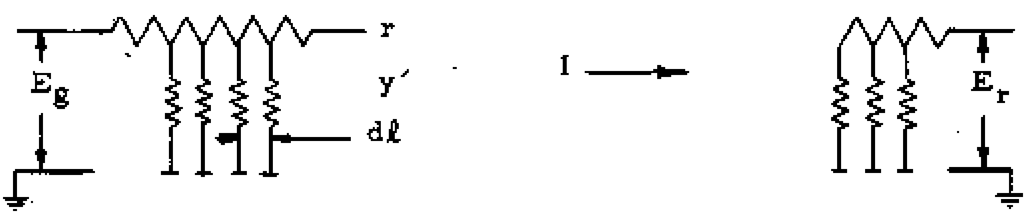
$(\Phi)_g$ is the flux at a distance ℓ from $(\Phi)_r$.

Fissionable material is distributed homogeneously along the reactor moderator.

Reactor material constants are Σ_a, $3\Sigma_t$

K = number of thermal neutrons produced per thermal neutron absorbed in an infinite reactor.

The equivalent electrical transmission line is:



In which volt g is the current generated per volt every dl distance along the line and is analogous to $k\Sigma_{ath}$ in the reactor.

$y' = \Sigma_a$ mohs per unit length.

$r = 3\Sigma_t$ ohms per unit length.

Then

I = Current in amps, is analogous to neutrons flowing at any point.

E = Voltage anywhere along the line.

E_r = At receiving end = $(nv)_r \equiv \Phi_r$

E_g = At sending end = $(nv)_g \equiv \Phi_g$

l = Length (cm) and will be measured from the receiving end.

$$\frac{dE}{dl} = rI$$

$$\frac{d^2E}{dl^2} = \frac{r\,dI}{dl}$$

Let the net conductance $y = y' - g$ (since g is a negative conductance).

Then:

$$\frac{dI}{dl} = yE$$

$$\frac{d^2I}{dl^2} = y\frac{dE}{dl}$$

$$\frac{d^2E}{dl^2} = ryE \tag{1}$$

$$\frac{d^2I}{dl^2} = ryI \tag{2}$$

Letting $D = \frac{d}{dl}$, the solution of these equations is

$$(D^2 - ry)\,E = 0$$

$$(D^2 - ry)\,I = 0$$

$$E = A \exp \ell \sqrt{ry} + B \exp - \ell \sqrt{ry}$$

$$I = C \exp \ell \sqrt{ry} + D \exp - \ell \sqrt{ry}$$

$$\frac{dE}{d\ell} = A \sqrt{ry} \quad \exp \ell \sqrt{ry} \quad - B \sqrt{ry} \quad \exp - \ell \sqrt{ry}$$

$$\frac{dI}{d\ell} = C \sqrt{ry} \quad \exp \ell \sqrt{ry} \quad - D \sqrt{ry} \quad \exp - \ell \sqrt{ry}$$

When $\ell = O$

$$\frac{dE}{d\ell} = r\, I_r \qquad (I_r = \text{receiving end current})$$

and

$$\frac{dI}{d\ell} = y\, E_r$$

Then

$$E = A + B = E_r$$

$$\frac{dE}{d\ell} = A \sqrt{ry} \quad - B \sqrt{ry} \quad = rI_r$$

$$I = C + D = I_r$$

$$\frac{dI}{d\ell} = C \sqrt{ry} \quad - D \sqrt{ry} = y\, E_r$$

Solving for A, B, C, and D substituting in E and I equations:

$$E = E_r \cosh \ell \sqrt{ry} - I_r \sqrt{\frac{r}{y}} \sinh \ell \sqrt{ry} \qquad (3)$$

$$I = I_r \cosh \ell \sqrt{ry} - E_r \sqrt{\frac{y}{r}} \sinh \ell \sqrt{ry} \qquad (4)$$

These are the usual transmission line equations. There are reactor constants to be inserted directly for the transmission line constants.

For use later the following relations can be set up.

With receiving end of line open. $I_r = O$

$$E_g = E_r \cosh \ell \sqrt{ry} \qquad (5)$$

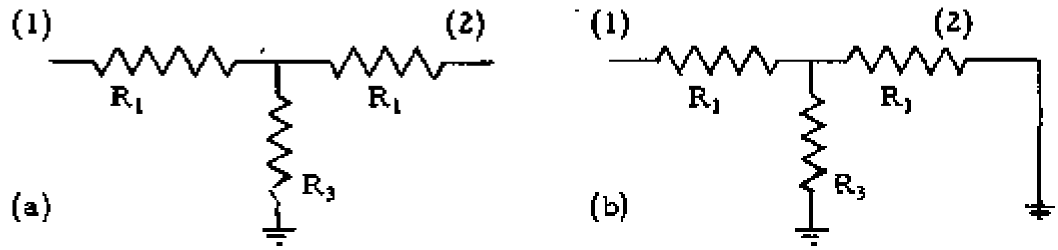
$$I_g = - E_r \sqrt{\frac{y}{r}} \sinh \ell \sqrt{ry} \qquad (6)$$

With receiving end short circuited. $E_r = O$

$$E_g = I_r\sqrt{\frac{r}{y}}\sinh \ell\sqrt{ry} \qquad (7)$$

$$I_g = I_r \cosh \ell\sqrt{ry} \qquad (8)$$

A lumped resistance network which is equivalent to the line of distributed constants must be found. This can be obtained in the following way:



Let R_f be the resistance to ground from (1) and (2) in (a)

Let R_g be the resistance to ground from (1) in (b)

<u>Then</u>

$$R_f = R_1 + R_3$$

$$R_g = R_1 + \frac{R_1R_3}{R_1 + R_3} = \frac{R_1^2 + R_1R_3 + R_1R_3}{R_1 - R_3}$$

Eliminate R_1

$$R_g = \frac{(R_f - R_3)^2 + 2\,(R_f - R_3)\,R_3}{R_f}$$

$$R_g = \frac{R_f^2 - 2R_f\,R_3 + R_3^2 + 2R_fR_3 - 2R_3^2}{R_f}$$

$$R_g = \frac{R_f^2 - R_3^2}{R_f}$$

$$R_3 = \sqrt{R_f^2 - R_fR_g} \qquad (9)$$

Solve for R_3 from (7) and (8)

$$R_g = \frac{E_g}{I_g} = \sqrt{\frac{r}{y}}\,\tanh \ell\,\sqrt{ry}$$

Likewise from (5) and (6)

$$R_f = \frac{E_g}{I_g} = +\sqrt{\frac{r}{y}}\,\coth \ell\,\sqrt{ry}$$

Putting these values in (9)

$$R_3 = \sqrt{\frac{r}{y}}\sqrt{\coth^2 \ell\sqrt{ry} - \tanh \ell\sqrt{ry} \cdot \coth \ell\sqrt{ry}}$$

$$R_3 = \sqrt{\frac{r}{y}} \cdot \frac{1}{\sinh \ell\sqrt{ry}}$$

$$R_1 = R_f - R_3$$

$$= \sqrt{\frac{r}{y}} \coth \ell \sqrt{ry} - \frac{1}{\sinh \ell\sqrt{ry}}$$

$$= \sqrt{\frac{r}{y}}\left[\frac{\cosh \ell\sqrt{ry} - 1}{\sinh \ell\sqrt{ry}}\right]$$

$$= \sqrt{\frac{r}{y}} \tanh \frac{\ell\sqrt{ry}}{2}$$

By division

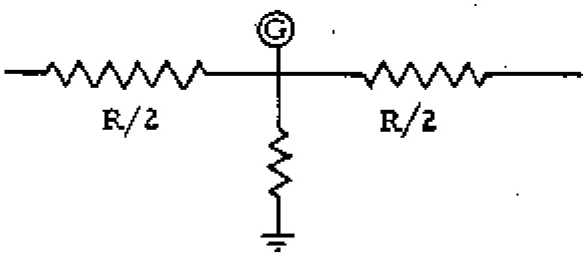
$$\frac{R_1}{R_3} = \tanh \frac{\ell\sqrt{ry}}{2} \cdot \sinh \ell\sqrt{ry}$$

Since R_1 is 1/2 of the resistance of the whole T section

$$\frac{R}{R_3} = 2 \tanh \frac{\ell}{2}\sqrt{ry} \cdot \sinh \ell\sqrt{ry}$$

In which the lumped generator = $g\ell$ is contained in R_3.

Let this lumped generator = G



Then

$$\frac{1}{R_3} = \frac{1}{R_2} - G$$

or

$$\frac{R}{R_2} - RG = 2 \tanh \frac{\ell}{2}\sqrt{ry} \cdot \sinh \ell\sqrt{ry}$$

Insert

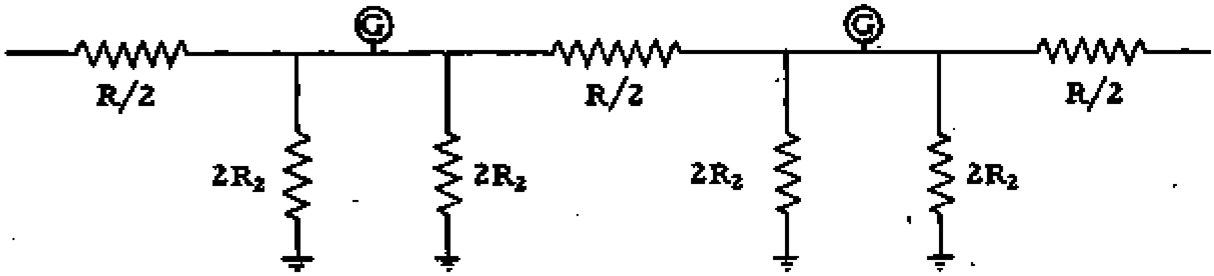
$$y = y' - g$$

The general relation is obtained

$$\frac{R}{R_2} - RG = 2 \tanh \frac{\ell}{2}\sqrt{r\,(y' - g)} \quad , \quad \sinh \ell \sqrt{r\,(y' - g)} \qquad (10)$$

in which G and g are any negative conductances, the values of which can be specified as desired.

The equivalent π section will be:



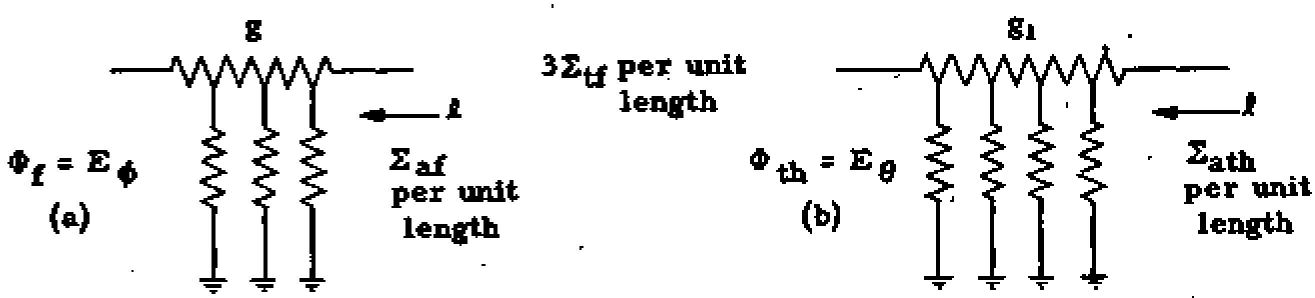
from which it is evident that the same relations will hold for the resistances.

This method can be used for three dimensions with equal results by simply considering that the neutrons flow only in the directions x, y, and z. Each direction is then a separate transmission line having the resistance relations of formula (10).

B. Two-Group Theory

This method is now extended into two-group theory by using the results of the preceding calculations.

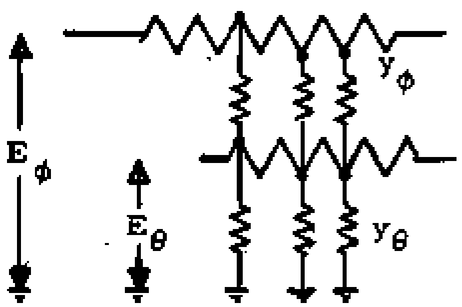
For simplicity a slab pile is again considered and an equivalent electrical transmission line is drawn for each neutron energy group.

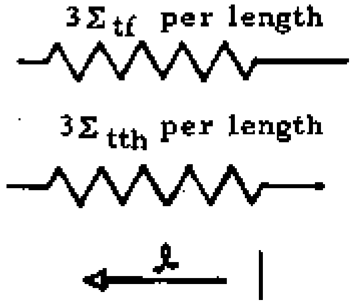


High Energy Group

Low Energy Group

The high energy group line is put on top of the low energy group line in such a way that $E_{\phi}\Sigma_{af}$ no longer feeds to ground but instead feeds into the low energy line in place of g_1 as follows:





We are interested in finding such values for y_{ϕ} and y_{θ} that with the lines combined, each will have the same current flow as it did with the lines separated.

For the separate lines if $g = k\Sigma_{ath}$ the following equations for current are obtained.

$$\frac{dI_{\phi}}{dl} = E_{\phi} \cdot \Sigma_{af} - k\Sigma_{ath} E_{\theta} \tag{11}$$

and

$$\frac{dI_{\theta}}{dl} = \Sigma_{ath} E_{\theta} - \Sigma_{af} E_{\phi} \tag{12}$$

For the combined lines

$$\frac{dI_{\phi}}{dl} = (E_{\phi} - E_{\theta})\, y_{\phi} - g E_{\theta}$$

$$\frac{dI_{\phi}}{dl} = E_{\phi}\, y_{\theta} - (g + y_{\phi})\, E_{\theta} \tag{13}$$

and

$$\frac{dI_{\theta}}{dl} = E_{\theta}\, y_{\theta} - (E_{\phi} - E_{\theta})\, y_{\phi}$$

$$\frac{dI_{\phi}}{dl} = (y_{\theta} - y_{\phi})\, E_{\theta} - y_{\phi} E_{\phi} \tag{14}$$

So that (11) and (12) will equal (13) and (14) respectively, the following conditions must exist:

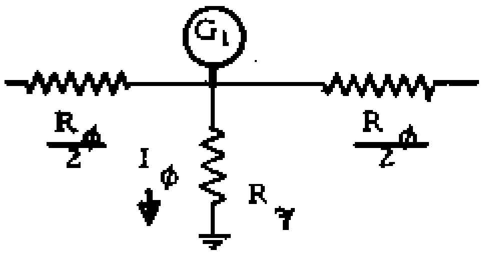
$$y_{\phi} = \Sigma_{af}$$
$$y_{\theta} = \Sigma_{ath} - \Sigma_{af} \tag{15}$$

$$g = k\,\Sigma_{ath} - \Sigma_{af}$$

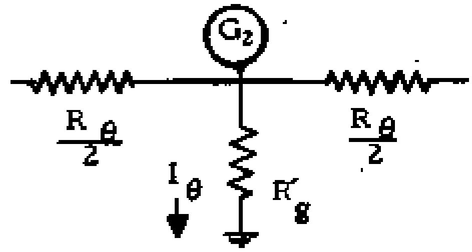
$$g' = \Sigma_{af} \qquad (15)$$

These relations will be used later.

A T network of resistors can be treated in a similar manner. The T sections are set up as follows:

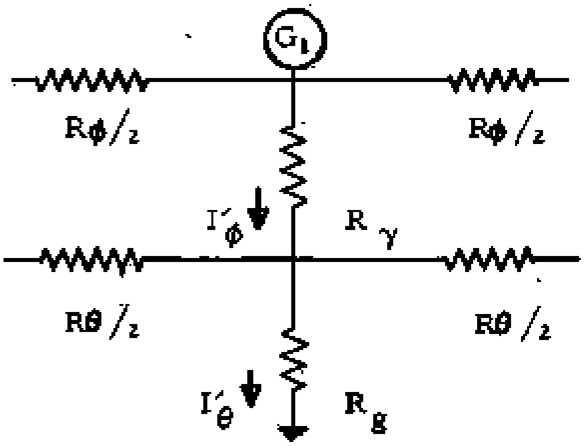


φ Network



θ Network

Put the φ network on top of the θ network



R_θ, R_ϕ and R_γ are the same as before and R_g is an adjusted value to be determined. The currents I'_ϕ I'_θ are those resulting from the rearrangement of the networks.

Then

$$I_\phi = \frac{E_\phi}{R_\gamma}$$

$$I'_\phi = \frac{E_\phi - E_\theta}{R_\gamma} = \frac{E_\phi}{R_\gamma} - \frac{E_\theta}{R_\gamma}$$

$$I_\phi - I'_\phi = \frac{E_\theta}{R_\gamma}$$

Since the currents in the resistors R_ϕ and R_θ are to be the same in the combined networks as in the separate networks, I'_θ is less by $\frac{E\theta}{R_\gamma}$ and,

$$I_\theta = \frac{E_\theta}{R'_g}$$

$$I'_\theta = \frac{E_\theta}{R_g} = \frac{E_\theta}{R'_g} - \frac{E_\theta}{R_\gamma}$$

or (16)

$$\frac{1}{R'_g} - \frac{1}{R_\gamma} = \frac{1}{R_g}$$

G_1 for the networks separately can be defined

$$G_1 = \frac{k}{R'_g} = \frac{k}{R_g} - \frac{k}{R_\gamma}$$

This conductance must be reduced by $\frac{1}{R_\gamma}$ mhos so that the currents in R_θ and R_ϕ will not change, and, as a result, the conductance for the combined networks is

$$G = \frac{k}{R_g} - \frac{k}{R_\gamma} - \frac{1}{R_\gamma} \qquad (17)$$

Using equation (10') to relate the distributed values to the lumped values the following are obtained for separate networks and lines:

For ϕ Network

$$\frac{R_\phi}{R_\gamma} - R_\phi G = 2 \tanh \frac{\ell}{2} \sqrt{3\Sigma_{tf}\Sigma_{af} - 3\Sigma_{tf\,g}} \quad \sinh \ell \sqrt{3\Sigma_{tf}\Sigma_{af} - 3\Sigma_{tf\,g}}$$

For θ Network

$$\frac{R_\theta}{R'_g} - R_\theta G_1 = 2 \tanh \frac{\ell}{2}\sqrt{3\Sigma_{tth}\Sigma_{ath} - 3\Sigma_{tth\,g}} \cdot \sinh \ell \sqrt{3\Sigma_{tth}\Sigma_{ath} - 3\Sigma_{tth\,g}}$$

Using relations 11 through 17 these can be converted into similar relations for combined networks and lines.

For ϕ Network portion

$$\frac{R_\phi(2-k)}{R_\gamma} - \frac{R_\phi k}{R_g} = 2 \tanh \frac{\ell}{2}\sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}} \cdot \sinh \ell \sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}} \qquad (18)$$

For θ Network Portion

$$\frac{R\theta}{R_g} = 2 \tanh \frac{\ell}{2}\sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}} \cdot \sinh \ell \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}} \qquad (19)$$

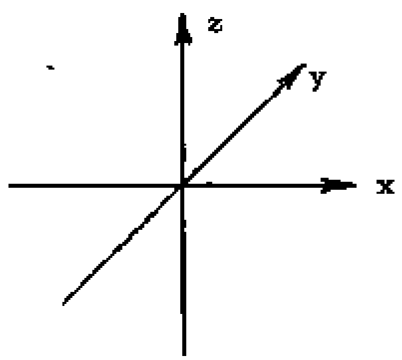
For small angles $\tanh \gamma = \sinh \gamma = \gamma$ and using this, reduction of (18) will give formula (10) of Section A.

$$\frac{R\phi}{R\gamma} = \frac{\ell^2}{T}$$

and for (19) formula (14) of section A is obtained.

$$\frac{R\theta}{R_g} - \frac{Re}{R\gamma} = \frac{\ell^2}{L^2}$$

To apply formulas (18) and (19) to a three dimensional network it is necessary to consider that the neutrons move in x, y and z directions of the cartesian coordinates. The analogy now calls for three transmission lines which meet at a point x, y, z.



If each line is then treated alone the formulas 18 and 19 will apply for each direction. If a, b, and c are made the lengths in the x, y and z direction respectively we will get:

$$\frac{R_{\phi x}}{R\gamma}[2-k] - \frac{R_{\phi x}}{R_g} = 2 \tanh \frac{a}{2}\sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}} \qquad \sinh a \sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}}$$

$$\frac{R_{\phi y}}{R\gamma}[2-k] - \frac{R_{\phi y}}{R_g} = 2 \tanh \frac{b}{2}\sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}} \qquad \sinh b \sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}}$$

$$\frac{R_{\phi z}}{R\gamma}[2-k] - \frac{R_{\phi z}}{R_g} = 2 \tanh \frac{c}{2}\sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}} \qquad \sinh c \sqrt{\frac{2}{T} - 3k\Sigma_{tf}\Sigma_{ath}}$$

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$$\frac{R_{\theta x}}{R_g} = 2 \tanh \frac{a}{2} \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}} \cdot \sinh a \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}}$$

$$\frac{R_{\theta y}}{R_g} = 2 \tanh \frac{b}{2} \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}} \cdot \sinh b \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}}$$

$$\frac{R_{\theta z}}{R_g} = 2 \tanh \frac{c}{2} \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}} \cdot \sinh c \sqrt{\frac{1}{L^2} - 3\Sigma_{tth}\Sigma_{af}}$$