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SUBJECT: PERMISSIBLE RATE OF TEMPERATURE RISE OF SHELLS

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PERMISSIBLE RATE OF TEMPERATURE RISE OF SHELLS

B. L. Greenstreet and A. S. Thompson

It is often desired to raise the temperature of apparatus from ambient to operating temperature by circulating a hot fluid over its surface. The rate at which the temperature can be raised is sometimes limited by the thermal distortions and stresses caused by temperature differences within the material.

Consider a material being heated only on one side by a fluid initially at the same temperature as the material. If it were required to maintain a constant thermal stress in the material during heating, the required rate of increase of the fluid temperature would be large at first, decreasing to some approximately constant minimum value after some time, t_1 . Thus, holding the rate of increase at a constant value will result in a nearly maximum thermal stress at time, t_1 , and lower thermal stresses at times less than t_1 .

From this consideration it seems possible to heat a material at a constant rate during the entire heating period, this constant rate being determined by the maximum allowable thermal stress. For a uniform material in which the properties are independent of the temperature, the temperature variation through the material is described by

$$(1) \quad \frac{\partial^2 T}{\partial x^2} = \frac{C_p \gamma}{k} \frac{\partial T}{\partial t}$$

where T is the temperature of the material at a distance x from the heated surface and at time t , C_p is the specific heat at constant pressure of the material, γ is its density, and k its thermal conductivity.

Here we have a material insulated on one side, whose temperature on the other side is some arbitrary function of time. Initially the material temperature is constant. For this case the boundary conditions to be used in Eq. (1) are:

$$\begin{aligned}
 T &= \phi(t) && \text{when } x = 0, \\
 \frac{dT}{dx} &= 0 && \text{when } x = l, \\
 \text{and } T &= T_1 && \text{when } t = 0.
 \end{aligned}$$

The temperature of the material for $x > 0$ is given by*

$$\begin{aligned}
 (2) \quad T &= \frac{2}{l} \sum_{n=0}^{\infty} e^{-\frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p l^2}} \cos \frac{(2n+1) \pi x}{2l} \left\{ \frac{(2n+1) \pi k (-1)^n}{2l \gamma C_p} \right. \\
 &\quad \left. \int_0^t e^{-\frac{k(2n+1)^2 \pi^2 \lambda}{4 \gamma C_p l^2}} \phi(\lambda) d\lambda + \int_0^l T_1 \cos \frac{(2n+1) \pi x'}{2l} dx' \right\}.
 \end{aligned}$$

Take $\phi(t)$ in the form $\phi(t) = A + Bt$.

For zero initial temperature, A becomes zero. Thus,

$$(3) \quad \phi(t) = Bt,$$

* H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, p. 87, Oxford University Press, London (1947).

and

$$(4) \quad T = \frac{16}{\pi^3} \frac{\ell^2 \gamma C_p}{k} B \sum_{n=0}^{\infty} \cos \frac{(2n+1) \pi x}{2\ell} \left[e^{-\frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p \ell^2}} + \frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p \ell^2} - 1 \right] \frac{(-1)^n}{(2n+1)^3}$$

The temperature difference across the material is obtained from Eqs. (3) and (4).

$$(5) \quad \Delta T = \phi(t) - T_{x=\ell} = \frac{16}{\pi^3} \frac{\ell^2 \gamma C_p}{k} B \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \left[1 - e^{-\frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p \ell^2}} \right]$$

When ΔT is a maximum, the thermal stress, σ , is also a maximum, since it is given by the following equation:

$$(6) \quad \sigma = D \frac{E \alpha \Delta T}{(1 - \mu)}$$

Here E is the modulus of elasticity and α is the coefficient of thermal expansion, μ is Poisson's ratio of lateral expansion, D is a factor which decreases from the value 1 at the beginning to $2/3$ near the end of the heating period.

Differentiating Eq. (5) with respect to time, we have

$$(7) \quad \frac{d(\Delta T)}{dt} = \frac{4}{\pi} B \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} e^{-\frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p \ell^2}}$$

Equation 7 indicates a maximum exists only at $t = \infty$. However, the exponential terms decrease very rapidly, and at some finite time, the derivative can be considered zero with little error.

In Eq. (5), when the term

$$e^{-\frac{k(2n+1)^2 \pi^2 t}{4 \gamma C_p l^2}} \leq 1/10 ,$$

it can be considered negligible in comparison to 1 for all practical purposes. The time, t_1 , mentioned earlier is then given by

$$(8) \quad \frac{k t_1}{\gamma C_p l^2} \approx 1 ,$$

and the temperature difference for $t \geq t_1$ is

$$(9) \quad \Delta T \approx \frac{\gamma C_p l^2}{2 k} B .$$

Substituting Eq. (9) into Eq. (6)

$$\sigma = \frac{E \alpha \gamma C_p l^2}{3 k(1-\mu)} B .$$

Thus, the allowable rate of increase, B , of the surface temperature becomes

$$(10) \quad \left. \frac{dT}{dt} \right|_{x=0} = B = \frac{3 \sigma (1-\mu)}{E \alpha} \frac{k}{\gamma C_p l^2} .$$

To heat a material rapidly with a minimum thermal stress requires a high thermal conductivity, a small thickness, and a low coefficient of thermal expansion. The thickness is very important since it alone enters the equation to the second power.

Equation (10) expresses a limitation on the rate of heating of a surface in terms of the stress imposed in the material. As an example consider the following.

$$k = 0.05 \text{ cal/sec} \cdot \text{cm} \cdot ^\circ\text{C}$$

$$C_p = 0.109 \text{ cal/gm} \cdot ^\circ\text{C}$$

$$\gamma = 8.51 \text{ gm/cc}$$

$$l = 5 \text{ cm}$$

$$E = 20 \times 10^6 \text{ psi}$$

$$\sigma = 30,000 \text{ psi}$$

$$\alpha = 10 \times 10^{-6} / ^\circ\text{F}$$

$$\mu = 0.3$$

Then from Eq. (10) the maximum allowable rate of temperature increase is

$$\left. \frac{dT}{dt} \right|_{x=0} = \frac{3 \times 30,000 \times 0.05 \times 0.7}{20 \times 10^6 \times 10 \times 10^{-6} \times 0.109 \times 8.51 \times 25} \sim 0.7^\circ\text{F/sec.}$$

NOTE: Since the writing of this paper, the following article on thermal stresses in spheres and cylinders has come to the attention of the authors: Clarence H. Kent, Thermal Stresses in Spheres and Cylinders Produced by Temperatures Varying with Time, Trans. ASME 54, 185-96 (1932).