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AEC Research and
Development Report

Knolls
Atomic Power
Laboratory

**Correlation of Experimental
Data with Theory for
Perforated Plates with a
Triangular Hole Array**

L. Deagle

January 7, 1958

Operated for the
United States Atomic
Energy Commission by

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CORRELATION OF EXPERIMENTAL DATA WITH
THEORY FOR PERFORATED PLATES WITH A
TRIANGULAR HOLE ARRAY

L. Deagle

January 7, 1958

W E Cooper

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Date

Knolls Atomic Power Laboratory
General Electric Company
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ABSTRACT

The design of the S3G and S4G reactor pressure vessel heads led to a study of perforated plates. This report presents a correlation of G. Horvay's perforated plate theory with experimental data. The test results and analysis used for the S3G model tests are contained in this report along with a review of two other experimental programs in which the experimental data were correlated with G. Horvay's theory.

The conclusions reached from the data presented are:

1. The perforated lattice does, in fact, behave as an equivalent plate of reduced stiffness when subjected to transverse loading.
2. The results obtained from the theory are sufficiently accurate for engineering applications for a range of plate sizes which extends from thin plates (radius-to-thickness ratio 7.5 to 1 for Duncan's tube sheets) to thick plates (radius-to-thickness ratio 1.6 to 1 for the reactor model head) with a variation in the number of penetrations ranging from the large number in Duncan's tube sheet tests down to the relatively small number (19 holes) in the reactor model head (3/8 scale model).

CORRELATION OF EXPERIMENTAL DATA WITH THEORY FOR PERFORATED PLATES WITH A TRIANGULAR HOLE ARRAY

L. Deagle

INTRODUCTION

The purpose of this paper was to investigate the adequacy and range of applicability of the equivalent solid plate analogy for the calculation of stresses and deflections in perforated plates, as developed by G. Horvay. This objective was to be met by comparing measured values of stresses and deflections with the calculated values obtained from Horvay's theory. The published literature, unpublished reports, and unpublished test results, such as the tests conducted by and for the Knolls Atomic Power Laboratory, were considered as a source of test information to be used to meet the stated objective. The results of unpublished tests were solicited from twenty-one manufacturers and users of pressure vessels.

The problem of determining stresses and deflections in a perforated plate is fundamental to the design of all pressure containing vessels which require multiple openings such as are found in the tube sheets of heat exchangers and the heads of nuclear reactor pressure vessels. The work of the designer is greatly simplified if the various parameters such as plate thickness, outside diameter, hole spacing, and hole size can be readily manipulated without recourse to empirical data. The theory developed by Horvay permits analytical treatment of the perforated plate by reducing it to an equivalent solid plate of the same geometry. The deflection is calculated by replacing Young's modulus E and Poisson's ratio μ with fictitious elastic constants E^* and μ^* that are based upon the hole size and spacing in the perforated plate. Similarly, the stress is calculated as if it were a solid plate and corrected by means of a stress multiplier, based on the hole size and spacing. These factors, as derived by Horvay, are presented in Figures 28 and 29, Appendix D, as a function of the ratio of the minimum ligament width to hole spacing.

Although the theory, derivations, results (including the curves in the appendix), and applications to various perforated plate problems are thoroughly treated by Horvay^{1,2,10-14} it is believed that a brief statement of the theory and derivation should be included here.

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FIGURE 1. Ligament Geometry and Loading for Horvay's Theory

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The method employed to obtain the equivalent elastic constants E^* and μ^* was to consider a triangular array of holes as a grid-work of rectangular beams subjected to an in-plane loading. The equivalent thickness of a rectangular beam which had the same resistance to tension and to bending as the actual curved ligament was derived for the cases shown in Figure 1(b), (c), and (d).¹

The type (d) loading is neglected in the subsequent derivations. An expression for the vertical and horizontal deflection of a ligament with type (b) and (c) loading that use the equivalent widths $2\bar{h}$ and $2h'$ in the elementary beam formulas is derived. An expression for the vertical and horizontal strain in a ligament is then obtained. Horizontal and vertical refer to directions along mutually perpendicular coordinates in the plane of the plate as shown in Figure 1(a).

The term μ^* is the ratio of the horizontal to vertical strains. The constant E^* is obtained by dividing the surface traction (load per unit area) by the strain in the same direction and then substituting the expression previously determined for the strain so that E^* becomes a function of μ^* , the actual Young's modulus E , and the ratio of minimum ligament width to hole spacing.² The fictitious constants E^* and μ^* are then a measure of the actual stiffness of the curved beams of real material which make up the plate.

The stress multiplier,^{1,2} used to correct the solid plate stress was obtained by considering a hexagonal array of beams subjected to in-plane perpendicular boundary tractions and shear loads, as shown in Figure 1, (a). The stresses in the ligaments were obtained from the stress distribution on the boundaries of the hexagonal array so that the maximum stress in a ligament was expressed as a function of this stress distribution, which corresponded to the principal stresses in a solid plate and the orientation of the ligament with respect to these principal stresses. Thus, when the ratio of solid plate principal stresses is unity (isotropic tension or compression), the maximum stress in the perforated plate occurs at the minimum ligament width and is equal to the solid plate stress multiplied by the ratio of the hole spacing to the minimum ligament width. When the ratio of the principal stresses is other than unity (this ratio can vary from 1 to -1), the maximum stress is increased over that for the isotropic case and is found to vary with the orientation of the ligament with respect to the larger of the principal stresses; the ligament nearest to an angle of 45 deg with the larger of the principal stresses then has the highest stress. The highest stress in a given plate is obtained when the ratio of the principal stresses is equal to -1 (pure shear). Thus, the stresses in the ligaments of a perforated plate are shown to be a function of ligament width, ratio of principal stresses, and the orientation of the ligament with respect to these "solid plate" principal stresses.

The increase in stress caused by the change in the ratio of principal stresses from unity is the result of bending moments of the type shown in Figure 1(b), which produce additional stresses away from the center of the ligament. Thus, the theory predicts that the maximum stress will occur in the center of the ligament only for an isotropic solid plate stress distribution and is shifted away from the center for other distributions. The curves in Figure 29, Appendix D, are plotted for ratios of the principal stresses of 1, 0, and -1 for a ligament oriented at 45 deg with the maximum principal stress.

The principal restrictions and assumptions aside from those normally associated with solid plates are:

1. The plate should have a large number of holes (several rows).
2. The holes must be arranged in an equilateral triangular array.
3. The plate is loaded in-plane.
4. Reinforcement of the plate by tubes inserted in the holes is negligible.
5. Use of the constants E^* and μ^* is restricted to plates whose hole spacing is smaller than the plate thickness.

NOTE: The foregoing restrictions are Horvay's.

TESTS PERFORMED

Flat Perforated Aluminum Plate Tested for Structural Efficiency

The purpose of this test was to determine the feasibility of extending the perforated plate theory, developed by Horvay, to plates whose thickness was large in comparison to the radius of the plate and in which the number of penetrations was relatively small. This test was made at the Knolls Atomic Power Laboratory, Schenectady, New York, and was undertaken as a preliminary step to the design of the closure head for the reactor pressure vessel for a submarine. With the foregoing in mind, the important features of the geometry, the number and size and skew angle of penetrations, were proportioned so that the plate became an $\sim 1/4$ -scale model of the head intended for the full-size vessel. The skew angle is formed by the intersection of the penetration centerline and a normal to the plate surface; this angle lies in a plane normal to a radius of the plate.

The test piece shown in Figure 2 was machined from 61ST Aluminum. The nineteen penetrations in the plate, 2.185-in. diam, are arranged in a triangular array with a maximum skew angle of 9 deg 52 min in the outer holes. This skewing of the penetrations, dictated by nuclear requirements of the core and external requirements for mechanical clearance, resulted in ligaments of different sizes on the two surfaces of the plate. A one-inch thick web was left in the center of the plate to provide a pressure seal for the hydraulic oil which is used to apply the load to the plate. The plate thickness in the perforated portion was five inches; this dimension was chosen as being scaled to the order of magnitude of the head that would be used on the full size vessel. Since the test piece was so thick, aluminum was chosen for its low modulus so that deflections, which are measurable with dial micrometers, could be obtained without having to use excessively high loads.

The aluminum plate was clamped into a heavy steel fixture for the duration of these tests, as shown in Figure 3. The cavity beneath the plate was filled with hydraulic oil and the plate was clamped against a gasket to form a seal against the pressure. Pressurization was accomplished by means of a hand pump and the hydraulic pressure was measured with a Bourdon-type pressure gage. Both strains and deflections were measured during the test. The general arrangement of gages is shown in Figure 2. The strains were measured with Baldwin A-19 electrical resistance strain gages of 1/8-in. gage length; stresses were then computed from these strain gage readings. The strain gages were mounted on the exposed surface only, at the narrowest part of three ligaments, 120 deg apart around the center hole. Three gages were mounted across each of the three ligaments; one gage in the center and one at each edge of the ligament. The deflections were measured by 0.0001-in.-dial micrometers of which eight were mounted at four different radii. The scribed circles show the locations of the dial gages, (see Figure 2).

FIGURE 2. Aluminum Plate with Top Side Up

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FIGURE 3 ALUMINUM PLATE AND
PRESSURE VESSEL

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The pressure was applied in 200-psi increments. After each increase in load, the dial indicator and strain gage readings were recorded. After all readings had been completed on one side, the test piece was reversed in the fixture, the gages and dial indicators were remounted on the exposed surface, and then the procedure was repeated. Thus, strain gage readings and dial indicator readings were made on both surfaces at the same location and the at same pressure loading. Since the holes in the test piece are skewed, the web thickness is larger on one surface than on the other; the surface with the larger web thickness is designated the "top" and the surface with the small web thickness the "bottom."

The mathematical calculation for stress and deflection in the model was made by using the general analysis given in Appendix A. The model was treated as a composite structure that consisted of a perforated inner plate and a solid outer annulus. The usual formulas for solid plates and rings, as set forth in Reference 18 and 19, with Horvay's equivalent elastic constants replacing the actual Young's modulus and Poisson's ratio where appropriate, were used to derive the formulas for stress and deflection in the test model. The outside radius of the perforated portion was chosen as being equal to twice the hole spacing plus one-half the hole radius. This approximation was judged to fit best the idealization of a perforated plate attached to a solid rim by means of

symmetrically spaced spokes. This approximation produces an outer solid annulus whose inner surface is conical as shown in Figures 14 and 18, Appendix A. This point of support of the plate was inferred from the experimentally determined deflection curve.

The details of the geometry and loading are given in Appendix B along with the experimental data. In general, the procedure made allowance for the effect of the skewed holes on the geometry of the perforated portion and the solid outer annulus and then the problem of interaction between the perforated portion, and the solid annulus was treated as though the holes were drilled normal to the surface. This was accomplished by obtaining the center of gravity of the cross section of the ring and its moment of inertia about a horizontal axis through the center of gravity by using the approximation that resulted in the conical inner surface, (see Figure 18, Appendix B). This conical effect was accounted for in the perforated portion by assuming that the neutral axis passed through the center of gravity of the trapezoidal section formed by the cross section at the narrowest part of the ligament. It was further assumed that the plate stiffness and stress intensification were proportional to the ratio of the average ligament width to the average hole spacing. This, in effect, tacitly assumes vertical holes of the same diameter, bored on centers equal to the hole spacing actually existing at the midplane. This last assumption made it possible to enter Horvay's curves and obtain the equivalent elastic constants and stress multiplier; also, it established an effective outer radius at the midplane of the perforated portion. It was then possible to calculate the discontinuity forces and moments which arise at the juncture of a plate and ring of known geometries and physical properties, provided the external loads were known.

The external loading consisted of a known uniform pressure load but unknown edge conditions. These unknown edge effects are the line of action of the vertical reaction and the magnitude of the horizontal friction force. The former was inferred by observing the point at which the deflection curve, drawn through the experimentally determined deflection points, went to zero, (see Figure 4).

The value for the coefficient of friction is based on judgment since no measurements under actual conditions were made to attempt to establish what this value might be. However, a value for dry friction of 0.61 for steel and aluminum had been measured in a very carefully controlled experiment,³ and this may be considered an upper limit. For lubricated or greasy surfaces for metal on metal, values as low as 0.01 or lower are quoted in various references. Since no effort was made to obtain either a maximum or minimum friction effect, it seems reasonable, in the absence of any other information, to choose a value half way between the extremes; specifically, the value of 0.3 was used for the coefficient of friction.

FIGURE 4. Deflection Curves - Aluminum Plates

The testing of the model consisted of two separate tests; (1) the strains and deflections were measured on the "top" surface with the largest ligament uppermost in the test fixture, and (2) another in which the measurements were made on the "bottom" surface with the minimum ligament width uppermost. This reversal of the plate in the fixture resulted in a change in the loading of the plate. This change in the actual loading caused what appears to be an anomaly in that the strains measured on the "top" surface were larger than those measured on the "bottom" surface while the deflections measured for the "top" were smaller than those measured for the "bottom" surface. The explanation for this is as follows.

When the measurements were made on the "top" surface, the pressure in the lower half of the nineteen holes produced an unbalanced force on the so-called solid outer annulus, as shown in Figure 18 Appendix B, (see Figure 3 also). This introduced a moment that opposed the rotation caused by the transverse pressure load while at the same time an in-plane tensile stress was produced. Thus, it was possible for the deflection to be reduced but to have strains on the tensile surface remain relatively high while the strains on the compression surface would undergo a marked reduction. Should this force be counterbalanced, an increase in deflection would be expected with a possible decrease in the tensile stresses. This counterbalancing actually takes place when the test piece is reversed in the fixture to allow the testing of the "bottom" surface since there is now fluid around the outer periphery of what is now the lower half of the plate relative to the fixture.

In addition, the position of the neutral axis of the plate is now such that the horizontal discontinuity force (T in the analysis) which results from the eccentricity of the neutral axis of the perforated portion with respect to the center of gravity (CG) of the solid outer annulus is now compressive and causes a further reduction in the tensile stresses but adds to the compressive stresses. Unfortunately, no gages were on the under side of the plate to verify these deductions. However, the same relative variation of stress and deflection was produced analytically when the effects mentioned were taken into account; therefore, the foregoing is judged to be a valid explanation of a phenomenon which, when it first appeared, was thought to be an error in recording the data but, when subsequently checked, it was found to be valid information.

The average stress across the narrowest part of the ligament around the central hole and the maximum deflections, as arrived at experimentally and analytically, are given in Table 1.

TABLE 1. Comparison of Experimental and Theoretical Results* -
Aluminum Model

		<u>Test Values</u>	<u>Calculated Values</u>
Deflection/Unit Load	Top	0.0050	0.0077
Mils/psi	Bottom	0.0066	0.0085
Stress/Unit Load	Top	6.4	6.3
Psi/psi	Bottom	5.0	5.3

*See Appendix B for details of calculation and experimental values.

Since the stresses as calculated are so close to the measured values, it would be expected that closer agreement between the measured and calculated deflections would have been obtained, and it is believed that such would have been the case had the deflections been measured relative to ground rather than relative to the fixture. The absolute motions are small; consequently, this failure in technique, which introduced the motion of the fixture into the measurement of plate deflection, probably accounts for a large part of the variation between measured and calculated deflections.

The theory that was used gives only average values of stress across the ligament, and it was with these average test values that the comparison was made; however, the three strain gages on each ligament showed a

variation in stress across the ligament with the higher stresses occurring near the edge of the hole. The following table shows these readings which were taken at the three different locations on the "top" and the "bottom" surface.

TABLE 2. Stress/Pressure Load, psi/psi

	<u>"Top" Surface</u>			
	<u>A*</u>	<u>B*</u>	<u>C*</u>	<u>Average</u>
Edge of Ligament	5.96	7.57	6.21	6.59
Center of Ligament	5.54	5.29	5.96	5.60
Edge of Ligament	6.94	6.94	6.94	6.94
	<u>"Bottom" Surface</u>			
	<u>A*</u>	<u>B*</u>	<u>C*</u>	<u>Average</u>
Edge of Ligament	5.37	4.70	5.37	5.13
Center of Ligament	4.81	4.60	4.70	4.70
Edge of Ligament	5.30	4.70	5.42	5.14

*A, B, and C refer to ligaments 120 deg apart around the center hole (see Figure 2).

The variation from maximum to minimum stress across the ligament is ~9% for the smaller ligament and 23% for the larger one. This trend is what might be expected with stresses approaching a uniform distribution over very thin ligaments and, on the other extreme, with those approaching the value of the stress concentration for a hole in an infinite plate when the ligament becomes very large.

It should be pointed out that Horvay's theory is derived for slender ligaments across which the stress is nearly uniform and, therefore, does not make allowance for the stress concentration effect observed in the test except for the approximation which was made by extrapolating the calculated curves in Figure 29, Appendix D, to the limiting stress concentration for a hole in an infinite plate. The comparison of test values with theory is shown in Table 1 on p. 9 for average values of stress across the ligament since this is all that the theory attempts to do. The significance of Table 2 (above) is the variation shown in the stress across the ligament which the designer would have to account for by other means when the ligaments are no longer slender.

FIGURE 5. The 3/8 Model Test Setup - Strain Gage Selector Switches

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FIGURE 6. The 3/8 Model Test Setup - Support Stand and Pressurizing Pump

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For static load-carrying capacity, only the average stress across the ligament is of interest; however, the stress concentration effect at the edge of the hole becomes important for fatigue considerations.

In general, it was concluded from these tests that it was permissible to extend Horvay's theory to the plate tested or to one of similar proportions which has the same type (skewed) and number of penetrations. Because of the numerous assumptions other than those in the perforated plate theory, it is not possible to assign quantitative value to the error that may exist when this theory is used; however, since the over-all error was found acceptable, it may be deduced that the error introduced by Horvay's approach to the perforated plate problem is small even for this extreme case which barely fulfills some of the assumptions on which Horvay derived his constants and stress multipliers.

The 3/8-Scale Model of the Pressure Vessel Head Tested for a Nuclear Reactor

The measurements of stress and deflection made on the pressure vessel head were part of a larger test program in which a 3/8-scale model, in steel, of the reactor pressure vessel, closure, and pipe nozzles were tested for pressure stresses, distortions, and pipe reaction stresses. This test work was done at Combustion Engineering Company, Chattanooga, Tennessee.

The purpose of this test was to provide experimental evidence of the structural adequacy of the design of a reactor pressure vessel and to provide a means of checking the analytical procedures used in the design before fabrication of the full size vessel was completed.

The general arrangement of the test setup showing the test stand and pressure vessel, electric strain gage selector switches, potentiometers, and pressure gage for measuring the hydraulic load is shown in Figure 5. The vessel was subjected to internal pressure; the working fluid is hydraulic oil. Pressurization was obtained from a motor driven pump, shown in Figure 6. Strains were measured by means of (1/4-in.) Baldwin electrical resistance strain gages, and dial micrometers (0.0001 in.) were used to measure the deflections.

The head has nineteen skewed penetrations to which nineteen standpipes are attached on the upper surface. Only three of these standpipes were made to scale for the model; these were: the center standpipe, one intermediate standpipe, and one of the outermost standpipes. The remainder of the holes were fitted with pipes which passed through the holes and were welded to the head by outside fillet welds on the upper surface. The penetrations are closed against pressure by the standpipes so that the head,

FIGURE 7. The 3/8 Model Head and Closure

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in effect, receives a uniform pressure load (see Figures 7 and 8). One of the fundamental assumptions in the analysis is that the standpipes, real or simulated, have a negligible effect on the head deflection. The stress field on the upper surface is, of course, altered by the presence of the standpipes. The nineteen penetrations are 3.54 in. in diameter with a maximum skew angle of 14 deg, 34.45 min. This results in the same general problem as that encountered in the aluminum test model; that is, the ligament varies continually in width from the "top" to the "bottom" surface; the greatest width is at the "top" surface.

The measurements made on the head were: strains on both surfaces and at the junction of the standpipes to the top of the head, and transverse deflections. The strain gages were mounted at the narrowest part of the ligament and at the outer periphery on the "bottom" surface of the head. These locations are shown in the sketch of Figure 26, Appendix C. Only the data provided by gages on the bottom of the head were useful in obtaining a correlation with theory for the stresses in ligaments since the presence of the standpipes eliminate the ligaments on the "top" surface. A view of the "top" surface of the head, taken with the head and closure blocks in place, is shown in Figure 8. Seven vertical deflection measurements were made, spaced symmetrically along a diameter of the head. The dial gages were hung from a truss which was supported from the ground as shown in Figure 5; thus, a relatively rigid framework was provided from which the absolute motion of the head was measured.

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Fig. 6 3/8 Model Head - Top View

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Strain gage readings and deflection readings were taken at 500-psi intervals, while the internal pressure was increased from 0 to 2500 psi. (See Appendix C for plotted data.)

Figures 7 and 8 show the manner in which the head was held in place by 30 shear blocks. A preload was imposed on the head by pulling down on the tension bolts, which forced the continuous wedge ring against the shear blocks which, in turn, were made to slide along the inclined upper surface of the pressure vessel flange; thus the interference between the pressure vessel and shear blocks was increased which resulted in a preload on the head. The pressure seal was obtained by the use of the pair of "O" rings, shown in the Figure 7. The head behaved, as nearly as possible, like a simply supported plate once the pressure had built up sufficiently to remove the preload. The friction force at the base of the shear block was the only force present which prevented the head from being truly simply supported. The surfaces in question were lubricated by Molykote (molybdenum disulphide) for which a friction coefficient of $f = 0.15$ had been obtained by bench tests. The measured data for stress and deflection were plotted versus pressure load in pounds per square inch and a straight line was fitted through the points. The slope of the line after the preload was removed was used to obtain the stress and deflection for correlation with the theory which makes no allowance for effects of preload.

The theoretical analysis of the steel plate was exactly the same as that used for the aluminum plate, the testing of which was described previously. The same assumptions were used in reference to the division of the perforated portion and solid outer annulus, location of the neutral axis, and method of determining Horvay's equivalent elastic constants. The general analysis as given in Appendix A was used to compute stresses and deflections which were to be compared with the measured ones. A comparison of experimental stresses with calculated values is shown in Figure 9. Although the stress values represent discrete points at the center of the respective ligaments rather than points on a continuous stress distribution, this method of presentation, rather than a tabular one, was chosen so that the manner in which the stress varied from point to point, as well as the individual magnitudes, could be compared. It can be seen that the calculated variation is very close to the measured one, which indicates that the head is, in fact, behaving like the analytical model. The ever present uncertainty of the edge conditions could account for the twenty per cent variation in the magnitude of the maximum stresses. In Figure 10 a calculated deflection curve is compared with a curve drawn through the test points. The calculated curve shows more curvature at a

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FIGURE 9. The 3/8 Model Head - Radial Variation of "Bottom" Surface Stress

point midway between the center and the outer edge than the curve through the measured points; however, it is probably close enough for most engineering purposes. The per cent of error between the maximum deflection as measured and the maximum deflection as calculated is approximately the same as the variation of the maximum stresses and could be accounted for in the unknown edge conditions.

The stresses that were measured and calculated were the maximum average stress across the ligament; i.e., the stress at the narrowest part of the ligaments and not the highest stress in the ligament as predicted by Horvay's theory. It is not possible to measure accurately this maximum stress by a strain gage technique; to obtain this stress a photoelastic analysis would be necessary. However, in the model which was investigated, this effect can be neglected since the variation in principal stresses, in the equivalent solid plate, from the center to the outermost ligament is small. In the ligaments around the center hole the ratio of principal stresses is essentially unity, and the maximum stress is at the center of the ligaments.

FIGURE 10. The 3/8 Model Head Deflections

In the outermost ligaments, according to theory, the maximum stress will have shifted from the narrowest part of the ligament and the value will be higher than the value at the minimum ligament width; however, since the variation in principal stress is small and hence their ratio is still close to unity, the stress at the narrowest part of the ligament will be very close to the maximum stress theoretically predicted. Consequently, from the viewpoint of the structural integrity of the head, the slight increase in stress over that which occurs at the narrowest point of the outer ligaments is of no concern since the point of maximum stress in the head is at the center ligaments. The significance of the foregoing, when an attempt is made to evaluate the usefulness of the perforated plate theory, is that no direct check of this phase of the theory is possible from this test. However, the close agreement between the radial variation of calculated stresses and the radial variation of the measured stresses, as shown in Figure 9, strongly indicates that the plate is behaving as predicted in theory and, hence, warrants this further extension of the theory.

In view of the complicated nature of the problem, the agreement obtained between the measured stresses and deflections and the calculated values is considered to be satisfactory. Since the mathematical model used is considered adequate and the perforated plate theory is only one of several approximations which was used, it is concluded that the extension of Horvay's theory to this particular problem is valid.

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FIGURE 11. Duncan's Test Setup

More generally it can be concluded that the perforated plate theory can be extended beyond the limits set by its author, to plates with a relatively small number of holes (down to three rows at least) and with a plate thickness that is not small when compared with the plate radius.

REVIEW OF OTHER DATA

J. P. Duncan's Paper "The Structural Efficiency of Tube-Plates for Heat Exchangers"

This paper describes the testing, test results, and correlation with various theories of perforated plates subjected to transverse loading. Three types of loading were employed on the plates: namely, a concentrated load in the center of the plate, uniform concentric loading at half the outer radius, and a uniform hydrostatic load. These loadings are referred to as central, concentric, and hydrostatic. An Olsen testing machine was used to apply the central and concentric loads as shown in the schematic sketch of Figures 11 (a) and (b). A special hydrostatic loading fixture was made [see Figure 11 (c)] which employed a steel diaphragm to close the holes in the perforated plate. The hydrostatic load was applied by a dead-weight testing machine.

FIGURE 12. Hole Patterns for Duncan's Test Piece

Testing began with an undrilled plate. Subsequently, tests were run with central and concentric loads on the plate which had been drilled, first, to a four-pass drilling pattern, then to a two-pass pattern, and finally to a one-pass pattern. The hydrostatic test piece was drilled to a two-pass pattern and then to a one-pass pattern. The holes were drilled on an equilateral pitch leaving a solid outer ring, as shown in Figure 12.

Under all the various conditions of loading, deflections were measured by means of dial micrometers. Stresses were calculated from strains measured by means of electrical resistance strain gages. By this means radial deflection curves and radial and tangential stress distributions were obtained. The dimensions of the plates that were subjected to central and concentric loading were as follows:

Outside diameter of plate, in.	16
Diameter of perforated portion, in.	11
Plate thickness, in.	1.068
Hole pitch, in.	7/16
Hole size, in.	5/16

The dimensions of the plate used in the hydrostatic test for first, a two-pass and then a one-pass pattern were:

Outside diameter, in.	8
Diameter of perforated portion, in.	5.5
Plate thickness, in.	0.534
Hole pitch, in.	7/16
Hole size, in.	5/32

In addition, hydrostatic tests were made on an 8-in.-OD plate whose initial 2-in. thickness was progressively reduced to 1.821 in., 1.590 in., 1.266 in., 1.104 in., and 1.000 in. Also, the effect of increasing hole size for a 2-in.-thick plate under hydrostatic load was investigated, with the hole size being increased from 3/32 in. to 1/8 in., to 5/32 in.

Duncan computed deflections of the one-pass plate for concentric loading and hydrostatic loading by using the fictitious constants E^* and μ^* (used by Horvay) for the perforated portion. In his procedure the slope and deflection of the perforated portion were matched with the solid outer annulus of the plate. These formulas are derived from Timoskenko's Plates and Shells and are given in the appendix to Duncan's paper. The comparison of calculated stress and deflection of simply supported undrilled plates with measured values made it possible to estimate the edge restraint caused by friction at the support. By applying this correction to calculations of deflection for the perforated plates, Duncan obtained a quantitative estimate of the error caused by using the fictitious constants in the "equivalent plate constant" analysis. The calculated and measured values of central deflection agree within 15% for the 0.534-in.-thick plate under hydrostatic load. The error between calculated values for central deflection and measured values was 13% for the plate subjected to concentric loading. In both cases the calculated deflections were larger than the measured deflections. Duncan concluded that "the tests on one-pass plates confirmed with fair accuracy the proposal that Horvay's thick plate theory might be applied to the central drilled area....."

Duncan points out in his reply to the discussion of his paper that "the essentially experimental study had been instituted to examine a particular case arising in practice, and had been extended as far as opportunity and equipment would allow to broaden its scope and usefulness to designers. It was, therefore, not always well suited to arbitrate some of the finer theoretical points raised." One such point that was neither proved nor disproved was the method proposed, by Horvay, for computing the maximum stress in the ligament.

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FIGURE 13. Guernsey's Perforated Plate

However, since Horvay computed the maximum ligament stress (a function of the ratio of principal stresses in the solid plate and the orientation of the ligament to these stresses) by using the same method and assumptions with which the equivalent elastic constants E^* and μ^* were obtained, it is not too rash to assume an indirect confirmation of the stress magnification factors to a similar degree of accuracy. (Note: This last statement is the writer's conclusion and not that of Duncan.)

"Photoelastic Investigation of Perforated Plates"*

This photoelastic study was undertaken, in part, for the purpose of testing the accuracy of Horvay's theory for the calculation of stresses in perforated plates subjected to loads in the plane of the plate. This test objective was fulfilled through the use of two models:

1. A solid circular plate,
2. A perforated circular plate (see Figure 13).

These models were subjected to the following loading conditions:

1. Two opposed diametral loads
2. Three equal radial loads.

*An unpublished report by R. Guernsey.

The solid plate was used to provide measured values of solid plate principal stresses for comparison with calculated values of principal stresses. The ratio of solid plate principal stresses is essential, in the theory, to the prediction of maximum ligament stresses. Curves which show the theoretical and measured solid plate stresses were drawn and very good agreement was obtained except, as was expected, at the point of application of the load. The maximum ligament stress was determined from the boundary fringe order at each of the holes (1 to 6); the location of these holes may be seen in Figure 13. The point of maximum stress was found in the inclined ligaments at a distance $\xi/L = 0.2$ from the center of the ligament; ξ variable distance along the ligament with the origin at the center and L is the ligament length measured along the center line of the ligament. The following tables show a comparison of theoretical ligament stress with observed stresses for solid plate stresses obtained experimentally and analytically:

TABLE 3. Comparison of Experimental with Theoretical
Stress for a Perforated Plate - Subjected to Two Diametral Loads

a. Solid Plate Stresses Based on Theoretical Solution of Solid Disk

Hole	p^*	x_p	x	Experimental	Horvay's Theory
				$\frac{\hat{\sigma}}{\bar{p}}$	$\frac{\hat{\sigma}}{\bar{p}}$
1	14.1	-4.7	-0.33	76	75
2	14.9	-4.7	-0.31	86	73
3	16.6	-4.7	-0.28	88	71
4	21.1	-4.7	-0.22	88	68
5	27.6	-4.7	-0.17	78	65
6	41.5	-4.7	-0.11	60	64

b. Solid Plate Stresses Based on Experimental Solution of Solid Disk

Hole	p^*	x_p	x	Experimental	Horvay's Theory
				$\frac{\hat{\sigma}}{\bar{p}}$	$\frac{\hat{\sigma}}{\bar{p}}$
1	13.7	-4.5	-0.33	78	75
2	14.4	-4.5	-0.31	90	73
3	16.3	-4.5	-0.28	90	71
4	20.2	-3.6	-0.18	92	66
5	29.6	-0.4	-0.01	72	60
6	52.3	9.0	0.17	48	52

Tables 3, a and b are data for the perforated plate subjected to two diametral loads. See p. 55 for list of symbols.

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TABLE 4. Comparison of Experimental with Theoretical
Stresses for a Perforated Plate - Subjected to Three Equal Radial Loads

a. Solid Plate Stresses Based on Theoretical Solution of Solid Disk

Hole	p	xp	x	Experimental	Horvay's Theory
				$\frac{\hat{\sigma}}{p}$	$\frac{\hat{\sigma}}{p}$
1	7.1	7.1	1.0	151	18
2	9.9	4.0	0.40	130	42
3	14.3	-0.9	0.06	93	57
4	19.2	-1.3	-0.07	86	63
5	27.1	-3.0	-0.11	78	64
6	42.1	-4.2	-0.10	56	64

b. Solid Plate Stresses Based on Experimental Solution of Solid Disk

Hole	p	xp	x	Experimental	Horvay's Theory
				$\frac{\hat{\sigma}}{p}$	$\frac{\hat{\sigma}}{p}$
1	7.1	7.1	1	151	18
2	9.7	3.7	0.38	133	42
3	13.5	0	0	98	60
4	18.1	-2.6	-0.14	91	66
5	25.5	-4.2	-0.16	83	67
6	37.8	0	0	61	60

Table 4, a and b, are data for the perforated plate - subjected to three equal radial loads.

where $\hat{\sigma}$ = maximum ligament stress,
p = largest principal solid plate stress, and
x = ratio of principal solid plate stresses.

Holes 1 to 6 are counted from the center radially outward.

Data in foregoing tables show that fairly good agreement between theory and experiment is obtained for the case of two diametral loads although the theory underestimates the stress in holes 2 through 5 by ~25%. For the case of three radial loads no agreement at all is obtained at the center (800% error) while fairly good agreement (15% error) is obtained at the outer hole. Guernsey attributes this discrepancy, in the central holes, to the presence of pure bending which Horvay's theory neglects.

Guernsey concludes that: "The approximate theory for calculation of stresses in perforated plates seems to be confirmed for the case of a plate loaded by two diametral loads." He further concludes that, "it is probable that the theory would be confirmed for any loading consisting of symmetrically placed pairs of loads..."

CONCLUSIONS

Based on the foregoing tests, with regard to the general validity of the equivalent plate constant theory and specifically to the usefulness of the theory as developed in detail by G. Horvay, these conclusions are made:

1. The perforated lattice does, in fact, behaves as an equivalent plate of reduced stiffness when it is subjected to transverse loading.
2. The results, as obtained from the Horvay's theory, are sufficiently accurate for engineering applications for a range of plate sizes extending from thin plates (radius-to-thickness ratio 7.5 to 1 for Duncan's tube sheets) to thick plates (radius-to-thickness ratio 1.6 to 1 for the reactor model head) with a variation in the number of penetrations ranging from the large number in Duncan's tube sheet tests down to the relatively small number (19 holes) in the reactor model head (3/8 scale model).

These conclusions apply only for Horvay's "thick plate" theory, which requires the hole spacing to be smaller than the plate thickness; the assumption of the derivation being that the plate was thin; i.e., the radius was large compared with the plate thickness. No conclusions regarding Horvay's "thin plate" theory (hole spacing larger than the plate thickness) can be drawn from these tests. Also nothing can be said about the utility of the method in the range, explicitly excluded by Horvay, for which

$$\frac{1}{2} < \frac{t}{2R} < 1 ,$$

where t = plate thickness, and
 $2R$ = hole spacing.

All of the tests which were reported have the same fundamental defect; i.e., they were intended primarily to establish the adequacy of a specific design only and secondly to supply information of a general character. However, the writer believes that the weight of the accumulated evidence is in favor of the theory.

In general, then, it is concluded that the equivalent plate theory as proposed by Horvay provides an analytical tool whereby a fairly wide range of perforated plate geometries with trigangular hole arrays may be handled by the designer with the same facility as that used for solid plates and with no more concern for the accuracy of the results than is normal for complicated problems where there are uncertain edge restraints or other statically indeterminate loads or supports.

RECOMMENDATIONS

Despite the encouraging results which were obtained in correlating the theory with test results, considerable work on perforated flat plates subjected to transverse loading still remains to be done. With regard to the triangular hole array, there is still uncertainty, from the experimental standpoint, concerning the variation and location of the maximum stress in the ligament that are caused by the offset bending which results from the difference in magnitude of the principal stresses and from the ligament orientation with respect to these stresses.

Also, the variation of stress across the ligament as the ligament becomes thick is uncertain. In addition, there are no published test results for plates with perforations such that the ratio of the hole spacing to plate thickness is less than unity. The writer knows of only one set of tests that was conducted with plates in which the hole spacing was very much larger than the plate thickness. These tests were conducted by the General Electric Company's Aircraft Nuclear Propulsion project at the Evendale Plant. The writer has seen some of the preliminary test results and there appeared to be fairly good agreement with theory; however, no details are available. To resolve the uncertainties concerning the stress distribution in flat plates with a triangular hole array, the following test program is suggested:

1. Make four plastic models of perforated plates for a photo-elastic study of the stress distribution under transverse load.
2. The models should have hole diameters such that the four plates have ratios of minimum ligament width to hole spacing of 0.5, 0.2, 0.1, and 0.05, respectively.
3. The plate diameters should be ~24 to 36 in.
4. The plate thickness should be such that the initial ratio of plate radius to thickness is 1.5 to 1; subsequently the thickness could be reduced in four or more steps until the thinnest plate practical was produced.

5. The type of loading and test procedure used by Duncan is recommended, that is, concentrated loading as well as uniform hydrostatic loads should be used. The initial use of an undrilled plate in the test fixture to obtain the effect of edge restraint caused by friction, in the manner of Duncan's test, is considered a sound method for obtaining a measurement of end effects.
6. Deflections should also be measured in sufficient number to furnish data for a deflection curve of the loaded plate.

It is believed that such a program, in outline, would produce sufficient information to evaluate, and correct or modify, if necessary, existing theory. It should be emphasized that whatever method of testing is followed, early in the design of the test, consideration should be given to the means for measuring the effect of edge restraints. If the edge conditions are not accurately known, quantitative evaluation of theory is not possible.

The square hole array in a flat plate is still an unknown quantity. While it appears that a method for the rational design of plates with triangular hole spacing could be advanced, there is neither theory nor experimental data available for plates with a square hole array.

The attack on the problem of the square hole should begin with the development of a theory or procedure useful in analytical design methods. The success of the equivalent plate constant method suggests this approach; however, should this not prove feasible, perhaps a code suitable for use on a digital computing machine could be developed for a gridwork of beams with a solution for the unknown forces and moments at the junction of the ligaments obtained by the usual slope-deflection analysis. No matter what procedure for rational design is put forward, it should be confirmed by means of photoelastic experiments.

For anything other than a pattern of uniform hole diameters, it does not seem possible that any over-all theory or rules could be developed, since the possible combinations are infinite and each is a special case.

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APPENDIX A

ANALYSIS OF PERFORATED PLATE WITH SKEWED PERFORATIONS AND SOLID
OUTER ANNULUS

A perforated plate with skewed holes is subjected to a uniform pressure load and a restraining edge load as shown in Figure 14.

The radius "b" to the OD of the perforated portion is arbitrarily chosen to give the best approximation to the outer boundary of the hole pattern.

FIGURE 14. Idealized Perforated Plate

The skewed hole array causes the neutral axis of the plate to be displaced by an amount "e" from the geometric centerline of the plate.

That portion of the plate outside the radius "b" is treated as a solid ring.

The perforated portion inside the radius "b" is treated as a solid plate by using Horvay's equivalent elastic constants for Young's modulus and Poisson's ratio.

The stress and deflection of the perforated portion (to be referred to as part "B") is determined by the applied load and the restraint caused by the ring (to be called part "A"). See Appendix E for the list of symbols.

FIGURE 15. Effective Solid Outer Annulus

Consider ring "A" and let θ_A = Rotation of "A";

then

$$\theta_A = \frac{M_I a_1^2}{EI}, \quad (\text{see Reference 19}), \quad (1)$$

where I = moment of inertia about axis through center of gravity (CG)
 E = Young's modulus, and
 M_I = sum of external moments;

therefore,

$$M_I = \frac{b}{a_1} M - \frac{c}{a_1} Wm - \frac{b}{a_1} T(e+e_1) + \frac{b}{a_1} P e_1 + \frac{b}{a_1} Q(a_1-b) + \frac{c}{a_1} R(c-a_1) \\ + p(a-b) \left(a_1 - \frac{a+b}{2} \right) \left(\frac{a+b}{2a_1} \right) + pg \left[\frac{a+c_1}{2a_1} \right] \left[\frac{t}{2} - e_1 - \frac{g}{2} \right]. \quad (2)$$

For equilibrium let

$$R = \frac{a^2 p}{2c}. \quad (3)$$

Q is the shear force caused by the pressure p on "B":

$$\pi b^2 p = 2\pi b Q; \quad (4)$$

therefore, it may be stated as,

$$Q = \frac{pb}{2}. \quad (5)$$

P , the resultant of the pressure p acting in the outer row of holes, is considered to be uniformly distributed;

therefore,

$$P = n \frac{\ell t}{2\pi b} p, \quad (6)$$

where ℓ = diameter of hole,
 t = thickness of plate, and
 n = number of holes.

Let T be the reaction of plate "B" to horizontal displacement of part "A" at neutral axis of part "B." Since the horizontal displacement of "A" must equal that of "B,"

$$K_B T = \Theta_A(e+e_1) + K_A P - K_A T - K_A p g \left(\frac{a+c_1}{2b} \right) - K_A \frac{c}{b} W; \quad (7)$$

therefore,

$$T = \frac{\Theta_A(e+e_1) + K_A \left[P - p g \left(\frac{a+c_1}{2b} \right) - \frac{c}{b} W \right]}{K_A + K_B}, \quad (8)$$

where

$$K_A = \frac{ba_1}{A_A E} \quad (9)$$

(the hoop flexibility of Ring "A" of cross section area A_A),

and

$$K_B = \frac{b}{t} \frac{1-\mu^*}{E^*} \quad (10)$$

(the flexibility of plate "B" using Horvay's equivalent elastic constants),

FIGURE 16. Effective Perforated Portion

Consider plate "B" and let θ_B = Rotation of "B" at $r = b$ (Reference 18)

then

$$\theta_B = \frac{b}{(1+\mu^*)D^*} \left[\frac{pb^2}{8} - M \right] \quad (11)$$

where

$$D^* = \frac{E^*t^3}{12(1-\mu^{*2})} \quad (12)$$

The bending stiffness is assumed to be proportional to Horvay's equivalent elastic constants E^* and μ^* based on the minimum ligament width at the midpoint of the plate and the moment of inertia of a rectangular section of unit width about an axis through its midpoint. (See References 1, 2, and 10 through 14.)

The shear deflection for "A" is assumed to be equal to "B" so that there is no mismatch of slope at $r = b$ due to different shear deflections.

The unknown moment M is obtained from the compatibility requirement that the slope of "B" at $r = b$ must equal the slope of "A";

therefore,

$$\theta_B = \theta_A, \quad (13)$$

and from Equations (1) and (10) we obtain

$$\frac{M_I a_1^2}{EI} = \frac{b}{(1+\mu^*)D^*} \left[\frac{pb^2}{8} - M \right] . \quad (14)$$

To solve for M, let

$$C_A = \frac{a_1^2}{EI} ; \quad (15)$$

$$C_B = \frac{b}{(1+\mu^*)D^*} . \quad (16)$$

Since T is a function of θ_A , eliminate θ from Equation (8) by substituting Equation (1) into (8); therefore,

$$T = \frac{C_A M_I (e+e_1) + K_A \left[P - pg \left(\frac{a+c_1}{2b} \right) - \frac{c}{b} W \right]}{K_A + K_B} \quad (17)$$

then substitute Equation (17) into (2) and we obtain

$$\begin{aligned} M_I = & \frac{1}{1 + \frac{b}{a_1} \frac{C_A(e+e_1)^2}{K_A + K_B}} \left\{ \frac{b}{a_1} M + \left[\frac{K_A(e+e_1)}{K_A + K_B} - m \right] \frac{c}{a_1} W + \left[e_1 - \frac{K_A(e+e_1)}{K_A + K_B} \right] \frac{b}{a_1} P \right. \\ & + pg \left[\frac{a+c_1}{2a_1} \right] \left[\frac{K_A(e+e_1)}{K_A + K_B} + \frac{t}{2} - e_1 - \frac{g}{2} \right] + \frac{b}{a_1} Q(a_1-b) \\ & \left. + \frac{c}{a_1} R(c-a_1) + p(a-b) \left(a_1 - \frac{a+b}{2} \right) \left(\frac{a+b}{2a_1} \right) \right\} ; \quad (18) \end{aligned}$$

and by substituting Equation (14) into (18) we obtain

$$\begin{aligned} M = & \frac{a_1(K_A+K_B) + bC_A(e+e_1)^2}{C_B \left[a_1(K_A+K_B) + bC_A(e+e_1)^2 \right] + bC_A(K_A+K_B)} \left\{ \left\{ C_B \frac{pb^2}{8} - \frac{C_A}{1 + \frac{b}{a_1} \frac{C_A(e+e_1)^2}{K_A + K_B}} \right\} \times \right. \\ & \left\{ \left[\frac{K_A(e+e_1)}{K_A + K_B} - m \right] \frac{c}{a_1} W + \left[e_1 - \frac{K_A(e+e_1)}{K_A + K_B} \right] \frac{b}{a_1} P + pg \left[\frac{a+c_1}{2a_1} \right] \right. \\ & \left. \left[\frac{K_A(e+e_1)}{K_A + K_B} + \frac{t}{2} - e_1 - \frac{g}{2} \right] + \frac{b}{a_1} Q(a_1-b) + \frac{c}{a_1} R(c-a_1) \right. \\ & \left. \left. \left. + p(a-b) \left(a_1 - \frac{a+b}{2} \right) \left(\frac{a+b}{2a_1} \right) \right) \right] \right\} \right\} . \quad (19) \end{aligned}$$

FIGURE 17: Cross Section through Ligament

The displacement of the neutral axis is determined as follows:
Consider a trapezoidal section taken at the minimum ligament thickness as seen in section A-A (Figure 17);

where

$$y = \frac{t(2d_1 + d_2)}{3(d_1 + d_2)} \quad (20)$$

and

$$e = y - \frac{t}{2} ; \quad (21)$$

therefore,

$$e = t \left[\frac{2d_1 + d_2}{3(d_1 + d_2)} - \frac{1}{2} \right] . \quad (22)$$

The equivalent elastic constants and stress multiplier are based on the ratio of the hole spacing to the minimum ligament width as measured at the center of the plate, i.e., at $t/2$.

Bending stresses in perforated plate "B" are as follows: in a solid circular plate the bending stress in both the radial and tangential direction is given by:

$$\sigma(r) = 6 \frac{M(r)}{t^2} \quad (23)$$

where $M(r)$ = the bending moment in lb-in./in. at any radius.

The bending stress in the plate "B" can be obtained by superimposing the stress caused by the uniform pressure load p and the edge moment M .

The bending moments caused by pressure on simply supported circular plates are expressed:

M_r = radial moment

$$M_r = \frac{p}{16} (13 + \mu^*)(b^2 - r^2), \quad (\text{see Reference 18}) \quad (24)$$

M_t = tangential moment

$$M_t = \frac{p}{16} \left[b^2(3 + \mu^*) - r^2(1 + 3\mu^*) \right]. \quad (25)$$

The solid plate stress in Equation 23 is converted to a perforated plate stress by using the Horvay stress multiplier $\bar{Y}$ based on the minimum ligament thickness at the midpoint of the plate and correcting for the shift in the neutral axis by multiplying by $(1 - 2e/t)$ for top surface stress and $(1 + 2e/t)$ for bottom surface stress.

Direct stress in plate "B" is given by:

$$\sigma_d = \left[\frac{T}{t} \right] \bar{Y}. \quad (26)$$

The total stress in "B" is the sum of the stresses caused by p , M , and T . Therefore, the top surface is expressed as:

$$\sigma_r(r) = \bar{Y} \left\{ \frac{6}{t^2} \left[1 - 2 \frac{e}{t} \right] \left[\frac{p}{16} (3 + \mu^*)(b^2 - r^2) - M \right] + \frac{T}{t} \right\} \quad (27)$$

$$\sigma_r(r) = \bar{Y} \left\{ \left\{ \frac{6}{t^2} \left[1 - 2 \frac{e}{t} \right] \left\{ \frac{p}{16} \left[b^2(3 + \mu^*) - r^2(1 + 3\mu^*) \right] - M \right\} + \frac{T}{t} \right\} \right\} \quad (28)$$

Bottom surface:

$$\sigma_r(r) = \bar{Y} \left\{ \frac{6}{t^2} \left[1 + 2 \frac{e}{t} \right] \left[- \frac{p}{16} (3 + \mu^*)(b^2 - r^2) + M \right] + \frac{T}{t} \right\} \quad (29)$$

$$\sigma_t(r) = \bar{Y} \left\{ \left\{ \frac{6}{t^2} \left[1 + 2 \frac{e}{t} \right] \left\{ - \frac{P}{16} \left[b^2(3+\mu^*) - r^2(1+3\mu^*) \right] + M \right\} + \frac{T}{t} \right\} \right\}, \quad (30)$$

where σ_r = radial stress,
 σ_t = tangential stress, and
 $\bar{Y}$ = Horvay's stress multiplier.

Stress in ring "A":

Because of the assumptions previously made, the stress in "A" will be hoop stress caused by rotation and direct radial displacement.

The bending stress is given by:

$$\sigma_b(r) = C_A M_I \frac{y}{r} E, \quad (31)$$

where y = vertical distance on "A" measured from neutral axis of "B", and
 r = radius where $b < r < a$.

The direct stress is given by:

$$\sigma_{dA}(r) = K_A \left[P - T - pg \left(\frac{a+c_1}{2b} \right) - \frac{c}{b} W \right] \frac{E}{r}. \quad (32)$$

Combined stress in the ring is

$$\sigma_{tA}(r) = \sigma_b(r) + \sigma_{dA}(r). \quad (33)$$

Top surface:

$$\sigma_{tA}(r) = \left\{ C_A M_I \left(\frac{t}{2} - e \right) + K_A \left[P - T - pg \left(\frac{a+c_1}{2b} \right) - \frac{c}{b} W \right] \right\} \frac{E}{r}. \quad (34)$$

Bottom surface:

$$\sigma_{tA}(r) = \left\{ - C_A M_I \left(\frac{t}{2} + e \right) + K_A \left[P - T - pg \left(\frac{a+c_1}{2b} \right) - \frac{c}{b} W \right] \right\} \frac{E}{r}. \quad (35)$$

Deflections:

The vertical displacement at any radius r is the sum of the displacements caused by the rotation of "A" and the bending of "B" plus the shear deflection of "B,"

where w_A = displacement at $r = b$ caused by rotation, and

$$w_A = (c-b)\theta_A. \quad (36)$$

$w_B(r)$ = Bending displacement at any radius of "B" due to pressure p and moment M

$$w_B(r) = \frac{(b^2-r^2)}{(1+\mu^*)D^*} \left\{ \frac{p}{64} \left[(5+\mu^*)b^2 - r^2(1+\mu^*) \right] - \frac{M}{2} \right\}. \quad (37)$$

$w_S(r)$ = Shear deflection (see Reference 18), and

$$w_S = \frac{3}{2} \cdot \frac{p}{4G^*t} (c^2-r^2), \quad (38)$$

where

$$G^* = \frac{2\bar{h}}{2R} G = \frac{2\bar{h}}{2R} \times \frac{E}{2(1+\mu)}, \quad (39)$$

where $2R$ = hole spacing,
 $2\bar{h}$ = average ligament width,
 w_T = total displacement,

$$w_T(r) = w_A + w_B + w_S \quad r < b \quad (40)$$

$$w_T(r) = (c-r)\theta_A + w_S \quad c < r < b. \quad (41)$$

Rotations:

The rotation of "B" at any radius r is given by:

$$\theta(r) = \frac{r}{(1+\mu^*)D^*} \left\{ \frac{p}{16} \left[b^2(3+\mu^*) - r^2(1+\mu^*) \right] - M \right\} \quad r < b \quad (42)$$

$$\theta(r) = \theta_A = C_A M_I \quad r > b. \quad (43)$$

To find the slope at any r , the first derivative of the shear deflection must be added to the equations for rotation; i.e.,

$$\frac{dw}{dr} = - \left(\theta(r) + \frac{3}{4} \frac{pr}{G^*t} \right). \quad (44)$$

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APPENDIX B

ALUMINUM TEST MODEL - "TOP" SURFACE GEOMETRY

FIGURE 18. Geometry and Loading of Outer Annulus
"Top" Surface

The geometry of the outer annulus used in the computation is as follows:

a	$= 9.25$ in.	plate outer radius at bottom surface
a_1	$= 7.49$ in.	radius to CG of outer annulus
b	$= 6.25$ in.	radius of perforated portion at midplane
b_1	$= 5.90$ in.	radius of perforated portion at bottom surface
b_2	$= 6.60$ in.	radius of perforated portion at top surface
b_3	$= 7.50$ in.	plate outer radius at top surface
c	$= 7.90$ in.	radius to point of support
d_1	$= 0.81$ in.	minimum ligament width at top surface
d_2	$= 0.49$ in.	minimum ligament width at bottom surface
e	$= 0.37$ in.	eccentricity of neutral axis of perforated plate with respect to its midplane
e_1	$= 0.65$ in.	eccentricity of CG of outer annulus with respect to midplane
e_2	$= 1.50$ in.	vertical position of P with respect to midplane
I	$= 19.60$ in. ⁴	moment of inertia of outer annulus about a horizontal axis through the CG

ℓ = 2.19 in. hole diameter
 m = e_1 in. vertical distance of point of support to CG of outer annulus
 n = 12 number holes in outer row
 t = 5.00 in. thickness of plate.

The hole spacing and minimum ligament width at the midplane of the perforated portion is given by:

$2R$ = 2.83 in. hole spacing at midplane
 $2h$ = 0.65 in. minimum ligament width at midplane
 $2\bar{h}$ = 0.81 in. average ligament width at midplane

$N = \frac{2h}{2R} = 0.23$ the solidity ratio

The equivalent plate constants from Figure 28 are:

$E^* = 0.14E$, $D^* = 0.175D$, $\mu^* = 0.52$.

The actual Young's modulus and Poisson's ratio for the plate material (61 ST aluminum) are:

$E = 10 \times 10^6$ psi; $\mu = 0.3$
 $G = 3.85 \times 10^6$ psi shear modulus.

The known forces applied to the ring are:

$Q/p = 3.13$ lb/in.-psi
 $R/p = 4.31$ lb/in.-psi reaction at support
 $P/p = 3.35$ lb/in.-psi pressure force from 12 outer holes
 $W/p = 1.30$ lb/in.-psi for coefficient of friction = 0.3.

The foregoing values are used in the general analysis in Appendix A; care must be taken to keep the proper signs. In this calculation the moment arm for P in Equation (2) of the general analysis is $(e_1 - e_2)$ and the dimension g is zero; with allowances made for these conditions there are no other variations from the general case. The solution was carried out such that stress per unit of pressure load and deflection per unit of pressure were obtained.

The stress multiplier ($\bar{Y}$) from Figure 29 is taken from the curve $\beta = +1$ for $\eta = 0.229$ and is $\bar{Y} = 4.4$. This is the ratio of ligament stress to solid plate stress at the narrowest part of the ligament.

Bottom Surface

FIGURE 19. Geometry and Loading of Outer Annulus -
"Bottom" Surface

The geometry used in this computation is the same as that for the "top" surface with the following exceptions:

c = 8.25 in.	radius to point of support
g = 2.50 in.	vertical outer surface exposed to pressure
m = 1.50 in.	vertical distance of point of support to CG of outer annulus.

The applied forces are the same as for the "top" surface with the following exceptions:

$$\frac{R}{p} = 4.135 \text{ lb/in.} \cdot \text{psi} \quad \text{reaction at support.}$$

Because of the altered loading condition some changes in the general case are required. These changes are given below.

Equation (8) is changed to read

$$T = \frac{\theta_A(e+e_1) - K_A \left[P - g \left(\frac{a+b_3}{2b} \right) p - W \frac{c}{b} \right]}{K_A + K_B}$$

Equation (17) becomes

$$T = \frac{C_A M_I (e+e_1) - K_A \left[P - g \left(\frac{a+b}{2b} \right) p - W \frac{c}{b} \right]}{K_A + K_B}$$

Equation (18) becomes

$$\begin{aligned} M_I = & \frac{1}{1 + \frac{b}{a_1} \frac{C_A (e+e_1)^2}{K_A + K_B}} \left\{ M \frac{b}{a_1} - \frac{c}{a_1} \left[\frac{K_A (e+e_1)}{K_A + K_B} + m \right] W \right. \\ & + pg \left[\frac{a+b}{2a_1} \right] \left[- \frac{K_A (e+e_1)}{K_A + K_B} + \frac{g}{2} + e_1 \right] + \frac{b}{a_1} \left[- (c+e_1) + \frac{K_A (e+e_1)}{K_A + K_B} \right] P \\ & \left. + Q(a_1-b) \frac{b}{a_1} + \frac{c}{a_1} R(c-a_1) + (a-b) \left(a_1 - \frac{a+b}{2} \right) \left(\frac{a+c}{2a_1} \right) \right\} \end{aligned}$$

Equation (19) becomes

$$\begin{aligned} M = & \frac{a_1 (K_A + K_B) + b C_A (e+e_1)^2}{C_B [a_1 (K_A + K_B) + b C_A (e+e_1)^2] + b C_A (K_A + K_B)} \left\{ C_B \frac{b^2 p}{8} - \frac{C_A}{1 + \frac{b}{a_1} \frac{C_A (e+e_1)^2}{K_A + K_B}} \right. \\ & - \frac{c}{a_1} \left[\frac{K_A (e+e_1)}{K_A + K_B} + m \right] W + pg \left[\frac{a+b}{2a_1} \right] \left[- \frac{K_A (e+e_1)}{K_A + K_B} + \frac{g}{2} + e_1 \right] \\ & + \frac{b}{a_1} \left[- (e_1+e_2) + \frac{K_A (e+e_1)}{K_A + K_B} \right] P + Q(a-b) \frac{b}{a_1} + \frac{c}{a_1} R(c-a_1) \\ & \left. + p(a-b) \left(a_1 - \frac{a+b}{2} \right) \left(\frac{a+c}{2a_1} \right) \right\} \end{aligned}$$

The remainder of the computation follows the general case, the foregoing being the only exceptions which were brought about by the difference in geometry and loading.

Experimental Data

The measured deflections and stresses were obtained from the experimental data plotted in Figures 20 and 22 for the "top" surface and Figures 21 and 23 for the "bottom" surface. The deflection measurements were duplicated at each radius for comparison and the readings were averaged to produce one set of data. The failure of the straight line, through the measured points, to pass through zero is attributed to the initial preload required to seal the gasket against the hydraulic load.

The deflections are given in the following list.

<u>Deflections/Pressure Load</u>			
<u>Gage</u>	<u>Radius, in.</u>	<u>Deflections, mils/psi</u>	
		<u>Top</u>	<u>Bottom</u>
1	1.5	5.00	6.60
2	3.4	4.15	5.45
3	4.5	2.90	4.60
4	7.0	0.80	1.70

The strains were measured at three of the ligaments 120 deg apart around the center hole. The gages were mounted at three points on a ligament: at the center and at each outer edge (as close as practicable). Some of the readings were identical so these were plotted as a single point.

The strains in each ligament are given in the following table.

<u>Strain/Pressure Load, in./in.-psi x 10⁻⁷</u>									
Ligament*	1			2			3		
	a	b	c	a	b	c	a	b	c
Bottom	5.37	4.70	5.42	4.70	4.60	4.70	5.30	4.80	5.37
Top	6.21	5.96	6.94	6.94	5.29	7.57	6.94	5.54	5.96

*Ligaments 1, 2, and 3 refer to three ligaments 120 deg apart around the center hole.

**The column headings a, b, and c refer to gage locations spaced radially outward from the center as shown in Figure 2.

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FIGURE 20. Aluminum Plate Deflection Data - "Top" Surface

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FIGURE 21. Aluminum Plate Deflection Data - "Bottom" Surface

FIGURE 22. Aluminum Plate Strain Gage Data - "Top" Surface

FIGURE 23. Aluminum Plate Strain Gage Data - "Bottom" Surface

The Young's modulus for the material is $E = 10 \times 10^6$ psi; therefore, the above strains when multiplied by 10^7 became the stresses in psi per psi of pressure load. It should be recalled that the measurements represent two separate tests; in both cases tension in the ligament was measured, the surface with the thickest ligament was designated the "top." The strains are presented in the above manner so that the strains at the same point on one surface can be more easily compared with those on the other surface. In both cases there is a variation across the ligament, the larger variation occurs with the larger ligament.

APPENDIX C

THE 3/8 MODEL TEST

FIGURE 24. Geometry and Loading of Outer Annulus 3/8 Model

The above figure is an idealization of the geometry of the actual head. The radii defining the boundary of the perforated portion was chosen in the same manner as that for the aluminum plate; i.e., the radii to the boundary was taken to be equal to twice the hole spacing plus one-half the hole radius.

The geometry used in the analysis is as follows:

a	$= 15.00$ in.	plate outer radius
a_1	$= 11.96$ in.	radius to CG of outer annulus
b	$= 10.20$ in.	radius of perforated portion at midplane
b_1	$= 9.30$ in.	radius of perforated portion at "bottom" surface
b_2	$= 11.10$ in.	radius of perforated portion at "top" surface
b_3	$= 12.45$ in.	plate outer radius at "top" surface
c	$= 14.40$ in.	radius to point of support
c_1	$= 12.00$ in.	average radius at which pressure on vertical outer surface acts.

$d_1 = 1.50$ in.	minimum ligament width at "top" surface
$d_2 = 0.58$ in.	minimum ligament width at "bottom" surface
$e = 0.68$ in.	eccentricity of natural axis of perforated plate with respect to its midplane
$e_1 = 0.54$ in.	eccentricity of CG of outer annulus with respect to midplane
$g = 4.75$ in.	vertical distance on outer surface exposed to pressure
$I = 162$ in. ⁴	moment of inertia of outer annulus about a horizontal axis through its CG
$\ell = 3.54$ in.	hole diameter
$m = 2.54$ in.	vertical distance of point of support to CG of outer annulus
$n = 12$	number of holes in outer row
$t = 9.38$ in.	thickness of plate.

The hole spacing and minimum ligament width at the midplane of the perforated portion are given by:

$2R = 4.67$ in.	hole spacing at midplane
$2h = 1.13$ in.	minimum ligament width at midplane
$2\bar{h} = 1.41$ in.	average ligament width at midplane

$$\eta = \frac{2h}{2R} = 0.24 \quad \text{the "solidity ratio."}$$

The equivalent plate constants from Figure 28 are:

$$E^* = 0.145E, \quad \mu^* = 0.56.$$

The actual Young's modulus and Poisson's ratio for the plate material (steel) are:

$$E = 30 \times 10^6 \text{ psi}, \quad \mu = 0.3$$

$$G = 11.5 \times 10^6 \text{ psi} \quad \text{shear modulus.}$$

The forces applied to the ring are:

$$Q = 12,800 \text{ lb/in.}$$

$$R = 19,500 \text{ lb/in.} \quad \text{reaction at support}$$

$$P = 15,600 \text{ lb/in.} \quad \text{pressure in 12 outer holes}$$

$$p = 2,500 \text{ lb/in.}^2 \quad \text{uniform pressure load}$$

$$W = 2,900 \text{ lb/in.} \quad (\text{for coefficient friction } f = 0.15).$$

The foregoing values are used in the general analysis as presented in Appendix A to compute the equivalent solid plate stress. The stress multiplier ($\bar{Y}$) from Figure 29 is taken from the curve for $\beta = +1$ for $\eta = 0.24$ and is $\bar{Y} = 4.3$.

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FIGURE 25. The 3/8 Model Deflection Gage Data

This is the ratio of ligament stress to solid plate stress at the narrowest part of the ligament. This stress is the maximum stress in a ligament, in accordance with the theory, only for those ligaments around the center hole. The variation in the principal solid plates stresses from the center to the outer ligaments is small so that the maximum stress in the plate is the stress in the ligaments around the center hole. Since this variation in the principal stresses is small, no attempt was made to find the location and magnitude of the theoretical maximum stresses in the ligaments other than those around the center hole. Therefore, the value of $\bar{Y} = 4.3$ was used to compare calculated stresses with the measured stresses at the center of the ligaments.

The 3/8 Model Experimental Data

The measured deflections shown in Figure 10 and the "measured" stresses in Figure 9 were obtained from the experimental data in Figure 25 and 26. There were many more strain gages than those shown (including duplicates of these gages for comparison purposes); however, the data from these strain gages are the only ones of interest for correlation with calculated ligament stresses. The deflection gage readings were also duplicated; those shown were taken from along one radius, from the center to the outer edge of the head.

FIGURE 26. The 3/8 Model Strain Gage Data

The break in the slope of the test points is caused by the removal of the initial preload. For comparison with theory, the slope, as determined from the last four points of deflection and strain data, was used to obtain the values shown in Figure 9 and 10. These values are also listed as follows:

<u>Gage</u>	<u>Radius, in.</u>	<u>Deflections</u>	
		<u>Total</u> <u>Deflections, in.</u>	<u>Deflections</u> <u>Relative to :</u> <u>Gage No. 4, in.</u>
1	0.0	0.0295	0.0167
2	6.2	0.0160	0.0109
3	12.5	0.0237	0.0032
4	15.0	0.0128	0.0000

At the edge of the head, gage No. 4 measured the lift of the head; therefore, deflections relative to gage No. 4 represent the net bending of the head.

Gage	Radius, in.	Stress	
		Strain, in./in.	Stress, psi
1	2.0	-850×10^{-6}	-25,500
2	3.5	-800×10^{-6}	-24,000
3	6.2	-713×10^{-6}	-21,400
4	7.5	-530×10^{-6}	-15,900
5	12.0	-425×10^{-6}	-12,700

Gage No. 5 is a circumferential gage at the outer edge of the "bottom" surface of the head. Gages No. 1 through 4 measured strains at the narrowest point of the ligament as shown schematically in Figure 26. The strains and corresponding stresses are for a pressure load of 2500 psi.

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APPENDIX D

EQUIVALENT ELASTIC CONSTANTS

The following curves are taken from the published works of G. Horvay and are included here because they are an integral part of the calculations for the aluminum perforated plate and the 3/8 scale model head. The symbols used are defined as follows:

- $2h$ = minimum ligament width
 $2h'$ = equivalent average ligament width for bending stiffness
 $2\bar{h}$ = equivalent average ligament width for tensile stiffness
 $2R$ = hole spacing or pitch
 E = Young's modulus
 D = flexural rigidity
 $\left. \begin{matrix} E^* \\ \mu^* \end{matrix} \right\}$ = Horvay's equivalent elastic constants
 $\bar{Y}$ = stress multiplier
 β = ratio of principal solid plate stresses.

FIGURE 27. Ratio of Equivalent Ligament Width to Minimum Ligament Width, in Bending and Tension

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FIGURE 28. Equivalent Elastic Constants

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FIGURE 29. Stress Multiplier

APPENDIX E

LIST OF SYMBOLS

a	=	plate outer radius,	in.
a_1	=	radius to center of gravity (CG) of outer annulus,	in.
b	=	radius of perforated portion at midplane,	in.
b_1	=	radius of perforated portion at "bottom" surface,	in.
b_2	=	radius of perforated portion at "top" surface,	in.
b_3	=	plate outer radius at "top" surface,	in.
c	=	radius to point of support,	in.
c_1	=	average radius at which pressure on vertical outer surface acts,	in.
d_1	=	minimum ligament width at "top" surface,	in.
d_2	=	minimum ligament width at "bottom" surface,	in.
e	=	eccentricity of neutral axis of perforated plate with respect to its midplane,	in.
e_1	=	eccentricity of CG of outer annulus with respect to midplane,	in.
e_2	=	vertical position of force P with respect to midplane,	in.
g	=	vertical distance on outer surface exposed to pressure,	in.
I	=	moment of inertia of outer annulus about a horizontal axis through its CG,	in. ⁴
ℓ	=	hole diameter of plate penetrations,	in.
m	=	vertical distance of point of support to CG of outer annulus,	in.
n	=	number of holes in outer row of penetrations,	
p	=	uniform pressure load,	psi
r	=	radial coordinate of plate,	in.
t	=	plate thickness,	in.
A_A	=	cross-section area of outer annulus "A,"	in. ²
$2R$	=	hole spacing at midplane,	in.

$2h$	=	minimum ligament width at midplane,	in.
$\bar{2h}$	=	average ligament width at midplane,	in.
η	=	Horvay's solidity ratio,	
E^*	=	Horvay's equivalent of Young's modulus,	psi
μ^*	=	Horvay's equivalent of Poisson's ratio,	
E	=	Young's modulus,	psi
G	=	shear modulus,	psi
μ	=	Poisson's ratio,	
R	=	vertical reaction at support,	$\frac{lb}{in.}$
P	=	resultant radial pressure load from 12 outer holes,	$\frac{lb}{in.}$
Q	=	shear force,	$\frac{lb}{in.}$
W	=	horizontal friction force at support,	$\frac{lb}{in.}$
T	=	horizontal force at neutral axis of perforated portion,	$\frac{lb}{in.}$
M	=	radial bending moment at edge of perforated portion,	$\frac{in.-lb}{in.}$
M_I	=	sum of external moments on outer annulus,	$\frac{in.-lb}{in.}$
$M_r(r)$	=	radial bending at any radius $r < b$ for perforated portion,	$\frac{in.-lb}{in.}$
$M_t(r)$	=	tangential bending moment at any radius $r > b$ for perforated portion,	$\frac{in.-lb}{in.}$
$\sigma_r(r)$	=	radial stress at any radius $r < b$ for perforated portion,	psi
$\sigma_t(r)$	=	tangential stress at any radius $r < b$ for perforated portion,	psi
$\sigma_{dA}(r)$	=	direct stress in outer annulus for any radius $b < r < a$,	psi

$\sigma_{tA}(r)$	= combined stress in outer annulus for any radius $b < r < a$,	psi
$\bar{Y}$	= Horvay's stress multiplier,	
β	= ratio of principal solid plate stresses,	
θ_A	= rotation of outer annulus about a horizontal axis through the CG,	radians
θ_B	= rotation at $r = b$ of perforated portion,	radians
w_A	= vertical deflection of points on outer annulus,	in.
w_B	= vertical deflection of perforated portion which results from bending,	in.
w_S	= shear deflection of plate,	in.

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