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Informal Report

THE RELATIONSHIP BETWEEN THE BEAM-BEAM TUNE SHIFT
AND THE NUMBER OF INTERACTION REGIONS

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ABSTRACT

The choice of the beam-beam tune shift for one interaction region (the beam-beam limit) is discussed. The effect of multiple crossings is then included. Because of radiation damping for electrons, these considerations are quite different for protons and electrons. Both are treated.

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1. Introduction

In considering the beam-beam interaction, we must bear in mind that beams themselves are current limited. This is what is referred to as the single beam limit. Since luminosity is proportional to the square of the current, we will take the current to be the maximal consistent with this single beam limit and take the tune-shift, $\Delta\nu$, to be an independent variable. We can therefore write for the luminosity,

$$L = \ell_0 \Delta\nu, \quad (1.1)$$

where ℓ_0 is taken to be constant, and one collision is assumed. The other performance parameter we must take account of is the beam lifetime. Let us assume that the lifetime due to the beam-beam interaction is a function of $\Delta\nu$ and denote it $\tau_{bb}(\Delta\nu)$. Whether a beam is actually governed by this lifetime depends on its value relative to the single beam lifetime, τ_{sb} , for protons; and, in the case of electrons, the relevant time scale to be compared with τ_{bb} is the radiation damping time, τ_r . We will treat the cases of protons and electrons separately in later sections.

In particular, we take τ_{bb} to be the lifetime due to the beam-beam interaction, in number of revolutions, for one interaction per revolution. If we assume that $\tau_{bb}(\Delta\nu)$ is a function only of $\Delta\nu$, the tune shift per collision, and specifically, independent of the machine tune, then we can show that the beam-beam lifetime (in units of number of revolutions) is inversely proportional to p , the number of collision points (taken to be identical) per revolution. That is,

$$\tau_{bb,p} = \tau_{bb}(p \text{ crossings}) = \frac{1}{p} \tau_{bb}(\Delta\nu). \quad (1.2)$$

We caution that the assumption that $\tau_{bb}(\Delta\nu)$ is independent of the machine tune is a strong one and not at all obvious. With such an assumption, however, we will be able to make conclusions concerning the performance of storage rings with p interaction regions.

The performance parameter we will use here is the product of total luminosity and beam lifetime, which we denote by $(L\tau)$. We are therefore overlooking the fact that p interaction regions could be considered better than 1 for reasons unrelated to the $(L\tau)$ product. For p collision per revolution, we write

$$(L\tau) = p \ell_0 \Delta\nu \tau, \quad (1.3)$$

where τ is the actual beam lifetime. With τ some function of Δv (as well as p), our optimization consists of finding the maximum of the $(L\tau)$ product.

2. Proton Performance

For protons, the $(L\tau)$ product is given by

$$(L\tau) = p \ell_o \Delta v \tau_{sb}, \text{ if } \tau_{bb,p} > \tau_{sb} \quad (2.1)$$

$$(L\tau) = p \ell_o \Delta v \tau_{bb,p}(\Delta v), \text{ if } \tau_{bb,p} < \tau_{sb} \quad (2.2)$$

If we formally assume that $\tau_{bb}(\Delta v)$ is related to Δv by¹

$$\tau_{bb}(\Delta v) = \tau_o (\Delta v)^{-m} \quad (2.3)$$

for some $m > 0$, then, if $\tau_{bb,p} < \tau_{sb}$,

$$(L\tau) = \ell_o \tau_o (\Delta v)^{1-m} \quad (2.4)$$

If $m > 1$, as is the case for protons,¹ the maximum $(L\tau)$ product clearly occurs for Δv computed from

$$\tau_{bb,p}(\Delta v_{opt}) = \tau_{sb} \quad (2.5)$$

or, using (1.2),

$$\tau_{bb}(\Delta v_{opt}) = p \tau_{sb} \quad (2.6)$$

Solving (2.6), using (2.3), we have

$$(\Delta v)_{opt} = \left(\frac{\tau_o}{p \tau_{sb}} \right)^{1/m} \quad (2.7)$$

Thus,

$$(L\tau)_{opt} = p \ell_o \left(\frac{\tau_o}{p \tau_{sb}} \right)^{1/m} \tau_{sb} \quad (2.8)$$

which means that for $m > 1$, the performance parameter $(L\tau)$ increases as $p^{(1-1/m)}$.

We therefore have the following two conclusions with respect to protons:

- (1) Choose the maximum Δv , but not to exceed the value where the beam-beam lifetime (per revolution) becomes smaller than the single beam lifetime.

(2) The performance parameter ($L\tau$) should increase with the number of crossings by an amount slightly less than linear, depending on the fall off of the beam-beam lifetime with Δv .

3. Electron Performance

For electrons, the dominating lifetime is the radiation damping lifetime, τ_r . If

$$\tau_{bb,p}(\Delta v) > \tau_r, \quad (3.1)$$

then the stability will be governed by the electron single beam lifetime, τ_{sb} , and

$$(L\tau) = p \ell_o \tau_{sb} \Delta v. \quad (3.2)$$

Notice the difference here from the proton case, in that (3.1) contains τ_r , a much shorter lifetime than τ_{sb} . The implication is clearly that larger values of Δv can be taken in (3.2) for the performance parameter, ($L\tau$). As Δv is increased, ($L\tau$) increases, but $\tau_{bb,p}(\Delta v)$ decreases. The optimum in Δv is then given by

$$\tau_{bb,p}(\Delta v_{opt}) = \tau_r. \quad (3.3)$$

Approximating $\tau_{bb,p}$, using (1.2) and (2.3), we have

$$(L\tau)_{opt} = p \ell_o \left(\frac{\tau_o}{p \tau_r} \right)^{1/m} \tau_{sb}. \quad (3.4)$$

Now we must make some assumption about m . In the region of small values of Δv (i.e., < 0.01), it appears that $\tau_{bb}(\Delta v)$ can be approximated by a function roughly proportional to $(\Delta v)^{-m}$, for m large.¹ In the region around $\Delta v \leq 0.1$, the value of m diminishes and it is conceivable that m is on the order of 1.² In this event we see from (3.4) that the optimum ($L\tau$) is independent of p , that is,

$$(L\tau)_{opt} = \ell_o \tau_o \frac{\tau_{sb}}{\tau_r}. \quad (3.5)$$

We can thus make the following conclusions in the case of electrons:

(1) Choose the maximum Δv , but not to exceed the value where the beam-beam lifetime (per revolution) becomes smaller than the radiation damping time. Since the radiation damping time is much less than the

single beam lifetime, this value will be larger than the Δv limit for protons.

(2) For electrons colliding near the beam-beam limit, the optimum $(L\tau)$ product is roughly independent of the number of collisions, p . Thus, no gain in performance can be expected. However, we can also conclude that if we operate below the beam-beam limit, we should regain the dependence of $(L\tau)$ on p . If we choose Δv such that

$$\tau_{bb,p}(\Delta v) > \tau_r \quad , \quad (3.6)$$

then

$$(L\tau) = p l_0 \Delta v \tau_{sb} \quad , \quad (3.7)$$

which is proportional to p . Note however that Δv must be chosen sufficiently small that (3.6) is satisfied for all p considered. We suggest that in electron machines where the number of interaction regions can be changed, one could test this p dependence of the performance parameter $(L\tau)$ both near and sufficiently far from the beam-beam limit.

References

1. E. Keil, VIII Int. Conf. on High Energy Accelerators, Geneva, Switzerland, p. 372 (1971).
2. E. Keil, Int. School of Applied Physics, Erice, Italy (June, 1973), (To be published.)

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