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CORRELATION OF EXPONENTIAL PILE
LATTICE MEASUREMENTS WITH THEORY

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CORRELATION OF EXPONENTIAL PILE LATTICE MEASUREMENTS
WITH THEORY

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February 8, 1955

HANFORD ATOMIC PRODUCTS OPERATION
RICHLAND, WASHINGTON

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ABSTRACT

Using diffusion theory type formulas for the calculation of thermal utilization, and resonance escape probabilities, and in particular those formulas of Rumsey and Volkoff (MT-221), an effort has been made to correlate experimentally measured bucklings (Exponential Pile Program at Hanford) with those calculated from theory. A simple method for the determination of the effective resonance cross section, σ_{eff} , and $\eta\epsilon$ that will permit the correlation of the experimental and theoretical buckling values for various lattice spacings with the 1.36 inch diameter fuel element is given. The values of $\eta\epsilon$ and σ_{eff} which best fit the dry lattices are 1.3465 and 1.693 barns, respectively, and those for the wet lattices are 1.3492 and 1.671 barns, respectively. An "effective" value of η is implied by these calculations (based on a calculated ϵ) which is 1.304 for the dry lattice and 1.306 for the wet lattice. A table showing the buckling correlation and values of the various lattice parameters is presented.

CORRELATION OF EXPONENTIAL PILE LATTICE MEASUREMENTS WITH THEORY

INTRODUCTION

The Exponential Pile Physics Unit has experimentally determined the material bucklings for a series of graphite-uranium lattices with the standard Hanford (1.36 inch diameter) four-inch fuel element.⁽¹⁾ The purpose of these experiments is not only the determination of the reactivity of the various lattices, but also the provision of a basis on which other reactor parameters (thermal utilization, resonance escape probability, migration area, the average number of fission neutrons emitted per thermal neutron captured in the fuel element, and fast effect) may be studied.

Using Project Handbook (CL-697)⁽²⁾ formulas for the calculation of Fermi age and fast effect, Rumsey-Volkoff (MT-221)⁽³⁾ formulas for the calculation of thermal utilization and resonance escape probability, and the formula for lattice diffusion length as presented in CRR-546⁽⁴⁾, an effort is made to correlate the experimentally measured bucklings with those calculated from theory. These formulas are presented in Appendix I and the various values utilized in the formulas are presented in Appendix III. A simple method for the determination of the effective resonance cross section, σ_{eff} , and an $\eta\epsilon$ that allows the experimental and theoretical buckling values to be correlated is given. A paper discussing the correlation of the experimental buckling values and the theoretical buckling values for the 0.926 inch and the 1.17 inch diameter fuel elements will be issued at a later date.

METHOD FOR CORRELATION OF THEORY AND EXPERIMENT

In effecting a correlation of the theoretical buckling values with those obtained from experiment, several assumptions were made, as follows: (1) the thermal neutron absorption cross sections for graphite,

uranium, aluminum, and water were assumed to be known; (2) the thermal utilization, the lattice diffusion length, and Fermi age could be calculated; (3) the effective resonance cross section necessary for the calculation of resonance escape probability was not known accurately and could be varied within reasonable limits to insure a fit; and (4) the product $\eta\epsilon$ was also available for adjustment.

Method for Determination of σ_{eff} and $\eta\epsilon$

The critical equation can be expressed approximately as

$$B = \frac{k_{\infty} - 1}{L_t^2 + k_{\infty} \tau} \quad (1)$$

where k_{∞} is the multiplication constant for a reactor of infinite dimensions, B is the material buckling, and $L_t^2 + k_{\infty} \tau$ is the migration area. Now substituting $\eta\epsilon p$ for k_{∞} and solving for $\eta\epsilon$, we obtain:

$$\eta\epsilon = \frac{1 + BL_t^2}{pf(1 - B\tau)} \quad (2)$$

Since $\eta\epsilon$ depends primarily on the properties of the slug we may reasonably assume that this product will be a constant value for a given size and insensitive to lattice spacing, i. e.

$$\eta_i\epsilon_i = \eta_j\epsilon_j$$

where i and j represent the i th and j th lattices, respectively. Therefore

$$\frac{1 + B_i L_{t_i}^2}{p_i f_i (1 - B_i \tau_i)} = \frac{1 + B_j L_{t_j}^2}{p_j f_j (1 - B_j \tau_j)} \quad (3)$$

Expressing equation (3) in logarithmic form results in

$$\ln \frac{1 + B_i L_t^2}{f_i(1 - B_i \tau_i)} - \ln p_i = \ln \frac{1 + B_j L_t^2}{f_j(1 - B_j \tau_j)} - \ln p_j. \quad (4)$$

Now expressing the resonance escape probability as*

$$\ln p = \frac{-N_u V_u \sigma_{\text{eff}}(1 + \delta)}{Z + N_u V_u \sigma_{\text{eff}}(X - \delta)}, \quad (5)$$

letting

$$\frac{1 + B L_t^2}{f(1 - B \tau)} = \xi,$$

and substituting these expressions in equation (4) and rearranging terms, results in

$$\begin{aligned} & N_u^2 V_u^2 \left\{ (X_j - \delta_j)(1 + \delta_i) - (X_i - \delta_i)(1 + \delta_j) - \left[\ln\left(\frac{\xi_j}{\xi_i}\right) \right] (X_i - \delta_i)(X_j - \delta_j) \right\} \sigma_{\text{eff}}^2 \\ & + N_u V_u \left\{ (1 + \delta_i)Z_j - (1 + \delta_j)Z_i - \left[\ln\left(\frac{\xi_j}{\xi_i}\right) \right] \left[(X_j - \delta_j)Z_i + (X_i - \delta_i)Z_j \right] \right\} \sigma_{\text{eff}} \\ & - \ln\left(\frac{\xi_j}{\xi_i}\right) Z_i Z_j = 0 \end{aligned} \quad (6)$$

which can easily be solved for σ_{eff} if the experimental buckling values are substituted into ξ along with the corresponding calculated parameters. The resonance escape probability can now be determined, and $\eta\epsilon$ can be obtained from equation (2).

CALCULATIONS AND RESULTS

If the values of σ_{eff} obtained by taking all combinations of lattices two at a time, first for the dry lattice case and then for the wet lattice case are not approximately constant, then it can be assumed that the

* Derived in Appendix II.

buckling values can not be calculated by an adjustment of σ_{eff} . It is also noted that once a value of σ_{eff} is obtained the value of $\eta\epsilon$ is fixed. The values of σ_{eff} as obtained by the above method are presented in Table I where 1, 2, 3, 4, 5, and 6 denote the 6-3/16, 7, 7-1/2, 8, 8-3/8, and 10-3/8 inch lattices, respectively.

TABLE I
VALUES OF EFFECTIVE RESONANCE CROSS SECTION

<u>i, j</u>	<u>Dry Lattice</u>	<u>Wet Lattice</u>
1, 2	1.665 barns	1.647 barns
1, 3	1.692	1.711
1, 4	1.686	1.654
1, 5	1.695	1.659
1, 6	1.573*	1.544*
2, 3	1.753*	1.845*
2, 4	1.713	1.662
2, 5	1.725	1.670
2, 6	1.524*	1.475*
3, 4	1.664	1.443*
3, 5	1.703	1.544*
3, 6	1.451*	1.364*
4, 5	1.763*	1.696
4, 6	1.671*	1.338*
5, 6	1.878*	1.236*
Avg.	1.693	1.671

*Not included in the average.

All the combinations which contain the 10-3/8 inch lattice give a widely varying value of σ_{eff} in both the wet and dry cases. As will be shown later the 10-3/8 inch lattice bucklings can not be correlated by this method. A majority of the 7-1/2 inch lattice combinations in the wet case also give a widely varying value of σ_{eff} . It will be shown that the difference between the theoretical and experimental bucklings for this lattice is greater than the differences of the other lattices but is just outside the $\pm 2 \mu$ buck uncertainty for the experimental bucklings. In the dry case,

the 2,3 and 4,5 combinations give values of σ_{eff} that are slightly higher than the other values. It was found that a better fit to the experimental bucklings can be made if these combinations were omitted from the average.

Table II gives the values of $\eta\epsilon$ as calculated for each lattice.

TABLE II
VALUES OF $\eta\epsilon$

<u>Lattice</u>	<u>Dry Lattice</u>	<u>Wet Lattice</u>
6-3/16	1.3467	1.3482
7	1.3457	1.3492
7-1/2	1.3468	1.3520*
8	1.3464	1.3496
8-3/8	1.3471	1.3498
10-3/8	1.3332*	1.3365*
Avg.	1.3465	1.3492

*Not included in the average.

As is evident from Table II, the product of $\eta\epsilon$ has a rather well-defined grouping except for the 7-1/2 inch wet lattice value and the 10-3/8 inch wet and dry lattice where the values are seen to deviate quite markedly from the rest of the group.

If one assumes that the calculated value of ϵ (1.0329) is correct, then an appropriate value of η is defined for the lattices above. On the other hand, if a calculated value of $\eta^{(5)}$ (1.30) is used then an effective value of ϵ is determined. Of course there is the possibility that the theoretical values of ϵ and η are both in error. However if we use the calculated value of ϵ as given above then the effective values of η are:

$$\eta_{\text{dry}} = 1.3036$$

$$\eta_{\text{wet}} = 1.3062$$

The slight difference between wet and dry lattice values of σ_{eff} and η could be attributed to either neutron energy differences or a small order error in calculating the water effect in either the thermal utilization or resonance escape probability. A calculated value of η for a Hanford slug based on hardening of the neutron spectra on the interior of the slug is about 1.30.⁽⁵⁾ The values which fit the exponential data are essentially the same, and are effective values which are correct only to the extent to which the diffusion theory is valid.

Table III gives the buckling correlation along with the values of the other lattice parameters determined from the theoretical formulas.

TABLE III
BUCKLING CORRELATION AND LATTICE PARAMETERS

Lattice	$\eta\epsilon$	f	p	k	L_t^2	τ	Calc. Buck- ling	Exp. Buck- ling	Diff.
Dry Lattice									
6-3/16	1.3465	0.9356	0.7945	1.0009	126	363	1.8	2.0	-0.3
7	1.3465	0.9227	0.8380	1.0411	165	356	76.7	75.5	+1.2
7-1/2	1.3465	0.9136	0.8588	1.0565	190	351	100.7	101.1	-0.4
8	1.3465	0.9037	0.8762	1.0669	218	347	112.6	112.7	-0.1
8-3/8	1.3465	0.8957	0.8876	1.0705	238	341	116.9	117.6	-0.7
10-3/8	1.3465	0.8502	0.9273	1.0617	364	333	85.9	71.5	+14.4
Wet Lattice									
6-3/16	1.3492	0.9102	0.8269	1.0155	145	331	32.2	30.6	+1.6
7	1.3492	0.8960	0.8592	1.0387	188	332	72.6	71.8	+0.8
7-1/2	1.3492	0.8860	0.8754	1.0464	217	331	82.4	85.3	-2.9
8	1.3492	0.8753	0.8893	1.0502	247	329	84.7	83.6	+1.1
8-3/8	1.3492	0.8666	0.8986	1.0507	270	326	82.7	81.8	+0.9
10-3/8	1.3492	0.8179	0.9323	1.0288	409	323	38.9	25.5	+13.4

L_t^2 and τ are in cm^2 units.

Bucklings are in 10^{-6} cm^{-2} units.

The theoretical buckling curves are given in Figure 1.

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The empirical formula for the effective resonance integral is usually written as

$$\int \sigma_{\text{eff}} \frac{dE}{E} = A(1 + \mu \frac{S}{M}).$$

The σ_{eff} which we solve for is actually $(\int \sigma_{\text{eff}} \frac{dE}{E}) / (\int \frac{dE}{E})$. It is therefore possible to calculate A, the volume part of the resonance integral, if a value of μ (coefficient of surface to mass term) has been correctly chosen and if one is satisfied with the form of the above empirical relation. If μ is taken to be 2.5 and $\ln \frac{E_0}{E}$ to be 5.6, A is found to be 8.12 barns for the dry lattice and 8.00 barns for the wet lattice. P. W. Mummery quotes⁽⁴⁾ a value of 7.3 barns as the value that best fits the Harwell experimental data and North American Aviation⁽⁶⁾ suggests that the best value might be 7.8 barns. The usually quoted value of 9.25 barns⁽²⁾ is suspected to be about 1.4 barns too high, since it includes the $1/v$ portion of the cross section.

DISCUSSION OF THE RESULTS

The values for the lattice parameters, that is σ_{eff} , $\eta\epsilon$, A, as obtained from this analysis are, at best, effective values which will apply when used with the diffusion theory expressions and nuclear constants given in this report. In the calculation of the thermal utilization, the disadvantage factor, G_0 , for the slug was obtained from measurement. The expressions themselves are, of course, almost entirely empirical in form. Nevertheless, it is felt that the results given here should be of value, not only as an indication of the usefulness of the diffusion theory expressions, but as an aid to those interested in lattices within the range of those discussed here. It is much easier to apply the simple expressions given here than to perform a P_3 type calculation, for example. The results of the parameters should carry over to other slug sizes provided these are not too different from the Hanford Standard Slug.

These calculations will be extended to other slug sizes as more experimental data becomes available, and the results will be given at a later date. However, it is very doubtful that any set of parameters can be determined which will permit calculation of the bucklings for all slug diameters and lattice spacings which might be of interest, since the diffusion expressions should not be rigorously valid anyway. P_3 type⁽⁷⁾ of calculations or new methods of approach⁽⁸⁾ might prove much more fruitful, although more difficult to carry out, and efforts should probably be directed along this line.

The fact that η is lower than the cross section values would indicate is not surprising. The inadequacies of the diffusion theory expressions to predict f , and p , may be taken up in this factor since the value obtained for η was just that value which agreed with the diffusion theory expressions for f and p . Recently some experimental evidence has been obtained which indicates more fission in 25 is taking place above the cadmium cut off than had previously been supposed. In the expressions used here no provisions were included to treat this effect separately.

SUMMARY AND CONCLUSIONS

A value of $\eta\epsilon$ and the effective resonance cross section may be found which enables one to calculate the buckling for the 1.36 inch diameter slugs within the experimental accuracy of measurements over a limited moderator-to-metal ratio range.

The values of $\eta\epsilon$ and σ_{eff} which best fit the dry lattices are 1.3465 and 1.693. The best wet lattice values are 1.3492 and 1.671 barns. The values of 1.693 barns and 1.671 barns for the effective resonance integral imply that the volume part of the resonance integral usually taken as 9.25 barns should be reduced to about 8 barns.

It should be pointed out that the authors are not quoting the above values as being the correct absolute values but rather as those values which best fit the data with the equations and constants used. Dr. D. E. Davenport has found that a series of values of $\eta\epsilon$ and σ_{eff} can be obtained which will give a good fit to the data, when the disadvantage factor-to-thermal cross section ratio is considered available for adjustment. We have restricted ourselves to accepted values of cross sections and measured quantities wherever possible.

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APPENDIX I

Thermal Utilization Equation⁽³⁾

The equation for thermal utilization as derived by V. H. Rumsey and G. M. Volkoff in MT-221 is

$$f_u = f_o (1 + \delta)$$

where

$$\frac{1}{f_o} - 1 = R_{al} + R_w + R_g + S_w + S_g + B_{w2} + B_{wg}$$

(the subscripts w, al, g, u, and w2 denote water, aluminum, graphite, uranium, and aluminum process tube, respectively)

$$\delta = \frac{q_w V_w}{q_g V_g} \left[X + \frac{1}{2} \sqrt{\frac{2}{w t_w}} \frac{N_g \sigma_g V_g}{N_w \sigma_w V_w} \right] / \left(1 + \frac{q_w V_w}{q_g V_g} \right).$$

The relative absorption terms R_i are defined as

$$R_i = \frac{N_i \sigma_i V_i}{N_u \sigma_u V_u} G_o$$

where G_o is the disadvantage factor of the uranium,

$$G_o = \frac{\mathcal{M}_u r_u}{2} \frac{I_o(\mathcal{M}_u r_u)}{I_1(\mathcal{M}_u r_u)}.$$

$\mathcal{M}_u$ is the effective inverse diffusion length of the metal and r_u is the radius of the uranium. B_{ij} denotes the "blocking" term in the j_{th} medium due to the neutron density across the i_{th} medium,

$$B_{w2} = \mathcal{M}_{w2} \sqrt{\frac{2}{w t_w}} \frac{N_{al} \sigma_{al} V_2}{N_w \sigma_w V_w}$$

and

$$B_{wg} = \mu_w^2 t_w^2 \left[\frac{N_g \sigma_g V_g}{N_w \sigma_w V_w} + R_g \left(\frac{1}{2} + \frac{N_{al} \sigma_{al} (V_{cap} + V_{can})}{N_w \sigma_w V_w} \right) \right].$$

μ_w is the inverse diffusion length of water, t_w is the water annulus thickness, V_2 is the volume of the aluminum in the process tube, and $V_{cap} + V_{can}$ is the volume of the aluminum in the slug can and the slug end cap. The "self blocking" terms S_w and S_g are defined as:

$$S_w = \frac{1}{2} \mu_w^2 t_w^2$$

and

$$S_g = X \left[1 + R_{al} + R_w + B_{w2} + S_w \right]$$

where X is the disadvantage factor of the moderator,

$$1 + X = \frac{\mu_g^2 (r_2^2 - r_1^2)}{2 \mu_g r_1} \frac{I_1(\mu_g r_2) K_0(\mu_g r_1) + K_1(\mu_g r_2) I_0(\mu_g r_1)}{I_1(\mu_g r_2) K_1(\mu_g r_1) - K_1(\mu_g r_2) I_1(\mu_g r_1)}.$$

μ_g is the inverse diffusion length in the moderator, r_1 is the inside radius of the graphite and r_2 is the effective outside radius of the graphite. q_w and q_g denote the rate of production of thermal neutrons per unit volume per second for water and graphite, respectively. The q_w/q_g is taken to be the ratio of the slowing down power of water to graphite, $q_w/q_g = 20$.

The thermal utilization of graphite, aluminum, and water can be calculated from the following equations:

$$f_g = f_g' - \frac{f_g'}{1 - f_o} (f_u - f_o),$$

$$f_{al} = f_{al}' - \frac{f_{al}'}{1 - f_o} (f_u - f_o).$$

and

$$f_w = f_w' - \frac{f_w'}{1 - f_o} (f_u - f_o)$$

where

$$f_g' = (R_m + B_{wm} + S_m)f_o,$$

$$f_{al}' = (R_1 + R_2 + B_{w2})f_o,$$

and

$$f_w' = (S_w + R_w)f_o.$$

Lattice Diffusion Length

The lattice diffusion length is determined from the expression:⁽⁴⁾

$$L_t^2 = \sum_i L_i^2 f_i,$$

where L_i is the diffusion length in component i and f_i the corresponding utilization in this component. The above equation is the result of weighting the transport mean free path as to flux and volume and the absorption cross section in the same manner.

Fermi Age

For a graphite-uranium lattice the Fermi age is determined by the equation:⁽²⁾

$$\tau_{dry} = \left[364(1 - P_i) + 231 P_i \right] \left[1 + \frac{V_w + V_u + V_{al}}{V_g} \right]^2 \left[\frac{1.6}{\rho} \right]^2$$

where ρ denotes the graphite density and

$$P_i = \left(\frac{P + P'}{2} \right) \frac{\sigma_i}{\sigma}.$$

P and P's are the probabilities of collision inside the uranium lump for thermal and flat source distributions, respectively, σ_i and σ are the cross sections for inelastic scattering and total cross section for fission neutrons. The Fermi age for the wet lattice is determined from the expression:⁽⁹⁾

$$\frac{V_{\text{total}}}{\tau_{\text{wet}}} = \frac{V_{\text{u} + \text{g} + \text{al}}}{\tau_{\text{dry}}} + \frac{V_{\text{water}}}{\tau_{\text{water}}}$$

where $\tau_{\text{water}} = 33 \text{ cm}^2$.

Fast Fission Factor

The fast effect is determined from the expression:⁽²⁾

$$\epsilon - 1 = \frac{(\nu - 1 \frac{\sigma_c}{\sigma_f}) (\frac{\sigma_f}{\sigma}) P}{1 - \frac{1}{\sigma} (\nu \sigma_f + \sigma_e) P'}$$

where $\sigma_i = 2.47$ barns, $\sigma_c = 0.04$ barns, $\sigma_e = 1.50$ barns, $\sigma_f = 0.29$ barns, $\nu = 2.55$, $\sigma = 4.30$ barns, and P and P' are the same probabilities of collision as utilized in the calculation of Fermi age. Therefore the fast effect becomes

$$\epsilon - 1 = \frac{0.095233 P}{1 - 0.52081 P'}$$

APPENDIX II

Resonance Escape Probability

The equation for resonance escape probability can be expressed⁽¹⁰⁾

$$P = \exp \left(\frac{-f_r}{1 - f_r} \right)$$

where f_r is the utilization of the fuel material for resonance neutrons. Therefore f_r has the same form as the thermal utilization expression

$$f_r = f_o (1 + \delta)$$

where

$$\frac{1}{f_o} - 1 = R_1 + R_2 + R_w + B_{w2} + S_w + R_m + B_{wm} + S_m.$$

Substituting in the expressions for the various terms and gathering terms

$$\begin{aligned} \frac{1}{f_o} - 1 = & \frac{G_o}{N_u V_u \sigma_{eff}} \left\{ N_g \sigma_g V_g \left[1 + \beta_{wt}^2 \left(\frac{1}{2} + \frac{N_{al} \sigma_{al} (V_{cap} + V_{can})}{N_w \sigma_w V_w} \right) \right] \right. \\ & + (1 + X) (N_{al} \sigma_{al} V_{al} + N_w \sigma_w V_w) \left. \right\} + \beta_{wt}^2 \left\{ \frac{1}{N_w \sigma_w V_w} \right. \\ & \left. \left[(X + 1) N_{al} \sigma_{al} V_2 + N_g \sigma_g V_g \right] + \frac{1}{2} (X + 1) \right\} + X. \end{aligned}$$

Letting

$$\begin{aligned} Z = G_o & \left\{ N_g \sigma_g V_g \left[1 + \beta_{wt}^2 \left(\frac{1}{2} + \frac{N_{al} \sigma_{al} (V_{cap} + V_{can})}{N_w \sigma_w V_w} \right) \right] \right. \\ & \left. + (X + 1) (N_{al} \sigma_{al} V_{al} + N_w \sigma_w V_w) \right\} \end{aligned}$$

and

$$X = \gamma_{wt}^2 \left\{ \frac{1}{N_w \sigma_w V_w} \left[(X+1) N_{al} \sigma_{al} V_2 + N_g \sigma_g V_g \right] + \frac{1}{2}(X+1) \right\} + X$$

results in

$$\frac{1}{f_o} - 1 = \frac{Z}{N_u V_u \sigma_{eff}} + X.$$

Now since

$$f_r = f_o (1 + \delta)$$

then

$$\frac{1}{f_r} - 1 = \frac{Z + N_u V_u \sigma_{eff} (1 + \delta)}{N_u V_u \sigma_{eff} (1 + \delta)}$$

and

$$\ln P = \frac{-N_u V_u \sigma_{eff} (1 + \delta)}{Z + N_u V_u \sigma_{eff} (1 + \delta)}.$$

APPENDIX III

Description of Material Used in the Graphite-Uranium Exponential Piles and Cross Section Values Used

Uranium

Volume of U/cm of length (cc/cm)	9.37
Slug length (cm)	10.27
Slug radius (cm)	1.73
Slug density (solid in g/cc)	18.75
Slug density (equivalent in g/cc)	17.11
Nuclei density (equivalent in atoms/cc)	0.0432×10^{24}
Slug disadvantage factor	1.27
Microscopic thermal neutron absorption cross section (barns) .	7.574
Effective diffusion length (cm)	1.27

Aluminum

Volume of process tube/cm of length (cc/cm)	2.31
Outside radius of Al process tube (cm)	2.20
Effective inside radius of Al process tube with the Al ribs taken into consideration (cm)	2.02
Volume of can/cm of length (cc/cm)	1.00
Outside radius of Al can (cm)	1.83
Inside radius of Al can (cm)	1.74
Double end cap thickness (cm)	0.94
Total volume of Al/cm of length (cc/cm)	4.10
Macroscopic thermal neutron absorption cross section (cm^{-1}) .	0.0175
Macroscopic resonance neutron absorption cross section (cm^{-1})	0.00098
Diffusion length (cm)	17.67

Water

Volume of water/cm of length (cc/cm)	2.33
Water annulus thickness (cm)	0.19
Macroscopic thermal neutron absorption cross section (cm^{-1}) .	0.0175
Macroscopic resonance neutron absorption cross section . . .	0.241
Diffusion length	2.85

Graphite

	<u>Lattice</u>					
	<u>6-3/16</u>	<u>7</u>	<u>7-1/2</u>	<u>8</u>	<u>8-3/8</u>	<u>10-3/8</u>
Density (g/cc)	1.656	1.647	1.648	1.650	1.657	1.658
Nuclei density (atoms/cc x 10 ²⁴)	0.0829	0.0825	0.0825	0.0826	0.0830	0.0830
Reciprocal diffusion length (cm ⁻¹)	0.0190	0.0189	0.0189	0.0189	0.0190	0.0190
Equivalent outside radius of cell (cm)	8.867	10.031	10.748	11.464	12.002	14.87
Microscopic thermal neutron absorption cross section (millibarns)	4.4 for all lattices					
Microscopic resonance neutron absorption cross section (barns)	0.1348 for all lattices					