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SURFACE FLUXES AND CURRENTS FOR VARIOUS SHIELDED RADIATION SOURCES

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ABSTRACT

The flux and current of radiation at the surface of shielded sources are calculated as a function of source radius and shield thickness. Curves are presented for source radius (or half-thickness) from 0 to 20 mean-free-paths and for shield thickness from 0 to 20 mean-free-paths for plane, cylindrical, and slab geometries. The curves are particularly useful for estimating flux leakage from large systems. Since Pe^T rather than P is the plotted function, the curves do not sacrifice accuracy.

I. INTRODUCTION

A situation of frequent occurrence in nuclear practice and design is that of a radiation source, distributed throughout a volume of absorbing material, and shielded by further layers of absorbing material. Knowing the distributed source strength, and the geometry of the various absorbers present, it is often desired to find the radiation intensity (flux, dose rate) at the surface of the outer shield.

Furthermore, it is often required to estimate the radiation intensity at more distant points outside the outer shield. This can usually be done with some accuracy merely from geometrical considerations, if the radiation current at the outer shield surface is known. This latter is equivalent to knowing the total source strength, and to knowing the probability that a typical particle emitted from the source escapes through the surface of the outer shield.

In the following, then, are presented some derivations and calculations of these two quantities, surface intensity and surface escape probability, in three simple geometries under certain simplifying conditions. The geometries considered have respectively, plane symmetry, spherical symmetry, and cylindrical symmetry. In each case we will only consider a uniformly distributed isotropic source strength throughout a central volume which is shielded by a concentric outer region of the same absorbing power.

The chief simplifying assumption will be that all emitted particles travel in straight (unbroken) lines; that is, we will assume that any scattering is either through negligibly small angles, or is very small in magnitude compared to the absorption present. This situation is of frequent occurrence in practical transport problems: The "particles" can be gamma rays — in which case our results are useful for obtaining gamma shield efficiencies, self-shielding factors of gamma sources, etc., or neutrons — in which case we might be studying escape from strong absorbers or calculating the fast-effect in solid fuel elements.

II. GENERAL CONSIDERATIONS

Consider now an isotropic source of particles, uniformly distributed within a volume v, which is in turn contained within a larger volume V;

V is bounded externally by area A. Particles will only propagate in straight lines, since we are neglecting scattering, but the number that penetrates a distance s within V will be attenuated by a factor exp $(-\Sigma s)$. Σ is the macroscopic absorption cross section, and we are confining ourselves to situations where Σ is constant throughout V (this last condition will be somewhat relaxed in the case of plane symmetry). Thus we will find it convenient to express all distances in units of a mean-free-path = Σ^{-1}, and with this convention we will expect the desired quantities to be a function only of the geometry of v and V.

Let then the "optical" distance (i.e., distance expressed in mean-free-paths) from an element dv of v, to an element dA of A be s. The angle between s and an outward normal to A is, say, θ. The particle current outward through dA, due to unit source density distribution within dv, is then the product of dv, the fractional solid angle subtended by dA at dv, and the exponential attenuation factor:

$$dv \cdot \frac{dA \cos\theta}{4\pi s^2} \cdot e^{-s} \quad .$$

If we integrate this over all of A and v, and divide by the total source strength, we then obviously get P, the probability of escape of a typical particle. And this is one of our desired quantities:

$$P = \frac{1}{v} \int_v dv \int_A dA \; \frac{\cos\theta}{4\pi s^2} \; e^{-s} \quad . \qquad \ldots(1)$$

If the body is sufficiently symmetrical every dA is equivalent, any fixed surface point (denoted A in the sketches, as symbolizing the total surface area A) may be used as a terminus for s in the volume integrations, and Eq. (1) reduces to:

$$P = \frac{A}{v} \int_v dv \; \frac{e^{-s} \cos\theta}{4\pi s^2} \quad . \qquad \ldots(2)$$

The other problem that we wish to consider is the determination of the flux on A—flux, it being remembered, is a scalar; the "track-length per unit volume" of neutron theory, proportionate to the "dose-rate" of health physicists. We might, indeed, visualize a small test body at A, being subjected to the radiation from the distributed source of unit intensity per unit volume throughout v. The projected area of such a test body along any direction s, multiplied by the collision probability Σ (test body)$\cdot$ds and summed over the test body, is constant: Σ(test body)$\cdot$ v(test body). Thus the total collision rate per unit volume of test body is determined by multiplying Σ(test body) and the flux at A, ϕ_A, where the projection factor $\cos\theta$ of Eq. (2) is now not present in the volume integration for this last:

$$\phi_A = \int_v dv \frac{e^{-s}}{4\pi s^2} . \qquad \ldots (3)$$

It will be convenient, however, for us to plot a dimensionless quantity instead of Eq. (3); so, in analogy with Eq. (2) we will define the quantity Ψ by setting

$$\Psi = \frac{A}{v}\phi_A = \frac{A}{v}\int_v dv \frac{e^{-s}}{4\pi s^2} . \qquad \ldots (4)$$

We will find below, P and Ψ for the three geometries: shielded slabs, spheres, and cylinders. The reader may also wish to consult related prior work that has become available. Castle, Ibser, Sacher, and Weinberg[1] give expressions and graphs of 1-P for unshielded spheres and cylinders, and also for hollow cylinders; these last involve integrals similar to some that we will find for shielded cylinders. Taylor and Obenshain[2] give expressions and extensive graphs of ϕ outside shielded cylindrical sources. They allow Σ to be different within v and the shielding region, and both concentric cylindrical and plane slab shields are considered. Case, de Hoffman, and Placzek[3] give an elegant formulation of the theory of P for unshielded sources, and give results for slabs, spheres, cylinders, and more complicated shapes.

III. SHIELDED SLABS

We consider a slab of thickness 2a, shielded on each side by thickness T of shield. This is shown in Fig. 1. Note that a and T are measured in mean-free-paths. We will compute the current density and flux at point A on the shield

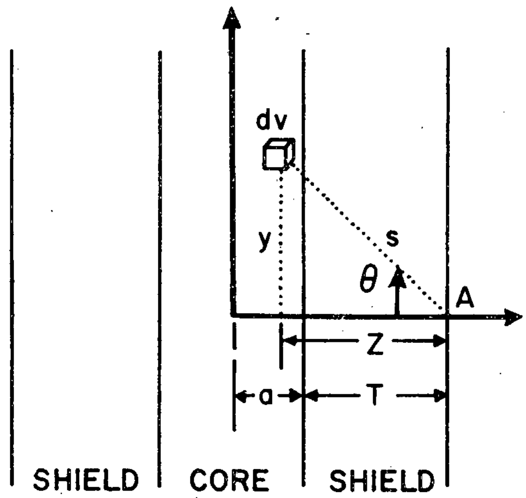

Fig. 1. Slab Source

surface due to unit source density within the slab; weighted with the ratio A/v = 1/a, these are then the desired dimensionless quantities P and Ψ:

$$P = \frac{1}{a} \int_T^{T+2a} dz \int_0^{\infty} \frac{e^{-s}}{4\pi s^2} \cos\theta \, 2\pi y dy, \qquad \ldots (5)$$

$$\Psi = \frac{1}{a} \int_T^{T+2a} dz \int_0^{\infty} \frac{e^{-s}}{4\pi s^2} \, 2\pi y dy, \qquad \ldots (6)$$

where $s^2 = y^2 + z^2$. Changing to integration on $\sec\theta = t$ gives

$$P = \frac{1}{2a}\int_{T}^{T+2a} dz \int_{1}^{\infty} \frac{e^{-zt}}{t^2} dt , \qquad \ldots(7)$$

and

$$\Psi = \frac{1}{2a}\int_{T}^{T+2a} dz \int_{1}^{\infty} \frac{e^{-zt}}{t} dt . \qquad \ldots(8)$$

The integrations on z are now immediately accomplished. We will, furthermore, introduce the family of exponential integrals

$$E_n(x) = \int_{1}^{\infty} \frac{e^{-xt}}{t^n} dt . \qquad \ldots(9)$$

These are discussed by Case, de Hoffman and Placzek[3], by Chandrasekhar[4], and many others, and have been extensively tabulated by Placzek[5]. So, finally,

$$P = \frac{1}{2a}\left\{E_3(T) - E_3(T + 2a)\right\} , \qquad \ldots(10)$$

and

$$\Psi = \frac{1}{2a}\left\{E_2(T) - E_2(T + 2a)\right\} . \qquad \ldots(11)$$

IV. SHIELDED SPHERES

We consider a sphere of radius a, surrounded by a concentric shield of outer radius R, and so of thickness $T = R - a$, see Fig. 2. We have uniform source

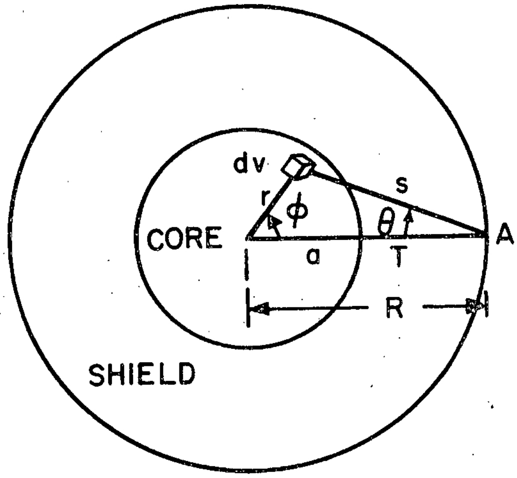

Fig. 2. Sphere Source

density throughout the central sphere. Requiring the mean-free-path to be the same in both sphere and shield allows us to measure all distances with this as the unit; the ratio $A/v = 3R^2/a^3$. The quantities we require are then given by:

$$P = \frac{3R^2}{a^3} \int_0^{\pi} d\phi \int_0^{a} \frac{e^{-s}}{4\pi s^2} \cos\theta \, 2\pi r \sin\phi \, r dr \quad , \qquad \ldots (12)$$

$$\Psi = \frac{3R^2}{a^3} \int_0^{\pi} d\phi \int_0^{a} \frac{e^{-s}}{4\pi s^2} \, 2\pi r \sin\phi \, r dr \quad , \qquad \ldots (13)$$

where $s^2 = r^2 + R^2 - 2rR\cos\phi$, and $r^2 = s^2 + R^2 - 2sR\cos\theta$. Integrating on s instead of ϕ, these become:

$$P = \frac{3}{4a^3}\int_0^a rdr \int_{R-r}^{R+r} \frac{e^{-s}}{s^2}(R^2 + s^2 - r^2)ds , \qquad \ldots(14)$$

and

$$\Psi = \frac{3R}{2a^3}\int_0^a rdr \int_{R-r}^{R+r} \frac{e^{-s}}{s} ds . \qquad \ldots(15)$$

These again integrate in terms of the exponential integrals. The results can be variously expressed by means of the recursion formula

$$(n-1)E_n(x) = e^{-x} - xE_{n-1}(x), \quad n > 1 ; \qquad \ldots(16)$$

perhaps the simplest are:

$$P = \frac{3}{8a^3}\Big[\frac{1}{2}\left(R^2 - a^2\right)^2\left\{E_1(R-a) - E_1(R+a)\right\} +$$

$$e^{-R}\left\{(R^3 + Ra^2 + R^2 + A^2 - 2R - 2)\sinh a +\right.$$

$$\left.(a^2 - R^2 + 2R + 2)a\cosh a\right\}\Big], \qquad \ldots(17)$$

and

$$\Psi = \frac{3R}{2a^3}\Big[-\frac{1}{2}(R^2 - a^2)\left\{E_1(R-a) - E_1(R+a)\right\} +$$

$$e^{-R}\left\{(R-1)\sinh a + a\cosh a\right\}\Big] . \qquad \ldots(18)$$

V. SHIELDED CYLINDERS

We consider a cylindrical source of radius a, surrounded by a concentric cylindrical shield of outer radius R, and so of thickness T = R-a, see Fig. 3.

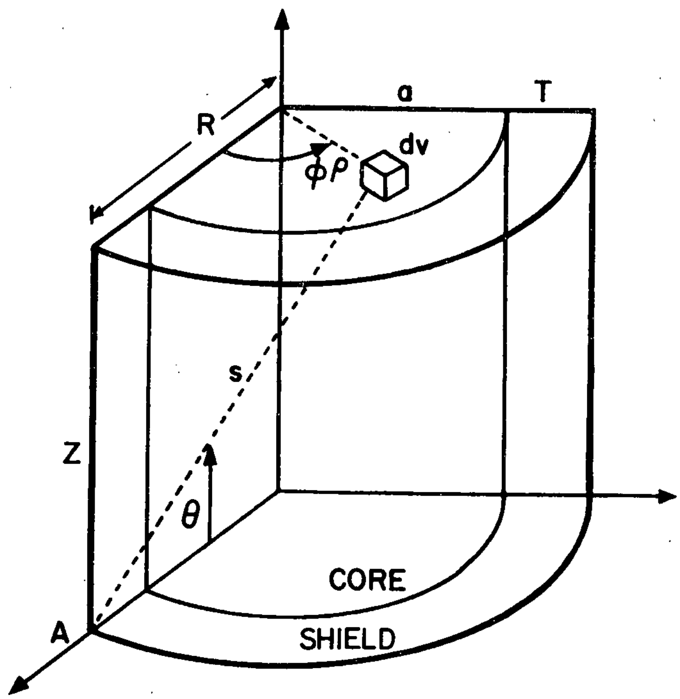

Fig. 3. Cylindrical Source

We have uniform source density throughout the central cylinder. Requiring the mean-free-path to be the same in both sphere and shield allows all distances to be measured in mean-free-paths. The ratio $A/v = 2R/a^2$.

The desired quantities on the shield surface are then given by

$$P = \frac{2R}{a^2} \int_0^a \rho d\rho \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz \; \frac{e^{-s}}{4\pi s^2} \cos\theta \quad , \qquad \ldots (19)$$

$$\Psi = \frac{2R}{a^2} \int_0^a \rho d\rho \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz \, \frac{e^{-s}}{4\pi s^2} \quad , \qquad \ldots (20)$$

where $s^2 = z^2 + R^2 + \rho^2 - 2R\rho\cos\phi$, and $\cos\theta = (R-\rho\cos\phi)/s$. We first may integrate on z by means of the substitutions $s^2 = f^2 + z^2 = \xi^2 f^2$. The partial integrals on z then readily can be transformed to more convenient forms:

$$\int_{-\infty}^{\infty} \frac{e^{-s}}{s^3} dz = \frac{2}{f^2} \int_1^{\infty} \frac{e^{-f\xi} \, d\xi}{\xi^2 \sqrt{\xi^2 - 1}} = \frac{2}{f} \int_1^{\infty} K_1(ft) \frac{dt}{t} \quad , \qquad \ldots (21)$$

and

$$\int_{-\infty}^{\infty} \frac{e^{-s}}{s^2} dz = \frac{2}{f} \int_1^{\infty} \frac{e^{-f\xi} \, d\xi}{\xi \sqrt{\xi^2 - 1}} = 2 \int_1^{\infty} K_o(ft) dt \quad . \qquad \ldots (22)$$

The ϕ integrations are now most easily accomplished by considering the projection of Fig. 3 on a horizontal plane, see Fig. 4. Introducing the angle χ between

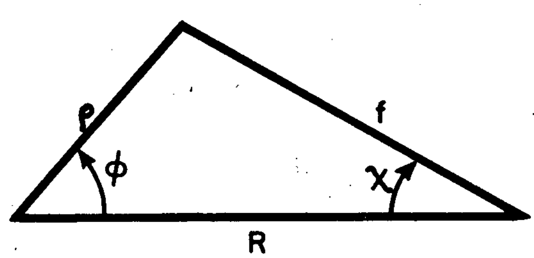

Fig. 4. Projection of Cylindrical Source on Horizontal Plane

R and f, and recognizing that $\cos\chi = (R-\rho\cos\phi)/f$, we see that our integrals have become

$$P = \frac{R}{\pi a^2} \int_0^a \rho \, d\rho \int_1^\infty \frac{dt}{t} \int_0^{2\pi} K_1(ft) \cos \chi \, d\phi \qquad \ldots (23)$$

and

$$\Psi = \frac{R}{\pi a^2} \int_0^a \rho \, d\rho \int_1^\infty dt \int_0^{2\pi} K_o(ft) \, d\phi \qquad \ldots (24)$$

The arguments may be expanded by the addition theorems of Graf and Neumann[6] and then immediately integrated:

$$P = \frac{2R}{a^2} \int_0^a \rho \, d\rho \int_1^\infty K_1(Rt) \, I_o(\rho t) \, \frac{dt}{t} \, , \qquad \ldots (25)$$

$$\Psi = \frac{2R}{a^2} \int_0^a \rho \, d\rho \int_1^\infty K_o(Rt) \, I_o(\rho t) \, dt \quad ; \qquad \ldots (26)$$

and so, finally

$$P = \frac{2R}{a} \int_1^\infty K_1(Rt) \, I_1(at) \, \frac{dt}{t^2} \, , \qquad \ldots (27)$$

and

$$\Psi = \frac{2R}{a} \int_1^\infty K_o(Rt) \, I_1(at) \, \frac{dt}{t} \, . \qquad \ldots (28)$$

The remaining integrations must, in general, be performed numerically. A compilation that is of assistance in the second case is an unclassified Hanford report by G. M. Muller[7]. In the special case of R = a, or T = 0, however, the integrals follow from results in Chapter V of reference 6:

$$P_{a=R} = \frac{2a}{3}\left\{ -2 + \left(2a + \frac{1}{a}\right) I_1(a) K_1(a) + I_o(a) K_1(a) \right.$$
$$\left. - I_1(a) K_o(a) + 2a\, I_o(a) K_o(a) \right\} , \qquad \ldots (29)$$

and

$$\Psi_{a=R} = 2\left\{ 1 - aI_1(a) K_1(a) - aI_o(a) K_o(a) + I_1(a) K_o(a) \right\} . \qquad \ldots (30)$$

Equation (29) is incorrectly given in reference 3, but has appeared elsewhere in the classified literature[8].

VI. RESULTS

Families of curves based on Eq. (10), (11), (17), (18), and (27), (28) are given in Fig. 5 through 10. Because of the large orders of magnitude that must be spanned for application to practical problems, it has been found convenient to plot the quantities Pe^T and Ψe^T versus a (in mean-free-paths), for various values of shield thickness T (in mean-free-paths).

Thus to obtain the escape probability P, knowing a and T (in mean-free-paths), the ordinates of Fig. 5, 7, and 9 should be directly read, then multiplied by e^{-T}.

To obtain the surface flux ϕ_A due to a given source strength per unit volume, we must remember Eq. (4): having read the ordinate of Fig. 6, 8, or 10 , it must then be multiplied by e^{-T}, by the source strength, and also by v/A. Almost always one wants the flux ϕ_A expressed in particles /cm^2-sec, so the source strength should be expressed in particles /cc-sec and the factor v/A must have the units [centimeters].

Finally it should be pointed out that the requirements for the constancy of the mean-free-path Σ^{-1} throughout V can be relaxed somewhat in the case of plane symmetry. In fact, so long as Σ is constant in the core region υ, it can be allowed to be a varying function of position z in the shield region without changing Eq. (10) and (11). In these the external factor 1/2a is unchanged; a is still to be measured in core mean-free-paths. The arguments of the exponential integral functions are now to be interpreted as total optical thicknesses ($\int\Sigma(z)\,dz$) of shield and shield-plus-core, respectively. Thus, if total optical shield thickness T is correctly computed, Fig. 5 and 6 can still be used. This remark may prove of convenience in computing plane shielding situations where the shield is laminated.

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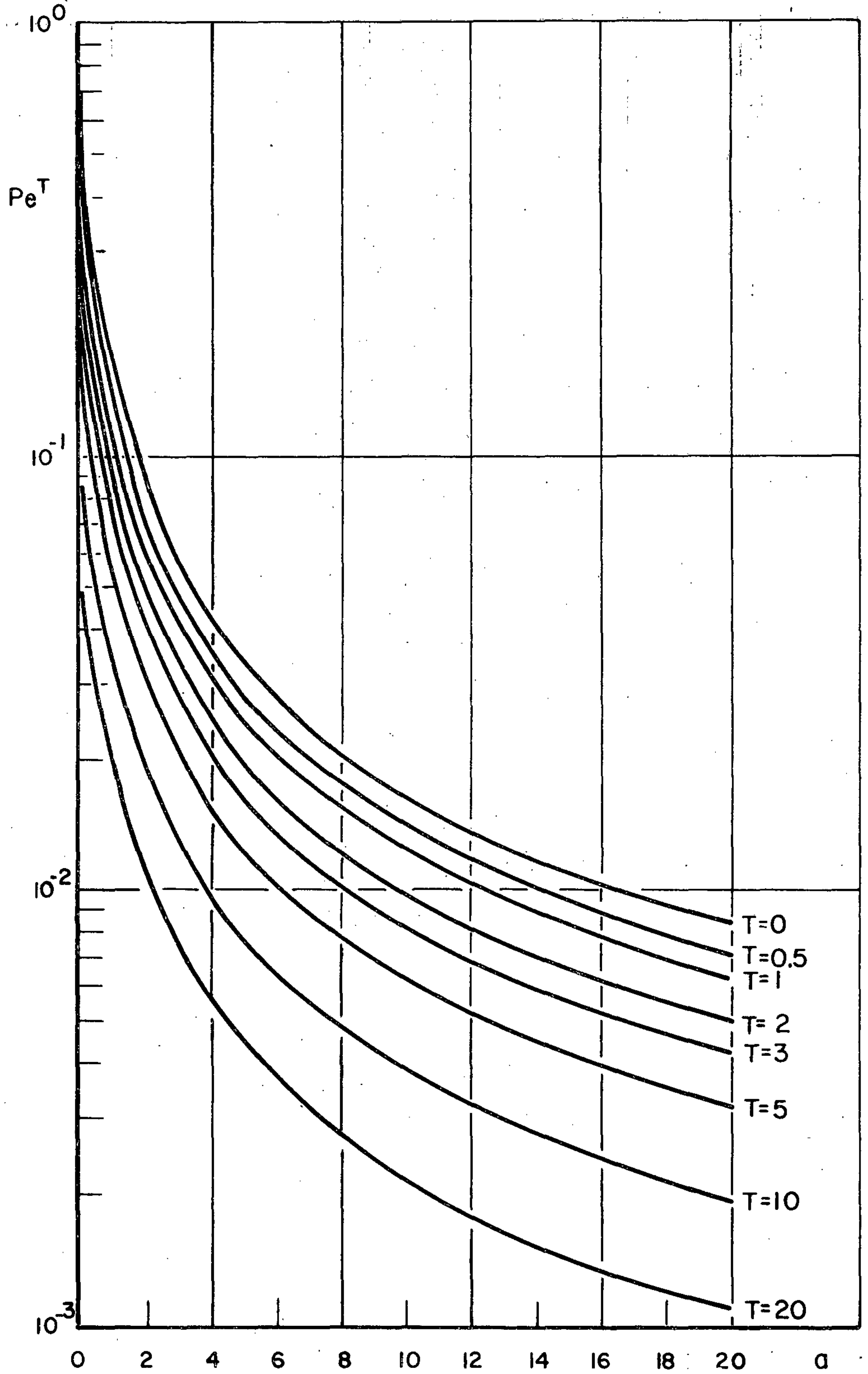

Fig. 5. Pe^T vs α for a Slab

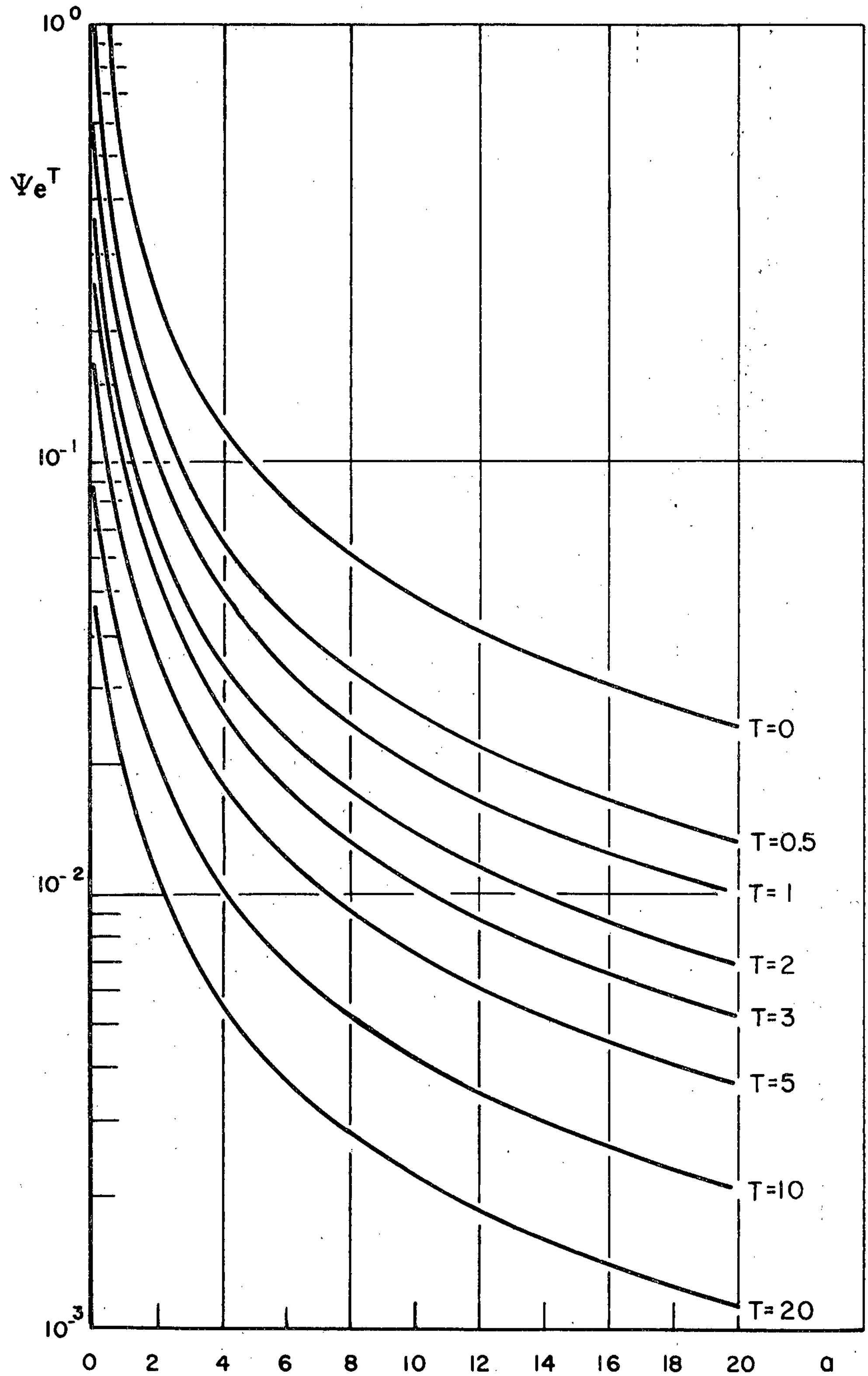

Fig. 6. Ψe^T vs α for a Slab

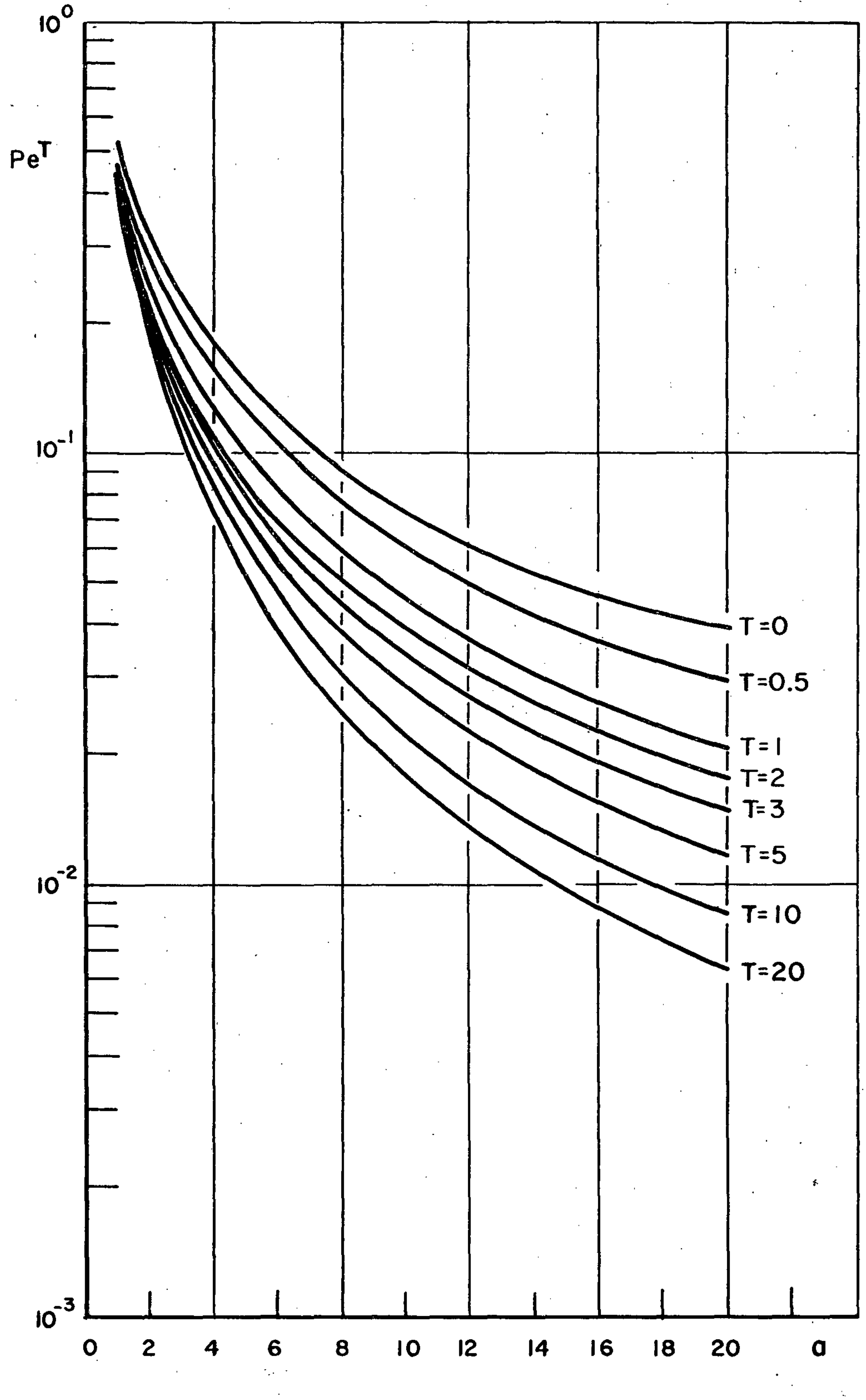

Fig. 7. Pe^T vs a for a Sphere

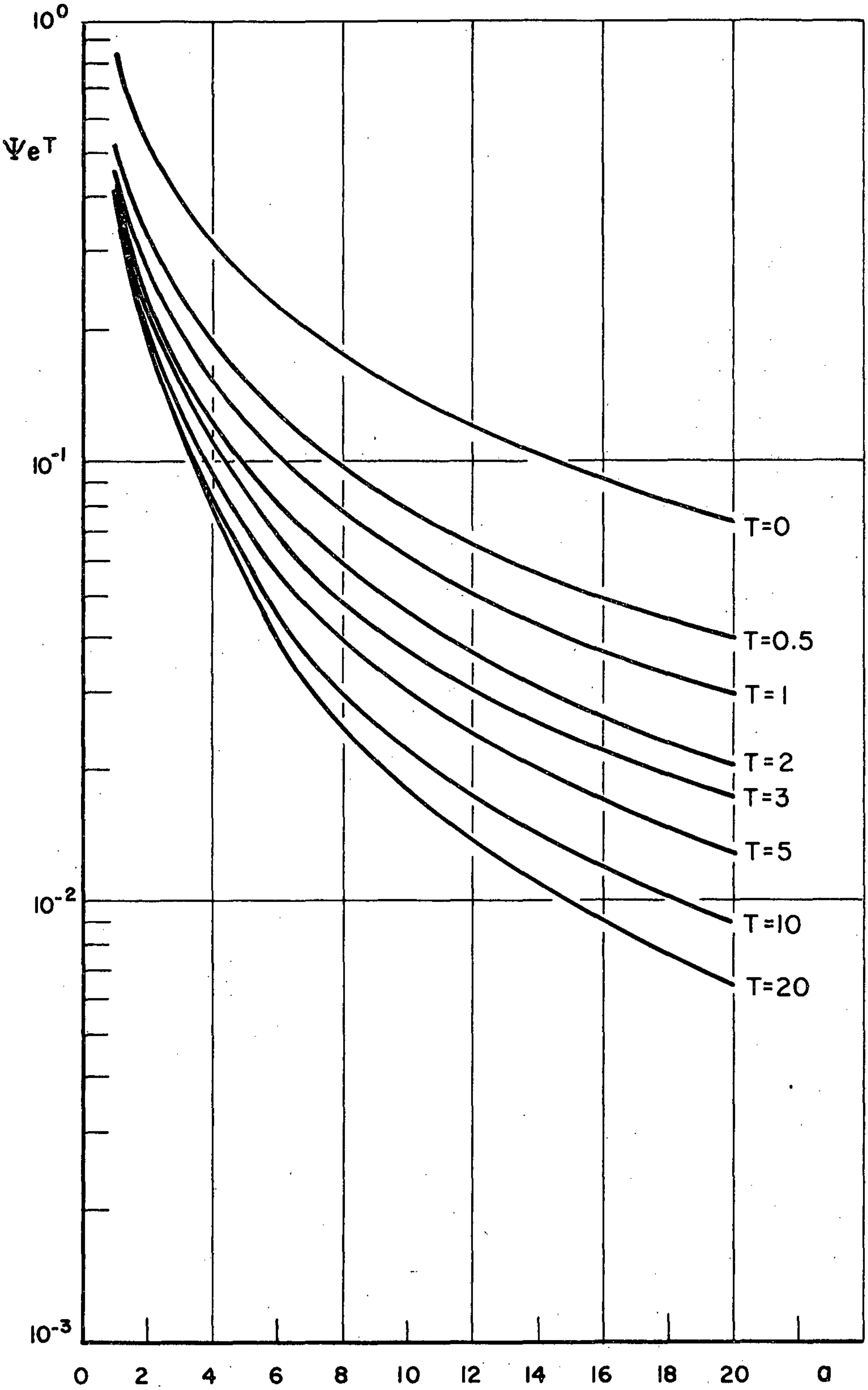

Fig. 8. Ψe^T vs α for a Sphere

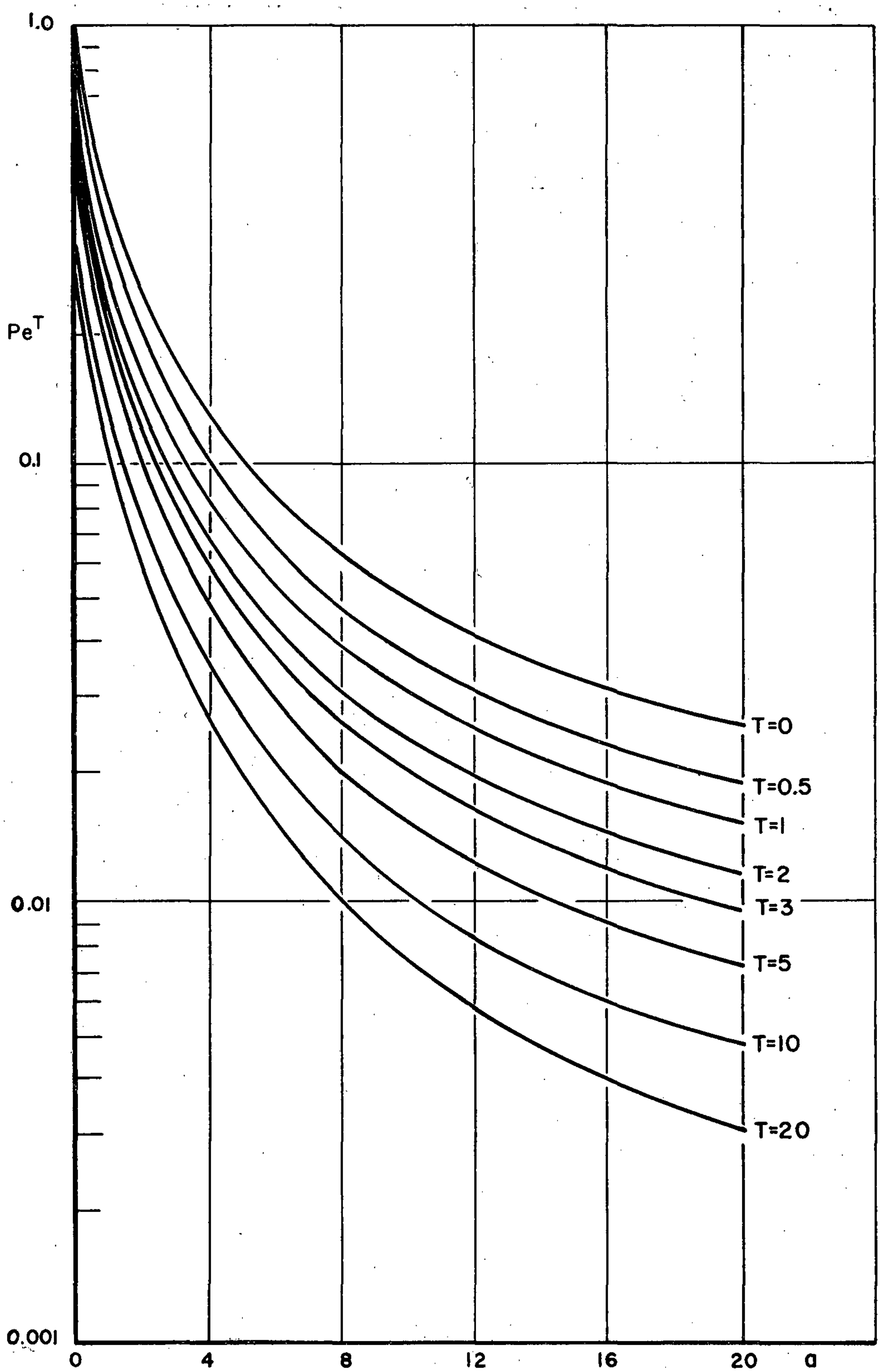

Fig. 9. Pe^T vs α for a Cylinder

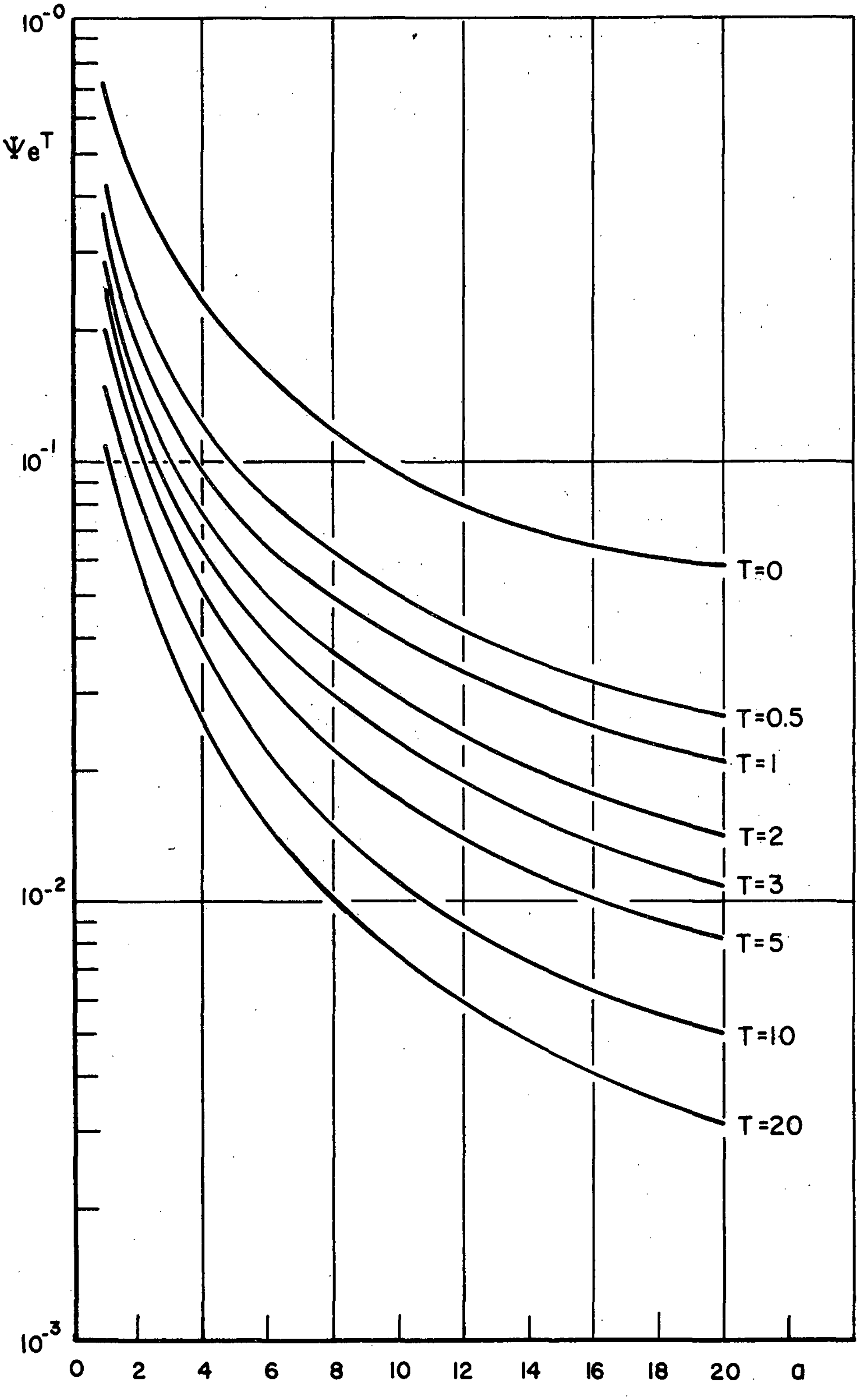

Fig. 10. Ψe^T vs α for a Cylinder