RESEARCH REPORT

BATTELLE MEMORIAL INSTITUTE

COLUMBUS LABORATORIES

TECHNICAL DOCUMENT CENTER
Nuclear Rocket Operations

DOC NO X - 11611

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

BATTELLE MEMORIAL INSTITUTE

COLUMBUS LABORATORIES • 505 KING AVENUE • COLUMBUS, OHIO 43201

FIELDS OF RESEARCH

Aeronautics — Astronautics
Agricultural Chemistry
Agricultural Economics
Alloy Development
Applied Mathematics
Area Economics
Biochemistry
Biophysics — Bionics
Catalysis — Surface Chemistry
Ceramics
Chemical Engineering
Chemical Processes
Communications Science
Computer Technology
Corrosion Technology
Earth — Atmospheric Sciences
Electrochemistry
Electronics
Energy Conversion
Engineering — Structural Materials
Environmental Systems
Extractive Metallurgy
Extreme-Temperature Technology
Ferrous Metallurgy
Food Technology

Foundry Practice
Fuels — Combustion
Glass Technology
Graphic Arts Technology
Immunology — Cancer Studies
Industrial Economics
Industrial Physics
Information Research
Inorganic Chemistry
Instrumentation
Light Alloys — Rare Metals
Lubricant Technology
Materials Separation — Concentration
Mechanical Engineering
Metal Fabrication Engineering
Metal Finishing
Metallurgical Processes
Microbiology
Microscopy — Mineralogy
Nondestructive Evaluation Technology
Nonferrous Metallurgy
Nucleonics
Ocean Engineering
Organic Chemistry

Organic Coatings
Packaging Research
Particle Dynamics
Petrochemicals
Petroleum Engineering
Pharmaceutical Chemistry
Physical Chemistry
Production Engineering
Psychological Sciences
Pulp — Paper Technology
Radioisotopes — Radiation
Reactor Technology
Refractories
Reliability Engineering
Rubber — Plastics
Semiconductors — Solid-State Devices
Sound — Vibration
Systems Engineering
Textiles — Fibers
Theoretical — Applied Mechanics
Thermodynamics
Transportation
Welding — Metals-Joining Technology
Wood — Forest Products

SPECIAL REPORT

on

THE DESIGN OF NUCLEAR-
RADIATION-EFFECTS EXPERIMENTS

to

SPACE NUCLEAR PROPULSION OFFICE

August 15, 1967

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

by

James E. Drennan, Roger W. Cote, and Donald J. Hamman

BATTELLE MEMORIAL INSTITUTE
Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

100-100000-100000

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
EXPERIMENTAL DESIGN PRINCIPLES	2
Experimental Purpose	4
Defining the Problem	4
Expressing the Problem	6
Experimental Objectives	7
Analysis Procedures	8
The Dependence of Analysis Procedures on Experimental Approach	9
Simple Graphical Analysis	10
Regression Analysis	11
Confidence Interval	11
Tolerance Band	12
Normalization	12
Engineering Interpretation of Regression Analysis Results	12
Other Analysis Techniques	13
Decisions Pertinent to Establishing Analysis Procedures	13
Requirements for Experimental Data	14
Defining Performance and Environment	14
Parametric Character of Data	15
Data Accuracy and Precision	15
Defining the Data Requirements for an Experiment	17
Experimental Procedures	18
Controlling the Experimental Conditions	19
Identification and Control of Systematic Error	20
Summary of Other Important Considerations	21
SUMMARY AND CONCLUSIONS	24
REFERENCES	26

APPENDIX A

MODEL EXPERIMENTAL DESIGN FOR NUCLEAR-RADIATION-EFFECTS EXPERIMENTS	A-1
---	-----

APPENDIX B

ILLUSTRATION OF RESULTS EXPECTED FROM EXPERIMENT RS/LDT-401	B-1
---	-----

APPENDIX C

SUMMARY OF STATISTICAL FORMULAS	C-1
---	-----

THE DESIGN OF NUCLEAR-RADIATION-EFFECTS EXPERIMENTS

by

James E. Drennan, Roger W. Cote, and Donald J. Hamman

ABSTRACT

Effective use of known experimental design principles in planning nuclear-radiation-effects experiments associated with engineering development programs assures that necessary, useful information will be obtained from the characteristically costly and time consuming radiation-effects tests. This report discusses the principles of experimental design and illustrates their compatibility with nuclear-radiation-effects experimentation by presenting a model experimental design for a hypothetical nuclear-radiation-effects experiment. Details of an analysis procedure that uses linear regression techniques are illustrated by numerical example. Although the model experiment design is directed toward specific problems and a specific instrument, its format and logic are applicable generally to nuclear-radiation-effects experimentation. It is believed that the data-reduction and analysis procedures documented in this report may be applicable in detail to a wide variety of nuclear-radiation-effects experiments that may be planned in support of system-development programs.

INTRODUCTION

In most cases, the experimental determination of how a device, structure, or material is affected by exposure to nuclear radiation is both time consuming and costly in comparison to non-nuclear experimentation. Yet, such determinations must be made as a part of engineering development programs to provide a basis for prediction of expected device, structure, or material performance for a specific application. Since engineering design decisions based upon such predictions may be crucial to the success and total cost of an entire system-development program, the basic experimental information must be both accurate and sufficient. However, the cost of the source of nuclear radiation and the necessary associated facilities for the experiment must be combined with the cost of planning, procurement of test specimens, developing test fixtures and measurement instrumentation, conducting the experiment, performing analyses of experimental results, and

reporting the experimental information to those who need it. These costs and the time required to accomplish the work must fit into the budget and time schedule of the over-all development program along with the other experimental work considered essential. The budgetary and time limitations are almost always such that program success depends upon the greatest use of available resources.

Another important consideration is that the engineering development program will generally proceed as a sequential set of decision points. Basic decisions will be made in sequence as the developmental program matures. If experimental information needed at the time of decision is not available, either the program must be delayed in order to accumulate this information or else the decision must be made without it. Since neither of these alternatives is really acceptable, it is obvious that another important criterion for nuclear-radiation-effects experiments is that they must produce the information needed to make decisions before the time for decision arrives.

It may be noted that, throughout history, progress has often been based on learning from mistakes. However, this doesn't mean that the maximum knowledge results from the maximum number of mistakes. Rather, the modern philosophy is to gain knowledge from both success and error with emphasis being placed upon minimizing the error. The goal of modern experimentation is to find the things that will work with a minimum of effort on unsuccessful studies; that is, modern experimentation must be effective and efficient. Thus, just as careful design is a necessary prerequisite to the fabrication of an effective and efficient machine, careful and complete experimental design is a necessary prerequisite to the efficient gathering of effective experimental information. Over many years, in all branches of science and engineering, techniques have been evolved that are pertinent to this task of designing efficient and effective experiments. The remainder of this report describes the application of some of these experimental design techniques to the problem of designing nuclear-radiation-effects experiments.

EXPERIMENTAL DESIGN PRINCIPLES

Generally speaking, there exists a continuum of types of nuclear-radiation-effects experiments. However, it is convenient to consider two distinct classifications: engineering and research experiments. The basic distinction between experiments in these two classifications is that, for engineering experiments, there is a specific application involved; for research experiments the purpose is to learn how and why a device, structure, or material is affected by radiation, and no direct effort to relate the observations to any specific application is intended.

The emphasis in this report will be on engineering nuclear-radiation-effects experiments with the primary purpose of determining whether or not the devices, structures, and materials being evaluated will adequately meet the requirements of a specific application. However, some aspects of the research-type experiments remain since the experimental results will be used, where necessary, to guide the selection or development of new devices and materials if those considered in the experiment are found to be inadequate.

Since the nuclear-radiation-effects experiments of interest in this report are primarily engineering experiments, they must be designed to provide the information needed to determine if the performance of the specimen is adequate for the application. Gathering research information is a secondary consideration. However, a good experimental design should achieve the engineering purpose and also produce some useful research information as well. Costs, time, and facility limitations require that a balance be maintained between these two somewhat divergent areas, with the first priority being given to the engineering purpose.

Assuming that the treatments (experimental stimulus environment), the experimental units (specimens), and the general nature of the observations have been determined, Cox* defines the requirements for a good experiment as follows: "The requirements for a good experiment are -- that the treatment comparisons should as far as possible be free from systematic error, that they should be made sufficiently precisely, that the conclusions should have a wide range of validity, that the experimental arrangement should be as simple as possible, and finally that the uncertainty in the conclusions should be accessible." The purpose of the many books and articles written about experimental design is to present the underlying principles and techniques that can be used prior to experimentation to assure that the experiment will be of high quality, i.e., that the experiment will meet the requirements quoted above.

The cardinal principle in designing an experiment is complete and detailed planning. The development of an experimental design involved preparing detailed answers to the following questions:

- (1) What is the problem?
- (2) What information is needed to solve the problem?
- (3) What analysis can be conducted to produce this information?
- (4) What experimental data are needed to perform this analysis?
- (5) What must be done experimentally to obtain these data?

Once an experimental design is prepared that answers these questions in detail, performing the experiment and analyzing the data appear to be almost routine since it is necessary only to follow the procedures already spelled out in the experimental design. Of course, experience has shown that nuclear-radiation-effects experiments are almost never routine, but a good experimental design will minimize the difficulties.

Since there will have been a concerted effort to take all eventualities into account during the development of the experimental design, the risk that the experiment will fail to solve the stated problem is minimized. A good experimental design will include an assessment of the expected error

* D. R. Cox, Planning of Experiments, John Wiley and Sons, Inc., New York (1958), p. 5 (Other general references are listed on page 26.)

of the experimental results, making it possible before the experiment is actually conducted to evaluate its merit in terms of whether or not the expected results will be worth the experimental costs.

It is seen from the above discussion that the documentation of an experimental design will consist of at least five parts corresponding to the five questions that must be answered. Thus, the experimental design will include a statement of the experimental purpose which defines the problem, statements of the experimental objectives which describe in detail the information to be obtained, a definition of the analysis procedures, a definition of the data to be obtained, and a detailed description of the experimental procedures to be followed. Some of the essential contents of these sections of an experimental design will be discussed in the following paragraphs.

Experimental Purpose

Perhaps the most important part of an experimental design is the statement of experimental purpose which is the definition of the problem. To conduct an experiment without a clear statement of experimental purpose is like trying to formulate an answer without a question. There is a basic difference between experimental purpose and experimental objectives. Experimental purpose is concerned with the "why" of the experiment. It relates the expected experimental information to an engineering or scientific need. Experimental objectives are concerned with the "what" of the experiment. They describe the specific information expected from the experiment. Thus, the experimental purpose pertains to solving an engineering or scientific problem using the information described by the experimental objectives. The connecting link that leads from the experimental measurements to meeting the experimental objectives and ultimately to satisfying the experimental purpose is, of course, the analysis procedure.

Logically, the experimental purpose depends on the problem to be solved, and the experimental objectives and analysis procedure depend on the experimental purpose. However, this is an oversimplification since the manner of stating the experimental purpose may be influenced by the information that can be practically obtained and the methods available for data analysis; that is, the experimental objectives and the analysis procedure may interact with the statement of experimental purpose. Because of these interactions, several iterations may be necessary in the definitions of purpose, objectives, and analysis procedure in order to achieve a practical experimental program. However, the planning must start with the problem and the definition of experimental purpose.

Defining the Problem

To develop a hardware system it is necessary to select and/or develop components and materials that can be assembled as required by the system designs to obtain hardware that represents the capabilities of the designs. Obviously

a component or material will be selected for use, or its development may be considered complete, when it has been shown that it is capable of performing as required by the system designs. In some cases, information already available from previous experiments or other sources will be a sufficient basis for decision. However, in many other cases supplemental information will be needed. The nuclear-radiation-effects experiment is intended to provide part of this supplemental information in support of the system development program.

The primary concern of the system designer is the question "Will the component or material work adequately in the system designs and under the expected environmental conditions?" This must also be the primary concern of the nuclear-radiation-effects experiment designer. However, the system designer also is concerned with improving his designs or perhaps modifying them to permit the use of previously unacceptable components or materials. In addition, it may be necessary or desirable to improve the component design or material formulation. Experimental information is needed to guide these efforts. Hence, the capabilities and limitations of components and materials need to be established for ranges of nuclear-radiation flux and fluence combined with other environmental factors that encompass present and anticipated system applications.

Note that the system designer and the experiment designer are usually both concerned with the capabilities and limitations of component designs and material formulations rather than specific individual components and materials. They want to know the performance to expect if at some future time a component or material nominally similar to those tested is purchased for use in a prototype or production system. It usually is not intended and probably will not be possible to use the specific experimental specimens in the system application. Thus, the experimental information desired is a prediction of the performance to be expected from nominally similar items, not just a report of performance observed for specific experimental specimens.

Realistically, it is recognized that an honest, absolute answer to the question "Will it work adequately in the system design?" will probably never be possible. Nor can honest, absolute statements about the performance of component types or material formulations be made. There will always be some uncertainty associated with the experimental information even though considerable effort has been expended to minimize the magnitudes of the uncertainties. However, the experimental results can be honestly and completely expressed in terms of probabilities with specified confidence levels; that is, probabilistic predictions of expected performance under specified conditions with stated confidence. Such predictions would be based on a "population inference" about the characteristics of the component type or material as indicated by statistical descriptions of the experimental data.

It is important to recognize that the ability to make predictions and population inferences and describe experimental data by statistics does not depend upon experimental sample size, upon the current existence of a "population" of the items, or even upon whether the items are "hand built"

or come from an established production line. The numerical values obtained may depend on the sample size and the assumptions made; however, the validity of the statistical approach cannot be questioned on these grounds. If the results indicate that the item should perform adequately, a population will probably be built if it doesn't already exist. Changes in production approaches may be implemented. The important fact is that, with this approach, the experimental results will be honestly evaluated and reported, their meaning interpreted in a form suitable for the system designers use, and the confidence that should be placed in the validity of the information numerically documented. A later section of this report will discuss statistical concepts and techniques in greater detail. The subject has been introduced here because it directly influences the format of the statement of experimental purpose.

In many cases, the problem with which the intended experiment is concerned is quite complex and actually consists of several simpler, related problems. In such cases, it is often better to consider conducting several experiments, each directed toward a separate aspect of the total problem, rather than attempt to consider the total problem in a single experiment. The actual experimentation for the several experiments may, of course, be conducted simultaneously. In any case, each aspect of the total problem must be considered in the complete experimental design. Thus, it is essential to break down the statement of the experimental purpose into its various parts to assure that every aspect of the total problem is clearly defined.

Expressing the Problem

The statement of the experimental purpose begins by showing the relation between the total problem and the specific aspect of this problem being considered. For engineering experiments the statement of experimental purpose includes definitions of the requirements imposed by the intended application (or references to documents where such definitions may be found). The remainder of the statement of experimental purpose should state explicitly how the problem will be solved with the experimental information to be generated.

Many formats for statements of experimental purpose are possible. However, it appears that the following format for stating experimental purpose is generally applicable to most nuclear-radiation-effects experiments.

"The purpose of this experiment is to

- (1) Establish with reasonable confidence, the probability that [identify specific component type or material] will satisfy the system performance requirements specified in [identify document where the required performance and environment is specified]
- (2) Determine the capabilities and limitations of performance for the specified item type over the ranges of radiation flux and fluence combined with [identify other environmental factors] from [identify specific ranges of radiation and other environmental factors included in this experiment]

- (3) Identify those failure modes and mechanisms associated with such nuclear-radiation exposure where failure is defined as [include specific definitions of failure or identify reference where such are given]."

The brackets in these statements enclose descriptions of specific information pertinent to the experiment or experimental specimens that should be inserted

It is seen that this kind of statement of experimental purpose relates directly to the intended use of the experimental information. Further, it requires expression of the experimental information in terms of the predicted performance of the component type or material as distinct from the observed performance of the specimens that are tested. It must be recognized that, in general, there is no intent to use the experimental specimens in the application but rather the concern is with predicting the expected performance of nominally similar devices, structures, and materials in the application environment. Thus, the system designer will be provided information expressing the performance he can expect from components or materials that he specifies in his design for future purchase and use in the system hardware. He is also provided a means of evaluating the risks associated with such use. Information is given that can guide system design changes and component or material improvement efforts. Finally, a direct relation between the system development program and the nuclear-radiation-effects program is provided as part of a basis for objective evaluation of the value of the specific experiment with respect to furthering the system development.

Experimental Objectives

The obvious general objective of any experiment is to gather the information needed to satisfy the experimental purpose. However, it is equally obvious that the information needed should be specifically defined. Thus, logically, the specific experimental objectives will define the information one expects to obtain from the experiment. This means that each specific experimental objective should describe in detail a specific kind of information to be obtained. In essence, this description will outline the format expected to be used in presenting the information obtained during the experiment. Experience has shown that a description of the presentation format for the experimental information is an effective means for stating each specific experimental objective with a minimum chance for misunderstanding.

Now, in most cases, the experiment being designed is not required to provide all of the information needed to satisfy the experimental purpose. Usually, information pertaining to the same or similar component designs or materials is available from manufacturer's tests, other tests outside the radiation environment, previous nuclear-radiation testing, and other literature sources such as the Radiation Effects Information Center (REIC). The section of the experimental design concerned with experimental objectives should logically

include a compilation of the pertinent information already available as well as specific descriptions of the supplemental information to be obtained experimentally. In other words, this section should describe what is already known as well as what additional information will be needed to satisfy the experimental purpose.

The amount of detail in the statement of a specific experimental objective will depend considerably on the analysis procedure to be employed as well as experiment, and experimental-specimen complexity. It is important to include sufficient detail to show the relevance of the expected experimental results to the experimental purpose. However, the objective must also be realistic. The statement of an experimental objective should be a clear, practical statement describing what is to be accomplished experimentally.

As stated previously, statements of specific experimental objectives should correspond to the presentation of experimental information planned for use in the experiment report. Thus, if it is expected that the experiment report will present a graph showing the variation of a particular component or material parameter as a function of nuclear-radiation flux or fluence, a brief description of the format of this graph would constitute an experimental objective. A set of such graphs that includes all of the important component or material parameters would establish the performance capabilities and limitations for the item.

In both this and the preceding discussion, satisfying the experimental purpose has been treated as if there were no chance of completing the experiment without satisfying the purpose. For a properly designed experiment with a realistically stated purpose, the only chance of an unsuccessful experiment lies in failure to obtain the data specified. Of course, instrumentation does fail, reactors do shut down, and environmental chambers do take unexpected excursions; but, barring such catastrophic occurrences, the properly stated purpose of a well-designed experiment will be satisfied.

It is not claimed that the numerical results obtained in the experiment will necessarily be those anticipated, but that, objectively, the probabilistic predictions, expressions of component or material capabilities and limitations, and determinations of failure modes and mechanisms required by the statement of experimental purpose will be obtained. If, in fact, they are not, the fault lies in failure to adequately conceive, express, and plan the experiment. The experiment should not have been conducted at all if there was any real doubt about successfully meeting the objectives or satisfying the purpose.

Analysis Procedures

Engineering experimentation can be considered from the viewpoint of the operations involved. First, some experimental specimens that are nominally representative of a component design or material are acquired. Then these specimens are exposed to a combination of environments while the values of various parameters believed pertinent to performance are measured. These recorded

parameter values and measurements of environmental stress levels constitute the experimental data. These data are now manipulated and presented in a manner intended to show the observed relationships between performance parameter values and environmental parameter values. The data may be reduced by statistical techniques to obtain a clearer picture of trends and experimental variability; that is, to obtain a separation of "signal" from "noise".

Technically, at this point the experimental analysis has been completed. However, the primary concern is not simply to describe the observed performance of one set of test specimens under specific test conditions, but rather to predict the expected behavior of nominally similar components or materials for various environmental conditions. Thus, the data and relationships resulting from the experimental analysis are the input to a process of synthesis which provides information about the population of components or materials for ranges of environmental conditions. This information, perhaps combined with other pertinent information already available, is then analyzed with respect to the conditions specified for each potential application to predict expected performance in the application.

The mathematical, graphical, logical, or other procedures employed to obtain the specific performance predictions, characteristic performance information, and experimental data presentations from the parameter values measured experimentally are normally called analysis procedures. However, both analysis and synthesis are actually involved.

The Dependence of Analysis Procedures on Experimental Approach

There are two courses of action available to establish the analysis procedures to be used for an experiment. The experimenter can gather the experimental data first and then consider how these data may be analyzed. However, in this case some vital data are often missing so that there may be no analysis procedure that will objectively produce the predictions and information desired without requiring a number of unrealistic or at least questionable assumptions. Since the experiment is already completed, it must be repeated or other experiments conducted to obtain analyzable data.

The alternative approach is to first establish analysis procedures that are capable of producing the desired information and predictions. The experiment is then planned and conducted to obtain all of the data required to implement these analysis procedures. This approach is recommended because it assures that the objective information and predictions desired as experimental results will be obtained within the technical, economic, and time constraints pertinent to the work.

The interaction between analysis procedures and experimental data is reasonably obvious; only the data obtained is available for analysis. The interaction between experimental objectives and analysis procedures is equally obvious; the information obtained depends on the analysis performed so the

analysis should be planned to produce the information desired. However, practical limits on the data that can be obtained experimentally may limit the choice of analysis procedures and, thus, limit the information that can be obtained. In such a case a modification of experimental objectives and perhaps experimental purpose would be necessary to assure a practical experimental design.

It is apparent that defining experimental objectives, establishing analysis procedures and defining data requirements are interdependent activities generally requiring one or more iterations to evolve a practical, compatible experimental design. In some cases, it will not be possible to obtain some of the information initially desired within the time and funds allocated. In other cases, more than one experiment will be needed to obtain the desired information. A major advantage of the practice of complete and detailed experimental design is that the merit of each proposed experiment can readily be weighed in relation to estimated time and costs before the experiment is performed. Thus, limited resources may be concentrated on experimentation that will have the greatest potential benefit to the system development program.

The establishment of analysis procedures for a given experiment starts with a statement of experimental purpose, a set of experimental objectives, and a number of available analysis techniques ranging in complexity from simple to very sophisticated. This basic information, augmented by the experience of the experiment designer and his associates as well as available literature, must be used to logically select appropriate analysis techniques and compile them into a comprehensive analysis procedure for the experiment.

A general rule for establishing analysis procedures is to select the simplest procedure that will do the job. The use of sophisticated procedures can be justified only where there are real advantages to be gained. However, it is essential that, regardless of its complexity, the analysis procedure should be established and documented in detail as part of the experimental design.

Simple Graphical Analysis

Perhaps the simplest available analysis procedure is one widely used in engineering and scientific work. This procedure involves graphically plotting the data for each parameter measured; fitting curves to the data points visually; and then making predictions based on these curves. This procedure is particularly appropriate where there is only one experimental specimen. However, it runs into difficulty when predictions must be based on multiple-specimen data and when there is a requirement to provide objective evaluations of the confidence that may be placed in the predictions. The major deficiency of this procedure is that it does not measure the experimental "noise" so that this "noise" can be separated from the information "signal". After all, the experiment is basically an attempt to communicate with nature and, like all communication links, the intelligibility of the reception depends on the signal-to-noise ratio. Where extraction of the maximum "signal" is desired (as is usual in experiments) special processing is needed to separate "signal" from "noise". The name of the body of mathematical techniques available for measuring experimental "noise"

and extracting the maximum possible information "signal" is statistics.

Regression Analysis

Instead of fitting a curve to the data by eye, a statistical technique known as the "method of least squares" can be used to find the curve which gives the "best possible fit" to the data. This analysis procedure is known as regression analysis. With this procedure the mathematical expression relating the dependent and independent variables is obtained where the sum of the squared deviations of data points from the regression curve is minimized. As part of the calculations made to determine the regression curve, values for a number of useful statistics are also obtained.

Regression analysis is not a complex or sophisticated procedure, and it becomes even simpler if a linear relation between variables exists. In fact, the simplicity of linear-regression-analysis techniques will justify efforts to make relationships linear by transforming variables or even the performance of separate analyses for regions where assumption of linearity is a reasonable approximation. Since many component or material parameters are related to radiation flux or fluence (or to other environmental stresses) by simple exponentials which may be made linear by a logarithmic transformation of data, linear regression analysis is expected to be appropriate for many of the nuclear-radiation-effects experiments.

Confidence Interval

The regression line that relates a parameter value for a component or material to an independent variable such as radiation flux or fluence is a prediction of the average parameter value expected for each specified value of the independent variable. Using statistics obtained during the regression-line computations, an interval about this regression line is easily computed that will contain the true relationship between the variables with a specified probability. This interval is known as the confidence interval and the associated probability is named the confidence level. Thus, the true average parameter value for a specified level of the independent variable will be contained within the confidence interval with a probability expressed by the confidence level.

The confidence interval expresses the limits of uncertainty of "signal" location resulting from experimental "noise". The confidence level expresses the probability that this interval contains the true "signal"; that is, the true relation of average parameter value to the independent variable.

Tolerance Band

Even if there were absolute assurance that the regression line accurately predicted the performance of the average component or material, it would not be expected that each component or material obtained from their respective populations for actual system use would be an average specimen. Rather, some variation in performance among the members of the population would be expected. It obviously is desirable to determine the amount by which the performance of an individual specimen might differ from the average performance of the population. Stating this another way, it is desirable to establish, with a specified confidence, the range of performance variation about the population average that will contain the performance of a specified proportion of the population. Using statistics already computed as part of the regression-analysis procedure, a tolerance band can be computed that gives just this desired information. The tolerance band for a specified tolerance level expresses the performance range that will contain a specified proportion (tolerance level) of the total range of population performance, symmetrical with respect to the average performance for the population.

Normalization

Another feature of regression-analysis techniques is that they are compatible with procedures for normalizing observations of performance among several test specimens. Such normalization is needed to remove experimental biases such as differences in initial specimen performance or unavoidable differences in applied environmental stress. The regression analysis can simplify the problem of normalization considerably because "noise" and "signal" for each specimen are separated. The "signal" information needed for normalization is thus readily available.

Engineering Interpretation of Regression Analysis Results

The performance of a linear regression analysis will provide (1) the best possible estimate of the average test-item performance as a function of environmental stress, (2) the confidence interval about this estimate that will contain, with a specified degree of confidence, the average performance of the population of items for which the test specimens are representatives, and (3) the tolerance band about this estimate that will contain, with a specified degree of confidence, the performance of a specified proportion of the population items for which the test specimens are representatives. Comparison of the tolerance-band limits with system-application performance requirements provides a simple means of predicting the probability of successful performance in system use at the specified confidence level. This information should satisfy the needs of many nuclear-radiation-effects experiments.

Of course, it is possible that, in some cases, suitable linearizing transformations of the data cannot be made. In such cases, nonlinear regression

analysis procedures could be employed, but some development work might be needed to extend the procedures for computing confidence intervals and tolerance bands to the nonlinear case. Detailed procedures for linear regression analysis are described and illustrated by example in Appendices A and B of this report. A summary of the statistical formulae for linear regression analysis is given in Appendix C.

Other Analysis Techniques

Some experimental purposes may require the use of one or more of the many other available analysis procedures. For example, combined environmental studies often require evaluation of interaction as well as main effects. In such cases analysis of variance techniques may be appropriate for use with matrix experiments (e.g., this approach is often used in material studies). Techniques such as power spectral analysis may be appropriate for experiments involving complex components or subsystems. Although considerable attention is given in this report to the utilization of regression analysis, the primary concern is that the analysis procedure (regardless of kind) should be established and specified in detail prior to experimentation and that this procedure should be that one necessary and sufficient to meet the experimental objectives and satisfy the experimental purpose.

A very important consideration in experimental analysis is the incorporation into the analysis procedures of any theoretical or empirical models that may already exist. Obviously, efficient experimentation requires that maximum use be made of information already available since the major experimental costs are involved in accumulating new information. Rather than adopting the philosophy that each experiment stands alone, it is necessary to design the experiments to supplement the information already available. Thus, the experiment may be concerned primarily with establishing whether or not a particular model is valid in the area of interest rather than trying each time to establish a new model.

Because this approach depends upon the kind of information that may already be available, it is difficult to generalize about the analysis procedures that might be used. However, Bayesian statistical techniques are available that have been developed specifically to cope with this kind of problem. A detailed discussion of these techniques is beyond the scope of this report. However, the Bayesian philosophy of using all pertinent prior information as well as new experimental data to obtain an expression of experimental results is certainly the approach recommended for the achievement of efficient experimentation.

Decisions Pertinent to Establishing Analysis Procedures

The pre-experimental planning of the analysis procedures provides the answers to the questions: "What information will be obtained?" "How will

the experimental data be used?" "What experimental data are necessary?" An experiment conducted without prior answers to these questions is certainly not scientific and has a very high probability of providing only inconclusive or meaningless results at considerable expense.

Certain decisions with respect to procedures for recording and analyzing data will be based on theoretical and empirical studies or nuclear-radiation effects on specific materials and components similar to those being investigated. These will include decisions with respect to

- (1) The operating characteristics of the component or material to be studied
- (2) The frequency with which these operating characteristics should be measured during the course of radiation exposure
- (3) The number of decimal places or significant digits to be recorded in making measurements, etc.

These are decisions which comprise the tactics of the experimental procedures. They will be affected by the particular type of nuclear-radiation facility available, the measuring equipment, the experimental equipment, dosimetry, etc. They may be modified during the course of the experiment.

Other decisions with respect to procedures for recording and analyzing experimental data will be designed to present information on nuclear-radiation effects for subsequent use in combination with other scientific or engineering information. Such decisions may be based on the general theory of nuclear-radiation effects in solids, liquids, or gases and/or conventional engineering methods for engineering analysis.

Experience has shown that the preparation of the analytical-procedures portion of an experimental design can best be accomplished with the support of statistical personnel. Engineering personnel, of course, have the primary responsibility and provide much of the essential knowledge and experience. However, the aid of statistical personnel is usually necessary at least on a consulting basis. With this approach, in nearly every case an experimental-analysis procedure can be developed prior to experimentation that, when implemented, will establish the degree of confidence that can be placed in the experimental results and will minimize the risk of obtaining inconclusive experimental results.

Requirements for Experimental Data

Defining Performance and Environment

The data recorded in a nuclear-radiation-effects experiment is the measure of performance of specimen components or materials before, during, and after exposure to a specific nuclear-radiation environment, usually in combination

with a variety of other environmental stresses. "Performance" and "environment" are general terms that may have many different meanings for different test items and experimental objectives. It is apparent that, at any instant of time, the measure of performance is the set of values measured at that instant for all parameters pertinent to the function that the component or material is expected to perform. The measure of environment at this instant is the set of values observed for all environmental stresses incident on the component or material.

Usually, in one experiment, the total performance or total environment cannot be considered because of practical limitations in facilities, time, and available funds. Hence, in each experiment, those performance and environmental parameters believed significant with respect to the experimental purpose and objectives must be selected for measurement and/or control. It is important to recognize that this selection of parameters constitutes defining "performance" and "environment" for the experiment. Also, when these definitions are made it is automatically assumed that no other performance or environmental parameters have significance with respect to the experimental purpose or objectives. It is obvious that the success of the experiment will depend on the validity of this assumption.

Parametric Character of Data

All basic experimental data consist of the measured values of a single parameter at specific instants of time. In most cases, these data are reduced by making time equivalencies for measurements of two or more parameters. In other words, the value of one parameter obtained at a specific time is related to that of another parameter obtained at essentially the same instant. An implicit assumption in this process is that the time factors cancel out leaving a true equivalency between the two (or more) parameters.

At least initially, most analysis procedures involve consideration of pairs of parameter values, one member being a performance parameter, the other being another performance parameter or an environmental parameter. It usually is required that, for this portion of the analysis, all other environmental parameters either remain constant or else that the relationships between the primary parameter pair and the uncontrolled environmental parameters have been established and the data adjusted to remove this effect. Subsequent steps in the analysis procedures may include means for combining these elementary relationships, but the analysis procedures, almost without exception, will start by pairing parameters to establish a set of points in a specific two-dimensional space.

Data Accuracy and Precision

The discussion so far has been based on an implied assumption that the exact value of any parameter can be measured at any desired instant of

time. However, the best that can be realistically expected is an ability to measure parameter values with a known degree of accuracy at any given time. Another desirable capability is that of measuring precisely; that is, if the true parameter value does not change, the same number should be obtained each time a measurement is made. Accuracy is concerned with how closely the measured value corresponds to the true value while precision pertains to an ability to measure something consistently.

There is a high correlation between measurement precision and the scatter of experimental data. By maintaining a high measurement precision, the random measurement errors are minimized. This minimizes the number of independent measurements needed to assure high confidence in the experimental results. The statistical techniques employed in the analysis procedures will take into account the random experimental errors and separate this "noise" from the information "signal".

However, achievement of precision in measurement does not guarantee that the measurement is accurate. It is quite easy to obtain a precise measurement that actually differs significantly from the true parameter value. This can occur through failure to take into account systematic experimental error. Such systematic experimental errors include measurement lead resistances, calibration error in measurement instruments, radiation or electromagnetically induced currents in measurement equipment wiring, specimen to specimen differences, and differences in environmental stresses applied to specimens. For each experiment, a determination of potential sources of systematic experimental error is essential, and the experimental plans should include methods for eliminating or, at least, measuring and controlling such errors. With such planning, the effects of systematic experimental error can be removed from the experimental results during the analysis providing the necessary data are obtained as part of the experiment.

The elimination and/or control of systematic experimental error is essential to the achievement of accurate experimental results and is an important justification for complete and detailed experimental planning. It is usually not possible to obtain the data needed to remove effects of systematic error from the experimental results after the experimental work is completed. Provisions for obtaining the required data must be integrated into the experimental procedures. This can only be accomplished by recognizing the need for such data and planning to obtain it as the experiment is conducted.

There is not enough money or time to completely eliminate experimental error from experimental results. Therefore, the experimenter must decide early how much error he can tolerate. The only rational basis for this decision is to assure obtaining data of sufficient accuracy and precision to meet the experimental objectives and satisfy the experimental purpose. Efforts to obtain data accuracies and precisions substantially greater than those absolutely necessary are usually an unjustifiable waste of time and money.

The accuracy and precision of radiation dosimetry presents a major problem in all nuclear-radiation-effects experiments. Since radiation flux and gamma exposure together with neutron fluence are independent experimental

variables, ideally the dosimetry should establish with high accuracy the rate at which gamma protons and neutrons of specified energies strike each specimen as well as the total number of these particles as a function of experiment duration. However, to be practical, the accuracy of present state-of-the-art neutron dosimetry ranges from 20 to 50 percent with precisions ranging from 2 to 5 percent. Furthermore, there is no satisfactory means for direct measurement of high-level gamma flux during the performance of an experiment. Calculations based on pre- or post-experimental flux mapping or calorimetric studies are much less desirable than direct measurements during the experiment although they are better than nothing. The dosimetry problem constitutes a major source of systematic error in radiation-effects experiments and thus may limit the accuracy attainable in the experimental results. However, by recognizing this limitation and designing experiments accordingly, high-confidence experimental information can be obtained.

Defining the Data Requirements for an Experiment

The section of the experimental design that defines the requirements for experimental data specifies the performance and environmental parameters to be measured in the experiment. In general, experimental data are required to accomplish four experimental functions:

- (1) Identify specimens, locations, dates, equipment, etc.
- (2) Define stimulus applied to specimens
- (3) Determine specimen response to applied stimulus
- (4) Establish degree of potential experimental error.

Thus, the experimental data obtained from a radiation-effects experiment can be separated into categories concerned with identification, stimulus, response, and experimental error.

In the identification category, it is important to include sufficient detailed data to assure that any additional information pertinent to the experiment can be located if needed. For example, the experimental results may indicate a significant difference between specimens, and it is then important to consider whether there were differences in the process or materials used to fabricate the specimens that would explain this observation. Proper identification of the specimens will facilitate reference to the manufacturer's records in an effort to answer this question. A second example is that, following the experiment, it is discovered that the calibration of one of the measurement instruments appears questionable. If the instrument in question has been properly identified as part of the experimental data, it is then possible to locate it and recheck its calibration.

The necessity of recording the input stimulus applied to the device or material being evaluated is obvious since this constitutes the independent variable for the experiment. However, it is not so obvious that the total environment to which the sample is exposed is the real stimulus, the intended

input variable being only one of many forces acting on the specimen. Thus, the data pertaining to all important environmental forces (including any electrical excitations) are included in the stimulus category.

It is obvious that the response category includes observed data pertaining to changes in specimen parameter values. However, it is seldom practical or desirable in a single experiment to measure all possible parameters for a specimen. The selection of which response parameters to measure depends upon the problem that the experiment is intended to solve and the data analysis that is planned.

The category concerned with experimental error will include data expressing the precision and accuracy of the experimental instrumentation. Precision is here taken to mean the capability of repeating successive measurements to a given number of decimal places. Absolute accuracy should be traceable through some standard to the National Bureau of Standards. Needless to say, the precision of a measurement instrument must be better than the absolute accuracy. It should also be noted that two kinds of accuracy may be involved. For example, a device that is excited electrically during the measurement of a parameter has an accuracy associated with the parametric measurement and also an accuracy associated with the control of the electrical excitation conditions.

It is obvious that the experimental-error category should include all data which may affect actual or potential uncertainties in the experimental results.

In preparing the experimental design it is necessary to establish the kinds and quantities of data that will be needed in each of the four categories described above. The identification necessary must be defined and procedures must be established to assure that the various identifications will be maintained throughout the experiment. The input stimulus and environmental parameters to be measured must be defined. The sample response parameters to be measured must be established on the basis of the analysis planned and the experimental problem to be solved in conjunction with the engineering knowledge of the known specimen characteristics. Finally, the questions of measurement precision and accuracy and the maintenance of experimental control must be resolved and the corresponding requirements defined.

Experimental Procedures

Examination of Webster's Dictionary definitions for "experiment" and "procedures" leads to a definition of "experimental procedures" as: "a particular way of accomplishing an operation under controlled conditions to discover an unknown effect or law". (Incidentally, "Test" is given as a synonym for "Experiment".) Examination of this definition shows that the key factors with respect to "experimental procedures" are

- (1) There must be a pre-established purpose and objectives related to discovery and application of an unknown effect or law.
- (2) The work is to be accomplished in a particular way where the conditions affecting the operation are controlled.

The previous discussions have described methods for satisfying the first of these factors as well as defining the major experimental conditions that must be controlled. Attention is now directed to the practical aspects of how to control the experimental conditions and how to determine the optimum procedure for experimentation.

The primary reason for preparation of an experimental design is to maximize the chances of successful experimentation. To accomplish this, it is necessary to first identify all the things that can go wrong and then define a course of action that reduces their number or their potential effect on experimental success as much as possible. Actually, the intent of the preceding discussions has been to describe means for eliminating any chance of things going wrong in the areas considered. Thus, the opportunity for experimental failure is essentially limited to the actual experimentation. The section of the experimental design that defines the experimental procedures is intended to further reduce the risk of experimental failure to the lowest possible level.

As far as the details of the experimental procedure for a given experiment are concerned, there may be many different procedures that will produce the data required. The selection of the actual procedure to be used will depend upon the availability and capabilities of facilities, equipment, and personnel. The final criteria for selection of the optimum experimental procedure will be economics and efficiency whenever a number of alternative procedures are available, each of which is capable of producing the data needed with the required degree of accuracy and precision.

It is important to recognize that the use of economy and efficiency as selection criteria for the experimental procedures should relate to the overall program of which the experiment being considered is just one part, and not relate to just the single experiment. Hence it is sometimes desirable to select an experimental procedure that is less than optimum for one experiment to achieve lower costs and greater efficiency for a group of experiments. This is the primary justification for the development and use of new facilities and equipment that have improved capabilities. The goal is to optimize the economics and efficiency of the experimental program, not simply a single experiment.

Controlling the Experimental Conditions

The section of the experimental design that expresses the requirements for experimental data will identify the major experimental conditions that must be measured and/or controlled. It is obvious that some consideration of the practical experimental procedures will have been needed in order to prepare these requirements. However, it is also very likely that other conditions pertaining to the detailed experimental techniques planned must be identified and means for control established as part of the planning of the experimental procedures. For example, calibration of the measurement equipment may depend upon secondary standards, and the chance of variation in these standards must be eliminated by establishing control procedures which reference the primary standards maintained by the National Bureau of Standards.

Actually, the experiment concerns an operation that takes place at a particular location in a particular facility. Performance parameters and environmental parameters must be measured and controlled at that location. However, in most nuclear-radiation-effects experiments this measurement and control function must be performed remotely. Thus, both data transmission links and control transmission links are required. There may or may not be automatic environmental control. Some combination of test specification, testing personnel, and auxiliary equipment will act as an "experiment programmer and controller". Both random and systematic error may be associated with each experimental function, data link, and control link.

Identification and Control of Systematic Error

The primary concern in the development of the experimental procedures is systematic experimental error. The success of the experiment will depend upon the identification and elimination or control of all sources of significant systematic error. The control of systematic error may be accomplished directly or through measurement coupled with corrective computations incorporated into the analysis procedures. The identification of sources of significant systematic error and the evaluation of error that cannot be completely eliminated is an engineering responsibility and cannot be postponed until the experimental work is finished if the risk of experimental failure is to be minimized.

Statistical techniques can be used to reduce the uncertainty in experimental results caused by random error to any desired extent at the cost of obtaining more data. It has been shown that the magnitude of uncertainty from this source varies inversely as the square root of the number of independent observations. However, as stated earlier, it is easy to be precisely wrong by neglecting sources of significant systematic experimental error.

The identification of sources of systematic experimental error and the development of means for control of these errors are not simple tasks. The major complication results from the fact that two or more normally insignificant environmental stresses may synergistically produce a significant source of error. For example, the ductility of a material may still be quite adequate at cryogenic temperatures and neutron bombardment of the material at room temperature or above may not change the ductility significantly. However, the material ductility may decrease intolerably because of neutron irradiation at cryogenic temperatures because the neutron damage is "frozen" into the material. But, if the material ductility is then measured at room temperature, it may be observed that it is adequate. Warming the irradiated specimens has allowed the damage to anneal. Thus, cryogenic temperatures and nuclear radiation may act synergistically to cause unacceptable reductions of material ductility.

Potential synergistic processes that can cause significant systematic error in experimental results must be recognized before the experiment is conducted.

Obviously, it is impossible to be completely successful in this effort, but it is equally obvious that every effort must be made to avoid a failure of the experiment because of synergy. The only general rule applicable to this problem is to employ common-sense judgment based on all available experience in planning complete and detailed experimental procedures.

Summary of Other Important Considerations

The philosophy required for planning experimental procedures is how to obtain the required data even when everything possible goes wrong. Obviously this can only be done by assuring that it is impossible for anything short of a major catastrophe to produce a significant uncontrolled systematic error in the experimental results. The experimental procedure will be satisfactory if it assures that, at every instant of measurement, the value of every independent experimental variable will be known either because it is measured at that instant or is controlled in such a way that there is virtual certainty of it being unchanged since the last time it was measured. (In this context the calibration of all measurement instrumentation is an independent variable for the experiment as is any power or excitation source upon which the function of the test specimens depends.) Following are some specific comments pertinent to the preparation of the experimental procedures section of the design for a nuclear-radiation-effects experiment.

One potential source of systematic error in an experiment is the human element. Control of human error requires that the personnel conducting the experimental work be given complete and detailed specifications for the work to be performed. There must be no ambiguity in these specifications. If the experiment will require decisions to be made during the course of the experiment, the decision criteria and the corresponding alternatives must be specified.

The preparation of a flow diagram for the experimental work can be a useful aid to the preparation of the detailed experimental procedures. Such a diagram expresses the critical points in the experimental work sequence and includes all of the alternative paths that may possibly occur. For example, at one initial point in the experiment observation of a specific parameter may show either no change, an increase in value, or a decrease in value. The flow diagram would show the various alternative courses of action depending upon what is actually observed at this point. The diagram would also take into account the effects of potential experimental problems (e.g. reactor power excursion or scrams). Such a flow diagram serves as a guide to the expression of action specifications for the experiment-operating personnel and helps to assure completeness of these specifications. Further, it helps to identify potential sources of systematic experimental error and establish the requirements for equipment reliability, accuracy, and precision.

The incorporation of redundant measurements into the experimental procedures provides a means for determining the actual measurement precision. A determination of measurement accuracy by using calibration checks during the experiment is also desirable. Preferably, such check measurements are programmed so that the experiment-operating personnel will not know if a measurement

is a real observation or if it is a calibration check. This is particularly important for measurements made manually with analog meters since humans tend to introduce a reading bias when judging the instrument readings. If possible, the measurement equipment should be selected or the measurement precision requirements established to minimize the need for operator judgment, thus minimizing this potential source of systematic experimental error.

The form of the data to be recorded has a decided effect on the choice of measurement equipment. If the data are to be manually recorded, then equipment with the required accuracy and precision and with adequate readability will suffice. However, if the data are to be recorded directly on punch cards, paper tape, or magnetic tape, then equipment with the appropriate electrical readout is required.

The choice of the method of data recording should be made with consideration of the long-range expense, as well as the requirements of measurement density or the method of data processing. Although equipment with electrical readout is initially more expensive, the time required to record the data will be less (with a corresponding reduction in the number of personnel required), operator judgment will be eliminated (thus reducing experimental error), and operator fatigue will be reduced.

Experience has shown that, if suitable commercial equipment is available, it definitely should be used. The cost of purchase of this equipment will almost invariably be less than the cost of in-house design and construction, even if the in-house equipment operates with no breakdowns. When considering the cost of commercial equipment, the cost of maintenance during the period of the experiment should be included. The best equipment available is very often the least costly in the long run.

All equipment should be tested in every way possible prior to its use for the experiment. In this way questions about the actual performance and capabilities of the equipment can be answered before the experiment is initiated. Thus, the possibility of such questions arising after initiation of the experiment (when it may not be possible to supply the answers) is minimized. It is important that equipment constructed "in-house" be tested and retested prior to its use for the experiment.

In developing the experimental procedures it must be determined whether each parameter to be measured is expected to increase or decrease with experimental-specimen degradation and at what level of degradation the measurements will be terminated. This can make an important difference in the measurement-equipment requirements. In some cases it may be desirable to continue measurements beyond some point of degradation with reduced accuracy and precision.

To summarize, preparation of the experimental-procedures section of an experimental design requires the establishment of potential sources of systematic experimental error and the determination of means for their elimination or control. Consideration must be given to the expected flow of the

experimental work and the possible alternative flow paths that may occur if the experiment does not proceed as expected. The procedures to be followed by the experiment-operating personnel in any eventuality should be specified in complete detail. Attention must be given to the calibration of equipment, and calibration checks should be an integral part of the experiment to provide a measure of experimental accuracy. Measurement equipment should be selected to meet the experimental accuracy and precision requirements while requiring a minimum of judgment from the equipment operator. A procedure for assuring, prior to experimentation, that the equipment reliability is adequate should be part of the experimental-procedures section of the experimental design.

SUMMARY AND CONCLUSIONS

The expensive and time-consuming nature of most nuclear-radiation-effects experiments requires that they be carefully planned with close attention being paid to budgets and schedules. This is particularly important for experiments that are to be conducted in support of system-development programs because the experimental information obtained must be timed to meet system design needs as well as fitting into the program budgets. The well-established principles of experimental design provide an approach to experimental planning that assures efficient gathering of effective experimental information.

Complete and detailed planning is the cardinal principle of experimental design. The experimental design will contain detailed answers to the following questions:

- (1) What is the problem?
- (2) What information is needed to solve the problem?
- (3) What analysis can be conducted to produce this information?
- (4) What experimental data are needed to conduct this analysis?
- (5) What must be done experimentally to obtain these data?

The statements of experimental purpose and objectives, definitions of analysis procedures and data requirements, and the description of experimental procedures that constitute an experimental design present the experimental plans in sufficient detail to permit evaluation of the merit of the experiment, estimation of expected costs, and estimation of the time that will be required to obtain the experimental results.

The experimental purpose expresses the application problem in terms of information that must be obtained experimentally and that already available. The experimental objectives describe the specific information that the experiment is expected to produce. Usually, the desired information relates to the expected performance of a component or material that may be obtained at some future date from a population of nominally similar items. Thus, the desired experimental results are often predictions of expected average performance and the expected range of performance variation resulting from inherent differences between nominally similar items selected from a production population. Often, these predictions must be based upon interpolations or extrapolations of the experimental data. Thus, another desired bit of information is the confidence that may be placed in these predictions.

The analysis procedure details the mathematical formulae, computational methods, and assumptions needed to transform the experimental data into component or material performance predictions, tolerance bands, and confidence statements. For many nuclear-radiation-effects experiments, it is believed that linear-regression-analysis techniques can be a powerful tool capable of adequately producing the information needed. However, utilization of this approach

requires careful and detailed planning involving a judicious blend of engineering and statistical insights.

The requirements for experimental data define "performance" and "environment" for the experiment. These are expressed in terms of parameters which are to be measured with specified accuracies and precisions. The performance and environmental parameters specified must be both sufficient for conducting the analysis procedures and compatible with the experimental measurement and control capabilities.

The experimental procedures detail the actual experimentation to be performed including the measurements of dependent and independent variables to be made and the methods to be used for maintaining control of experimental variables. Planning of these procedures should be conducted on the basis of identifying all potential sources of systematic experimental error and either eliminating or controlling them. Requirements for the exercise of human judgment should be minimized, and, where decision will be required, the alternatives and consequences should be documented.

It is concluded that the principles of experimental design are necessary to the planning of nuclear-radiation-effects experiments and that their use provides an approach that best assures efficient and effective experimentation. Documented experimental designs provide a basis for scheduling and budgeting a nuclear-radiation effects experimental program in a manner that optimizes both time and cost. The chances of conducting inconclusive or inappropriate experiments will be minimized by this approach.

The application of experimental design principles to nuclear-radiation-effects experiments is illustrated by example in Appendix A. Appendix B provides a numerical example of the analysis and results that might be obtained by conducting the experiment described in Appendix A.

REFERENCES

The following references are of interest with respect to designing nuclear-radiation-effects experiments:

- (1) Acton, F. S., Analysis of Straight-Line Data, John Wiley and Sons, Inc., New York (1959).
- (2) Beers, Y., Introduction to the Theory of Error, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts (1958)
- (3) Burington, R. S., and May, D. C., Jr., Handbook of Probability and Statistics With Tables, Handbook Publishers, Inc., Sandusky, Ohio (1953).
- (4) Cox, D. R., Planning of Experiments, John Wiley & Sons, Inc., New York (1958).
- (5) Dixon, W. J., and Massey, F. J., Jr., Introduction to Statistical Analysis, McGraw-Hill Book Company, Inc., New York (1957).
- (6) Jones, D. C., Fairand, B. P., Gruber, H. T., and Mace, A. E., "Radiation-Effects Experimental Procedures", Radiation Effects Information Center Report No. 35, Battelle Memorial Institute, Columbus, Ohio (1964).
- (7) Mace, Arthur E., Sample-Size Determination, Reinhold Publishing Corporation, New York (1964)
- (8) Owens, D. L., Handbook of Statistical Tables, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts (1962).

APPENDIX A

MODEL EXPERIMENTAL DESIGN FOR
NUCLEAR-RADIATION-EFFECTS EXPERIMENTS

APPENDIX A

MODEL EXPERIMENTAL DESIGN FOR
NUCLEAR-RADIATION-EFFECTS EXPERIMENTS

The preceding sections of this report have described the design of nuclear-radiation-effects experiments in general terms. To illustrate the specific application of the principles discussed, a model experimental design for nuclear-radiation-effects experiments is presented in the following paragraphs. This model experimental design pertains specifically to a linear displacement transducer intended for use in a high-flux nuclear-radiation environment. However, as well as demonstrating the application of the principles of experimental design to nuclear-radiation-effects experiments, this model should be useful as a guide in the preparation of experimental designs for other types of nuclear-radiation-effects experiments. The reactor system, transducer designations and reference documents employed in this model design are hypothetical.

DESIGN OF AN EXPERIMENT TO EVALUATE THE
EFFECTS OF NUCLEAR RADIATION ON
LINEAR DISPLACEMENT TRANSDUCER
PERFORMANCE IN THE REACTOR SYSTEMIntroduction

In order to analyze the capabilities and limitations of the reactor system design, it is necessary to obtain linear-displacement (position) measurements at points inside the pressure vessels of the prototype reactors. The general requirements for the instrumentation needed to obtain these data are given in the System Specifications.* The information available pertaining to candidate transducers including manufacturers specifications and the results of transducer design evaluations, laboratory tests, and radiation-screening tests are summarized in the System Data Book.** Evaluation of this information has led to the selection of a linear-variable-differential-transformer (LVDT)-type transducer as being most suitable for the application. This experiment is concerned with the evaluation of the performance to be expected for a commercially available LVDT transducer model in the high-flux nuclear-radiation environment associated with the intended reactor-system application.

-
- * The System Specifications are assumed to be a set of documents that would include descriptions of the transducer performance requirements for the desired application.
- **The System Data Book is assumed to be a set of documents that includes a summary of the transducer performance information already available.

Purpose

The purpose of this experiment is to

- (1) Establish, with reasonable confidence,* the probability that the XYZ Corporation linear displacement transducer, Model 001, will satisfy the reactor-system performance requirements specified in the System Specifications, Code No. Pt-10**
- (2) Determine the capabilities and limitations of performance for this transducer type over ranges of radiation flux and fast-neutron fluence that extend to at least 125 percent of the maximum values expected in the application with the ambient temperature maintained at $125 \pm 2^{\circ}\text{F}$, the no-load excitation voltage maintained at 5 ± 0.1 volts, and transducer displacements of 0, 50, and 100 percent of full travel
- (3) Identify those failure modes and mechanisms associated with such nuclear-radiation exposure where failure is defined as erratic changes in coil resistance during periods of constant radiation flux and/or observation of a coil-insulation resistance value less than 10 megohms.

Objectives

The available information concerning LVDT-type linear displacement transducers, including the results of nonirradiation tests of the XYZ Corporation Model 001 transducer, is contained in the System Data Book. This information shows that a major problem area with LVDT-type linear displacement transducers (LDT's) results from excessive and/or nonuniform internal temperatures caused by radiation heating. The high localized internal temperatures resulting from exposure to high-flux radiation environments have been observed to cause catastrophic degradation of insulating materials and/or localized melting in the LDT coil windings. Neutron radiation may cause LDT output errors because of changes in the phase shift from the primary to the secondary coil windings. These phase-shift changes may cause the transducer sensitivity to change and will produce a zero shift.

* For this experiment reasonable confidence is defined to be 99 percent.

**The reference defines the expected levels of radiation flux and neutron fluence for the application. (If this is not true in an actual case, these levels should be defined at this point in the experimental design.)

The design of the XYZ Corporation, Model 001 LDT, contains measures expected to correct these deficiencies. The general objective of this experiment is to evaluate the effectiveness of these LDT design changes with respect to the LDT performance in the high-flux radiation environment as related to the requirements specified for applications in the Reactor System.

Specific experimental objectives are

- (1) Determine with 99 percent confidence the probability that coil-winding temperatures in the XYZ Corporation Model 001 LDT, will not exceed the manufacturer's specified maximum of 900 F during exposure to the radiation flux expected in the Reactor System application
- (2) Determine with 99 percent confidence the probability that neutron-radiation induced changes in LDT sensitivity do not exceed the limits specified as acceptable for the Reactor System application in the System Specifications
- (3) Determine with 99 percent confidence the probability that neutron-irradiation induced increase in LDT Zero-position output signal (zero shift) will be correctable with a residual error of less than ± 2 percent of the full-scale transducer output
- (4) Establish with 99 percent confidence the expected neutron-irradiation induced increase in LDT zero-position output signal (zero shift) as a function of neutron fluence and the population 99 percentile range over a range that includes the neutron fluence expected in the Reactor System application. (Plot of expected value of zero shift versus neutron fluence with 99 percent confidence band and 99 percent tolerance band)
- (5) Establish with 99 percent confidence the expected coil-winding temperature as a function of radiation-heating rate and the population 99 percentile range for a flux range that includes the range expected in the Reactor System application (Plot of expected value of average coil-winding temperature versus gamma heating rate with 99 percent confidence band and 99 percent tolerance band)
- (6) Establish with 99 percent confidence the expected relationship of LDT sensitivity to neutron fluence and the population 99 percentile range for a range that includes the neutron fluence expected in the Reactor System application (Plot of expected value of LDT sensitivity versus neutron fluence with 99 percent confidence band and 99 percent tolerance band)

- (7) Determine the modes and mechanisms of LDT failures that are observed because of the irradiation. (Description of failure symptoms and the observations made during dissection of failed specimens. Discussion of mechanisms believed to have caused the failures).

Analysis Procedure

Development of Mathematical Models, Definition of Parameters, and Basic Assumptions

The primary dependent variables for the experiment are LDT internal temperature, zero-shift, and sensitivity. The experimental independent variables are gamma flux* and fast-neutron fluence. Ambient temperature and transducer excitation voltage are important environmental factors.

The actual control of the radiation flux is achieved by controlling the reactor power level. However, the radiation flux experienced by each experimental sample will depend upon the location of the sample within the test facility. Thus, the relationship between radiation flux and reactor power level must be established for each sample LDT in the experiment. It is expected that the reactor facility can provide the results of flux mapping studies which can be used to calculate the expected radiation flux for any point in the test area as a function of reactor power level.

The computations of expected radiation flux are checked experimentally by the dosimetry packets included in the experiment. Although the gamma flux cannot be verified in this way, some assurance of the validity of the computed values is gained by computing the neutron fluence for each dosimetry mounting point using flux mapping data supplied by the facility and then comparing this value with the dosimetry data following the experiment. Also, the experimental results themselves may indicate the variation of radiation flux from specimen to specimen.

The relating of the results of the analysis of experimental data to the questions posed by the experimental purpose and objectives involves the inference of characteristics of the transducer population from which the specimens are taken. On the basis of these inferred characteristics of the population, predictions are made about the expected performance of the transducers selected from this population for use in the Reactor System. The inferences and predictions are necessarily based on the following assumptions:

- (1) The transducers employed as experimental specimens as well as those selected for Reactor System use are representative samples of a population of nominally similar transducers fabricated by a controlled repetitive manufacturing process.

*Gamma flux is the primary contributor to material heating caused by radiation. The contribution of neutron flux is negligible for the case being considered.

- (2) Any differences between the nonnuclear portion of the Reactor System environment and that of the experiment which are not already taken into account in the analysis produce insignificant effects.

The validity of any inferences or predictions made will, of course, depend upon the validity of these assumptions.

The analysis procedure is required to predict values expected for certain parameters of XYZ Corporation Model 001 linear displacement transducers. The values of these parameters are determined by measurements made during the exposure of specimen transducers to various radiation flux levels or various integrated neutron exposures. In the following discussion, y will denote an arbitrary value of the parameter being measured and x will denote an arbitrary integrated neutron exposure or radiation-flux level. The following assumptions define the mathematical regression model for the experiment being considered:

- (1) For each x in the range of admissible experimental values there is a random variable Y_x whose distribution depends on x ; in particular, if μ_Y denotes the mean of the distribution of the random variable Y then the model asserts that

$$\mu_{Y_x} = \alpha_0 + \alpha_1 x.$$

In other words, the line

$$y = \alpha_0 + \alpha_1 x$$

passes through the means of the distributions of the Y_x 's.

- (2) For each x , Y_x is normally distributed with mean μ_{Y_x} and variance σ^2 and Y_{x_1} and Y_{x_2} , $x_1 \neq x_2$, are statistically independent random variables.

Let x_i , $i = 1, \dots, n$ be n specified values of integrated neutron exposure on radiation flux and let y_i denote the observed value of Y_{x_i} .

It is assumed that any random errors in x are so small that they may be ignored when compared with random errors in y , so that the values of x_i specified are actually the values observed. Thus, given the data (y_i, x_i) , $i = 1, \dots, n$, the least squares estimates of α_1 and α_0 are, respectively,

$$A_1 = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)^2}$$

$$A_0 = \frac{\sum y_i - A_1 \sum x_i}{n} = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{n \sum x_i^2 - (\sum x_i)^2},$$

where $\sum$ means $\sum_{i=1}^n$. The predicted y -value, $\hat{y}_x$ for a specified x -value, x , is

$$\hat{y}_x = (A_0) + (A_1)_x.$$

If the transducer specimens are nominally identical and are exposed to identical levels of radiation, the regression lines obtained for each sample should not be significantly different. However, if the transducers are not nominally identical or are exposed to different levels of radiation, then the regression lines may be different. For the intended experiment, an effort will be made to select nominally identical specimens as representatives of a production population of linear displacement transducers. However, experience has shown that the thermal characteristics of the specimens may vary and also the radiation flux will probably be different for each specimen owing to the problems of locating the specimens with respect to the radiation source.

For the portion of the experiment concerned with transducer internal temperature, before irradiation it will be necessary to establish the relationships between coil winding resistance and internal temperature for each specimen intended to be used in the irradiation experiment. From coil winding resistance measurements, $(R')_{\lambda i}$, obtained when the transducer specimens are heated to an equilibrium temperature, $(T')_{\lambda i}$, in an oven, where λi denotes the i th measurement of the λ th specimen, $\lambda = 1, 2, \dots, K$, the least-squares estimates of the regression coefficients $(A'_0)_\lambda$ and $(A'_1)_\lambda$ can be computed:

$$(A'_0)_\lambda = \frac{\left[\sum_{i=1}^n (T')_{\lambda i} \right] \left[\sum_{i=1}^n (R')_{\lambda i}^2 \right] - \left[\sum_{i=1}^n (R')_{\lambda i} \right] \left[\sum_{i=1}^n (R')_{\lambda i} (T')_{\lambda i} \right]}{n \left[\sum_{i=1}^n (R')_{\lambda i}^2 \right] - \left[\sum_{i=1}^n (R')_{\lambda i} \right]^2}$$

$$(A'_1)_\lambda = \frac{n \left[\sum_{i=1}^n (R')_{\lambda i} (T')_{\lambda i} \right] - \left[\sum_{i=1}^n (T')_{\lambda i} \right] \left[\sum_{i=1}^n (R')_{\lambda i} \right]}{n \left[\sum_{i=1}^n (R')_{\lambda i}^2 \right] - \left[\sum_{i=1}^n (R')_{\lambda i} \right]^2},$$

where n denotes the total number of measurements for the λ th one of the K specimens. Thus, for the λ th specimen, the estimate of the internal temperature corresponding to a coil winding resistance $R_{\lambda i}$ is

$$\hat{T}_{\lambda i} = (A'_0)_{\lambda} + (A'_1)_{\lambda} R_{\lambda i}.$$

Now if the specimens intended for irradiation are immersed in a cooling bath of constant temperature (T_A'') and electrical power $W_{\lambda i}$ is applied to the coil windings with the applied voltage $V_{\lambda i}$ and current $I_{\lambda i}$ being measured for each power after equilibrium is reached, a set of n values* of coil winding resistance $(R'')_{\lambda i}$ corresponding to the $W_{\lambda i}$ may be computed:

$$(R'')_{\lambda i} = \frac{V_{\lambda i}}{I_{\lambda i}}$$

$$W_{\lambda i} = (V_{\lambda i}) (I_{\lambda i}) ; i = 1, 2, \dots, n.$$

Corresponding internal temperatures $(T'')_{\lambda i}$ may now be computed:

$$(T'')_{\lambda i} = (A'_0)_{\lambda} + (A'_1)_{\lambda} R'_{\lambda i} ,$$

using the results of the calibration experiment described above. For each specimen, an average thermal resistance, H_{λ} , is now computed by:

$$H_{\lambda} = \frac{\sum_{i=1}^n \left[\frac{(T'')_{\lambda i} - (T_A'')}{W_{\lambda i}} \right]}{n} .$$

Selecting the largest average thermal resistance observed for the specimens and denoting this factor H^* , a relative thermal characteristic correction factor $Q_{\lambda} = \frac{H_{\lambda}}{H^*}$ is computed for each specimen.

It is seen that before the irradiation is initiated, there will be available for each specimen included in the experiment numerical data for the factors: $(A'_0)_{\lambda}$, $(A'_1)_{\lambda}$ and Q_{λ} . The irradiation experiment will produce for each measurement of each specimen values of the coil winding resistance, $R_{\lambda i}$, the specimen ambient temperature, $(T_A)_{\lambda i}$, and the reactor power level, $P_{\lambda i}$. These data are reduced and corrected for thermal characteristics and ambient temperature fluctuations by the relation:

$$y_{\lambda i} = Q_{\lambda} (A'_0)_{\lambda} + (1 - Q_{\lambda}) (T_A)_{\lambda i} + [Q_{\lambda} (A'_1)_{\lambda}] R_{\lambda i} ; i = 1, 2, \dots, n, \lambda = 1, 2, \dots, K,$$

where it is assumed that the equivalency between internal heat generated by applying electrical power to the coil windings and the heat generated by exposure to a given radiation flux is the same for all specimens. The reduced data then are a set of n points $(y_{\lambda i}, P_{\lambda i})$ for each of the K specimens.

The set of reduced data will still include effects resulting from nonuniformity of radiation flux within the irradiation facility and/or the different specimen locations employed. Before attempting to remove this effect from the experimental data, it is first desirable to establish that a significant radiation effect exists. This may be

*It may be sufficient to make measurements at only one temperature; that is, it seems reasonable to expect the thermal resistance to be nearly constant over the temperature range of interest. In this case, n for the calculations becomes one.

accomplished by considering the hypothesis that the slopes of the regression lines are not significantly different from zero. Letting K denote the number of specimens and λ denote a generic integer in the interval $1 \leq \lambda \leq K$, first estimate the regression line for each specimen:

$$(y_x)_\lambda = (A_0)_\lambda + (A_1)_\lambda x ,$$

where x denotes an arbitrary value of the independent variable (at this point, x denotes reactor power level). $(A_0)_\lambda$ and $(A_1)_\lambda$ are, respectively, estimates of $(\alpha_0)_\lambda$ and $(\alpha_1)_\lambda$. It is desired to test the hypothesis that $(\alpha_1)_\lambda = 0$ against one of the three alternatives (1) $(\alpha_1)_\lambda < 0$, (2) $(\alpha_1)_\lambda \neq 0$, or (3) $(\alpha_1)_\lambda > 0$. The choice of which alternative is to be considered depends on the experimenters a priori knowledge or opinion of the physical phenomenon being investigated, and this choice must be made before the data are observed. For example if the physical hypothesis is such that it is not possible for $(\alpha_1)_\lambda$ to be greater than zero, then Alternative (1) would apply.

In order to test the hypothesis that $(\alpha_1)_\lambda = 0$ against one of the alternatives stated above, use the fact that

$$\frac{(A_1)_\lambda - (\alpha_1)_\lambda}{S_{y \cdot x}} \sqrt{(S_x^2)_\lambda (n_\lambda - 1)} \quad (1)$$

has Student's t -distribution with $n_\lambda - 2$ degrees of freedom where

$$(S_x^2)_\lambda = \frac{\sum [x_{\lambda i} - \bar{x}_\lambda]^2}{n_\lambda - 1} = \frac{(n_\lambda) \sum (x_{\lambda i})^2 - [\sum x_{\lambda i}]^2}{(n_\lambda) (n_\lambda - 1)}$$

$$(S_{y \cdot x})_\lambda = \sqrt{\left(\frac{n_\lambda - 1}{n_\lambda - 2} \right) \left\{ 1 - (A_1)_\lambda^2 \left[\frac{(S_x^2)_\lambda}{(S_y^2)_\lambda} \right] \right\} (S_y^2)_\lambda}$$

$$(S_y^2)_\lambda = \frac{\sum [y_{\lambda i} - \bar{y}_\lambda]^2}{n_\lambda - 1} = \frac{(n_\lambda) \sum (y_{\lambda i})^2 - [\sum y_{\lambda i}]^2}{(n_\lambda) (n_\lambda - 1)} .$$

If the alternative is $(\alpha_1)_\lambda < 0$, we reject the hypothesis, $(\alpha_1)_\lambda = 0$, at level of significance γ , $0 < \gamma < 1$, in favor of the alternative if $(A_1)_\lambda < -B_\gamma$ where B_γ is determined by

$$P \left\{ (A_1)_\lambda < -B_\gamma \mid (\alpha_1)_\lambda = 0 \right\} = \gamma . \quad (2)$$

The above equality is read: The probability that $(A_1)_\lambda < -B_\gamma$, given that $(\alpha_1)_\lambda = 0$, equals γ . From Expressions (1) and (2) one finds that

$$B_\gamma = \frac{S_{y \cdot x}}{\sqrt{(S_x^2)_\lambda (n_\lambda - 1)}} t_{1-\gamma}^{(n_\lambda - 2)} ,$$

where $t_{1-\gamma}^{(n_\lambda - 2)}$ is the 100 $(1 - \gamma)$ th percentile of the t -distribution with $n_\lambda - 2$ degrees of freedom.

For the alternative $(\alpha_1)_\lambda > 0$, the rejection region (of level γ) is given by

$$(A_1)_\lambda > B_\gamma ,$$

and for the two-sided alternative $(\alpha_1)_\lambda \neq 0$, the region is given by

$$\left| (A_1)_\lambda \right| > B_{\frac{\gamma}{2}} .$$

If the test discussed above indicates that there exists a significant radiation effect for at least one specimen, it is then justifiable to proceed with the analysis.

As previously indicated, it is expected that an assumption of uniform radiation flux for all the specimens would not be justified. It is necessary to remove the effect of nonuniformity of radiation flux from the results of the experiment. This may be accomplished by selecting one specimen as a reference unit and making linear transformations on the reactor power levels associated with the remaining specimens to yield effective reactor power levels. Data obtained from the radiation facility giving the relation between radiation flux and reactor power for the reference specimen location may then be used to compute effective radiation flux for each measurement of every specimen. This complete procedure is accomplished in a stepwise manner as follows:

- (1) Letting (λ) denote one of the specimens, $(\lambda) = 1, 2, \dots, q, \dots, K$, where the q^{th} specimen is the one selected as the reference, there is a nominal reactor power level, P , for each data point measured. These data are used to compute $(ERP)_{\lambda i}$ = the effective reactor power level for the i^{th} measurements of unit (λ) from the relation:

$$(ERP)_{\lambda i} = \left[\frac{(A_1)_\lambda}{(A_1)_q} \right] P_i ; \lambda = 1, 2, \dots, K .$$

- (2) Using the data provided by the reactor facility determine the radiation flux per unit of reactor power, ϕ_q , at the location of the reference sample, q . Since the radiation flux at a specified location is directly proportional to reactor power, the effective radiation flux, $x_{\lambda i}$ for each sample data point may be computed by the relation:

$$x_{\lambda i} = (\phi_q) (ERP)_{\lambda i} .$$

- (3) Normalize the specimen temperature data, $y_{\lambda i}$, to give a common zero power temperature. (This temperature may be arbitrarily selected but should probably be reasonably close to the nominal facility ambient temperature.) The normalization is accomplished by computing:

$$y'_{\lambda i} = y_{\lambda i} - (A_o)_\lambda + T ,$$

where T denotes the common zero power temperature. It is seen that

there will be $n = \sum_{\lambda=1}^K n_\lambda$ values of $y'_{\lambda i}$.

- (4) Tabulate the $\left(n = \sum_{\lambda=1}^K n_{\lambda}\right)$ values of $(y'_{\lambda i}, x_{\lambda i})$ in order of increasing $x_{\lambda i}$,

intermixing the values for the various specimens.* (n denotes the total number of measurements.) These pooled data will now be denoted

$$(y_i, x_i), \text{ where } i = 1, 2, \dots, n, \text{ and } n = \sum_{\lambda=1}^K n_{\lambda}.$$

- (5) Utilize the model and the pooled data of Step (4) to obtain least-squares estimates A_1 and A_0 of the coefficients α_1 and α_0 , thus obtaining a single regression line for all the data:

$$\hat{y}_x = A_0 + A_1 x.$$

It is recognized that the experiment actually involves two distinct parts. One part is concerned with the effect of radiation flux; the other is concerned with the effect of fast - neutron fluence. Thus, there will be a separate set of data for each parameter for each part of the experiment. Having pooled the data for each part of the experiment according to the above method, in each case a single regression line is obtained, employing the basic regression model discussed previously. The subsequent analysis is described in the following paragraphs and will be the same for both parts of the experiment.

To evaluate the regression line as a predictor of the transducer parameter value, confidence bands for the true line are generated from the data as follows. The interval $[Y_c(x), Y^c(x)]$ is a P_c -per cent confidence interval for the mean value of the ordinate, μ_{y_x} , determined by the abscissa, x . This interval is defined by:

$$Y_c(x) = \hat{y}_x - \left[t_{P_c}^{(n-2)} \right] \sqrt{MS_{y \cdot x}^2} \leq \mu_{y_x} \leq \hat{y}_x + \left[t_{P_c}^{(n-2)} \right] \sqrt{MS_{y \cdot x}^2} = Y^c(x)$$

where**

$$M = \left[\frac{1}{n} + \frac{(x - \bar{x})^2}{(n-1)S_x^2} \right];$$

and $t_{P_c}^{(n-2)}$ is the $\left(\frac{100 + P_c}{2} \right)^{\text{th}}$ percentile of the Student's t-distribution with $n - 2$ degrees of freedom; i. e., $t_{P_c}^{(n-2)}$ is the number such that

$$P_r \left\{ -t_{P_c}^{(n-2)} < T < t_{P_c}^{(n-2)} \right\} = P_c = \frac{P_c}{100},$$

where T is a random variable which has Student's t-distribution with $n - 2$ degrees of freedom.

Next, for a given percentage, P_T , and a given confidence coefficient, ϵ , a tolerance interval $[Y_T(x), Y^T(x)]$ is determined from the data as a function of x so that for any selected value of x it can be stated with confidence $(1 - \epsilon)$ that the interval

*The ordering of the data is not important to the subsequent calculations but is convenient for plotting the data points.

**Computational formulas for S_x^2 , $S_{y \cdot x}$ and other statistical values are given in Appendix B.

$[Y_T(x), Y^T(x)]$ includes P_T per cent of the population of ordinates, y . Approximate values for $Y_T(x)$ and $Y^T(x)$ are computed from:

$$Y_T(x) = \hat{y}_x - B S_{y \cdot x}$$

and

$$Y^T(x) = \hat{y}_x + B S_{y \cdot x}$$

where

$$B = Z_{P_T/2} \left\{ 1 + \frac{M}{2} - \frac{[2(Z_{P_T/2})^2 - 3]M^2}{24} \right\} \sqrt{F_{\epsilon}(\infty, n-2)}$$

$$= \left[\left(1 + \frac{M}{2} + \frac{M^2}{8} \right) Z_{P_T/2} - \left(\frac{M^2}{12} \right) Z_{P_T/2}^3 \right] \sqrt{F_{\epsilon}(\infty, n-2)} ;$$

$Z_{P_T/2}$ is the number such that $P_r \{ Z < Z_{P_T/2} \} = \frac{1 + P_T}{2}$ where Z is a random variable with the standard normal distribution and $P_T = \frac{P_T}{100}$; $F_{\epsilon}(\infty, n-2)$ is the $100(1 - \epsilon)^{th}$ percentile of the F -distribution with ∞ and $(n - 2)$ degrees of freedom (i. e., F_{ϵ} is a number such that $\int_0^{F_{\epsilon}} [h_{m_1}, m_2(F)] dF = 1 - \epsilon$, where $[h_{m_1}, m_2(F)]$ is the

probability density function of the F -distribution with, in this case, $m_1 = \infty$ and $m_2 = n - 2$ degrees of freedom for the desired confidence coefficient (usually $\epsilon = 0.01$ or $\epsilon = 0.05$). The value of $F_{\epsilon}(\infty, n - 2)$ is obtained from tables of the F -distribution. The confidence level is expressed as a percentage given by $100(1 - \epsilon)$ and is usually 99 or 95 per cent, corresponding to the usual values of ϵ given above. A 99 per cent confidence level ($\epsilon = 0.01$) will be used in this experiment.

The Reactor System requirements and physical considerations both impose limits on the acceptable values of the transducer parameters. Thus the j^{th} of m transducer parameters considered in the experiment may be required to have a value less than a specified maximum, greater than a specified minimum, or between a specified minimum and a specified maximum. If $[(P_s)_j]^*$ denotes the probability that the parameter value will be less than the specified maximum, $(Y_j)^*$, and $[(P_s)_j]_*$ denotes the probability that the parameter value will be greater than the specified minimum, $(Y_j)^*$, the probability, $(P_s)_j$, that the Reactor System requirements are met is given by

$$(P_s)_j = \theta_1 [(P_s)_j]^* + \theta_2 [(P_s)_j]_* + \theta_3 ,$$

where

$$\theta_1 = 0.5, \theta_2 = 0, \theta_3 = 0.5 \text{ if only a maximum limit is specified}$$

$$\theta_1 = 0, \theta_2 = 0.5, \theta_3 = 0.5 \text{ if only a minimum limit is specified}$$

$$\theta_1 = 0.5, \theta_2 = 0.5, \theta_3 = 0 \text{ if both maximum and minimum limits are specified.}$$

The values of $[(P_s)_j]^*$ and/or $[(P_s)_j]_*$ are obtained by substituting the value of the maximum limit $(Y_j)^*$ and/or the minimum limit $(Y_j)_*$ into the tolerance limit equation and solving for the value of $[Z_{(P_s)j/2}]^*$ and/or $[Z_{(P_s)j/2}]_*$ as appropriate.

For the internal temperature, a maximum limit can be established from the experimental data by observing the temperatures (indicated on the regression line plot of the pooled data) at which the transducers appeared to fail. The maximum acceptable internal-temperature limit is then set at a value sufficiently less than this to provide a reasonable margin of safety. If no such transducer failures are observed, the maximum acceptable internal temperature is then the maximum temperature observed or a lesser value as may be deemed desirable on the basis of supplemental data. In this experiment, engineering considerations have established a maximum acceptable limit for internal temperature of 900 F as indicated by the winding resistance of the linear displacement transducers.

To determine the values of $[Z_{(P_s)j/2}]^*$ and/or $[Z_{(P_s)j/2}]_*$, the tolerance limit equation is rewritten as follows:

$$\left[Z_{(P_s)j/2} \right]^3 + a \left[Z_{(P_s)j/2} \right] + b = 0,$$

where

$$a = - \left[\frac{12 \left(1 + \frac{M}{2} + \frac{M^2}{8} \right)}{M^2} \right]$$

$$b = \frac{12(Bj)^*}{M^2 \sqrt{F_{\epsilon, (\infty, n_j - 2)}}$$

$$\text{or } b = \frac{12(Bj)_*}{M^2 \sqrt{F_{\epsilon, (\infty, n_j - 2)}}$$

$$(Bj)^* = \frac{(Y_j)^* - (\hat{y}_x)_j}{(S_{y \cdot x})_j}$$

$$\text{or } (Bj)_* = \frac{(y_x)_j - (Y_j)_*}{(S_{y \cdot x})_j}$$

Before attempting to solve this equation, the following test should be performed. Compute the value of M for the value of x_j specified in the Reactor System requirements corresponding to $(Y_j)^*$ and $(Y_j)_*$. Next compute the values of $(Bj)^*$ and $(Bj)_*$ and compute:

$$C = (5.13) \left(1 + \frac{M}{2} + \frac{M^2}{8} \right) - (7.78)M^2.$$

If $(Bj)^* \geq C$, $[(P_s)_j]^* \geq 0.99999$ and if $(Bj)_* \geq C$, $[(P_s)_j]_* \geq 0.99999$ at a confidence level $(1 - \epsilon) = 0.99$. Thus it is a reasonable approximation to set $[(P_s)_j]^* \doteq 1$ or $[(P_s)_j]_* \doteq 1$ and proceed with the analysis of other parameters.

If $(B_j)^* < C$ or $(B_j)_* < C$, it is desirable to compute the value of $[(P_s)_j]^*$ or $[(P_s)_j]_*$ by solving for $[Z_{(P_s)j/2}]^*$ or $[Z_{(P_s)j/2}]_*$. The first step in this computation utilizes the values of a and b defined above to compute*:

$$D = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3.$$

If $D > 0$,

$$\left[Z_{(P_s)j/2}\right] = \left[2\sqrt{\frac{a}{3}}\right] \operatorname{ctn} 2\phi,$$

where

$$2\phi = 2 \tan^{-1} \sqrt[3]{\tan \psi}$$

$$\psi = 1/2 \operatorname{ctn}^{-1} \sqrt{\frac{(b/2)^2}{(a/3)^3}}.$$

If $D = 0$, there are two possible values of $[Z_{(P_s)j/2}]$ given by

$$\left[Z_{(P_s)j/2}\right]_1 = -2 \sqrt{-\left(\frac{a}{3}\right)}$$

$$\left[Z_{(P_s)j/2}\right]_2 = \sqrt{-\left(\frac{a}{3}\right)}.$$

If $D < 0$ (which is the most likely case), there are three possible values of $[Z_{(P_s)j/2}]$ given by:

$$\left[Z_{(P_s)j/2}\right]_1 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)}\right] \cos \left(\frac{\phi}{3}\right)$$

$$\left[Z_{(P_s)j/2}\right]_2 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)}\right] \cos \left(\frac{\phi}{3} + 120^\circ\right)$$

$$\left[Z_{(P_s)j/2}\right]_3 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)}\right] \cos \left(\frac{\phi}{3} + 240^\circ\right),$$

where

$$\phi = \cos^{-1} \sqrt{\frac{(b/2)^2}{-(a/3)^3}}.$$

The appropriate value of $[Z_{(P_s)j/2}]$ is the one for which $0 < [Z_{(P_s)j/2}] < 4.3$. The tables of the normal distribution function are now employed to determine $(P_s)_j^*$ corresponding to $[Z_{(P_s)j/2}]^*$ and $[(P_s)_j]_*$ corresponding to $[Z_{(P_s)j/2}]_*$. The value of $(P_s)_j$ may now be computed as described previously.

*For this and subsequent calculations, the confidence level $(1 - \epsilon) = 0.99$ should be employed to obtain $\sqrt{F_{\epsilon}(\infty, n-2)} = \sqrt{F_{0.01}(\infty, n-2)}$.

Once all of the m values of $(P_s)_j$ have been established for the m transducer parameters considered in the experiment, the combined probability of the transducer's meeting the Reactor System requirements, P_s , may be computed. This probability is given by*:

$$P_s = \prod_{j=1}^m (P_s)_j,$$

where $\prod_{j=1}^m$ denotes taking the product of all m values of $(P_s)_j$. The confidence level associated with this value of P_s is $(0.99)^m$. The following statement may now be made.

"For transducers nominally similar to those tested in this experiment, it is $[100 (0.99)^m]$ percent certain that the probability is $[(100) P_s]$ percent that all of the m parameters considered will have values within the limits specified by the Reactor System requirements for the specified environment, assuming that the nonnuclear environmental conditions of the experiment are a realistic simulation of the corresponding system environments."

A similar statement with a 99 percent confidence level and probability $(P_s)_j$ can be made for each of the m individual parameters considered in the experiment. These statements will show the relative importance of the parameters with respect to the over-all capability of the transducer in the Reactor System application, thus guiding attention to any critical problem areas that may exist.

To establish the maximum radiation flux that can be tolerated with respect to radiation-heating-induced internal temperature, the graph of the upper 99 percent tolerance limit (with 99 percent confidence level) is used to determine $x(\max)$, the value of radiation flux for which $Y^T(x) = 900$ F. It may then be stated: " $x(\max)$ is the maximum radiation flux that can be tolerated with respect to radiation-induced internal temperature where the risk that the maximum acceptable internal temperature will be exceeded is no greater than 1 percent at the 99 percent confidence level." The validity of this statement is, of course, dependent on the validity of the previously stated assumptions with respect to the population of transducers and the similarity of the test environment to the Reactor System environment.

In evaluating the performance of the linear displacement transducers with respect to fast-neutron fluence, it is, of course, possible that no significant effects will be observed. If significant effects are observed, the analysis described above will provide the information needed to make the following statement.

"For transducers nominally similar to those tested in this experiment, it is $[(100) (0.99)^m]$ percent certain that the probability is $[(100) P_s]$ percent that all of the m parameters considered will have values within the limits specified by the Reactor System requirements for the specified expected fast-neutron fluence, assuming that the nonnuclear environmental conditions of the experiment are a realistic simulation of the corresponding system environment."

The maximum fast-neutron fluence that can be tolerated by the transducers is established by referring to the 99 percent tolerance limit plots (99 percent confidence level) for each parameter. For each parameter, determine the

*This provides a conservative useful value for P_s .

fast-neutron fluence, $x(\max)$, at which the tolerance value and the Reactor System requirement limit are coincident. The smallest of these values, $[x(\max)]'$ is the value used for the statement:

" $[x(\max)]'$ is the maximum fast-neutron fluence that can be tolerated with respect to transducer performance where the risk of at least one of the transducer parameters exceeding the limits of the Reactor System requirements is no greater than 1 percent at the 99 percent confidence level."

It is seen that the above statements resulting from the analysis will answer the questions associated with the experimental purpose and objectives. Following is a summary of the analysis presented as a stepwise procedure.

Summary of Analysis Procedure by Steps

The analysis of data resulting from the experimental irradiation of linear displacement transducers selected as potentially suitable for use in the Reactor System application will proceed as follows:

- (1) Tabulate the data obtained for each parameter of each specimen transducer with the corresponding observed value of reactor power level, $P_{\lambda i}$, given by the reactor-power-level computer or other reactor instrumentation or with the corresponding fast-neutron fluence as appropriate.
- (2) Obtain from the preirradiation studies of each specimen values of $(A'_0)_\lambda$, $(A'_1)_\lambda$, and Q_λ and reduce the internal temperature data $[R_{\lambda i}, (T_A)_{\lambda i}, P_{\lambda i}]$ by the relation:

$$Y_{\lambda i} = Q_\lambda (A'_0)_\lambda + (1 - Q_\lambda) (T_A)_{\lambda i} + Q_\lambda (A'_1)_\lambda R_{\lambda i}; \lambda = 1, 2, \dots, K; \\ i = 1, 2, \dots, n.$$

Perform appropriate data reduction on other measured parameters.

- (3) For each parameter of each specimen, compute least-square estimates of the regression coefficients:

$$(A_1)_\lambda = \frac{n \left(\sum_{i=1}^n x_i y_i \right) - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right)}{n \left(\sum_{i=1}^n x_i^2 \right) - \left(\sum_{i=1}^n x_i \right)^2}$$

$$(A_0)_\lambda = \frac{\left(\sum_{i=1}^n y_i \right) \left(\sum_{i=1}^n x_i^2 \right) - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n x_i y_i \right)}{n \left(\sum_{i=1}^n x_i^2 \right) - \left(\sum_{i=1}^n x_i \right)^2}$$

where x_i denotes the observed value of the independent variable, reactor power level, or integrated reactor power as appropriate.

- (4) For each parameter, predetermine which is appropriate of the following possibilities: $(A_1)_\lambda < 0$; $(A_1)_\lambda \neq 0$; $(A_1)_\lambda > 0$ where $\lambda = 1, 2, \dots, K$, identifies the specimen being considered. Then conduct the appropriate test:

$$(A_1)_\lambda < -B_\gamma; \left| (A_1)_\lambda \right| > B_{\gamma/2}; \text{ or } (A_1)_\lambda > B_\gamma \text{ where } B_\gamma = \frac{S_{y \cdot x}}{\sqrt{S_x^2 (n-1)}} t_{1-\gamma}^{(n-2)}$$

$$\text{and } B_{\gamma/2} = \frac{S_{y \cdot x}}{\sqrt{S_x^2 (n-1)}} t_{1-\gamma/2}^{(n-2)}. \text{ If the test shows the effect is not significant}$$

at the γ significance level, a statement to this effect is made and the analysis of the parameter is terminated. Otherwise proceed to Step (5).

- (5) (a) For the portion of the experiment concerned with internal temperature, select one specimen to be used as a reference. This should usually be the one located closest to the reactor core and showing the largest apparent effect of radiation. Also select an arbitrary zero-power temperature, T , which should be approximately equal to the nominal facility ambient temperature. For each data point of each parameter of each of the K specimens compute

$$ERP_{(\lambda)i} = \left[\frac{(A_1)_\lambda}{(A_1)_q} \right] P_{(\lambda)i}; \lambda = 1, 2, \dots, K$$

where $(A_1)_q$ is the value obtained for the referenced specimen. Then compute $x_i = (\phi_q) (ERP_{(\lambda)i})$, ϕ_q being the radiation flux per megawatt reactor power at the location of the referenced specimen. Normalize the temperature data, $y_{\lambda i}$, by computing $y_i = y_{\lambda i} - (A_0)_\lambda + T$. Tabulate the paired data values (y_i, x_i) in order of increasing x_i .

- (b) For the portion of the experiment concerned with **fast-neutron**

fluence, compute $x_i = \int_0^t \phi_i dt$. This may be approximated by

$$x_i = \sum_{t=0}^t \left[\frac{(\phi_i)_t + (\phi_i)_{(t-1)}}{2} \right] (\Delta t),$$

where Δt is the time interval between the observations of $(\phi_i)_{(t-1)}$ and $(\phi_i)_t$. The final value of x_i computed for each specimen for the time of termination of reactor operation is compared with the **fast-neutron** fluences determined from the dosimetry attached to each specimen. Each value of x_i is adjusted proportionally to give agreement between the final x_i value and the dosimetry data.

- (6) The values of x_i obtained in Step (5) are employed to compute the least-squares estimates of the pooled-data regression coefficients using the formulas of Step (3). (The Subscripts, λ , do not apply in this case).

- (7) For each parameter compute the P_c -percent confidence interval for the mean value of the ordinate, μ_x , determined by the abscissa, x , using the formulas

$$Y_c(x) = \hat{y}_x - \left[t_{p_c}^{(n-2)} \right] \sqrt{M S_{y \cdot x}^2} \leq \mu_{y_x} \leq \hat{y}_x + \left[t_{p_c}^{(n-1)} \right] \sqrt{M S_{y \cdot x}^2} = Y^c(x),$$

$$\text{where } p_c = \frac{P_c}{100}.$$

- (8) For each parameter compute the P_T -percent tolerance interval for a confidence level $(1 - \epsilon)$, using the formulas:

$$Y_T(x) = \hat{y}_x - B S_{y \cdot x}$$

$$Y^T(x) = \hat{y}_x + B S_{y \cdot x},$$

where

$$B = \left[\left(1 + \frac{M}{2} + \frac{M^2}{8} \right) Z_{p_{T/2}} - \left(\frac{M^2}{12} \right) Z_{p_{T/2}}^3 \right] \sqrt{F_{\epsilon}(\infty, n-2)},$$

and $p_T = \frac{P_T}{100}$. The same confidence level should be used for each parameter value $(1 - \epsilon) = 0.99$ being recommended.

- (9) Compute $C = (5.13) \left(1 + \frac{M}{2} + \frac{M^2}{8} \right) - (7.78)M^2$ and compute for each parameter:

$$(B_j)^* = \frac{(Y_j)^* - (\hat{y}_x)_j}{(S_{y \cdot x})_j} \quad \text{and} \quad (B_j)_* = \frac{(\hat{y}_x)_j - (Y_j)_*}{(S_{y \cdot x})_j},$$

where $(Y_j)^*$ and $(Y_j)_*$ are, respectively, the maximum and minimum limits for the parameter value specified by the Reactor System requirements and $(\hat{y}_x)_j = (A_0) + (A_1)x_j$; (A_0) and (A_1) being the regression coefficients computed in Step (6) and x_j being the flux or exposure value specified in the Reactor System requirements. Test $(B_j)^* \geq C$ and $(B_j)_* \geq C$ and if so set $[(P_s)_j]^*$ and/or $[(P_s)_j]_* = 1$. If $(B_j)^* < C$ or $(B_j)_* < C$ compute the value of $[(P_s)_j]^*$ or $[(P_s)_j]_*$ as follows:

$$(a) \text{ Compute } a = - \left[\frac{12 \left(1 + \frac{M}{2} + \frac{M^2}{8} \right)}{M^2} \right] \text{ and}$$

$$b = \frac{12(B_j)^*}{M^2 \sqrt{F_{\epsilon}(\infty, n-2)}} \quad \text{or} \quad b = \frac{12(B_j)_*}{M^2 \sqrt{F_{\epsilon}(\infty, n-2)}}.$$

(b) Compute $D = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3$.

(c) If $D > 0$,

$$\left[Z_{(P_s)j/2} \right] = \left[2 \sqrt{\left(\frac{a}{3}\right)} \right] \text{ctn } 2 \phi \text{ ,}$$

where

$$2 \phi = 2 \tan^{-1} \left[\sqrt[3]{\tan \Psi} \right]$$

$$\Psi = \frac{1}{2} \text{ctn}^{-1} \left[\sqrt{\frac{(b/2)^2}{(a/3)^3}} \right] \text{ .}$$

If $D = 0$,

$$\left[Z_{(P_s)j/2} \right]_1 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)} \right]$$

$$\left[Z_{(P_s)j/2} \right]_2 = \sqrt{-\left(\frac{a}{3}\right)}$$

If $D < 0$,

$$\left[Z_{(P_s)j/2} \right]_1 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)} \right] \cos \left(\frac{\phi}{3} \right)$$

$$\left[Z_{(P_s)j/2} \right]_2 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)} \right] \cos \left(\frac{\phi}{3} + 120^\circ \right)$$

$$\left[Z_{(P_s)j/2} \right]_3 = \left[-2 \sqrt{-\left(\frac{a}{3}\right)} \right] \cos \left(\frac{\phi}{3} + 240^\circ \right) \text{ ,}$$

where

$$\phi = \cos^{-1} \sqrt{\frac{(b/2)^2}{-(a/3)^3}} \text{ .}$$

(d) Select appropriate value of $Z_{(P_s)j/2}$ on the basis:

$$0 < Z_{(P_s)j/2} < 4.3 \text{ .}$$

(e) Using the tables of the normal distribution function determine the values of $\left[(P_s)_j \right]^*$ and $\left[(P_s)_j \right]_*$ corresponding to $\left[Z_{(P_s)j/2} \right]^*$ and $\left[Z_{(P_s)j/2} \right]_*$.

- (10) For each parameter, j , of the m transducer parameters measured, compute separately for the two portions of the experiment the value of $(P_s)_j$ given by:

$$(P_s)_j = \theta_1 [(P_s)_j]^* + \theta_2 [(P_s)_j]_* + \theta_3 \quad ,$$

where

$\theta_1 = 0.5$, $\theta_2 = 0$, $\theta_3 = 0.5$ if only a maximum limit is specified;
 $\theta_1 = 0$, $\theta_2 = 0.5$, $\theta_3 = 0.5$ if only a minimum limit is specified;
 and $\theta_1 = 0.5$, $\theta_2 = 0.5$, $\theta_3 = 0$ if both maximum and minimum limits are specified.

- (11) For each portion of the experiment compute the probability, P_s , that all of the parameters will have values within the NERVA requirement limits:

$$P_s = \prod_{j=1}^m (P_s)_j$$

and compute $(0.99)^m$ which is the confidence level for each P_s .

- (12) Use the values of P_s , and $(P_s)_j$ with their associated confidence levels, and plots of the regression line, confidence interval, and tolerance limits as functions of x to prepare the statistical statements of the experimental results which answer the questions of the problem statement. These statements must be accompanied by statements of the assumptions on which the results given are dependent.

Requirements for Experimental Data

Preirradiation Experiments

Knowledge of the relation existing between coil winding resistance and internal temperature for each transducer specimen is a prerequisite for reduction of the irradiation-experiment data. Thus, a preirradiation experiment* will be required to obtain this information. The precision of these determinations must be sufficiently high that the errors in the relationships established can be safely neglected when they are used to reduce the irradiation data.

Assuming, on the basis of engineering considerations, that a temperature error in the radiation experimental results of ± 1 F is acceptable, then the preirradiation-experiment temperature should be measured to ± 0.1 F or better. This means that the corresponding resistances should be measured to at least ± 0.001 ohm with the regression coefficients $(A_0')_\lambda$ and $(A_1')_\lambda$ being determined with accuracies of ± 0.01 and ± 0.001 units respectively.

Another prerequisite to the radiation experiment is knowledge of the relative thermal characteristics of the specimens, Q_λ . A precision error in the determination of Q_λ will be reflected in the experimental results as an error in the accuracy of the radiation-flux values. Since it is expected that the state of the art of radiation dosimetry will not permit determining the radiation flux to better than ± 20 per cent accuracy, a relative thermal characteristic error of as much as 1 per cent can be considered negligible. The error associated with Q_λ will be less than 1 per cent if the voltage and current measurements in the preirradiation experiment conducted to determine Q_λ have accuracies of ± 0.001 v and ± 0.001 amp, respectively, and if the ambient temperature is measured to an accuracy of ± 0.1 F.

Irradiation Experiment

The specific engineering parameters of interest in the irradiation experiment are the internal temperature, electrical-insulation resistance, output-signal zero shift, sensitivity, and primary to secondary phase shift for the transducers as functions of

*It should be noted that many other basic programs might also be considered as necessary preirradiation experiments. For example, the validity of accepting equivalency between internal heat generated by applying electrical power (or oven heat) and heat generated by exposure to a given radiation field may be sufficiently questionable to warrant preliminary experimentation. Also, the materials and structure of the test specimens, the materials and structures of the test mounting fixtures, the means of mounting the specimens, etc., may require experimental investigation before the radiation experiment is conducted. For the purpose of this report, it will be assumed that there already exists sufficient information so that no preliminary experiments other than those described will be required. Of course, the validity of any experimental results would depend upon the validity of this assumption.

radiation flux and integrated neutron exposure. The experiment will be identified by the number RS/LDT401 and is to be sponsored by ABC Company. It is planned that the experiment would be conducted February 30, 1967, at The ABC Company Reactor Facility. Three XYZ Corporation Model 001 LVDT-type position transducers (System Specifications Code No. PT-10) will be employed in this experiment.

Stimulus Measurements. The accuracy* of reactor power measurements is expected to be ± 5 percent. Assuming that the accuracy of the value for radiation flux per megawatt at the location of the reference specimen is about ± 20 percent (which includes uncertainties associated with establishing the specimen location within the facility and the perturbation associated with mounting fixtures, etc.), the values obtained for radiation flux in the experimental results are expected to have an accuracy of ± 26 percent.**

In the portion of the experiment concerned with the effects of fast-neutron fluence, it is expected that the reactor power level will be recorded at 30-minute intervals and that the power level existing at the time a given parameter for a given specimen is measured will be computed by a linear interpolation. The accuracy of the resulting power-level value is expected to still be ± 5 percent. The time of measurement of each parameter of each specimen should be recorded to the minute so that the accuracy associated with integrated power values will be ± 8.2 percent. Since it is expected that the value of fast-neutron fluence per megawatt-hour at the location of the reference specimen can be established with an accuracy of ± 20 percent (including uncertainties associated with actual specimen location, fixture perturbations, etc.), the values of integrated fast-neutron fluence expressed in the experimental results should have accuracies of about ± 30 percent.

* It is recognized that the experimental accuracy must be consistent with the most limited measurements and that there is a considerable economic question involved in obtaining increased accuracies. As a general rule, the accuracy that should be required is only that sufficient to satisfy the experimental purpose. In this Model Experimental Design, no effort has been made to optimize the experimental accuracy and precision requirements since this would involve engineering considerations pertaining to the linear-displacement-transducer use and such detail is not relevant in a hypothetical example. The significant point is that the question of accuracy and precision of measurement is an integral part of any experimental design and involves not only the experimental purpose and analytical procedures but also depends considerably upon economic factors.

**Computation of the accuracy involves an error cross product which, when considered in a worse case manner, leads to an estimated accuracy of ± 26 percent rather than the additive value of ± 25 percent. (Actual values are $+ 26$ percent, -24 percent)

As a general rule, the precision* of the measurements made in both parts of the experiment should be at least an order of magnitude better than the required accuracies. Thus, if the temperature-measurement accuracy is required to be ± 1 F, the required precision is (at least) ± 0.1 F. The number of significant digits recorded for the measurements should be based on the precision rather than the accuracy. Thus for the temperature-measurement example stated above, data would be recorded to a tenth of a degree.

The accuracy and precision associated with the position data (a-c coil winding voltages and d-c output of the demodulator) will depend upon the accuracy and precision of the electrical excitation applied to the transducer and the precision with which the transducer core travel is controlled. The transducer excitation should be controlled to an accuracy of at least ± 2 percent of the nominal 5 v, rms, and 3000 cps specified. If the data obtained are to be meaningful, the steps employed to control the transducer core-position settings should ideally have a precision corresponding to at least the limit of transducer resolution; i.e., 0.0001 in.** It is noted that there may be considerable difficulty in obtaining this precision for the 50 percent travel position. Also, mechanical precisions of this order are usually temperature sensitive so that means should be devised for experimental compensation or data correction to remove the effect of temperature variations.

Ambient temperature in this experiment should be measured with an accuracy of at least ± 1 F and a precision of at least ± 0.1 F. Pressure measurements are not critical so that an accuracy of ± 2 psig is adequate. As previously indicated, specimen and dosimeter locations are factors in the determination of radiation flux and should be established with an accuracy of ± 0.1 in.

Response Measurements. To assure that the accuracy of internal temperature determinations will be at least ± 1 F, and with a transducer ambient temperature measurement accuracy of ± 1 F, the accuracy of the coil winding resistance measurements is required to be ± 0.01 ohm. The measurement precision should therefore be ± 0.001 ohm. The required accuracy for insulation resistance measurements is ± 10 percent. The a-c voltages and d-c output of the demodulator are required to be measured with a precision of at least ± 0.1 percent of the full-scale output with a measurement accuracy of at least ± 1 percent. The accuracy required for measurements of input-output phase is ± 0.25 deg and the precision required is at least ± 0.1 deg.

* The probability statements made to express the experimental results automatically include the measurement precision. The measurement accuracy in most cases acts to translate the experimental plots, confidence intervals, and tolerance bands along the x- and/or y-axis and thus must be taken into account in a worst-case manner when preparing the statements of experimental results.

**The test for significance of radiation effects in the analysis procedure depends upon the experimental precision achieved. Thus, fluctuations in the data caused by imprecision of the mechanical stops will limit the ability of the experiment to detect small radiation effects.

Maintenance of Equipment Calibration Records

Prior to the initiation of experimental work, all instrumentation and equipment should be calibrated in accordance with standard procedures at the ABC Company Standards Laboratory. Suitable standards for checking equipment and instrumentation calibration should be made available at the radiation-facility location. The instrumentation and equipment calibration will then be checked and the measurements recorded at the times specified in the test specification. It is required that the standards employed for these calibration checks have a traceability to the National Bureau of Standards. Recalibration of instrumentation at the facility site is not permitted, but data pertaining to the calibration of the instrumentation and equipment will be used to correct experimental data if this is found to be necessary.

Summary of Data Requirements

The data requirements for the preirradiation experiments are summarized and listed in Table A-1. A detailed tabular summary of data requirements for the radiation experiment is given in Table A-2.

Experimental Procedures

Preirradiation Experiments

Coil Winding Resistance - Internal Temperature Calibration. In order to express coil-winding-resistance measurements in terms of internal transducer temperature, it is necessary to establish the relation existing between internal temperature and coil winding resistance. The necessary data will be obtained by the following experimental procedure. The specimens to be employed in the radiation experiment are installed in a mounting fixture and this assembly is placed in an oven. Material (such as iron, etc.) should be added to the oven to provide a high thermal capacity so that the oven temperature will drop approximately 0.2 degrees in the time required to make coil resistance measurements with the oven heating power turned off. After the calibration of the instrumentation is checked and the measured values of the calibration standards recorded, the specimens are allowed to soak with the oven door closed and oven power off for a time sufficient for the specimens and fixture to reach equilibrium at room temperature. The values of oven temperature, fixture temperature, and the d-c resistances of the primary and secondary coil windings of the transducers may now be measured and recorded. The oven temperature is now raised to slightly above 100 F. When the fixture temperature is slightly higher than 100 F, the oven heating power is turned off. As the fixture temperature cools between 100.1 and 100.0 F, the values of oven temperature, fixture temperature, and the d-c resistances of the primary and secondary coil windings of the transducers are measured and recorded. The oven temperature may now be raised to slightly higher than 150 F and the measurement procedure repeated as the fixture cools between 150.1 and 150 F. This procedure is repeated at 50-degree intervals until data are obtained for a temperature of 850 F.

TABLE A-1. SUMMARY OF DATA REQUIREMENTS FOR PREIRRADIATION EXPERIMENTSExperiment Identifications

(A)	Experiment identification number	RS/LDT 401
(B)	Sponsor	ABC Company
(C)	Testing agency	ABCRF
(D)	Test data	January-February, 1967

Drawings

None

Equipment List*

(A) To be provided by ABC Company Instrumentation Development Laboratory:

- (1) Three specimen linear displacement transducers
- (2) Test fixture for internal temperature - coil winding resistance calibration
- (3) Irradiation test fixture
- (4) Controlled-temperature oven (range ambient to 900 F) and material to insert in oven to provide high thermal capacity
- (5) Oven-internal-temperature sensor and readout equipment
- (6) Fixture-temperature sensor and readout equipment
- (7) Kelvin Bridge for coil-resistance measurements
- (8) Controlled-temperature water bath
- (9) Water-data and fixture-temperature sensors and readout equipment
- (10) Two precision d-c power supplies
- (11) Two precision d-c voltmeters
- (12) Two precision d-c ammeters

Facility Requirements

(A)	Electrical	115-VAC, 60-cps, 30-amp service
(B)	Pneumatic	None
(C)	Hydraulic	Temperature-controlled water bath and circulating system
(D)	Special fluids	None
(E)	Special shielding	None

Test-Specimen Identification

<u>Number</u>	<u>Manufacturer</u>	<u>Model Number</u>	<u>Type</u>	<u>System Specifications</u> <u>Code Number</u>
3	XYZ Corp.	001	LVDT	PT-10

*Manufacturer's name and equipment model number should be included in this list where pertinent. Such detailed data are not relevant for use in this Model Experimental Design.

TABLE A-1. (Continued)

Definition of Stimulus

(A) Internal temperature - coil winding resistance calibration

(1) Temperature	100 F to 850 F in 50 F steps
(2) Pressure	Atmospheric
(3) Humidity	Ambient
(4) Vibration	None
(5) Nuclear radiation	None
(6) Mechanical stimulus	None
(7) Fluid environment	Air
(8) Electrical excitation	None
(9) Duration	Not applicable

(B) Thermal-Characteristics Determination

(1) Temperature	100 $\pm$ 1 F (water-bath ambient temperature)
(2) Pressure	As required
(3) Humidity	Immersed in water bath
(4) Vibration	None
(5) Nuclear radiation	None
(6) Mechanical stimulus	None
(7) Fluid environment	H ₂ O
(8) Electrical excitation	As required to produce internal temperature of 200 $\pm$ 1 F
(9) Duration	Not applicable

Stimulus Parameters

(A) Internal temperature - coil winding resistance calibration

- (1) Ambient temperature

(B) Thermal-characteristics determination

- (1) Ambient temperature
- (2) Electrical-power dissipation

Stimulus Measurements Required

(A) Internal temperature - coil winding resistance calibration

	<u>Accuracy</u>	<u>Precision</u>
(1) Ambient temperature	± 0.1 F	± 0.01 F

(B) Thermal-characteristics determination

	<u>Accuracy</u>	<u>Precision</u>
(1) Water bath temperature	± 0.1 F	± 0.01 F
(2) Fixture temperature	± 0.1 F	± 0.01 F
(3) Coil winding voltage	± 0.001 v	± 0.0001 v
(4) Coil winding current	± 0.001 amp	± 0.0001 amp

TABLE A-1. (Continued)

Definition of Response

- (A) Internal temperature - coil winding resistance calibration
 - (1) Coil winding resistance
- (B) Thermal-characteristics determination
 - (1) Internal temperature

Response Parameters

- (A) Internal temperature - coil winding resistance calibration
 - (1) Primary coil winding resistance
 - (2) Secondary coil winding resistance
- (B) Thermal-characteristics determination
 - (1) Primary coil winding resistance
 - (2) Secondary coil winding resistance

Response Measurements

- (A) Internal temperature - coil winding resistance calibration

	<u>Accuracy</u>	<u>Precision</u>
(1) Primary coil winding resistance	± 0.001 ohm	± 0.0001 ohm
(2) Secondary coil winding resistance	± 0.001 ohm	± 0.0001 ohm

- (B) Thermal-characteristics determination

	<u>Accuracy</u>	<u>Precision</u>
(1) Voltage across primary coil	± 0.001 v	± 0.0001 v
(2) Voltage across secondary coil	± 0.001 v	± 0.0001 v
(3) Current through Primary coil	± 0.001 amp	± 0.0001 amp
(4) Current through secondary coil	± 0.001 amp	± 0.0001 amp

TABLE A-2. SUMMARY OF DATA REQUIREMENTS FOR RADIATION EXPERIMENT

Experiment Identifications

(A)	Experiment identification number	RS/LDT 401
(B)	Sponsor	ABC Company
(C)	Testing agency	ABCRF
(D)	Test date	February 30, 1967

Drawings*

(A)	Test item	See note
(B)	Pneumatic single line	See note
(C)	Electrical single line	See note
(D)	Dosimeters	See note
(E)	Hydraulic single line	Not applicable

Equipment List**

(A) To be provided by ABC Company Instrumentation Development Laboratory:

- (1) Test fixture containing three linear displacement transducers
- (2) One dual-beam oscilloscope, Tektronix Model 502A
- (3) One demodulator, Crescent Model 8303
- (4) One phase meter, AD-YU Model 202A
- (5) Four thermocouples and associated readout equipment for monitoring specimen temperature
- (6) One 115 VAC line-voltage regulator, Sorenson Model 1000-S
- (7) One Wheatstone Bridge, Rubicon Model 1071
- (8) One vacuum-tube voltmeter, Hewlett Packard Model 412A
- (9) One a-c voltmeter, Hewlett Packard Model 400H
- (10) One megohm bridge, General Radio Type 1862-C
- (11) Standards as required for instrumentation calibration checks
- (12) Hook-up cable, 75 ft long, with shielded leads for instruments and extension thermocouple wires, enclosed in watertight Tygon tubing. (The insulation resistance of this cable must be at least 100,000 megohms per ft.)

(B) To be provided by ABCRF:

- (1) None

Facility Requirements

(A)	Electrical	115-VAC, 60-cps, 30-amp service
(B)	Pneumatic	Reactor off-gas system
(C)	Hydraulic	Primary coolant water
(D)	Special fluids	None
(E)	Special shielding	None

*Because such detail is irrelevant to a hypothetical example, the drawings are not included in this Model Experimental Design though they would be in a real design.

**Manufacturers' names and model numbers in this equipment list are given for illustrative purposes only and no recommendation for their actual use is expressed or implied.

TABLE A-2. (Continued)

Definition of Stimulus

Test Environment

(A) Temperature	125 F (ambient water temperature)
(B) Pressure	130 psi (nominal capsule water pressure)
(C) Humidity	Test items submerged in coolant water
(D) Vibration	None
(E) Fast-neutron flux*	3.2×10^{13} n/(cm ²)(sec) (E > 1.0 Mev)
(F) Thermal-neutron flux*	1.4×10^{14} n/(cm ²)(sec) (E < 0.48 ev)
(G) Gamma flux*	9.7×10^{10} ergs/(g)(C)(hr) maximum
(H) Integrated exposures	
Neutron (fast)*	1.2×10^{18} n/cm ² (E > 1.0 Mev) maximum
Neutron (thermal)*	5.0×10^{18} n/cm ² (E < 0.48 ev) maximum
Gamma exposure*	9.7×10^{11} ergs/(g)(C) maximum
(I) Mechanical stimulus	Displace transducer core 100 percent of travel
(J) Fluid environment	Water
(K) Electrical excitation	5 v rms at 3000 cps (accuracy ± 2 percent)
(L) Duration	8 hr at 50 Mw maximum

Preirradiation Analysis

(A) Heating rate

The heating rate produced in the fixture, transducers, and dosimeters, with a total mass of 3190 g, will be 7.178 kw (at 50-Mw reactor power level).

(B) Activation**

The activation resulting from an 8-hr irradiation at 50 Mw of the fixture, transducers, thermocouples, and dosimeters will be:

Radioisotope CuriesRadioisotope CuriesRadioisotope Curies

Na-24	Sc-46	Mn-56
Al-26	Ca-47	Co-57
Mg-27	Sc-47	Co-58m
Al-28	SC-48	Co-58
Si-31	Ti-51	Fe-59
P-32	Cr-51	Co-60m
P-33	V-52	Co-60
S-35	Mn-54	Ni-63
S-37	Cr-55	Cu-64
Ni-65	Nb-92	Cd-107
Zn-65	Nb-95	Ag-110
Cu-66	Nb-97	Cd-113m
Cu-67	Mo-99	Cd-115m
Zn-69m	Tc-99	Cd-115
Zn-69	Mc-101	In-115m
Zn-71	Tc-101	Cd-117m
Zr-89		In-117

*The radiation fluxes and integrated exposures given are hypothetical values.

**In a real experimental design the curie values expected for the various radioisotopes would be given.

TABLE A-2. (Continued)

Stimulus Parameters

(A) Gamma flux	Determine from reactor power level, specimen location and flux mapping data
(B) Neutron flux	Determine as above and also by differentiating dosimetry data
(C) Transducer core displacement	Determine from actuator pressure and known position of stops
(D) Ambient temperature	Monitor facility temperature recorders and record test fixture thermocouple outputs

Stimulus Measurements Required

	<u>Accuracy</u>	<u>Precision</u>
(A) Reactor power level	± 5 percent	± 0.5 percent
(B) Specimen location	± 0.1 in.	± 0.01 in.
(C) Fixture stop location	± 0.001 in.	± 0.0001 in.
(D) Dosimeter location	± 0.1 in.	± 0.01 in.
(E) Accumulative irradiation time	± 1 min	± 0.1 min
(F) Actuator pressures	± 2 psig	± 0.2 psig
(G) Ambient water pressures	± 2 psig	± 0.2 psig
(H) Specimen thermocouple temp.	± 1 F	± 0.1 F
(I) Ambient water temperature (at test specimen location)	± 1 F	± 0.1 F
(J) Electrical excitation	± 2 percent	± 0.2 percent
(K) Neutron dosimetry	± 20 percent	± 2 percent
(1) Aluminum-cobalt (bare)		
(2) Aluminum-cobalt (CD covered)		
(3) Sulfur pellet		
(4) Nickel wire		
(L) Gamma dosimetry	± 20 percent	± 2 percent
(1) None attached to experiment (flux to be computed from reactor power level and gamma flux mapping data)		

Definition of ResponseResponse Parameters

(A) Transducer internal temperature	Determine from coil-winding-resistance change
(B) Electrical insulation resistance	Direct measurement with 100 v applied
(C) Transducer sensitivity	Determine from core displacement, excitation voltage, primary-secondary voltage difference, and demodulator output

TABLE A-2. (Continued)

(D) Primary-secondary phase Relationship	Direct measurement
(E) Input and output signal wave shape	Monitor on oscilloscope

Response Measurements Required

	<u>Accuracy</u>	<u>Precision</u>
(A) Primary and secondary winding d-c resistances	± 0.01 ohm	± 0.001 ohm
(B) Electrical insulation resistance	± 10 percent	± 1 percent
(C) Primary and secondary a-c voltages for nominal core positions of 0, 50 and 100 percent travel	± 0.1 percent full scale	± 0.01 percent full scale
(D) D-C output of demodulator for the three core positions	± 0.1 percent full scale	± 0.01 percent full scale
(E) Primary-secondary phase relationship	± 0.25 deg	± 0.1 deg
(F) Monitor input and output signal wave shapes and note any abnormalities as they occur	--	--
(G) Internal temperature v s primary d-c resistance calibration data*		
(H) Internal temperature v s secondary d-c resistance calibration data*		
(I) Internal-external temperature difference v s internal heat dissipation*		

*Obtained from preirradiation experiments.

Determination of Specimen Thermal Characteristics. To establish the thermal resistance for each of the specimens that will be used in the radiation experiment, the specimens are installed in the irradiation-test fixture and this assembly is placed in a controlled-temperature water bath. The water bath should be of sufficient size to permit free circulation of the water around the irradiation-test fixture with the use of either external pumps or several stirring motors. After the calibration of all instrumentation has been checked and the measured values of the calibration standards recorded, the temperature of the water bath is adjusted to 100 F. The water temperature should be measured at a point close to the irradiation-test fixture but not in contact with this fixture. When the fixture and the water bath have reached equilibrium, power is applied to the transducer windings individually by passing d-c current through the windings.* The current is increased in steps while the average winding temperature is monitored. The average winding temperature is determined from the coil resistances as computed from the voltage-current ratios. The input power is increased until the average temperature of each winding is 200 ± 1 F. The winding temperature and the water bath temperature are now allowed to stabilize. (It may be necessary to provide for cooling of the water bath.) When equilibrium has been reached, the water-bath temperature close to the irradiation fixture, the current through and voltage across the primary coil winding, and the current through and voltage across the secondary coil winding are measured and the data recorded. The coil winding resistances are now computed from the voltage current ratios and the data obtained in the preceding experiment are employed to determine the primary and secondary winding temperatures. These temperatures should not differ by more than 1 F. The power inputs to the primary and secondary coil windings are now computed by taking the products of the appropriate voltages and currents.

The winding temperatures should not be increased appreciably above 200 F because the winding temperatures measured are averages as indicated by the coil resistances. Heating the coils by passing current through them is likely to cause local hot spots as the result of local-high-resistance sections of the windings. In order to minimize the possibility of these local-hot-spot temperatures exceeding 900 F, it is advisable to limit the average winding temperatures to 200 F.

Irradiation Experiment

System Description. The linear displacement transducers are installed in a test fixture that permits the displacement of the transducer cores from null position in two fixed steps (0.050 and 0.100 in) while the bodies remain unmoved. The displacement of the cores is obtained with a pneumatically operated actuator to which the cores are attached. Three transducers will be mounted on the test fixtures. The test fixture is shown in Figure ().**

*Note that the windings of a real transducer may be incapable of carrying sufficient power without damage for this technique to be employed. If such is the case another means for obtaining this information would have to be developed.

**These drawings are not included in this Model Experimental Design because they would provide unnecessary details. Of course, a real experimental design would include such drawings.

The gas system for the test fixture is shown in Figure ().* Gas is supplied from a standard cylinder via a two-stage pressure regulator and a three-way valve to a small accumulator or charge bottle.

The charge bottle serves as the source of gas used to pressurize the bellows. Gas is admitted stepwise in small quantities by cracking and then closing the gas-supply needle valve. The intermediate stop is actuated by opening its control valve. The bellows are returned to the original zero position by exhausting the helium via the gas-outlet needle valve to the radiation-monitored reactor-tank off-gas system. Overpressure relief for the bellows, hose, and tubing is provided by the gas safety valve. A valved-off vacuum connection is provided to initially evacuate the gas system of air prior to filling the system with helium. The pressure gage is mounted at the experiment rack and indicates the gas pressure there.

The electrical signal cable connections are shown in Figure ().* Each cable will be identified with a letter of the alphabet, and each conductor in the cable with a number. Each cable will be securely mounted on the test jig with a clamp for mechanical rigidity. Both ends of the cable will be identified with the same letter. Each of the linear motion transducers will be provided with a 13-ft long stainless steel-sheathed, four-conductor, MgO-insulated copper lead wire. The 13-ft length is sufficient to bring the lead wire through the Conax fitting located on the back plate of the capsule. The end of the sheathed wire will be spliced to a shielded, four-conductor extension wire insulated with irradiated polyolefin. The splice joint will be covered with a piece of shrinkable, irradiated polyolefin tubing. All of the extension wires will be enclosed within Tygon tubing in order to run them through the quadrant water.

Laboratory Check. Before the specimen transducers, test fixture, and measurement instrumentation are shipped to the radiation-facility site the system should be checked in the laboratory. First, the calibration of all instrumentation should be checked, recording the measured values of the calibration standards. The instruments are now connected to the appropriate jacks on the test panel. The linear displacement transducers are set at the null position by releasing the gas pressure.

The procedure for making resistance measurements is as follows. Set the rotary switch to Position I and measure the primary and secondary winding d-c resistances with the Wheatstone bridge. Secondary resistance is measured from S1 to S2, while the primary resistance is measured from P1 to P2. Next, measure the insulation resistance from S1 to G, P1 to G, and S1 to P1. These measurements are made with the megohm bridge with 100 v applied. Perform the above measurements for the remaining transducers by positioning the rotary switch at Positions II and III. Positions I, II, and III correspond, respectively, to Numbers 1, 2, and 3 transducers.

Positions IV through VI on the rotary switch are utilized to measure sensitivity and phase shift of the linear displacement transducers. Positions IV, V, and VI on the rotary switch correspond, respectively, to Numbers 1, 2, and 3 transducers. With the rotary switch set on Position IV, the primary and secondary voltages are measured with the a-c voltmeter. The d-c output of the demodulator is measured with

*See footnote on preceding page.

the vacuum-tube voltmeter. Next the phase relationship between the primary and secondary windings is measured with the phase meter. These sensitivity and phase measurements are repeated for the Number 2 and Number 3 transducers. While the above measurements are being made, the input and output signals should be monitored on the oscilloscope. The oscilloscope should be on a-c differential position.

To position the transducer core to 50 percent of full travel, pressurize the intermediate stop bellows and then pressurize the main bellows to 35 psig, driving the core against the intermediate stop. The above test should now be repeated. For 100 percent core travel, release the intermediate-stop-bellows and the main-bellows pressure to allow the stop to withdraw, and then pressurize the main bellows to 70 psig, driving the core against the end stop. The above measurements are then repeated for this position.

Preirradiation-Test Procedure. Following the laboratory check, the experimental equipment and specimen transducers are transported to the reactor facility. The test items are installed in the test position. The calibration of all instrumentation is checked and the measured values of the calibration standards recorded. The reactor coolant water is then brought to the operating pressure, and the helium pressure in the pneumatic system is adjusted so that it just balances the reactor-coolant-water pressure for the null position of the test transducer cores. The series of measurements performed in the laboratory checkout are now repeated. The pneumatic-system pressure needed to position the test transducer cores to 50 percent of full travel will be at least 35 psig greater than the reactor-coolant-water pressure; the pressure needed to obtain 100 percent core travel will be at least 70 psig greater than the reactor-coolant-water pressure.

Irradiation-Test Procedure. Before the irradiation is started, all equipment is put into operating condition and the calibration of all instrumentation is checked by measuring and recording the values of the calibration standards. The quantity of helium in the helium supply is checked to assure that it will be sufficient to maintain the internal pressure of the bellows at least at the same level as the reactor-coolant-water pressure.

The tests performed in the laboratory checkout are now repeated at reactor power levels of 1, 5, 10, 20, 30, 40, and 50 Mw, remaining at each power level for about 1 hour. Data cycles should be taken twice for each power level, a complete data cycle for all three test transducers being completed before starting the second data cycle. However, the internal temperature of the transducers should not be permitted to exceed 900 F as indicated by the coil winding resistance of the linear displacement transducers. If the internal temperatures of the specimens reach the 900 F limit at a reactor power level below 50 Mw, this level should be maintained for 10 hours and test data should be taken every hour during this interval as indicated above. On the other hand, if the transducer temperatures do not exceed the 900 F limit at the 50 Mw power level, the test should be run for 8 hours with data taken every hour as indicated above.

Following reactor shutdown, a recheck should be made of the calibration of all instrumentation and the measured values of the calibration standards recorded.

Postirradiation-Test Procedure. Following the irradiation, the capsule is removed from the irradiation facility and located in the quadrant water. The tests performed in the laboratory checkout are now repeated. It should be noted that the helium pressure in the pneumatic system must be vented as the reactor-coolant-water pressure is reduced following reactor shutdown to avoid possible damage to the bellows. Upon conclusion of the postirradiation test, the test fixture is placed in dry storage. The test specimens and fixture should be held in dry storage pending a postirradiation examination. Following this postirradiation examination, the readout equipment supplied by the ABC Company will be returned.

Environmental Control Data. At the time measurements are made on each specimen during the laboratory checkout, the preirradiation test, the irradiation test, and the postirradiation test, the temperature indicated by each of the specimen thermocouples should be recorded. The time at which each measurement is made during the preirradiation test, the irradiation test, and the postirradiation test must be recorded so that the data on reactor-coolant-water pressure and temperature and the reactor power level pertinent to each measurement can be obtained from the reactor log. The reactor-coolant-water pressure should be measured and recorded each time the transducer core position is changed. The pneumatic-system pressure should also be measured and recorded at these same times.

Test Specification

For convenience of experimentation, the experimental design has been used as the basis for preparing a test specification.* This test specification is the document controlling the actual experimental work to be performed.

*For a real experimental design the test specification would be included as an appendix to the experimental design. However, since it is basically a repetition in summarized form of material already given in the Model Experimental Design, it will not be included for this hypothetical example.

APPENDIX B

ILLUSTRATION OF RESULTS EXPECTED FROM EXPERIMENT RS/LDT-401

APPENDIX B

ILLUSTRATION OF RESULTS EXPECTED
FROM EXPERIMENT RS/LDT-401

The application of experimental design principles to nuclear-radiation-effects experiments has been illustrated by the model experimental design presented in Appendix A. Of course, the example is purely hypothetical, but it is interesting to consider the results that might be obtained if this experiment could actually be conducted. Also, a numerical example of the analysis would be beneficial to illustrate further the effectiveness of experimental design principles. Since an LVDT-type pressure transducer actually consists of an LDT which measures the pressure-induced displacement of a diaphragm, available pressure-transducer nuclear-radiation-effects data have been transformed to be compatible with the hypothetical experiment RS/LDT-401. Following is an analysis and expression of experimental results using these data.

The data to be employed relate the winding resistance to reactor power for four specimen transducers, and relate the output voltages for three positions of two transducers to the duration of exposure at a nominally constant reactor power level. Hypothetical values for $(T_A)_{\lambda i}$, $(A_o)_{\lambda}$, $(A_1)_{\lambda}$ and Q_{λ} have been assumed for purposes of this illustration. The analysis step designations correspond to those of the Summary of Analysis Procedure by Steps given in Appendix A.

Example Analysis for Effects of Radiation Flux

Step 1. The first portion of the analysis is concerned with the relation of internal temperature, as indicated by coil winding resistance, to radiation flux. Raw data for four specimens are tabulated with respect to reactor power in Table 1. Values of ambient temperature have been assumed for purposes of this illustration. The reactor power is the same for all specimens, i.e., $P_{1i} = P_{2i} = P_{3i} = P_{4i} = P_i$. (Where the data indicated apparent failure of the device, the subsequent data are deleted.)

TABLE 1. COIL WINDING RESISTANCE AND AMBIENT TEMPERATURE FOR VARIOUS NOMINAL REACTOR POWER LEVELS

Reactor Power ^(a) , P _i , Mw	Coil Winding Resistance, ohms				Ambient Temperature ^(a) , (T _A) _i , F
	R _{1i}	R _{2i}	R _{3i}	R _{4i}	
1	25.02	25.21	25.09	26.88	120.0
1	25.23	25.44	25.28	27.01	120.2
5	27.11	28.65	28.25	32.95	120.5
5	27.46	28.99	28.43	33.07	120.3
10	29.34	32.52	31.29	38.79	120.5
10	29.24	32.90	31.19	38.22	120.6
15	31.19	36.47	33.90	43.06	120.8
15	31.54	36.47	33.30	43.14	120.9
20	33.41	39.54	35.50	47.91	121.0
20	33.40	39.70	35.72	48.30	121.2
20	33.37	39.72	35.52	48.53	120.9
20	33.08	39.58	35.38	--	121.3
25	35.32	43.22	37.92	52.82	121.7
25	35.32	43.90	37.90	51.80	121.5
25	35.52	44.42	37.95	51.81	121.6
25	35.66	44.52	38.11	51.46	121.8
30	37.12	47.20	39.99	↑	122.2
30	37.38	47.28	40.01	Failure	122.0
30	37.38	47.29	40.21	↓	122.1
30	37.31	47.31	40.23	↓	122.4
35	38.89	49.90	42.10	↓	122.6
35	38.90	49.82	42.40	↓	122.8
35	38.90	49.80	42.52	↓	122.7
35	38.92	49.91	42.71	↓	122.8
40	40.13	↑	44.44	↓	123.4
40	40.55	Probable	45.22	↓	123.6
40	40.72	Failure	45.40	↓	124.0
40	40.60	↓	45.30	↓	123.8

(a) $P_{1i} = P_{2i} = P_{3i} = P_{4i} = P_i$; $(T_A)_{1i} = (T_A)_{2i} = (T_A)_{3i} = (T_A)_{4i} = (T_A)_i$.

Step 2. In the absence of real data for $(A'_0)_\lambda$, $(A'_1)_\lambda$, and Q_λ , the following values have been assumed:

$$(A'_0)_1 = -817.50; (A'_1)_1 = 37.5; Q_1 = 1.00$$

$$(A'_0)_2 = -864.60; (A'_1)_2 = 39.3; Q_2 = 0.90$$

$$(A'_0)_3 = -895.05; (A'_1)_3 = 40.5; Q_3 = 0.95$$

$$(A'_0)_4 = -857.28; (A'_1)_4 = 37.6; Q_4 = 0.90.$$

These hypothetical values will be employed to illustrate the application of the analysis procedure. Utilizing these data it is seen that

$$y_{1i} = (37.50) R_{1i} - 817.50$$

$$y_{2i} = (35.37) R_{2i} + (0.10) (T_A)_{2i} - 778.14$$

$$y_{3i} = (38.48) R_{3i} + (0.05) (T_A)_{3i} - 850.30$$

$$y_{4i} = (33.84) R_{4i} + (0.10) (T_A)_{4i} - 771.55.$$

The reduced data are tabulated in Table 2.

Step 3. The results of computations leading to the least squares estimates of the regression coefficients are listed in Table 3. The data and the regression lines are plotted in Figure 1.

Step 4. Now a priori it is seen that α_1 , must be greater than zero if there is an effect of radiation. The computation of B_γ for the 0.01 significance level is given in Table 3 for each specimen; in each case it is seen that $(A_1)_\lambda > B_\lambda$, so that in each case the hypothesis $\alpha_{1\lambda} = 0$ is rejected at the level of significance $\gamma = 0.01$.

Step 5. The data plots of Figure 1 indicate that the greatest effect of radiation flux on coil winding resistance is observed for Specimen 4. Specimen 4 will be used as a reference unit to compute the effective radiation flux so $q = 4$. First, the effective reactor power $(ERP)_{\lambda i}$ is computed for each data point of each specimen by

$$(ERP)_{\lambda i} = \left[\frac{(A_1)_\lambda}{(A_1)_4} \right] P_{\lambda i} .$$

The values obtained are given in Table 4.

For this example, it will be assumed that the radiation flux per megawatt reactor power at the location of Specimen 4 is $\phi_4 = 6 \times 10^8$ ergs/(g)(C)(hr)(Mw) The effective radiation flux for each specimen data point is then

$$x_i = (6 \times 10^8) (ERP)_{\lambda i} \text{ ergs/(g) (C) (hr) } .$$

TABLE 2. CORRECTED INTERNAL TEMPERATURE^(a) FOR VARIOUS REACTOR POWER LEVELS

Reactor Power ^(b) , P_i , Mw	Corrected Internal Temperature, $y_{\lambda i}$, F			
	y_{1i}	y_{2i}	y_{3i}	y_{4i}
1	120.8	125.5	121.2	150.1
1	128.6	133.7	128.5	154.5
5	199.1	247.3	242.8	355.5
5	212.3	259.3	249.7	359.6
10	282.8	384.1	359.8	553.2
10	279.0	397.6	355.9	533.9
15	352.1	523.9	460.2	697.7
15	365.3	523.9	437.1	700.4
20	435.4	632.5	521.8	861.8
20	435.0	638.2	530.3	875.0
20	433.9	638.8	522.6	882.8
20	423.0	633.9	517.2	--
25	507.0	762.7	614.9	1028.0
25	507.0	786.8	614.2	993.5
25	514.5	805.2	616.1	993.9
25	519.8	808.7	622.3	982.0
30	574.5	903.5	694.6	<div style="text-align: center;"> <div style="border-top: 1px solid black; width: 50px; margin: 0 auto; margin-bottom: 5px;"></div> <div style="margin: 0 auto; margin-bottom: 5px;">↑</div> <div style="margin: 0 auto; margin-bottom: 5px;">Failure</div> <div style="margin: 0 auto; margin-bottom: 5px;">↓</div> </div>
30	584.3	906.4	695.4	
30	584.3	906.7	703.1	
30	581.6	907.5	703.9	
35	640.9	999.1	775.8	
35	641.3	996.3	787.4	
35	641.3	995.6	792.0	
35	642.0	999.5	799.3	
40	687.4	<div style="text-align: center;"> <div style="border-top: 1px solid black; width: 50px; margin: 0 auto; margin-bottom: 5px;"></div> <div style="margin: 0 auto; margin-bottom: 5px;">↑</div> <div style="margin: 0 auto; margin-bottom: 5px;">Probable failure</div> <div style="margin: 0 auto; margin-bottom: 5px;">↓</div> </div>	865.9	
40	703.1		895.9	
40	709.5		902.9	
40	705.0		899.0	

(a) The values are hypothetical but are based on real data.

(b) $P_{1i} = P_{2i} = P_{3i} = P_{4i} = P_i$.

TABLE 3. ESTIMATION OF REGRESSION COEFFICIENTS AND TEST FOR SIGNIFICANT RADIATION EFFECT

Statistic(a)	Computed Values			
	$\lambda=1$	$\lambda=2$	$\lambda=3$	$\lambda=4$
n_{λ}	28	24	28	15
$\sum x_{\lambda i}^2$	19,702	13,302	19,702	4,402
$\sum x_{\lambda i}$	662	502	662	222
$(\sum x_{\lambda i})^2$	438,244	252,004	438,244	49,284
$n_{\lambda}(\sum x_{\lambda i}^2) - (\sum x_{\lambda i})^2$	113,412	67,244	113,412	16,746
$\sum y_{\lambda i}^2$	7.28993×10^6	12.40151×10^6	11.04748×10^6	8.15417×10^6
$\sum y_{\lambda i}$	13,410.8	15,916.7	16,429.8	10,121.9
$\sum x_{\lambda i} y_{\lambda i}$	376,172	404,670	463,720	188,050
$(A_o)_{\lambda}$	133.969	127.589	147.403	167.772
$(A_1)_{\lambda}$	14.59163	25.60669	18.58385	34.25822
$(S_x^2)_{\lambda}$	150.01587	121.81884	150.01587	79.74285
$(S_y^2)_{\lambda}$	32,101.1	80,244.3	52,104.6	94,569.9
$r^2 = (A_1)_{\lambda}^2 \frac{(S_x^2)_{\lambda}}{(S_y^2)_{\lambda}}$	0.99500	0.99542	0.99433	0.98961
$(S_{y \cdot x})_{\lambda}$	166.6785	384.2225	306.7955	1,058.162
$(S_{y \cdot x})_{\lambda}$	12.91040	19.60160	17.51561	32.52940
$t_{0.99}^{(n_{\lambda}-2)}$	2.479	2.508	2.479	2.650
$(n_{\lambda}-1)(S_x^2)_{\lambda}$	4,050.42849	2,801.83332	4,050.42849	1,116.39990
$\sqrt{(n_{\lambda}-1)(S_x^2)_{\lambda}}$	63.6430	52.9324	63.6430	33.4127
$\frac{(S_{y \cdot x})_{\lambda}}{\sqrt{(n_{\lambda}-1)(S_x^2)_{\lambda}}}$	0.20286	0.37031	0.27522	0.97356
$B_{\gamma}; (\gamma = 0.01)$	0.503	0.929	0.682	2.580

(a) $\sum$ means $\sum_{i=1}^{n_{\lambda}}$.

FIGURE 1. CORRECTED INTERNAL TEMPERATURE VERSUS REACTOR POWER

TABLE 4. EFFECTIVE REACTOR POWER FOR EACH SPECIMEN

Reactor Power ^(a) , P_i , Mw	Effective Reactor Power, $(ERP)_{\lambda i}$, Mw			
	Specimen 1	Specimen 2	Specimen 3	Specimen 4
	$(\lambda = 1)$ $(ERP)_{1i} = (0.42593)P_{1i}$	$(\lambda = 1)$ $(ERP)_{2i} = (0.74746)P_{2i}$	$(\lambda = 3)$ $(ERP)_{3i} = (0.54246)P_{3i}$	$(\lambda = 4)$ $(ERP)_{4i} = (1.00000)P_{4i}$
1	0.43	0.75	0.54	1
5	2.13	3.74	2.71	5
10	4.26	7.47	5.42	10
15	6.39	11.21	8.14	15
20	8.52	14.95	10.85	20
25	10.65	18.69	13.56	25
30	12.78	22.42	16.27	
35	14.91	26.16	18.99	
40	17.04		21.70	

(a) $P_{1i} = P_{2i} = P_{3i} = P_{4i} = P_i$.

It may be assumed that at zero reactor power [corresponding to the intercepts, $(A_o)_{\lambda}$, of the regression lines] the specimens would all have the same temperature. Thus, the corrected internal temperature values, $y_{\lambda i}$, may be normalized to a common zero power temperature, T , by computing

$$y_i = y_{\lambda i} + [T - (A_o)_{\lambda}] ,$$

where $y_{\lambda i}$ is the temperature value for the i^{th} measurement of the λ^{th} specimen and T is the nominal facility ambient temperature. The paired values (y_i, x_i) are tabulated in Table 5.

Step 6. The data of Table 5 are used to compute least-squares estimates of the pooled data regression line and other pertinent statistics. The results of these computations are tabulated in Table 6. The data and regression line are plotted in Figure 2.

Step 7. The statistics of Table 6 are used to compute the 99 percent confidence interval about the regression line. The computational results are tabulated in Table 7 and the interval is plotted in Figure 2.

Step 8. The statistics of Table 6 are also used to compute 99 percent tolerance band about the regression line for a 99 percent confidence level. The results of these computations are tabulated in Table 8 and the tolerance band is plotted in Figure 2.

Step 9. Since this example is intended solely to illustrate the application of the experimental design analytical procedures, the following hypothetical requirements for transducer performance will be assumed. It is assumed that transducer performance will be considered acceptable if the internal temperature remains less than 900 F when the transducer is exposed to a radiation flux of 9.52×10^9 ergs/(g) (C) (hr) and all other environmental conditions are the same as those encountered in the experiment. It is also assumed that the only parameter of interest is internal temperature.

Reference to the upper 99 percent tolerance limit plotted in Figure 2 for a radiation flux of 1.2×10^{10} ergs/(g) (C) (hr), which is the worst case condition considering a possible ± 26 percent error in the experimental determination of radiation flux, $(1.26 \times 9.52 \times 10^9 \text{ } 1.2 \times 10^{10})$ shows

TABLE 5. POOLED INTERNAL-TEMPERATURE DATA VERSUS RADIATION FLUX

Radiation Flux, x_1 , ergs/(g) (C) (hr)	Internal Temperature, y_1 , F	Radiation Flux, x_1 , ergs/(g) (C) (hr)	Internal Temperature, y_1 , F	Radiation Flux, x_1 , ergs/(g) (C) (hr)	Internal Temperature, y_1 , F	Radiation Flux, x_1 , ergs/(g) (C) (hr)	Internal Temperature, y_1 , F
2.58×10^8	106.8	5.11×10^9	421.4	8.95×10^9	626.9	1.20×10^{10}	814.0
2.58×10^8	114.6	5.11×10^9	421.0	8.95×10^9	627.3	1.20×10^{10}	827.2
3.24×10^8	93.8	5.11×10^9	419.9	8.95×10^9	627.3	1.20×10^{10}	835.0
3.24×10^8	101.1	5.11×10^9	409.0	8.95×10^9	628.0	1.30×10^{10}	838.5
4.50×10^8	117.9	6.00×10^9	505.4	8.97×10^9	624.9	1.30×10^{10}	868.5
4.50×10^8	126.1	6.00×10^9	486.1	8.97×10^9	630.6	1.30×10^{10}	875.5
6.00×10^8	102.3	6.39×10^9	493.0	8.97×10^9	631.2	1.30×10^{10}	871.6
6.00×10^8	106.7	6.39×10^9	493.0	8.97×10^9	626.3	1.35×10^{10}	895.9
1.28×10^9	185.1	6.39×10^9	500.5	9.00×10^9	649.9	1.35×10^{10}	898.8
1.28×10^9	198.3	6.39×10^9	505.8	9.00×10^9	652.6	1.35×10^{10}	899.1
1.63×10^9	215.4	6.51×10^9	494.4	9.76×10^9	667.2	1.35×10^{10}	899.1
1.63×10^9	222.3	6.51×10^9	502.9	9.76×10^9	668.0	1.50×10^{10}	980.2
2.24×10^9	239.7	6.51×10^9	495.2	9.76×10^9	675.7	1.50×10^{10}	945.7
2.24×10^9	251.7	6.51×10^9	489.8	9.76×10^9	676.5	1.50×10^{10}	956.1
2.56×10^9	268.8	6.73×10^9	516.3	1.02×10^{10}	673.4	1.50×10^{10}	934.2
2.56×10^9	265.0	6.73×10^9	516.3	1.02×10^{10}	689.1	1.57×10^{10}	991.5
3.00×10^9	307.7	7.67×10^9	560.5	1.02×10^{10}	695.5	1.57×10^{10}	988.7
3.00×10^9	311.8	7.67×10^9	570.3	1.02×10^{10}	691.0	1.57×10^{10}	988.0
3.25×10^9	332.4	7.67×10^9	570.3	1.12×10^{10}	755.1	1.57×10^{10}	991.9
3.25×10^9	328.5	7.67×10^9	567.6	1.12×10^{10}	779.2		
3.83×10^9	338.1	8.14×10^9	587.5	1.12×10^{10}	797.6		
3.83×10^9	351.3	3.14×10^9	586.8	1.12×10^{10}	801.1		
4.48×10^9	376.5	8.14×10^9	588.7	1.14×10^{10}	748.4		
4.48×10^9	390.0	8.14×10^9	594.9	1.14×10^{10}	760.0		
4.88×10^9	432.8			1.14×10^{10}	764.6		
4.88×10^9	409.7			1.14×10^{10}	771.9		

TABLE 6. REGRESSION COEFFICIENT ESTIMATES
AND STATISTICS FOR POOLED DATA

Statistic ^(a)	Computed Value
n	95.
n(n-1)	8930.
n ²	9025.
$\frac{(n-1)}{(n-2)}$	1.01075
$\sum x_i^2$	7.63567×10^{21}
$\sum x_i$	7.43024×10^{11}
$\bar{x}$	7.82131×10^9
$n(\sum x_i^2) - (\sum x_i)^2$	1.73304×10^{23}
$\sum x_i y_i$	5.25081×10^{14}
$\sum y_i^2$	3.64698×10^7
$\sum y_i$	5.38206×10^4
Ao	120.007
A1	5.70829×10^{-3}
$\hat{y}_x$	$(5.70829 \times 10^{-8})x + 120.007$
S_x^2	1.94069×10^{19}
S_y^2	6.36029×10^4
$r^2 = (A_1)^2 (S_x^2/S_y^2)$	0.99424
$r = (A_1) \sqrt{S_x^2/S_y^2}$	0.99712
$S_{y.x}^2$	370.29100
$S_{y.x}$	19.24295

(a) $\sum$ means $\sum_{i=1}^n$, where $n = \sum_{\lambda=1}^K n_{\lambda}$.

TABLE 7. RESULTS OF CONFIDENCE-INTERVAL COMPUTATIONS

$x,$ ergs/(g)(C)(hr)	$\hat{y}_x,$ F	M	$\left[\sqrt{MS^2_{y \cdot x}} \right]^{(n-2)}_{t_{0.99}}$	(99% Confidence)	(99% Confidence)
1.0×10^8	125.72	1.10793×10^{-2}	5.22	130.9	120.5
1.0×10^9	177.09	1.10077×10^{-2}	5.20	182.3	171.9
2.0×10^9	234.17	1.09384×10^{-2}	5.18	239.4	229.0
4.0×10^9	348.34	1.08326×10^{-2}	5.16	353.5	343.2
6.0×10^9	462.50	1.07708×10^{-2}	5.14	467.6	457.4
8.0×10^9	576.67	1.07528×10^{-2}	5.14	581.8	571.5
1.0×10^{10}	690.84	1.07786×10^{-2}	5.14	696.0	685.7
1.2×10^{10}	805.00	1.08483×10^{-2}	5.16	810.2	799.8
1.4×10^{10}	919.17	1.09619×10^{-2}	5.19	924.4	914.0
1.6×10^{10}	1033.33	1.11193×10^{-2}	5.23	1038.6	1028.1
1.8×10^{10}	1147.50	1.13205×10^{-2}	5.27	1152.8	1142.2

(a) For this computation $t_{0.99}^{(93)} \doteq Z_{0.495}$ (from normal distribution table) = 2.575.

FIGURE 2. TRANSDUCER INTERNAL TEMPERATURE AS A FUNCTION OF RADIATION FLUX

TABLE 8. RESULTS OF TOLERANCE-BAND COMPUTATIONS

$x,$ ergs/(g)(C)(hr)	$\hat{y}_x, F$	M	$(1 + \frac{M}{2} + \frac{M^2}{8})$	B(a)	B $S_{y \cdot x}$	Y^T, F [$p_T = 0.99,$ (1- ϵ) = 0.99]	Y_T, F [$p_T = 0.99,$ (1- ϵ) = 0.99]
1.0×10^8	125.72	0.0110793	1.00556	3.11746	59.99	185.7	65.7
1.0×10^9	177.09	0.0110077	1.00552	3.11732	59.99	237.1	117.1
2.0×10^9	234.17	0.0109384	1.00549	3.11724	59.98	294.2	174.2
4.0×10^9	348.34	0.0108326	1.00544	3.11708	59.98	408.3	288.4
6.0×10^9	462.50	0.0107708	1.00541	3.11699	59.98	522.5	402.5
8.0×10^9	576.67	0.0107528	1.00540	3.11696	59.98	636.7	516.7
1.0×10^{10}	690.84	0.0107786	1.00541	3.11699	59.98	750.8	630.9
1.2×10^{10}	805.00	0.0108483	1.00544	3.11708	59.98	865.0	745.0
1.4×10^{10}	919.17	0.0109619	1.00550	3.11726	59.99	979.2	859.2
1.6×10^{10}	1033.33	0.0111193	1.00558	3.11752	59.99	1093.3	973.3
1.8×10^{10}	1147.50	0.0113205	1.00568	3.11783	60.00	1207.5	1087.5

(a) Computation is for $P_T = 99$ per cent; $p_T = 0.99$; $Z_{p_T/2} = 2.575$; $(1-\epsilon) = 0.99$; $(n-2) = 93$; $\sqrt{F_{\epsilon}(\infty, n-2)} = 1.204$.

that there is a 99 percent certainty that the probability is greater than 99 percent ($P_s > 0.99$) that the transducer performance will be acceptable for a radiation flux of $(9.52 \times 10^9 \pm 26\%) \text{ ergs/(g) (C) (hr)}$; i. e., the probability is greater than 99 percent that the internal temperature will not exceed the 900 F limit (at the 99 percent confidence level) for a radiation flux of $9.52 \times 10^9 \pm 26\% \text{ ergs/(g) (C) (hr)}$.

To compute a more precise value for P_s , the limit of acceptable internal temperature, 900 F, is employed to compute $(B_j)^*$ for $x = 1.2 \times 10^{10} \text{ ergs/(g) (C) (hr)}$:

$$(B_j)^* = \frac{(Y_s)^* - (\hat{y}_x)_j}{S_{y \cdot x}} = \frac{900 - 805.00}{19.24295} = 4.93687.$$

$$\text{Now } C = (5.13) \left(1 + \frac{M}{2} + \frac{M^2}{8} \right) - (7.78)M^2 = (5.13) (1.00544) - (7.78) (0.00012) =$$

5.15697, so $(B_j)^* < C$. It is seen that

$$a = - \frac{12 \left(1 + \frac{M}{2} + \frac{M^2}{8} \right)}{M^2} = - \frac{12 (1.00544)}{0.00012} = -1.00544 \times 10^5$$

$$b = \frac{12(B_j)^*}{M^2 \sqrt{F_{0.99(\infty, 93)}}} = \frac{12(4.93687)}{(0.00012) (1.204)} = 4.10039 \times 10^5,$$

and

$$\begin{aligned} D &= \left(\frac{b}{2} \right)^2 + \left(\frac{a}{3} \right)^3 = (2.05019 \times 10^5)^2 - (3.35146 \times 10^4)^3 \\ &= (4.20328 \times 10^{10}) - (3.76445 \times 10^{13}) = -3.76025 \times 10^{13}. \end{aligned}$$

Since $D < 0$,

$$\phi = \cos^{-1} \sqrt{\frac{(b/2)^2}{-(a/3)^3}} = \cos^{-1} \sqrt{\frac{4.20323 \times 10^{10}}{3.76445 \times 10^{13}}} = \cos^{-1} \sqrt{11.16572 \times 10^{-4}}$$

$$= \cos^{-1} (0.03342) = 88^\circ 5.1'$$

$$\text{and } \frac{\phi}{3} = 29^\circ 21.7'.$$

Also

$$\begin{aligned} \left[-2 \sqrt{\left(\frac{a}{3} \right)} \right] &= -2 \sqrt{(3.35146 \times 10^4)} = -2 (1.83070 \times 10^2) \\ &= -366.140. \end{aligned}$$

Now

$$\cos (29^\circ 21.7') = 0.87154$$

$$\cos (149^\circ 21.7') = -0.86041$$

$$\cos (269^\circ 21.7') = -0.01114.$$

Hence, it is seen that the only solution where $0 < [Z(P_s)_{j/2}]^* < 4.3$ is

$$\begin{aligned} [Z(P_s)_{j/2}]^* &= (-366.140) (-0.01114) \\ &= 4.07880. \end{aligned}$$

Reference to tables of the normal distribution gives

$$[(P_s)_j]^* = 0.99997.$$

Step 10. Since only a maximum limit is specified:

$$\theta_1 = 0.5, \theta_2 = 0, \text{ and } \theta_3 = 0.5,$$

giving $(P_s)_j = (0.5)(0.99997) + 0.5 = 0.99999$.

Step 11. Since $m = 1$,

$$P_s = \prod_{j=1}^1 (P_s)_j = 0.99999 \text{ and } (0.99)^m = 0.99.$$

Step 12. With a 99 per cent confidence level, the probability that the transducer will perform successfully in the assumed application is 99.999 per cent.

Reference to the plots of Figure 2 shows that for the conditions of the assumed application, 1.26×10^{10} ergs/(g)(C)(hr) is the absolute maximum radiation flux that can be tolerated with respect to radiation-induced internal temperature where the risk of obtaining an internal temperature of 900 F or more is no greater than 1 percent at the 99 percent confidence level. This absolute maximum value does not provide for any safety factor or uncertainty in radiation flux.

Example Analysis for Effects of Integrated Neutron Exposure

Step 1. The remaining portion of the analysis is concerned with the relation of transducer sensitivity and zero shift to integrated fast-neutron exposure. The raw data are tabulated in Table 9.

Step 2. The data are reduced by computing the sensitivity and zero shift. The sensitivity, $(y_1)_{\lambda i}$ is computed from:

$$(y_1)_{\lambda i} = \frac{(y_1)_{\lambda i}' + (y_1)_{\lambda i}''}{2},$$

where

$$(y_1)_{\lambda i}' = \frac{(V_1)_{\lambda i} - (V_0)_{\lambda i}}{50} \text{ and } (y_1)_{\lambda i}'' = \frac{(V_2)_{\lambda i} - (V_0)_{\lambda i}}{100},$$

the units being millivolts per per cent of travel. The zero shift $(y_2)_{\lambda i}$ is computed from the null position data, $(V_0)_{\lambda i}$, by subtracting the 0000 time output value from the other measurements [i. e., $(y_2)_{1i} = (V_0)_{1i} - 118.8$ and $(y_2)_{2i} = (V_0)_{2i} - 159.8$]. The reduced data are tabulated against integrated reactor power in Table 10.

TABLE 9. MEASURED TRANSDUCER OUTPUT FOR VARIOUS POSITIONS

Time ^(a) , hr	Reactor Power Level, Mw	Specimen 1 Output, mv			Specimen 2 Output, mv		
		Null (V ₀) _{1i}	50% Travel (V ₁) _{1i}	100% Travel (V ₂) _{1i}	Null (V ₀) _{2i}	50% Travel (V ₁) _{2i}	100% Travel (V ₂) _{2i}
0000	47.6	118.8	129.3	139.3	159.8	170.0	180.6
0100	47.6	121.9	132.8	143.6	163.2	173.6	183.9
0200	47.9	123.1	133.5	144.0	163.2	173.6	184.0
0300	48.1	125.2	136.0	146.2	165.5	175.6	186.3
0400	48.1	128.2	138.7	149.3	166.3	176.3	187.0
0500	48.1	129.2	140.8	151.6	166.0	176.3	187.1
0600	48.1	135.0	144.6	155.8	162.9	173.1	183.3
0700	49.1	134.0	144.9	155.5	167.3	177.6	188.3
0800	49.1	135.7	147.0	156.4	167.1	175.3	187.5
0900	49.5	136.0	146.8	156.8	168.1	178.3	188.7
1000	49.5	135.9	146.7	157.0	169.6	179.3	189.8
1100	50.7	138.5	148.8	159.3	168.8	179.3	189.8

(a) Time measured from start of nominally constant reactor power.

TABLE 10. SENSITIVITY AND ZERO SHIFT FOR VARIOUS INTEGRATED REACTOR POWERS

Integrated Reactor Power ^(a) , (IP) _i , Mw-hr	Specimen 1 ($\lambda = 1$)				Specimen 2 ($\lambda = 2$)			
	Sensitivities			Zero Shift, (Y ₂) _{1i} , mv	Sensitivities			Zero Shift, (Y ₂) _{2i} , mv
	(Y ₁) _{1i} , mv/%	(Y ₁) _{1i} , mv/%	(Y ₁) _{1i} , mv/%		(Y ₁) _{2i} , mv/%	(Y ₁) _{2i} , mv/%	(Y ₁) _{2i} , mv/%	
173.79	0.210	0.211	0.2105	0.0	0.204	0.208	0.2060	0.0
221.39	0.218	0.217	0.2175	3.1	0.208	0.207	0.2075	3.4
269.14	0.208	0.209	0.2085	4.3	0.208	0.208	0.2080	5.7
317.14	0.216	0.210	0.2130	6.4	0.202	0.208	0.2050	5.7
365.24	0.210	0.211	0.2105	9.4	0.200	0.207	0.2035	6.5
413.34	0.232 ^(b)	0.224 ^(b)	0.2280 ^(b)	10.4 ^(b)	0.206 ^(b)	0.211 ^(b)	0.2085 ^(b)	6.2 ^(b)
461.94	0.192 ^(b)	0.208 ^(b)	0.2000 ^(b)	16.2 ^(b)	0.204 ^(b)	0.204 ^(b)	0.2040 ^(b)	3.1 ^(b)
511.04	0.218	0.215	0.2165	15.2	0.206	0.210	0.2080	7.5
560.34	0.226 ^(b)	0.207 ^(b)	0.2165 ^(b)	16.9 ^(b)	0.164 ^(b)	0.204 ^(b)	0.1890 ^(b)	7.3 ^(b)
609.84	0.216	0.208	0.2120	17.2	0.204	0.206	0.2050	8.3
659.34	0.216	0.211	0.2135	17.1	0.194	0.202	0.1980	9.8
709.44	0.206	0.208	0.2070	19.7	0.210	0.210	0.2100	9.0

(a) $(IP)_i = (IP)_{\lambda i} = (IP)_{1i} = (IP)_{2i}$.

(b) The values indicated are deleted from subsequent computations because of apparent measurement discrepancies.

Step 3. The results of computations leading to the least-squares estimates of the regression coefficients are listed in Table 11. The data and the regression lines are plotted in Figures 3 and 4 for sensitivity and zero shift, respectively. For these computations, the data for 413.34, 461.94, and 560.34-Mw-hr integrated exposure have been deleted because the outlying values indicate possible measurement discrepancies.

Step 4. There does not appear to be an a priori basis for predicting the direction of any neutron-exposure-induced change in sensitivity. Hence, $\alpha_1 \neq 0$ is selected as the alternative hypothesis to $\alpha_1 = 0$. From Table 11 it is seen that for both specimens $|(A_1)_1| < B_{\gamma}/2$, so that there is no reason to reject the hypothesis $\alpha_1 = 0$ at the significance level $\gamma = 0.01$. Accordingly no further analysis of sensitivity data is justified and the following statement may be made:

"The transducer sensitivity did not change significantly (at the 0.01 significance level) as a result of fast-neutron fluence under the conditions of this experiment."

"Thus, with 99 percent confidence, the probability of successful transducer performance with respect to sensitivity is at least 99 percent for neutron fluences up to $4.7 \times 10^{17} \text{ n cm}^{-2}$."

With respect to transducer zero shift, it will be assumed for the purpose of illustration that there is a priori justification for expecting an increase in transducer zero-position output signal with increasing fast-neutron fluence if there is a significant effect observed. Thus the alternative to the hypothesis $\alpha_1 = 0$ is $\alpha_1 > 0$. The data of Table 11 show that for both specimens $(A_1)_2 > B_{\gamma}$; ($\gamma = 0.01$), so that the alternative hypothesis must be accepted. Since this means that there is a significant increase in zero shift with increasing fast-neutron fluence, further analysis of the zero-shift data is warranted.

Step 5. The data plots of Figure 4 show that the largest change in zero-displacement output voltage is observed for Specimen 1. Assuming that the two specimens are nominally identical, all observed differences between specimens may be attributed to different experimental radiation-flux values and Specimen 1 may be used as a reference unit to compute the effective fast-neutron fluence. First, the effective integrated reactor power is computed for each data point of both specimens by

$$(\text{IERP})_{\lambda i} = \left[\frac{(A_1)_{\lambda}}{(A_1)_1} \right] (\text{IP})_{\lambda i} .$$

The values obtained are given in Table 12.

For this example, it will be assumed that the fast-neutron flux at the location of Specimen 1 is $3 \times 10^{13} \text{ n/(cm)}^2 \text{ (sec)}$ for a reactor power of 50 Mw. This is equivalent to a fast-neutron fluence per megawatt hour of $\phi_1 = 2.2 \times 10^{15} \text{ n/(cm}^2 \text{)(Mw-hr)}$.^{*} Using this value for ϕ_1 , the effective fast-neutron fluence for each specimen data point is

$$x_{\lambda i} = (2.2 \times 10^{15}) (\text{IERP})_{\lambda i} \text{ n/cm}^2 .$$

^{*}The dosimetry data from the experiment should be used to obtain an experimental value for ϕ_1 . However, these data are not available and the above value for ϕ_1 is used for the example calculations.

TABLE 11. ESTIMATION OF REGRESSION COEFFICIENTS AND TEST FOR SIGNIFICANT EFFECT OF INTEGRATED NEUTRON EXPOSURE

Statistic(a)	Computed Values - Sensitivity (j = 1)		Computed Values - Zero Shift (j = 2)	
	Specimen 1, ($\lambda = 1$)	Specimen 2, ($\lambda = 2$)	Specimen 1, ($\lambda = 1$)	Specimen 2, ($\lambda = 2$)
$n_{j\lambda}$	9	9	9	9
$\Sigma x_{\lambda i}^2$	1.95673×10^6	1.95673×10^6	1.95673×10^6	1.95673×10^6
$\Sigma x_{\lambda i}$	3.83636×10^3	3.83636×10^3	3.83636×10^3	3.83636×10^3
$(\Sigma x_{\lambda i})^2$	14.71766×10^6	14.71766×10^6	14.71766×10^6	14.71766×10^6
$n_{j\lambda} (\Sigma x_{\lambda i}^2) - (\Sigma x_{\lambda i})^2$	2.89291×10^6	2.89291×10^6	2.89291×10^6	2.89291×10^6
$\Sigma (y_{j\lambda})^2$	0.40502	0.38079	1.36480×10^3	0.40004×10^3
$\Sigma y_{j\lambda}$	1.90900	1.85100	92.4	53.6
$\Sigma [x_{\lambda i}(y_{j\lambda})_{\lambda i}]$	8.12834×10^2	7.87906×10^2	5.08143×10^4	2.75905×10^4
$(A_0)_{j\lambda}$	0.21331	0.20713	-4.88784	-0.33405
$(A_1)_{j\lambda}$	-2.80176×10^{-6}	-3.43887×10^{-6}	3.55521×10^{-2}	1.47552×10^{-2}
$(S_x^2)_{j\lambda}$	4.01793×10^4	4.01793×10^4	4.01793×10^4	4.01793×10^4
$(S_y^2)_{j\lambda}$	1.24861×10^{-5}	1.26250×10^{-5}	52.02000	10.10277
$(r_{j\lambda})^2 = (A_1)_{j\lambda}^2 \left[\frac{(S_x^2)_{j\lambda}}{(S_y^2)_{j\lambda}} \right]$	2.52602×10^{-2}	3.76359×10^{-2}	0.97625	0.86586
$(S_{y \cdot x}^2)_{j\lambda}$	1.39093×10^{-5}	1.38854×10^{-5}	1.41182	1.54875
$(S_{y \cdot x'})_{j\lambda}$	3.72952×10^{-3}	3.72631×10^{-3}	1.18820	1.24449
Significance alternative	$\alpha_1 \neq 0$	$\alpha_1 \neq 0$	$\alpha_1 > 0$	$\alpha_1 > 0$
$t_{0.99}^{(n_{j\lambda}-2)}$	--	--	2.998	2.998
$t_{0.995}^{(n_{j\lambda}-2)}$	3.499	3.499	--	--
$\sqrt{(S_x^2)_{j\lambda} (n_{j\lambda} - 1)}$	5.66952×10^2	5.66952×10^2	5.66952×10^2	5.66952×10^2
$(S_{y \cdot x'})_{j\lambda} \sqrt{(S_x^2)_{j\lambda} (n_{j\lambda} - 1)}$	6.57819×10^{-6}	6.57253×10^{-6}	2.09577×10^{-3}	2.19505×10^{-3}
$(B_{\gamma})_{j\lambda}; (\gamma = 0.01)$	--	--	0.62831×10^{-2}	0.65076×10^{-2}
$(B_{\gamma/2})_{j\lambda}; (\gamma = 0.01)$	23.0171×10^{-6}	22.9973×10^{-6}	--	--

(a) For these computations $x_{\lambda i} = (IP)_{\lambda i}$ and $\Sigma = \sum_{i=1}^{n_{j\lambda}}$.

FIGURE 3. TRANSDUCER SENSITIVITY VERSUS INTEGRATED REACTOR POWER

FIGURE 4. TRANSDUCER ZERO SHIFT VERSUS INTEGRATED REACTOR POWER

TABLE 12. EFFECTIVE INTEGRATED REACTOR POWER FOR EACH SPECIMEN

Nominal Integrated Reactor Power ^(a) , (IP) λ_i , Mw-hr	Effective Integrated Reactor Power, (IERP) λ_i , Mw-hr	
	Specimen 1 ($\lambda = 1$)	Specimen 2 ($\lambda = 2$)
173.79	173.79	72.13
221.39	221.39	91.88
269.14	269.14	111.70
317.14	317.14	131.62
365.24	365.24	151.59
511.04	511.04	212.10
609.84	609.84	253.10
659.34	659.34	273.65
709.44	709.44	294.44

(a) (IP) $_{1i}$ = (IP) $_{2i}$.

It is desirable to normalize the zero-shift values to give a value of approximately zero for zero fast-neutron fluence. This is done by algebraically subtracting the intercept values, $(A_0)_{2\lambda}$, of the zero-shift regression lines from each data value to obtain a normalized data value, $y_{\lambda i}$; that is:

$$y_{\lambda i} = (y_2)_{\lambda i} - (A_0) \quad ,$$

where $(y_2)_{\lambda i}$ is the zero-shift value observed for the i^{th} measurement of the λ^{th} specimen. The paired values $(y_{\lambda i}, x_{\lambda i})$ are tabulated in Table 13 in order of increasing value of $x_{\lambda i}$, disregarding the λ subscript which now may be dropped so that $(y_{\lambda i}, x_{\lambda i}) = (y_i, x_i)$.

Step 6. The data of Table 13 are used to compute least-squares estimates of the pooled data regression line and other pertinent statistics. The results of these computations are tabulated in Table 14. The data and regression line are plotted in Figure 5.

Step 7. The statistics of Table 14 are used to compute the 99 percent confidence interval about the regression line. The results of the computations are listed in Table 15 and the interval is plotted in Figure 5.

Step 8. The statistics of Table 14 are also used to compute the 99 percent tolerance band about the regression line for a 99 percent confidence level. The results of these computations are tabulated in Table 16 and the tolerance band is plotted in Figure 5.

Step 9. Since this example is intended solely to illustrate the application of the experimental-design analytical procedures, the following hypothetical requirements for transducer performance will be assumed. First, it is assumed that the position data in the application will be corrected for zero-shift error so that the criterion of transducer-performance acceptability is the amount of error introduced into the corrected data because of uncertainty associated with predicting the value to use in correcting the zero-shift data. It will be assumed that the transducer will be considered acceptable if the error in the corrected output signal data due to uncertainty about the zero-shift

TABLE 13 TABULATION OF ADJUSTED DATA(a)

Effective Fast-Neutron Fluence, x_i , n/cm ²	Normalized Zero Shift, y_i , mv
1 587 x 10 ¹⁷	0 334
2 021 x 10 ¹⁷	3 734
2 457 x 10 ¹⁷	3 734
2 896 x 10 ¹⁷	6 034
3 350 x 10 ¹⁷	6 834
3 823 x 10 ¹⁷	4 888
4 666 x 10 ¹⁷	7 834
4 871 x 10 ¹⁷	7 988
5 568 x 10 ¹⁷	8 634
5 921 x 10 ¹⁷	9 188
6 020 x 10 ¹⁷	10 134
6 478 x 10 ¹⁷	9 334
6 977 x 10 ¹⁷	11 288
8 035 x 10 ¹⁷	14 288
1 124 x 10 ¹⁸	20 088
1 342 x 10 ¹⁸	22 088
1 451 x 10 ¹⁸	21 988
1 561 x 10 ¹⁸	24 588

(a) $(y_i, x_i) = (y_{\lambda i}, x_{\lambda i})$ TABLE 14 REGRESSION COEFFICIENT ESTIMATES
AND STATISTICS FOR POOLED ZERO-
SHIFT DATA

Statistic(a)	Computed Value
n	18
n (n-1)	306
$\left(\frac{n-1}{n-2}\right)$	1 06250
Σx_i	1 19450 x 10 ¹⁹
Σx_i^2	1 11051 x 10 ³⁷
$\bar{x}$	6 63611 x 10 ¹⁷
$n(\Sigma x_i^2) - (\Sigma x_i)^2$	5 72088 x 10 ³⁷
$\Sigma x_i y_i$	1 79435 x 10 ²⁰
Σy_i	1 92998 x 10 ²
Σy_i^2	2 91998 x 10 ³
A_0	-0 00156
A_1	1 61596 x 10 ⁻¹⁷
$\hat{y}_x$	$(1 61596 \times 10^{-17}) x + (0 00156)$
S_x^2	1 86957 x 10 ³⁵
S_y^2	50 03729
$r^2 = (A_1)^2 \left(\frac{S_x^2}{S_y^2} \right)$	0 97569
r	0 98777
$S_y^2 x$	1 29243
$S_y x$	1 13685

(a) Except where otherwise noted $\Sigma = \sum_{i=1}^n$ where $n = \sum_{\lambda=1}^k$ and $k = 2$

TABLE 15 RESULTS OF CONFIDENCE-INTERVAL COMPUTATIONS

x , n/cm ²	$\hat{y}_x$, mv	M	$\left[\sqrt{MS_y^2 x} \right] t_{0.99}^{(n-2)(a)}$	y^c	y_c
1 x 10 ¹⁷	1.61440	0.10050	1.05273	2.667	0.562
2 x 10 ¹⁷	3.23036	0.06818	0.86710	4.097	2.363
4 x 10 ¹⁷	6.46228	0.02242	0.49727	6.960	5.965
6 x 10 ¹⁷	9.69420	0.00183	0.14219	9.836	9.552
8 x 10 ¹⁷	12.92612	0.00641	0.26578	13.192	12.660
1.0 x 10 ¹⁸	16.15804	0.03616	0.63143	16.789	15.527
1.2 x 10 ¹⁸	19.38996	0.09108	1.00217	20.392	18.388
1.4 x 10 ¹⁸	22.62188	0.17117	1.37389	23.996	21.248
1.6 x 10 ¹⁸	25.85380	0.27644	1.74597	27.600	24.108
1.8 x 10 ¹⁸	29.08572	0.40687	2.11819	31.204	26.968

(a) For this computation $t_{0.99}^{(18)} = 2.921$ for $P_c = 99$ per cent

TABLE 16. RESULTS OF TOLERANCE-BAND COMPUTATION

$x, n/\text{cm}^2$	$\hat{y}_x, \text{mv}$	M	$\left(1 + \frac{M}{2} + \frac{M^2}{8}\right)$	$B(a)$	$BS_{y \cdot x}^{(a)}$	$y_T^{(a)},$ mv	$y_T^{(a)},$ mv	$B(b)$	$BS_{y \cdot x}^{(b)}$	$y_T^{(b)},$ mv	$y_T^{(b)},$ mv
1×10^{17}	1.61440	0.10050	1.05151	4.46628	5.07749	6.692	-3.463	0.91018	1.03474	2.649	0.580
2×10^{17}	3.23036	0.06818	1.03467	4.40724	5.01037	8.241	-1.780	0.89570	1.01828	4.126	2.335
4×10^{17}	6.46228	0.02242	1.01127	4.31708	4.90787	11.370	+1.554	0.87552	0.99533	7.458	5.467
6×10^{17}	9.69420	0.00183	1.00092	4.27408	4.85899	14.553	4.835	0.86725	0.98593	10.680	8.708
8×10^{17}	12.92612	0.00641	1.00322	4.28380	4.87004	17.796	8.056	0.86857	0.98743	13.914	11.939
1.0×10^{18}	16.15804	0.03616	1.01824	4.34495	4.93956	21.098	11.218	0.88154	1.00218	17.160	15.156
1.2×10^{18}	19.38996	0.09108	1.04658	4.44948	5.05839	24.448	14.332	0.90595	1.02993	20.420	18.360
1.4×10^{18}	22.62188	0.17117	1.08925	4.58213	5.20919	27.831	17.413	0.94248	1.07146	23.693	21.550
1.6×10^{18}	25.85380	0.27644	1.14777	4.72084	5.36689	31.221	20.487	0.99222	1.12801	26.982	24.726
1.8×10^{18}	29.08572	0.40687	1.22413	4.83661	5.49850	34.584	23.587	1.05659	1.20118	30.287	27.885

(a) Calculation is for $P_T = 99$ per cent, $p_T = 0.99$; $Z_{p_T/2} = 2.575$; $(1 - \epsilon) = 0.99$; $(n - 2) = 16$; $\sqrt{F_\epsilon(\infty, n - 2)} = 1.65831$.

(b) Calculation is for $P_T = 39.8$ per cent; $p_T = 0.398$; $Z_{p_T/2} = 0.52209$; $(1 - \epsilon) = 0.99$; $(n - 2) = 16$; $\sqrt{F_\epsilon(\infty, n - 2)} = 1.65831$.

FIGURE 5. ZERO SHIFT AS A FUNCTION OF FAST-NEUTRON FLUENCE

correction factor is less than ± 2 per cent of the full-scale transducer output. Assuming a nominal full-scale transducer output of 50 mv, it is seen that the maximum acceptable zero-shift uncertainty is ± 1.0 mv.

Reference to the 99 per cent tolerance band plotted in Figure 5 shows that the zero-shift uncertainty associated with the 99th percentile of the transducer population is about a factor of 5 larger than the maximum acceptable uncertainty. This means that on the basis of the available data, the probability that the transducer successfully meets the assumed requirement, $(P_s)_j$, is less than 99 per cent; i. e., $(P_s)_j < 0.99$.

To compute an actual value of $(P_s)_j$, the value of acceptable zero-shift uncertainty, ± 1.0 mv, is employed to compute $(B_j)^*$ and $(B_j)_*$. For this computation, it will be assumed that the maximum fast-neutron fluence expected for the application is $x_j = 4 \times 10^{17}$ n/cm². Hence:

$$(B_j)^* = \frac{1.0}{S_{y \cdot x}} = \frac{1}{1.13685} = 0.87962 .$$

Now since $(P_s)_j < 0.99$, it is not necessary to compute C (which corresponds to a $(P_s)_j = 0.99999$) to establish that $(B_j)^* < C$. It is seen that

$$a = - \frac{12 \left(1 + \frac{M}{2} + \frac{M^2}{8} \right)}{M^2} = - \frac{12(1.01127)}{(0.02242)^2} = - 2.41422 \times 10^4$$

$$b = \frac{12(B_j)^*}{M^2 \sqrt{F_{0.99(\infty, 16)}}} = \frac{12(0.87962)}{(0.02242)^2 (1.65831)} = 1.26631 \times 10^4,$$

and

$$\begin{aligned} D &= \left(\frac{b}{2} \right)^2 + \left(\frac{a}{3} \right)^3 = (0.63315 \times 10^4)^2 - (0.80474 \times 10^4)^3 \\ &= 4.00879 \times 10^7 - 5.21158 \times 10^{11} = -5.21118 \times 10^{11} . \end{aligned}$$

Since $D < 0$,

$$\begin{aligned} \phi &= \cos^{-1} \sqrt{\frac{(b/2)^2}{-(a/3)^3}} = \cos^{-1} \sqrt{\frac{4.00879 \times 10^7}{5.21158 \times 10^{11}}} = \cos^{-1} \sqrt{7.69208 \times 10^{-5}} \\ &= \cos^{-1} (0.00877) = 89^\circ 29.86', \end{aligned}$$

and

$$\phi/3 = 29^\circ 50' .$$

Also,

$$\begin{aligned} \left[-2 \sqrt{-\left(\frac{a}{3} \right)} \right] &= -2 \sqrt{(0.80474 \times 10^4)} = -2 (0.89707 \times 10^2) \\ &= -179.414. \end{aligned}$$

Now,

$$\cos (29^\circ 50') = 0.86748$$

$$\cos (149^\circ 50') = -0.86457$$

$$\cos (269^\circ 50') = -0.00291.$$

It is seen that the only solution where $0 < [Z(P_s)_{j/2}]^* < 4.8$ is:

$$[Z(P_s)_{j/2}]^* = (-179.414) (-0.00291) = 0.52209$$

Reference to tables of the normal distribution gives $[(P_s)_j]^* = 0.398$. Since the assumed requirement limits are symmetrical, it is seen that $[(P_s)_j]^* = 0.398$ also.

Step 10. Since both the upper and lower limit are specified:

$$\theta_1 = 0.5, \theta_2 = 0.5, \text{ and } \theta_3 = 0,$$

giving $(P_s)_j = (0.5)(0.398) + (0.5)(0.398) = 0.398$.

Step 11. This portion of the analysis has considered the two parameters; sensitivity and zero shift. Thus, $m = 2$ and

$$P_s = \prod_{j=1}^2 (P_s)_j = (1.00000)(0.398) = 0.398.$$

The confidence level is $(0.99)^2 = 0.98$. To show the relation between the zero-shift uncertainty and integrated neutron exposure, the tolerance band for $P_T = 39.8$ per cent at a 99 per cent confidence level is computed and tabulated in Table 16 and plotted in Figure 6.

FIGURE 6. 39.8 PERCENT TOLERANCE BAND ON ZERO SHIFT AS A FUNCTION OF FAST-NEUTRON FLUENCE

Step 12. The minimum probability that the transducer will perform successfully in the assumed application is 39.8 percent with a 98 percent confidence level.

The high correlation ($r = 0.9877$) between zero shift and integrated neutron exposure indicates that the effect should be quite predictable and that correction factors can be determined and used to correct the data in the application, assuming that the neutron flux as a function of time at the use location of the transducer can be established. The relatively high uncertainty associated with the prediction of a zero-shift correction factor shown by this example analysis may be a result of the relatively small sample size and possible excessive experimental error. Further experimentation would be required to establish whether the true probability of success for the transducer is greater than 39.8 percent.*

It is seen from Figures 5 and 6 that the random uncertainties associated with the prediction of zero shift as a function of fast-neutron fluence are nearly independent of fluence. (The small variation with fluence is inherent to the statistics involved in computing tolerance bands, and as long as the experimental data covers the value of fast-neutron fluence expected in the application so that extrapolation is not required, the variation should be small if the correlation coefficient is close to one.) Thus, no statement about maximum permissible fast-neutron fluence can be deduced from these data. The implication is that if the random uncertainty pertaining to the prediction of zero-shift correction factors can be reduced to an acceptable level, there may be no limitation on fast-neutron fluence within the range of interest.

It is important to recognize that, in addition to the random error associated with prediction of the zero-shift correction factor, there is also a systematic error associated with measurement of radiation flux and hence fast-neutron fluence. The experimental value of fast-neutron fluence thus may be in error by about ± 30 percent. This means that the uncertainty about the absolute value of zero shift from this source and hence the uncertainty about any zero-shift correction factor will be a function of the fast-neutron fluence. For the example, this uncertainty amounts to $\pm(4.84788 \times 10^{-18}) x$; i.e., $Y_x = Y_x \pm (4.84788 \times 10^{-18}) x$. This can result in a substantial error in correcting for zero shift far exceeding that to be expected from random error. Hence, it is concluded that the correction of zero shift by determining the fast-neutron fluence and applying a correction factor is not too promising an approach. A change in transducer design that eliminates or at least substantially reduces the amount of zero shift resulting from fast-neutron fluence appears to be essential. It is seen that the probability of meeting Reactor System requirements is very small when the uncertainty due to the inherent inaccuracy of neutron-fluence determination is considered.

*This example analysis was performed using data from pressure transducers rather than position transducers and the results are expected to show only that the analysis procedure is workable. However, the results may also give an indication of potential problems in the position-transducer experiment. It is seen that sample size and experimental error are critical factors with respect to making high-confidence predictions of zero-shift correction factors.

Summary of Analysis Results

The relationship of the results of this analysis to the experimental purpose and objectives listed in Appendix A on pages A-2 and A-3 respectively, is shown in the following summary.* The portion of the analysis contributing each result is shown by reference to the part, step and page numbers that are pertinent.

Experimental objective 1 is met by the results of Part 1, Steps 9 and 10, Pages B-7, B-13 and B-14, which lead to the statement:

"There is a 99 percent certainty that the probability that coil winding temperatures in the XYZ Corporation Model 001 LDT will not exceed the manufacturers specified maximum of 900 F during exposure to radiation fluxes of up to 9.52×10^9 ergs / g (c)(hr) is at least 99.997 percent."

Experimental objectives 2 and 6 are met by the results of Part 2, Step 4, Page B-17 which leads to the statement:

"There is a 99 percent certainty that neutron induced changes in LDT sensitivity will not be significant in the Reactor System Application."*

Experimental objective 3 is met by the results of Part 2, Step 10, Page B-25 which leads to the statement:

"There is a 99 percent certainty that the probability is 39.8 percent that neutron-irradiation induced increase in LDT zero-position output signal (zero shift) will be correctable with a residual error of less than ± 2 percent of the full-scale transducer output."

Experimental objective 4 is met by Figure 5, Page B-23, which graphically shows the results of Part 2, Steps 6, 7, and 8, Page B-20.

Experimental objective 5 is met by Figure 2, Page B-11, which graphically shows the results of Part 1, Steps 6, 7, and 8, Page B-7.

This example analysis does not include an examination of failed specimens so no consideration has been given to objective 7 or Experimental Purpose (3).

The results of Step 12, Page B-14, and Step 12, Page B-26, satisfy the Experimental Purpose (1) leading to the statement:

"There is a 99 percent certainty that the XYZ Corporation linear displacement transducer, Model 001, will satisfy the reactor-system performance requirements specified in the System Specifications, Code No. P -10, with a probability of at least 99 percent with respect to radiation-flux-induced internal temperature increase. With a 98 percent certainty, the probability of satisfying the requirements pertaining to neutron fluence is 39.8 percent."

*It is assumed that the requirements of the application involve a radiation flux of 9.52×10^{10} ergs/g(c)(hr) and a neutron fluence less than 4.7×10^{17} n cm⁻².

Uncertainty associated with connections for neutron-irradiation-induced increase in zero shift is the prime cause of this low probability of successful performance."

Figures 2 and 5, as listed for objectives 4 and 5, together with Figure 6, Page B-25 satisfy Experimental Purpose (2).

APPENDIX C

SUMMARY OF STATISTICAL FORMULAS

APPENDIX C

SUMMARY OF STATISTICAL FORMULAS

The key statistical formulas employed in the analysis procedure are given below in a form suitable for performing the computations. This appendix has been included to provide a means for ready reference to these formulas as the computations are performed. In this appendix, the symbol Σ will in all cases mean the summation over an

interval from $i = 1$ to n ; i. e., $\Sigma = \sum_{i=1}^n$. With each formula is included a reference to the page number where the formula is introduced in the analytical procedure.

A_0 with or without subscripts or primes is equal to the estimate of the ordinate intercept, α_0 , of a regression line. The value for A_0 is computed by

$$A_0 = \frac{(\Sigma y_i)(\Sigma x_i) - (\Sigma x_i)(\Sigma x_i y_i)}{n(\Sigma x_i^2) - (\Sigma x_i)^2}.$$

A_1 with or without subscripts or primes is equal to the estimate of the slope, α_1 , of a regression line. The value for A_1 is computed by

$$A_1 = \frac{n(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{n(\Sigma x_i^2) - (\Sigma x_i)^2}.$$

Values of statistics pertaining to the regression line $\hat{y}_x = A_0 + (A_1)x$ are computed by

$$S_x^2 = \frac{n(\Sigma x_i^2) - (\Sigma x_i)^2}{n(n-1)}$$

$$S_y^2 = \frac{n(\Sigma y_i^2) - (\Sigma y_i)^2}{n(n-1)}$$

$$\begin{aligned} S_{y \cdot x}^2 &= \left(\frac{n-1}{n-2} \right) \left[1 - (A_1)^2 \frac{(S_x^2)}{(S_y^2)} \right] S_y^2 \\ &= \left(\frac{n-1}{n-2} \right) (1-r^2) S_y^2, \end{aligned}$$

where r denotes the correlation coefficient

$$r = \sqrt{(A_1)^2 \frac{(S_x^2)}{(S_y^2)}}.$$

$\hat{y}_x$ denotes the predicted y-value associated with a specified x-value, x:

$$\hat{y}_x = (A_0) + (A_1)x$$

$Y^C(x)$ and $Y_C(x)$ denote the upper and lower limits of a P_C -per cent confidence interval $[Y^C(x), Y_C(x)]$ for the mean value of the ordinate, Y_x :

$$Y^C(x) = \hat{y}_x + \left[t_{p_c}^{(n-2)} \right] \sqrt{MS_{y \cdot x}} \geq Y_x \geq \hat{y}_x - \left[t_{p_c}^{(n-2)} \right] \sqrt{MS_{y \cdot x}} = Y_C(x),$$

where

$$p_c = P_c / 100$$

$$M = \left[\frac{1}{n} + \frac{(x - \bar{x})^2}{(n-1)S_x^2} \right]$$

$$\bar{x} = \left(\frac{1}{n} \right) \sum x_i,$$

and $t_{\alpha}^{(n)}$ equals the $100\alpha^{\text{th}}$ percentile of the t-distribution with n degrees of freedom, n being an integer and α being a number in the interval $0 < \alpha < 1$

$Y^T(x)$ and $Y_T(x)$ denote the upper and lower limits of a P_T -per cent tolerance interval $[Y^T(x), Y_T(x)]$ on the population of y-values associated with the x-value, x:

$$Y^T(x) = \hat{y}_x + BS_{y \cdot x} \geq Y_x \geq \hat{y}_x - BS_{y \cdot x} = Y_T(x),$$

where

$$B = \left[\left(1 + \frac{M}{2} + \frac{M^2}{8} \right) Z_{PT/2} - \left(\frac{M^2}{12} \right) Z_{PT/2}^3 \right] \sqrt{F_{\epsilon}(\infty, n-2)}$$

$$p_t = \frac{P_T}{100}$$

$$M = \left[\frac{1}{n} + \frac{(x - \bar{x})^2}{(n-1)S_x^2} \right]$$

Z_{α} = the $100\alpha^{\text{th}}$ percentile of the standard normal distribution,

and

$F_{\epsilon}(\infty, n-2)$ = that number, F_{ϵ} , such that $\int_0^{F_{\epsilon}} \left[h_{m_1, m_2}(F) \right] dF = 1-\epsilon$, $m_1 = \infty$,

$m_2 = n-2$, where $h_{m_1, m_2}(F)$ is the density function of the F -distribution with m_1 and m_2 degrees of freedom.