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Calibration of Tandem Analyzing Magnet

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by

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CALIBRATION OF TANDEM ANALYZING MAGNET

Norman H. Magee
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ABSTRACT

A particle scattering method has been used to help establish the energy calibration of the analyzing magnet at the LASL tandem Van de Graaff. The method and experimental procedures are described in some detail and a fairly complete presentation of the results is given. A least-squares fit to a large number of values obtained for the magnet "constant" K {defined for protons by $f = K [E(1 + E/2M)]^{\frac{1}{2}}$ } gives $K = (6.057 \pm 0.004) - (9 \pm 18)10^{-16} f$ in the proton energy range of 10-20 MeV. Although K is found to be approximately constant over the range of the measurements, the value obtained does not agree with the value, 6.065 ± 0.002 , derived from precise measurements of the $^{27}\text{Al}(p,n)$ threshold at 5.797 MeV.

INTRODUCTION

The analyzing magnet on the LASL tandem Van de Graaff has previously been calibrated only by (p,n) threshold measurements for ^{27}Al at approximately 5.800 MeV and ^{58}Ni at 9.48 MeV. The results of these two measurements implied significantly different "magnet constants" at the two energies. The purpose of this experiment is to check these calibrations by a different method, to extend them to higher energies, and to verify or disprove the variation of the magnet constant.

The method used here has been referred to as the crossover technique and has been treated non-relativistically by Heymann and Keizer¹ and by Bardin and Richey,² and relativistically by Smythe.³ The method is kinematically exact; the energy of the incident beam is dependent solely on

the angle for which beam particles scattered inelastically from a low mass constituent of the target and elastically from hydrogen in the same target have the same energy.

THEORY

As one goes toward backward angles, the energy of a light, elastically scattered particle decreases much more rapidly than the energy of the same particle inelastically scattered from a heavier nucleus. There is, in many cases, a unique angle at which the energies of two such particles are equal. Since both particles came from a monoenergetic beam, they must have lost equal amounts of energy in the scattering process. We can then determine an exact expression for the energy of the incident beam from the angle at which this crossover takes place.

Kinematics (Nonrelativistic)

From the standard conservation laws, the energy of a resultant light particle from a nuclear reaction as a function of lab angle θ is:

$$\sqrt{E_3} = \frac{\sqrt{M_1 M_3 E_1} \cos \theta \pm \left\{ M_1 M_3 E_1 \cos^2 \theta + (M_3 + M_4) \left[E_1 (M_4 - M_1) + Q M_4 \right] \right\}^{\frac{1}{2}}}{M_3 + M_4} \quad (1)$$

For inelastic scattering, $M_3 = M_1 = M$, the mass of the incident particle, and $M_2 = M_4 = M_t$, the mass of the target nucleus, Eq. (1) reduces to

$$\sqrt{E} = \frac{M \sqrt{E_1} \cos \theta \pm \left[\frac{M^2 E_1}{M_t} - M^2 E_1 \sin^2 \theta - (M + M_t) M_t E_x \right]^{\frac{1}{2}}}{M + M_t} \quad (2)$$

where E_1 is the energy of the incident particle, E is the energy of the scattered particle, and E_x is the excitation energy of the target nucleus.

For elastic scattering of identical particles, the expression simplifies still further to

$$\sqrt{E} = \sqrt{E_1} \cos \theta \quad (3)$$

Equating the two expressions for the energy of the scattered particle, we obtain a mathematically unique angle, θ , at which the two energies are equal

$$\theta_c = \cos^{-1} \left\{ \left[1 - \frac{M_t E_x}{(M_t - M) E_1} \right]^{\frac{1}{2}} \right\} \quad (4)$$

or

$$E_1 = \frac{M_t E_x}{(M_t - M) \sin^2 \theta_c} \quad (5)$$

where θ_c is the critical angle at which the crossover occurs.²

Kinematics (Relativistic)

The nonrelativistic expressions for the incident energy as a function of the crossover angle are 0.3% in error at 12 MeV and 1% at 24 MeV.³ The nonrelativistic expressions have therefore been used only in the evaluation of errors, while the relativistic expressions have been used in actual energy calculations.

According to the relativistic conservation laws for two identical particles emergent from a nuclear process with the same angle and energy, (one inelastically scattered from a heavy nucleus, the other elastically scattered from an identical nucleus), the momentum transferred in the reaction is³

$$P^2 = \frac{E_x \left(M_t - M + \frac{E_x}{2} \right) \left[2M_t M + E_x \left(M + M_t + \frac{E_x}{2} \right) \right]}{(M_t - M)^2} \quad (6)$$

where all terms have been defined in the previous section:

From conservation of momentum

$$\cos \theta = \frac{E_1^2 + 2ME_1 + P_2^2 - P^2}{2(E_1^2 + 2E_1 M)^{\frac{1}{2}} P_2} \quad (7)$$

where P_2 is found from conservation of momentum.

$$P_2^2 = (E_1 - E_r) (E_1 - E_r + 2M) \quad (8)$$

where E_r is the energy transferred in the reaction

$$E_r = (M^2 + P^2)^{\frac{1}{2}} - M \quad (9)$$

so that

$$\theta = \cos^{-1} \frac{E_1(E_1 + 2M) + (E_1 - E_r)(E_1 - E_r + 2M) - P^2}{2 \left[(E_1 - E_r)(E_1 - E_r + 2M)(E_1^2 + 2E_1 M) \right]^{\frac{1}{2}}} \quad (10)$$

The dependence of E_1 on θ is plotted in Graph 1 for inelastic scattering from the 4.433- and the 9.629-MeV states of ^{12}C .

Graph 1. Crossover Angle vs Energy (Theoretical) for $^{12}\text{C}(p,p')^{12}\text{C}^*$ and p,p Elastic.

EXPERIMENTAL PROCEDURE

The prime consideration of this experiment was to define the angle of the scattered particles as closely as possible, since a difference of 0.05° corresponds to an energy shift of 40 keV at 15 MeV. To achieve the desired accuracy, very tight collimation was used throughout the experimental setup,

which caused the counting rate to be low, no more than 3600 counts being taken for any point. The experimental setup is shown in Fig. 1. The beam collimators are all arranged so that when the incident beam passes through the target, it passes directly through the center of rotation of the detector. This was arranged by use of a leveled transit by which the first two sets of slits were optically aligned with the quadrupole on the beam tube. Then the center of the chamber (marked by a pin placed on the target holder) and the third set of slits were aligned on the same line. Each of the eight slit edges of the second and third set of slits was insulated so that currents produced by the beam striking them could be monitored to give information on the beam's location.

A TMC lithium drifted detector, type W80-2, 2-mm thick and capable of stopping 18-MeV protons, was used. The 0.014-in.-wide by 3/8-in.-high slit placed in front of the detector and located 8.75 in. from the target gave an angular spread at the detector of only 6 minutes. An approximate calibration of the absolute angle of the detector to the beam was determined by reading the angle at which the detector was located when the slit was aligned with the transit line. The error was found to be no greater than 0.4° for any run, usually much less. The signal generated in the detector was amplified and then counted using the electronic setup shown in Fig. 2.

The operational procedure was relatively simple once the apparatus was set up. After obtaining the expected value of the crossover angle from

Fig. 1 Experimental Setup.

Fig. 2 Electronics.

Smythe,⁴ an angular distribution was taken in 0.5° to 1° steps in a range of 3° to 4° on both sides of the critical angle. For each incident energy, the crossover angle was determined on both sides of the beam, the average angle giving the true crossover angle. Several times, when both sides had been taken, some points were repeated on the first side to make sure that the detector angle could be repeated accurately, and no significant deviations were observed.

Two computer programs were used in the data analysis. The first determined the exact channel number of the proton peaks in the energy spectrum, and the second took this information and determined the energy of the incident beam, uncorrected for energy loss in the target.

The program used to calculate the peak location was written by W. S. Hall (P-12) and performed the calculation by fitting a skewed gaussian curve to the experimental points. The other program took the peak locations at different angles for the hydrogen and carbon peaks and fit a curve to each set of data, using a least-squares fit. The intersection of these curves gave the crossover angle. After both crossover angles for each energy had been obtained, they were averaged and the incident proton energy was calculated, using the expressions developed in the theory. A complete writeup of this program is listed in the Appendix. It is recommended that the program be run on a computer with greater than eight-digit accuracy or errors will result from rounding off.

EXPERIMENTAL RESULTS

The data in this experiment were taken on four different days, each data run requiring all of the experimental apparatus to be set up from scratch. The first day consisted of one 10-MeV run, Case I, and the 12.5-MeV run, Case II. Case I of 12.5 MeV was thrown out because the back collimators were taken out before the run, the beam spot on target was found to be off center by about $1/4$ in., and the resultant value for the energy was in error by more than 100 keV. The second group of data included the other two 10-MeV cases and two 15-MeV runs, Cases 4.433 I and 9.629. On the third day, one more 15-MeV run, Case 4.433 II, was taken along with all the 17.5-MeV and 20-MeV runs. All of the runs on the last day were taken with the vertical slits just before the chamber set at $1/32$ in. Run Number 15 MeV A from last year was also included in the data, as it was the only run taken which had the collimators set as in Fig. 1. The other runs had the final slits either removed or set twice as wide as shown, which apparently caused the inconsistencies that made the data unusable.

In all but the first day's runs this year, one of the previous energies was repeated to ensure that the experimental setup used would produce consistent values. The results of all the data runs are shown in Graphs 2 to 13 and are listed in Table I. All of the data were analyzed with the carbon curve considered to be a straight line, because the curvature was very slight over the angular distribution shown in each graph. The uncorrected ener-

Graph 2. 10 MeV, Case I, Right and Left Crossovers.

Graph 3. 10 MeV, Case II, Right and Left Crossovers.

Graph 4. 10 MeV, Case III, Right and Left Crossovers.

Graph 5. 12.5 MeV, Case II, Right and Left Crossovers.

Graph 6. 15 MeV, Case A, Right and Left Crossovers.

Graph 7. 15 MeV, 4.433 Cross I, Right and Left Crossovers.

Graph 8. 15 MeV, 9.629 Cross, Right and Left Crossovers.

Graph 9. 15 MeV, 4.433 Cross II, Right and Left Crossovers.

Graph 10. 17.5 MeV, 4.433 Cross, Right and Left Crossovers.

Graph 11. 17.5 MeV, 9.629 Cross, Right and Left Crossovers.

Graph 12. 20 MeV, 4.433 Cross, Right and Left Crossovers.

Graph 13. 20 MeV, 9.629 Cross, Right and Left Crossovers.

TABLE I

DATA

Approx. Energy (MeV)	Frequency (MHz)	θ_{Lab} (Deg)	Channel p, p'	Channel $^{12}\text{C}(p, p')^{12}\text{C}^*$	Data Run	Approx. Energy (MeV)	Frequency (MHz)	θ_{Lab} (Deg)	Channel p, p'	Channel $^{12}\text{C}(p, p')^{12}\text{C}^*$	Data Run						
10	19.229	40	348.386	187.299	① I	12.5	21.5130	42	207.370	180.593	① II						
		41	304.962	183.069				42.5	200.413	179.626							
		42	261.379	179.797				45	155.565	176.783							
		43	216.650	174.265				45.5	146.211	174.797							
		45	118.113	166.627				46	137.490	175.608							
		45.5	89.621	164.307				46.5	120.236	175.190							
		46	60.939	162.485				47	118.462	174.713							
		314	60.506	162.910				313	129.916	173.747							
		314.5	88.821	164.980				314	148.906	175.756							
		315	117.991	166.776				314.5	157.159	176.532							
		317	216.412	174.723				317	201.311	100.080							
		317.5	238.212	177.212				317.5	210.401	180.693							
		318	260.324	179.649				318	218.943	181.624							
		318.5	281.937	181.173				319	236.206	183.010							
								320	254.396	184.458							
								322	289.916	187.549							
		10	19.2289	40				246.588	186.780	② II		12.5	21.5130	33.5	346.103	191.988	① III
				41				229.380	185.349					34	329.616	190.728	
				42				211.618	183.738					34.5	316.469	189.607	
				43				193.908	182.081					35	298.514	187.944	
45	158.996			179.674	35.5	285.656	187.072										
45.5	149.944			178.875	36	267.502	185.566										
46	140.675			178.042	36.5	253.224	184.428										
47	121.008			176.117	37	235.491	182.483										
48	98.215			174.746	37.5	221.465	101.203										
313	132.577			177.267	38	204.522	180.436										
314	150.489			178.987	40	139.453	174.644										
314.5	160.121			179.993	40.5	122.454	173.135										
317	204.139			183.714	41	103.321	171.966										
317.5	212.230			184.291	41.5	82.014	170.531										
318	221.362			185.227	319.5	85.190	170.269										
319	238.838			186.375	320	106.855	171.615										
10	19.2289			38	279.464	186.613	② III	12.5	21.5130		320.5			125.184	173.184		
				39	262.262	185.771					321			142.525	174.507		
				40	244.700	184.397					323			205.908	180.065		
				41	227.141	183.089					323.5			222.682	180.838		
					324	246.074				185.393							

TABLE I (Contd)

Approx. Energy (MeV)	Frequency (MHz)	θ_{Lab} (Deg)	Channel P, P'	Channel $^{12}\text{C}(p, p')^{12}\text{C}^*$	Data Run	Approx. Energy (MeV)	Frequency (MHz)	θ_{Lab} (Deg)	Channel P, P'	Channel $^{12}\text{C}(p, p')^{12}\text{C}^*$	Data Run
		324.5	254.212	184.086				321	224.612	275.678	
		325	278.522	187.677				322	236.962	276.560	
		325.5	287.018	186.764				323	249.393	277.708	
		326	305.234	189.343				324	261.863	278.710	
		326.5	318.665	189.630				327	298.128	281.838	
		327	333.663	191.313				328	309.559	282.563	
								329	321.068	283.500	
								330	332.176	284.154	
15	23.581	31	228.6	199.1	(A) Masterson	17.5	25.4785	28	343.763	232.536	(3) 4.4
		32	218.0	198.2				29	311.217	230.290	
		33	208.8	197.4				30	276.828	228.460	
		33.5	203.0	196.7				30.5	264.638	228.873	
		34	198.2	196.3				33	171.746	219.425	
		35	188.0	195.7				34	135.840	216.971	
		35.5	183.3	195.3				34.5	117.692	215.079	
		36	178.0	194.8				35	99.529	213.605	
		37	166.5	193.8							
		38	155.2	192.8							
		322	162.2	193.3							
		323	173.1	194.2				325	99.049	214.365	
		324	184.0	195.1				325.5	117.347	215.869	
		324.5	189.0	195.6				326	136.027	217.445	
		326	203.8	197.0				327	172.442	220.433	
		326.5	208.8	197.4				329.5	261.359	228.368	
		327	213.5	198.1				330	278.321	229.619	
		328	223.0	198.6				331	312.199	232.046	
								332	345.463	234.185	
15	23.5819	30	344.462	263.019	(2) 4.4	17.5	25.4785	47.5	322.833	176.718	(3) 9.6
		31	325.036	260.909				48	298.135	175.432	
		32	304.506	259.473				49	249.064	171.907	
		33	282.427	257.470				52.5	79.242	157.382	
		35	233.573	253.093				53	59.678	154.994	
		35.5	217.193	251.293							
		36	204.003	251.354							
		37	165.030	248.936							
								307	56.711	154.745	
								307.5	76.600	154.261	
		323	187.974	249.204				311	243.622	169.981	
		323.5	206.923	250.638				311.5	268.665	169.852	
		324	221.471	251.936				312	292.206	171.276	
		324.5	235.143	253.212							
		326.5	284.437	257.560							
		327	295.295	258.712		20	27.2657	25	279.793	228.479	(3) 4.4
		328	315.853	260.380				26	267.765	227.355	
		329	335.484	262.124				27	254.563	226.393	
		330	355.281	264.084				27.5	248.778	226.212	
								31	198.272	221.779	
								32	182.163	220.251	
								33	166.274	219.396	
15	23.5819	53	280.404	189.247	(2) 9.6						
		54	251.291	187.222							
		55	222.218	185.197							
		55.5	208.540	183.279				325	131.394	217.022	
		57.5	152.643	178.518				327	167.346	219.529	
		58	137.584	176.896				328	183.161	220.072	
		58.5	122.776	176.575				329	198.593	221.518	
		59	105.769	175.111				332.5	249.076	226.149	
		60	66.093	172.873				333	255.421	226.091	
								334	268.298	227.681	
		301	127.389	177.305							
		301.5	143.462	178.391							
		302	156.082	178.420		20	27.2657	42	217.327	157.384	(3) 9.6
		304	211.873	184.609				43	201.604	155.895	
		305	241.887	186.898				44	186.605	155.092	
		306	270.280	188.986				45	171.297	154.346	
		307	299.978	190.855				48	126.026	149.668	
								49	110.777	148.691	
								50	95.961	147.100	
15	23.5818	30	331.386	283.574	(3) 4.4 II						
		31	319.708	282.787				310	96.616	147.212	
		32	308.548	282.049				311	111.233	148.446	
		32.5	303.372	282.401				312	126.488	149.473	
		33	296.832	281.053				315	171.671	154.534	
		36	260.787	278.137				316	187.463	154.967	
		37	248.616	277.250				317	202.856	156.223	
		38	236.025	276.290				318	217.966	157.445	
		39	223.634	275.308							

gies for each case are listed in Table II. The values for the 10-MeV cases differ by only 20 keV, and the values for the 15-MeV cases agree within 50 keV. This shows that the results are reasonably reproducible.

TABLE II

RESULTS

Approx. Energy (MeV)	Frequency (MHz)	Av Cross-over Angle (Deg)	Uncorrected Energy (MeV)	Av Loss in Target (MeV)	Energy (MeV)	Error (keV)
10-MeV Case I	19.2290	43.962	10.017	0.021	10.039	±12
10-MeV Case II	19.2289	44.005	10.001	0.021	10.022	12
10-MeV Case III	19.2289	44.017	9.997	0.021	10.018	12
12.5-MeV Case II	21.5130	38.368	12.512	0.018	12.530	17
15-MeV Masterson	23.5810	34.473	15.028	0.015	15.043	23
15-MeV 4.4 Cross I	23.5819	34.458	15.038	0.015	15.053	21
15-MeV 9.6 Cross	23.5819	36.786	14.988	0.015	15.003	17
15-MeV 4.4 Cross II	23.5818	34.477	15.024	0.015	15.039	21
17.5-MeV 4.4 Cross	25.4785	31.564	17.546	0.014	17.560	26
17.5-MeV 9.6 Cross	25.4785	50.642	17.524	0.014	17.538	20
20-MeV 4.4 Cross	27.2657	29.305	20.041	0.012	20.053	30
20-MeV 9.6 Cross	27.2657	46.285	20.027	0.012	20.039	24

The energy loss of incident protons through the target was found by summing the contributions of each element in the 994-ug/cm² Mylar target, using the tables by W. Barkas and M. Berger.⁴ The average energy loss of the particles that interacted in the target was assumed to be one-half of the total energy loss. For 20-MeV protons, this average loss was 12 keV, giving a corrected value of 20.041 MeV for the 4433 crossover. All the corrected energies are listed in Table II. Energy losses of scattered particles in leaving the detector and any losses in a dead layer on the surface of the detector are equal for the elastically and inelastically scattered protons at the critical angle, and nearly so in the small angular interval of interest.

The magnet constant for protons was calculated for each energy, using the relativistic formula.

$$K = \frac{f}{\sqrt{E \left(1 + \frac{E}{2mc^2} \right)}}, \quad (11)$$

where f is the nuclear magnetic resonance (NMR) frequency, E is the corrected incident energy, and

mc^2 the mass energy of the proton. Using this constant, it is easier to compare the relative errors of the different energies. The values are listed in Table III and shown on Graph 14, along with a least-squares fit to the different constants. The fit is almost horizontal and disagrees with the earlier value of 6.0646, obtained by (p,n) threshold experiments, by only 0.12%.

TABLE III

MAGNET CONSTANTS

Approx. Energy (MeV)	Frequency (MHz)	Magnet Constant MHz/(MeV) ^{1/2}	Difference from Mean
10	19.2290	6.0528	-0.0042
	19.2289	6.0579	+0.0009
	19.2289	6.0592	+0.0022
12.5	21.5130	6.0573	+0.0003
15	23.5810	6.0551 (a)	+0.0013
	23.5819	6.0539	-0.0031
	23.5819	6.0542 (b)	+0.0072
	23.5818	6.0567	-0.0003
17.5	25.4785	6.0519	-0.0051
	25.4785	6.0556 (b)	-0.0014
20	27.2657	6.0565	-0.0005
	27.2657	6.0586 (b)	+0.0025

(a) Acceptable data from 1965.

(b) Inelastic scattering from 9.6 MeV level.

LEAST-SQUARES FIT OF MAGNET CONSTANTS

Frequency	Magnet Constant MHz/(MeV) ^{1/2}
0	6.05719
19.229	6.05702
21.513	6.05700
23.582	6.05698
25.479	6.05696
27.266	6.05695

Graph 14. Measured Values of the Magnet Constant at Various Frequencies.

DISCUSSION OF PROBABLE SOURCES OF ERROR

There are four major sources of error in this experiment that contribute to the total estimated error. First, the uncertainty in the Q value of the 4.433- and 9.629-MeV levels in carbon is ± 0.005 and ± 0.010 MeV, respectively. By differentiation of Eq. 5 with respect to E_x , the uncertainty in the incident energy, dE_1 , is

$$dE_1 = \frac{dE_x}{E_x} E_1, \quad (12)$$

which is approximately 22 and 21 keV for the 4.433 and 9.629 crossovers, respectively, at 20 MeV.

Differentiation of the same equation with respect to θ will give the uncertainty in the incident energy, dE_1 , as

$$dE_1 = -2 \cot \theta d\theta E_1 \quad (13)$$

due to inaccuracy in setting and reading the detector angle. An error of 0.015° corresponds to uncertainties in energy at 20 MeV of 19 and 10 keV for the 4.433 and 9.629 crossovers, respectively.

A third source of error arises from the positioning of the target and slits. If, when the beam passes through the target, it does not pass directly through the center of rotation of the detector arm, and if the face of the target does not happen to be perpendicular to the beam, the observed crossover angle will be different from the actual crossover angle. Consider Fig. 3, where point A is the center of rotation of the detector and point B is the location of the beam when it hits the target. With the use of the following relations,

$$\frac{\rho}{\sin \gamma} = \frac{\sqrt{\epsilon^2 + \Delta^2}}{\sin \alpha}$$

and

$$\frac{\rho}{\sin (\gamma + 2\theta)} = \frac{\sqrt{\epsilon^2 + \Delta^2}}{\sin \beta}, \quad (14)$$

$$\phi + 2\theta = 180^\circ$$

$$2\theta + (\delta - \alpha) + (\delta + \beta) = 180^\circ, \quad (15)$$

it is possible to find an expression relating ϕ (twice the actual crossover angle) and θ .

Fig. 3 The Scattering Geometry, with target rotated so not normal to beam direction and beam not striking target at center of rotation of detector arm.

$$\sin \beta = \frac{1}{\rho} [\Delta \cos \theta + \epsilon \sin \theta], \quad (16)$$

$$\sin \alpha = \frac{1}{\rho} [\Delta \cos \theta - \epsilon \sin \theta], \quad (17)$$

and

$$\theta = \frac{1}{2} [\phi + \beta - \alpha]. \quad (18)$$

It is evident that if $\epsilon \neq 0$, $\theta \neq \phi/2$. (With the detector arm, ρ , approximately equal to 8.75 in., if the lateral displacement of the beam $\Delta = \frac{1}{16}$ in. and the angle between the plane of the target and the beam direction $\psi = 88^\circ$, one gets errors of 9 and 7 keV for the 4.433 and 9.629 crossovers, respectively, at 20 MeV.)

Finally, we discovered that the Mylar film used was not exactly 1/4-mil thick, as supposed, and therefore had to be measured. Two methods were used: weighing a piece of Mylar of known area, and measuring the shift in energy of α particles as they passed through the film. The answers obtained for the thickness differed by 50%, so the more accurate weight measurement was used, with an error of 2.2%, which gave an error of 1 keV for 20 MeV.

Summing all these uncorrelated errors and taking the square root gives final answers of 20.041 ± 0.030 MeV and 20.027 ± 0.024 MeV for the 4.433 and 9.629 crossovers, respectively, at an NMR frequency of 27.2657 MHz. The same procedure was used for the other energies, and the results are listed in Table II.

CONCLUSION

This calibration experiment was quite successful. All the results are remarkably consistent and also close enough to previously assumed calibrations so that it is reasonable that others have not noticed any large errors in their experiments. The largest noted difference was only 60 keV at 15 MeV, a 0.4% difference which lies within the statistical error of most experiments. The equation of the least-squares fit to the magnet constant values, K , as a function of NMR frequency, f , is:

$$K = (6.0572 \pm 0.0042) - [(9.075 \pm 18) \times 10^{-6}] f \quad (19)$$

where the uncertainty in the slope could only be estimated. Although this uncertainty is larger than the slope value, the slope is still very small, and the magnet constant is almost constant with changing frequency, as it should be. The constant term in the equation is to be compared with the value 6.065 ± 0.002 , derived from measurements of the $^{27}\text{Al}(p,n)$ threshold. If there are no appreciable systematic errors, this would imply that the magnet constant in the 10- to 20-MeV region may be somewhat smaller than the value now used for the calibration. There is obviously a need for still another independent measurement, such as the $d(^{16}\text{O},n)^{17}\text{F}$ experiment which will be conducted at the LASL Van de Graaff in the near future.

APPENDIX

CROSSO

ORIGIN: Los Alamos Scientific Laboratory

TITLE: Energy Calibration of Monoenergetic Proton Beams Using the Crossover Method

USE: The program determines the precise energy of a beam of protons (discounting target loss, etc.) by examining the protons scattered from the hydrogen and carbon components of a Mylar target.³ The energy of the protons elastically scattered from the hydrogen has a different angular dependence from the energy of the protons inelastically scattered from the carbon. Above an incident energy of around 5 MeV, the two energies are equal at one, and only one, angle. The program takes the values of the energies at different angles above and below the crossover angle and determines the exact crossover angle, and then the energy of the incident beam. The equations which best fit the hydrogen and carbon curves are also given.

RELEVANT FORMULAE: The data fed into the program

consist of two sets of experimental points which form two curves. The equations of the solid curves, upon which these points most closely lie, are determined through least-squares analysis of the points. The curves are simple enough that they can be represented by the following formulas,

$$Y_H = a + bx + cx^2 + dx^3 \quad (A1)$$

$$Y_C = e + fx \quad , \quad (A2)$$

where subscripts H and C refer to the hydrogen and carbon curves, respectively. The coefficients of the hydrogen curve are determined through use of a generalized gaussian method⁵ for solving simultaneous linear equations because this method was found to reduce the loss of significant figures which caused other methods, such as determinants (used for the carbon coefficients), to give inaccurate answers.

The crossover angle is found by equating Y_H

and Y_c giving

$$dx^3 + cx^2 + x(b - f) + (a - e) = 0, \quad (A3)$$

which is solved numerically by use of the Newton-Raphson method. For each case, two crossover angles are found, one in the 0° to 90° quadrant, and one in the 270° to 360° quadrant. After both angles are found, they are averaged together (this eliminates most beam orientation errors) to find the final values of the crossover angle.

The energy of the incident protons is now found by solving the relativistic equations of motion for the incident protons and the target nuclei at the average angle determined above. Using Reference 3 for the details of the derivation, one gets

$$\cos \theta = \frac{(T - E_r)(2M_p + T - E_r) - P_r^2 + T(T + 2M_p)}{2[(T - E_r)(2M_p + T - E_r) T(T + 2M_p)]^{\frac{1}{2}}}, \quad (A4)$$

where

$$P_x^2 = \frac{x(M_2 - M_1 + \frac{1}{2}x)[2M_1M_2 + x(M_1 + M_2 + \frac{1}{2}x)]}{(M_2 - M_1)^2}, \quad (A5)$$

$$E_r = (M_1^2 + P_r^2)^{\frac{1}{2}} - M_1 \quad (A6)$$

In the above, θ is the crossover angle, T is the incident energy, M_p is the proton mass, M_1 is the lighter target mass, M_2 is the mass of the carbon, and x is the excitation energy of the excited carbon atom. All of the above quantities are known except T , and Eq. (A4) is solved for T using the Newton-Raphson method.

INPUT FORMAT: Card #1 (5A6, 5X, I2, 3X, I1, 4X, F7.3, 3X, F7.3)

Column	Symbol	Identification
1 - 30	Z3,Z4,Z5,Z6,Z7	Alphanumeric ident. for the case
35	N1	No. of points in each curve (same for both)
40	NE	1 for 4.433-MeV excitation, 2 for 9.624-MeV excitation
45	AP	Approximate value for crossover angle
55	EP	Approximate value for beam energy (MeV)

Card #2, 3, 4, 5 [9(F7.3, 1X)]

Each "card" may consist of several cards, depending on the number of points being considered.

Column	Card #2	Card #3	Card #4	Card #5
1	X1(1) Angles at	Y1(1) Energies of	X2(1) Angles at	Y2(1) Energies of
9	X1(2) which energy readings were taken for	Y1(2) protons scattered from	X2(2) which energy readings were taken for	Y2(2) protons scattered from
65	X1(9) hydrogen	Y1(9) hydrogen	X2(9) carbon	Y2(9) carbon

NOTE: When calculating the angle in the 270° - 360° quadrant, use the complement of the angle ($315^\circ = 45^\circ$).

<u>TEST CASE:</u> INPUT	15-MeV Case	9.6 Left Cross	<u>X1 = X2</u>	<u>Y1</u> *	<u>Y2</u> *
NI	5		58.50	143.462	178.391
NE	2		58.00	156.082	178.420
AP	56.05		56.00	211.873	184.609
			54.00	270.280	188.986
EP	15.000		53.00	299.978	190.855

*Energies used in this case were peak locations in a pulse height analyzer.

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