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WEAK INELASTIC PRODUCTION AND LEPTONIC DECAYS
OF HEAVY LEPTONS[†]

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ABSTRACT

We examine here reactions of the type,

$$\begin{array}{lcl} \nu(\bar{\nu})N \longrightarrow L^{\pm} + \text{"anything"} & \text{and} & l^{\pm}N \longrightarrow L^0(\bar{L}^0) + \text{"anything"} \\ \downarrow & & \downarrow \\ \nu(\bar{\nu}) + l^{\pm} + \nu_{l^{\mp}} & & l^{\pm} + l'^{\mp} + \nu_{l'^{\pm}} \end{array}$$

where L is a lepton heavier than the muon, and derive expressions for the inclusive cross sections of the outgoing charged leptons. The phase space integration is simplified by neglecting the electron and muon masses and the differential cross section, with respect to the transverse momentum of the charged leptons in the final state, are calculated. Using a simple parameterization for the hadronic structure functions, results are presented for several incident lepton energies, between 50 and 500 GeV in the lab frame, and, for heavy lepton masses ranging from 2 to 11 GeV, assuming the coupling constant of the heavy lepton to be identical to that of the muon. A simple relation between the differential cross section of the outgoing charged leptons along the beam axis and the total cross section is obtained. Throughout these calculations the effects produced on the spectrum of the final charged lepton by the polarization of the heavy lepton are fully incorporated.

1. Introduction

The existence of leptons, both charged and neutral, heavier than the muon has been predicted by several recent theories of weak interactions¹. A number of experiments² for detection of these leptons are currently in progress. Since these particles are expected to have a very short lifetime ($\lesssim 10^{-11}$ sec.) experimentally one has to look for their signatures in their decay products. The characteristic effect of such a heavy lepton on some aspects of the inclusive ℓN spectrum has been examined by a number of authors, but effects due to polarization of the heavy lepton have been neglected^{3,4}.

In this work we take into account the polarization of the heavy lepton produced weakly in ℓN inelastic reactions and derive expressions for the cross section of the out going charged leptons. We simplify the phase space integration by neglecting the electron and muon masses and use a simple parameterization for the hadronic structure functions⁵. The differential cross section $\frac{d\sigma}{dQ_t^2}$, where Q_t is component of the final charged lepton's momentum that is transverse to the direction of the incident lepton beam in the laboratory frame, is computed for heavy lepton masses of 2, 5, 8 and 11 GeV, taking the incident lepton energy $E = 50, 100, 200$ and 500 GeV and assuming that the coupling constant of the heavy lepton is identical to that of the muon. In the decay of a neutral heavy lepton where there are two charged leptons in the final state, we calculate the differential cross sections for both. Results are displayed graphically taking the incident beam to be μ^-, μ^+, ν or $\bar{\nu}$ and assuming that the heavy lepton couples via $V - A$ or $V + A$. These results are compared with a similar calculation where the polarization of the heavy lepton is ignored.

We also show that as $Q_t \rightarrow 0$, the remaining variables of the outgoing lepton can be integrated analytically yielding a simple relation for $\left. \frac{d\sigma}{dQ_t^2} \right|_{Q_t=0}$

given by

$$\left. \frac{d\sigma}{dQ_t^2} \right|_{Q_t=0} = \frac{8}{m_L^2} \sigma_p R \quad (1)$$

where σ_p is the cross section for production of heavy leptons of mass m_L and R is the branching ratio for the leptonic decay mode under consideration.

II. Calculations

For definiteness, we concentrate on the reaction⁶ (see Fig. 1) :

$$\begin{aligned} \nu(k) + N(p) &\rightarrow L^-(k') + \text{"anything"} \\ &\quad \searrow \\ &\quad \nu(K) + \ell^-(Q) + \bar{\nu}_\ell(\bar{k}) \end{aligned} \quad (2)$$

where L is a heavy lepton and ℓ may be e or μ . In the limit when the electron and the muon masses, $m_e, m_\mu \rightarrow 0$, the expressions derived for (2) would also apply to the reaction

$$\begin{aligned} \mu^-(k) + N(p) &\rightarrow L^0(k') + \text{"anything"} \\ &\quad \searrow \\ &\quad \mu^-(K) + \mu^+(Q) + \nu_\mu(\bar{k}) \end{aligned} \quad (3)$$

$$\mu^-(K) + e^+(Q) + \nu_e(\bar{k}) \quad (4)$$

The corresponding expressions for incident $\bar{\nu}$ or μ^+ producing charged and neutral heavy leptons can be trivially obtained from expressions for reaction (2).

a). The Production Process:

The amplitude, $A_{\lambda\lambda'}$, for the production of a heavy lepton of helicity λ' from an incident neutrino of helicity λ can be expressed as⁷

$$A_{\lambda\lambda'} = \frac{G_1}{\sqrt{2}} \bar{U}_L(k', \lambda') \not{J} (1 + h \gamma_5) U_\nu(k, \lambda) \quad (5)$$

where J is the hadronic current, G_1 is the analog of the Fermi constant for the coupling of the heavy lepton to the nucleon at the production vertex (see Fig. 1), and $h = \pm 1$ if the heavy lepton couples via $(V \mp A)$ respectively. We describe the hadronic vertex of the reaction in terms of the structure functions defined by⁴

$$\begin{aligned}
 W_{\mu\nu} = & -\delta_{\mu\nu} W_1 - \frac{1}{M^2} p_\mu p_\nu W_2 - \frac{1}{2M^2} \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta W_3 - \frac{1}{M^2} q_\mu q_\nu W_4 \\
 & - \frac{1}{2M^2} (p_\mu q_\nu + p_\nu q_\mu) W_5
 \end{aligned} \quad (6)$$

where M is the mass of the target nucleon, and $q = k - k'$, the momentum transfer from the leptons to the hadrons. The state of the produced lepton L in its rest frame can then be described by a (2×2) density matrix of the form

$$\rho_o = \frac{1}{2} \left(1 + \vec{\sigma} \cdot \frac{\vec{B}_o}{A_o} \right) \quad (7)$$

where σ 's are 2×2 Pauli matrices, and the vector $\frac{\vec{B}_o}{A_o}$ gives the polarization of L in its rest frame. In this frame, where $\vec{k}' = \vec{k} - \vec{q} = 0$, $\vec{B}_o$ is expressed, for convenience in subsequent generalization to a covariant frame, in terms of the dependent 3-vectors, $\vec{k}$, $\vec{q}$, and $\vec{p}$ as

$$\begin{aligned}
 \vec{B}_o = & -\frac{2h G_1^2 m_L}{8\pi^2 k \cdot p} \left\{ \vec{k} \left[2W_1 + \frac{W_2}{M^2} p^2 + \frac{W_4}{M^2} q^2 + \frac{W_5}{M^2} q \cdot p \right] \right. \\
 & + \vec{p} \left[-2 \frac{W_2}{M^2} k \cdot p - \frac{1+h^2}{2h} \frac{W_3}{M^2} q \cdot k - \frac{W_5}{M^2} q \cdot k \right] \\
 & \left. + \vec{q} \left[\frac{1+h^2}{2h} \frac{W_3}{M^2} - 2 \frac{W_4}{M^2} k \cdot q - \frac{W_5}{M^2} k \cdot p \right] \right\} \quad (8)
 \end{aligned}$$

and

$$A_o = -\frac{G_1^2 (1+h^2) m_L}{8\pi^2 k \cdot p} \left\{ 2k_o W_1 + \frac{W_2}{M^2} (k_o p^2 - 2p_o k \cdot p) - \frac{W_3}{M^2} \frac{2h}{1+h^2} (p_o k \cdot q - q_o k \cdot p) \right. \\ \left. + \frac{W_4}{M^2} (q^2 k_o - 2q_o k \cdot q) + \frac{W_5}{M^2} (k_o p \cdot q - p_o q \cdot k - q_o k \cdot p) \right\} \quad (9)$$

where k_o , q_o and p_o are the o^{th} components of k , q and p and m_L is the mass of heavy lepton L .

The cross section for production of heavy leptons is directly related to the function A_o and is given in the lab frame by

$$\frac{d\sigma_p}{dq^2 d\nu} = -\frac{G_1^2 (1+h^2)}{8ME^2 \pi} \left\{ 2k' \cdot k W_1 + \frac{W_2}{M^2} (k' \cdot k p^2 - 2k' \cdot p k \cdot p) \right. \\ \left. + \frac{2h}{1+h^2} \frac{W_3}{M^2} (k' \cdot q k \cdot p - k' \cdot p k \cdot q) + \frac{W_4}{M^2} (k' \cdot k q^2 - 2k' \cdot q k \cdot q) \right. \\ \left. + \frac{W_5}{M^2} (k' \cdot k p \cdot q - k' \cdot p k \cdot q - k' \cdot q k \cdot p) \right\} \quad (10)$$

where E is the incident lepton energy in the lab frame, and $\nu = -q \cdot p/M$.

b). Decay of the Heavy Lepton:

The amplitude for the decay of a heavy lepton L of helicity λ' to a ν of helicity λ_f via a reaction of the type

$$L^-(k', \lambda') \longrightarrow \nu(k, \lambda_f) + \ell^-(q) + \bar{\nu}_\ell(\bar{k})$$

is expressed as

$$d_{\lambda' \lambda_f} = \frac{G_2}{\sqrt{2}} \left[\bar{U}_\nu(K, \lambda_f) \gamma_\mu (1 + g \gamma_5) U_L(k', \lambda') \right] \\ \times \left[\bar{U}_l(Q) \gamma_\mu (1 + \gamma_5) V_{\nu_l}(\bar{k}) \right] \quad (11)$$

where again $g = \pm 1$ depending on whether L couples via $(V \mp A)$ respectively.⁸

In the rest frame of the heavy lepton, the decay matrix for the process can be written as

$$D_o = \gamma_o \left[1 + \vec{\sigma} \cdot \frac{\vec{\beta}_o}{\gamma_o} \right] \quad (12)$$

where

$$\gamma_o = -32 G_2^2 m_L \left[(1+g)^2 \vec{k}_o \cdot K \cdot Q + (1-g)^2 Q_o \cdot K \cdot \vec{k} \right] \quad (13)$$

and

$$\vec{\beta}_o = -32 G_2^2 m_L \left[(1+g)^2 \vec{k} \cdot K \cdot Q + (1-g)^2 \vec{Q} \cdot K \cdot \vec{k} \right] \quad (14)$$

The rate for the decay of a polarized heavy lepton to leptons is then given by

$$\Gamma_{L \rightarrow l} = \frac{1}{(2\pi)^5} \frac{1}{2m_L} \int \frac{d^3 Q}{2Q_o} \frac{d^3 K}{2K_o} \frac{d^3 \bar{k}}{2\bar{k}_o} \delta^4 [k' - K - Q - \bar{k}] \frac{1}{2} \text{Tr} [\rho D_o] \quad (15)$$

where ρ is the density matrix specifying the polarization state of the heavy lepton.

c). Production and Subsequent Decay of the Heavy Lepton:

Taking the trace of the matrix $(\rho_o D_o)$ and generalizing to a covariant form we get the required expression for the cross section for process of Fig. 1 as

$$\sigma = \frac{G_1^2 G_2^2}{8(2\pi)^7 \sqrt{(k \cdot p)^2 - m_\ell^2 \cdot M^2}} \frac{1}{m_\ell \Gamma} \quad (16)$$

where

$$I = \int \frac{d^3 k'}{2k'_0} \frac{d^3 Q}{2Q_0} \frac{d^3 \bar{k}}{2\bar{k}_0} \frac{d^3 K}{2K_0} \delta^4(k' - \bar{k} - K - Q) \quad T \quad (17)$$

m_ℓ is the mass of the incident lepton, Γ the total width⁹ of L, and

$$\begin{aligned} T = & 64(1+g)^2 K \cdot Q \left\{ 2W_1 \left[2hm_L^2 k \cdot \bar{k} + (1+h)^2 k' \cdot k k' \cdot \bar{k} \right] \right. \\ & + \frac{W_2}{M^2} \left[2hm_L^2 (p^2 k \cdot \bar{k} - 2k \cdot p \bar{k} \cdot p) + (1+h)^2 k' \cdot \bar{k} (k' \cdot k p^2 - 2k' \cdot p k \cdot p) \right] \\ & - \frac{W_3}{M^2} \left[(1+h)^2 k' \cdot \bar{k} (k' \cdot p k \cdot q - k' \cdot q k \cdot p) + (1+h^2) m_L^2 (k \cdot q \bar{k} \cdot p - k \cdot p \bar{k} \cdot q) \right] \\ & + \frac{W_4}{M^2} \left[2hm_L^2 (q^2 k \cdot \bar{k} - 2k \cdot q \bar{k} \cdot q) + (1+h)^2 k' \cdot \bar{k} (k' \cdot k q^2 - 2k' \cdot q k \cdot q) \right] \\ & + \frac{W_5}{M^2} \left[2hm_L^2 (q \cdot p k \cdot \bar{k} - q \cdot k p \cdot \bar{k} - q \cdot \bar{k} k \cdot p) + (1+h)^2 k' \cdot \bar{k} (k' \cdot k p \cdot q - k' \cdot p k \cdot q - k' \cdot q k \cdot p) \right] \Big\} \\ & + 64(1-g)^2 K \cdot \bar{k} \left\{ 2W_1 \left[-2hm_L^2 k \cdot Q + (1-h)^2 k' \cdot k k' \cdot Q \right] \right. \\ & + \frac{W_2}{M^2} \left[-2hm_L^2 (p^2 k \cdot Q - 2k \cdot p p \cdot Q) + (1-h)^2 k' \cdot Q (k' \cdot k p^2 - 2k' \cdot p k \cdot p) \right] \\ & - \frac{W_3}{M^2} \left[(1-h)^2 k' \cdot Q (k' \cdot q k \cdot p - k' \cdot p k \cdot q) + (1+h^2) m_L^2 (k \cdot p q \cdot Q - k \cdot q p \cdot Q) \right] \\ & + \frac{W_4}{M^2} \left[-2hm_L^2 (q^2 k \cdot Q - 2k \cdot q q \cdot Q) + (1-h)^2 k' \cdot Q (q^2 k' \cdot k - 2k' \cdot q k \cdot q) \right] \\ & + \frac{W_5}{M^2} \left[-2hm_L^2 (q \cdot p Q \cdot k - k \cdot p q \cdot Q - k \cdot q p \cdot Q) + (1-h)^2 k' \cdot Q (k' \cdot k q \cdot p - k' \cdot p k \cdot q - k' \cdot q k \cdot p) \right] \Big\} \quad (18) \end{aligned}$$

It may be worth mentioning that expressions (16) to (18), for the cross section for production and decay of charged heavy leptons, $L^\pm$, via reactions of the type (2) are exact. However, for the production and decay of neutral heavy leptons, L^0 or $\bar{L}^0$, via reactions of types (3) and (4), the expressions for the cross section obtained above is correct only to the extent that the incident lepton mass $m_L = 0$.

d). Phase Space Integration and the Differential Cross Section:

Neglecting the electron and muon mass, the phase space integration, over the unobserved pair of leptons in the decay products, is carried out analytically yielding I of the general form:

$$I = \int \frac{d^3 k'}{2k'_0} \frac{d^3 Q}{2Q_0} F(k' \cdot Q) \theta [-(k' - Q)^2] \quad (19)$$

where F is a quadratic function of $(k' \cdot Q)$ and is given in the appendix. We next introduce a coordinate system in the lab frame, shown in Fig. 2, in which the incident lepton beam with 3-momenta $\vec{k}$ is along the Z axis, $\vec{k}'$ the 3-momentum of the produced heavy lepton lies in the ZX plane, making an angle θ' with the Z axis and θ and ϕ are the polar and azimuthal angles of $\vec{Q}$, the 3-momentum of the charged lepton which is under observation. Since I , in equation (19) is quadratic in $\cos \phi$, the integration over ϕ can be done trivially to yield:

$$\begin{aligned} \frac{d\sigma}{dq^2 d\nu dQ_\parallel^2 dQ_\perp^2} &= \frac{G_1^2 G_2^2}{48(2\pi)^4 m_L \Gamma_{ME^2 Q_0}} \left[\left(A_{0Q} + \frac{A_{2Q}}{2} \right) \Phi_Q \right. \\ &\quad \left. + A_{1Q} \sin \Phi_Q + \frac{A_{2Q}}{4} \sin (2 \Phi_Q) \right] \quad (20) \end{aligned}$$

where Q_ℓ , Q_t are the longitudinal and transverse components of $\vec{Q}$, A_{1Q} , A_{2Q} are the coefficients of $\cos \theta$ and $(\cos \theta)^2$ and A_{0Q} is the term in (19) that is independent of θ . These are given in the appendix and Φ_Q , the upper limit on θ , is given by¹⁰

$$\Phi_Q = \cos^{-1} \left(\frac{2k'_0 Q_0 - 2k'_\ell Q_\ell - m_L^2}{2k'_t Q_t} \right) \quad (21)$$

For the decay of a neutral heavy lepton into two charged leptons and a neutrino, as in (3) and (4), one may also be interested in the differential cross section of the other charged particle of 3-momentum $\vec{K}$. This is the same in form as (20), that is,

$$\begin{aligned} \frac{d\sigma}{dq^2 d\nu dK_t^2 dK_\ell} = & \frac{G_1^2 G_2^2}{48(2\pi)^5 m_L^2 E^2 MK_0} \left[\left(A_{0K} + \frac{A_{2K}}{2} \right) \Phi_K \right. \\ & \left. + A_{1K} \sin \Phi_K + \frac{A_{2K}}{4} \sin(2\Phi_K) \right] \end{aligned} \quad (22)$$

where again,

$$\Phi_K = \cos^{-1} \left(\frac{2k'_0 K_0 - 2k'_\ell K_\ell - m_L^2}{2k'_t K_t} \right) \quad (23)$$

The coefficients A_{0K} , A_{1K} and A_{2K} are again given in the appendix.

e). Differential Cross Section Neglecting Polarization of L :

The cross section, $d\sigma_0$, neglecting polarization of the heavy lepton, is obtained by taking the product of $A_0 \gamma_0$ instead of $\rho_0 D_0$ from equations (7) and (12) and is given by

$$\frac{d\sigma_0}{dq^2 d\nu dQ_f^2 dQ_l^2} = \frac{G_1^2 G_2^2}{48(2\pi)^5 m_L \Gamma M E^2 Q_0} \left[\left(A_{00} + \frac{A_{02}}{2} \right) \Phi_Q + A_{01} \sin \Phi_Q + \frac{A_{02}}{4} \sin (2\Phi_Q) \right] \quad (24)$$

where A_{00} , A_{01} , and A_{02} are again given in the appendix.

f). Differential Cross Section along the Beam Axis:

Along the beam axis Q_f , (or K_f) = 0 and Φ_Q , (or Φ_K) = π ; and since $Q_0 = |Q_f|$, $K_0 = |K_f|$, the differential cross section (20) and (22) becomes linear functions of Q_f (or K_f) which can be analytically integrated to yield,

$$\left. \frac{d\sigma}{dq^2 d\nu dQ_f^2} \right|_{Q_f=0} = C_- (1-g)^2 + C_+ (1+g)^2 \quad (25)$$

where C_- and C_+ are given in the appendix. On the other hand

$$\left. \frac{d\sigma}{dq^2 d\nu dK_f^2} \right|_{K_f=0} = \frac{G_2^2 (1+g^2) m_L^3}{48\pi^3 \Gamma} \frac{d\sigma_p}{dq^2 d\nu} \quad (26)$$

So that for both $V + A$ and $V - A$ couplings

$$\left. \frac{d\sigma}{dK_f^2} \right|_{K_f=0} = \frac{G_2^2 m_L^3}{24\pi^3} \frac{\sigma_p}{\Gamma} \quad (27)$$

or using (15) we get¹¹

$$\left. \frac{d\sigma}{dK_f^2} \right|_{K_f=0} = \frac{8}{m_L^2} \sigma_p R \quad (28)$$

where R is the branching ratio defined as

$$R = \frac{\Gamma_{L \rightarrow l}}{\Gamma} \quad (29)$$

From (24) and the appendix we also notice that only for $V - A$, i.e. for $g = 1$,

$$\left. \frac{d\sigma}{dQ_f^2} \right|_{Q_f=0} \xrightarrow{g \rightarrow 1} \frac{8}{m_L^2} \sigma_p R \quad (30)$$

Finally, from (24) we get

$$\left. \frac{d\sigma_o}{dQ_f^2} \right|_{Q_f=0} = \frac{[5(1+g^2) - 2g]}{m_L^2} \sigma_p R \quad (31)$$

So that for $V - A$

$$\left. \frac{d\sigma_o}{dQ_f^2} \right|_{Q_f=0} \xrightarrow{g \rightarrow 1} \frac{8}{m_L^2} \sigma_p R \quad (32)$$

and for $V + A$

$$\left. \frac{d\sigma_o}{dQ_f^2} \right|_{Q_f=0} \xrightarrow{g \rightarrow -1} \frac{12}{m_L^2} \sigma_p R \quad (33)$$

III. Numerical Computations

Numerical computations are done for $E = 50, 100, 200$ and 500 GeV. The couplings G_1 and G_2 are taken identical to that for the muon. The branching ratio, R , into any one of the leptonic decay modes of L is taken to be ¹² 15%.

Introducing $\omega' = (2M\nu + M^2)/q^2$ the structure functions are parameterized as follows ^{4,5}:

$$\left. \begin{aligned} F_2(\omega') &= 1 \cdot 1 \left(1 - \left(\frac{1}{\omega'}\right)^2\right)^3 \\ F_1(\omega') &= \frac{\omega'}{2} F_2(\omega') \\ F_3(\omega') &= -|B| \omega' F_2(\omega') \\ F_4(\omega') &= 0 \\ F_5(\omega') &= 2F_1(\omega') \end{aligned} \right\} \quad (34)$$

where $F_1 = W_1$ and $F_i = \frac{\nu}{M} W_i$ with $i = 2, 3, 4$ and 5 . As indicated by the CERN Gargamelle data on $\bar{\nu}, \nu$ cross section ratio we take $B = 1$. ⁵ The total cross section σ_p as a function of the heavy lepton mass obtained by this parameterization, is shown in Fig. 3, for $E = 50, 100, 200$ and 500 GeV. We find that σ_p for $m_L = m_\mu$ in Fig. 3 compares very well with the experimental result.

$$\sigma_p^\nu = (0.8 \pm 0.2) \times 10^{-38} E \text{ cm}^2 \quad (35)$$

Further, our parameterization (34) with $|B| = 1$ also yields $\frac{\sigma_p(\bar{\nu})}{\sigma_p(\nu)} = \frac{1}{3}$ in agreement with the observed values.⁵

To obtain $\frac{d\sigma}{dQ_t^2}$, $\frac{d\sigma}{dK_t^2}$, $\frac{d\sigma_o}{dQ_t^2}$, three dimensional numerical integration of (20), (22) and (24) is done using the IBM systems 360, Scientific Subroutine Package, RANDU, for generating random numbers. 3×10^4 points are taken for each value of Q_t or K_t . Statistical fluctuation in the differential cross sections are estimated to be $\lesssim 2\%$.

IV. Discussion of Results

To get a feeling for the cross section for production of heavy leptons we draw a graph for σ_p versus m_L (Fig. 3) for incident lepton energies of $E = 50, 100, 200$ and 500 GeV in the lab frame. They exhibit a fast drop in the cross section with an increase in m_L . For example, for $E = 100$ GeV, when the kinematical limit is $m_L \lesssim 13$ GeV, the cross section drops from $5.7 \times 10^{-37} \text{ cm}^2$ for $m_L = 2$ GeV to $3.0 \times 10^{-41} \text{ cm}^2$ for $m_L = 12$ GeV, a drop of ~ 4 orders of magnitude over a 10 GeV increase in mass.

The differential cross section distribution with respect to (transverse momentum)², assuming (V - A) coupling for L, is shown in graphs 4, 5, 6 and 7. In figures 4 and 5 the incident particle is ν or a left handed μ^- and in figures 6 and 7 the incident particle is $\bar{\nu}$ or a right handed μ^+ . These curves exhibit a very sharp drop with increase in transverse momentum. In fact, a numerical estimate of $\langle Q_t^2 \rangle$ shows that for $E = 100$ GeV and $m_L = 5$ GeV, $\langle Q_t^2 \rangle = 3.8 \text{ GeV}^2$ when the kinematical limit is $Q_t^2 \lesssim 45 \text{ GeV}^2$. The cross section $\frac{d\sigma}{dQ_t^2}$ again drops by 5 orders magnitude over an increase of Q_t^2 from 0 to 35 GeV^2 for $E = 100$ GeV. For (V - A) coupling of L, $\frac{d\sigma}{dQ_t^2}$ and $\frac{d\sigma}{dK_t^2}$ are identical as exhibited by (18)⁸. Again, it may be worth reminding that the branching ratio, R, throughout these computations has been taken to be 15%, and, in adapting these graphs to various models for heavy leptons, one must normalize them to the appropriate value of R given by the individual models.¹²

In Figs. 8 and 9 we redraw Figs. 4 and 5 by normalizing $\frac{d\sigma}{dQ_t^2}$ to $\frac{d\sigma}{dQ_t^2} \bigg|_{Q_t=0}$

using Eq. (30). We plot $P_{Q_f} = \left(\frac{d\sigma/dQ_f^2}{\left. \frac{d\sigma}{dQ_f^2} \right|_{Q_f=0}} \times 100 \right) \% \text{ versus } Q_f^2$.

These graphs are therefore independent of the branching ratio and hence can be directly compared to an experimental curve of $\left(\frac{\text{number of events with } Q_f^2}{\text{number of events with } Q_f=0} \right)$ versus Q_f^2 . In Fig. 8 we keep the energy fixed at $E = 100 \text{ GeV}$ and vary the mass m_L from 2 to 11 GeV, and, in Fig. 9 we keep the mass fixed at 5 GeV and vary the energy from 50 to 500 GeV. Fig. 8 exhibits a very strong dependence of P_{Q_f} on m_L whereas Fig. 9 shows practically no dependence of P_{Q_f} on E , for the kinematically allowed values of Q_f^2 for each E .

If the $(V + A)$ coupling is instead assumed for L then the differential cross sections for incident ν or right handed μ^- is shown in Figures 10 and 11, and, for incident $\bar{\nu}$ or left handed μ^+ is shown in Figs. 12 and 13. For $(V + A)$ coupling $\frac{d\sigma}{dQ_f^2} \neq \frac{d\sigma}{dK_f^2}$ and we therefore draw both the distributions.

In our calculation we have assumed L^- to have the same lepton number as ℓ^- , and ν_ℓ . If instead L^+ is taken to have the same lepton number as ℓ^- and ν_ℓ , as is done in a number of gauge theories, then our graphs for the differential cross sections of the emerging charged leptons will get interchanged for the $(V + A)$ and $(V - A)$ cases.

The effects due to polarization of the heavy lepton are shown in Figures 14 and 15. The density matrix of the heavy lepton, expressed in Equations 7, 8 and 9, is very convenient for this purpose. The polarization vector $\hat{S}$ in the rest frame of L , given by

$$\hat{S} = \frac{\vec{B}_o}{A_o}$$

where B_o , A_o are given in Equations (8) and (9), is used to obtain the degree of left handedness, P_{LH} , of L defined as:

$$P_{LH} = \frac{1 - \hat{S} \cdot \hat{k}'}{2} \times 100\% \quad (36)$$

Fig. 14 shows P_{LH} as a function of the heavy lepton energy, k'_o , keeping the scattering angle θ^i fixed in the lab frame. The figure shows effects of changing the scattering angle θ^i , which, incidentally, has to be very small because of the constraint $q^2 \leq 2M\nu$, the incident lepton energy E and the mass m_L of the heavy lepton.

The consequence of all these dependences of $\hat{S}$ on k'_o , θ^i and E on the distributions of the emerging charged lepton is shown in Fig. 15. For this purpose we plot two functions D_o and D_L where

$$D_o = \frac{d\sigma/dQ_t^2 - d\sigma_o/dQ_t^2}{d\sigma/dQ_t^2} \quad (37)$$

and

$$D_L = \frac{d\sigma/dQ_t^2 - d\sigma_L/dQ_t^2}{d\sigma/dQ_t^2} \quad (38)$$

Here $\frac{d\sigma}{dQ_t^2}$ is the distribution which incorporates all the polarization effects of the heavy lepton, $\frac{d\sigma_o}{dQ_t^2}$ is obtained by assuming the heavy lepton to be completely unpolarized and $\frac{d\sigma_L}{dQ_t^2}$ assuming the heavy lepton to be completely left handed. In the region of experimental interest, $Q_t^2 \gtrsim 1.5 \text{ GeV}^2$, one

finds that the polarization effects are rather significant.

Acknowledgement

I am deeply indebted to Professor Norman Christ for his sponsorship of this problem and for constant encouragement and advice in the course of this work.

Appendix

a). Coefficients A_{0Q} , A_{1Q} , and A_{2Q} of equation (20):

From (19):

$$I = \int \frac{d^3 k'}{k'_o} \frac{d^3 Q}{Q_o} F(k' \cdot Q) \theta [-(k' - Q)^2] \quad (A.1)$$

where $F(k' \cdot Q)$ is a quadratic function of $(k' \cdot Q)$. Hence we may rewrite this as a quadratic function of $\cos \vartheta$. We write, for convenience,

$$F(k' \cdot Q) = B_Q - m_L^2 A_Q \quad (A.2)$$

where

$$B_Q = B_0 + B_1 \cos \vartheta + B_2 (\cos \vartheta)^2 \quad (A.3)$$

and

$$A_Q = A_0 + A_1 \cos \vartheta \quad (A.4)$$

with

$$B_0 = (k'_l Q_l - k'_o Q_o) \left[C_o + C_k k(Q_l - Q_o) - C_p M Q_o + C_{k'} (k'_l Q_l - k'_o Q_o) \right] \quad (A.5)$$

$$B_1 = k'_t Q_t \left[C_o + C_k k(Q_l - Q_o) - C_p M Q_o + C_{k'} (k'_l Q_l - k'_o Q_o) \right] \quad (A.6)$$

$$B_2 = C_{k'} k'^2_t Q_t^2 \quad (A.7)$$

$$A_0 = k A_k (Q_l - Q_o) - A_p M Q_o + A_{k'} (k'_l Q_l - k'_o Q_o) \quad (A.8)$$

and

$$A_1 = k'_t Q_t A_{k'} \quad (A.9)$$

The subscripts k, p, k' in the coefficients C 's and A 's imply that the latter are coefficients of $k \cdot Q, p \cdot Q$, and $k' \cdot Q$ respectively and subscript o denotes the term independent of Q in $F(k'Q)$.

The coefficients that appear in equation (20) are then given as

$$\begin{aligned} A_{0Q} &= B_0 - m_L^2 A_0 \\ A_{1Q} &= B_1 - m_L^2 A_1 \\ A_{2Q} &= B_2 \end{aligned} \quad (A.10)$$

So we need to know $C_o, C_k, C_p, C_{k'}$ and $A_k, A_p, A_{k'}$ of equations (A.5) to (A.9) above. These are given below.

$$C_o = -2(1+g)^2 (1+h^2) m_L^2 C_{ot} \quad (A.11)$$

where

$$\begin{aligned} C_{ot} &= 2W_1 k' \cdot k + \frac{W_2}{M^2} (k' \cdot k p^2 - 2k' \cdot p k \cdot p) - \frac{W_3}{M^2} (k' \cdot p k \cdot q - k' \cdot q k \cdot p) \frac{2h}{1+h^2} \\ &\quad + \frac{W_4}{M^2} (k' \cdot k q^2 - 2k' \cdot q k \cdot q) + \frac{W_5}{M^2} [(k' \cdot k p \cdot q - k' \cdot p k \cdot q - k' \cdot q k \cdot p)] \end{aligned} \quad (A.12)$$

$$C_p = 8h m_L^2 [(1+g)^2 - 3(1-g)^2] C_{pt} \quad (A.13)$$

where

$$C_{pt} = 2 \frac{W_2}{M^2} k \cdot p + \frac{W_3}{M^2} \frac{1+h^2}{2h} k \cdot q + \frac{W_5}{M^2} k \cdot q \quad (A.14)$$

$$C_k = -8m_L^2 [(1+g)^2 - 3(1-g)^2] C_{kt} \quad (A.15)$$

where

$$C_{kt} = 2W_1 + \frac{W_2}{M^2} p^2 + \frac{W_3}{M^2} \frac{1+h^2}{2h} k \cdot p + \frac{W_4}{M^2} (q^2 - 2q \cdot k) + \frac{W_5}{M^2} (q \cdot p - k \cdot p) \quad (A.16)$$

$$\begin{aligned}
C_{k'} = & -4 \left[3(1-g)^2 (1-h)^2 + (1+g)^2 (1+h)^2 \right] C_{k't_1} \\
& -4 \left[(1+g)^2 (1+h)^2 - 3(1-g)^2 (1-h)^2 \right] C_{k't_2} \\
& + 8h m_L^2 \left[3(1-g)^2 - (1+g)^2 \right] C_{k't_3}
\end{aligned} \tag{A.17}$$

where

$$C_{k't_1} = C_{ot} + \frac{W_3}{M^2} (k' \cdot pk \cdot q - k' \cdot qk \cdot p) \frac{2h}{1+h^2} \tag{A.18}$$

$$C_{k't_2} = \frac{W_3}{M^2} (k' \cdot qk \cdot p - k' \cdot pk \cdot q) \tag{A.19}$$

$$C_{k't_3} = 2 \frac{W_4}{M^2} k \cdot q + \frac{W_5}{M^2} k \cdot p - \frac{W_3}{M^2} \frac{1+h^2}{2h} k \cdot p \tag{A.20}$$

Next,

$$A_k = 2h m_L^2 \left[(1+g)^2 - 6(1-g)^2 \right] C_{kt} \tag{A.21}$$

$$A_p = -2h m_L^2 \left[(1+g)^2 - 6(1-g)^2 \right] C_{pt} \tag{A.22}$$

and

$$\begin{aligned}
A_{k'} = & \left[6(1-g)^2 (1-h)^2 + (1+g)^2 (1+h)^2 \right] C_{k't_1} \\
& + \left[(1+h)^2 (1+g)^2 - 6(1-g)^2 (1-h)^2 \right] C_{k't_2} \\
& + 2h m_L^2 \left[(1+g)^2 - 6(1-g)^2 \right] C_{k't_3}
\end{aligned} \tag{A.23}$$

b). Coefficients A_{0K} , A_{1K} and A_{2K} of equation (22):

These are obtained in the same general way outlined in the last section.

$$\begin{aligned} A_{0K} = & (k'_\ell K_\ell - k'_o K_o) C_{0K} - (k'_\ell K_\ell - k'_o K_o) B_{k'} \left[2(k'_\ell K_\ell - k'_o K_o) + m_L^2 \right] \\ & + B_p M K_o \left[2(k'_\ell K_\ell - k'_o K_o) + m_L^2 \right] \\ & - E K_o (\cos \theta' - 1) B_k \left[2(k'_\ell K_\ell - k'_o K_o) + m_L^2 \right] \end{aligned} \quad (A.24)$$

$$\begin{aligned} A_{1K} = & 2k'_t K_t \left[C_{0K} - 4B_{k'} (k'_\ell K_\ell - k'_o K_o) + 2M K_o B_p - 2B_k E K_o (\cos \theta' - 1) \right] \\ & - m_L^2 B_{k'} k'_t K_t \end{aligned} \quad (A.25)$$

$$A_{2K} = - 4 k_t'^2 K_t^2 B_{k'} \quad (A.26)$$

where C_{0K} , $B_{k'}$, B_p , and B_k are given below.

$$C_{0K} = - (1+h^2) m_L^2 \left[(1+g)^2 + (1-g)^2 \right] C_{ot} \quad (A.27)$$

and C_{ot} is given in equation (A.12).

$$B_k = 2h \left[(1+g)^2 - (1-g)^2 \right] m_L^2 C_{kt} \quad (A.28)$$

$$B_p = - 2h \left[(1+g)^2 - (1-g)^2 \right] m_L^2 C_{pt} \quad (A.29)$$

$$\begin{aligned} B_{k'} = & - \left[(1+g)^2 (1+h)^2 + (1-g)^2 (1-h)^2 \right] C_{k't_1} \\ & + \left[(1-g)^2 (1-h)^2 - (1+g)^2 (1+h)^2 \right] C_{k't_2} \\ & + 2h m_L^2 \left[(1-g)^2 - (1+g)^2 \right] C_{k't_3} \end{aligned} \quad (A.30)$$

where C_{kt} , C_{pt} , $C_{k't_1}$ and $C_{k't_3}$ are given in part a) of this section.

c). A_{00} , A_{01} and A_{02} of equation (24):

These are again obtained the same way as those in section a).

$$A_{00} = -(1+h^2) C_{ot} (k'_\lambda Q_\lambda - k'_o Q_o) \left\{ 9(1+g^2 - \frac{2}{3}g) m_L^2 + 16(1+g^2 - g) (k'_\lambda Q_\lambda - k'_o Q_o) \right\} \quad (A.31)$$

$$A_{01} = -(1+h^2) C_{ot} k'_t Q_t \left\{ 9(1+g^2 - \frac{2}{3}g) m_L^2 + 32(1+g^2 - g) (k'_\lambda Q_\lambda - k'_o Q_o) \right\} \quad (A.32)$$

$$A_{02} = -16(1+h^2) C_{ot} k'^2_t Q_t^2 (1+g^2 - g) \quad (A.33)$$

where C_{ot} is given in (A.12).

d). C_+ , C_- occurring in equation (25):

$$C_+ = +2C (1+h^2) m_L^4 C_{ot} \quad (A.34)$$

where

$$C = \frac{G_1^2 G_2^2 \pi}{48(2\pi)^5 m_L^7 ME^2} \quad (A.35)$$

and C_{ot} is given in (A.12).

$$C_- = C m_L^4 \left\{ - \frac{6k m_L^2 h}{k'_o + k'_L} C_{kt} + \frac{6h k'_o m_L^2}{k'^2_t + m_L^2} C_{pt} + 3(1-h)^2 (C_{k't_1} - C_{k't_2}) - 6m_L^2 h C_{k't_3} \right\} \quad (A.36)$$

where C_{kt} , C_{pt} , $C_{k't_1}$, $C_{k't_2}$ and $C_{k't_3}$ are given in equations (A.16), (A.14) (A.18), (A.19) and (A.20).

FOOTNOTES

1. Some of the more prominent papers on this topic are H. Georgi, and S.L. Glashow, Phys. Rev. Letters 28, 1494 (1972); B.W. Lee, Phys. Rev. D6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972).
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2. R. Cester, et al., a proposal to NAL on "Feasible Search for Heavy Neutral Muon Predicted by Gauge Theories", Princeton University (unpublished - 1973). W. Lee, et al., "Proposal to Search for Heavy Leptons and Intermediate Boson from Photon-Nucleon and Photon-Nuclei Collisions", NAL Proposal No. 87 (1970). M. Bernadini, et al., paper submitted to the 6th International Symposium on Electron and Photon Interaction at High Energy, Bonn, Germany, August 1973. B. C. Barish, et al., preprint, "Mass Limit on a Positively Charged Heavy Muon", Calif. Inst. of Tech. (unpublished - 1973).
3. J.D. Bjorken and C.H. Llewellyn Smith, Phys. Rev. D7, 887 (1973).
4. C. H. Albright, Phys. Rev. Letters 28, 1150 (1972).
5. C. Baltay, invited paper at Los Angeles Meeting of the American Physical Society, December 27, 1972.
6. We are assuming L^- to have the same lepton number as ℓ^- or ν_ℓ . If instead L^+ has the same lepton number as ℓ^- or ν_ℓ , as happens in certain gauge theories, our graphs, for (V-A) and (V+A) couplings, for the differential cross section of the outgoing charged lepton will just get interchanged.

7. We use $p \cdot q = \vec{p} \cdot \vec{q} - p_0 q_0$ and in our notation $\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$.

8. Note that we are assuming (V-A) coupling for the (e, ν_e) or (μ, ν_μ) currents even though L may couple either via (V-A) or (V+A) currents.

9. We use the narrow width approximation, that is,

$$\left| \frac{1}{k'^2 + m_L^2 - im_L \Gamma} \right|^2 = \frac{\pi}{m_L \Gamma} \delta(k'^2 + m_L^2) .$$

10. Φ_Q is the upper limit on ϕ . This results from the constraint $(k' - Q)^2 \leq 0$.

11. From (15),

$$\Gamma_{L \rightarrow \ell} = \frac{G_2^2 m_L^5}{4 \times 192 \pi^3} \left[(1+g)^2 + (1-g)^2 \right] .$$

12. See Ref. 3. Assuming L^+ to have the same lepton number as ν_ℓ and ℓ^-

they estimate $R(L_e^+ \rightarrow \mu^+ \nu_\mu \nu_e) \simeq 15\%$ and

$$R(L_e^+ \rightarrow e^+ \nu_e \nu_e) \simeq 30\% ,$$

the difference coming from the identity of the two neutrinos in the final state of the second reaction. Throughout our numerical computations we are using $R = 15\%$.

distributions of the final charged lepton by keeping the energy of the incident lepton beam fixed at 100 GeV and taking the mass of the heavy lepton $m_L = 2, 5, 8$ and 11 GeV. In Fig. 5, m_L is held fixed and we take the incident lepton energy $E = 50, 100, 200$ and 500 GeV.

Figures 6 and 7: Here the incident particle is taken to be $\bar{\nu}$ or ℓ_{RH}^+ producing L^+ or $\bar{L}^0$ and decaying via $L^+ \rightarrow \bar{\nu}(K) + \ell^+(Q) + \nu_\ell(\bar{k})$ or $\bar{L}^0 \rightarrow \ell^+(K) + \ell^-(Q) + \bar{\nu}_\ell(\bar{k})$. Again (V-A) coupling is assumed so that

$$\frac{d\sigma}{dQ_t^2} = \frac{d\sigma}{dK_t^2}.$$

Figures 8 and 9: These are the normalized versions of Figs. 4 and 5. Since, for (V-A) coupling of L, $\frac{d\sigma}{dQ_t^2} = \frac{8}{m_L^2} \sigma_P R$ such distributions are essentially normalized to the total cross section for a fixed value of m_L and are free of the value of R, unlike Figs. 4 and 5, and are, therefore, more readily comparable to an experimental graph of $\left(\frac{\text{number of events with } Q_t^2}{\text{number of events with } Q_t^2=0} \right)$ versus Q_t^2 .

Figures 10 and 11: Here (V+A) coupling for L is assumed, so the incident lepton is right handed and $\frac{d\sigma}{dQ_t^2} \neq \frac{d\sigma}{dK_t^2}$. The two charged leptons in the final state of decay of L^0 have significantly different distributions. In Fig. 10 we compare $\frac{d\sigma}{dQ_t^2}$ and $\frac{d\sigma}{dK_t^2}$ for fixed energy $E = 100$ GeV and for $m_L = 2$ and 8 GeV. In Fig. 11, we hold $m_L = 5$ GeV fixed and compare $\frac{d\sigma}{dQ_t^2}$ and $\frac{d\sigma}{dK_t^2}$ for $E = 50, 200$ and 500 GeV.

Figures 12 and 13: These are for (V+A) coupling assuming a left handed incident anti-lepton. Again in Fig. 12, we compare $\frac{d\sigma}{dQ_t^2}$ and $\frac{d\sigma}{dK_t^2}$ for fixed E and

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6.

Figure 7.

(V-A) COUPLING, ν (or μ_{LH}^-) INCIDENT.

$$P_{Q_{\uparrow}} = \left(\frac{\frac{d\sigma}{dQ_{\uparrow}^2}}{\left. \frac{d\sigma}{dQ_{\uparrow}^2} \right|_{Q_{\uparrow}=0}} \times 100 \right) \%$$

$$\left. \frac{d\sigma}{dQ_{\uparrow}^2} \right|_{Q_{\uparrow}=0} = \frac{8}{m_L^2} \sigma_P R, \quad R = \text{BRANCHING RATIO} = 15\%$$

$E = 100 \text{ GeV}$

Figure 8.

(V-A) COUPLING, ν (or μ_{LH}^-) INCIDENT.

$$P_{Q_{\uparrow}} = \left(\frac{\frac{d\sigma/dQ_{\uparrow}^2}{\left. \frac{d\sigma}{dQ_{\uparrow}^2} \right|_{Q_{\uparrow}=0}} \times 100 \right) \%$$

$$\left. \frac{d\sigma}{dQ_{\uparrow}^2} \right|_{Q_{\uparrow}=0} = \frac{8}{m_L^2} \sigma_P R, \quad R = \text{BRANCHING RATIO} = 15\%$$

$$m_L = 5 \text{ Gev}/c^2$$

Figure 9.

Figure 10.

Figure 11.

(V + A) COUPLING

$\bar{\nu}$ (or μ_{LH}^+) INCIDENT

BRANCHING RATIO = 15%

E = 100 Gev

$I(\kappa) \cdots \frac{d\sigma}{dK_{\uparrow}^2}$ for $m_L = 2 \text{ Gev}/c^2$

$I(Q) \cdots \frac{d\sigma}{dQ_{\uparrow}^2}$ for $m_L = 2 \text{ Gev}/c^2$

$II(\kappa) \cdots \frac{d\sigma}{dK_{\uparrow}^2}$ for $m_L = 8 \text{ Gev}/c^2$

$II(Q) \cdots \frac{d\sigma}{dQ_{\uparrow}^2}$ for $m_L = 8 \text{ Gev}/c^2$

Figure 12.

Figure 13.

$$P_{LH} = \frac{1 - \hat{S} \cdot \hat{k}'}{2}$$

$E = 100 \text{ Gev}$ { I --- $m_L = 5 \text{ Gev}/c^2$ $\theta' = 2^\circ$
 II --- $m_L = 5 \text{ Gev}/c^2$ $\theta' = 5^\circ$
 III --- $m_L = 8 \text{ Gev}/c^2$ $\theta' = 2^\circ$
 $E = 200 \text{ Gev}$ IV --- $m_L = 5 \text{ Gev}/c^2$ $\theta' = 2^\circ$

Figure 14.

$$D_0 = \frac{\left(\frac{d\sigma}{dQ_{\uparrow}^2} - \frac{d\sigma_0}{dQ_{\uparrow}^2} \right)}{\frac{d\sigma}{dQ_{\uparrow}^2}} \times 100\%$$

$$D_L = \frac{\left(\frac{d\sigma}{dQ_{\uparrow}^2} - \frac{d\sigma_L}{dQ_{\uparrow}^2} \right)}{\frac{d\sigma}{dQ_{\uparrow}^2}} \times 100\%$$

(V-A) COUPLING

E = 100 GeV

I (O) --- D_0 for $m_L = 5 \text{ GeV}/c^2$

I (L) --- D_L for $m_L = 5 \text{ GeV}/c^2$

II (O) --- D_0 for $m_L = 8 \text{ GeV}/c^2$

II (L) --- D_L for $m_L = 8 \text{ GeV}/c^2$

Figure 15.