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A PRELIMINARY NUCLEAR DESIGN STUDY OF DIPHENYL
MODERATED AND COOLED REACTORS

Price \$ 0.70

by

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	7
I. SCOPE OF DESIGN STUDY	7
II. STAINLESS STEEL-DIPHENYL REACTORS	8
A. Calculation of Fast Reactor Constants	8
1. Fermi Age	8
2. Fast Diffusion Constant	8
B. Unborated Stainless Steel-Diphenyl Reactors	8
C. Borated Stainless Steel-Diphenyl Reactor Using Temperature Override of Maximum Xenon and Samarium	19
D. Unborated Stainless Steel Reactor Having an Infinite Diphenyl Reflector	26
E. Temperature Override of Maximum Xenon and Samarium for a 60-mw Reactor	26
III. SLIGHTLY ENRICHED DIPHENYL REACTOR	29
A. Description of Fuel Element Assembly	29
1. Wire Fuel Elements	29
2. Flat Plate Fuel Elements	31
B. Resonance Escape Probability	35
1. Wire Fuel Elements	36
2. Flat Plate Fuel Elements	38
C. Thermal Fission Utilization	40
D. Neutron Reproduction Factor	40
E. Fast Fission Factor	44
F. Fermi Age and Fast Diffusion Constant	44
G. Critical Reactor Loading	46

DECLASSIFIED

4

TABLE OF CONTENTS

	<u>Page</u>
IV. SUMMARY OF NUCLEAR DESIGN DATA	49
A. 20-mw Fully Enriched Zirconium-Diphenyl Test Reactor	49
B. 60-mw Fully Enriched Stainless Steel-Diphenyl Reactor	50
C. 60-mw Fully Enriched Stainless Steel-Diphenyl Reactor (Temperature Override of Maximum Xenon) . . .	53
D. 140-mw Slightly Enriched Zirconium-Diphenyl Reactor	54
V. CONCLUSIONS	56

DECLASSIFIED

LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	Fermi Age for Zirconium and Stainless Steel in Diphenyl at 600F	9
2	Fast Diffusion Coefficient for Zirconium and Stainless Steel in Diphenyl at 500F	10
3	Critical Core Radii vs Design Full Power for Various Stainless Steel:Diphenyl Volume Ratios (n_g)	16
4	Critical U^{235} Loading vs Design Full Power	17
5	Average Neutron Flux in Old Core (at end of 2000 hr) vs Design Full Power for Various Stainless Steel:Diphenyl Volume Ratios (n_g)	18
6	Override Temperature of Maximum Xenon and Samarium for Borated Fuel Element-Diphenyl-Cooled Reactor	24
7	Override Temperature of Maximum Xenon and Samarium for Borated Fuel Element-Diphenyl-Cooled Reactor	25
8	Radial Neutron Flux at Midplane of Core (60-mw Reactor)	27
9	Axial Neutron Flux at Center of Core (60-mw Reactor)	28
10	Neutron Shielding by Neighboring Fuel Elements	30
11	Typical Flat-Plate Fuel Assembly	32
12	Neutron Shielding Analysis	37
13	Resonance Escape Probability for Wire and Plate Fuel Elements as a Function of Atomic Enrichment ($n_g = 0.50$)	39
14	Thermal Utilization Factor for Wire and Plate Fuel Elements as a Function of Atomic Enrichment ($n_g = 0.50$)	41
15	Neutron Reproduction Factor for Wire and Plate Fuel Elements as a Function of Atomic Enrichment	43
16	Multiplication Constant for Wire and Plate Fuel Elements as a Function of Atomic Enrichment	45
17	Radial Neutron Flux at Midplane of Core	47

6
LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
18	Axial Neutron Flux at Center of Core	48
19	Radial Neutron Flux at Midplane of Core	51
20	Axial Neutron Flux at Center of Core	52

LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
I.	Thermal Cross Sections	35
II.	Resonance Cross Sections	35
III.	Design Parameters	42

A PRELIMINARY NUCLEAR DESIGN STUDY OF DIPHENYL MODERATED AND COOLED REACTORS

by

E. F. Fricke

ABSTRACT

Studies have been made on the basic nuclear characteristics of diphenyl moderated and cooled reactors for an average operating temperature of 600F. Calculations include the determination of fast diffusion constants, Fermi Age, critical loadings, and radial and axial fast and slow flux distributions, for different types of reactors with power levels ranging from zero to 140 mw.

I. SCOPE OF DESIGN STUDY

The design study was carried out along the following routes:

1. A broad study of stainless steel, fully enriched reactors assuming a reflector savings of 7 cm.
2. A study of temperature override and burnable poisons for reactor control and a reflector savings of 7 cm.
3. A study of a 60-mw stainless, fully enriched reactor having an infinite diphenyl reflector.
4. A study of a 20-mw zirconium, fully enriched reactor having an infinite diphenyl reflector.
5. A study of a U^{238} -zirconium reactor, slightly enriched with U^{235} , and having an infinite diphenyl reflector.

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II. STAINLESS STEEL REACTORS

A. Calculation of Fast Reactor Constants

1. Fermi Age

The calculated values of the Fermi Age, τ , for various stainless steel to diphenyl volume ratios are shown in Fig. 1. These were calculated by using Marshak's method, following procedures outlined in a publication by Oak Ridge.¹ The scattering cross sections were obtained from the latest revised supplements to AECU-2040.² The averaging was carried out to 10 mev, in which case the cross section for chromium was extrapolated. Since there are no experimental data for the Fermi Age for diphenyl, the age for pure diphenyl was compared to τ for a zirconium-light water mixture, normalized to conform with τ equal to 31 cm² for pure water at 68F, having the same scattering per hydrogen atom at identical temperatures. This comparison indicates that the calculated value of τ for pure diphenyl, which is 83 cm² at 600F, should be multiplied by a normalization factor of 1.09. All of the calculated values of τ were multiplied by 1.09 to obtain the normalized curves.

2. Fast Diffusion Constant

The fast diffusion constants D_f shown in Fig. 2 were calculated at 500F using procedures outlined in ANL-4337.³

The scattering cross sections were those used for calculating τ . In order to check these calculations, τ and D_f for pure light water were recalculated at 68F using the most recent cross section data for hydrogen. Indications are that existing values for D_f for light water-zirconium mixtures should be recalculated and that the values of D_f plotted in ANL-4337 are slightly too low. Therefore, a normalization factor was not applied to D_f for diphenyl.

B. Unborated Stainless Steel-Diphenyl Reactors

These reactors have the common design features:

1. Average design temperature: 600F
2. Maximum xenon and samarium override

¹Reactor Design and Feasibility Problem: "MTR-Type Power Producer for a Remote Location," ORNL-CR-52-8-220 (August 20, 1952).

²Neutron Cross Sections, AECU-2040 (May 15, 1952).

³Naval Reactor Division Quarterly Report, ANL-4337 p. 13 (September 15, 1949).

FIG. 1
FERMI AGE FOR ZIRCONIUM AND STAINLESS STEEL IN DIPHENYL AT 600 F
(NORMALIZATION FACTOR = 1.09)

ANL-EFF-16405-A

10

FIG. 2
FAST DIFFUSION COEFFICIENT FOR ZIRCONIUM AND STAINLESS
STEEL IN DIPHENYL AT 500 F. (NOT NORMALIZED)

ANL-EFF-16406-A

3. Reflector savings of 7 cm
4. Metal to diphenyl volume ratios of 1:4, 1:3 and 1:2
5. Fission product buildup for 2000 hr of full-power operation
6. Design power from zero to 120 mw
7. Core height equal to core diameter
8. Wire fuel elements

When these studies were made the fast diffusion constant D_f was not yet calculated and the possibility of having to use a poor reflector instead of diphenyl was not out of order since it seemed reasonable to eliminate as much diphenyl as possible from radiation damage.

Two-group critical equations were used to calculate the U^{235} loading.

$$(1 + L^2 B^2) (1 + \tau B^2) = k_{\infty} = \eta f \quad (1)$$

where

L = diffusion length of thermal neutrons in the core (cm)

τ = Fermi Age of neutrons (cm^2)

B^2 = geometric buckling (cm^{-2})

$$= \frac{8.250460}{(R + RS)^2} \quad (2)$$

R = radius of core (cm)

RS = reflector savings (cm). It was assumed that the reflector savings on an end of the core is equal to the radial value.

Also

$$L^2 = \frac{D_s}{\Sigma_a^{U^{235}} + \Sigma_a^{\text{mod}} + \Sigma_a^{\text{Sm}} + \Sigma_a^{\text{Xe}} + \Sigma_a^{\text{F}}} \quad (3)$$

where

D_s = thermal neutron diffusion constant (cm)

$\Sigma_a^{U^{235}}$ = macroscopic absorption cross section of U^{235} (cm^{-1})

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Σ_a^{mod} = macroscopic absorption cross section of moderator (cm^{-1})

Σ_a^{Sm} = macroscopic absorption cross section of samarium (cm^{-1})

Σ_a^{Xe} = macroscopic absorption cross section of xenon (cm^{-1})

Σ_a^{F} = macroscopic absorption cross section of fission products (cm^{-1}).

The thermal fission utilization is:

$$f = \frac{\Sigma_a^{\text{U}^{235}}}{\Sigma_a^{\text{U}^{235}} + \Sigma_a^{\text{mod}} + \Sigma_a^{\text{Sm}} + \Sigma_a^{\text{Xe}} + \Sigma_a^{\text{F}}} \quad (4)$$

η = neutron reproduction factor = 2.09.

k_{∞} = neutron multiplication constant.

Σ_a^{Xe} has been calculated from the following equation, assuming complete instantaneous shutdown from full power:

$$\Sigma_a^{\text{Xe}} = \sigma_a^{\text{Xe}} C_0 \left[\left(\frac{\gamma_1 + \gamma_2}{\lambda_2 + \phi_0 \sigma_a^{\text{Xe}}} - \frac{\gamma_1}{\lambda_2 - \lambda_1} \right) e^{-\lambda_2 t} + \frac{\gamma_1}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} \right] \quad (5)$$

$$C_0 = \frac{\Sigma_a^{\text{U}^{235}}}{1,184} \phi_0 = \text{rate of fission (fissions/cc-sec)}$$

$$= 3.16865 \times 10^{16} P_0/V$$

V = volume of the core (cc)

P_0 = power produced in core (mw)

λ_1 = decay constant for I^{135} (sec^{-1}) 2.9×10^{-5}

λ_2 = decay constant for Xe^{135} (sec^{-1}) 2.1×10^{-5}

γ_1 = fractional yield of I^{135} 0.064

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13

$$\begin{aligned}
 \gamma_2 &= \text{fractional yield of Xe}^{135} \text{ by direct fission} & 0.003 \\
 \sigma_a^{\text{Xe}} &= \text{microscopic absorption cross section for} \\
 &\quad \text{xenon (cm}^2\text{)} & 2.4 \times 10^{-18} \\
 \phi_0 &= \text{average thermal neutron flux in the core} \\
 &= \frac{3.751682 \times 10^{16}}{\sum_a U^{235}} \frac{P_0}{V} & (6)
 \end{aligned}$$

$$\begin{aligned}
 t &= \text{time for xenon to reach a maximum (sec)} \\
 &= \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1} \left(1 - \frac{\lambda_2 - \lambda_1}{\lambda_2 + \sigma_a^{\text{Xe}} \phi_0} \frac{\gamma_1 + \gamma_2}{\gamma_1} \right) . & (7)
 \end{aligned}$$

The macroscopic cross section of samarium at the time of maximum xenon concentration was calculated from the equation:

$$\sum_a^{\text{Sm}} = \frac{\gamma_3 \sum_a U^{235}}{1.184} + \frac{\gamma_3}{\lambda_3} \sigma_a^{\text{Sm}} (1 - e^{-\lambda_3 t}) C_0 \quad (8)$$

where

$$\begin{aligned}
 \gamma_3 &= \text{fractional yield of promethium} & 0.014 \\
 \lambda_3 &= \text{decay constant for promethium (sec}^{-1}\text{)} & 4.1 \times 10^{-6} \\
 \sigma_a^{\text{Sm}} &= \text{microscopic absorption cross section for} \\
 &\quad \text{samarium (cm}^2\text{)} & 8.12 \times 10^{-21}
 \end{aligned}$$

The fission product macroscopic cross section for any age of the core, t_c , was calculated as (60 barns/fission) (fissions/cc-days) (time in days):

$$\begin{aligned}
 \sum_a^{\text{F}} &= (60 \times 10^{-24}) (\sum_f U^{235} \phi_0) (8.64 \times 10^4 t_c) \\
 &= 5.184 \times 10^{-18} C_0 t_c \text{ (cm}^{-1}\text{)} . & (9)
 \end{aligned}$$

$$t_c = \text{equivalent age of core in days if operated at full design power.}$$

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14

The macroscopic cross section of the moderator was calculated from the equation:

$$\Sigma_a^{\text{mod}} = \frac{n_g}{1+n_g} \Sigma_a^{\text{SS}} + \frac{1}{1+n_g} \Sigma_a^{\text{Di}} \quad (10)$$

where

n_g = stainless steel to diphenyl volume ratio

Σ_a^{SS} = macroscopic cross section of stainless steel at 600F
 $= 0.141351 \text{ cm}^{-1}$

Σ_a^{Di} = macroscopic cross section of diphenyl at the design 600F
 $= 0.006440 \text{ cm}^{-1}$

$\Sigma_{\text{tr}}^{\text{SS}}$ = macroscopic transport cross section of stainless steel at 600F
 $= 0.897444 \text{ cm}^{-1}$

$\Sigma_{\text{tr}}^{\text{Di}}$ = macroscopic transport cross section of diphenyl at 600F
 $= 0.475094 \text{ cm}^{-1}$

The macroscopic cross section of U^{235} is based upon the following equation, which was calculated for a number of temperatures with averaging over each Maxwellian flux:

$$\sigma_a^{\text{U}^{235}} = 582.215 - 0.17304 T \text{ (barns)} \quad (11)$$

T = degrees Rankin

It is noted that Eq. (1) can be written as:

$$\left(\frac{\eta}{1 + \tau B^2} - 1 \right) \Sigma_a^{\text{U}^{235}} = \Sigma_a^{\text{mod}} + \Sigma_a^{\text{Xe}} + \Sigma_a^{\text{Sm}} + \Sigma_a^{\text{F}} + D_s B^2 \quad (12)$$

or

$$\left(\frac{\eta}{1 + \tau B^2} - 1 \right) \frac{3.751682 \times 10^{16}}{\phi_0} \frac{P_0}{V} = \Sigma_a^{\text{mod}} + \Sigma_a^{\text{Xe}} + \Sigma_a^{\text{Sm}} + \Sigma_a^{\text{F}} + D_s B^2 \quad (13)$$

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15

It is noted that for a given design power, P_0 , the flux ϕ_0 in Eq. (13) is a function of the radius of the reactor, through V and B^2 . For each selected value of P_0 , a bell-shaped curve results from a plot of ϕ_0 vs R . The radius at which ϕ_0 is a maximum yields a core design having a minimum U^{235} loading. Equation (13) assumes that the control rods are fully withdrawn, the burnable U^{235} has about been used up, and the fission poisons are a maximum; that is, the reactor is operating its last day, but can still override maximum xenon.

Equation (13) can be used to calculate the loading for a new core provided Σ_a^P , an equivalent poison macroscopic cross section which compensates for additional macroscopic cross section of the U^{235} , is added to the right side of the equation.

For a zero power reactor Σ_a^{Xe} , Σ_a^{Sm} and Σ_a^F are deleted from Eq. (12) and the critical loading W is plotted as a function R .

$$\Sigma_a^{U^{235}} = \frac{W}{235} \frac{6.023 \times 10^{23}}{V} \sigma_a^{U^{235}} \text{ (cm}^{-1}\text{)} \quad (14)$$

For one particular value of R the loading W will be a minimum. If the radius of the reactor has been set by other requirements, then the loading for which no xenon or samarium production can be tolerated also can be calculated from Eq. (12).

Figure 3 indicates the critical radii for stainless steel to diphenyl volume ratios of 1:4, 1:3 and 1:2 as a function of the design full power.

Figure 4 shows the loading of U^{235} for the critical radii given in Fig. 3.

Figure 5 gives the average old-core flux that would be expected at the end of 2000 hr of full-power operation.

A reflector having a higher reflector savings than 7 cm would reduce the U^{235} loading, but the thermal flux would also increase, and it is assumed that some of the thermal flux aids the fast flux in the decomposition of the diphenyl.

FIG. 3
CRITICAL CORE RADIUS VS DESIGN FULL POWER FOR
VARIOUS STAINLESS STEEL:DIPHENYL VOLUME RATIOS (n_g)
(REFLECTOR SAVINGS = 7 cm)

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FIG. 4
CRITICAL U^{235} LOADING VS DESIGN FULL POWER

ANL-EFF-16408-A

FIG. 5
 AVERAGE NEUTRON FLUX IN OLD CORE (AT END OF
 2000 hr) VS DESIGN FULL POWER FOR VARIOUS
 STAINLESS STEEL:DIPHENYL VOLUME RATIOS (n_g)
 REFLECTOR SAVINGS = 7 cm

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19

C. Borated Stainless Steel-Diphenyl Reactor Using Temperature Override of Maximum Xenon and Samarium

A preliminary study has been made to determine the possibility of using temperature override of maximum xenon and samarium. B^{10} in the form of borated fuel elements was used as poison to hold down the U^{235} added for burnup and fission product compensation.

The general equation used for these calculations is given by Eq. (1), from which:

$$\left(\frac{\eta}{1 + \tau B^2} - 1 \right) \Sigma_a^{U^{235}} = \Sigma_a^{mod} + \Sigma_a^{Xe} + \Sigma_a^{Sm} + \Sigma_a^F + \Sigma_a^{B^{10}} + D_s B^2 \quad (15)$$

In this equation Σ_a^{Xe} and Σ_a^{Sm} are equilibrium macroscopic cross sections, since it is intended to override the maximum values by lowering the temperature of the reactor. $\Sigma_a^{B^{10}}$ is the macroscopic boron cross section and is a maximum for a new core. It is assumed that as the U^{235} burns up the boron does also according to the equations:

$$\frac{d N^{B^{10}}}{dt} = - \phi_0 N^{B^{10}} \sigma_a^{B^{10}} \quad (16)$$

and

$$\phi_0 = \frac{C_0}{\Sigma_f^{U^{235}}} = \frac{1.184 C_0}{\Sigma_{a0}^{U^{235}} - r_0 t} \quad (17)$$

and

$$\frac{d (\sigma_a^{B^{10}} N^{B^{10}})}{\sigma_a^{B^{10}} N^{B^{10}}} = - \phi_0 \sigma_a^{B^{10}} dt \quad (18)$$

or

$$\int_{\Sigma_{a0}^{B^{10}}}^{\Sigma_{ae}^{B^{10}}} \frac{d \Sigma_a^{B^{10}}}{\Sigma_a^{B^{10}}} = - 1.184 \sigma_a^{B^{10}} C_0 \int_0^{t_c} \frac{dt}{\Sigma_{a0}^{U^{235}} - r_0 t} \quad (19)$$

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$$\ln \frac{\left(\sum_{ae} B^{10} \right)}{\left(\sum_{a0} B^{10} \right)} = \frac{1.184 \sigma_a^{B^{10}} C_0}{r_0} \ln \left(1 - \frac{r_0}{\sum_{a0} U^{235}} t_c \right) \quad (20)$$

or

$$\sum_{ae} B^{10} = \sum_{a0} B^{10} \left(1 - \frac{r_0 t}{\sum_{a0} U^{235}} \right)^y ; \quad (21)$$

$$y = \frac{1.184 C_0 \sigma_a^{B^{10}}}{r_0} \quad (22)$$

$\sum_{ae} B^{10}$ = macroscopic absorption cross section of B^{10} at the end of the life of the core

$\sum_{a0} B^{10}$ = macroscopic absorption cross section of B^{10} for a new core

r_0 = rate of change of U^{235} macroscopic cross section per unit time

$\sum_{a0} U^{235}$ = macroscopic U^{235} absorption cross section for a new core

C_0 = number of fissions per unit time per cc

$\sigma_a^{B^{10}}$ = microscopic absorption cross section of boron B^{10} .

For a new core, assuming equilibrium xenon and samarium, Eq. (15) becomes:

$$\sum_{a0} B^{10} = \left(\frac{\eta}{1 + \tau B^2} - 1 \right) \sum_{a0} U^{235} - \left(\sum_a^{mod} + D_s B^2 + \sum_{a0}^{Xe} + \sum_{a0}^{Sm} \right). \quad (23)$$

For an old core,

$$\sum_{ae} B^{10} = \left(\frac{\eta}{1 + \tau B^2} - 1 \right) \sum_{ae} U^{235} - \left(\sum_a^{mod} + D_s B^2 + \sum_{ae}^{Xe} + \sum_{ae}^{Sm} + \sum_{ae}^F \right) \quad (24)$$

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21

and

$$\Sigma_{ae}^{U^{235}} = \Sigma_{a0}^{U^{235}} - r_0 t_c \quad (25)$$

$$\left(1 - \frac{r_0 t_c}{\Sigma_{a0}^{U^{235}}}\right)^y = \frac{\left(\frac{\eta}{1 + \tau B^2} - 1\right) \left(\Sigma_{a0}^{U^{235}} - r_0 t_c\right) - \left(\Sigma_a^{\text{mod}} + D_s B^2 + \Sigma_{ae}^{Xe} + \Sigma_{ae}^{Sm} + \Sigma_{ae}^F\right)}{\left(\frac{\eta}{1 + \tau B^2} - 1\right) \Sigma_{a0}^{U^{235}} - \left(\Sigma_a^{\text{mod}} + D_s B^2 + \Sigma_{a0}^{Xe} + \Sigma_{a0}^{Sm}\right)} \quad (26)$$

where

$$\Sigma_{ae}^{Xe} = \frac{C_0 \sigma_a^{Xe} (\gamma_1 + \gamma_2) \left(\Sigma_{a0}^{U^{235}} - r_0 t_c\right)}{\lambda_2 \left(\Sigma_{a0}^{U^{235}} - r_0 t_c\right) + 1.184 C_0 \sigma_a^{Xe}} \quad (27)$$

= equilibrium macroscopic absorption cross section of xenon at end of life of the core.

$$\Sigma_{ae}^{Sm} = 0.011824 \left(\Sigma_{a0}^{U^{235}} - r_0 t_c\right) \quad (28)$$

= equilibrium macroscopic cross section of samarium at end of life of core.

t_c = life of core and the units depend upon the units of r_0 .

Σ_{ae}^F = macroscopic cross section of fission product at end of life of the core.

The other terms have been defined previously.

For a particular radius and height of the core and reflector savings, a solution of Eq. (26) yields $\Sigma_{a0}^{U^{235}}$. A solution is obtained by plotting the worth of the two sides of Eq. (26) against $\Sigma_{a0}^{U^{235}}$ for particular

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values of R . For each design power there will be one value of R for which the U^{235} loading is a minimum. From this minimum value of the loading, the initial B^{10} loading, $\Sigma_{a0}^{B^{10}}$, can be calculated from Eq. (24) and $\Sigma_{ae}^{B^{10}}$ from Eq. (25).

The easiest method of solving for the U^{235} loading is to assume a reasonable value of the reflector savings, calculate Eq. (26) and then apply a two-group, two-region calculation and repeat the iteration process until the guessed and actual reflector savings are identical.

It is noted from Eq. (21) there will be unburned B^{10} in the core at the end of the life of the reactor. The fraction of this unburned boron is:

$$\frac{\Sigma_{ae}^{B^{10}}}{\Sigma_{a0}^{B^{10}}} = \left(1 - \frac{r_0 t_c}{\Sigma_{a0}^{U^{235}}} \right)^y \quad (29)$$

The unburned boron can tie up a considerable amount of uranium, causing a high initial inventory of U^{235} . This is the cost of the easier control made possible by using a burnable poison.

In order to estimate the override temperature, the critical size, loading and reflector savings are first calculated at the design temperature of the reactor. All of the terms of Eq. (23) are varied from the design values by placing perturbations upon the temperature. Once the reactor has been designed, the only real transient terms in Eq. (23) and (24) are Σ_a^{Xe} and Σ_a^{Sm} . These two terms will vary greatly from equilibrium values to maximum values when the reactor is shut down. Therefore, the transient and non-transient terms have been separated in Eq. (23) and (24).

In general:

$$\left(\Sigma_a^{Xe} + \Sigma_a^{Sm} \right)_{\max} = \left(\frac{\eta}{1 + \tau B^2} - 1 \right) \Sigma_a^{U^{235}} - \left(D_s B^2 + \Sigma_a^{B^{10}} + \Sigma_a^F + \Sigma_a^{\text{mod}} \right) \quad (30)$$

For a new core Eq. (30) becomes Eq. (23) and for an old core it becomes Eq. (24).

The left side of Eq. (30) can be calculated from Eq. (5) and (8), for which the variables depend upon the operating conditions of the reactor before shutdown; it is assumed that it is desired to start up the reactor exactly at the time when the buildup of xenon is a maximum value. If the

shutdown reactor is to be made critical, the temperature of the core must be lowered and as the temperature drops the macroscopic absorption cross sections for xenon and samarium increase as shown in Fig. 6 and 7. Likewise, the worth of the right side of Eq. (30) increases as the average temperature in the core is lowered, but the slope of this temperature curve is greater than that for the transients and the temperature at which the worths of the two sides of an equation are equal is the override temperature.

Figures 6 and 7 indicate that for the design conditions listed thereon the override temperatures are quite reasonable. It is noted that as the core ages the slope of the non-transient part of Eq. (30) decreases. One must make a systematic variation of all of the terms associated with temperature when plotting the worth of the terms of Eq. (30). When one compares the critical loading of borated and unborated 60-mw reactors it is found that the two types of reactors have about the same loadings, although the borated ones are temperature controlled to override the maximum xenon.

n_g	Borated		Unborated	
	U^{235} Loading, kg	R, cm	U^{235} Loading, kg	R, cm
1:4	37.5	33	43.6	33.5
1:2	57.5	34	59.4	34

The reason for this is that in the unborated reactor no excess uranium is needed to produce criticality for unburned boron at the end of the life of the core, and we can say, in general, that the uranium required is for clean critical, for burnout, for maximum xenon and samarium, and for fission product buildup. For a borated reactor, using temperature override, we must add uranium for clean critical, for burnout, for equilibrium xenon and samarium, for fission product buildup and for unburned boron.

Although additional calculations to determine the override temperatures for unborated reactors were not made, indications are that the price, in kg of uranium, to pay for the elimination of control rods can be estimated from Eq. (29).

For a metal to diphenyl volume ratio of 1:2, the ratio of $\Sigma_{ae}^{B^{10}}$ to $\Sigma_{a0}^{B^{10}}$ is 0.48 and approximately 7 kg of U^{235} must be added to compensate for this $\Sigma_{ae}^{B^{10}}$ alone. On the other hand, for a metal to diphenyl volume ratio of 1:4 the ratio of $\Sigma_{ae}^{B^{10}}$ to $\Sigma_{a0}^{B^{10}}$ is 0.309 and $\Sigma_{ae}^{B^{10}}$ is holding down approximately 3.5 kg of U^{235} .

24

FIG. 6
OVERRIDE TEMPERATURE OF MAXIMUM XENON AND SAMARIUM
FOR BORATED FUEL ELEMENT-DIPHENYL-COOLED REACTOR

ANL-EFF-16410-B

25

FIG. 7
OVERRIDE TEMPERATURE OF MAXIMUM XENON AND SAMARIUM
FOR BORATED FUEL ELEMENT-DIPHENYL COOLED REACTOR

ANL-EFF-16411-B

D. Unborated Stainless Steel Reactor Having an Infinite Diphenyl Reflector

A calculation to obtain the fast diffusion constant D_f for various stainless steel-diphenyl volume ratios was completed (see Fig. 2) and the critical size, loading, reflector savings and axial and radial fast and slow flux were obtained for a 60-mw reactor at a design temperature of 600F, including the override of maximum xenon or samarium for a core 2000 hr old. The core was fully enriched with U^{235} , the metal to diphenyl volume ratio was 1:2, and an infinite diphenyl reflector was used.

A two-group, two-region calculation was made to determine the loading for a core of 35-cm radius and a height of 70 cm. The critical loading is 39 kg of U^{235} for a core 2000 hr old. The radial reflector savings is 13.22 cm and is approximately the same for the slab end of the core.

Figure 8 shows a radial flux plot (in arbitrary flux units) through the midplane of the core. The maximum to average slow neutron flux is 1.16 while the maximum to average fast flux is 1.531.

Figure 9 shows the axial flux plot at the center of the core. The maximum to average slow neutron flux is 1.11 while the maximum to average fast flux is 1.291.

The volume average slow flux is 5.7×10^{13} neutrons/(cm²)(sec). The volume average fast flux is approximately 1.1×10^{15} . The volume average of the fast flux is only indicative since fast flux based upon a two-group theory is more or less hypothetical. A multigroup calculation would probably prove that the fast flux is less than this.

E. Temperature Override of Maximum Xenon and Samarium for a 60-mw Reactor

A 60-mw reactor having a stainless steel to diphenyl volume ratio of 1:2, a radius of 35 cm, height of 70 cm, 2000 hr old, a reflector savings of 13.2 cm, and an infinite diphenyl reflector, was found to be able to override maximum xenon and samarium at 250F when the loading at 2000 hr was approximately 32 kg of highly enriched U^{235} , the average design temperature being 600F. The non-transient terms in the buckling equation have a worth of zero at 725F, indicating a subcritical point at this temperature.

FIG. 8
RADIAL NEUTRON FLUX AT MIDPLANE OF CORE
(60-mw REACTOR)

ANL-EFF-16765-A

FIG. 9
AXIAL NEUTRON FLUX AT CENTER OF CORE
(60-mw REACTOR)

ANL-EFF-16766-A

III. SLIGHTLY ENRICHED DIPHENYL REACTOR

The critical uranium loading for a core four feet in diameter and four feet in height has been calculated. Two types of fuel elements, using U^{238} slightly enriched with U^{235} , were investigated. The first type considered was a wire-type element. The second type was an ordinary flat plate element. The four constants needed to calculate the multiplication constant k_{∞} were calculated for each type of fuel element.

$$k_{\infty} = \eta f p \epsilon \quad (31)$$

η = neutron reproduction factor

f = thermal fission utilization

p = resonance escape probability

ϵ = fast fission factor

A. Description of Fuel Element Assembly

1. Wire Fuel Elements

The wire elements have an over-all radius of 30 mils. The core of the wire has a radius of 20 mils and is surrounded by a zirconium clad 10 mils thick. In order to simplify calculations, an assumption was made that the fuel elements can be spaced as shown in Fig. 10.

If

$$n_g = \frac{\text{Total metal volume}}{\text{Liquid volume}},$$

then

$$\frac{n_g}{1 + n_g} = \text{volume of metal/cc of core}$$

and

$$\frac{1}{1 + n_g} = \text{volume of liquid/cc of core.}$$

If

$$p_u = \frac{\text{Volume of uranium meat}}{\text{Volume of wire}} = 0.4444$$

NEAREST NEIGHBOR TO ELEMENT "I" = $d = 0.234$ cm
 NEXT NEAREST NEIGHBOR = $1.414 d$
 NEXT NEAREST NEIGHBOR = $2 d$
 NEXT NEAREST NEIGHBOR = $2.24 d$

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FIG. 10
 NEUTRON SHIELDING BY
 NEIGHBORING FUEL ELEMENTS

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$$a = \frac{\text{Volume of zirconium in end plates}}{\text{total volume of metal}} = 0.13$$

$$x = \frac{\text{atoms of } U^{235}}{\text{atoms of uranium}}$$

$$\rho_m = \text{density of uranium meat}$$

$$\rho_{Zr} = \text{density of zirconium}$$

then

$$V_{Zr} = \frac{n_g}{1+n_g} [1 - p_u (1 - a)] = \text{volume of zirconium/cc of core} \quad (32)$$

$$V_{Di} = \frac{1}{1+n_g} = \text{volume of diphenyl/cc of core} \quad (33)$$

$$V_{U^{238}} = 0.87 p_u \frac{n_g}{1+n_g} (1 - 0.9874x) \quad (34)$$

$$= \text{volume of } U^{238}/\text{cc of core}$$

$$V_{U^{235}} = 0.859 p_u \frac{n_g}{1+n_g} x \quad (35)$$

$$= \text{volume of } U^{235}/\text{cc of core}$$

2. Flat Plate Fuel Elements

A typical cell is shown in Fig. 11. The symbols are defined as follows:

$$t_a = 0.10 \text{ in.}$$

$$t_c = \text{thickness of fuel plate clad}$$

$$t_1 = \text{thickness of end plates}$$

$$t_2 = \text{total thickness of fuel plates}$$

$$t_3 = \text{thickness of uranium alloy meat}$$

$$\ell_1 = \text{total width of assembly}$$

$$\ell_2 = \text{length of fuel plate}$$

FIG. 11
TYPICAL FLAT-PLATE
FUEL ASSEMBLY

n = number of fuel plates in assembly = $\ell_1/(y + t_2)$

n_g = total metal to diphenyl volume ratio

$$= (t_2/\ell_2) \frac{(\ell_2 + 2t_1)}{y} + (2t_1/\ell_2) \quad (36)$$

$$y = \text{spacing between the fuel plates} = t_2 \left(\frac{\ell_2 + 2t_1}{n_g \ell_2 - 2t_1} \right) \quad (37)$$

$$p_u = \frac{\text{volume of meat in a plate}}{\text{volume of the fuel plate}} = \frac{t_3}{t_2} \left(\frac{\ell_2 - 2t_a}{\ell_2} \right) \quad (38)$$

t_1 is set by mechanical feasibility and is 0.125 in. t_2 is determined by the allowable maximum temperature at the center of the fuel plate and therefore, is dependent upon the heat output per unit surface area of the plate and heat of conduction through the metal and the liquid film. n_g is set by both heat transfer and nuclear physics. The value of ℓ_2 is generally set by column buckling (because all of the fuel plates are not at the same temperature), and if the thickness of a plate is set, the length is then dependent upon the mechanical slenderness ratio. For these studies ℓ_2 was set equal to 3.5 in. The calculated values for different types of assemblies are tabulated below:

$n_g = 0.50; t_c = 0.005 \text{ in.}$						
$t_1, \text{ in.}$	$t_2, \text{ in.}$	$\ell_2, \text{ in.}$	$y, \text{ in.}$	n	$\ell_1, \text{ in.}$	p_u
0.125	0.125	3.5	0.3125	8	3.5	0.86742
0.125	0.1875	3.5	0.46875	6	3.9375	0.89257
0.125	0.250	3.5	0.625	4	3.5	0.90514
$n_g = 1; t_c = 0.005 \text{ in.}$						
0.125	0.125	3.5	0.14423	13	3.5	0.86742
0.125	0.1875	3.5	0.21635	9	3.6347	0.89257
0.125	0.250	3.5	0.28846	7	3.7692	0.90514

The volume fractions have the following values:

V_{Zr} = volume of zirconium/cc of core

$$= \frac{n_g}{1 + n_g} \left[1 - \frac{t_2}{y} \frac{p_u}{n_g} \left(1 - z \frac{\rho_m}{\rho_{Zr}} \right) \right] \quad (39)$$

$$V_{Di} = \frac{1}{1 + n_g} = \text{volume of diphenyl/cc of core} \quad (40)$$

$$V_U = \frac{1}{1+n_g} \left[\frac{t_2}{y} p_u \left(1 - z \frac{\rho_m}{\rho_{Zr}} \right) \right] = \text{volume of uranium/cc of core} \quad (41)$$

$$\frac{\rho_m}{\rho_{Zr}} = \frac{\rho_U}{\rho_{Zr} + z (\rho_U - \rho_{Zr})} \quad (42)$$

$$1 - z \frac{\rho_m}{\rho_{Zr}} = \frac{1}{1 + \frac{z}{1-z} \frac{\rho_U}{\rho_{Zr}}} = 0.86727 \quad (43)$$

$$\frac{\rho_U}{\rho_{Zr}} = \frac{18.9}{6.5} = 2.9077$$

$$\frac{z}{1-z} = \frac{0.05}{0.95} = 0.052632$$

$$V_{U^{238}} = \frac{1}{1+n_g} \left[\frac{t_2}{y} p_u \left(1 - z \frac{\rho_m}{\rho_{Zr}} \right) \right] (1 - 0.9874 x) \quad (44)$$

= volume of U^{238} /cc of core

$$V_{U^{235}} = \frac{1}{1+n_g} \left[\frac{t_2}{y} p_u \left(1 - z \frac{\rho_m}{\rho_{Zr}} \right) \right] (0.9874 x) \quad (45)$$

z = weight fraction of Zr in uranium.

In order to calculate the four factors in k_∞ , the cross sections shown in Tables I and II were used. These are based upon a neutron temperature of approximately 1200F.

Σ_a = macroscopic absorption cross sections for the pure elements of materials (cm^{-1})

Σ_{tr} = macroscopic transport cross sections (cm^{-1})

Σ_{ar} = macroscopic resonance cross sections (cm^{-1})

$\xi \Sigma_s$ = slowing down power for the pure elements (cm^{-1})

Table I

THERMAL CROSS SECTIONS

	$(\Sigma_a), \text{ cm}^{-1}$	$(3\Sigma_{tr}), \text{ cm}^{-1}$
U^{235}	18.18	1.415
U^{238}	0.06464	1.1745
Zr	0.004172	1.0124
$\text{C}_{12}\text{H}_{10}$	0.005619	1.42528

Table II

RESONANCE CROSS SECTIONS

	$(\xi\Sigma_s), \text{ cm}^{-1}$	$(\Sigma_{ar}), \text{ cm}^{-1}$
$\text{C}_{12}\text{H}_{10}$	0.651245	0
Zr	0.00748	0
U^{235}	0.00397	0
U^{238}	0.0033	$1.1969 \left[0.31180 + \frac{1}{0.1 + \frac{M}{S(1-C)}} \right]$

B. Resonance Escape Probability

The general equation used to calculate the resonance escape probability is:

$$p = \exp \left[- \frac{V_{\text{U}^{238}} \Sigma_{\text{ar}}^{\text{U}^{238}}}{V_{\text{U}^{238}} (\xi_{\text{U}^{238}} \Sigma_s^{\text{U}^{238}}) + V_{\text{U}^{235}} (\xi_{\text{U}^{235}} \Sigma_s^{\text{U}^{235}}) + V_{\text{Zr}} (\xi_{\text{Zr}} \Sigma_s^{\text{Zr}}) + V_{\text{Di}} (\xi_{\text{Di}} \Sigma_s^{\text{Di}})} \right] \quad (46)$$

The terms in this equation have been defined above for wire and flat plate fuel elements.

The general equation for calculating Σ_{ar} is:⁴

$$\begin{aligned} \Sigma_{\text{ar}} &= N^{\text{U}^{238}} \sigma_{\text{ar}}^{\text{U}^{238}} \\ &= 7.5 D_r N^{\text{U}^{238}} \left\{ (1 + \beta \Delta T_m) + \frac{3.34}{0.1 + \frac{M}{S[(1-C)(1+2\alpha\Delta T_m)]}} \right\} \quad (47) \end{aligned}$$

⁴ "Light Water Moderated Reactors in Cross Flow," NYO-3924, Eq. 2.2, (Sept. 30, 1953)

where

$N^{U^{238}}$ = atoms of U^{238} /cc of pure U^{238}

= 0.0473×10^{24} atoms/cc

D_r = disadvantage factor = 1

$\beta \Delta T_m$ = Doppler correction for temperature broadening of the resonance peaks

β = Doppler coefficient = $1.4 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$

ΔT_m = $316\text{C} - 20\text{C} = 296\text{C}$

$\frac{M}{S}$ = $\frac{\text{mass of fuel element}}{\text{surface area of element}}$

C = neutron shielding factor accounting for shielding of one fuel element by a neighboring element

α = coefficient of expansion of metal

$2\alpha\Delta T_m$ can be neglected for these calculations.

1. Wire Fuel Elements

Figure 10, page 30, illustrates the neutron shielding effect by neighboring rods. The calculation of the factor C was carried out according to the method by Dancoff.⁵ The closest spacing of the fuel elements is:

$$d = 1.7724 r_0 \sqrt{\frac{1+n_g}{n_g}}, \text{ cm} \quad (48)$$

where r_0 = outside radius of an element = 0.0762 cm

n_g = metal to diphenyl volume ratio

d_i = spacing between any fuel elements and element i .

The factor C was calculated according to the formula:

$$C_i = \frac{r}{\pi^2} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \left\{ \frac{d_i \cos \theta - r [1 + \cos(\theta + \theta')]}{d_i^2 - 2rd_i (\cos \theta + \cos \theta') + 2r^2 [1 + \cos(\theta + \theta')]} \right\} d\theta d\theta' \cdot K_{i3} \frac{(\rho - \rho')}{\lambda_s} \quad (49)$$

⁵S. M. Dancoff, M. Ginsberg, "Surface Resonance Absorption in a Close Packed Lattice," CP-2157.

where

$$\begin{aligned} \frac{\rho - \rho'}{\lambda_s} &= \text{distance in mean free paths from two points on the surface} \\ &\quad \text{of two neighboring fuel elements} \\ &= \frac{\sqrt{d_i^2 - 2rd_i (\cos \theta + \cos \theta') + 2r^2 [1 + \cos (\theta + \theta')]} }{\lambda_s} \end{aligned} \quad (50)$$

and

$$K_{i3} \left(\frac{\rho - \rho'}{\lambda_s} \right) = K_{i3}(x) = \int_0^\infty e^{-x} \frac{\cosh u \, du}{\cosh nu \, du} \quad (\text{see standard tables}) \quad (51)$$

Figure 12 illustrates the constants involved.

FIG. 12
NEUTRON SHIELDING ANALYSIS

Calculations were made for the following assembly:

$$r_o = 0.0762 \text{ cm}$$

$$r = 0.0508 \text{ cm}$$

$$z = 0.13$$

$$n_g = 0.50.$$

The following C_i 's were calculated:

$$C_1 = 0.2576 \quad \text{Nearest neighbors to Element I} \quad (4 \text{ elements})$$

$$C_2 = 0.1888 \quad \text{Next nearest neighbors} \quad (4 \text{ elements})$$

$$C_3 = 0 \quad \text{Next nearest neighbors} \quad (4 \text{ elements})$$

$$C_4 = 0.0184 \quad \text{Next nearest neighbors} \quad (8 \text{ elements})$$

$$C = \sum_{i=1}^{i=4} C_i = 0.4648$$

The term $M/S = r \rho_m / 2 = 0.480 \text{ gm/cm}^2$. The calculated value of Σ_{ar} is 1.5744 cm^{-1} .

2. Flat Plate Fuel Elements

A similar procedure was used for calculating the value of C for flat plates. It can be shown that

$$C = 2E_3(y')$$

where⁶

$$E_3(y') = \int_1^{\infty} \frac{e^{-y'u}}{u^3} du \quad (52)$$

$$y' = \frac{y + 2t_c}{\lambda_s} = \text{effective fuel element spacing in mean free paths} \quad (53)$$

$$= \Sigma_s (y + 2t_c)$$

Σ_s = macroscopic scattering coefficient of diphenyl-zirconium mixture surrounding the uranium meat.

For a metal to diphenyl volume ratio of 0.5 and 8 plates per assembly (see page 33), C has a value of 0.566 and M/S has a value of $t_2 \rho_m / 2 = 1.0925 \text{ gm/cm}^2$. For this particular assembly Σ_{ar} has a value of 0.8486 cm^{-1} .

Figure 13 shows the resonance escape probabilities for wire and plate elements as a function of atomic enrichment.

⁶National Research Council of Canada, Division of Atomic Energy, MT-1

39

FIG. 13
RESONANCE ESCAPE PROBABILITY FOR WIRE AND PLATE
FUEL ELEMENTS AS A FUNCTION OF ATOMIC ENRICHMENT
($\eta_g = 0.50$)

ANL-EFF-16769-A

40

C. Thermal Fission Utilization

Using the cross sections given in Table I and the specific volumes defined above, one can calculate the thermal utilization from the following equation:

$$f = \frac{V_{U^{235}} \Sigma_a^{U^{235}} + V_{U^{238}} \Sigma_a^{U^{238}}}{V_{U^{238}} \Sigma_a^{U^{238}} + 1.0325 V_{U^{235}} \Sigma_a^{U^{235}} + V_{Zr} \Sigma_a^{Zr} + V_{Di} \Sigma_a^{Di} + V_F \Sigma_a^F + 0.00718} \quad (54)$$

The disadvantage factor of 1 for the flux in the moderator is approximately the same as that in the fuel element. This equation includes maximum xenon and samarium based upon an operating power level of 140 mw before complete, instantaneous shutdown. From Eq. (5) and (8) one finds that:

$$V_{Xe} \Sigma_a^{Xe} + V_{Sm} \Sigma_a^{Sm} = 0.032538 V_{U^{235}} \Sigma_a^{U^{235}} + 0.00718 \quad (55)$$

$$V_F \Sigma_a^F = (60 \text{ barns/fission}) (\text{fission/cc day}) (\text{time in days})$$

$$= 60 \times 10^{-24} \Sigma_F \phi_0 \times 8.64 \times 10^4 t_c$$

$$= 518.4 \times 10^{-20} C_0 t_c$$

$$= 0.1643 \frac{P_{mw}}{V_{cc}} t_c = 0.002738 \text{ cm}^{-1} \quad (56)$$

$$t_c = \text{life of core} = 167 \text{ days}$$

Figure 14 indicates the variation of the thermal utilization for both wire and plate elements as a function of the atomic enrichment of U^{235} in uranium. The design parameters are the same as those listed in Table III.

D. Neutron Reproduction Factor

The formula used to calculate the neutron reproduction factor, η , is:

$$\eta = \frac{2.0893 V_{U^{235}} \Sigma_a^{U^{235}}}{V_{U^{235}} \Sigma_a^{U^{235}} + V_{U^{238}} \Sigma_a^{U^{238}}} \quad (57)$$

Figure 15 shows the variation of η with the atomic enrichment, being the same for wire and plate fuel elements.

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41

FIG. 14
THERMAL UTILIZATION FACTOR FOR WIRE AND PLATE
FUEL ELEMENTS AS A FUNCTION OF ATOMIC ENRICHMENT
($n_g = 0.50$)

ANL-EFF-16770-A

42

Table III

DESIGN PARAMETERS

Age of the Core, hr	4000
Design Temperature, F	600
Metal to diphenyl volume ratio	0.50
Structural Material	Zirconium
Core Diameter, ft	4
Core Height, ft	4
U ²³⁸ slightly enriched with U ²³⁵	
Design Power, mw	140
Fuel Elements	
a. <u>Wires</u>	
Fuel Element Core Radius, cm	0.0508
Thickness of Zr clad, cm	0.0254
Outside Radius of Wire, cm	0.0762
Fraction of Metal in end plates	0.13
No Zirconium Alloyed with Uranium in Meat	
Maximum Xenon and Samarium	
b. <u>Plates (See tabulation on page 33)</u>	
Over-all thickness of plates, in.	0.125
Plate spacing, in.	0.3125
Thickness of uranium meat, in.	0.115
Clad thickness on fuel plates, in.	0.005
End plate thickness, in.	0.125
Meat contains 5 wt-% Zirconium in Uranium	
Maximum Xenon and Samarium	

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FIG. 15
NEUTRON REPRODUCTION FACTOR FOR WIRE AND PLATE
FUEL ELEMENTS AS A FUNCTION OF ATOMIC ENRICHMENT

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E. Fast Fission Factor

The fast fission factor, ϵ , for a diphenyl-uranium mixture was estimated by extrapolation from light water data.⁷

$$\epsilon_{Di} = 1.02 + (\epsilon' - 1) \frac{(N_H)_{68F}^{water}}{(N_H)_{600F}^{diphenyl}} \quad (58)$$

ϵ' = fast fission factor for a light water-uranium mixture having the same volume ratio as that for a diphenyl-uranium mixture.

The fast fission factor ϵ_{Di} , using the above crude approximation, is 1.063 for wire elements and 1.113 for plates. The multiplication constants for wire and for plate fuel elements are shown in Figure 16.

F. Fermi Age and Fast Diffusion Constant

The Fermi Age for a mixture of zirconium, uranium and diphenyl was calculated according to the procedures outlined in CF-1895.⁸ The Fermi Age for pure diphenyl was also calculated by the same procedure and a normalization factor was then applied to bring this age into agreement with that calculated by the Marshak method (see Section II-A-1). The lattice was assumed to have the following volume fractions:

$$V_{Di} = 0.666667 \text{ cc/cc of core}$$

$$V_{Z,r} = 0.132723 \text{ cc/cc of core}$$

$$V_{II} = 0.200610 \text{ cc/cc of core.}$$

The volume ratios correspond to a total metal to diphenyl volume ratio of 0.5, assuming flat plate elements as outlined in Table III. The normalized value of τ is 95 cm² at 600F. Figure 1 shows the variation of τ for the above mentioned volume ratio of uranium to zirconium of 1.511.

The fast diffusion constant D_f was calculated according to Section II-A-2; however, the cross sections were the same as those used for calculating the Fermi Age, and also accounting for the inelastic scattering in uranium. The value for D_f at 600F is 1.202 for the same lattice defined for the Fermi Age calculation. A normalization factor was not applied.

⁷North American Aviation Report, NAA-SR-Memo-1094.

⁸H. Soodak, S. M. Forman, "Age of Fission Neutrons in Uranium Water Lattices," CF-1895 (August 24, 1944.)

45

FIG. 16
MULTIPLICATION CONSTANT FOR WIRE
AND PLATE FUEL ELEMENTS AS A
FUNCTION OF ATOMIC ENRICHMENT

ANL-EFF-16772-A

46

G. Critical Reactor Loading

A two-group, two-region calculation was made for a reactor using flat fuel plates and the design parameters given in Table III. The critical loading was obtained by iterating on x , the fraction of U^{235} atoms in the total number of atoms of uranium. An infinite diphenyl reflector was assumed.

From this iteration the critical loading was approximately 106 kg of U^{235} and 5292 kg of U^{238} at the end of the life of the core, after 4000 hr of full-power operation. The radial flux curves through the center of the core are shown in Fig. 17. The ratio of the maximum to average fast flux is 1.75 and the corresponding ratio for the slow flux is 1.55. The reflector savings amounts to 14.1 cm. The atomic enrichment is 1.99%.

The axial flux curves through the center of the core are shown in Fig. 18. The ratio of the maximum to average fast flux is 1.32 while the same ratio for the slow flux is 1.28.

The volume average of the slow flux is approximately 5×10^{13} while the volume average of the fast flux is 6×10^{14} neutrons/(cm²) (sec). This fast flux is probably too high since the two-group theory was used in its evaluation. The above analysis of this reactor is qualitative only since many of the constants have no experimental proof. The control problem has not been studied and the method of holding down a large inventory of U^{235} necessary for a new core is a separate problem not yet undertaken.

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47

FIG. 17
RADIAL NEUTRON FLUX AT MIDPLANE OF CORE

ANL-EFF-16773-A

48

FIG. 18
AXIAL NEUTRON FLUX AT CENTER OF CORE

ANL-EFF-16774-A

49

IV. SUMMARY OF NUCLEAR DESIGN DATA

A. 20-mw Fully Enriched Zirconium-Diphenyl Test Reactor

Zirconium to diphenyl volume ratio	1:1
Reactor height = diameter, ft	2.5
Equilibrium xenon and samarium	
Design temperature, F	600
Life of core, days	25
Maximum design power, mw	20
Infinite diphenyl reflector	
Volume of core, cc	347,500
Plate-type fuel elements	
τ core, cm^2	147.8
τ reflector, cm^2	91
D_f core	1.2182
D_f reflector	1.4936
$\Sigma_a^{U^{235}}$ (cold core), cm^{-1}	0.025743
Σ_a^X (equilibrium), cm^{-1}	0.001398
Σ_a^{Sm} (equilibrium), cm^{-1}	0.000304
Σ_a^F (end of 2000 hrs), cm^{-1}	0.000788
Σ_a^{mod} , cm^{-1}	0.005608
Loading (Old core), kg	8.8
Loading (New core), kg	9.4
Reflector savings, cm	12.57
Power Density, $\frac{P}{V}$, watts/cc	57.55

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50

The volume fractions for the core materials can be calculated by Eqs. (39) to (45), except that all the uranium is U^{235} . A typical fuel plate assembly is shown in Fig. 11 (page 32), where

$$t_a = 0.10 \text{ in.}$$

$$t_2 = 0.060 \text{ in.}$$

$$t_3 = 0.040 \text{ in.}$$

$$l_2 = 2.6 \text{ in.}$$

$$t_1 = 0.20 \text{ in.}$$

$$y = 0.08182 \text{ in.}$$

$$p_u = 0.61538$$

$$l_1 = 3.4 \text{ in.}$$

$$n = 24 \text{ fuel plates/assembly}$$

The radial and axial fast and slow flux distributions are shown in Figs. 19 and 20.

From Fig. 19, the maximum to average radial slow flux in the core is 1.4. The maximum to average radial fast flux is 2.77. From Fig. 20 the maximum to average axial slow flux in the core is 1.117 and the corresponding maximum to average fast flux is 1.262.

The volume average of the slow flux is 8.3×10^{13} neutrons/(cm²)(sec). The volume average of the fast flux is 3×10^{14} neutrons/(cm²)(sec).

B. 60-mw Fully Enriched Stainless Steel-Diphenyl Reactor

Total stainless steel to diphenyl volume ratio	0.50
Reactor height = diameter, ft	2.2966
Design temperature, F	600
Maximum xenon and samarium	
Wire fuel elements	
Over-all diameter, mil	30
Clad thickness, mil	10

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57

FIG. 19
RADIAL NEUTRON FLUX AT MIDPLANE OF CORE

ANL-EFF-16775-A

52

FIG. 20
AXIAL NEUTRON FLUX AT CENTER OF CORE

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Life of core (basis for fission products), hr	2,000
Infinite diphenyl reflector	
Volume of core, cc	269,391
τ core, cm^2	104.6
τ reflector, cm^2	91
D_f core	1.1167
D_f reflector	1.4936
Total reflector savings	13.22
Total U^{235} loading (Old core), kg	39
Thermal flux, neutrons/ $(\text{cm}^2)(\text{sec})$	5.7×10^{13}
Fast flux, neutrons/ $(\text{cm}^2)(\text{sec})$	1.1×10^{15}
Power density, $\frac{P}{V}$, watts/cc	223

C. 60-mw Fully Enriched Reactor (Temperature Override of Maximum Xenon)

Total stainless steel to diphenyl volume ratio	0.50
Reactor height = diameter, ft	2.2966
Design temperature, F	600
Equilibrium xenon and samarium	
Life of core (basis for fission products), hr	2,000
Override temperature, F	250
Infinite diphenyl reflector	
Volume of core, cc	269,391
τ core, cm^2	104.6

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τ reflector, cm^2	91
D_f core	1.1167
D_f reflector	1.4936
Total reflector savings	13.2
Total U^{235} loading, kg	32
Thermal flux, neutrons/ $(\text{cm}^2)(\text{sec})$	6.85×10^{13}
Fast flux	- -
Power density, $\frac{P}{V}$, watts/cc	223
Wire fuel elements	
Over-all diameter, mils	30
Clad thickness, mils	10

D. 140-mw Slightly Enriched Zirconium-Diphenyl Reactor

Total metal to diphenyl volume ratio	0.50
Total uranium to diphenyl volume ratio	0.300915
Total uranium to zirconium volume ratio	1.51
Reactor height = diameter, ft	4
Design temperature, F	600
Maximum xenon and samarium	
Life of core (basis for fission products), hr	4,000
Infinite diphenyl reflector	
Volume of core, cc	1.425×10^6
Plate-type fuel elements	
Over-all thickness, in.	0.125
Clad thickness, in.	0.005
Plate spacing, in.	0.3125

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τ core, cm ²	95
τ reflector, cm ²	91
D_f core	1.202
D_f reflector	1.4936
Reflector savings, cm	14.1
Total loading U ²³⁸ (Old core), kg	5,292
Total loading U ²³⁵ (Old core), kg	106
Thermal flux, neutrons/(cm ²)(sec)	5×10^{13}
Fast flux, neutrons/(cm ²)(sec)	6×10^{14}
Power density, $\frac{P}{V}$, watts/cc	98.4

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V. CONCLUSIONS

1. The negative temperature coefficient is sufficiently high to make temperature override of maximum xenon and samarium feasible.
2. Indications are that the fast flux probably will not exceed 10^{15} neutrons/(cm²)(sec), although volume averages of the fast flux assuming only two groups give a slightly higher value for some of the cores studied.
3. The problem of reactor control becomes more complex at power levels beyond 60 mw owing to the high temperature coefficient and excess reactivity required to compensate for fission product poisons and fuel burnup.
4. Further investigations of temperature override reactors, using A-70 (Firebox Grade) steel for core structural material and reflector, should be made with particular attention to thermal stresses in the reflector basket anticipated during rapid burnup of xenon.
5. The use of zirconium, and perhaps aluminum, as structural material should be re-studied with a view toward diminishing the U²³⁵ inventory for high-power reactors.
6. The cost of replacing or cleaning up radiation-damaged diphenyl must be weighed against the cost of U²³⁵, control systems, and simplicity of design, before a definite metal to diphenyl ratio, as well as design power level, can be set.
7. The fast diffusion constant and Fermi Age of diphenyl must be determined, experimentally, before the core design of a high-power reactor can be undertaken.

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