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TO

AEC-NASA SPACE NUCLEAR PROPULSION OFFICE

TRANSFER FUNCTION ANALYSIS PROGRAM

NERVA PROGRAM

CONTRACT SNP-1

MASTER

SEPTEMBER 1967

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AEROJET-GENERAL CORPORATION
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REPORT NO. RN-S-0429

TRANSFER FUNCTION ANALYSIS PROGRAM

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ABSTRACT

This report presents the Aerojet-General transfer function program to be used with the X-engine test program. A two-phase verification program has been performed and the results indicate excellent correlation with expected values. The first phase of the verification program was to evaluate the analytical transfer function program by using numerical tapes to simulate the equipment transfer function. The second phase consisted of utilizing an analog computer to simulate the expected X-engine transfer function using sine, step, and pseudo-random inputs similar to those in the X-engine test program. In addition, the Aerojet-General processing capability is evaluated. The results of the second phase also showed excellent correlation and provided comparative data relative to the transfer function obtained from the three types of inputs.

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I. INTRODUCTION

Computer programs for analyzing transfer functions for the XE test program have been developed by Aerojet-General. This report describes the test data demonstration programs used to prove the validity and applicability of these programs. The Aerojet transfer function analysis system accepts 32- and 14-channel FM analog tape from NTO AW tapes and through an integrated processing and analytical system is able to produce transfer functions (phase and gain) as plots and listings in a short period of time to satisfy the needs for the XE test program.

Two separate programs are used for Aerojet transfer function analyses. The random transfer function program, Job 2907, accepts random inputs, such as random noise, pseudo-random functions, or other appropriate functions, and the deterministic transfer function program, Job 2909, accepts deterministic functions such as step, sine, impulse, or arbitrary functions. The primary difference between the two programs is that for finite-time increments random noise or pseudo-random inputs (Job 2907) require a larger number of points and also spectral-smoothing methods to assure valid results. The programs were designed specifically for the XE tests and the NERVA program, and several options were built into the program to enable the analyses to be sufficiently flexible and useful to both design and analysis engineers. The demonstration program was also designed to check out these options.

The objective of the demonstration program is to verify that the analytical programs are correct and that the transfer functions can be produced in the 3-day period proposed to satisfy SPEAR data requirements. The method used to verify the analytical programs was to numerically program a known second-order transfer function on analog tape. A known input is utilized, and by comparing output with input a transfer function is obtained that can be compared to the true or known transfer function. In all cases in the demonstration test the Aerojet programs performed as required.

The transfer functions are obtained through the use of an integrated processing and analysis system at the Aerojet Sacramento Test Area. The processing system has been used for more than 15 years by Aerojet solid and liquid rocket operations. The system includes extensive hardware and software that enable flexibility and utility in the application to the NERVA program. Aerojet Reports No. RN-TM-0427 "Wide-Band Data Processing and Analysis Capability" and RN-TM-0440 "Hybrid Spectral Analysis System" describe some of the existing facilities to be used for NERVA programs. With adequate definition and planning and establishment of the necessary controls based on design tests similar to those defined in this report, transfer functions needed for the SPEAR team can be completed within 3 days after receipt of the AW tapes and well within the 7 days to meet Aerojet engineering requirements. By satisfying these time requirements the transfer function data can be analyzed and appropriate decisions can be made between experimental plans of the XE test program.

In addition to using numerical tapes for checking out the transfer function programs, an XE simulation transfer function test and analysis was made. A second-order transfer function similar to that typical of XE equipment was simulated with an analog computer. Sine, steps, and pseudo-random inputs were applied to simulate the actual inputs of the XE tests. The XE simulated-transfer function tests enable evaluation of the processing portion of the transfer function analysis and a better understanding of the use of the different data types.

II. TECHNICAL DISCUSSION

A. RANDOM TRANSFER FUNCTION PROGRAM, JOB 2907

The random transfer function program was primarily designed to accept random inputs and, in particular, to accept pseudo-random inputs to be used in the XE tests. Several options have been included in the program to provide flexibility and to facilitate interpretation of the XE data. To verify the random transfer function program a number of tests were performed. One test included a second-order transfer function system that was numerically programmed on an analog tape. This second-order system had a damping ratio of 0.05 and a natural frequency of 1 cps. A random input from a uniform distribution was selected at 10 samples/sec for a total of 4096 points.

Figure 1 shows the resultant phase and gain plot. The trace is almost perfect for the .024 cps band-width selected for this test. In addition, a coherence function is plotted to indicate the quality of the data. As expected, coherence shows the value of 1.00 at all frequencies, except at 1 cps, which shows a small departure. Figure 1 shows the format that will be used to present all transfer functions for the XE test program. The plotting routines at Aerojet include different scales for plotting. These are arithmetic and semilog with 2-1/2-, 6-, and 7-1/2-in. cycles.

Several additional tests were made to demonstrate the validity of the random transfer function program and to obtain further information relative to interpretation of transfer function data. In addition, a verification of some of the available options was performed. These options include varying the resolution (N/m , where N is the number of points and m is the equivalent lags), evaluation of three spectral windows (trapezoidal, Parzen, and Hanning-Tukey), and mixing different inputs to evaluate varying coherence functions. Mathematical explanations are presented in Section II,A,2.

Figure 1 - Frequency Response Characteristics for Uniform Noise Input Through a Second-Order Filter

The random transfer function program uses the IBM Cooley-Tukey algorithm subroutine, also called Fast Fourier Transform, which enables transfer functions to be computed directly rather than calculating correlation functions first. Use of this procedure was recommended by Professor Ronald R. Mohler, Aerojet consultant in the field of controls and dynamic analysis.

1. Verification of Random Transfer Function Program

Random data from a uniform distribution and from a Gaussian distribution were used as inputs for the demonstration program. A known second-order transfer function with a damping ratio of 0.05 and a natural frequency of 1 cps was programmed on analog tape. Results almost identical to those shown in Figure 1 were obtained. This series of tests demonstrated that the random transfer function program, Job 2907, can determine transfer functions well within the resolution requirements of the XE test program. To improve understanding of the random transfer function program, sine functions with 0.25 and 0.75 cps frequencies were superimposed on the uniform and Gaussian noise-input functions. The sine functions plus a Gaussian noise distribution were combined as the input and the sine functions plus a uniform noise distribution were combined as the output. The time traces of these data are shown in Figure 1, Appendix A. Visual inspection alone does not reveal that sine components exist in the data as shown in Figure 1, Appendix A. However, by applying the spectral analysis, the capability of detecting the sine components is excellent as shown by sharp peaks in Figures 2 through 10, Appendix A.

Three spectral windows, the Parzen, trapezoidal, and Hanning-Tukey, were evaluated. The definition and interpretation of spectral windows are described in the section on mathematical description. Each spectral window was evaluated for different levels of resolution. The Parzen spectral window was evaluated at three resolution levels, $\frac{N}{m}$ values: 10, 20, and 40. Figures 2 through 4, Appendix A, show the plots of the power spectral density function of the input for the three resolution levels. The .25 and .75 cps frequencies are shown as sharp peaks. The precision obtained by increasing the band width

from .024 to .098 cps is graphically shown. The precision must be compared to the resolution of data to arrive at an optimum setting for XE tests. Similar sets of output power spectral density plots using uniform noise data are shown in Figures 5, 6, and 7, Appendix A. Sharp discrimination of the .25 cps and .75 cps is evident and a comparison of precision and resolution shows similar results to those for the input Gaussian noise data. The cross-power spectral density plots are shown in Figures 8, 9, and 10, Appendix A. The cross-power spectral plots relate the output and input data and show results similar to those presented in the other figures relative to resolution and precision.

To compute a transfer function, it is necessary to obtain the ratio of the cross-power spectral density function and the input power spectral density function. For these curves, it is apparent that only frequencies at .25 and .75 cps would be meaningful. A comparison of these two frequencies shows an amplitude ratio of approximately 1.005 or 0.04 db, compared with 1.00 and 0 db.

Figures 11, 12, and 13, Appendix A, show the coherence values for these functions, and resolution vs precision is evaluated. Figure 11, Appendix A, shows that for a bandwidth of .024 cps there is a coherence of 0.98 for the 0.25 and 0.75 cps frequencies and values of coherence as high as 0.4 for the random noise. However, with the broader bandwidth of .098 cps, the coherence is lowered to 0.92 at the sine frequencies and the noise is well below 0.2. The meaning of coherence as applied to XE tests is interesting in that coherence values as high as .35 to .40 must be considered questionable if finer bandwidths are used. Also, it may be necessary to establish standards prior to an XE test to define the expected accuracy in terms of precision and resolution.

A similar study was made for the trapezoidal spectral window. Only two levels of resolution were evaluated. As indicated in Figures 14 through 21, Appendix A, the results obtained were similar to those of the Parzen spectral window. One reason for the similarity is that normalized smoothing is used causing all the smoothing coefficients to be equal to one.

Also, the spectral windows are applied so that only the major lobe is considered and the side lobes are eliminated. Similar results were also obtained with the Hanning-Tukey spectral window so none of these plots were included. It should be emphasized that spectral windows are essentially weighting functions and are, therefore, more meaningful to particular types of data. Also, provisions are available for modifying the shape of the spectral windows. For example, with the trapezoidal window the slope and width of base can be specified. In the preceding example, the trapezoid was formed in the shape of a triangle.

A final series of tests with the random transfer function program was made to gain further knowledge and improve interpretation of the coherence and gain functions. One test that appears to exemplify these functions best was a comparison of gain and coherence where the input was a uniform noise distribution (R_1) and the output was the sum of two functions ($f(R_1) + R_2$). The first, $f(R_1)$, is the direct output function from a second-order system with a 0.05 damping ratio and 1 cps natural frequency; and the second is a uniform noise distribution (R_2), uncorrelated with the first input uniform noise distribution. The amplitudes were the same for all functions. The predicted results of gain and coherence would be expected to show approximately equal contributions of $f(R_1)$ and R_2 compared with R_1 at low frequencies; and with increasing frequencies, the $f(R_1)$ would show greater influence and have the characteristics of a second-order system with a maximum at 1 cps. At higher frequencies, $f(R_1)$ would not contribute significantly and R_2 would predominate so that no correlation would be expected between the noncoherent signals R_2 with R_1 . Figure 22, Appendix A, shows the gain and coherence as predicted. Also, the coherence has more than 0.25 at higher frequencies when the coherence is estimated to be zero. Figures 23 and 24, Appendix A, show that finer resolution results in improved precision. For example, as bandwidth varied from .024 to .098 cps, the peak gain at 1 cps is 19.6 db (true value is 20.0 db) for .024 cps bandwidth, compared with 16.0 db for .098 cps bandwidth. However, the variation in gain, especially at the higher frequencies, decreases by a factor of two for the broader bandwidth. This type of data will assist greatly in interpreting XE transfer-function data.

The same test at two resolution levels for the trapezoidal spectral windows shown on Figures 25 and 26, Appendix A, shows similar results.

A similar set of tests was made with Gaussian noise and with the Hanning-Tukey spectral window. Comparable results were obtained and are not presented in this report.

Additional options are available for the random transfer function program. Plotting options include use of 2-1/2-, 6-, and 7-in. cycle semilog paper as well as arithmetic paper with capability for automatic or manual scaling of the x and y axes. A decimation procedure and a numerical filter are available and used when the total number of points exceeds computer storage capability. The capability of selecting the sampling rate and the total number of points to 4096 is part of the random transfer function program capability.

A feature of the transfer function analysis programs is that multiple transfer functions can be computed. This requirement is necessary to enable a large number of transfer functions to be computed in a relatively short (3-day) period. Currently eight transfer functions can be computed in one pass. In addition to transfer function listings and plots for XE tests, the transfer function package will include an engineering unit time trace of the input and output similar to that shown in Figure 1 of Appendix A, a plot of power density and cross-power density plots (if requested), and a multiple plot of gain, phase, and coherence. In addition, a visirecord analog plot, which is a low-cost, self-developing, immediately available oscillograph, is made for all wide-band variables for quality control purposes. Finally, a listing is made when requested for all variables. The format of the listings that will be used for XE tests is shown in Table 1, Appendix A.

2. Mathematical Description of Random Transfer Function Program

Aerojet uses a direct Fourier spectral rather than the commonly used correlation method. The Fourier transform approach to spectral

density computation, if it is used in conjunction with the spectral window method, is well suited to computing transfer functions from stationary dynamic data. The advantage of the Fourier spectral method is advantageous in that shape and width of the spectral window can be controlled more precisely than with the correlation method to achieve the desired resolution and statistical confidence of the spectral density estimates. Also, recent development of a fast digital procedure for obtaining Fourier transforms (the Cooley-Tukey fast Fourier algorithm) has significantly reduced time required to compute transfer functions using random inputs.

The transfer function, consisting of gain and phase, is determined from the frequency response function, which can be obtained from measured spectral densities by use of the following relationship:

$$H_{ij}(f) = \frac{G_{ij}(f)}{G_{ii}(f)} \quad (1)$$

where $H_{ij}(f)$ = the frequency response for input $x_i(t)$ and output $x_j(t)$
 $G_{ij}(f)$ = one-sided cross-spectral density between $X_i(t)$ and $X_j(t)$
 $G_{ii}(f)$ = one-sided power spectral density of input, $X_i(t)$

The classical definition of the cross-spectral density is given by:

$$G_{ij}(f) = \lim_{T \rightarrow \infty} \frac{2}{T} [X_i^*(f) \cdot X_j(f)] \quad (2)$$

where

$$X_j(f) = \int_0^T x_j(t) e^{-j2\pi ft} dt \quad (3)$$

$$X_i^*(f) = \int_0^T x_i(t) e^{j2\pi ft} dt \quad (4)$$

The frequency response function is valid only if the signals X_i and X_j are coherent. This property is measured by the ordinary coherence function $\gamma_{ij}^2(f)$ which is defined by the expression:

$$\gamma_{ij}^2(f) = \frac{|G_{ij}|^2}{G_{ii}(f) G_{jj}(f)} \quad (5)$$

The theoretical properties are such that at any frequency the coherence function is bounded by zero and one. That is,

$$0 \leq \gamma_{ij}^2(f) \leq 1 \quad (6)$$

When $\gamma_{ij}^2(f) = 1$, the signals are said to be coherent and the frequency response is valid. The coherence for all frequencies, as shown in Figure 1 and Table 1, Appendix A, will be listed and plotted for all transfer functions using random type inputs. If the coherence is less than one, the reason can be explained by three possible sources of error: uncorrelated noise, feedback of the original or other signals, or nonlinearity in the system components.

Applying the preceding equations an infinitesimally small effective bandwidth would be needed, resulting in poor statistical accuracy for the raw spectral-density estimates. The spectral window method is used to obtain a wider analysis bandwidth and to reduce the variability of the results by weighing the function in the frequency being considered. Three spectral windows are used in the random transfer function program to provide for proper resolution and statistical precision. These spectral windows are described as a function of the dimensionless frequency, Ω^* , defined as:

$$\Omega = \frac{\Delta}{2Tf} = \frac{2mk}{N} \quad (7)$$

where T = time interval

f = sampling frequency

m = total number of lags

k = integer 1, 2,

N = total number of samples

*A detailed discussion is presented in Aerojet Report No. AMDR 9635-045.

The expressions for the spectral windows shown on Figure 2 are given below:

Hanning-Tukey spectral window:

$$Q_T(\Omega) = \frac{1}{1 - \Omega^2} \frac{\sin \pi \Omega}{\pi \Omega} \quad (8)$$

Parzen spectral window:

$$Q_P(\Omega) = \frac{3}{4} \left[\frac{\sin(\frac{\pi}{4} \Omega)}{\frac{\pi}{4} \Omega} \right]^4 \quad (9)$$

Trapezoidal spectral window:

$$Q_t(\Omega) = \begin{cases} 1 & 0 \leq |\Omega| < \Omega_1 \\ \frac{\Omega_2 - |\Omega|}{\Omega_2 - \Omega_1} & \Omega_1 \leq |\Omega| < \Omega_2 \\ 0 & \Omega_2 \leq |\Omega| \end{cases} \quad (10)$$

Plots of these spectral windows indicate that undesirable negative side lobes exist for the Hanning-Tukey window, although this spectral window is the most popular in industrial spectral analysis. The Parzen and trapezoidal spectral windows are always non-negative and must be given particular consideration.

The discrete expressions for the spectral windows are as follows:

$$Q_T(q) = \frac{\frac{m}{N}}{1 - 4 \left(\frac{m}{N} q\right)^2} \left[\frac{\sin 2\pi \frac{m}{N} q}{2\pi \frac{m}{N} q} \right] \quad (11)$$

Figure 2 - Spectral Windows Available for Random Transfer Function Program, Job 2907

$$Q_p = \frac{3}{4} \frac{m}{N} \left[\frac{\sin \frac{\pi}{2} \frac{m}{N} q}{\frac{\pi}{2} \frac{m}{N} q} \right]^4 \quad (12)$$

$$Q_t = \begin{cases} \frac{m}{N} & , \quad 0 \leq q \leq \frac{N}{2m} (1 - C) \\ \frac{m}{N} \frac{1+C}{2C} - \frac{mq}{NC}, & \frac{N}{2m} (1 - C) \leq q \leq \frac{N}{2m} (1 + C) \\ 0 & , \quad \frac{N}{2m} (1 + C) \leq q \end{cases} \quad (13)$$

where C = trapezoidal filter coefficient.

The effective bandwidth for each of the above discrete spectral windows is determined by the input parameter m . The effective bandwidth is given as:

$$B_e \approx \frac{f}{m} = \frac{1}{m \Delta t} = \frac{N}{m} \frac{1}{T} \quad (14)$$

Therefore, $\frac{N}{m}$ is a measure of the resolution of the system.

B. DETERMINISTIC TRANSFER FUNCTION PROGRAM, JOB 2909

The deterministic transfer function program is a program that produces transfer functions from deterministic types of inputs such as sine, step, impulse, or arbitrary functions. To verify the deterministic transfer function, a known transfer function of the form $G(s) = \frac{1}{s^2 + s + 1}$ was programmed on an analog tape by numerical methods. The different inputs were evaluated by computing the ratio of the output to the input to obtain the test data transfer functions and comparing these results to the known transfer function. The results of Figures 3 through 6 show excellent correlation for this program. Supplementary test results are presented in Appendix B.

The deterministic transfer function program consists of two separate methods of analysis. One method uses a direct Fourier transform for a given function. This program has proven to be extremely useful for evaluating transfer functions of sine inputs and complex wave forms. A second analysis program is based on the impulse approximation or Guillemin's method. This method converts a function into straight line approximations and differentiates each of these straight lines twice to obtain impulse functions. Fourier transform are then taken of these impulses considering the double differentiation that was performed. This program is very useful for true steps and impulse inputs in addition to arbitrary inputs used for computing transfer functions.

1. Verification of Deterministic Transfer Function Program

The verification of the deterministic transfer function program was performed in two phases. The first phase was a duplication of WANL verification as defined in WANL Report WANL-TME-1338, "Digital Programs for Obtaining System Frequency Response from Time Responses for Impulse, Step, or Arbitrary Outputs," to verify the impulse approximation method for step, impulse, and arbitrary function inputs. The second phase was the verification

Figure 3 - Verification of Impulse Approximation Method Transfer Function for Impulse Function

Figure 4 - Verification of Impulse Approximation Method Transfer Function for Step Function

Figure 5 - Comparison Program - Impulse Approximation Method
vs Theoretical

Figure 6 - Comparison Program - Fourier Transform Method
vs Theoretical

of the direct Fourier transfer-function method for sine and arbitrary function inputs. In addition, several options, which were added to both methods to provide for a more flexible program, were verified.

The impulse approximation method, described in the mathematical description was used by Los Alamos Scientific Laboratory (LASL) for the KIWI tests. The LASL program was revised by WANL by inserting open- and closed-loop unity feedback systems. To ensure compatibility with WANL's verification, the same data points as those used by WANL were utilized as shown in Figures 1 through 4, Appendix B. Among these, Figure 1 represents the output from a true impulse function imposed on a transfer function of the form

$$F(s) = 1 / (s^2 + s + 1)$$

while Figure 2 is the output of a true step function through the same transfer function. Figures 3 and 4 represent the input (control) and the output (response) such that the ratio of output to input should yield the preceding transfer function. The analysis of the data also included unity feedback for both the input and output. The results are identical to those found in WANL report WANL-TME-1338 and are presented in Figures 13 through 18, Appendix B. This would be expected, since the identical computer program was used (except for input and plotting routines) and the same points were used for verification. For verification of the impulse approximation method, a step input and an impulse input were utilized and compared to the true transfer functions. The results which are shown in Figures 3 and 4 are comparable to the true transform function and demonstrate the validity of the program. The program is presently programmed to shift the phase to 180° as it passes the -180° phase lag as shown in Figures 3 through 6.

The comparison of the Fourier transform and impulse approximation methods was made using the arbitrary function as the signal for analysis. The results in Figures 5 and 6 show that both methods give equally valid results for the arbitrary function input.

Since sine functions are also to be used for determining the transfer function for XE tests and are not part of the WANL program, four additional sets of data to compare the two analysis methods were used as shown in Figures 5 through 8, Appendix B. Among these, Figure 5 is a 0.25 cps sine wave; Figure 6 is a 0.75 cps sine wave; and Figure 7 is the sum of 0.25 and 0.75 cps sine waves. Figure 8 is the sum of a 0.25 cps cosine function of the form of Figure 5 and a ramp function defined by linear ramp $y = 26.5 + 1.57t$. To increase insight into the deterministic transfer function program, peak amplitude vs frequency is plotted in Figures 9 through 12, Appendix B, to correspond to the sine time function of Figures 5 to 8. The sinex/x lobe characteristics resulting from taking a Fourier transform over a finite time interval are clearly indicated.

Since the transfer function of a single sine frequency represents a single point on the frequency scale, only single points can be evaluated. To evaluate the direct Fourier transform program, six conditions were established by varying phase of the sine function defined in Figures 5, 6, and 7, Appendix B, and interchanging the inputs and outputs. The amplitude was ± 20 units for each sine frequency. These six cases were also evaluated by use of the impulse approximation methods. The results are shown in Table 1. Large errors in phase (greater than 10°) observed in both methods led to the possibility that incomplete cycles were being analyzed. An option was developed that would consider only complete cycles. That is, partial cycles were dropped or truncated. Table 1 presents the results of the test. The results indicate that Fourier-truncated is most applicable to sine inputs for XE tests especially relative to phase, while major discrepancies in phase may occur in the other two methods. The gain error is also lower for the Fourier-truncated method of analysis although no appreciable differences are evident from the other methods.

TABLE 1

COMPARISON OF JOB 2909 FOR SINE INPUTS

	Sine Output Frequency, cps and Phase, Degrees	Sine Input Frequency, cps and Phase, Degrees	Expected Value			Observed DEVIATION from Expected					
			Gain*	Phase	Freq	Impulse		Fourier		Fourier-Truncated	
						Gain	Phase	Gain	Phase	Gain	Phase
			db	deg	cps	db	deg	db	deg	db	deg
1.	.25 (0°) + .75 (0°)	.75 (45°)	0	45°	.75	-.05	2.3	.81	-1.3	-.61	3.9
2.	.25 (-90°) + .75 (-30°)	.75 (45°)	0	15°	.75	.04	-0.1	1.02	1.6	.96	3.7
3.	.25 (-90°) + .75 (-30°)	.25 (0°) + .75 (0°)	0	-90°	.25	.40	-15.0	-2.89	37.3	.00	0.0
4.	.25 (-90°) + .75 (-30°)	.25 (0°) + .75 (0°)	0	-30°	.75	.90	-2.4	.20	-2.9	1.57	-0.2
5.	.25 (0°) + .75 (0°)	.25 (15°)	0	15°	.25	1.73	12.8	2.34	-3.3	.00	0.0
6.	.25 (-90°) + .75 (-30°)	.25 (15°)	0	-75°	.25	2.13	-2.2	-.54	34.0	.00	0.0

*Amplitudes were +20 units for all tests.

Evaluation of data using Figure 8, Appendix B, having a linear ramp showed no influence on the transfer function, so results are not included.

Several options in addition to truncation were developed for the deterministic transfer function program. The plotting routine, which is performed in conjunction with the processing and analysis program, has been standardized with that of the random transfer function program. The same scales and same plots are used to enable comparison of different computed transfer functions. Provision also exists to plot results from both programs on the same chart. The manual or automatic options for scaling are available with the same options for paper: arithmetic and 2-1/2-, 6-, and 7-1/2-in. semilog.

A searching option has been programmed for the sine input portion of the deterministic transfer function. The program computes vector magnitudes of the input variable at specified frequency increments and selects the maximum vector for given frequency intervals. By this method the actual frequency is used rather than accept a stated frequency for a given input. The program can then select an integral number of cycles to preclude any truncation effects. The searching option was demonstrated on NRX-EST-EP-4 data and on the XE simulated transfer function test described in this report. The NRX-EST-EP-4 tape consisting of eight sine frequencies was searched for optimum input. For this demonstration, each computation was printed out as shown in Table 1, Appendix B. Deterministic transfer functions are performed on several characteristics at one time. Table 2, Appendix B, shows a typical listing for a multiple transfer-function computation. These data listings are identical to the graphical plots. At the present time, eight transfer functions can be computed on a single pass.

A time demodulator has been designed and constructed by Aerojet. Although not specifically a part of the transfer function program, a unit has been constructed that will accept the 1000-cps time code used in XE tests and convert it to an unmodulated time signal to make all time analyses compatible. For NRX test analyses, an unmodulated signal was used for transfer-function computations. Time corrections are updated each second, and interpolated between each second using an Aerojet precision, 10 kc time base, so that when data are obtained from two tapes the error will be less than 0.4 millisecc. This device was checked out and qualified with ETS-1 facility test data.

2. Mathematical Description of Deterministic Transfer Functions

The Job 2909 program makes use of two algorithms for computing transfer functions. For the Fourier transfer algorithm the time function $f(t)$ is converted to the frequency domain with the Fourier transform conversion where:

$$F(f) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad (1)$$

where $\omega = 2\pi f$ and f is the frequency in cps. $e^{-j\omega t}$ can be replaced by its trigonometric identity:

$$e^{j\omega t} = \cos \omega t - j \sin \omega t \quad (2)$$

or

$$F(f) = \int_{-\infty}^{\infty} f(t) \cos \omega t dt - j \int_{-\infty}^{\infty} f(t) \sin \omega t dt \quad (3)$$

Define a real (Re) part and an imaginary (I) part as

$$\text{Re} = \int_{-\infty}^{\infty} f(t) \cos \omega t dt \quad \text{and} \quad I = \int_{-\infty}^{\infty} f(t) \sin \omega t dt \quad (4)$$

Then the amplitude (A) is defined as $\sqrt{\text{Re}^2 + \text{I}^2}$ and the phase angle (θ) is defined as $\tan^{-1} \frac{\text{I}}{\text{Re}}$. The transfer function can thus be defined as

$$\begin{aligned} \text{Gain} &= \frac{A_o}{A_i} \quad \frac{(\text{output amplitude})}{(\text{input amplitude})} \\ \text{Phase} &= \theta_o - \theta_i \end{aligned} \tag{5}$$

where the subscripts o and i refer to output and input.

The transform method solves the integral equation (3) by taking digital summations of the product of $f(t)$ and $\cos \omega t$ and $f(t)$ and $\sin \omega t$.

The impulse approximation method approximates $g(t)$ as a sequence of straight lines. This approximate function is differentiated two times to form a sequence of impulses. The transform is then taken of each of the impulses in the following manner:

$$G(j\omega) = \int_0^{\infty} g(t) e^{-j\omega t} dt \tag{6}$$

Differentiation of $g(t)$ with respect to t corresponds to multiplication of $G(j\omega)$ by $j\omega$. Therefore:

$$G(j\omega) = \frac{1}{(j\omega)^n} \int_0^{\infty} g^{(n)}(t) e^{-j\omega t} dt \tag{7}$$

where n refers to the number of derivatives made and $g^{(n)}(t)$ is approximated by the train of n impulses occurring at $t_1, t_2, \dots, t_n$, or

$$g^{*(n)}(t) = \sum_{K=1}^n a_K U_o(t - t_K) \quad (8)$$

Substituting equation (8) in equation (7) yields:

$$G^*(j\omega) = \sum_{K=1}^n a_K e^{-j\omega t_K} / (j\omega)^n \quad (9)$$

The real and imaginary parts and the gain and phase are computed by expanding the exponential similar to that described in equations (2) through (5).

C. XE SIMULATION TRANSFER FUNCTION PROGRAM

A test and analysis program was conducted to simulate the transfer functions that will be obtained during the XE tests. At the present time sine, step, and pseudo-random inputs will be used for the XE tests at several performance and operating conditions. An analog computer was used to simulate engine performance and a typical second-order transfer function of the form:

$$H(s) = \frac{1}{(0.1 s + 1) (1.0 s + 1)}$$

was designed for the computer simulation. Step, sine, and pseudo-random input data were fed into the computer and the output and input for each signal were recorded on a 14-track FM analog tape. The pseudo-random generator had nine flip-flops to duplicate the 511 values ($2^N - 1$) in a given period as is currently done for power tests at NTO. The results of the three input transfer functions are compared with the true transfer function in Figure 7. Figure 7 indicates that each type of input has a particular use in transfer function analysis and that Aerojet-Sacramento has an excellent analysis and processing system for evaluating transfer functions.

The complete data set to be used in the XE test program is presented to show the type of plots and type of analysis that will be made. The first step in the analysis is to prepare Visicord voltage traces for quality control purposes. Since strip charts were available, no Visicord records were made.

Figure 7 - Comparison of Transfer Functions from Sine, Step, and Pseudo-Random Inputs from XE Transfer Function Simulation Program

1. Step-Input Transfer Functions

The transfer function plots for the step data are shown in Figures 1 and 2, Appendix C. The curves represent the transfer functions for increasing and decreasing step functions. The two curves agree well with the true value to approximately 4 cps. To enable further interpretation, time plots for the input and output are presented in Figures 3 through 6, a polarity reversal existed that is evident in the charts and was accounted for by modifying the phase scale. The adequacy of the step input and responses are readily discernible. A page of the transfer function listing for step inputs is shown in Table 1, Appendix C.

2. Sine Input Transfer Function

The sine data transfer function is shown in Figure 7 in the main text and in Figure 7, Appendix C, compared with the true function. Correlation of the data transfer function is excellent, with a standard deviation of less than 0.5 db compared with the true transfer function. The sine function is useful for evaluating transfer functions because phase and gain can be observed from the time plots, and can be used to investigate discrepancies. Figures 4 through 17, Appendix C, show representative input and output of the sine functions. For example, in Figures 14 through 17, Appendix C, a ratio of input to the output is 50 and 100 to one, a range in which the amplifier sensitivity is extended as shown by noise and an apparent integration evidenced by the exponentially increasing output signal. However, transfer functions within 1 db were still obtained.

The searching subroutine was applied to the sine data as shown in Table 2, Appendix C. The searching subroutine selects the maximum vector amplitude obtained by searching and sampling amplitudes at different incremental frequencies about the expected input frequency. For example, the 0.16 and 0.25 cps were found to be 0.14 and 0.23 cps, respectively, and three frequencies were off by 0.01 cps.

3. Pseudo-Random Inputs

Figure 7 shows the transfer function determined from pseudo-random input superimposed on the sine and step inputs transfer function results. As indicated by this figure, the transfer function using a pseudo-random input shows variations in excess of ± 3 db at the higher frequencies (> 4 cps). The time plot for the input and output for the pseudo-random test is shown in Figure 18, Appendix C. The transfer function results consisting of gain, phase, and coherence function are shown in Figure 19, Appendix C. As already indicated, excessive variation occurs compared with the sine input at frequencies greater than 4 cps. Secondly, the coherence function, which showed almost 1.0 values for the numerical tape test, shows values of 0.7 to 0.8 at lower frequencies and values much lower at the higher frequencies. The power spectral density functions of the input and output and the cross power spectral density function are shown on Figure 20, Appendix C. A page of the listing for pseudo-random spectral characteristics is shown in Table 3, Appendix C. Since the polarity was reversed for the output, adjustments were made for the phase values.

A total of 76.5 sec of data was used in this analysis, which was sampled at 375 samples/sec, resulting in 28,687 points. Since the capacity of the random transfer function program is 4096, it was necessary to use the decimation and numerical filter subroutine and to decimate the data by one-seventh. The subroutine was successfully demonstrated.

APPENDIX A

SUPPLEMENTARY FIGURES FOR RANDOM
TRANSFER FUNCTION PROGRAM VERIFICATION

TABLES

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TABLE 1

INPUT R1		DFGO	FREQUENCY RESPONSE ANALYSIS PROGRAM										JOB 2907
OUTPUT F(R1)	DFGO												
START TIME	0.0		TEST CASE										
DATA DURATION 409.5999													
POINTS 4096 N/M 10 N			COMPUTER GENERATED TEST TAPE USED AS INPUT										
SMOOTHING PARZEN													
BANDWIDTH 0.024			COOLEY-TUKEY FOURIER TRANSFORM CALCULATION METHOD USED										
CONSTANTS L=10 C=.0													
PHASE - FOR OUTPUT LAG													
(00)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)		
FREQUENCY	INPUT	P.S.D.	OUTPUT	P.S.D.	CROSS SPEC.	DEN.	GAIN (DB)		PHASE (DEG)		COHERENCE		
HERTZ	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	SMOOTHED		
.02686	28.7619	26.9244	25.1382	34.4726	26.8891	35.6636	-58.484	-30.177	-1.3198	-2.8686	.999221		
.05127	15.3403	49.8426	14.8053	46.8804	15.0705	48.3239	-154.16	-26.879	2.29649	-1.0946	.999381		
.07568	27.5810	55.3213	26.5225	53.0715	27.0466	54.1723	-16.995	-18.231	-2.5231	-4.6990	.999540		
.10010	31.9997	40.5559	29.3138	38.4642	30.6273	39.4840	-38075	-23267	-828389	-49255	.999380		
.12451	3.55674	35.7392	4.02922	34.0691	3.78562	34.8817	541683	-21094	-1.6597	-7.4059	.999286		
.14893	49.7700	38.2363	50.3042	37.2688	50.0364	37.7386	046355	-11379	-2.2167	-1.6803	.999427		
.17234	98.2490	22.3538	93.2561	31.8300	95.7200	32.0784	-22651	-07426	-93036	-1.5868	.999223		
.19775	87.3303	41.6011	92.5842	42.8398	89.9188	42.2022	253718	124597	-74805	-1.4159	.999349		
.22217	12.9447	52.4732	13.6300	54.0652	13.2830	53.2516	224045	127912	430935	-1.6537	.999564		
.24658	14.6748	48.4162	14.1717	51.0819	14.4211	49.7145	-15151	229831	-2.1182	-1.7170	.999326		
.27100	24.7860	42.5531	27.6228	48.4727	26.1660	45.9550	470609	458304	-3.0596	-1.7153	.999426		
.29541	22.9372	33.6316	25.1901	37.5634	24.0373	35.5312	406881	477245	-53756	-1.9364	.999324		
.31982	124.661	46.7725	141.450	55.5108	132.470	52.0185	569678	559665	-2.3733	-1.9199	.999456		
.34424	74.7423	28.5490	93.7203	34.5507	83.6951	31.3912	982666	824324	-1.4577	-2.1260	.999003		
.36865	9.73189	45.7534	11.5668	57.6995	10.6098	51.3888	750157	1.00130	-5.4297	-3.0035	.999445		
.39307	31.3004	36.4280	38.9345	47.6655	34.9094	41.6444	947835	1.16244	-1.5665	-2.5631	.998791		
.41748	14.7028	45.2055	19.2899	63.3340	16.8409	53.4908	1.17929	1.46176	-3.0872	-3.2636	.999378		
.44189	15.1869	44.1694	20.4027	63.7180	17.6026	53.0328	1.28218	1.58845	-4.7649	-3.5176	.999322		
.46631	12.3108	29.3756	17.4991	45.5392	14.6775	36.5540	1.52731	1.89897	-2.9898	-3.2280	.998844		
.49072	38.8733	44.4462	67.3924	73.5605	51.1837	57.1581	2.38959	2.18487	-3.2407	-4.1449	.999257		
.51514	11.1626	36.2833	19.9421	62.0715	14.9200	47.4391	2.52005	2.32858	-1.7043	-4.3142	.999247		
.53955	31.5244	27.6669	57.9261	52.5047	42.7327	38.0803	2.64228	2.77479	-5.3233	-4.3215	.998254		
.56396	14.1880	26.4219	27.7561	53.8465	19.8445	37.6976	2.91437	3.08699	-7.4034	-5.0445	.998862		
.58838	38.4867	27.6650	80.2701	60.5545	55.5818	40.8984	3.19242	3.39550	-4.6794	-5.0419	.998471		
.61279	35.1026	45.1875	88.5007	110.945	55.7370	70.7666	4.01607	3.89620	-3.9763	-5.1611	.998882		
.63721	52.5567	57.9596	141.830	155.417	86.3372	94.8683	4.31140	4.27991	-5.3396	-6.4056	.999120		
.66162	7.13724	34.7458	19.0712	102.377	11.6669	59.5796	4.26848	4.68391	-3.9765	-6.9132	.997907		
.68604	8.85090	36.0803	28.2684	125.849	15.8177	67.3381	5.04312	5.41981	-5.2697	-7.7076	.998625		
.71045	37.7553	45.6872	147.920	173.561	74.7352	89.0105	5.93002	5.79293	-7.4597	-8.4451	.999159		
.73486	31.6426	23.7183	148.240	101.057	68.4888	48.8719	6.70694	6.27950	-9.0065	-9.1237	.996482		
.75928	38.7512	24.6255	199.095	191.385	87.8360	81.3197	7.10775	7.41600	-9.6191	-10.829	.997881		
.78369	19.5996	48.7405	116.031	304.275	47.6880	121.657	7.72326	7.94492	-10.451	-11.929	.997952		
.80811	46.2841	37.7377	367.856	285.935	130.483	103.741	9.00246	8.78349	-13.755	-12.993	.997357		
.83252	10.0910	41.6369	108.068	417.105	33.0228	131.489	10.2976	9.98819	-16.491	-16.161	.995526		
.85693	20.2660	48.2015	255.677	598.433	71.9830	169.334	11.0092	10.9136	-20.143	-18.343	.994050		
.88135	111.696	55.9462	1893.36	973.338	459.869	232.558	12.2919	12.3752	-22.070	-21.811	.993183		
.90576	126.673	56.5899	3023.21	1382.30	617.370	278.469	13.7984	13.8408	-27.300	-27.248	.991317		
.93018	22.0984	41.6877	773.618	1514.84	130.751	248.843	15.4417	15.5184	-33.235	-35.128	.980568		
.95459	50.5744	38.1030	2964.62	2394.81	388.741	296.443	17.6461	17.8196	-45.764	-50.902	.963057		
.97900	59.1813	54.5040	5168.34	4748.36	553.054	499.192	19.4117	19.2368	-68.763	-69.487	.962658		
1.0034	2.76672	57.0734	319.167	5398.50	29.7160	544.796	20.6205	19.5960	-93.813	-93.325	.963298		
1.0278	143.279	47.9991	10576.0	3643.62	1230.98	410.825	18.6814	18.6485	-119.47	-115.60	.965047		
1.0522	25.3167	39.3219	1205.12	1764.65	174.671	258.578	16.7762	16.3592	-135.93	-134.61	.963587		
1.0767	24.7140	49.5923	685.097	1427.02	130.121	264.277	14.4281	14.5329	-143.94	-144.68	.986906		
1.1011	31.5687	31.4320	563.683	596.754	133.397	136.032	12.5178	12.7254	-151.44	-151.47	.986546		

Appendix A

Figure 1 - Time Traces of Random Data Plus Sine Components

Figure 2 - Power Spectral Density Plot, Gaussian Noise Plus Sine Functions
Input, Parzen Spectral Window, Resolution $N/M = 10$

Figure 3 - Power Spectral Density Plot, Gaussian Noise Plus Sine Functions Input, Parzen Spectral Window, Resolution N/M = 20

Figure 4 - Power Spectral Density Plot, Gaussian Noise Plus Sine Functions Input, Parzen Spectral Window, Resolution N/M = 40

Figure 5 - Power Spectral Density Plot, Uniform Noise Plus Sine Functions
Output, Parzen Spectral Window, Resolution N/M = 10

Figure 6 - Power Spectral Density Plot, Uniform Noise Plus Sine Functions Output, Parzen Spectral Window, Resolution N/M = 20

Figure 7 - Power Spectral Density Plot, Uniform Noise Plus Sine Functions
Output, Parzen Spectral Window, Resolution N/M = 40

Figure 8 - Cross Spectral Density Plot, Parzen Spectral Window,
Resolution N/M = 10

Figure 9 - Cross Spectral Density Plot, Parzen Spectral Window,
Resolution $N/M = 20$

Figure 10 - Cross Spectral Density Plot, Parzen Spectral Window,
Resolution N/M = 40

Figure 11 - Coherence Function, Parzen Spectral Window, Resolution N/M = 10

Figure 12 - Coherence Function, Parzen Spectral Window, Resolution N/M = 20

Figure 13 - Coherence Function, Parzen Spectral Window, Resolution N/M = 40

Figure 14 - Power Spectral Density Plot, Gaussian Noise Plus Sine Functions
 Input, Trapezoidal Spectral Window, Resolution N/M = 10

Figure 15 - Power Spectral Density Plot, Gaussian Noise Plus Sine Functions
Input, Trapezoidal Spectral Window, Resolution N/M = 40

Figure 16 - Power Spectral Density Plot, Uniform Noise Plus Sine Functions
Output, Trapezoidal Spectral Window, Resolution N/M = 10

Figure 17 - Power Spectral Density Plot, Uniform Noise Plus Sine Functions
Output, Trapezoidal Spectral Window, Resolution N/M = 40

Figure 18 - Cross Spectral Density Plot, Trapezoidal Spectral Weindow,
Resolution N/M = 10

Figure 19 - Cross Spectral Density Plot, Trapezoidal Spectral Window,
 Resolution N/M = 40

Figure 20 - Coherence Function, Trapezoidal Spectral Window,
 Resolution $N/M = 10$

Figure 21 - Coherence Function, Trapezoidal Spectral Window,
 Resolution N/M = 40

Figure 22 - Multiple Output Transfer Function Analysis, Parzen Spectral Window, Resolution $N/M = 10$

Figure 23 - Multiple Output Transfer Function Analysis, Parzen Spectral Window, Resolution $N/M = 20$

Figure 24 - Multiple Output Transfer Function Analysis, Parzen Spectral Window, Resolution N/M = 40

Figure 25 - Multiple Output Transfer Function Analysis, Trapezoidal Spectral Window, Resolution $N/M = 10$

Figure 26 - Multiple Output Transfer Function Analysis, Trapezoidal Spectral Window, Resolution N/M = 40

APPENDIX B

SUPPLEMENTARY FIGURES FOR DETERMINISTIC
TRANSFER FUNCTION PROGRAM VERIFICATION

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Output Function Resulting from True Impulse Function Through $1/(s^2 + s+1)$ Transfer Function	2
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Response Output Function for Comparison Program	4
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0.75 cps Sine Input Data	6
0.25 cps Plus 0.75 cps Sine Input Data	7
0.25 cps Plus DC Ramp Input Data	8
Spectral Plot of 0.25 cps Sine Function	9
Spectral Plot of 0.75 cps Sine Function	10
Spectral Plot of 0.25 cps Plus 0.75 cps Sine Functions	11
Spectral Plot of 0.25 cps Sine Function Plus Ramp	12
Open Loop Impulse Response Derived from Impulse Approximation Method Assuming Original Transient Input Was Measured from a Closed Loop System with Unity Feedback	13
Closed Loop Impulse Response with Unity Feedback Derived from Impulse Approximation Method Assuming Original Transient Input Was Measured from an Open Loop System	14

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Closed Loop Step Response with Unity Feedback Derived from Impulse Approximation Method Assuming Original Transient Input was Measured from an Open Loop System	16
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Closed Loop Response with Unity Feedback Derived from Impulse Approximation Method Assuming Original Transient Input was Measured from an Open Loop System	18

TABLE 1

MULTIPLE CHANNEL TRANSFORM ANALYSIS PROGRAM

NRX-EST-EP-IV

DISK PACK NAME = JB2909.DD0019

05-12-67 COMPUTER PROGRAM 2909

REQUEST NO. 5123

START TIME	(00) FREQUENCY	(08) GAIN-DB	(09) PHASE-DEG	(10) GAIN-DB	(11) PHASE-DEG	(12) GAIN-DB	(13) PHASE-DEG
SECONDS	HERTZ	03/01 F	03/01 F	04/01 F	04/01 F	05/01 F	05/01 F
11364.0	.66000	45.3094	232.532	19.4527	-85.281	16.4699	246.299
11364.0	.68000	45.9261	233.805	19.9738	-84.690	17.1558	233.817
11364.0	.70000	46.4998	232.699	20.1026	-87.015	16.9102	217.319
11380.0	1.4000	31.7680	46.0395	7.45581	-15.050	20.0053	107.521
11380.0	1.4500	28.5851	36.1670	7.55882	-23.656	17.0825	82.8209
11380.0	1.5000	22.7112	-8.6485	7.99020	-35.586	11.6643	48.0179
11380.0	1.5500	26.7213	91.0675	10.0687	-38.235	15.8199	75.0231
11380.0	1.6000	22.8385	124.243	10.4093	-47.578	14.1032	49.3551
11380.0	1.6500	23.8831	160.603	10.6568	-54.096	15.8845	30.5143
11380.0	1.7000	27.2284	181.449	10.9887	-59.247	16.1698	11.5860
11380.0	1.7500	29.4856	191.441	11.3742	-69.440	14.8625	-10.798
11380.0	1.8000	30.2725	197.091	12.1012	-71.914	16.9356	-24.668
11393.0	2.2000	21.3083	126.084	2.59496	84.6180	21.5967	154.794
11393.0	2.2500	22.0897	159.540	.964189	67.7911	17.8466	133.439
11393.0	2.3000	23.0149	188.045	-1.2155	56.9688	16.1864	114.325
11393.0	2.3500	25.2288	208.811	-2.9924	52.7555	16.4371	100.770
11393.0	2.4000	27.2856	213.183	-2.3525	57.3636	18.2601	87.6744
11393.0	2.4500	28.9044	214.230	-6.4198	31.3914	15.5431	69.2130
11393.0	2.5000	29.7919	210.479	-6.1712	32.0497	15.0042	54.0921
11393.0	2.5500	34.2085	203.579	-3.8720	-11.063	16.8436	46.7810
11393.0	2.6000	34.6041	206.512	-2.1992	-30.593	17.1091	26.2699
11393.0	2.6500	35.1335	209.167	-1.1812	-56.799	19.6136	4.82254
11393.0	2.7000	36.0637	209.850	2.48863	-52.597	18.1418	-23.464
11399.0	6.0000	30.8335	70.4968	-7.5920	153.512	17.6913	165.082
11399.0	6.1000	29.0707	109.863	-2.1932	169.947	18.2942	159.379
11399.0	6.2000	25.4228	105.072	-2.5780	154.805	18.4436	140.949
11399.0	6.3000	22.3608	108.565	-10.513	183.258	13.4742	122.615
11399.0	6.4000	19.7574	116.663	-8.0426	165.296	16.9191	103.565
11399.0	6.5000	21.2117	124.983	-5.7147	119.332	14.8959	85.9272
11399.0	6.6000	20.8624	147.867	-15.574	119.964	20.3904	71.5209

Demonstration of Searching Subroutine for
NRX/EST EP-IV Sine Input Test Data

Appendix B

MULTIPLE CHANNEL TRANSFORM ANALYSIS PROGRAM

TEST CASE

COMPUTER GENERATED TEST TAPE USED AS INPUT

DATA DURATION TRUNCATED TO EVEN PERIODS AT EACH ANALYSIS FREQUENCY

(00)	(08)	(09)	(10)	(11)	(12)	(13)	(14)
START TIME	FREQUENCY	PHASE-DEG	GAIN-DB	PHASE-DEG	GAIN-DB	PHASE-DEG	GAIN-DB
SECONDS	HERTZ	02/01 I	02/01 F	02/01 F	IMP. 04	IMP. 04	04/03 I
.000000	.00210	.690235	.000000	.000000	.090914	.773222	-.34715
.000000	.00320	1.05247	.000000	.000000	-.08694	1.09493	-.25146
.000000	.00480	1.57756	.000000	.000000	-.16118	1.56171	-.21781
.000000	.00640	2.10609	.000000	.000000	-.18048	2.03745	-.18270
.000000	.00800	2.63559	.000000	.000000	-.18525	2.52273	-.20713
.000000	.00950	3.13514	.000000	.000000	-.18098	2.98587	-.19972
.000000	.01110	3.67140	.000000	.000000	-.17128	3.48534	-.17761
.000000	.01270	4.21088	.000000	.000000	-.15859	3.99547	-.15265
.000000	.01430	4.75477	.000000	.000000	-.14325	4.51281	-.15889
.000000	.01590	5.30368	.000000	.000000	-.12523	5.03849	-.13792
.000000	.02070	6.98198	.000000	.000000	-.05961	6.67276	-.06339
.000000	.02550	8.71732	.000000	.000000	.021249	8.39795	.022007
.000000	.03180	11.0998	.000000	.000000	.144486	10.8225	.143861
.000000	.03980	14.3194	.000000	.000000	.319424	14.1920	.319501
.000000	.05090	19.1838	.000000	.000000	.573592	19.4348	.574431
.000000	.06370	25.3878	.000000	.000000	.846134	26.2802	.846847
.000000	.07960	33.9339	18.7060	8.36268	1.10038	35.8296	1.10237
.000000	.10030	46.2299	18.8486	16.4151	1.21829	49.5724	1.22107
.000000	.12570	62.5748	19.7630	27.5957	.956407	67.6006	.960772
.000000	.15920	84.9989	19.0105	43.0786	-.14145	91.6093	-.13423
.000000	.20050	111.316	16.9771	60.1937	-2.6564	118.172	-2.6448
.000000	.25150	134.226	16.7747	74.2964	-6.8854	139.352	-6.8673
.000000	.31830	136.554	11.4809	76.3212	-11.091	150.631	-11.062
.000000	.39950	154.528	12.2024	81.5443	-15.626	162.830	-15.580
.000000	.50290	145.257	8.62048	78.7010	-19.547	170.890	-19.475
.000000	.63340	149.366	6.26180	77.2600	-23.696	177.058	-23.581
.000000	.79740	149.428	3.48065	72.8751	-28.171	184.348	-27.989
.000000	1.0040	142.208	.930475	66.4858	-32.242	187.113	-31.953
.000000	1.2640	173.707	-1.3584	59.1045	-36.753	193.319	-36.294
.000000	1.5920	173.096	-3.2453	49.4583	-40.615	203.961	-39.884

Appendix B

TABLE 2

Illustration of Deterministic Transfer
Function Program Listings

Figure 1 - Output Function Resulting from True Impulse Function
Through $1/(s^2 + s + 1)$ Transfer Function

Figure 2 - Output Function Resulting from True Impulse Function
Through $1/(s^2 + s + 1)$ Transfer Function

Figure 3 - Control Input Function for Comparison Program

Figure 4 - Response Output Function for Comparison Program

Figure 5 - 0.25 cps Sine Input Data

Figure 6 - 0.75 cps Sine Input Data

Figure 7 - 0.25 cps Plus 0.75 cps Sine Input Data

Figure 8 - 0.25 cps Plus DC Ramp Input Data

Figure 9 - Spectral Plot of 0.25 cps Sine Function

Figure 10 - Spectral Plot of 0.75 cps Sine Function

Figure 11 - Spectral Plot of 0.25 cps Plus 0.75 cps Sine Functions

Figure 12 - Spectral Plot of 0.25 cps Sine Function Plus Ramp

Figure 13 - Open Loop Impulse Response Derived from Impulse Approximation Method Assuming Original Transient Input Was Measured from a Closed Loop System with Unity Feedback

Figure 14 - Closed Loop Impulse Response with Unity Feedback Derived from Impulse Approximation Method Assuming Original Transient Input Was Measured from an Open Loop System

Figure 15 - Open Loop Step Response Derived from Impulse Approximation Method Assuming Original Transient Input Was Measured from a Closed Loop System with Unity Feedback

Figure 16 - Closed Loop Step Response with Unity Feedback Derived from Impulse Approximation Method Assuming Original Transient Input was Measured from an Open Loop System

APPENDIX C

SUPPLEMENTARY FIGURES FOR XE SIMULATION
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MULTIPLE CHANNEL TRANSFORM ANALYSIS PROGRAM

ACL-001 STEP

ACL MODEL

STEP DATA ANALYSIS USING IMPULSE APPROXIMATION

PROGRAM 2909 REQ.5196

000073-00

START TIME SECONDS	(00) FREQUENCY HERTZ	(01) AMPLITUDE	(02) AMPLITUDE	(03) GAIN-DB 02/01 I	(04) PHASE-DEG 02/01 I
0.000000	9.6599	.621555	1.89726	-34.444	54.7988
0.000000	9.7499	.729892	1.90309	-29.078	265.747
0.000000	9.7599	.543528	1.88967	-41.231	57.3176
0.000000	9.8499	.310625	1.84897	-30.145	92.4328
0.000000	9.8599	.502736	1.81947	-25.332	224.295
0.000000	9.9499	.224253	1.82914	-34.360	221.732
0.000000 STEP #2	9.9599	.538537	1.92758	-24.647	-43.395
20.0000	10.000	4.10261	136.528	.246315	200.041
20.0000	15.000	5.79823	176.040	-4.1259	241.017
20.0000	20.000	4.38718	100.100	-2.5319	233.729
20.0000	25.000	5.47997	107.311	-7.2616	247.748
20.0000	30.000	3.88149	75.1102	-5.1138	255.299
20.0000	35.000	3.56924	58.2430	-9.1231	251.583
20.0000	40.000	4.54271	58.8775	-8.2073	269.986
20.0000	45.000	3.75172	47.5869	-9.6682	254.981
20.0000	50.000	4.02450	48.1937	-11.024	-81.144
20.0000	55.000	3.59048	40.4584	-10.296	266.553
20.0000	60.000	4.03559	40.7223	-13.758	-79.924
20.0000	65.000	3.63260	34.9686	-11.517	-80.897
20.0000	70.000	3.52558	30.8968	-15.010	-82.469
20.0000	75.000	3.95493	30.6481	-13.818	-68.705
20.0000	80.000	3.67241	27.2730	-15.209	-84.696
20.0000	85.000	3.86842	27.1590	-16.262	-64.221
20.0000	90.000	3.60655	24.5553	-15.023	-76.341
20.0000	95.000	3.51889	22.3193	-18.488	-64.636
20.0000	1.00000	3.76344	22.1786	-16.261	-65.247
20.0000	1.0500	3.45555	20.3147	-18.979	-75.690
20.0000	1.1000	3.65548	20.3873	-17.581	-54.746
20.0000	1.1500	3.58302	18.8111	-18.577	-72.028
20.0000	1.2000	3.70799	18.7402	-20.221	-50.721
20.0000	1.2500	3.60184	17.4083	-18.318	-63.756
20.0000	1.3000	3.44616	16.2369	-21.844	-55.817
20.0000	1.3500	3.65968	16.2324	-19.245	-49.340
20.0000	1.4000	3.49075	15.1825	-21.905	-63.987
20.0000	1.4500	3.80828	15.2319	-20.860	-33.957
20.0000	1.5000	3.67721	14.1461	-22.405	-61.026
20.0000	1.5500	3.63703	13.2863	-25.332	-33.699
20.0000	1.6000	3.53972	13.2572	-21.535	-60.488
20.0000	1.6500	3.49905	12.6727	-25.069	-50.235
20.0000	1.7000	3.62573	12.6566	-22.056	-37.858
20.0000	1.7500	3.63081	11.9093	-26.365	-57.051
20.0000	1.8000	3.65030	11.8699	-25.294	-32.883
20.0000	1.8500	3.50124	11.3472	-23.432	-60.527
20.0000	1.9000	3.56607	10.8840	-26.349	-21.969
20.0000	1.9500	3.59554	10.6921	-25.310	-47.449
20.0000	2.0000	3.51834	10.3172	-28.526	-50.947
20.0000	2.0500	3.58026	10.2733	-25.427	-35.803
20.0000	2.1000	3.52960	9.88798	-26.590	-55.439

TABLE 1

Transfer Function Listing for Step Input

MULTIPLE CHANNEL TRANSFORM ANALYSIS PROGRAM

ACL-001 SINE

ACL MODEL

SINE DATA ANALYSIS USING FOURIER COEFFICIENTS
PROGRAM 2909 REQ.5196

DD0071-00

START TIME SECONDS	(00) FREQUENCY HERTZ	(01) AMPLITUDE	(02) AMPLITUDE	(03) GAIN-DB 02/01 F	(04) PHASE-DEG 02/01 F
.000000	.04000	737.132	714.931	-.26563	195.393
61.0000	.06400	740.646	684.609	-.68335	203.429
100.000	.10200	737.414	631.107	-1.3522	214.165
130.000	.14000	751.162	492.087	-3.6738	228.752
150.000	.23000	744.977	392.711	-5.5614	242.805
160.000	.41000	741.618	255.843	-9.2441	261.531
170.000	.62000	733.610	168.396	-12.783	-82.711
180.000	1.0200	735.828	95.0881	-17.773	-67.961
190.000	1.5800	732.209	47.8545	-23.694	-51.039
200.000	2.4800	730.985	24.8980	-29.355	-37.765
210.000	4.0000	725.382	10.3945	-36.875	-22.735
220.000	6.3000	713.768	4.85867	-43.341	-12.964
230.000	9.9000	687.981	1.79942	-51.649	-5.3523

TABLE 2

Results of Searching Subroutine and Transfer
Function Listing for Sine Input

INPUT OUTPUT		M.V.	FREQUENCY RESPONSE ANALYSIS PROGRAM						JOB 2907		
START TIME		M.V.	ACL-001 PSEUDO-RAND.								
DATA DURATION		76.4586	ACL MODEL								
POINTS 4096		N/M 10 N	PSEUDO-RANDOM DATA ANALYSIS								
SMOOTHING		HANNING									
BANDWIDTH		0.131	REQFST 5196						DD0072-00		
CONSTANTS		L=10 C=.									
PHASE + FOR OUTPUT LAG											
(00)	(01)	(02)	(03)	(04)	(05)	(06)	(07)	(08)	(09)	(10)	(11)
FREQUENCY	INPUT	P.S.D.	OUTPUT	P.S.D.	CROSS	SPC. DEN.	GAIN	(DB)	PHASE	(DEG)	COHERENCE
HERTZ	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	RAW	SMOOTHED	SMOOTHED
.14387	118493	36927.7	57125.6	21916.3	82273.8	24936.9	-3.1686	-3.4102	242.153	225.014	.768364
.27466	11778.8	39110.4	208.928	9853.18	1568.73	17356.9	-17.511	-7.0564	210.833	243.089	.781765
.40545	2113.69	37024.2	2191.65	6302.84	2152.31	14078.5	.157287	-8.3986	269.814	260.868	.849364
.53624	97753.8	36772.5	6233.07	3154.53	24684.1	9627.19	-11.954	-11.640	-70.726	269.010	.798988
.66703	23283.9	48861.9	210.452	1849.89	2213.62	8581.22	-20.439	-15.108	259.294	-84.918	.814670
.79782	1757.52	39835.0	463.516	1355.76	901.288	6612.00	-5.7760	-15.599	249.174	-76.824	.809500
.92861	141419	42119.6	3855.70	1187.97	23350.9	6508.23	-15.644	-16.220	-80.689	-72.409	.846519
1.0594	3829.29	43665.0	39.9415	735.343	391.080	5041.23	-19.817	-18.752	56.4284	-68.220	.791497
1.1902	10381.7	42605.9	237.433	597.526	1570.01	4537.88	-16.407	-19.452	-16.815	-60.440	.808870
1.3210	125186	44509.5	565.332	398.107	8412.57	3665.71	-23.452	-21.686	-47.558	-60.361	.758340
1.4518	3867.64	45264.0	40.6281	251.093	396.403	2932.48	-19.786	-23.770	44.8933	-56.393	.756628
1.5826	1229.04	40314.5	105.619	213.050	360.292	2533.15	-10.658	-24.036	-39.361	-61.884	.747099
1.7133	55049.1	40980.9	463.875	152.873	5053.30	2103.47	-20.743	-25.793	-54.155	-53.528	.706253
1.8441	10914.5	39184.1	1.23395	175.354	111.164	2262.17	-39.093	-24.772	128.886	-45.330	.744777
1.9749	2207.76	38703.6	72.6488	115.233	400.489	1773.03	-14.827	-26.781	-48.570	-54.599	.704860
2.1057	62548.7	43742.2	284.207	133.156	4216.25	2151.28	-23.426	-26.164	-45.444	-54.223	.794567
2.2365	25506.2	43886.5	1.31722	106.186	183.296	1872.79	-42.870	-27.397	-60.383	-48.543	.752626
2.3673	481.212	38722.1	18.9014	79.3914	95.3707	1426.12	-14.058	-28.676	161.146	-37.176	.661573
2.4981	17254.6	46397.2	82.0053	62.9125	1189.52	1462.53	-23.231	-30.028	-23.931	-16.424	.732788
2.6299	78205.3	42860.1	127.441	64.9277	3156.99	1391.19	-27.879	-29.773	-13.553	-26.103	.695485
2.7597	835.799	41309.7	15.1854	67.2893	112.659	1441.05	-17.407	-29.147	205.307	-32.335	.747063
2.8904	2876.74	39793.8	21.9606	37.5019	251.347	928.184	-21.173	-32.644	-88.266	-20.491	.577298
3.0212	67712.6	33896.9	16.3709	37.6007	1052.86	871.383	-36.166	-31.799	-67.739	-24.624	.595746
3.1520	9758.06	37053.6	34.7473	35.4948	582.293	888.076	-24.484	-32.408	-15.114	-25.501	.599661
3.2828	902.528	39129.8	9.09377	27.2879	90.5946	755.545	-19.967	-34.285	160.513	-31.178	.534618
3.4136	134700	43425.9	79.0911	32.6908	3263.98	950.152	-32.312	-33.199	-49.019	-25.508	.635933
3.5444	3425.07	35107.4	9.69462	20.0392	182.222	582.692	-25.481	-35.599	67.5674	-13.865	.482612
3.6752	814.992	35654.3	9.92904	16.5338	89.9560	495.703	-19.142	-37.138	69.7236	-13.301	.416830
3.8060	160126	40048.2	21.1814	14.6007	1841.65	490.084	-38.785	-38.246	-51.133	-34.481	.410755
3.9367	7309.65	36849.7	6.86667	15.3453	224.038	501.156	-30.271	-37.329	240.789	-35.569	.444157
4.0675	19478.4	42006.6	15.3803	19.4631	547.342	703.948	-31.026	-35.516	-68.032	-33.822	.606110
4.1983	131922	40029.1	49.6868	15.9569	2560.23	578.727	-34.241	-36.798	-39.032	-31.827	.524352
4.3291	2485.99	42892.0	9.47639	13.0654	153.487	532.289	-24.189	-38.125	-65.923	-42.327	.505587
4.4599	20048.5	42305.6	17.4301	15.7776	591.142	636.873	-30.608	-36.447	-10.628	-12.071	.607669
4.5907	136767	42495.7	25.4176	10.5191	1864.48	443.402	-37.308	-39.631	8.36163	-2.5312	.439816
4.7215	6540.11	41159.9	3.80631	10.0184	157.777	411.350	-32.351	-40.005	97.3410	-8.1924	.410345
4.8523	2198.65	37224.2	6.96850	10.5848	123.779	443.632	-24.990	-38.476	-79.599	-1.3909	.499505
4.9831	118044	47414.7	31.1545	14.2851	1917.71	690.649	-35.785	-36.733	-27.249	-13.536	.704239
5.1138	8738.88	40751.6	9.53641	8.93033	288.682	397.950	-29.621	-40.206	-22.303	-5.4353	.435156
5.2446	7855.78	39766.0	2.17844	7.47500	130.818	334.633	-35.570	-41.499	167.082	-38.323	.376715
5.3754	56545.1	39949.6	15.2183	9.22586	927.643	437.820	-35.700	-39.204	-37.716	-22.340	.520081
5.5062	36138.3	38832.0	.781543	8.36032	168.059	393.808	-46.650	-39.878	106.776	-16.745	.477701
5.6370	100.929	41869.9	4.38365	7.21426	21.0342	345.662	-13.622	-41.665	-73.920	-20.711	.395557
5.7678	38506.6	42143.2	9.47807	7.08107	604.126	369.463	-36.088	-41.143	-48.674	-31.858	.457421
5.8986	81300.3	44759.8	12.1363	7.40212	993.319	397.817	-38.260	-41.024	-47.167	-15.887	.477665

TABLE 3

Transfer Function Listing for Pseudo-Random Input

Figure 1 - Transfer Function Characteristics for Increasing Step Input

Figure 2 - Transfer Function Characteristics for Decreasing Step Input

Figure 3 - Time Plot of Increasing Step Input

Figure 4 - Time Response for Increasing Step Input

Figure 5 - Time Plot for Decreasing Step Input

Figure 6 - Time Response for Decreasing Step Input

Figure 7 - Transfer Function Characteristics for Multiple Sine Inputs

Figure 8 - Time History 0.040 cps Sine Output

Figure 9 - Time History 0.040 cps Sine Input

Figure 10 - Time History 0.25 cps Sine Output

Figure 11 - Time History 0.25 cps Sine Input

Figure 12 - Time History 1.0 cps Sine Output

Figure 13 - Time History 1.0 cps Sine Input

Figure 14 - Time History 6.3 cps Sine Output

Figure 15 - Time History 6.3 cps Sine Input

Figure 16 - Time History 10.0 cps Sine Output

Figure 17 - Time History 10.0 cps Sine Input

Figure 18 - Time History of Pseudo-Random Input and Output

Figure 19 - Transfer Function Characteristics for Pseudo-Random Inputs

Figure 20 - Power Spectral and Cross Spectral Density Plots