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ONE-DIMENSIONAL COMPRESSIBLE FLOWS
OF MAGNETO-HYDRODYNAMICS

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ABSTRACT

The systematic description of the physical state of electrically conducting fluid-like media (liquids, gases) is the concern of magneto-hydrodynamics. For such an incompressible media, H. Alfvén has obtained explicit one-dimensional plane wave solutions. Here the Alfvén problem is reinvestigated but now the assumption of incompressibility is not necessarily retained. By a constructive process, all simple wave solutions of this generalized Alfvén problem are determined. The method is essentially free of difficulties, once the basic non-linear, ordinary-differential equation of the theory has been integrated.

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ONE-DIMENSIONAL COMPRESSIBLE FLOWS
OF MAGNETO-HYDRODYNAMICS

O. G. Owens

Introduction. The equations of magneto-hydrodynamics (more briefly, hydromagnetics) describe the physical state of any inviscid medium which has the usual fluid (gas) dynamic properties and which in addition can bear an electric current. H. Alfvén has shown [1] that for an incompressible medium of this type the equations of hydromagnetics admit plane-wave solutions for which all physical quantities depend on the same single argument

$$\varphi(x, t) \equiv x - v_A t,$$

where $v_A = (\mu H_0^2 / \rho_0)^{1/2}$ is called the Alfvén velocity, μ is the magnetic permeability, ρ_0 the density, and H_0 the component of the magnetic field along the wave front normal.

The desire of extending Alfvén's method so as to make accessible a treatment of compressible inviscid media has led us to investigate a class of simple-waves which contains the Alfvén wave. By a simple-wave solution we mean one for which all the physical quantities are defined by means of the same single argument $\varphi(x, t)$, which in our investigation has the property that the curves $\varphi = \text{constant}$ are straight lines. The fact that these lines are characteristic curves for the equations of hydromagnetics will then follow by making use of a criterion derived by K. O. Friedrichs [2]. It is evident that the envelope of these lines must also be a characteristic curve and thus may carry discontinuous data; that is, the envelope can be a shock-front. Hence, if the values of a single pertinent physical quantity is prescribed over the side of the envelope immediately away from the flow, then the remaining physical variables on this same side of the envelope are determined by means of the magneto-hydrodynamic shock relations [2], [3].

The essential difficulty in applying the theory to a specific medium with a known equation of state is the necessity of integrating a certain non-linear ordinary differential equation. Although no explicit analytical formula is known for the solution of this equation, the integral curves are always obtainable by numerical methods. For the two special cases

$$P = \rho^{5/3}, \quad P = \rho^{7/5},$$

and with various values for the initial data, the integral curves are given graphically.

1. The Magneto-Hydrodynamic Equations. In Georgi (mks) units the equations of Maxwell are:

$$(1.1) \quad \operatorname{div} \vec{D} = \rho_c, \quad \operatorname{div} \vec{B} = 0$$

$$(1.2) \quad \operatorname{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \operatorname{curl} \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j},$$

where: $\vec{E}$ is the electric field vector, $\vec{D}$ the electric displacement, $\vec{H}$ the magnetic field, $\vec{B}$ the magnetic induction, ρ_c the charge density, $\vec{j}$ the current per unit cross-section.

The force exerted on unit volume of current by the external magnetic field is:

$$(1.3) \quad \vec{F} = \vec{j} \times \vec{B} + \rho_c \vec{E} \quad (\rho_c = 0)$$

where the operation denoted in (1.3) is that of forming the vector cross-product. Moreover, the current is given by:

$$(1.4) \quad \vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}),$$

where $\vec{v}$ is the flow velocity and σ is the electrical conductivity.

The fluid properties of the medium are expressed by the equations expressing the conservation of mass and that of momentum; therefore,

$$(1.5) \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0,$$

$$(1.6) \quad \rho \frac{d\vec{v}}{dt} = -\operatorname{grad} P + \vec{F},$$

where ρ is the mass-density and $P = P(\rho)$ is the equation of state.

We shall now impose certain additional conditions on the media in order to derive the equations of hydromagnetics in a form that will be basic for our investigation. By reason of (1.4), the assumption $\sigma = \infty$ ($\vec{j}$ being finite) implies:

$$(1.7) \quad \vec{E} = - \vec{v} \times \vec{B}.$$

Finally, by reason of (1.2), the assumption $\frac{\partial \vec{D}}{\partial t} = 0$ implies:

$$(1.8) \quad \vec{j} = \text{curl } \vec{H}.$$

Using (1.7) and (1.8), the equations (1.1), (1.2), (1.5) and (1.6) reduce to:

$$(1.9) \quad \frac{\partial \vec{H}}{\partial t} = \text{curl} (\vec{v} \times \vec{H}), \quad \text{div } \vec{H} = 0$$

$$(1.10) \quad \rho \frac{d\vec{v}}{dt} = - \text{grad } P - \mu \vec{H} \times \text{curl } \vec{H}, \quad \frac{\partial \rho}{\partial t} + \text{div} (\rho \vec{v}) = 0$$

where μ is the constant magnetic permeability. Under the above assumptions the equations (1.9), (1.10) are called the equations of magneto-hydrodynamics. Although this system consists of eight equations for seven unknowns, the system is not overdetermined since (1.9) implies that $\text{div } \vec{H} \equiv 0$ if it vanishes initially.

2. The Alfvén Assumptions. Let $\vec{a}_x, \vec{a}_y, \vec{a}_z$ denote unit vectors having, respectively, the directions of the positive X, Y, Z -- axes of a cartesian coordinate system. Furthermore, let the components of the magnetic and velocity field be defined by

$$\vec{H} = H_x \vec{a}_x + H_y \vec{a}_y + H_z \vec{a}_z \quad \text{and} \quad \vec{v} = v_x \vec{a}_x + v_y \vec{a}_y + v_z \vec{a}_z.$$

Then the assumptions made by Alfvén are that the flow is one-dimensional (that is, all physical quantities depend only on the variables (x, t)) and

$$(2.1) \quad H_y(x, t) \equiv 0 \quad v_y(x, t) \equiv 0.$$

Under these hypotheses (1.9) and (1.10) reduce to the following set of equations:

$$(2.2) \quad \frac{\partial \vec{H}}{\partial t} = \vec{a}_z \frac{\partial}{\partial x} (\vec{v}_z H_x - \vec{v}_x H_z)$$

$$(2.3) \quad \frac{\partial H_x}{\partial x} = 0$$

$$(2.4) \quad \rho \frac{d\vec{v}}{dt} = - \text{grad } P - \vec{a}_x \mu H_z \frac{\partial H_z}{\partial x} + \vec{a}_z \mu H_x \frac{\partial H_z}{\partial x}$$

$$(2.5) \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) = 0.$$

From (2.2) and (2.3) we obtain the relation:

$$(2.6) \quad H_x(x, t) = H_0 \quad (\text{a constant}).$$

Using this result, we can write (2.2) - (2.5) in the following form:

$$(2.7) \quad \frac{\partial H_z}{\partial t} = H_0 \frac{\partial v_z}{\partial x} - \frac{\partial}{\partial x} (v_x H_z)$$

$$(2.8) \quad \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} - \mu H_z \frac{\partial H_z}{\partial x}$$

$$(2.8') \quad \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} = \frac{\mu H_0}{\rho} \frac{\partial H_z}{\partial x}$$

$$(2.9) \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) = 0.$$

-- In order to solve this system Alfven made an additional assumption; namely, he assumed that

$$(2.10) \quad \rho = \rho_0 \quad (\text{a constant}), \quad v_x(x, t) = 0.$$

Thus, the system (2.7) - (2.9) becomes

$$(2.11) \quad \frac{\partial H_z}{\partial t} = H_0 \frac{\partial v_z}{\partial x}$$

$$(2.12) \quad \frac{\partial P}{\partial x} = - \mu H_z \frac{\partial H_z}{\partial x}$$

$$(2.13) \quad \frac{\partial v_z}{\partial t} = \frac{\mu H_0}{\rho_0} \frac{\partial H_z}{\partial x}.$$

Eliminating v_z from (2.11) and (2.13) yields the one-dimensional wave equation

$$(2.14) \quad \frac{\partial^2 H_z}{\partial t^2} = \frac{\mu H_0^2}{\rho_0} \frac{\partial^2 H_z}{\partial x^2},$$

which has the solution

$$(2.15) \quad H_z = H_z(x - v_A t) \quad v_A = (\mu H_0^2 / \rho_0)^{1/2},$$

where H_z is an arbitrary function of its argument. Hence, because of (2.12) and (2.13),

$$(2.16) \quad P = -\frac{\mu}{2} H_z^2 \quad P(0) = 0$$

$$(2.17) \quad v_z = -\sqrt{\frac{\mu}{\rho_0}} H_z \quad v_z(0) = 0.$$

Therefore, in summary, the Alfvén solution of the magneto-hydrodynamic equations is given by the following ordered 8-tuple:

$$(2.18) \quad (H_x, H_y, H_z, v_x, v_y, v_z, \rho, P) \equiv (H_0, 0, H_z(x - v_A t), 0, 0, -\sqrt{\frac{\mu}{\rho_0}} H_z, \rho_0, -\frac{\mu}{2} H_z^2).$$

3. A Generalization of Alfvén's Method. We retain all of Alfvén's hypotheses except those of (2.10). Now, the relations (2.16) and (2.17) suggest looking for solutions of the system, (2.7) - (2.9), having the form

$$(3.1) \quad v_x = v_x(H_z), \quad v_z = v_z(H_z), \quad \rho = \rho(H_z).$$

Substituting these expressions into (2.7) - (2.9) yields

$$(3.2) \quad 1 \cdot \frac{\partial H}{\partial t} + (H v_x^{(1)} + v_x - H_0 v_z^{(1)}) \frac{\partial H}{\partial x} = 0$$

$$(3.3) \quad \rho v_x^{(1)} \frac{\partial H}{\partial t} + (\rho v_x v_x^{(1)} + P^{(1)} + \mu H) \frac{\partial H}{\partial x} = 0$$

$$(3.4) \quad \rho v_z^{(1)} \frac{\partial H}{\partial t} + (\rho v_x v_z^{(1)} - \mu H_0) \frac{\partial H}{\partial x} = 0$$

$$(3.5) \quad \rho^{(1)} \frac{\partial H}{\partial t} + (\rho v_x^{(1)} + v_x \rho^{(1)}) \frac{\partial H}{\partial x} = 0,$$

where the notation $H \equiv H_z$ and $v^{(1)} \equiv \frac{dv}{dH}$ is being used.

The system, (3.2) - (3.5), consists of four homogeneous equations in the two unknowns $\frac{\partial H}{\partial t}$, $\frac{\partial H}{\partial x}$. Thus, for a non-identically vanishing solution, it is necessary and sufficient that all second orders determinants formable from the system vanish. Since three of these six determinants are linearly dependent, it suffices to require that the following three determinants vanish:

$$(3.6) \quad P^{(1)} + \mu H - \rho H (v_x^{(1)})^2 + \rho H_0 v_x^{(1)} v_z^{(1)} = 0$$

$$(3.7) \quad H_0 \rho (v_z^{(1)})^2 - \rho H v_x^{(1)} v_z^{(1)} - \mu H_0 = 0$$

$$(3.8) \quad \rho v_x^{(1)} - H \rho^{(1)} v_x^{(1)} + \rho^{(1)} H_0 v_z^{(1)} = 0.$$

If the Alfvén assumption (2.10) be applied to this system, we get

$$(3.9) \quad P^{(1)} + \mu H = 0$$

$$(3.10) \quad v_z^{(1)} - \sqrt{\frac{\mu}{\rho_0}} = 0.$$

Thus,

$$(3.11) \quad P = -\frac{\mu H^2}{2}, \quad v_z = -\sqrt{\frac{\mu}{\rho_0}} H, \quad (P(0) = v_z(0) = 0)$$

which coincides with the Alfvén relations (2.16) and (2.17).

4. The Basic Differential Equation. In what follows we shall assume that $\rho^{(1)}$ is not zero; that is, we shall confine ourselves to the compressible case. Because of this assumption, the system (3.6) - (3.8) is expressible as:

$$(4.1) \quad v_z^{(1)} = \frac{1}{H_0} \frac{(H \rho^{(1)} - \rho)}{\rho^{(1)}} v_x^{(1)}$$

$$(4.2) \quad H_0 (v_z^{(1)})^2 - H v_x^{(1)} v_z^{(1)} - \mu H_0 = 0$$

$$(4.3) \quad H_0 v_x^{(1)} v_z^{(1)} - H (v_x^{(1)})^2 + \frac{1}{\rho} (P^{(1)} + \mu H) = 0.$$

By means of the first equation $v_z^{(1)}$ can be eliminated from the last two. This yields:

$$(4.4) \quad (v_x^{(1)})^2 = -\mu H_0^2 \left(\frac{\rho^{(1)}}{\rho}\right)^2 \frac{1}{H \rho^{(1)} - \rho}$$

and

$$(4.5) \quad -\frac{\rho}{\rho^{(1)}} (v_x^{(1)})^2 + \frac{1}{\rho} (P^{(1)} + \mu H) = 0.$$

Now, by means of (4.4), $v_x^{(1)}$ can be eliminated from (4.5). This gives

$$(4.6) \quad \mu H_0^2 \rho^{(1)} + (H \rho^{(1)} - \rho) (P^{(1)} + \mu H) = 0,$$

a differential equation for the unknown-function $\rho(H)$. For a given equation of state $P = P(\rho)$, the equation (4.6) can be written in the form

$$(4.7) \quad H a^2 (\rho^{(1)})^2 + \rho^{(1)} (\mu H^2 + \mu H_0^2 - a^2 \rho) - \mu H \rho = 0,$$

where $P^{(1)} = a^2 (\rho) \rho^{(1)}$. We see from (4.7) that the initial-value

$$(4.8) \quad \rho_0 = \rho(0)$$

is arbitrarily prescribable as long as

$$(4.9) \quad \rho_0 a^2 (\rho_0) - \mu H_0^2 \neq 0,$$

which implies that $\rho^{(1)}(0) = 0$.

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$H_0 = 1$
 $P = \rho^{1.4}$
 $\mu = 4\pi \times 10^{-7} \text{ (mks)}$
 $\rho_0 \frac{dP}{d\rho_0} = \mu H_0^2$
 $\rho_0 \times 10^4 = .479$

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If $\rho = \rho(H)$ denotes the solution of (4.7) subject to (4.8) and (4.9), then, because of (4.1) and (4.4),

$$(4.10) \quad v_x^{(1)} = \frac{H_0 \sqrt{\mu} \rho^{(1)}}{\rho \sqrt{\rho - H} \rho^{(1)}},$$

$$(4.11) \quad v_z^{(1)} = \frac{-\sqrt{\mu} \sqrt{\rho - H} \rho^{(1)}}{\rho}.$$

Therefore, the quantities v_x , v_z , ρ , $P(\rho)$ are now known functions of H . It now remains to determine H as a function, $\varphi(x, t)$, of the variables (x, t) .

5. Determination of $\varphi(x, t)$: Introducing the notation

$$(5.1) \quad g(H) \equiv \frac{\rho v_x^{(1)}}{\rho^{(1)}} = \frac{H_0 \sqrt{\mu}}{\sqrt{\rho - H} \rho^{(1)}}$$

and recalling equation (3.5) leads to the next equation

$$(5.2) \quad \frac{\partial H}{\partial t} + (g(H) + v_x(H)) \frac{\partial H}{\partial x} = 0.$$

Now, if the Alfvén hypothesis (2.10) is invoked, (5.2) assumes the simpler form

$$(5.3) \quad \frac{\partial H}{\partial t} + v_A \frac{\partial H}{\partial x} = 0,$$

which has the previously given solution

$$(5.4) \quad H = H(x - v_A t) \quad (H \text{ arbitrary}).$$

The integration of (5.2) can be effected by means of the characteristic theory for first-order partial differential equations. This theory requires that

$$(5.5) \quad \frac{dH}{d\alpha} = 0, \quad \frac{dx}{d\alpha} = g(H) + v_x(H), \quad \frac{dt}{d\alpha} = 1.$$

Therefore,

$$\begin{aligned}
 (5.6) \quad H(\alpha, \beta) &= H_I(\beta) \\
 x(\alpha, \beta) &= [g(H_I) + v_x(H_I)]\alpha + x_I(\beta) \\
 t(\alpha, \beta) &= \alpha + t_I(\beta),
 \end{aligned}$$

where H_I, x_I, t_I are the initial values assumed, respectively, by H, x, t for $\alpha = 0$. We now determine $\varphi(x, t)$ as follows. Making the assumption that the jacobian

$$(5.7) \quad \frac{\partial(x, t)}{\partial(\alpha, \beta)} \neq 0$$

allows us to solve for (α, β) in terms of (t, x) ; i.e.,

$$(5.8) \quad \alpha = \alpha(t, x), \quad \beta = \beta(t, x).$$

Thence,

$$(5.9) \quad \varphi(x, t) \equiv H(\alpha, \beta) = H_I(\beta(t, x)).$$

6. Hydro-Magnetic Shock Fronts. The envelope of the one-parameter family of curves

$$(6.1) \quad x = x(\alpha, \beta), \quad t = t(\alpha, \beta)$$

is obtained by eliminating one of the variables (α, β) from the equations (6.1) and

$$(6.2) \quad 0 = \frac{\partial t}{\partial \alpha} \frac{\partial \alpha(x, \beta)}{\partial \beta} + \frac{\partial t}{\partial \beta} \quad (\text{or } \frac{\partial(x, t)}{\partial(\alpha, \beta)} = 0).$$

For the special case (5.6) it follows that the equation of the envelope is

$$(6.3) \quad x(\beta) = x_I(\beta) + \frac{[g(H_I) + v_x(H_I)] [t_I^{(1)}(g(H_I) + v_x(H_I)) - x_I^{(1)}]}{H_I^{(1)} [g^{(1)}(H_I) + v_x^{(1)}(H_I)]}$$

$$t(\beta) = t_I(\beta) + \frac{[t_I^{(1)}(g(H_I) + v_x(H_I)) - v_I^{(1)}]}{H_I^{(1)} [g^{(1)}(H_I) + v_x^{(1)}(H_I)]}.$$

If this envelope is to be capable of representing a shock-front, then it must be a characteristic manifold for the magneto-hydrodynamic equations. Recently, K. O. Friedrichs has shown [2] that these characteristic manifolds are the integral-surfaces of the following partial differential equation:

$$(6.4) \quad \rho c^2 (\rho c^2 - \mu H_n^2) \left\{ \rho c^4 - (\rho a^2 + \mu (H_x^2 + H_y^2 + H_z^2)) c^2 + \mu a^2 H_n^2 \right\} = 0,$$

where H_n is the component of $\vec{H}$ along the normal to the manifold in XYZ-space at time t ; c is the difference between the velocity of the surface in its normal direction, and the normal component of flow velocity; $a^2 \equiv dP/d\rho$ is the square of the sound-speed.

We shall now establish the fact that the envelope is a characteristic curve by showing that the lines of the family (5.6) are characteristic lines. For any such line

$$(6.5) \quad c = \frac{dx}{dt} - v_x = (g(H_I) + v_x(H_I)) - v_x(H_I) = g(H_I).$$

Now, recalling (5.1) the definition of g , and setting

$$(6.6) \quad \mathcal{F} \equiv \rho c^4 - (\rho a^2 + \mu (H_o^2 + H_z^2)) c^2 + \mu a^2 H_o^2 \quad (H_n^2 = H_o^2),$$

we find that

$$(6.7) \quad \frac{[\rho - H\rho^{(1)}]^2 \Phi}{\mu H_0^2} \equiv \rho \mu H_0^2 - [\rho - H\rho^{(1)}] \left[\rho \frac{dP}{d\rho} + \mu(H_0^2 + H^2) \right] + \frac{dP}{d\rho} [\rho - H\rho^{(1)}]^2$$

$$= \rho \mu H_0^2 - \mu(H_0^2 + H^2)(\rho - H\rho^{(1)}) - H \frac{dP}{d\rho} \rho^{(1)} (\rho - H\rho^{(1)}) .$$

Therefore,

$$(6.8) \quad \frac{[\rho - H\rho^{(1)}]^2 \Phi}{\mu H_0^2} \equiv \mu H_0^2 \rho + (H\rho^{(1)} - \rho) [H P^{(1)} + \mu(H_0^2 + H^2)] \quad H \equiv H_z$$

$$= \mu H_0^2 \rho + H(H\rho^{(1)} - \rho)(P^{(1)} + \mu H) + \mu H_0^2 (H\rho^{(1)} - \rho)$$

$$= \mu H_0^2 H\rho^{(1)} + H(H\rho^{(1)} - \rho)(P^{(1)} + \mu H) .$$

Consequently,

$$(6.9) \quad \frac{[\rho - H\rho^{(1)}]^2 \Phi}{\mu H_0^2 H} = \mu H_0^2 \rho^{(1)} + (H\rho^{(1)} - \rho)(P^{(1)} + \mu H) .$$

Because of (4.6) the right member of (6.9) must vanish, and this implies that $\Phi = 0$, if $\rho - H\rho^{(1)} \neq 0$. This shows that the envelope is a characteristic curve.

7. Special Solution of (4.6). For an incompressible fluid

$$(7.1) \quad P^{(1)} + \mu H = 0 .$$

This suggests looking for solutions of (4.6) with

$$(7.2) \quad P^{(1)} + \mu H = c_0 H^{k-1} \quad (k, \text{ constant}) .$$

Under this assumption, the equation (4.6) becomes

$$(7.3) \quad \mu H_0^2 \rho^{(1)} + (H\rho^{(1)} - \rho) c_0 H^{k-1} = 0 .$$

This linear equation has the solution

$$(7.4) \quad \rho = c_1 (c_0 H^k + \mu H_0^2)^{1/k} \quad (c_1 > 0).$$

Since (7.2) has the solution

$$(7.5) \quad P + \mu \frac{H^2}{2} = \frac{c_0}{k} H^k,$$

it follows that the equation of state is given by

$$(7.6) \quad P(\rho) = \frac{1}{k} \left[\left(\frac{\rho}{c_1} \right)^k - \mu H_0^2 \right] - \frac{\mu}{2 c_0^{2/k}} \left[\left(\frac{\rho}{c_1} \right)^k - \mu H_0^2 \right]^{2/k}.$$

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