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REACTOR PHYSICS CONSTANTS

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PREFACE

This compilation of reactor physics constants is the first publication of the Reactor Physics Constants Center (RPCC) located at the Argonne National Laboratory.

The primary objective of the RPCC is the periodic compilation of the latest and best values of the constants, recipes, formulae, etc., which are necessary to calculate reactor characteristics. It is hoped that in addition to providing a unified compilation of pertinent data, the activities of the RPCC will (1) encourage critical comparison of experiment and theory; (2) encourage critical analysis of experimental techniques; (3) encourage critical evaluations between contradictory data; (4) call attention at areas where further experimental and theoretical work will be most profitable; (5) promote standardization in definitions of reactors physics parameters; and (6) serve as a clearing house for reactor physics constants.

The ultimate success of subsequent editions of the compilation largely depends on the extent to which direct contributions are made to the RPCC. On the basis of this consideration, it was decided that rather than critically evaluate each individual item, with a consequent delay and reduction in scope, the purposes of the RPCC would best be served by a less critically evaluated initial compilation. It is hoped that subsequent editions will become progressively more authoritative. To assist in the preparation of future editions, contributions of new data, criticism of reported results, results of new analysis, comments on the form of presentation and on content of the compilation will be welcomed by the RPCC.

✓ This compilation is based on data available up to March 15, 1958. Effort was made to include information that became available later, if it could be done easily, up to the time of publication, July 1, 1958.

Section 1 pertains to data associated with the fission process, and as such is not specialized to any particular reactor type; for example, values for the delayed neutron parameters do not include any weighting factors for relative effectiveness. Effective values for the delayed neutron fractions are properties of the specific reactor under consideration, and hence do not properly belong in Section 1, but are considered in the section appropriate to the reactor.

Section 2 contains selected cross section data not specialized to any particular reactor. For convenience, some slight overlap exists with, say, the BNL series on neutron cross sections. In general it is not intended that such overlap be appreciable. In this category are the fission cross sections and the angular distribution cross sections for various materials and energies which are given in a form particularly suitable for reactor calculations. A large part of the section pertains to microscopic cross section data averaged over particular neutron spectra which are of interest in reactor physics; for example, the average of the thermal cross section over a Maxwellian distribution, or the resonance integral, proportional to the average of the resonance energy cross section over a $\frac{1}{E}$ spectrum. Cross sections averaged over particular spectra such as fission spectra and natural uranium equilibrium spectra are also included. The scattering law at thermal energies is discussed because of its extreme importance in reactor problems.

Sections 3 and 4 cover thermal homogeneous and heterogeneous reactors, respectively. It is in these sections that the problem of delineating the scope of the compilation becomes most acute, since theory is much entwined with the constants necessary for calculation of such reactors. In general, theory has been included only to the extent considered necessary for exposition of the data. Further problems exist in the arrangement and

possible overlap of material between the two sections. Unless specifically pertinent to the lattice structure of the reactor, the material is included in Section 3 on homogeneous systems. For example, discussions of long-term changes are included in Section 3, even though they are equally important in heterogeneous systems. Some topics, for example resonance capture, are discussed in both sections. Some of the considerations are not related to the lumping of the fuel, and are therefore given in Section 3. Considerations on resonance capture which primarily result from the lumping of fuel, are given in Section 4. The related material in Section 3 is not repeated, although this material is clearly also important in the treatment of resonance capture in lumps.

Sections 5 and 6 pertain to intermediate and fast reactors, respectively. Some theoretical methods which are also applicable to other reactor types are considered in these sections because they are used primarily in connection with these reactors. For example, the S_n method is discussed in the section on fast reactors, even though its application to other reactor types is straightforward.

Section 7 on shielding constants is included even though more comprehensive manuals on shielding are readily available. Although somewhat marginal within the scope of the compilation, this material is included mainly for the convenience of reactor physicists whose primary interests may not be shielding, and who therefore may find the more compact and less detailed compilation of use.

Section 8 is mainly intended to be a discussion of the corrections which the experimentalist must apply to his data in order to eliminate factors not relevant to the basic quantity being measured. For example, in measuring the flux at a certain point in a reactor, the experimentalist is not concerned with the local depression of the flux caused by a foil, other than to correct for the effect. Brief discussions of some general experimental techniques commonly used in reactor measurements and not compactly covered elsewhere are also included.

Section 9 contains information on the elements and various materials and other data that does not properly fall within any of the other sections.

The compilation represents the efforts of many people. Unless otherwise noted the following are from Argonne National Laboratory. The compilation was edited by R. Avery. The original plans for the format and guiding principles of the compilation were made by R. Avery, H. Greenspan and B. I. Spinrad.

Section 1 was prepared by H. Greenspan. A. B. Smith contributed much of the data and evaluation in Section 1.1. S. Katcoff (BNL) contributed the data on fission product yields in the form of a compilation prepared for publication in the Addison-Wesley "Handbook of Nuclear Engineering." G. R. Keepin (LASL) provided information on delayed neutrons in addition to his published work.

Section 2 was written by P. J. Persiani. D. Meneghetti contributed data on the natural uranium equilibrium spectrum.

Much of Section 3 was written by D. H. Shaftman. Section 3.9 on integral data was prepared by L. J. Templin. C. H. Westcott (Chalk River) contributed some of his more recent data on effective cross sections (also used in Section 2). J. T. Roberts and A. M. Perry (ORNL) cooperated in making available the Ocusol A set of multigroup cross sections. Other principal contributors to Section 3 are P. J. Persiani, A. D. Rossin, B. J. Toppel, H. Greenspan, W. A. Reardon and C. N. Kelber.

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SECTION 1

FISSION PROCESS

1.1 NUMBER OF PROMPT NEUTRONS EMITTED IN FISSION

The following discussion is confined to the emission of prompt fission neutrons; more specifically, to binary events ($>10^6$ times as probable as the ternary event⁽¹⁾). Experiment has shown that prompt fission neutrons are emitted within 10^{-14} seconds of the actual fission of the nucleus.⁽²⁾ Experimentally, the prompt neutrons are categorized as the number of neutrons counted simultaneously with the fission event, within the resolving time of the experimental apparatus ($\sim 30\mu$ sec for a large liquid scintillation detector). Independent experiments have shown less than 1% of the fission neutrons are emitted between 10^{-12} and 10^{-4} sec after fission.^(2,3,4)

Within the coarse limits of experiment, the emission is isotropic in the reference frame of the fragment, and is 1.3 times as prevalent from the light fragment as from the heavy fragment.^(2,5,6)

Since the emission is from highly excited fragments, theoretical treatments are generally of a statistical nature.⁽⁷⁾ Such an approach has been used successfully to predict the number of neutrons emitted in fission.⁽⁸⁾

1.1.1 Average Number and Dependence on Energy

Many experimental techniques have been applied to measurements of ν_p , the average number of prompt neutrons emitted per fission. The accuracy of these measurements has been improved substantially by the development of large scintillation detectors.⁽⁹⁾ The most recent data are listed in Table 1-1. These data can also be used to estimate the variation of ν_p with energy of the incident neutron causing fission in U^{233} , U^{235} , U^{238} , and Pu^{239} .

$$\frac{d\nu_p}{dE} (U^{233}) = 0.095 \quad \text{neutrons/Mev}$$

$$\frac{d\nu_p}{dE} (U^{235}) = 0.145 \quad \text{neutrons/Mev}$$

$$\frac{d\nu_p}{dE} (U^{238}) = 0.144 \quad \text{neutrons/Mev}$$

$$\frac{d\nu_p}{dE} (Pu^{239}) = 0.105 \quad \text{neutrons/Mev}$$

Table 1-1

AVERAGE NUMBER OF PROMPT NEUTRONS PER FISSION

Isotope	Neutron Energy	ν_p	Ref.
U^{233}	Thermal ^a	2.51 ± 0.03	10
	80 kev	2.58 ± 0.062	11,12
	Fission Spectrum	2.70 ± 0.06	12
U^{235}	Thermal	2.47 ± 0.03	10
	80 kev	2.47 ± 0.03	11
	1.25 Mev	2.65 ± 0.07	13
	4.8 Mev	3.20 ± 0.08	13
U^{238}	1.5 Mev	2.65 ± 0.09	13
	14.0 Mev	4.45 ± 0.10	14
Pu^{239}	Thermal	2.91 ± 0.04	10
	80 kev	3.048 ± 0.079	11
Pu^{240}	Spontaneous Fission	2.257 ± 0.045	11,15
	Fission Spectrum	3.42 ± 0.2	12
Pu^{241}	Thermal	2.91 ± 0.1	16
	Thermal	3.17 ± 0.12	17
Th^{232}	1.45 Mev	2.4	b
	3.5 Mev	2.35 ± 0.07	18

^aCorresponds to a neutron velocity of 2200 meter/sec.

^bResults of preliminary measurements by A. B. Smith, ANL

The above slopes for the energy-dependence of ν_p are superimposed on the experimental data in Fig. 1-1. In addition, several relative measurements of ν_p have been performed:

 U^{233}

$$\frac{\nu_p (\text{fission spectrum})}{\nu_p (\text{thermal})} = 1.075 \pm 0.01 \quad (\text{Ref. 12})$$

 U^{235}

$$\frac{\nu_p (0.7 \text{ Mev})}{\nu_p (\text{thermal})} = 1.02 \pm 0.02 \quad (\text{Ref. 19})$$

$$\frac{\nu_p (1 \text{ Mev})}{\nu_p (\text{thermal})} = 1.15 \pm 0.14 \quad (\text{Ref. 19})$$

$$\frac{\nu_p (\text{fission spectrum})}{\nu_p (\text{thermal})} = 1.10 \pm 0.01 \quad (\text{Ref. 12})$$

 Pu^{239}

$$\frac{\nu_p (\text{fission spectrum})}{\nu_p (\text{thermal})} = 1.11 \pm 0.01 \quad (\text{Ref. 12})$$

FIG. 1-1
AVERAGE PROMPT NEUTRONS PER FISSION (ν_p)
FOR VARIOUS ISOTOPES AS A FUNCTION
OF INCIDENT NEUTRON ENERGY (Mev)

Using a detailed statistical theory, Leachmann⁽⁸⁾ calculates

$$\frac{d\nu_p}{dE}(\text{U}^{235}) = \begin{matrix} 0.147 \\ 0.126 \\ 0.116 \end{matrix} \quad \text{neutrons/Mev}$$

for nuclear temperatures corresponding to 1.4 Mev and obtains a spectrum in agreement
with experiment when the nuclear temperature is in the range 1.0-1.4 Mev.

In addition to the correlation between ν_p and the incident neutron energy outlined above, experiments show that the value of ν_p is related to the total fragment energy of a given fission event,⁽²⁰⁾ (ν_p decreases as the fragment energy increases) which is in qualitative agreement with theory.⁽²¹⁾

1.1.2. Distribution Function

The distribution function, P_ν , is the probability of emission of ν prompt neutrons per fission. Recently, a simple cumulative distribution function of P_ν has been proposed,⁽¹³⁾ which is in excellent agreement with experiment:

$$\sum_{n=0}^{\nu} P_n = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\nu - \nu_p + 0.5 + b)/\sigma} \exp(-t^2/2) dt \quad (1-1)$$

where

$b = \sim 10^{-2}$ for all fission processes

$\sigma = 1.08$, a constant.

Except for the anomalous case of Cf²⁵² fission, Eq. (1-1) gives, to good accuracy, the distribution in P_ν for most applications.

The P_ν values for U²³³, U²³⁵, Pu²³⁹ and Pu²⁴⁰ obtained with large liquid scintillation detectors are listed in Table 1-2. Also included in the table is a measure of the relative width of the neutron multiplicity distribution:

$$\frac{(\nu_p^2)_{\text{avg}} - \nu_p}{\nu_p^2} \quad (1-2)$$

where ν_p and $(\nu_p^2)_{\text{avg}}$ are, respectively, the average and average squared number of prompt neutrons per fission. The quantity is unity for a Poisson distribution.

Table 1-2

DISTRIBUTION FUNCTION (P_ν) FOR EMISSION OF
PROMPT NEUTRONS IN VARIOUS ISOTOPES

(Reproduced from Data by Diven, et al.⁽¹¹⁾)

Neutron Energy

Pu²⁴⁰ = Corresponding to Spontaneous Fission

Others = 80 kev

	U ²³³	U ²³⁵	Pu ²³⁹	Pu ²⁴⁰
P_0	0.010 ± 0.008	0.027 ± 0.004	-0.01 ± 0.01	0.049 ± 0.006
P_1	0.151 ± 0.024	0.158 ± 0.010	0.11 ± 0.03	0.214 ± 0.012
P_2	0.326 ± 0.037	0.339 ± 0.014	0.13 ± 0.06	0.321 ± 0.014
P_3	0.301 ± 0.044	0.305 ± 0.015	0.56 ± 0.08	0.282 ± 0.017
P_4	0.176 ± 0.041	0.133 ± 0.013	0.11 ± 0.08	0.112 ± 0.013
P_5	0.042 ± 0.028	0.038 ± 0.009	0.06 ± 0.09	0.021 ± 0.008
P_6	-0.010 ± 0.017	-0.001 ± 0.003	0.05 ± 0.08	0.001 ± 0.003
P_7	0.006 ± 0.009	0.001 ± 0.002	0.00 ± 0.06	0.000 ± 0.002
P_8	-0.002 ± 0.002	0.000 ± 0.000	-0.01 ± 0.03	0.000 ± 0.000
$(\nu_p^2)_{\text{avg}}$	7.84 ± 0.34	7.32 ± 0.15	10.62 ± 0.53	6.37 ± 0.21
$(\nu_p^2)_{\text{avg}} - \nu_p$				
ν_p^2	0.786 ± 0.013	0.795 ± 0.007	0.815 ± 0.017	0.807 ± 0.008

1.2 FISSION PRODUCT YIELDS

Unless otherwise noted, the data in this section have been taken from a compilation by Katcoff.⁽²²⁾ References used in the compilation are repeated for convenience.

In general, very few fission yields have been measured on an absolute basis. In most cases, yields have been determined relative to Ba^{140} whose absolute yield has been averaged to 6.44%. Where the reported data have been based on an assumed yield for Ba^{140} other than 6.44%, the values given in the tables have been re-normalized to 6.44%. Yields which have been determined relative to an isotope other than Ba^{140} are indicated by footnotes in the tables.

1.2.1 Fission Product Chains

The fission product chains are shown schematically in Table 1-3.

1.2.2 Total Chain Yields from U^{235} Thermal Fission

Table 1-4 lists the total chain yield for each mass number from thermal neutron fissions in U^{235} . The fission product uniquely characterizes the total chain yield for the respective mass number. Where an actively decaying isotope is indicated, no other isotope further along the chain is formed directly in fission.

1.2.3 Direct Yield of Some Primary U^{235} Fission Products

Table 1-5 gives the direct yield of some primary fission products from U^{235} fission induced by both thermal and 14-Mev neutrons. Also given is the fraction of the total chain yield represented by these primary products.

1.2.4 Cumulative Yields of Various Fission Products from Thermal Fissions

The cumulative yield of a fission product is the probability that, in one fission, the fission product is formed either directly or later in the decay chain.

Table 1-6 lists the cumulative fission yields of various isotopes from thermal fission of U^{233} , U^{235} , and Pu^{239} . The data in Table 1-4, which also represent cumulative yields, are not repeated.

The cumulative yields in Table 1-6 do not necessarily represent total chain yields since a later member of the chain may also be formed directly from fission. In addition, as a result of isomerism, the yields for some U^{235} fission products are lower than their counterparts listed in Table 1-4 for the total chain yield. In the absence of isomerism, any discrepancy between values for the same isotope in Tables 1-4 and 1-6 (for U^{235} fission) is due to experimental error. The values in Table 1-4 are considered more reliable.

TABLE 1-3
FISSION PRODUCT CHAINS

NOTE: WHEN BRANCHING OCCURS, THE FRACTION IN EACH BRANCH, IF KNOWN, IS INDICATED BY THE ARROW. DASHED ARROWS ARE USED WHEN ONLY THE UPPER LIMITS HAVE BEEN ESTABLISHED. BRACKETS DENOTE THAT THE GIVEN NUCLIDE PROBABLY OCCURS IN FISSION BUT THAT IT HAS NOT BEEN SO OBSERVED.

s = second m = minute h = hour d = day y = year * = metastable

TABLE 1-3 (cont'd)
FISSION PRODUCT CHAINS

TABLE 1-3 (cont'd)
FISSION PRODUCT CHAINS

125	$\text{Sn}^{125}(9.6 \text{ m}) \rightarrow \text{Sb}^{125}(2.0 \text{ y}) \xrightarrow{0.25} \text{Te}^{125*}(58 \text{ d}) \xrightarrow{0.75} \text{Te}^{125}(\text{stable})$ $\text{Sn}^{125}(9.6 \text{ d}) \rightarrow \text{Sb}^{125}(2.0 \text{ y})$	$\text{I}^{138}(5.8 \text{ s}) \xrightarrow{\text{Xe}^{137}(3.9 \text{ m}) + \text{neutron}} \text{Xe}^{138}(17 \text{ m}) \rightarrow \text{Cs}^{138}(32.2 \text{ m}) \rightarrow \text{Ba}^{138}(\text{stable})$
126	$\text{Sn}^{126}(50 \text{ m}) \rightarrow \text{Sb}^{126}(9 \text{ h}) \rightarrow \text{Te}^{126}(\text{stable})$	139 $\text{I}^{139}(2.7 \text{ s}) \rightarrow \text{Xe}^{139}(41 \text{ s}) \rightarrow \text{Cs}^{139}(9.5 \text{ m}) \rightarrow \text{Ba}^{139}(84 \text{ m}) \rightarrow \text{La}^{139}(\text{stable})$
127	$\text{Sn}^{127}(1.9 \text{ h}) \rightarrow \text{Sb}^{127}(91 \text{ h}) \xrightarrow{0.22} \text{Te}^{127*}(105 \text{ d}) \xrightarrow{0.02} \text{I}^{127}(\text{stable})$ $\text{Te}^{127*}(105 \text{ d}) \xrightarrow{0.98} \text{Te}^{127}(9.3 \text{ h}) \xrightarrow{0.78} \text{Te}^{127}(\text{stable})$	140 $\text{Xe}^{140}(16 \text{ s}) \rightarrow \text{Cs}^{140}(66 \text{ s}) \rightarrow \text{Ba}^{140}(12.8 \text{ d}) \rightarrow \text{La}^{140}(40.2 \text{ h}) \rightarrow \text{Ce}^{140}(\text{stable})$
128	$\text{Sn}^{128}(57 \text{ m}) \rightarrow \text{Sb}^{128}(10.3 \text{ m}) \rightarrow \text{Te}^{128}(\text{stable})$	141 $\text{Xe}^{141}(1.7 \text{ s}) \rightarrow (\text{short Cs}) \rightarrow \text{Ba}^{141}(18 \text{ m}) \rightarrow \text{La}^{141}(3.7 \text{ h}) \rightarrow \text{Ce}^{141}(33 \text{ d}) \rightarrow \text{Pr}^{141}(\text{stable})$
129	$\text{Sb}^{129}(4.6 \text{ h}) \xrightarrow{0.35} \text{Te}^{129*}(37 \text{ d}) \xrightarrow{0.64} \text{Te}^{129}(72 \text{ m}) \rightarrow \text{I}^{129}(1.7 \times 10^7 \text{ y}) \rightarrow \text{Xe}^{129}(\text{stable})$ $\text{Te}^{129*}(37 \text{ d}) \xrightarrow{0.64} \text{Te}^{129}(72 \text{ m})$	142 $\text{Ba}^{142}(6 \text{ m}) \rightarrow \text{La}^{142}(75 \text{ m}) \rightarrow \text{Ce}^{142}(\text{stable})$
130	$\text{Sn}^{130}(2.6 \text{ m}) \rightarrow \text{Sb}^{130}(10.0 \text{ m}) \rightarrow \text{Te}^{130}(\text{stable})$	143 $\text{Xe}^{143}(1 \text{ s}) \rightarrow (\text{short Cs}) \rightarrow \text{Ba}^{143}(0.5 \text{ m}) \rightarrow \text{La}^{143}(18 \text{ m}) \rightarrow \text{Ce}^{143}(33 \text{ h}) \rightarrow \text{Pr}^{143}(3.7 \text{ d}) \rightarrow \text{Nd}^{143}(\text{stable})$
131	$\text{Sn}^{131}(3.4 \text{ m}) \rightarrow \text{Sb}^{131}(23 \text{ m}) \xrightarrow{0.15} \text{Te}^{131*}(30 \text{ h}) \xrightarrow{0.80} \text{I}^{131}(8.05 \text{ d}) \xrightarrow{0.008} \text{Xe}^{131*}(12 \text{ d}) \xrightarrow{0.992} \text{Xe}^{131}(\text{stable})$ $\text{Te}^{131*}(30 \text{ h}) \xrightarrow{0.20} \text{Te}^{131}(24 \text{ m}) \xrightarrow{0.85} \text{Te}^{131}(\text{stable})$	144 $\text{Ce}^{144}(285 \text{ d}) \rightarrow \text{Pr}^{144}(17.4 \text{ m}) \rightarrow \text{Nd}^{144}(5 \times 10^{15} \text{ y})$
132	$\text{Sn}^{132}(2.2 \text{ m}) \rightarrow \text{Sb}^{132}(2.1 \text{ m}) \rightarrow \text{Te}^{132}(77 \text{ h}) \rightarrow \text{I}^{132}(2.30 \text{ h}) \rightarrow \text{Xe}^{132}(\text{stable})$	145 $\text{Ce}^{145}(3.0 \text{ m}) \rightarrow \text{Pr}^{145}(5.95 \text{ h}) \rightarrow \text{Nd}^{145}(\text{stable})$
133	$\text{Sb}^{133}(4.1 \text{ m}) \xrightarrow{0.72} \text{Te}^{133*}(63 \text{ m}) \xrightarrow{0.28} \text{Te}^{133}(2 \text{ m}) \rightarrow \text{I}^{133}(20.8 \text{ h}) \xrightarrow{0.024} \text{Xe}^{133*}(2.3 \text{ d}) \xrightarrow{0.976} \text{Xe}^{133}(5.27 \text{ d}) \rightarrow \text{Cs}^{133}(\text{stable})$ $\text{Te}^{133*}(63 \text{ m}) \xrightarrow{0.28} \text{Te}^{133}(2 \text{ m})$	146 $\text{Ce}^{146}(13.9 \text{ m}) \rightarrow \text{Pr}^{146}(24.4 \text{ m}) \rightarrow \text{Nd}^{146}(\text{stable})$
134	$\text{Sb}^{134}(0.8 \text{ m}) \rightarrow \text{Te}^{134}(44 \text{ m}) \rightarrow \text{I}^{134}(52.5 \text{ m}) \rightarrow \text{Xe}^{134}(\text{stable})$	147 $\text{Nd}^{147}(11.1 \text{ d}) \rightarrow \text{Pm}^{147}(2.65 \text{ y}) \rightarrow \text{Sm}^{147}(1.3 \times 10^{11} \text{ y})$
135	$\text{Te}^{135}(0.5 \text{ m}) \rightarrow \text{I}^{135}(6.7 \text{ h}) \xrightarrow{0.30} \text{Xe}^{135*}(15.3 \text{ m}) \xrightarrow{0.70} \text{Xe}^{135}(9.2 \text{ h}) \rightarrow \text{Cs}^{135}(2.6 \times 10^6 \text{ y}) \rightarrow \text{Ba}^{135}(\text{stable})$ $\text{I}^{135}(6.7 \text{ h}) \xrightarrow{0.70} \text{Xe}^{135}(9.2 \text{ h})$	148 $\text{Nd}^{148}(\text{stable})$
136	$\text{I}^{136}(68 \text{ s}) \rightarrow \text{Xe}^{136}(\text{stable})$	149 $\text{Nd}^{149}(2.0 \text{ h}) \rightarrow \text{Pm}^{149}(54 \text{ h}) \rightarrow \text{Sm}^{149}(\text{stable})$
137	$\text{I}^{137}(24.0 \text{ s}) \xrightarrow{0.08} \text{Xe}^{136}(\text{stable}) + \text{neutron}$ $\text{I}^{137}(24.0 \text{ s}) \xrightarrow{0.92} \text{Xe}^{137}(3.9 \text{ m}) \rightarrow \text{Cs}^{137}(29 \text{ y}) \xrightarrow{0.32} \text{Ba}^{137*}(2.57 \text{ m}) \xrightarrow{0.68} \text{Ba}^{137}(\text{stable})$ $\text{Cs}^{137}(29 \text{ y}) \xrightarrow{0.68} \text{Ba}^{137}(\text{stable})$	150 $\text{Nd}^{150}(\text{stable})$
		151 $\text{Nd}^{151}(13 \text{ m}) \rightarrow \text{Pm}^{151}(27.5 \text{ h}) \rightarrow \text{Sm}^{151}(80 \text{ y}) \rightarrow \text{Eu}^{151}(\text{stable})$
		152 $\text{Sm}^{152}(\text{stable})$
		153 $\text{Sm}^{153}(47 \text{ h}) \rightarrow \text{Eu}^{153}(\text{stable})$
		154 $\text{Sm}^{154}(\text{stable})$
		155 $\text{Sm}^{155}(24 \text{ m}) \rightarrow \text{Eu}^{155}(1.9 \text{ y}) \rightarrow \text{Gd}^{155}(\text{stable})$
		156 $\text{Sm}^{156}(10 \text{ h}) \rightarrow \text{Eu}^{156}(15.4 \text{ d}) \rightarrow \text{Gd}^{156}(\text{stable})$
		157 $\text{Eu}^{157}(15.4 \text{ h}) \rightarrow \text{Gd}^{157}(\text{stable})$
		158 $\text{Eu}^{158}(60 \text{ m}) \rightarrow \text{Gd}^{158}(\text{stable})$
		159 $\text{Gd}^{159}(18.0 \text{ h}) \rightarrow \text{Tb}^{159}(\text{stable})$
		161 $[\text{Gd}^{161}(3.6 \text{ m})] \rightarrow \text{Tb}^{161}(6.9 \text{ d}) \rightarrow \text{Dy}^{161}(\text{stable})$

Table 1-4

TOTAL CHAIN YIELD FROM THERMAL NEUTRON FISSIONS IN U^{235}

s = second
m = minute
h = hour

d = day
y = year
* = metastable

Mass No.	Fission Product	% Yield	Ref.	Mass No.	Fission Product	% Yield	Ref.
72	Zn ⁷² (49h)	0.000016	23, 24	125	Sb ¹²⁵ (2.0y)	0.021	45
73	Ga ⁷³ (5.0h)	0.00011	24	126	Sb ¹²⁶ (9h)	0.05 ^b	46
77	As ⁷⁷ (38.7h)	0.0083	25, 26	127	Sb ¹²⁷ (91h)	0.13 ^a	40, 47
78	As ⁷⁸ (91m)	0.021	25	128	Sn ¹²⁸ (57m)	0.37	39
79	As ⁷⁹ (9.0m)	0.056	27	129	I ¹²⁹ (1.7 x 10 ⁷ y)	0.9	48
81	Se ⁸¹ (17.6m)	0.14	28	130	Sb ¹³⁰ (10m)	2.0	49
83	Kr ⁸³ (stable)	0.544	29, 30, 31	131	Xe ¹³¹ (stable)	2.93 2.88 ^b	31
84	Kr ⁸⁴ (stable)	1.00	29, 30, 31	132	Xe ¹³² (stable)	4.38 4.31 ^b	31
85	Rb ⁸⁵ (stable)	1.30	29	133	Cs ¹³³ (stable)	6.59 6.49 ^b	29
86	Kr ⁸⁶ (stable)	2.02	29, 30, 31	134	Xe ¹³⁴ (stable)	8.06 7.9 ^b	31
87	Rb ⁸⁷ (6 x 10 ¹⁰ y)	2.49	29	135	Cs ¹³⁵ (2.6 x 10 ⁶ y)	6.41 6.31 ^b	29
88	Sr ⁸⁸ (stable)	3.57 ^a	29, 32	136	Xe ¹³⁶ (stable)	6.46 6.36 ^b	31
89	Sr ⁸⁹ (51d)	4.79	33	137	Cs ¹³⁷ (29y)	6.15 6.05 ^b	31
90	Sr ⁹⁰ (28y)	5.77 ^a	29, 32	138	Ba ¹³⁸ (stable)	5.74	32
91	Zr ⁹¹ (stable)	5.84	32	139	Ba ¹³⁹ (84m)	6.55 ^a	33, 50
92	Zr ⁹² (stable)	6.03	32	140	Ce ¹⁴⁰ (stable)	6.44 ^{a, c}	29, 32
93	Zr ⁹³ (1.1 x 10 ⁶ y)	6.45	32	141	Ce ¹⁴¹ (33d)	~6.0	51
94	Zr ⁹⁴ (stable)	6.40	32	142	Ce ¹⁴² (stable)	5.95	52
95	Mo ⁹⁵ (stable)	6.27	32	143	Nd ¹⁴³ (stable)	5.98 ^a	29, 32
96	Zr ⁹⁶ (stable)	6.33	32	144	Nd ¹⁴⁴ (5 x 10 ¹⁵ y)	5.67 ^a	29, 32
97	Mo ⁹⁷ (stable)	6.09	32	145	Nd ¹⁴⁵ (stable)	3.95 ^a	29, 32
98	Mo ⁹⁸ (stable)	5.78	32	146	Nd ¹⁴⁶ (stable)	3.07 ^a	29, 32
99	Mo ⁹⁹ (66h)	6.06 ^a	33, 34	147	Sm ¹⁴⁷ (1.3 x 10 ¹¹ y)	2.38	29
100	Mo ¹⁰⁰ (stable)	6.30	32	148	Nd ¹⁴⁸ (stable)	1.70 ^a	29, 32
101	Ru ¹⁰¹ (stable)	5.0	32	149	Sm ¹⁴⁹ (stable)	1.13	29
102	Ru ¹⁰² (stable)	4.1	32	150	Nd ¹⁵⁰ (stable)	0.67 ^a	29, 32
103	Ru ¹⁰³ (39.7d)	3.0	35, 36	151	Sm ¹⁵¹ (80y)	0.45	29
104	Ru ¹⁰⁴ (stable)	1.8	32	152	Sm ¹⁵² (stable)	0.285	29
105	Ru ¹⁰⁵ (4.45h)	0.90 ^a	37, 38	153	Sm ¹⁵³ (47h)	0.15 ^a	40, 52
106	Ru ¹⁰⁶ (1.01y)	0.38	32, 35, 36	154	Sm ¹⁵⁴ (stable)	0.077	29
107	Rh ¹⁰⁷ (22m)	0.19	39	155	Sm ¹⁵⁵ (24m)	0.033	53
109	Pd ¹⁰⁹ (13.4h)	0.030	40	156	Eu ¹⁵⁶ (15.4d)	0.014 ^a	40, 52, 54
111	Ag ¹¹¹ (7.6d)	0.019	40	157	Eu ¹⁵⁷ (15.4h)	0.0078	55
112	Pd ¹¹² (21h)	0.010 ^a	40, 41	158	Eu ¹⁵⁸ (60m)	0.002	55
115	Cd ¹¹⁵ (53h) + Cd ^{115*} (43d)	0.011	42	159	Gd ¹⁵⁹ (18h)	0.00107 ^a	52, 56
117	Cd ^{117*} (3.0h)	0.011	43	161	Tb ¹⁶¹ (6.9d)	0.000076	52, 56
121	Sn ¹²¹ (27.5h)	0.015	44				

a = Average of values in references cited.

b = E. P. Steinberg, ANL, Private Communication. Based on a yield of (6.15 + 5.94)/2 = 6.05 for Cs¹³⁷.

c = Measured absolute yield is 6.32%. The number 6.44% is used to normalize other yields.

Table 1-5

DIRECT YIELD OF PRIMARY FISSION PRODUCTS FOR U^{235}

s = second

d = day

m = minute

y = year

h = hour

* = metastable

Mass No.	Fission Product	Thermal Neutrons			14-Mev Neutrons		
		% Yield	Fraction of Chain Yield	Ref.	% Yield	Fraction of Chain Yield	Ref.
78	As ⁷⁸ (91m)	0.0019	9×10^{-2}	26			
82	Br ⁸² (36h)	4×10^{-5}	1.2×10^{-4} ^a	32,57	0.003	3×10^{-3}	60
86	Rb ⁸⁶ (19d)	3×10^{-5}	1.5×10^{-5} ^a	32,58			
90	Y ⁹⁰ (6.4h)	<0.0013	$<2.3 \times 10^{-4}$	59			
91	Y ⁹¹ (58d)	<0.05	$<9 \times 10^{-3}$	59			
96	Nb ⁹⁶ (23h)	5.7×10^{-4}	9×10^{-5} ^a	32,60	0.0034	6.4×10^{-4}	60
102	Rh ¹⁰² (210d)	$<5 \times 10^{-7}$	$<1.2 \times 10^{-7}$	61			
131	Te ^{131*} (30h)	<0.15	$<5 \times 10^{-2}$	62	1.6	0.35	62
131	Te ¹³¹ (24m)	<0.09	$<3 \times 10^{-2}$	62	<1.4	<0.30	62
131	I ¹³¹ (8.05d)	<0.03	$<1 \times 10^{-2}$	62	<0.36	$<8 \times 10^{-2}$	62
132	I ¹³² (2.3h)	<0.044	$<1 \times 10^{-2}$	62	0.8	0.16	62
133	Te ¹³³ (2m)	<0.66	<0.1	62	<2.2	<0.40	62
133	I ¹³³ (20.8h)	<0.33	$<5 \times 10^{-2}$	62	<2.2	<0.40	62
133	Xe ¹³³ (5.27d)	<0.0066	$<1 \times 10^{-3}$	63			
134	I ¹³⁴ (52.5m)	0.89	0.11	62	2.5	0.43	62
135	Xe ¹³⁵ (9.2h)	0.19	0.03	63,64			
136	Xe ¹³⁶ (stable)	3.4	0.53	65			
136	Cs ¹³⁶ (13d)	0.006	9×10^{-4} ^a	32,58	0.22	4×10^{-2}	60
140	La ¹⁴⁰ (40.2h)	<0.2	$<3 \times 10^{-2}$	66			
141	La ¹⁴¹ (3.7h)	~0.1	$\sim 2 \times 10^{-2}$	67			
148	Pm ¹⁴⁸ (5.3d)	$<2 \times 10^{-4}$	$<1 \times 10^{-4}$	32			

a = Average of values in references cited.

Table 1-6

CUMULATIVE YIELDS OF VARIOUS FISSION PRODUCTS FROM THERMAL FISSION

s = second
m = minute
h = hour
d = day
y = year
* = metastable

		U ²³³		U ²³⁵		Pu ²³⁹				U ²³³		U ²³⁵		Pu ²³⁹	
Mass No.	Fission Product	% Yield	Ref.	% Yield	Ref.	% Yield ^{b,c,d}		Mass No.	Fission Product	% Yield	Ref.	% Yield	Ref.	% Yield ^{b,c,d}	
72	Zn ⁷² (49h)					0.00012		127	Sb ¹²⁷ (91h)						0.39
77	Ge ⁷⁷ (12h)	0.010	32	0.0031 ^a	25,26			127	Te ^{127*} (105d)			0.035	82		
77	As ⁷⁷ (38.7h)	0.019	32					129	Te ^{129*} (37d)			0.35	47		
78	Ge ⁷⁸ (86m)			0.020 ^a	25,26			131	Sb ¹³¹ (23m)			2.6	49		
81	Se ^{81*} (57m)			0.0084	28			131	Te ^{131*} (30h)			0.44	47		
83	Se ⁸³ (25m)			0.22	72			131	I ¹³¹ (8.05d)	2.7	32	3.1 ^a	40,83	3.8	
83	Br ⁸³ (2.4h)	0.79	32	0.51	73	0.085		131	Xe ¹³¹ (stable)	3.74	68			2.87	
83	Kr ⁸³ (stable)	1.14	68					132	Te ¹³² (77h)			4.7	47	5.2	
84	Br ⁸⁴ (31.8m)			0.90	74			132	Xe ¹³² (stable)	5.10	68			4.02	
84	Kr ⁸⁴ (stable)	1.90	68					133	Sb ¹³³ (4.1m)			4.0	47		
85	Kr ⁸⁵ (10.3y)	0.56	68	0.293 ^a	29,30,31			133	Te ^{133*} (63m)			4.9	47		
86	Kr ⁸⁶ (stable)	3.18	68					133	I ¹³³ (21h)			6.9	47	5.3	
87	Br ⁸⁷ (55s)			3.1	75			133	Xe ¹³³ (5.27d)			6.62	63	5.27	
89	Sr ⁸⁹ (51d)	6.5	32			1.9		133	Ca ¹³³ (stable)	6.18	68			5.27	
90	Sr ⁹⁰ (28y)			5.8 ^a	29,32 59,76			134	Te ¹³⁴ (44m)			6.9 ^a	47,84		
91	Sr ⁹¹ (9.7h)			5.8 ^a	32,59,76	2.4		134	Xe ¹³⁴ (stable)	6.54	68			5.69	
91	Sr ⁹¹ (2.7h)			5.3	77			135	I ¹³⁵ (6.7h)	5.1	32	6.1 ^a	47,85,86	5.8	
91	Y ⁹¹ (58d)			5.35	59	3.0		135	Ca ¹³⁵ (2.6 × 10 ⁶ y)	>4.9	68			5.53	
91	Zr ⁹¹ (stable)	6.53	69					136	I ¹³⁶ (86s)	1.7	65	3.1	65	2.1	
92	Zr ⁹² (stable)	6.70	69					136	Xe ¹³⁶ (stable)	<8.9	68			5.06	
93	Zr ⁹³ (1.1 × 10 ⁶ y)	7.10	69					137	Ca ¹³⁷ (29y)	7.16	32	6.15	29,87	5.24	
94	Y ⁹⁴ (16.5m)			5.4	78			138	Ca ¹³⁸ (32m)			5.74	32,88		
94	Zr ⁹⁴ (stable)	6.82	69					139	Ba ¹³⁹ (84m)					5.7	
95	Zr ⁹⁵ (65d)	5.9	32	6.2	79	5.9		140	Ba ¹⁴⁰ (12.8d)	6.0	32	6.44 ^e	32,33,89	5.68	
95	Mo ⁹⁵ (stable)	6.10	69					140	Ce ¹⁴⁰ (stable)	5.6	70			5.68	
96	Zr ⁹⁶ (stable)	5.60	69					141	Ce ¹⁴¹ (33d)					5.2	
97	Zr ⁹⁷ (17h)			5.9 ^a	33,79	5.7		142	Ce ¹⁴² (stable)	5.6	70			6.69	
97	Mo ⁹⁷ (stable)	5.35	69					143	Ce ¹⁴³ (33h)			5.7	29,32,90	5.4	
98	Mo ⁹⁸ (stable)	5.18	69					143	Nd ¹⁴³ (stable)	5.2	70,71			6.31	
99	Mo ⁹⁹ (66h)	4.8	32			5.9		144	Ce ¹⁴⁴ (285d)	4.1	70,71	6.0	29,32	5.28	
100	Mo ¹⁰⁰ (stable)	4.40	69					144	Nd ¹⁴⁴ (stable)	4.0	70,71			5.29	
101	Mo ¹⁰¹ (14.6m)			~5.6	38			145	Nd ¹⁴⁵ (stable)	3.0	70,71			4.24	
101	Ru ¹⁰¹ (stable)	3.00	69					146	Nd ¹⁴⁶ (stable)	2.3	70,71			3.53	
102	Mo ¹⁰² (11.5m)			4.3	38			147	Nd ¹⁴⁷ (11d)			2.7	29,91		
102	Ru ¹⁰² (stable)	2.37	69					147	Sm ¹⁴⁷ (stable)	1.71	71			2.92	
103	Ru ¹⁰³ (39.7d)	1.6	32			5.8		148	Nd ¹⁴⁸ (stable)	1.15	70,71			2.28	
104	Ru ¹⁰⁴ (stable)	0.96	69					149	Pm ¹⁴⁹ (5.6h)			1.4	29,91		
105	Rh ¹⁰⁵ (35.3h)					3.9		149	Sm ¹⁴⁹ (stable)	0.61	71			1.89	
106	Ru ¹⁰⁶ (1.01y)	0.28	32			5.0		150	Nd ¹⁵⁰ (stable)	0.48	70,71			1.38	
109	Pd ¹⁰⁹ (13.4h)	0.040	32	0.030	40	1.5		151	Sm ¹⁵¹ (80y)	0.27	71			1.17	
111	Ag ¹¹¹ (7.6d)	0.025	32			0.27		152	Sm ¹⁵² (stable)	0.17	71			0.83	
112	Pd ¹¹² (21h)	0.016	32			0.10		153	Sm ¹⁵³ (47h)	0.095	32			0.41	
115	Ag ¹¹⁵ (21m)			0.0077	42			154	Sm ¹⁵⁴ (stable)	0.037	71			0.32	
115	Cd ^{115*} (43d)	0.001	32	0.0007	42	0.003		155	Sm ¹⁵⁵ (24m)					0.22	
115	Cd ¹¹⁵ (53h)	0.019	32	0.0097	42	0.038		155	Eu ¹⁵⁵ (1.9y)			0.03	53,54		
121	Sn ¹²¹ (27.5h)	0.018	32			0.044		156	Sm ¹⁵⁶ (10h)			0.013	41,52,54		
123	Sn ¹²³ (136d)			0.0013	80			156	Eu ¹⁵⁶ (15.4h)					0.12	
125	Sn ¹²⁵ (9.6d)	0.050	32	0.013	81	0.072									

^aAverage of values in references cited.^bThe Ca, Nd, and Sm yields were measured by Wiles, et al. (93) and corrected to 5.6% yield for Ba¹⁴⁰. For some of the Ce and Nd isotopes, the ratios of yields per Ref. 94 were used.^cMost values are from Ref. 92, corrected to a Ba¹⁴⁰ yield of 1.06 × 5.32% = 5.68%.^dThe xenon isotope yields were determined by Fleming, et al. (95). These were normalized to corrected Ca yields at mass 133.^eMeasured value is 6.32%. The factor 1.06 is introduced to bring the total yield for light and heavy groups to 100% each.

1.2.5

Cumulative Yields of Various Fission Products from Fast Fissions

The cumulative fission product yields from fission spectrum neutron-induced fissions in Pu^{239} , U^{238} , and Th^{232} are summarized in Table 1-7. Table 1-8 lists the yields from 14-Mev-induced fissions in U^{235} and 8-Mev-induced fissions in Th^{232} .

Table 1-7

PERCENTAGE YIELDS FROM FISSION SPECTRUM NEUTRON FISSIONS IN Pu^{239} , U^{238} , AND Th^{232}

s = second		m = minute		h = hour		d = day		y = year		* = metastable	
Mass No.	Fission Product	Pu ^{239a}	U ^{238b}	Th ^{232c}	Mass No.	Fission Product	Pu ^{239a}	U ^{238b}	Th ^{232c}		
72	Zn ⁷² (49 h)			0.00033	111	Ag ¹¹¹ (7.6 d)		0.073	0.052		
73	Ga ⁷³ (5.0 h)			0.00045	112	Pd ¹¹² (21 h)	0.14	0.046	0.057		
77	Ge ⁷⁷ (12 h)			0.009	115	Cd ^{115*} (43 d)		0.003	0.003		
77	As ⁷⁷ (39 h)		0.0038	0.020	115	Cd ¹¹⁵ (53 h)	0.069	0.037	0.072		
83	Br ⁸³ (2.4 h)			1.9	127	Sb ¹²⁷ (93 h)		0.12			
83	Kr ⁸³ (stable)		0.40	1.99	131	I ¹³¹ (8.05 d)			1.2		
84	Kr ⁸⁴ (stable)		0.85	3.65	131	Xe ¹³¹ (stable)		3.2 ^d	1.62		
85	Kr ⁸⁵ (10.3 y)		0.153	0.87	132	Te ¹³² (77 h)		4.7	2.4		
86	Kr ⁸⁶ (stable)		1.38	6.0	132	Xe ¹³² (stable)		4.7 ^d	2.87		
89	Sr ⁸⁹ (51 d)		2.9	6.7	133	Cs ¹³³ (stable)		5.5 (8.08) ^e			
90	Sr ⁹⁰ (28 y)	2.2	3.2	6.8	134	Xe ¹³⁴ (stable)		6.6 ^d	5.38		
91	Sr ⁹¹ (9.7)			7.2	135	Cs ¹³⁵ (2.6 x 10 ⁶ y)		6.0 ^e			
95	Zr ⁹⁵ (65 d)		5.7		136	Xe ¹³⁶ (stable)		5.9 ^d	5.65		
97	Zr ⁹⁷ (17 h)	5.2		5.2	137	Cs ¹³⁷ (29 y)	6.6	6.2 (7.11) ^e	6.3		
99	Mo ⁹⁹ (67 h)	5.9	6.3	2.7	140	Ba ¹⁴⁰ (12.8 d)	5.0	5.7	6.2		
103	Ru ¹⁰³ (40 d)		6.6	0.16	141	Ce ¹⁴¹ (33 d)			9.0		
105	Rh ¹⁰⁵ (35 h)			0.07	144	Ce ¹⁴⁴ (290 d)		4.9	7.1		
106	Ru ¹⁰⁶ (1.0 y)		2.7	0.042	153	Sm ¹⁵³ (47 h)	0.48				
109	Pd ¹⁰⁹ (13.4 h)	1.9	0.32	0.055	156	Eu ¹⁵⁶ (15.4 d)		0.066			

^aValues from Ref. 96

^bMost values are averages of data in Ref. 97 and 98, normalized to absolute yield of Mo^{99} in Ref. 99.

^cMost values from Ref. 100. Kr and Xe yields measured by T. J. Kennett and H. G. Thode, and reported by private communication to S. Katcoff.

^dReference 101

^e Cs^{133} and Cs^{135} yields derived from ratios to Cs^{137} measured by R. H. Tomlinson and reported by private communication to S. Katcoff.

Table 1-8

CUMULATIVE PERCENTAGE YIELDS FROM 14-MEV NEUTRON-INDUCED FISSIONS IN U^{235} AND 8-MEV NEUTRON-INDUCED FISSIONS IN Th^{232}

s = second h = hour y = year m = minute d = day * = metastable							
Mass No.	Fission Product	Cum. % Yield in		Mass No.	Fission Product	Cum. % Yield in	
		U^{235}	Th^{232}			U^{235}	Th^{232}
77	Ge^{77} (12h)		0.022	131	I^{131} (8.05d)	4.3	
83	Br^{83} (2.4h)	1.30		132	Te^{132} (77h)	4.3	2.0
83	Kr^{83} (stable)		2.7	132	I^{132} (2.3h)	5.0	
89	Sr^{89} (51d)	4.38	6.7	133	I^{133} (20.8h)	5.4	
91	Sr^{91} (9.7h)	4.19	5.6	133	Cs^{133} (stable)	5.6	
97	Zr^{97} (17h)	4.87	5.0	134	I^{134} (52.5m)	5.3	
99	Mo^{99} (66h)	5.17	3.1	134	Xe^{134} (stable)	5.9	
103	Ru^{103} (39.9d)	3.31	0.5	135	I^{135} (6.7h)	4.5	
105	Rh^{105} (35.3h)	1.86		135	Cs^{135} (2.6×10^6 y)	5.7	
106	Ru^{106} (1.01y)	1.41	0.53	139	Ba^{139} (84m)	4.87	9.0
111	Ag^{111} (7.6d)	0.92	0.63	140	Ba^{140} (12.8d)	4.58	
115	Cd^{115} (53h)	0.89	0.76	143	Ce^{143} (33h)	3.72	
115	Total ¹¹⁵ chain	0.94		144	Ce^{144} (285d)	2.75	7.2
117	Cd^{117*} (3.0h)		0.4	156	Eu^{156} (15.4d)	0.11	
127	Sb^{127} (91h)	1.9					

Note: Most of the data by Ford and Gilmore,⁽¹⁰²⁾ and Turkevich, et al.⁽¹⁰³⁾
Iodine data by Wahl.⁽¹⁰⁴⁾ All values normalized to Mo^{99} as reported by Terrell.⁽⁹⁹⁾

where

$$y_1 = 1.01797 \sqrt{E} + 0.74328$$

$$y_2 = 1.01797 \sqrt{E} - 0.74328$$

FIG. 1-3
COMBINED RESULTS SHOWING FISSION NEUTRON SPECTRUM
OF U^{235} OVER THE ENERGY RANGE 0.18 TO 12 Mev
(REPRODUCED FROM DATA OF CRANBERG, et al¹⁰⁶)

In some applications it is convenient to express the fission spectrum $F(u)$, in terms of the lethargy, u .

For $u = \ln(10/E)$:

$$F(u) = f(10 e^{-u}) \left| \frac{dE}{du} \right|$$

and satisfies the normalization condition:

$$\int_{-\infty}^{\infty} F(u) du = 1$$

The integral of $F(u)$,

$$I_F(u) = \int_{-\infty}^u F(x) dx = \int_E^{\infty} f(x) dx = I_f(E)$$

where E is the energy value corresponding to a given u .

Table 1-10 lists the values of u , $f(E)$, and $I_f(E)$ for $0.05 \leq E \leq 20$ in intervals $\Delta E = 0.05$ for $0.05 \leq E \leq 6.0$, and in intervals $\Delta E = 1$ for $6 \leq E \leq 20$.

Table 1-11 lists the values of E , $F(u)$, and $I_F(u)$ for $-0.7 \leq u \leq 20$ in intervals $\Delta u = 0.1$.

Table 1-10
VALUES OF u , $f(E)$ and, $I_f(E)$ VS E

E	u	$f(E)$	$I_f(E)$	E	u	$f(E)$	$I_f(E)$
5.000 (-2)	5.2983	1.4824 (-1)	9.9499 (-1)	3.550	1.0356	9.8615 (-2)	1.4357 (-1)
1.000 (-1)	4.6052	2.0285	9.8611	3.600	1.0217	9.5540	1.3872
1.500 (-1)	4.1997	2.4036	9.7498	3.650	1.0079	9.2548	1.3402
2.000 (-1)	3.9120	2.6847	9.6223	3.700	9.9425 (-1)	8.9637	1.2946
2.500 (-1)	3.6889	2.9031 (-1)	9.4824 (-1)	3.750	9.8083 (-1)	8.6806 (-2)	1.2505 (-1)
3.000 (-1)	3.5066	3.0755 (-1)	9.3328 (-1)	3.800	9.6758 (-1)	8.4052 (-2)	1.2078 (-1)
3.500	3.3524	3.2121	9.1755	3.850	9.5451	8.1376	1.1664
4.000	3.2189	3.3200	9.0120	3.900	9.4161	7.8774	1.1264
4.500	3.1011	3.4041	8.8439	3.950	9.2887	7.6245	1.0877
5.000 (-1)	2.9957	3.4683 (-1)	8.6720 (-1)	4.000	9.1629 (-1)	7.3789 (-2)	1.0502 (-1)
5.500 (-1)	2.9004	3.5157 (-1)	8.4973 (-1)	4.050	9.0387 (-1)	7.1403 (-2)	1.0139 (-1)
6.000	2.8134	3.5485	8.3206	4.100	8.9160	6.9086	9.7874 (-2)
6.500	2.7334	3.5687	8.1427	4.150	8.7948	6.6836	9.4476 (-2)
7.000	2.6593	3.5780	7.9640	4.200	8.6750	6.4652	9.1189 (-2)
7.500 (-1)	2.5903	3.5778 (-1)	7.7850 (-1)	4.250	8.5567 (-1)	6.2533 (-2)	8.8010 (-2)
8.000 (-1)	2.5257	3.5692 (-1)	7.6063 (-1)	4.300	8.4397 (-1)	6.0476 (-2)	8.4935 (-2)
8.500 (-1)	2.4651	3.5534	7.4282	4.350	8.3241	5.8480	8.1962
9.000 (-1)	2.4079	3.5311	7.2511	4.400	8.2098	5.6545	7.9086
9.500 (-1)	2.3539	3.5031	7.0752	4.450	8.0968	5.4667	7.6306
1.000	2.3026	3.4703 (-1)	6.9008 (-1)	4.500	7.9851 (-1)	5.2846 (-2)	7.3619 (-2)
1.050	2.2538	3.4331 (-1)	6.7282 (-1)	4.550	7.8746 (-1)	5.1081 (-2)	7.1021 (-2)
1.100	2.2073	3.3921	6.5576	4.600	7.7653	4.9370	6.8509
1.150	2.1628	3.3478	6.3891	4.650	7.6572	4.7712	6.6083
1.200	2.1203	3.3006	6.2229	4.700	7.5502	4.6105	6.3737
1.250	2.0794	3.2510 (-1)	6.0591 (-1)	4.750	7.4444 (-1)	4.4547 (-2)	6.1471 (-2)
1.300	2.0402	3.1993 (-1)	5.8978 (-1)	4.800	7.3397 (-1)	4.3039 (-2)	5.9282 (-2)
1.350	2.0025	3.1458	5.7392	4.850	7.2361	4.1577	5.7167
1.400	1.9661	3.0908	5.5833	4.900	7.1335	4.0162	5.5123
1.450	1.9310	3.0346	5.4301	4.950	7.0320	3.8791	5.3150
1.500	1.8971	2.9773 (-1)	5.2798 (-1)	5.000	6.9315 (-1)	3.7464 (-2)	5.1244 (-2)
1.550	1.8643	2.9193 (-1)	5.1324 (-1)	5.050	6.8320 (-1)	3.6179 (-2)	4.9403 (-2)
1.600	1.8326	2.8607	4.9879	5.100	6.7334	3.4935	4.7625
1.650	1.8018	2.8017	4.8463	5.150	6.6359	3.3731	4.5908
1.700	1.7720	2.7423	4.7077	5.200	6.5393	3.2566	4.4251
1.750	1.7430	2.6829 (-1)	4.5721 (-1)	5.250	6.4436 (-1)	3.1439 (-2)	4.2651 (-2)
1.800	1.7148	2.6235 (-1)	4.4394 (-1)	5.300	6.3488 (-1)	3.0348 (-2)	4.1107 (-2)
1.850	1.6874	2.5642	4.3097	5.350	6.2549	2.9293	3.9616
1.900	1.6607	2.5052	4.1830	5.400	6.1619	2.8272	3.8177
1.950	1.6348	2.4465	4.0592	5.450	6.0697	2.7285	3.6788
2.000	1.6094	2.3882 (-1)	3.9383 (-1)	5.500	5.9784 (-1)	2.6330 (-2)	3.5448 (-2)
2.050	1.5847	2.3304 (-1)	3.8204 (-1)	5.550	5.8879 (-1)	2.5407 (-2)	3.4154 (-2)
2.100	1.5606	2.2732	3.7053	5.600	5.7982	2.4515	3.2906
2.150	1.5371	2.2165	3.5931	5.650	5.7093	2.3652	3.1702
2.200	1.5141	2.1606	3.4836	5.700	5.6212	2.2817	3.0541
2.250	1.4917	2.1054 (-1)	3.3770 (-1)	5.750	5.5339 (-1)	2.2011 (-2)	2.9420 (-2)
2.300	1.4697	2.0509 (-1)	3.2731 (-1)	5.800	5.4473 (-1)	2.1232 (-2)	2.8339 (-2)
2.350	1.4482	1.9973	3.1719	5.850	5.3614	2.0479	2.7296
2.400	1.4271	1.9445	3.0733	5.900	5.2763	1.9751	2.6291
2.450	1.4065	1.8925	2.9774	5.950	5.1919	1.9048	2.5321
2.500	1.3863	1.8415 (-1)	2.8841 (-1)	6.000	5.1083 (-1)	1.8368 (-2)	2.4386 (-2)
2.550	1.3665	1.7913 (-1)	2.7933 (-1)	7.000	3.5667 (-1)	8.7725 (-3)	1.1376 (-2)
2.600	1.3471	1.7421	2.7049	8.000	2.2314 (-1)	4.1099 (-3)	5.2211 (-3)
2.650	1.3280	1.6938	2.6190	9.000	1.0536 (-1)	1.8877 (-3)	2.3638 (-3)
2.700	1.3093	1.6464	2.5355	1.000(1)	0.000	8.5618 (-4)	1.0577 (-3)
2.750	1.2910	1.6000 (-1)	2.4544 (-1)	1.100(1)	-9.5310 (-2)	3.8368 (-4)	4.6862 (-4)
2.800	1.2730	1.5546 (-1)	2.3755 (-1)	1.200(1)	-1.8232 (-1)	1.7016 (-4)	2.0576 (-4)
2.850	1.2553	1.5101	2.2989	1.300(1)	-2.6236 (-1)	7.4777 (-5)	8.9680 (-5)
2.900	1.2379	1.4666	2.2245	1.400(1)	-3.3647 (-1)	3.2597 (-5)	3.8830 (-5)
2.950	1.2208	1.4241	2.1522	1.500(1)	-4.0547 (-1)	1.4107 (-5)	1.6690 (-5)
3.000	1.2040	1.3825 (-1)	2.0821 (-1)	1.600(1)	-4.7000 (-1)	0	0.7130 (-5)
3.050	1.1874	1.3418 (-1)	2.0140 (-1)	1.700(1)	-5.3063 (-1)	0	0
3.100	1.1712	1.3022	1.9479	1.800(1)	-5.8779 (-1)	0	0
3.150	1.1552	1.2634	1.8837	1.900(1)	-6.4185 (-1)	0	0
3.200	1.1394	1.2256	1.8215	2.000(1)	-6.9315 (-1)	0	0
3.250	1.1239	1.1887 (-1)	1.7612 (-1)				
3.300	1.1087	1.1527 (-1)	1.7026 (-1)				
3.350	1.0936	1.1177	1.6459				
3.400	1.0788	1.0835	1.5908				
3.450	1.0642	1.0502	1.5375				
3.500	1.0498	1.0177 (-1)	1.4858 (-1)				

Table 1-11

VALUES OF E, F(u) AND $I_F(u)$ VS u

u	E	F(u)	$I_F(u)$	u	E	F(u)	$I_F(u)$	u	E	F(u)	$I_F(u)$
-7.000 (-1)	2.0138 (1)	3.5090 (-6)	2.1000 (-7)	6.000	2.4788 (-2)	2.6304 (-3)	9.9823 (-1)	1.300 (1)	2.2603 (-5)	0.0000	1.0000
-6.000 (-1)	1.8221 (1)	1.6610 (-5)	1.0800 (-6)	6.100	2.2429	2.2675	9.9848	1.310	2.0452		
-5.000 (-1)	1.6487 (1)	6.6442 (-5)	4.6900 (-6)	6.200	2.0294	1.9544	9.9869	1.320	1.8506		
-4.000 (-1)	1.4918 (1)	2.2543 (-4)	1.7890 (-5)	6.300	1.8363	1.6843	9.9887	1.330	1.6745		
-3.000 (-1)	1.3499 (1)	6.6786 (-4)	5.9120 (-5)	6.400	1.6616 (-2)	1.4514 (-3)	9.9903 (-1)	1.340 (1)	1.5151 (-5)	0.0000	1.0000
-2.000 (-1)	1.2214 (1)	1.7442 (-3)	1.7237 (-4)	6.500	1.5034 (-2)	1.2505 (-3)	9.9916 (-1)	1.350 (1)	1.3710 (-5)	0.0000	1.0000
-1.000 (-1)	1.1052 (1)	4.0667 (-3)	4.4918 (-4)	6.600	1.3604	1.0773 (-3)	9.9928	1.360	1.2405 (-5)		
0.000	1.0000 (1)	8.5618 (-3)	1.0577 (-3)	6.700	1.2309	9.2805 (-4)	9.9938	1.370	1.1224 (-5)		
1.000 (-1)	9.0484	1.6445 (-2)	2.2742 (-3)	6.800	1.1138	7.9939 (-4)	9.9947	1.380	1.0156 (-5)		
2.000 (-1)	8.1873	2.9086 (-2)	4.5053 (-3)	6.900	1.0078 (-2)	6.8852 (-4)	9.9954 (-1)	1.390 (1)	9.1898 (-6)	0.0000	1.0000
3.000 (-1)	7.4082	4.7769 (-2)	8.2930 (-3)	7.000	9.1188 (-3)	5.9299 (-4)	9.9960 (-1)	1.400 (1)	8.3153 (-6)	0.0000	1.0000
4.000 (-1)	6.7032	7.3399 (-2)	1.4291 (-2)	7.100	8.2510	5.1068	9.9966	1.410	7.5240		
5.000 (-1)	6.0653	1.0624 (-1)	2.3214 (-2)	7.200	7.4659	4.3977	9.9971	1.420	6.8080		
6.000 (-1)	5.4881	1.4573 (-1)	3.5762 (-2)	7.300	6.7554	3.7869	9.9975	1.430	6.1601		
7.000 (-1)	4.9659	1.9052 (-1)	5.2538 (-2)	7.400	6.1125 (-3)	3.2608 (-4)	9.9978 (-1)	1.440 (1)	5.5739 (-6)	0.0000	1.0000
8.000 (-1)	4.4933	2.3854 (-1)	7.3974 (-2)	7.500	5.5308 (-3)	2.8077 (-4)	9.9981 (-1)	1.450 (1)	5.0435 (-6)	0.0000	1.0000
9.000 (-1)	4.0657	2.8732 (-1)	1.0027 (-1)	7.600	5.0045	2.4174	9.9984	1.460	4.5635		
1.000	3.6788	3.3426 (-1)	1.3138 (-1)	7.700	4.5283	2.0813	9.9986	1.470	4.1292		
1.100	3.3287	3.7697	1.6698	7.800	4.0973	1.7919	9.9988	1.480	3.7363		
1.200	3.0119	4.1345	2.0656	7.900	3.7074 (-3)	1.5427 (-4)	9.9990 (-1)	1.490 (1)	3.3807 (-6)	0.0000	1.0000
1.300	2.7253	4.4227	2.4942	8.000	3.3546 (-3)	1.3281 (-4)	9.9991 (-1)	1.500 (1)	3.0590 (-6)	0.0000	1.0000
1.400	2.4660	4.6264 (-1)	2.9473 (-1)	8.100	3.0354	1.1434 (-4)	9.9992	1.510	2.7679		
1.500	2.2313	4.7436 (-1)	3.4165 (-1)	8.200	2.7465	9.8430 (-5)	9.9993	1.520	2.5045		
1.600	2.0190	4.7773	3.8933	8.300	2.4852	8.4734 (-5)	9.9994	1.530	2.2662		
1.700	1.8268	4.7346	4.3695	8.400	2.2487 (-3)	7.2943 (-5)	9.9995 (-1)	1.540 (1)	2.0505 (-6)	0.0000	1.0000
1.800	1.6530	4.6252	4.8380	8.500	2.0347 (-3)	6.2791 (-5)	9.9996 (-1)	1.550 (1)	1.8554 (-6)	0.0000	1.0000
1.900	1.4957	4.4606 (-1)	5.2927 (-1)	8.600	1.8411	5.4052	9.9996	1.560	1.6788		
2.000	1.3534	4.2525 (-1)	5.7286 (-1)	8.700	1.6659	4.6528	9.9997	1.570	1.5191		
2.100	1.2246	4.0123	6.1421	8.800	1.5073	4.0051	9.9997	1.580	1.3745		
2.200	1.1080	3.7509	6.5304	8.900	1.3639 (-3)	3.4476 (-5)	9.9998 (-1)	1.590 (1)	1.2437 (-6)	0.0000	1.0000
2.300	1.0026	3.4774	6.8919	9.000	1.2314 (-3)	2.9676 (-5)	9.9998 (-1)	1.600 (1)	1.1254 (-6)	0.0000	1.0000
2.400	9.0718 (-1)	3.2000 (-1)	7.2257 (-1)	9.100	1.1167 (-3)	2.5544 (-5)	9.9998	1.610	1.0183 (-6)		
2.500	8.2085 (-1)	2.9251 (-1)	7.5320 (-1)	9.200	1.0104 (-3)	2.1988 (-5)	9.9999	1.620	9.2136 (-7)		
2.600	7.4274	2.6578	7.8110	9.300	9.1424 (-4)	1.8926 (-5)	9.9999	1.630	8.3368 (-7)		
2.700	6.7206	2.4020	8.0639	9.400	8.2724 (-4)	1.6291 (-5)	9.9999 (-1)	1.640 (1)	7.5435 (-7)	0.0000	1.0000
2.800	6.0810	2.1603	8.2919	9.500	7.4852 (-4)	1.4022 (-5)	9.9999 (-1)	1.650 (1)	6.8256 (-7)	0.0000	1.0000
2.900	5.5023 (-1)	1.9345 (-1)	8.4965 (-1)	9.600	6.7729	1.2070 (-5)	9.9999	1.660	6.1761		
3.000	4.9787 (-1)	1.7256 (-1)	8.6793 (-1)	9.700	6.1283	1.0389 (-5)	9.9999	1.670	5.5883		
3.100	4.5049	1.5338	8.8422	9.800	5.5452	8.9422 (-6)	9.9999	1.680	5.0565		
3.200	4.0762	1.3591	8.9867	9.900	5.0175 (-4)	7.6969 (-6)	9.9999 (-1)	1.690 (1)	4.5753 (-7)	0.0000	1.0000
3.300	3.6883	1.2008	9.1145	1.000 (1)	4.5400 (-4)	6.6250 (-6)	1.0000	1.700 (1)	4.1399 (-7)	0.0000	1.0000
3.400	3.3373 (-1)	1.0583 (-1)	9.2274 (-1)	1.010	4.1080	5.7024 (-6)		1.710	3.7460		
3.500	3.0197 (-1)	9.3054 (-2)	9.3267 (-1)	1.020	3.7170	0.0000		1.720	3.3895		
3.600	2.7324	8.1650	9.4139	1.030	3.3633	0.0000		1.730	3.0669		
3.700	2.4724	7.1510	9.4904	1.040 (1)	3.0432 (-4)	0.0000	1.0000	1.740 (1)	2.7751 (-7)	0.0000	1.0000
3.800	2.2371	6.2523	9.5573	1.050 (1)	2.7536 (-4)	0.0000	1.0000	1.750 (1)	2.5110 (-7)	0.0000	1.0000
3.900	2.0242 (-1)	5.4583 (-2)	9.6158 (-1)	1.060	2.4916			1.760	2.2720		
4.000	1.8316 (-1)	4.7586 (-2)	9.6668 (-1)	1.070	2.2545			1.770	2.0558		
4.100	1.6573	4.1435	9.7113	1.080	2.0400			1.780	1.8602		
4.200	1.4996	3.6039	9.7499	1.090 (1)	1.8458 (-4)	0.0000	1.0000	1.790 (1)	1.6832 (-7)	0.0000	1.0000
4.300	1.3569	3.1313	9.7836	1.100 (1)	1.6702 (-4)	0.0000	1.0000	1.800 (1)	1.5230 (-7)	0.0000	1.0000
4.400	1.2277 (-1)	2.7183 (-2)	9.8128 (-1)	1.110	1.5112			1.810	1.3781		
4.500	1.1109 (-1)	2.3578 (-2)	9.8381 (-1)	1.120	1.3674			1.820	1.2469		
4.600	1.0052 (-1)	2.0436 (-2)	9.8601	1.130	1.2373			1.830	1.1283		
4.700	9.0953 (-2)	1.7701	9.8791	1.140 (1)	1.1195 (-4)	0.0000	1.0000	1.840 (1)	1.0209 (-7)	0.0000	1.0000
4.800	8.2297 (-2)	1.5323	9.8956	1.150 (1)	1.0130 (-4)	0.0000	1.0000	1.850 (1)	9.2374 (-8)	0.0000	1.0000
4.900	7.4466 (-2)	1.3257 (-2)	9.9099 (-1)	1.160	9.1661 (-5)			1.860	8.3584		
5.000	6.7379 (-2)	1.1463 (-2)	9.9222 (-1)	1.170	8.2938 (-5)			1.870	7.5630		
5.100	6.0967	9.9083 (-3)	9.9329	1.180	7.5046 (-5)			1.880	6.8433		
5.200	5.5166	8.5608 (-3)	9.9421	1.190 (1)	6.7904 (-5)	0.0000	1.0000	1.890 (1)	6.1920 (-8)	0.0000	1.0000
5.300	4.9916	7.3938 (-3)	9.9501	1.200 (1)	6.1442 (-5)	0.0000	1.0000	1.900 (1)	5.6028 (-8)	0.0000	1.0000
5.400	4.5166 (-2)	6.3838 (-3)	9.9569 (-1)	1.210	5.5595			1.910	5.0696		
5.500	4.0868 (-2)	5.5102 (-3)	9.9629 (-1)	1.220	5.0305			1.920	4.5872		
5.600	3.6979	4.7548	9.9680	1.230	4.5517			1.930	4.1507		
5.700	3.3460	4.1019	9.9724	1.240 (1)	4.1186 (-5)	0.0000	1.0000	1.940 (1)	3.7557 (-8)	0.0000	1.0000
5.800	3.0276	3.5380	9.9762	1.250 (1)	3.7267 (-5)	0.0000	1.0000	1.950 (1)	3.3983 (-8)	0.0000	1.0000
5.900	2.7394 (-2)	3.0509 (-3)	9.9795 (-1)	1.260	3.3720			1.960	3.0749		
				1.270	3.0511			1.970	2.7823		
				1.280	2.7608			1.980	2.5175		
				1.290 (1)	2.4981 (-5)	0.0000	1.0000	1.990	2.2779		
								2.000 (1)	2.0612 (-8)	0.0000	1.0000

Where appearing in the tables, the quantity in parentheses indicates the power of 10 by which the given value should be multiplied.

With the fission spectrum given above, the average fission neutron energy is:

$$\int_0^{\infty} E f(E) dE = 1.98 \text{ Mev}$$

The evaporation model of fission has been applied with some success to calculations⁽⁸⁾ of fission spectra. These calculations and measurements⁽¹⁰⁶⁾ indicate the difference in incident energy does not affect the fission spectrum.

1.3.2 Fission Neutron Spectrum for Pu^{239}

The fission neutron spectrum for Pu^{239} was measured by Nereson⁽¹⁰⁷⁾ in 1949. The data may be fitted with an expression of the form

$$f(E) = \sqrt{2/\pi} e \exp(-E) \sinh \sqrt{2E}$$

as shown in Fig. 1-4.

FIG. 1-4
FISSION NEUTRON SPECTRUM FOR Pu^{239}
(REPRODUCED FROM DATA OF NERESON¹⁰⁷)

1.3.3 Fission Neutron Spectra for Other Fissionable Isotopes

The fission neutron spectra for U^{233} , Pu^{241} , and the spontaneous fission spectrum for Ci^{252} have been measured recently by A. B. Smith of ANL. The measurements are in the range 0.3 to 0.8 Mev. Time-of-flight techniques were employed to determine the lower energy portions of the spectra, while proton recoils in emulsions were used to study the higher energy neutrons. As a check the U^{235} spectrum was determined and agreed well with the established U^{235} spectrum. The unnormalized data for the above spectra are shown in Fig. 1-5.

FIG. 1-5
FISSION NEUTRON SPECTRA OF VARIOUS FISSIONABLE ISOTOPES
(DATA OF A.B. SMITH, ANL)

Attempts have been made to fit the spectra with a "Watt" -type expression:

$$f(E) = c \exp(-aE) \sinh \sqrt{bE}$$

The tentative values for the constants are:

U ²³³	Pu ²⁴¹
$a = 1.05 \pm 0.03$	1.0 ± 0.05
$b = 2.3 \pm 0.10$	2.2 ± 0.05
$c = 0.46534$	0.43892

For Cf²⁵² the following constants have been reported:⁽¹⁰⁸⁾

$$a = 0.88 \pm 0.05; b = 2.0 \pm 0.2; c = 0.37317$$

In order that

$$\int_0^{\infty} f(E) dE = 1$$

the value of the constant, c , in the "Watt" - type expression is given by:

$$c = \frac{2(a)^{3/2}}{\sqrt{\pi b}} \exp(-b/4a)$$

1.4 ENERGY RELEASED IN FISSION

Several types of energy release are involved in the fission process.

- (1) Kinetic energy of fission fragments $\equiv E_k$ (Mev)
- (2) Instantaneous gamma-ray energy accompanying fission $\equiv E_{\gamma I}$ (Mev)
- (3) Energy of fission neutrons $\equiv E_{fn}$ (Mev)
- (4) Beta decay energy from fission products $\equiv E_{\beta}$ (Mev)
- (5) Gamma-ray decay energy from fission products $\equiv E_{\gamma d}$ (Mev)

1.4.1 Distribution of Energy from Fission

The distribution of energy released from fissions of U^{233} , U^{235} , U^{238} , and Pu^{239} , is listed in Table 1-12.^(109,110)

Table 1-12

ENERGY RELEASE FROM FISSIONS OF U^{233} , U^{235} , U^{238} , and Pu^{239}
(in Mev)

Isotope	E_k^a		$E_{\gamma I}^b$	E_{fn}^c	E_{β}^d	$E_{\gamma d}^b$	Total
	Light Fragments	Heavy Fragments					
U^{233}	97	66	~7	5.0	~9	~7	191
U^{235}	98	67	7.8	4.9	9	7.2	194
U^{238}	95.3	67.4		5.2			
Pu^{239}	100	72	~7	5.8	~9	~7	201

^a U^{233} , U^{235} , Pu^{239} data by Stein;⁽¹⁰⁹⁾ U^{238} data by Wahl⁽¹¹⁰⁾ (average values), and includes ionization chamber correction to data by Leachman.⁽¹¹¹⁾

^b U^{235} values are averages obtained from data in TID-7004, p. 34.
 U^{233} and Pu^{239} values are estimates.

^cValues obtained by multiplying ν by the average energy (2 Mev) of fission neutrons.

^dValues for U^{235} reported by Muehlhause and Oleksa;⁽¹¹²⁾ others are estimates.

The energy distribution of fragments from fast fission of U^{235} , U^{238} , and Pu^{239} was measured by Wahl.⁽¹¹⁰⁾ There was no significant difference between the energy distribution from thermal fission and 14-Mev neutron fission of U^{235} and Pu^{239} . For U^{238} the experiment was carried out with 2.5-Mev and 14-Mev neutrons.

For purposes of estimating the power generated in a reactor per fission, note that the energy release due to capture gammas is not included in the above data.

1.4.2

Energy Release Dependence on Time After Fission

The dependence of fission product energy release as a function of time after fission is plotted in Fig. 1-6.(113)

FIG. 1-6
BETA AND GAMMA DECAY RATES AFTER
THERMAL NEUTRON - INDUCED FISSION OF U^{235}
(REPRODUCED FROM REF. 113)

1.5 DELAYED NEUTRONS1.5.1 Yields and Half-Lives of Delayed Neutron Groups

Tables 1-13 and 1-14 list the results of a series of measurements on delayed neutrons by Keepin, Wimett and Ziegler⁽¹¹⁴⁾ of LASL.

Table 1-13 gives the total absolute yield in delayed neutrons per thermal fission in U^{233} , U^{235} , and Pu^{239} . Corresponding to the decomposition of the delayed neutrons into six groups, the half-lives, T_i , decay constants, λ_i , relative abundances, a_i ,

where $\sum a_i = 1$, and absolute yields per thermal fission are given for each group. Table 1-14 gives the same information for fast neutron-induced fissions in U^{233} , U^{235} , U^{238} , Pu^{239} , Pu^{240} , and Th^{232} .

Table 1-13

DELAYED NEUTRON YIELD FROM THERMAL FISSION IN U^{233} , U^{235} , AND Pu^{239}

Isotope	Delayed Neutron/ Fission	Group Index i	Half-Life T_i	Decay Constant λ_i	Relative Abundance, a_i	Absolute Group Yield %
U^{233}	0.0066 ± 0.0003	1	55.00 ± 0.54	0.0126 ± 0.0002	0.086 ± 0.003	0.057 ± 0.003
		2	20.57 ± 0.38	0.0337 ± 0.0006	0.299 ± 0.004	0.197 ± 0.009
		3	5.00 ± 0.21	0.139 ± 0.006	0.252 ± 0.040	0.166 ± 0.027
		4	2.13 ± 0.20	0.325 ± 0.030	0.278 ± 0.020	0.184 ± 0.016
		5	0.615 ± 0.242	1.13 ± 0.40	0.051 ± 0.024	0.034 ± 0.016
		6	0.277 ± 0.047	2.50 ± 0.42	0.034 ± 0.014	0.022 ± 0.009
U^{235}	0.0158 ± 0.0005	1	55.72 ± 1.28	0.0124 ± 0.0003	0.033 ± 0.003	0.052 ± 0.005
		2	22.72 ± 0.71	0.0305 ± 0.0010	0.219 ± 0.009	0.346 ± 0.018
		3	6.22 ± 0.23	0.111 ± 0.004	0.196 ± 0.022	0.310 ± 0.036
		4	2.30 ± 0.09	0.301 ± 0.012	0.395 ± 0.011	0.624 ± 0.026
		5	0.61 ± 0.083	1.13 ± 0.15	0.115 ± 0.009	0.182 ± 0.015
		6	0.23 ± 0.025	3.00 ± 0.33	0.042 ± 0.008	0.066 ± 0.008
Pu^{239}	0.0061 ± 0.0003	1	54.28 ± 2.34	0.0128 ± 0.0005	0.035 ± 0.009	0.021 ± 0.006
		2	23.04 ± 1.67	0.0301 ± 0.0022	0.298 ± 0.035	0.182 ± 0.023
		3	5.60 ± 0.40	0.124 ± 0.009	0.211 ± 0.048	0.129 ± 0.030
		4	2.13 ± 0.24	0.325 ± 0.036	0.326 ± 0.033	0.199 ± 0.022
		5	0.618 ± 0.213	1.12 ± 0.39	0.086 ± 0.029	0.052 ± 0.018
		6	0.257 ± 0.045	2.69 ± 0.47	0.044 ± 0.016	0.027 ± 0.010

Within the limits of experimental error,⁽¹¹⁴⁾ no dependence of the total yield on the energy of the neutron causing fission was observed. However, there appear to be small differences in the relative group yields.

The value of the delayed neutron fraction, β , is equal to the total number of delayed neutrons per fission divided by ν , the total number of neutrons per fission for the given material and energy. The values for β are listed in Table 1-15.⁽¹¹⁵⁾

The above delayed neutron data supercede the previously widely used delayed neutron data of Hughes, et al.⁽¹¹⁶⁾, for U^{235} .

1.5.2 Mean Energy of Delayed Neutrons

The mean energy of the various groups of delayed neutrons are given in Table 1-16.

Table 1-14

DELAYED NEUTRON YIELD FROM FAST FISSION IN U^{233} , U^{235} , U^{238} , Pu^{239} , Pu^{240} , and Th^{232}

Isotope	Delayed Neutron/ Fission	Group Index i	Half-Life T_i	Decay Constant λ_i	Relative Abundance, a_i	Absolute Group Yield, %
U^{233}	0.0070 ± 0.0004	1	55.11 ± 1.86	0.0126 ± 0.0004	0.086 ± 0.003	0.06 ± 0.003
		2	20.74 ± 0.86	0.0334 ± 0.0014	0.274 ± 0.005	0.192 ± 0.009
		3	5.30 ± 0.19	0.131 ± 0.005	0.227 ± 0.035	0.159 ± 0.025
		4	2.29 ± 0.18	0.302 ± 0.024	0.317 ± 0.011	0.222 ± 0.012
		5	0.546 ± 0.108	1.27 ± 0.266	0.073 ± 0.014	0.051 ± 0.010
		6	0.221 ± 0.042	3.13 ± 0.675	0.023 ± 0.007	0.016 ± 0.005
U^{235}	0.0165 ± 0.0005	1	54.51 ± 0.94	0.0127 ± 0.0002	0.038 ± 0.003	0.063 ± 0.005
		2	21.84 ± 0.54	0.0317 ± 0.0008	0.213 ± 0.005	0.351 ± 0.011
		3	6.00 ± 0.17	0.115 ± 0.003	0.188 ± 0.016	0.310 ± 0.028
		4	2.23 ± 0.06	0.311 ± 0.008	0.407 ± 0.007	0.672 ± 0.023
		5	0.496 ± 0.029	1.40 ± 0.081	0.128 ± 0.008	0.211 ± 0.015
		6	0.179 ± 0.017	3.87 ± 0.369	0.026 ± 0.003	0.043 ± 0.005
U^{238}	0.0412 ± 0.0017	1	52.38 ± 1.29	0.0132 ± 0.0003	0.013 ± 0.001	0.054 ± 0.005
		2	21.58 ± 0.39	0.0321 ± 0.0006	0.137 ± 0.002	0.564 ± 0.025
		3	5.00 ± 0.19	0.139 ± 0.005	0.162 ± 0.020	0.667 ± 0.087
		4	1.93 ± 0.07	0.358 ± 0.014	0.388 ± 0.012	1.599 ± 0.081
		5	0.49 ± 0.023	1.41 ± 0.067	0.225 ± 0.013	0.927 ± 0.060
		6	0.172 ± 0.009	4.02 ± 0.214	0.075 ± 0.005	0.309 ± 0.024
Pu^{239}	0.0063 ± 0.0003	1	53.75 ± 0.95	0.0129 ± 0.0002	0.038 ± 0.003	0.024 ± 0.002
		2	22.29 ± 0.36	0.0311 ± 0.0005	0.280 ± 0.004	0.176 ± 0.009
		3	5.19 ± 0.12	0.134 ± 0.003	0.216 ± 0.018	0.136 ± 0.013
		4	2.09 ± 0.08	0.331 ± 0.012	0.328 ± 0.010	0.207 ± 0.012
		5	0.549 ± 0.049	1.26 ± 0.115	0.103 ± 0.009	0.065 ± 0.007
		6	0.216 ± 0.017	3.21 ± 0.255	0.035 ± 0.005	0.022 ± 0.003
Pu^{240}	0.0088 ± 0.0006	1	53.56 ± 1.21	0.0129 ± 0.0004	0.028 ± 0.003	0.022 ± 0.003
		2	22.14 ± 0.38	0.0313 ± 0.0005	0.273 ± 0.004	0.238 ± 0.016
		3	5.14 ± 0.42	0.135 ± 0.011	0.192 ± 0.053	0.162 ± 0.044
		4	2.08 ± 0.19	0.333 ± 0.031	0.350 ± 0.020	0.315 ± 0.027
		5	0.511 ± 0.077	1.36 ± 0.205	0.128 ± 0.018	0.119 ± 0.018
		6	0.172 ± 0.033	4.04 ± 0.782	0.029 ± 0.006	0.024 ± 0.005
Th^{232}	0.0496 ± 0.0020	1	56.03 ± 0.95	0.0124 ± 0.0002	0.034 ± 0.002	0.169 ± 0.012
		2	20.75 ± 0.66	0.0334 ± 0.0011	0.150 ± 0.005	0.744 ± 0.037
		3	5.74 ± 0.24	0.121 ± 0.005	0.155 ± 0.021	0.769 ± 0.108
		4	2.16 ± 0.08	0.321 ± 0.011	0.446 ± 0.015	2.212 ± 0.110
		5	0.571 ± 0.042	1.21 ± 0.090	0.172 ± 0.013	0.853 ± 0.073
		6	0.211 ± 0.019	3.29 ± 0.297	0.043 ± 0.006	0.213 ± 0.031

Table 1-15

DELAYED NEUTRON FRACTION (β)
FOR FAST AND THERMAL FISSIONS
IN VARIOUS ISOTOPES

Isotope	Fast Fission	Thermal Fission
U^{233}	0.0027 ± 0.0002	0.0026 ± 0.0002
U^{235}	0.0065 ± 0.0003	0.0064 ± 0.0003
U^{238}	0.0157 ± 0.0012	
Pu^{239}	0.0021 ± 0.0002	0.0021 ± 0.0002
Pu^{240}	0.0026 ± 0.0003	
Th^{232}	0.022	

Table 1-16

MEAN ENERGY OF DELAYED NEUTRONS
(Reproduced from the data of Goldstein⁽¹¹⁷⁾)

Group	Half-Life, sec	Mean Energy, kev
1	~ 55	250
2	~ 22	460
3	~ 6	405
4	~ 2	450
5	~ .6	420
6	~ .2	-

1.5.3 Relation between Reactivity and Asymptotic Period

In the space independent, one-energy group model the time behavior of the neutron density in a reactor is related to the reactivity, prompt neutron lifetime, and delayed neutron precursors, by the following set of first-order differential equations:

$$\frac{dn}{dt} = [k(1 - \beta) - 1] \frac{n}{\ell} + \sum_i \lambda_i C_i \quad (1-3)$$

$$\frac{dC_i}{dt} = \frac{k \beta_i n}{\ell} - \lambda_i C_i$$

where

- n = neutron density
- C_i = density of precursors for the i th delayed neutron group
- k = multiplication constant
- β = delayed neutron fraction
- a_i = relative yield of i th delayed neutron group $\sum a_i = 1$
- $\beta_i = \beta a_i$, delayed neutron fraction for i th delayed neutron group
- λ_i = decay constant for i th delayed neutron group
- ℓ = prompt neutron lifetime

Further notation used is:

- $\Delta k = k - 1$
- $\rho = \Delta k / k \beta$, reactivity in dollars
- T = period (e-folding time) of the neutron density

In the above equations, no account is taken of the possible variation in effectiveness of the delayed neutrons relative to prompt neutrons. Such variation is dependent on the details of the reactor and is discussed for various reactor types in the appropriate section.

An alternate, commonly used version of the kinetics equations⁽¹¹⁸⁾ is obtained from Eqs. (1-3) by replacing ℓ by ℓk . The equations then become:

$$\begin{aligned}\frac{dn}{dt} &= (\rho - 1) \beta \frac{n}{\ell} + \sum_i \lambda_i C_i \\ \frac{dC_i}{dt} &= \beta_i \frac{n}{\ell} - \lambda_i C_i\end{aligned}\quad (1-4)$$

The following discussion and numerical values are based on Eqs. (1-3).

For a constant value of k , the following relation exists between the reactivity and period of the neutron density:

$$\rho = \frac{\ell}{\beta(\ell + T)} + \frac{T}{\ell + T} \sum_i \frac{a_i}{1 + \lambda_i T}$$

The relationship between reactivity and asymptotic periods for prompt neutron lifetimes in the range $10^{-8} \text{ sec} \leq \ell \leq 10^{-3} \text{ sec}$ are presented in both tabular and graphical form. The numerical results for positive reactivities using the thermal fission delayed neutron data for U^{233} , U^{235} , and Pu^{239} are given in Tables 1-17, 1-18, and 1-19, respectively. The corresponding results for negative reactivities are given in Table 1-20. Figures 1-7 through 1-10 show the data in graphical form.

1.5.4 Reactor Transfer Function

The reactor transfer function (zero power transfer function) is a measure of the response of the neutron density to a sinusoidal time variation in reactivity.

For

$$k - 1 = \Delta k = C \sin \omega t$$

when C is small, set

$$n = n_1(t) + n_0$$

$$C_i = C_{i,1}(t) + C_{i,0}$$

If

$$n_1(t) \ll n_0$$

linearizing Eqs. (1-3), and taking the Laplace Transform, one obtains

$$\frac{N(s)}{n_0 R(s)} = \frac{1}{\beta} M(s) = \frac{1 - s \sum \frac{\beta_i}{s + \lambda_i}}{s \left(\ell + \sum \frac{\beta_i}{s + \lambda_i} \right)} \quad (1-5)$$

where

$$N(s) = \text{Laplace Transform of } n_1(t)$$

$$R(s) = \text{Laplace Transform of } \Delta k(t)$$

TABLE I-17
REACTIVITY VS ASYMPTOTIC PERIOD (T)
FOR VARIOUS NEUTRON LIFETIMES (ℓ) FOR U²³³

		REACTIVITY, dollars					
T (sec)	ℓ (sec)=10 ⁻⁸	10 ⁻⁷	10 ⁻⁶	10 ⁻⁵	10 ⁻⁴	10 ⁻³	
1.00 (-7)	3.5874 (1)						
2.00 (-7)	1.9267 (1)						
4.00 (-7)	1.0356 (1)						
6.00 (-7)	7.2888						
8.00 (-7)	5.7360						
1.00 (-6)	4.7982						
2.00 (-6)	2.9085						
4.00 (-6)	1.9566						
6.00 (-6)	1.6383	7.2888					
8.00 (-6)	1.4789	5.7360					
1.00 (-5)	1.3832	4.7982					
2.00 (-5)	1.1917	2.9085					
4.00 (-5)	1.0959	1.9566					
6.00 (-5)	1.0639	1.6383	7.2888				
8.00 (-5)	1.0479	1.4789	5.7360				
1.00 (-4)	1.0383	1.3832	4.7981				
2.00 (-4)	1.0191	1.1917	2.9085				
4.00 (-4)	1.0095	1.0958	1.9565				
6.00 (-4)	1.0062	1.0638	1.6381	7.2886			
8.00 (-4)	1.0046	1.0477	1.4787	5.7358			
1.00 (-3)	1.0036	1.0381	1.3830	4.7979			
2.00 (-3)	1.0014	1.0186	1.1912	2.9080			
4.00 (-3)	9.9985 (-1)	1.0085	1.0948	1.9555			
6.00 (-3)	9.9898 (-1)	1.0047	1.0623	1.6366	7.2871		
8.00 (-3)	9.9827 (-1)	1.0026	1.0457	1.4767	5.7338		
1.00 (-2)	9.9762 (-1)	1.0011	1.0356	1.3805	4.7954		
2.00 (-2)	9.9473 (-1)	9.9646 (-1)	1.0137	1.1863	2.9031		
4.00 (-2)	9.8939 (-1)	9.9026 (-1)	9.9889 (-1)	1.0852	1.9460		
6.00 (-2)	9.8432 (-1)	9.8490 (-1)	9.9065 (-1)	1.0482	1.6226	7.2733	
8.00 (-2)	9.7945 (-1)	9.7988 (-1)	9.8420 (-1)	1.0274	1.4583	5.7156	
1.00 (-1)	9.7475 (-1)	9.7509 (-1)	9.7855 (-1)	1.0131	1.3580	4.7731	
2.00 (-1)	9.5329 (-1)	9.5347 (-1)	9.5529 (-1)	9.7246 (-1)	1.1450	2.8620	
4.00 (-1)	9.1749 (-1)	9.1757 (-1)	9.1844 (-1)	9.2707 (-1)	1.0134	1.8743	
6.00 (-1)	8.8769 (-1)	8.8775 (-1)	8.8832 (-1)	8.9408 (-1)	9.5163 (-1)	1.5262	
8.00 (-1)	8.6186 (-1)	8.6190 (-1)	8.6234 (-1)	8.6665 (-1)	9.0982 (-1)	1.3409	
1.00	8.3893 (-1)	8.3897 (-1)	8.3931 (-1)	8.4277 (-1)	8.7730 (-1)	1.2223	
2.00	7.5104 (-1)	7.5106 (-1)	7.5123 (-1)	7.5296 (-1)	7.7023 (-1)	9.4287 (-1)	
4.00	6.4059 (-1)	6.4060 (-1)	6.4069 (-1)	6.4155 (-1)	6.5019 (-1)	7.3656 (-1)	
6.00	5.6909 (-1)	5.6909 (-1)	5.6915 (-1)	5.6973 (-1)	5.7549 (-1)	6.3308 (-1)	
8.00	5.1698 (-1)	5.1698 (-1)	5.1703 (-1)	5.1746 (-1)	5.2178 (-1)	5.6498 (-1)	
1.00 (1)	4.7642 (-1)	4.7643 (-1)	4.7646 (-1)	4.7681 (-1)	4.8027 (-1)	5.1483 (-1)	
2.00 (1)	3.5396 (-1)	3.5396 (-1)	3.5398 (-1)	3.5415 (-1)	3.5588 (-1)	3.7317 (-1)	
4.00 (1)	2.4430 (-1)	2.4430 (-1)	2.4431 (-1)	2.4440 (-1)	2.4526 (-1)	2.5391 (-1)	
6.00 (1)	1.8947 (-1)	1.8947 (-1)	1.8948 (-1)	1.8954 (-1)	1.9011 (-1)	1.9588 (-1)	
8.00 (1)	1.5558 (-1)	1.5558 (-1)	1.5559 (-1)	1.5563 (-1)	1.5606 (-1)	1.6039 (-1)	
1.00 (2)	1.3230 (-1)	1.3230 (-1)	1.3230 (-1)	1.3234 (-1)	1.3269 (-1)	1.3615 (-1)	
2.00 (2)	7.6335 (-2)	7.6335 (-2)	7.6337 (-2)	7.6354 (-2)	7.6527 (-2)	7.8257 (-2)	
4.00 (2)	4.1618 (-2)	4.1618 (-2)	4.1619 (-2)	4.1627 (-2)	4.1714 (-2)	4.2579 (-2)	
6.00 (2)	2.8645 (-2)	2.8645 (-2)	2.8646 (-2)	2.8652 (-2)	2.8709 (-2)	2.9286 (-2)	
8.00 (2)	2.1845 (-2)	2.1845 (-2)	2.1845 (-2)	2.1850 (-2)	2.1893 (-2)	2.2325 (-2)	
1.00 (3)	1.7655 (-2)	1.7656 (-2)	1.7656 (-2)	1.7659 (-2)	1.7694 (-2)	1.8040 (-2)	
2.00 (3)	9.0153 (-3)	9.0153 (-3)	9.0155 (-3)	9.0172 (-3)	9.0345 (-3)	9.2076 (-3)	
4.00 (3)	4.5567 (-3)	4.5567 (-3)	4.5568 (-3)	4.5576 (-3)	4.5663 (-3)	4.6528 (-3)	
6.00 (3)	3.0489 (-3)	3.0489 (-3)	3.0489 (-3)	3.0495 (-3)	3.0553 (-3)	3.1130 (-3)	
8.00 (3)	2.2909 (-3)	2.2909 (-3)	2.2909 (-3)	2.2913 (-3)	2.2957 (-3)	2.3389 (-3)	
1.00 (4)	1.8347 (-3)	1.8347 (-3)	1.8348 (-3)	1.8351 (-3)	1.8386 (-3)	1.8732 (-3)	

TABLE 1-18
REACTIVITY VS ASYMPTOTIC PERIOD (T)
FOR VARIOUS NEUTRON LIFETIMES (ℓ) FOR U²³⁵

T(sec)	REACTIVITY, dollars					
	ℓ (sec) 10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
1.00 (-7)	1.5114 (1)					
2.00 (-7)	8.3929					
4.00 (-7)	4.7866					
6.00 (-7)	3.5451					
8.00 (-7)	2.9167					
1.00 (-6)	2.5371					
2.00 (-6)	1.7724	8.3929				
4.00 (-6)	1.3872	4.7866				
6.00 (-6)	1.2583	3.5451				
8.00 (-6)	1.1938	2.9167				
1.00 (-5)	1.1551	2.5371				
2.00 (-5)	1.0776	1.7724	8.3928			
4.00 (-5)	1.0388	1.3871	4.7866			
6.00 (-5)	1.0258	1.2583	3.5451			
8.00 (-5)	1.0194	1.1938	2.9166			
1.00 (-4)	1.0155	1.1551	2.5371			
2.00 (-4)	1.0077	1.0775	1.7723	8.3928		
4.00 (-4)	1.0037	1.0386	1.3870	4.7864		
6.00 (-4)	1.0023	1.0256	1.2581	3.5448		
8.00 (-4)	1.0016	1.0191	1.1935	2.9163		
1.00 (-3)	1.0011	1.0151	1.1547	2.5367		
2.00 (-3)	9.9997 (-1)	1.0070	1.0768	1.7716	8.3921	
4.00 (-3)	9.9878 (-1)	1.0023	1.0372	1.3855	4.7850	
6.00 (-3)	9.9785 (-1)	1.0002	1.0235	1.2559	3.5427	
8.00 (-3)	9.9699 (-1)	9.9873 (-1)	1.0162	1.1906	2.9135	
1.00 (-2)	9.9616 (-1)	9.9756 (-1)	1.0115	1.1511	2.5332	
2.00 (-2)	9.9219 (-1)	9.9289 (-1)	9.9903 (-1)	1.0697	1.7645	8.3853
4.00 (-2)	9.8466 (-1)	9.8501 (-1)	9.8851 (-1)	1.0234	1.3718	4.7716
6.00 (-2)	9.7751 (-1)	9.7774 (-1)	9.8007 (-1)	1.0034	1.2358	3.5229
8.00 (-2)	9.7068 (-1)	9.7086 (-1)	9.7260 (-1)	9.9007 (-1)	1.1645	2.8877
1.00 (-1)	9.6413 (-1)	9.6427 (-1)	9.6567 (-1)	9.7964 (-1)	1.1192	2.5016
2.00 (-1)	9.3476 (-1)	9.3485 (-1)	9.3555 (-1)	9.4254 (-1)	1.0124	1.7075
4.00 (-1)	8.8745 (-1)	8.8748 (-1)	8.8783 (-1)	8.9133 (-1)	9.2628 (-1)	1.2749
6.00 (-1)	8.4939 (-1)	8.4941 (-1)	8.4964 (-1)	8.5197 (-1)	8.7528 (-1)	1.1060
8.00 (-1)	8.1724 (-1)	8.1725 (-1)	8.1743 (-1)	8.1916 (-1)	8.3666 (-1)	1.0113
1.00	7.8928 (-1)	7.8930 (-1)	7.8944 (-1)	7.9083 (-1)	8.0482 (-1)	9.4459 (-1)
2.00	6.8645 (-1)	6.8646 (-1)	6.8653 (-1)	6.8723 (-1)	6.9423 (-1)	7.6419 (-1)
4.00	5.6524 (-1)	5.6524 (-1)	5.6528 (-1)	5.6563 (-1)	5.6913 (-1)	6.0415 (-1)
6.00	4.9085 (-1)	4.9085 (-1)	4.9088 (-1)	4.9111 (-1)	4.9345 (-1)	5.1631 (-1)
8.00	4.3854 (-1)	4.3854 (-1)	4.3856 (-1)	4.3873 (-1)	4.4049 (-1)	4.5801 (-1)
1.00 (1)	3.9691 (-1)	3.9692 (-1)	3.9893 (-1)	3.9907 (-1)	4.0047 (-1)	4.1450 (-1)
2.00 (1)	2.8488 (-1)	2.8488 (-1)	2.8489 (-1)	2.8496 (-1)	2.8566 (-1)	2.9268 (-1)
4.00 (1)	1.8966 (-1)	1.8966 (-1)	1.8966 (-1)	1.8970 (-1)	1.9005 (-1)	1.9356 (-1)
6.00 (1)	1.4436 (-1)	1.4436 (-1)	1.4436 (-1)	1.4438 (-1)	1.4462 (-1)	1.4696 (-1)
8.00 (1)	1.1711 (-1)	1.1711 (-1)	1.1711 (-1)	1.1713 (-1)	1.1731 (-1)	1.1906 (-1)
1.00 (2)	9.8734 (-2)	9.8734 (-2)	9.8736 (-2)	9.8750 (-2)	9.8890 (-2)	1.0030 (-1)
2.00 (2)	5.5734 (-2)	5.5734 (-2)	5.5734 (-2)	5.5741 (-2)	5.5812 (-2)	5.6515 (-2)
4.00 (2)	2.9945 (-2)	2.9946 (-2)	2.9946 (-2)	2.9949 (-2)	2.9985 (-2)	3.0336 (-2)
6.00 (2)	2.0495 (-2)	2.0495 (-2)	2.0495 (-2)	2.0497 (-2)	2.0521 (-2)	2.0755 (-2)
8.00 (2)	1.5582 (-2)	1.5582 (-2)	1.5582 (-2)	1.5584 (-2)	1.5601 (-2)	1.5777 (-2)
1.00 (3)	1.2570 (-2)	1.2570 (-2)	1.2570 (-2)	1.2572 (-2)	1.2586 (-2)	1.2726 (-2)
2.00 (3)	6.3935 (-3)	6.3935 (-3)	6.3936 (-3)	6.3943 (-3)	6.4013 (-3)	6.4716 (-3)
4.00 (3)	3.2249 (-3)	3.2249 (-3)	3.2250 (-3)	3.2253 (-3)	3.2286 (-3)	3.2640 (-3)
6.00 (3)	2.1563 (-3)	2.1563 (-3)	2.1563 (-3)	2.1566 (-3)	2.1589 (-3)	2.1824 (-3)
8.00 (3)	1.6196 (-3)	1.6196 (-3)	1.6197 (-3)	1.6198 (-3)	1.6216 (-3)	1.6392 (-3)
1.00 (4)	1.2969 (-3)	1.2969 (-3)	1.2969 (-3)	1.2970 (-3)	1.2984 (-3)	1.3125 (-3)

TABLE I-19
REACTIVITY VS ASYMPTOTIC PERIOD (T)
FOR VARIOUS NEUTRON LIFETIMES (ℓ) FOR Pu^{239}

REACTIVITY, dollars						
T (sec)	ℓ (sec) = 10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
1.00 (-7)	4.4199 (1)					
2.00 (-7)	2.3628 (1)					
4.00 (-7)	1.2590 (1)					
6.00 (-7)	8.7900					
8.00 (-7)	6.8665					
1.00 (-6)	5.7049					
2.00 (-6)	3.3641					
4.00 (-6)	2.1850					
6.00 (-6)	1.7907	8.7900				
8.00 (-6)	1.5932	6.8665				
1.00 (-5)	1.4747	5.7049				
2.00 (-5)	1.2375	3.3641				
4.00 (-5)	1.1188	2.1850				
6.00 (-5)	1.0792	1.7906	8.7900			
8.00 (-5)	1.0594	1.5932	6.8665			
1.00 (-4)	1.0475	1.4747	5.7048			
2.00 (-4)	1.0237	1.2374	3.3641			
4.00 (-4)	1.0117	1.1186	2.1849			
6.00 (-4)	1.0077	1.0790	1.7905	8.7898		
8.00 (-4)	1.0057	1.0591	1.5930	6.8663		
1.00 (-3)	1.0044	1.0472	1.4744	5.7045		
2.00 (-3)	1.0017	1.0230	1.2368	3.3634		
4.00 (-3)	9.9977 (-1)	1.0105	1.1173	2.1836		
6.00 (-3)	9.9867 (-1)	1.0058	1.0771	1.7885	8.7879	
8.00 (-3)	9.9777 (-1)	1.0031	1.0566	1.5904	6.8638	
1.00 (-2)	9.9695 (-1)	1.0012	1.0440	1.4712	5.7014	
2.00 (-2)	9.9328 (-1)	9.9542 (-1)	1.0168	1.2305	3.3572	
4.00 (-2)	9.8654 (-1)	9.8761 (-1)	9.9830 (-1)	1.1052	2.1715	
6.00 (-2)	9.8017 (-1)	9.8088 (-1)	9.8801 (-1)	1.0593	1.7708	8.7704
8.00 (-2)	9.7408 (-1)	9.7461 (-1)	9.7996 (-1)	1.0334	1.5673	6.8409
1.00 (-1)	9.6824 (-1)	9.6866 (-1)	9.7294 (-1)	1.0157	1.4429	5.6734
2.00 (-1)	9.4496 (-1)	9.4421 (-1)	9.4431 (-1)	9.6570 (-1)	1.1794	3.3064
4.00 (-1)	8.9932 (-1)	8.9943 (-1)	9.0050 (-1)	9.1119 (-1)	1.0181	2.0846
6.00 (-1)	8.6483 (-1)	8.6490 (-1)	8.6561 (-1)	8.7274 (-1)	9.4403 (-1)	1.6557
8.00 (-1)	8.3556 (-1)	8.3562 (-1)	8.3615 (-1)	8.4450 (-1)	8.9497 (-1)	1.4290
1.00	8.1002 (-1)	8.1006 (-1)	8.1049 (-1)	8.1477 (-1)	8.5755 (-1)	1.2849
2.00	7.1522 (-1)	7.1525 (-1)	7.1546 (-1)	7.1760 (-1)	7.3899 (-1)	9.5284 (-1)
4.00	6.0146 (-1)	6.0148 (-1)	6.0158 (-1)	6.0265 (-1)	6.1335 (-1)	7.2033 (-1)
6.00	5.3013 (-1)	5.3014 (-1)	5.3021 (-1)	5.3092 (-1)	5.3806 (-1)	6.0939 (-1)
8.00	4.7902 (-1)	4.7902 (-1)	4.7907 (-1)	4.7961 (-1)	4.8496 (-1)	5.3847 (-1)
1.00 (1)	4.3966 (-1)	4.3966 (-1)	4.3971 (-1)	4.4013 (-1)	4.4442 (-1)	4.8723 (-1)
2.00 (1)	3.2254 (-1)	3.2254 (-1)	3.2257 (-1)	3.2278 (-1)	3.2492 (-1)	3.4633 (-1)
4.00 (1)	2.1942 (-1)	2.1942 (-1)	2.1943 (-1)	2.1954 (-1)	2.2061 (-1)	2.3132 (-1)
6.00 (1)	1.6851 (-1)	1.6852 (-1)	1.6852 (-1)	1.6859 (-1)	1.6931 (-1)	1.7645 (-1)
8.00 (1)	1.3736 (-1)	1.3736 (-1)	1.3736 (-1)	1.3742 (-1)	1.3795 (-1)	1.4331 (-1)
1.00 (2)	1.1614 (-1)	1.1614 (-1)	1.1614 (-1)	1.1618 (-1)	1.1661 (-1)	1.2090 (-1)
2.00 (2)	6.5911 (-2)	6.5911 (-2)	6.5913 (-2)	6.5935 (-2)	6.6149 (-2)	6.8291 (-2)
4.00 (2)	3.5492 (-2)	3.5492 (-2)	3.5493 (-2)	3.5503 (-2)	3.5611 (-2)	3.6682 (-2)
6.00 (2)	2.4305 (-2)	2.4305 (-2)	2.4305 (-2)	2.4312 (-2)	2.4384 (-2)	2.5098 (-2)
8.00 (2)	1.8483 (-2)	1.8483 (-2)	1.8484 (-2)	1.8489 (-2)	1.8543 (-2)	1.9078 (-2)
1.00 (3)	1.4913 (-2)	1.4913 (-2)	1.4913 (-2)	1.4917 (-2)	1.4960 (-2)	1.5389 (-2)
2.00 (3)	7.5867 (-3)	7.5867 (-3)	7.5870 (-3)	7.5891 (-3)	7.6105 (-3)	7.8248 (-3)
4.00 (3)	3.8272 (-3)	3.8272 (-3)	3.8273 (-3)	3.8283 (-3)	3.8391 (-3)	3.9462 (-3)
6.00 (3)	2.5591 (-3)	2.5591 (-3)	2.5591 (-3)	2.5599 (-3)	2.5670 (-3)	2.6384 (-3)
8.00 (3)	1.9222 (-3)	1.9222 (-3)	1.9222 (-3)	1.9228 (-3)	1.9281 (-3)	1.9817 (-3)
1.00 (4)	1.5391 (-3)	1.5391 (-3)	1.5392 (-3)	1.5396 (-3)	1.5439 (-3)	1.5867 (-3)

The quantity on the right hand side of Eq. (1-5) is known as the reactor transfer function. Substituting $s = j\omega$, the response of the system to a sinusoidal input is obtained.

Table 1-20

NEGATIVE REACTIVITY VS. NEGATIVE ASYMPTOTIC PERIOD (T)
FOR U^{233} , U^{235} AND Pu^{239}

Reactivity, dollars				Reactivity, dollars			
T, sec	U^{233}	U^{235}	Pu^{239}	T, sec	U^{233}	U^{235}	Pu^{239}
-7.86 (1)			-9.66	-9.60 (1)	-5.74 (-1)	-3.19 (-1)	-3.44 (-1)
-7.87			-7.26	-9.80 (1)	-5.26	-2.95	-3.23
-7.88			-5.83	-1.00 (2)	-4.86	-2.76	-3.05
-7.89			-4.89	-1.10 (2)	-3.59	-2.13	-2.43
-7.90 (1)			-4.22	-1.20 (2)	-2.90 (-1)	-1.77 (-1)	-2.05 (-1)
-7.91 (1)			-3.72	-1.30 (2)	-2.45 (-1)	-1.53 (-1)	-1.78 (-1)
-7.92			-3.33	-1.40	-2.13	-1.35 (-1)	-1.58
-7.93			-3.02	-1.50	-1.89	-1.22 (-1)	-1.43
-7.94			-2.76	-1.75	-1.49	-9.74 (-2)	-1.15 (-1)
-7.95 (1)			-2.55	-2.00 (2)	-1.23 (-1)	-8.16 (-2)	-9.64 (-2)
-7.96 (1)			-2.37	-2.25 (2)	-1.05 (-1)	-7.03 (-2)	-8.31 (-2)
-7.97			-2.22	-2.50	-9.15 (-2)	-6.18	-7.31
-7.98			-2.09	-2.75	-8.12	-5.51	-6.53
-7.99			-1.97	-3.00	-7.30	-4.98	-5.90
-8.00 (1)			-1.87	-3.50 (2)	-6.09 (-2)	-4.17 (-2)	-4.95 (-2)
-8.01 (1)	-9.50		-1.78	-4.00 (2)	-5.22 (-2)	-3.59 (-2)	-4.26 (-2)
-8.02	-8.39		-1.70	-4.50	-4.57	-3.16	-3.75
-8.03	-7.51		-1.62	-5.00	-4.06	-2.81	-3.34
-8.04	-6.81		-1.56	-6.00	-3.33	-2.31	-2.74
-8.05 (1)	-6.23		-1.50	-7.50 (2)	-2.62 (-2)	-1.83 (-2)	-2.17 (-2)
-8.06 (1)	-5.74		-1.44	-1.00 (3)	-1.93 (-2)	-1.35 (-2)	-1.60 (-2)
-8.07	-5.32	-8.64	-1.39	-2.00	-9.43 (-3)	-6.63 (-3)	-7.87 (-3)
-8.08	-4.97	-6.60	-1.35	-3.00	-6.24	-4.39	-5.21
-8.09	-4.66	-5.35	-1.30	-4.00	-4.66	-3.28	-3.90
-8.10 (1)	-4.38	-4.51	-1.26	-5.00 (3)	-3.72 (-3)	-2.62 (-3)	-3.11 (-3)
-8.20 (1)	-2.80	-1.83	-9.83 (-1)	-6.00 (3)	-3.09 (-3)	-2.18 (-3)	-2.59 (-3)
-8.30	-2.08	-1.20	-8.20	-7.00	-2.65	-1.87	-2.22
-8.40	-1.67	-9.15 (-1)	-7.13	-8.00	-2.32	-1.63	-1.94
-8.50	-1.41	-7.53 (-1)	-6.37	-9.00	-2.06	-1.45	-1.72
-8.60 (1)	-1.22	-6.47 (-1)	-5.79 (-1)	-1.00 (4)	-1.85 (-3)	-1.31 (-3)	-1.55 (-3)
-8.70 (1)	-1.08	-5.73 (-1)	-5.34 (-1)				
-8.80	-9.76 (-1)	-5.17	-4.98				
-9.00	-8.21	-4.39	-4.42				
-9.20	-7.14	-3.87	-4.01				
-9.40 (1)	-6.35 (-1)	-3.48 (-1)	-3.70 (-1)

Fig. 1-7. Reactivity vs asymptotic period (T) for various neutron lifetimes (ℓ) for U^{235} .

Fig. 1-8. Reactivity vs asymptotic period (T) for various neutron lifetimes (ℓ) for U^{235} .

Fig. 1-9. Reactivity vs asymptotic period (T) for various neutron lifetimes (ℓ) for Pu^{239} .

FIG. 1-10
NEGATIVE REACTIVITY VS NEGATIVE
ASYMPTOTIC PERIOD (T) FOR U²³³, U²³⁵, AND Pu²³⁹

Tables 1-21, 1-22, and 1-23, give, for U²³³, U²³⁵ and Pu²³⁹, respectively, the magnitude and phase angle of the reactor transfer function for the frequency range $10^{-3} \leq \omega \leq 10^{-6}$ radians per second, for prompt neutron lifetimes in the range $10^{-8} \text{ sec} \leq \ell \leq 10^{-3} \text{ sec}$. Figures 1-11 through 1-16 give the corresponding data in graphical form.

1.5.5 Delayed Photoneutrons

The yield of photoneutrons for U²³⁵ fission products has been measured in D₂O by Bernstein, Preston, Wolfe and Slattery⁽¹¹⁹⁾ and in Be by Bernstein, Talbot, Leslie and Stanford.⁽¹²⁰⁾ Implicit in both of these measurements are the details of yield and half-life of delayed neutrons from U²³⁵ fissions. The delayed neutron data for U²³⁵ of Hughes, Dabbs, Cahn and Hall⁽¹¹⁶⁾ were used in the above experiments.

For the D₂O measurement it was possible to resolve the data into eight groups of photoneutrons.⁽¹¹⁹⁾ To each of these groups there corresponds a half-life T_j , and a yield relative to the total yield of delayed neutrons. These are listed in Table 1-24.

The experimental data for the yield of photoneutrons from U²³⁵ fission products in Be could not be resolved into sums of exponentials⁽¹²⁰⁾ as was the case in the D₂O data.

TABLE I-21
MAGNITUDE (IMI) AND PHASE ANGLE (θ)
OF ZERO POWER TRANSFER FUNCTION FOR U^{233}

ω rad/sec	$L(\text{sec}) = 10^{-3}$		10^{-4}		10^{-5}		10^{-6}		10^{-7}		10^{-8}	
	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI	θ
(1/10) (-2)	5.3247 (1)	-87.509	5.4246 (1)	-87.462	5.4347 (1)	-87.457	5.4358 (1)	-87.457	5.4359 (1)	-87.457	5.4359 (1)	-87.457
(1/8)	4.2639 (1)	-86.890	4.3439 (1)	-86.832	4.3521 (1)	-86.826	4.3529 (1)	-86.825	4.3530 (1)	-86.825	4.3530 (1)	-86.825
(1/6)	3.2046 (1)	-85.866	3.2648 (1)	-85.788	3.2710 (1)	-85.780	3.2716 (1)	-85.779	3.2716 (1)	-85.779	3.2716 (1)	-85.779
(1/4)	2.1490 (1)	-83.848	2.1895 (1)	-83.731	2.1936 (1)	-83.719	2.1940 (1)	-83.718	2.1941 (1)	-83.718	2.1941 (1)	-83.718
(1/2)	1.1064 (1)	-78.179	1.1276 (1)	-77.949	1.1297 (1)	-77.926	1.1299 (1)	-77.924	1.1300 (1)	-77.923	1.1300 (1)	-77.923
(1/10) (-1)	6.0465	-69.117	6.1670	-68.681	6.1792	-68.636	6.1805	-68.632	6.1806	-68.631	6.1806	-68.631
(1/8)	5.0803	-65.636	5.1838	-65.105	5.1944	-65.051	5.1955	-65.045	5.1956	-65.045	5.1956	-65.045
(1/6)	4.1272	-60.927	4.2146	-60.249	4.2235	-60.180	4.2244	-60.173	4.2245	-60.172	4.2245	-60.172
(1/4)	3.1852	-54.067	3.2575	-53.118	3.2648	-53.020	3.2656	-53.010	3.2656	-53.010	3.2656	-53.009
(1/2)	2.2588	-43.327	2.3201	-41.461	2.3263	-41.178	2.3269	-41.161	2.3270	-41.159	2.3270	-41.159
(1/10) (0)	1.7312	-36.534	1.7930	-33.670	1.7992	-33.373	1.7998	-33.343	1.7998	-33.340	1.7998	-33.339
(1/8)	1.5965	-34.899	1.6593	-31.516	1.6655	-31.164	1.6662	-31.128	1.6662	-31.125	1.6662	-31.124
(1/6)	1.4444	-32.897	1.5089	-28.696	1.5152	-28.256	1.5158	-28.212	1.5159	-28.208	1.5159	-28.207
(1/4)	1.2717	-30.291	1.3397	-24.533	1.3461	-23.924	1.3468	-23.863	1.3468	-23.857	1.3468	-23.856
(1/2)	1.0740	-27.741	1.1571	-17.505	1.1639	-16.402	1.1646	-16.291	1.1647	-16.280	1.1647	-16.278
(1/10) (1)	9.3844 (-1)	-31.110	1.0696	-12.578	1.0776	-10.486	1.0783	-10.276	1.0784	-10.254	1.0784	-10.252
(1/8)	8.9140 (-1)	-33.912	1.0528	-11.660	1.0605	-9.078	1.0612	-8.817	1.0613	-8.791	1.0613	-8.789
(1/6)	8.2072 (-1)	-38.732	1.0327	-10.954	1.0427	-7.560	1.0435	-7.217	1.0436	-7.182	1.0436	-7.179
(1/4)	6.9757 (-1)	-47.358	1.0110	-10.894	1.0241	-5.889	1.0250	-5.382	1.0251	-5.331	1.0251	-5.326
(1/2)	4.5157 (-1)	-63.398	9.7882 (-1)	-13.867	1.0056	-4.113	1.0067	-3.115	1.0068	-3.015	1.0068	-3.005
(1/10) (2)	2.4937 (-1)	-75.570	9.2459 (-1)	-22.453	9.9823 (-1)	-3.779	9.9991 (-1)	-1.795	1.0000	-1.596	1.0000	-1.576
(1/8)	2.0224 (-1)	-78.328	8.9249 (-1)	-26.741	9.9689 (-1)	-4.021	9.9899 (-1)	-1.543	9.9909 (-1)	-1.295	9.9910 (-1)	-1.270
(1/6)	1.5335 (-1)	-81.168	8.3399 (-1)	-33.362	9.9526 (-1)	-4.622	9.9824 (-1)	-1.323	9.9836 (-1)	-0.992	9.9837 (-1)	-0.959
(1/4)	1.0306 (-1)	-84.074	7.2516 (-1)	-44.222	9.9220 (-1)	-6.128	9.9768 (-1)	-1.190	9.9782 (-1)	-0.695	9.9783 (-1)	-0.645
(1/2)	5.1780 (-2)	-87.025	4.5912 (-1)	-62.597	9.7854 (-1)	-11.196	9.9722 (-1)	-1.422	9.9750 (-1)	-0.431	9.9751 (-1)	-0.331
(1/10) (3)	2.5922 (-2)	-88.511	2.5081 (-1)	-75.437	9.3007 (-1)	-21.178	9.9658 (-1)	-2.363	9.9741 (-1)	-0.381	9.9743 (-1)	-0.182
(1/8)	2.0740 (-2)	-88.809	2.0302 (-1)	-78.256	8.9814 (-1)	-25.781	9.9616 (-1)	-2.880	9.9740 (-1)	-0.404	9.9742 (-1)	-0.156
(1/6)	1.5557 (-2)	-89.107	1.5370 (-1)	-81.136	8.3906 (-1)	-32.729	9.9526 (-1)	-3.764	9.9738 (-1)	-0.463	9.9741 (-1)	-0.133
(1/4)	1.0372 (-2)	-89.404	1.0316 (-1)	-84.063	7.1856 (-1)	-43.910	9.9272 (-1)	-5.556	9.9735 (-1)	-0.615	9.9740 (-1)	-0.119
(1/2)	5.1864 (-3)	-89.702	5.1793 (-2)	-87.024	4.6005 (-1)	-62.533	9.7935 (-1)	-10.916	9.9721 (-1)	-1.134	9.9740 (-1)	-0.142
(1/10) (4)	2.5932 (-3)	-89.851	2.5923 (-2)	-88.511	2.5096 (-1)	-75.427	9.3083 (-1)	-21.051	9.9665 (-1)	-2.219	9.9739 (-1)	-0.236
(1/8)	2.0746 (-3)	-89.881	2.0741 (-2)	-88.808	2.0310 (-1)	-78.251	8.9883 (-1)	-25.687	9.9624 (-1)	-2.765	9.9739 (-1)	-0.288
(1/6)	1.5559 (-3)	-89.911	1.5558 (-2)	-89.106	1.5373 (-1)	-81.134	8.3963 (-1)	-32.668	9.9535 (-1)	-3.677	9.9738 (-1)	-0.377
(1/4)	1.0373 (-3)	-89.940	1.0372 (-2)	-89.404	1.0317 (-1)	-84.063	7.1892 (-1)	-43.880	9.9281 (-1)	-5.499	9.9735 (-1)	-0.557
(1/2)	5.1865 (-4)	-89.970	5.1864 (-3)	-89.702	5.1795 (-2)	-87.023	4.6014 (-1)	-62.526	9.7944 (-1)	-10.889	9.9721 (-1)	-1.105
(1/10) (5)	2.5932 (-4)	-89.985	2.5932 (-3)	-89.851	2.5924 (-2)	-88.511	2.5098 (-1)	-75.426	9.3091 (-1)	-21.039	9.9666 (-1)	-2.204
(1/8)	2.0746 (-4)	-89.988	2.0746 (-3)	-89.881	2.0741 (-2)	-88.808	2.0311 (-1)	-78.250	8.9890 (-1)	-25.678	9.9625 (-1)	-2.754
(1/6)	1.5559 (-4)	-89.991	1.5559 (-3)	-89.911	1.5558 (-2)	-89.106	1.5373 (-1)	-81.133	8.3968 (-1)	-32.662	9.9536 (-1)	-3.669
(1/4)	1.0373 (-4)	-89.994	1.0373 (-3)	-89.940	1.0372 (-2)	-89.404	1.0317 (-1)	-84.063	7.1895 (-1)	-43.877	9.9282 (-1)	-5.493
(1/2)	5.1865 (-5)	-89.997	5.1865 (-4)	-89.970	5.1864 (-3)	-89.702	5.1795 (-2)	-87.023	4.6015 (-1)	-62.526	9.7945 (-1)	-10.886
(1/10) (6)	2.5932 (-5)	-89.999	2.5932 (-4)	-89.985	2.5932 (-3)	-89.851	2.5924 (-2)	-88.511	2.5098 (-1)	-75.426	9.3092 (-1)	-21.038
(1/8)	2.0746 (-5)	-89.999	2.0746 (-4)	-89.988	2.0746 (-3)	-89.881	2.0741 (-2)	-88.808	2.0311 (-1)	-78.250	8.9891 (-1)	-25.677
(1/6)	1.5559 (-5)	-89.999	1.5559 (-4)	-89.991	1.5559 (-3)	-89.911	1.5558 (-2)	-89.106	1.5373 (-1)	-81.133	8.3969 (-1)	-32.661
(1/4)	1.0373 (-5)	-89.999	1.0373 (-4)	-89.994	1.0373 (-3)	-89.940	1.0372 (-2)	-89.404	1.0317 (-1)	-84.063	7.1896 (-1)	-43.877
(1/2)	5.1865 (-6)	-90.000	5.1865 (-5)	-89.997	5.1865 (-4)	-89.970	5.1864 (-3)	-89.702	5.1795 (-2)	-87.023	4.6015 (-1)	-62.526
(1/10) (7)	2.5932 (-6)	-90.000	2.5932 (-5)	-89.999	2.5932 (-4)	-89.985	2.5932 (-3)	-89.851	2.5924 (-2)	-88.511	2.5098 (-1)	-75.426

TABLE I-22
MAGNITUDE (IMI) AND PHASE ANGLE (θ)
OF ZERO POWER TRANSFER FUNCTION FOR U^{235}

L (sec) = ω (rad/sec)	10^{-3}		10^{-4}		10^{-5}		10^{-6}		10^{-7}		10^{-8}	
	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI	θ
(1/10) (-2)	7.6017 (1)	-87.970	7.6837 (1)	-87.948	7.6921 (1)	-87.946	7.6929 (1)	-87.945	7.6930 (1)	-87.945	7.6930 (1)	-87.945
(1/8)	6.0856 (1)	-87.465	6.1513 (1)	-87.438	6.1580 (1)	-87.435	6.1587 (1)	-87.435	6.1587 (1)	-87.435	6.1587 (1)	-87.435
(1/6)	4.5711 (1)	-86.628	4.6205 (1)	-86.592	4.6255 (1)	-86.588	4.6260 (1)	-86.588	4.6261 (1)	-86.588	4.6261 (1)	-86.588
(1/4)	3.0603 (1)	-84.976	3.0935 (1)	-84.922	3.0969 (1)	-84.916	3.0972 (1)	-84.916	3.0972 (1)	-84.916	3.0972 (1)	-84.916
(1/2)	1.5630 (1)	-80.284	1.5801 (1)	-80.176	1.5818 (1)	-80.165	1.5820 (1)	-80.164	1.5820 (1)	-80.164	1.5820 (1)	-80.164
(1/10) (-1)	8.3462	-72.448	8.4406	-72.242	8.4501	-72.221	8.4511	-72.219	8.4512	-72.219	8.4512	-72.219
(1/8)	6.9336	-69.255	7.0134	-69.004	7.0215	-68.979	7.0223	-68.976	7.0224	-68.976	7.0224	-68.976
(1/6)	5.5458	-64.751	5.6117	-64.429	5.6184	-64.396	5.6190	-64.393	5.6191	-64.392	5.6191	-64.392
(1/4)	4.1958	-57.965	4.2488	-57.509	4.2542	-57.463	4.2547	-57.459	4.2548	-57.458	4.2548	-57.458
(1/2)	2.8793	-47.584	2.9227	-46.787	2.9271	-46.706	2.9275	-46.698	2.9276	-46.697	2.9276	-46.697
(1/10)	2.1053	-40.601	2.1461	-39.283	2.1502	-39.148	2.1507	-39.135	2.1507	-39.133	2.1507	-39.133
(1/8)	1.9130	-38.619	1.9533	-37.075	1.9573	-36.918	1.9578	-36.902	1.9578	-36.900	1.9578	-36.900
(1/6)	1.6944	-36.013	1.7393	-34.115	1.7432	-33.920	1.7436	-33.901	1.7437	-33.899	1.7437	-33.899
(1/4)	1.4598	-32.113	1.4992	-29.542	1.5031	-29.277	1.5035	-29.251	1.5035	-29.248	1.5035	-29.248
(1/2)	1.2011	-25.677	1.2429	-21.135	1.2467	-20.664	1.2471	-20.616	1.2472	-20.612	1.2472	-20.611
(1/10) (1)	1.0631	-22.952	1.1171	-14.583	1.1214	-13.704	1.1218	-13.615	1.1219	-13.607	1.1219	-13.606
(1/8)	1.0285	-23.241	1.0902	-13.035	1.0948	-11.954	1.0952	-11.846	1.0953	-11.835	1.0953	-11.834
(1/6)	9.8558 (-1)	-24.552	1.0620	-11.383	1.0669	-9.969	1.0674	-9.827	1.0674	-9.813	1.0674	-9.812
(1/4)	9.2105 (-1)	-28.434	1.0321	-9.700	1.0378	-7.627	1.0383	-7.418	1.0383	-7.397	1.0383	-7.395
(1/2)	7.6367 (-1)	-41.056	1.0005	-8.775	1.0089	-4.729	1.0095	-4.321	1.0095	-4.280	1.0095	-4.276
(1/10) (2)	5.2699 (-1)	-58.176	9.8021 (-1)	-11.137	9.9746 (-1)	-3.175	9.9814 (-1)	-2.367	9.9819 (-1)	-2.286	9.9820 (-1)	-2.278
(1/8)	4.4738 (-1)	-63.353	9.7219 (-1)	-12.849	9.9579 (-1)	-2.959	9.9654 (-1)	-1.950	9.9660 (-1)	-1.849	9.9660 (-1)	-1.839
(1/6)	3.5343 (-1)	-69.216	9.5747 (-1)	-15.918	9.9435 (-1)	-2.881	9.9525 (-1)	-1.538	9.9531 (-1)	-1.403	9.9532 (-1)	-1.390
(1/4)	2.4518 (-1)	-75.716	9.2109 (-1)	-22.158	9.9299 (-1)	-3.168	9.9430 (-1)	-1.155	9.9437 (-1)	-0.953	9.9437 (-1)	-0.933
(1/2)	1.2602 (-1)	-82.718	7.8002 (-1)	-38.295	9.9015 (-1)	-4.932	9.9370 (-1)	-0.914	9.9379 (-1)	-0.512	9.9379 (-1)	-0.471
(1/10) (3)	6.3444 (-2)	-86.341	5.3463 (-1)	-57.451	9.8113 (-1)	-9.109	9.9346 (-1)	-1.129	9.9364 (-1)	-0.323	9.9365 (-1)	-0.243
(1/8)	5.0797 (-2)	-87.071	4.5223 (-1)	-62.928	9.7161 (-1)	-11.232	9.9338 (-1)	-1.306	9.9362 (-1)	-0.299	9.9363 (-1)	-0.198
(1/6)	3.8123 (-2)	-87.802	3.5590 (-1)	-69.012	9.6098 (-1)	-14.728	9.9322 (-1)	-1.632	9.9361 (-1)	-0.289	9.9362 (-1)	-0.155
(1/4)	2.5427 (-2)	-88.534	2.4632 (-1)	-75.647	9.4249 (-1)	-21.418	9.9279 (-1)	-2.330	9.9359 (-1)	-0.317	9.9361 (-1)	-0.116
(1/2)	1.2717 (-2)	-89.267	1.2614 (-1)	-82.707	7.8267 (-1)	-38.028	9.9052 (-1)	-4.514	9.9357 (-1)	-0.494	9.9360 (-1)	-0.091
(1/10) (4)	6.3589 (-3)	-89.633	6.3459 (-2)	-86.338	5.3551 (-1)	-57.388	9.8163 (-1)	-8.903	9.9347 (-1)	-0.919	9.9360 (-1)	-0.113
(1/8)	5.0872 (-3)	-89.707	5.0805 (-2)	-87.069	4.5276 (-1)	-62.892	9.7511 (-1)	-11.069	9.9340 (-1)	-1.138	9.9360 (-1)	-0.131
(1/6)	3.8154 (-3)	-89.780	3.8126 (-2)	-87.801	3.5616 (-1)	-68.995	9.6117 (-1)	-14.610	9.9326 (-1)	-1.506	9.9360 (-1)	-0.163
(1/4)	2.5436 (-3)	-89.853	2.5428 (-2)	-88.534	2.4641 (-1)	-75.641	9.2545 (-1)	-21.345	9.9284 (-1)	-2.246	9.9359 (-1)	-0.233
(1/2)	1.2718 (-3)	-89.927	1.2717 (-2)	-89.267	1.2615 (-1)	-82.706	7.8295 (-1)	-38.002	9.9058 (-1)	-4.472	9.9357 (-1)	-0.452
(1/10) (5)	6.3590 (-4)	-89.963	6.3589 (-3)	-89.633	6.3460 (-2)	-86.338	5.3559 (-1)	-57.381	9.8168 (-1)	-8.883	9.9348 (-1)	-0.898
(1/8)	5.0872 (-4)	-89.971	5.0872 (-3)	-89.707	5.0806 (-2)	-87.069	4.5282 (-1)	-62.888	9.7517 (-1)	-11.053	9.9341 (-1)	-1.121
(1/6)	3.8154 (-4)	-89.978	3.8154 (-3)	-89.780	3.8126 (-2)	-87.801	3.5618 (-1)	-68.993	9.6153 (-1)	-14.598	9.9326 (-1)	-1.493
(1/4)	2.5436 (-4)	-89.985	2.5436 (-3)	-89.853	2.5428 (-2)	-88.534	2.4641 (-1)	-75.641	9.2549 (-1)	-21.338	9.9284 (-1)	-2.238
(1/2)	1.2718 (-4)	-89.993	1.2718 (-3)	-89.927	1.2717 (-2)	-89.267	1.2615 (-1)	-82.706	7.8298 (-1)	-37.999	9.9058 (-1)	-4.468
(1/10) (6)	6.3590 (-5)	-89.996	6.3590 (-4)	-89.963	6.3589 (-3)	-89.633	6.3461 (-2)	-86.338	5.3560 (-1)	-57.381	9.8169 (-1)	-8.881
(1/8)	5.0872 (-5)	-89.997	5.0872 (-4)	-89.971	5.0872 (-3)	-89.707	5.0806 (-2)	-87.069	4.5282 (-1)	-62.888	9.7517 (-1)	-11.052
(1/6)	3.8154 (-5)	-89.998	3.8154 (-4)	-89.978	3.8154 (-3)	-89.780	3.8126 (-2)	-87.801	3.5618 (-1)	-68.993	9.6153 (-1)	-14.597
(1/4)	2.5436 (-5)	-89.999	2.5436 (-4)	-89.985	2.5436 (-3)	-89.853	2.5428 (-2)	-88.534	2.4642 (-1)	-75.641	9.2550 (-1)	-21.337
(1/2)	1.2718 (-5)	-89.999	1.2718 (-4)	-89.993	1.2718 (-3)	-89.927	1.2717 (-2)	-89.267	1.2615 (-1)	-82.706	7.8298 (-1)	-37.999
(1/10) (7)	6.3590 (-6)	-90.000	6.3590 (-5)	-89.996	6.3590 (-4)	-89.963	6.3589 (-3)	-89.633	6.3461 (-2)	-86.338	5.3560 (-1)	-57.381

TABLE I-23
MAGNITUDE (IMI) AND PHASE ANGLE (θ)
OF ZERO POWER TRANSFER FUNCTION FOR P239

$L(\text{sec}) =$	10^{-3}		10^{-4}		10^{-5}		10^{-6}		10^{-7}		10^{-8}	
	ω (rad/sec)	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI	θ	IMI
(1/10) (-2)	6.2872 (1)	-87.987	6.4612 (1)	-87.931	6.4791 (1)	-87.925	6.4809 (1)	-87.925	6.4811 (1)	-87.925	6.4811 (1)	-87.925
(1/8)	5.0329 (1)	-87.486	5.1722 (1)	-87.446	5.1866 (1)	-87.409	5.1880 (1)	-87.408	5.1882 (1)	-87.408	5.1882 (1)	-87.408
(1/6)	3.7798 (1)	-86.655	3.8845 (1)	-86.562	3.8953 (1)	-86.552	3.8964 (1)	-86.551	3.8965 (1)	-86.551	3.8965 (1)	-86.551
(1/4)	2.5295 (1)	-85.010	2.5996 (1)	-84.871	2.6069 (1)	-84.857	2.6076 (1)	-84.855	2.6077 (1)	-84.855	2.6077 (1)	-84.855
(1/2)	1.2893 (1)	-80.294	1.3254 (1)	-80.020	1.3291 (1)	-79.992	1.3295 (1)	-79.989	1.3295 (1)	-79.988	1.3295 (1)	-79.988
(1/10) (-1)	6.8563	-72.190	7.0534	-71.660	7.0737	-71.605	7.0757	-71.600	7.0759	-71.599	7.0759	-71.599
(1/8)	5.6897	-68.790	5.8557	-68.139	5.8728	-68.072	5.8745	-68.065	5.8747	-68.065	5.8747	-68.065
(1/6)	4.5510	-63.914	4.6873	-63.069	4.7013	-62.982	4.7027	-62.973	4.7029	-62.972	4.7029	-62.972
(1/4)	3.4601	-56.462	3.5697	-55.250	3.5809	-55.125	3.5820	-55.112	3.5821	-55.111	3.5821	-55.111
(1/2)	2.4383	-45.202	2.5303	-43.010	2.5397	-42.782	2.5406	-42.759	2.5407	-42.757	2.5407	-42.756
(1/10)	1.8532	-38.878	1.9463	-35.150	1.9557	-34.756	1.9566	-34.717	1.9567	-34.713	1.9567	-34.712
(1/8)	1.7020	-37.493	1.7968	-33.107	1.8062	-32.642	1.8072	-32.595	1.8073	-32.590	1.8073	-32.590
(1/6)	1.5297	-35.905	1.6272	-30.404	1.6368	-29.916	1.6377	-29.858	1.6378	-29.852	1.6378	-29.851
(1/4)	1.3285	-33.991	1.4316	-26.672	1.4414	-25.880	1.4424	-25.800	1.4425	-25.792	1.4425	-25.791
(1/2)	1.0852	-32.582	1.2106	-19.930	1.2211	-18.518	1.2221	-18.376	1.2222	-18.361	1.2222	-18.360
(1/10) (1)	9.0612 (-1)	-37.111	1.0971	-15.040	1.1095	-12.403	1.1106	-12.136	1.1107	-12.109	1.1107	-12.106
(1/8)	8.4480 (-1)	-40.213	1.0726	-14.124	1.0861	-10.883	1.0873	-10.554	1.0874	-10.522	1.0874	-10.518
(1/6)	7.5728 (-1)	-45.250	1.0460	-13.435	1.0616	-9.197	1.0628	-8.767	1.0630	-8.724	1.0630	-8.720
(1/4)	6.1834 (-1)	-53.694	1.0153	-13.486	1.0356	-7.279	1.0370	-6.646	1.0371	-6.583	1.0372	-6.576
(1/2)	3.7849 (-1)	-68.084	9.6882 (-1)	-17.120	1.0096	-5.162	1.0113	-3.922	1.0115	-3.798	1.0115	-3.786
(1/10) (2)	2.0370 (-1)	-78.279	8.9177 (-1)	-27.171	9.9894 (-1)	-4.733	1.0016	-2.276	1.0017	-2.030	1.0017	-2.005
(1/8)	1.6459 (-1)	-80.538	8.4864 (-1)	-32.013	9.9696 (-1)	-5.023	1.0002	-1.956	1.0004	-1.649	1.0004	-1.618
(1/6)	1.2443 (-1)	-82.852	7.7436 (-1)	-39.224	9.9452 (-1)	-5.754	9.9912 (-1)	-1.674	9.9931 (-1)	-1.264	9.9932 (-1)	-1.223
(1/4)	8.3431 (-2)	-85.209	6.3756 (-1)	-50.327	9.9888 (-1)	-7.597	9.9830 (-1)	-1.499	9.9852 (-1)	-0.884	9.9854 (-1)	-0.823
(1/2)	4.1862 (-2)	-87.597	3.8545 (-1)	-67.283	9.9936 (-1)	-13.782	9.9761 (-1)	-1.773	9.9804 (-1)	-0.546	9.9806 (-1)	-0.423
(1/10) (3)	2.0950 (-2)	-88.798	2.0494 (-1)	-78.150	8.9975 (-1)	-25.631	9.9664 (-1)	-2.931	9.9791 (-1)	-0.478	9.9794 (-1)	-0.232
(1/8)	1.6762 (-2)	-89.038	1.6525 (-1)	-80.469	8.5443 (-1)	-30.884	9.9599 (-1)	-3.570	9.9789 (-1)	-0.505	9.9792 (-1)	-0.198
(1/6)	1.2572 (-2)	-89.278	1.2472 (-1)	-82.821	7.8084 (-1)	-38.513	9.9462 (-1)	-4.660	9.9787 (-1)	-0.578	9.9791 (-1)	-0.168
(1/4)	8.3820 (-3)	-89.519	8.3519 (-2)	-85.199	6.4439 (-1)	-50.004	9.9074 (-1)	-6.870	9.9782 (-1)	-0.764	9.9790 (-1)	-0.150
(1/2)	4.1911 (-3)	-89.759	4.1874 (-2)	-87.595	3.8632 (-1)	-67.224	9.7061 (-1)	-13.431	9.9760 (-1)	-1.405	9.9790 (-1)	-0.177
(1/10) (4)	2.0956 (-3)	-89.880	2.0951 (-2)	-88.797	2.0507 (-1)	-78.141	9.0084 (-1)	-25.480	9.9675 (-1)	-2.747	9.9789 (-1)	-0.293
(1/8)	1.6765 (-3)	-89.904	1.6762 (-2)	-89.038	1.6532 (-1)	-80.464	8.5738 (-1)	-30.775	9.9612 (-1)	-3.423	9.9788 (-1)	-0.357
(1/6)	1.2574 (-3)	-89.928	1.2573 (-2)	-89.278	1.2475 (-1)	-82.819	7.8156 (-1)	-38.445	9.9476 (-1)	-4.550	9.9787 (-1)	-0.467
(1/4)	8.3824 (-4)	-89.952	8.3821 (-3)	-89.519	8.3528 (-2)	-85.199	6.4480 (-1)	-49.973	9.9089 (-1)	-6.797	9.9783 (-1)	-0.690
(1/2)	4.1912 (-4)	-89.976	4.1911 (-3)	-89.759	4.1875 (-2)	-87.595	3.8641 (-1)	-67.218	9.7075 (-1)	-13.396	9.9762 (-1)	-1.368
(1/10) (5)	2.0956 (-4)	-89.988	2.0956 (-3)	-89.880	2.0951 (-2)	-88.797	2.0508 (-1)	-78.140	9.0095 (-1)	-25.465	9.9677 (-1)	-2.728
(1/8)	1.6765 (-4)	-89.990	1.6765 (-3)	-89.904	1.6762 (-2)	-89.038	1.6533 (-1)	-80.463	8.5748 (-1)	-30.764	9.9614 (-1)	-3.408
(1/6)	1.2574 (-4)	-89.993	1.2574 (-3)	-89.928	1.2573 (-2)	-89.278	1.2475 (-1)	-82.819	7.8164 (-1)	-38.438	9.9477 (-1)	-4.539
(1/4)	8.3824 (-5)	-89.995	8.3824 (-4)	-89.952	8.3821 (-3)	-89.519	8.3529 (-2)	-85.198	6.4484 (-1)	-49.970	9.9090 (-1)	-6.790
(1/2)	4.1912 (-5)	-89.998	4.1912 (-4)	-89.976	4.1911 (-3)	-89.759	4.1875 (-2)	-87.595	3.8642 (-1)	-67.218	9.7076 (-1)	-13.393
(1/10) (6)	2.0956 (-5)	-89.999	2.0956 (-4)	-89.988	2.0956 (-3)	-89.880	2.0951 (-2)	-88.797	2.0509 (-1)	-78.140	9.0096 (-1)	-25.464
(1/8)	1.6765 (-5)	-89.999	1.6765 (-4)	-89.990	1.6765 (-3)	-89.904	1.6762 (-2)	-89.038	1.6533 (-1)	-80.463	8.5749 (-1)	-30.763
(1/6)	1.2574 (-5)	-89.999	1.2574 (-4)	-89.993	1.2574 (-3)	-89.928	1.2573 (-2)	-89.278	1.2475 (-1)	-82.819	7.8164 (-1)	-38.437
(1/4)	8.3824 (-6)	-90.000	8.3824 (-5)	-89.995	8.3824 (-4)	-89.952	8.3821 (-3)	-89.519	8.3529 (-2)	-85.198	6.4484 (-1)	-49.970
(1/2)	4.1912 (-6)	-90.000	4.1912 (-5)	-89.998	4.1912 (-4)	-89.976	4.1911 (-3)	-89.759	4.1875 (-2)	-87.595	3.8642 (-1)	-67.218
(1/10) (7)	2.0956 (-6)	-90.000	2.0956 (-5)	-89.999	2.0956 (-4)	-89.988	2.0956 (-3)	-89.880	2.0951 (-2)	-88.797	2.0509 (-1)	-78.140

Table 1-24

PHOTONEUTRON HALF-LIVES AND YIELDS FROM U^{235}
FISSION PRODUCTS IN D_2O (119)

Group j	Half-Life ^a T_j	Yield%, Relative to Total Delayed Neutron Yield ^b
1	53 h	0.0163
2	4.4 h	0.0515
3	1.65 h	0.370
4	27 m	0.328
5	7.7 m	0.534
6	2.4 m	1.11
7	41 s	3.24
8	2.5 s	10.31

^am = minute; h = hour; s = sec

^bBased on delayed neutron data of
Hughes, et al.⁽¹¹⁶⁾

Corresponding to an exposure time t_e , the corrected experimental yield relative to the 22-second delayed neutron half-life is plotted as a function of time after irradiation. Figure 1-17 shows the relative yield of photoneutrons from U^{235} fission products in Be for $t_e = 15$ minutes. Figure 1-18 shows the relative yield for $t_e = 74$ hours. Figure 1-19 compares yields for $t_e = 15$ minutes, 5 hours, 6 hours, and 74 hours. Finally, Fig. 1-20 compares the yields of photoneutrons from U^{235} fission products in D_2O with those in Be for $t_e = 6$ hours. (Figures 1-7 through 1-20 are taken from Ref. 120.) W. K. Ergen⁽¹²¹⁾ has correlated the above results on the photoneutrons from D_2O and Be with known hard gamma emitters among the fission fragments.

FIG. 1-17
RELATIVE YIELD OF PHOTONEUTRONS FROM U^{235}
FISSION PRODUCTS IN Be AFTER 15-MINUTE EXPOSURE
(REPRODUCED FROM REF. 120)

FIG. 1-18
RELATIVE YIELD OF PHOTONEUTRONS FROM
 U^{235} FISSION PRODUCTS IN Be
(REPRODUCED FROM REF. 120)

FIG. 1-19
RELATIVE YIELD OF PHOTONEUTRONS FROM U^{235} FISSION
PRODUCTS IN Be FOR VARIOUS EXPOSURE TIMES
(REPRODUCED FROM REF. 120)

FIG. 1-20
COMPARISON OF PHOTONEUTRON ACTIVITIES
OF D_2O AND Be AFTER 6-HOUR EXPOSURE
(REPRODUCED FROM REF. 120)

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SECTION 2

SELECTED CROSS SECTION DATA

2.1 DEPENDENCE OF η , AND α , AND FISSION CROSS SECTION ON ENERGY2.1.1 Measurements of η and α

The values of η (average neutron yield per neutron absorbed) as a function of neutron energy are presented for the fissionable isotopes U^{233} , U^{235} and Pu^{239} . In the cases where η is computed from the measured value of α (capture to fission cross section ratio), the ν used in the calculation is indicated in the text. The available measured values of α for U^{235} and Pu^{239} are also given.

The compilation includes the data in BNL-325, Supplement No. 1,⁽¹⁾ along with recently published and unpublished measurements. The α data in the BNL-325 Supplement were converted to η by assuming ν to be a constant in the ev energy region. The values of ν used for each of the materials were the world weighted averaged values at 0.0253 ev.

Uranium

The thermal values of η for slightly enriched uranium are listed in Table 2-1. These values were obtained experimentally by Kouts and Sher⁽²⁾ in their study of slightly enriched uranium, water moderated lattices.

Table 2-1

 U^{233} THERMAL VALUES OF η FOR SLIGHTLY ENRICHED URANIUM⁽²⁾(Based on $\eta_{nat. U} = 1.309 \pm 0.006$)

<u>Enrichment</u> %	<u>η</u>
1.3	1.5674
1.15	1.5182
1.0	1.4745

Figures 2-1 and 2-2 show the variation of η for U^{233} as a function of energy. A value of $\nu = 2.54 \pm .03$ was used to convert the BNL-325 α values to η values.

The value of η was recently measured by Sanders, et al,⁽³⁾ at Harwell using the crystal spectrometer. The crystals used were KCl, and Cu, with the Cu crystal giving better resolution.

The measurements of η in the ev region reported by the USSR groups, Spivak, et al,^(4,5) were made by using a system of filters on the incident neutron beam. This resulted in an integrated value of η and is represented as a block in the figures. The η values at 30, 140, and 250 kev neutrons were obtained by using photoneutron sources (Sb + Be, Ga + D_2O , Na + D_2O , respectively). A Na + Be source was used for the 900-kev neutrons. The latest measurements of η in the 0.1 to 10 ev range by Smith and Magelby,⁽⁶⁾ using the crystal spectrometer at the MTR, are included. Errors associated with the measurements are not reported.

FIG. 2-1
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR U^{233} AS A FUNCTION OF ENERGY

FIG. 2-2
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR U^{233} AS A FUNCTION OF ENERGY

U²³⁵

Figures 2-3, 2-4 and 2-5 show the variation of η for U²³⁵ as a function of energy. Table 2-2 lists measured values of α other than BNL-325 data. A value of $\nu = 2.47 \pm 0.03$ was used to convert the BNL-325 α values to η values.

FIG. 2-3
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR U²³⁵ AS A FUNCTION OF ENERGY
(OTHER DATA IN THE 0.1 TO 1 ev RANGE ARE
GIVEN IN FIG. 2-4)

Additional data for η in the 0.001-0.05 ev range were supplied by Bollinger and Soplakaglu at ANL (unpublished) using the fast chopper. The value η/ν was inferred from direct measurements of σ_f and σ_t , using $\sigma_s \sim 12$ barns (constant). The cross sections reported were relative measurements ($< 3\%$ error); η was normalized to 2.07 at 0.0251 ev.

The KAPL measurements of α were made by Kanne, Stewart, and White(7) with appropriately shielded foils and mass spectrometer analysis. A theoretical analysis of α data has been made by Oleksa.(8) η was computed using $\nu = 2.47$ except for the 215 and 300-kev values where a correction of ν was made using Terrell's relation:(9)

$$\frac{d\nu}{dE} = 0.145 \pm 0.011 \text{ neutron/Mev}$$

(see Sect. 1.1.1)

Diven's(10) experiment in the 100 kev - 1 Mev region measured α directly by studying the capture and fission gamma rays.

The value for α was again calculated by taking the dependence of ν in energy as given by Terrell.

Skarsgard and Kenward(11) used a crystal spectrometer to measure η in the energy region 0.006 - 0.050 ev. Within experimental error, η appeared to be constant in this range.

Also included are the latest MTR crystal spectrometer measurements(6) of η in the 0.1 - 10 ev range. Errors associated with the measurements are not reported.

Pu²³⁹

Figures 2-6 through 2-9 show the variation of η for Pu²³⁹ as a function of energy.

With reference to Fig. 2-9, the set of data on η/η_0 (η relative to thermal η) in the energy region 6 - 60 ev was obtained by Farley(12) from a transmission-type experiment on two different thicknesses of sample after correcting for scattering.

FIG. 2-4
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR U^{235} AS A FUNCTION OF ENERGY

(OTHER DATA IN THE 0.1 TO 1 ev RANGE ARE
GIVEN IN FIG. 2-3)

FIG. 2-5
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR U^{235} AS A FUNCTION OF ENERGY

Table 2-2

VALUES OF α FOR U^{235} AS A FUNCTION OF NEUTRON ENERGY

Energy	Kanne(7)	KAPL(8) ($\pm 20\%$)	Diven(10)
Thermal	0.17		
30 ev	0.65		
100 ev	0.52	0.52	
150 ev		0.42	
1200 ev	0.47		
1500 ev		0.45	
3000 ev		0.42	
15 kev	0.41		
175 kev			0.173 ± 0.021
215 kev		0.1	
250 kev			0.157 ± 0.021
300 kev		0.17	
400 kev			0.140 ± 0.013
504 kev			0.131 ± 0.013
602 kev			0.132 ± 0.016
705 kev			0.106 ± 0.014
800 kev			0.090 ± 0.013
903 kev			0.090 ± 0.010
1.006 Mev			0.070 ± 0.012

FIG. 2-6
NEUTRON YIELD PER NEUTRON ABSORBED (η) FOR Pu^{239} AS A FUNCTION OF ENERGY

FIG. 2-7
NEUTRON YIELD PER NEUTRON ABSORBED (η)
FOR Pu^{239} AS A FUNCTION OF ENERGY

2.1.1

Values of α reported by Kanne, Stewart, and White,⁽⁷⁾ are listed in Table 2-3. η was computed using $\nu = 2.91 \pm 0.04$. Errors associated with these results were not reported.

Table 2-3 includes values of α reported by Egelstaff and Sanders.⁽¹⁴⁾ Because of the uncertainty of $\pm 30\%$, the η values were not computed for this range of energies.

Table 2-3

VALUES OF α FOR Pu^{239} AS A FUNCTION OF NEUTRON ENERGY

Energy	Kanne	Egelstaff & Sanders ($\pm 30\%$)
Thermal	0.42	
0.30 ev		0.8
7.85 ev		2.0
10.95 ev		1.1
11.95 ev		2.7
14.3 ev } 14.75 ev }		1.0
17.7 ev		.6
30 ev	0.65	
100 ev	0.81	
1200 ev	0.60	
15 kev	0.45	

2.1.2 Dependence of Fission Cross Sections on Energy

Figures 2-10 through 2-13 show the fission cross section of U^{233} , U^{235} , and Pu^{239} for neutron energies 0.01 - 10 Mev. The data are reproduced from a summary compiled by Henkel,⁽¹⁵⁾ and are included since some of the values are not available in BNL-325. Figure 2-14 is a composite fission cross section curve of other fissionable isotopes.

2.2 CROSS SECTIONS FOR ENERGY TRANSFER

2.2.1 Energy Transfer in the Thermal Region

In the neutron energy region where the scattering nuclei do not act independently, the neutron may gain or lose energy to the system of scatterers. Weinstock,⁽¹⁶⁾ utilizing the Debye model, developed the theory of inelastically scattered neutrons for a monatomic polycrystalline substance. He shows that the processes to consider as a first approximation, other than elastic scattering, are the inelastic interactions in which the neutron gains or loses a single quantum (phonon) of vibrational energy from or to the lattice. A summary and extension of the theory to include spin and isotope effects has been published by Cassels.⁽¹⁷⁾

FIG. 2-10
FISSION CROSS-SECTION OF U^{233} FROM 0.01 TO 10 Mev
(REPRODUCED FROM DATA OF HENKE⁽¹⁵⁾)

FIG. 2-11
FISSION CROSS-SECTION OF U^{235} FROM 0.01 TO 10 Mev
(REPRODUCED FROM DATA OF HENKE⁽¹⁵⁾)

FIG. 2-12
FISSION CROSS-SECTION OF U^{238} FROM 1 TO 10 MeV
(REPRODUCED FROM DATA OF HENKEL (15))

FIG. 2-13
FISSION CROSS-SECTION OF Pu^{239} FROM 0.01 TO 10 MeV
(REPRODUCED FROM DATA OF HENKEL (15))

FIG. 2-14
COMPOSITE FISSION CROSS SECTION CURVE
(BASED ON DATA FROM REF. 15)

The single phonon approximation has been found in good agreement with experiments⁽¹⁸⁾ on total cross-section for lead at 80°K and 295°K in the neutron energy range 0.0013-0.02 ev. Also total cross section measurements by Palevsky, Hughes and Smith^(19,20,21) on Fe, C and Be in the range 0.0002-0.003 ev are in good agreement with theory.

For high temperatures of the scatterer and also at high neutron energies, the contribution due to multi-phonon processes becomes appreciable. Squires,⁽²²⁾ Placzek,⁽²³⁾ and Kothari and Singwi⁽²⁴⁾ generalize Weinstock's theory to include such higher order processes.

Singwi and Kothari show that a two-phonon process, in addition to retaining certain correction terms for the inelastic coherent scattering neglected by Placzek, gives a temperature dependence for total cross section in good agreement with experiments (in the energy region below the Bragg cutoff).

The expression for the total inelastic scattering cross section (in the incoherent approximation) derived by Kothari and Singwi is:

$$\sigma = \sum_{\ell} \sigma_{\ell} \quad (2-1)$$

$$\sigma_{\ell}(k_1^2) = \frac{(S+s)}{2k_1^2 \ell!} \left(\frac{3}{2M}\right)^{\ell} \frac{\ell!}{\pi} \int_{\max(-1, -k_1^2)}^{+1} dE_j E_j \left[\coth \frac{E_j}{2T} - 1 \right] \int_{k_2-k_1}^{k_2+k_1} dt e^{-\frac{6F}{M} t^2} t^{2\ell+1} \quad (2-2)$$

where ℓ ($= 0, 1, 2, \dots$) denotes the number of phonons exchanged between the neutron and lattice. All quantities are expressed in dimensionless units: unit length $(\hbar^2/2m_0 k_0^2 \theta)^{1/2}$, energy $k_0 \theta$, where k_0 is Boltzmann's constant, m_0 is the neutron mass, $\hbar$ is Planck's constant divided by 2π ; and θ is the Debye temperature of the solid. The quantity M is the ratio of nucleus to neutron mass m/m_0 ; k_1 and k_2 are the wave numbers associated with the initial and final state of the neutron respectively; and $T = T_s/\theta$ is the ratio of the temperature of the solid, T_s , to the Debye temperature. The phonon energy is denoted by E_j and may take positive as well as negative values which correspond to a phonon emission and absorption process respectively.

The exponential term in the integrand of Eq. (2-2) is the Debye-Waller factor which, in the Debye approximation, is

$$F = \frac{1}{2} \int_0^1 dE E \coth \frac{E}{2T} = \frac{1}{4} T^2 \int_0^{T^{-1}} d\beta \frac{\beta}{(e\beta - 1)} \quad (2-3)$$

The terms s and S are the incoherent and coherent scattering, respectively;

$$S = 4\pi \left[\sum_i \rho_i (\omega_+ a_+^i + \omega_- a_-^i) \right]^2 \quad (2-4)$$

$$s = 4\pi \left\{ \sum_i \rho_i (\omega_+ a_+^i + \omega_- a_-^i)^2 - \left[\sum_i \rho_i (\omega_+ a_+^i + \omega_- a_-^i) \right]^2 \right\} \quad (2-5)$$

where ρ_i is the abundance of the i^{th} isotope, having scattering amplitude a_+^i for total spin $I + \frac{1}{2}$ and a_-^i for total spin $I - \frac{1}{2}$. The weights ω_+ and ω_- are

$$\omega_+ = \frac{I+1}{2I+1} \quad (2-6)$$

$$\omega_- = \frac{I}{2I+1}$$

The correction term to the coherent inelastic scattering is

$$\sigma'_\ell(k_1^2) = \frac{S}{2k_1^2 \ell!} \left(\frac{3}{2M} \right)^\ell \sum_r \frac{Z_r}{r} \prod_{j=1}^{\ell} \int_{-1}^{+1} dE_j E_j \left[\coth \left(\frac{E_j}{2T} \right) - 1 \right] \int_{k_2-k_1}^{k_2+k_1} dt e^{-\frac{6F}{M} t^2} t^{2\ell} \sin rt \quad (2-7)$$

where Z_r is the number of nuclei at the lattice distance r .

As the energy of the incident neutron increases, but is still below the binding energy, the amount of energy exchanged in the emission process should be greater than that for a two-phonon process. Usage of the Debye Model to account for such processes becomes complex. An alternate theory, developed by Finkelstein,⁽²⁵⁾ in which the Einstein model is used, can be considered as a multiphonon process in the Debye model. Singwi and Kothari⁽²⁶⁾ in the study of the slowing down of neutrons obtain Finkelstein's results as a special case. The expression for the cross section in which ℓ phonons are absorbed and m phonons emitted is

$$\sigma_{\ell,m} = \frac{(S+s) (1 - e^{1/T})^{-m} (e^{1/T} - 1)^{-\ell}}{2k_1^2 (\ell+m)! M^{\ell+m}} \int_{k_2-k_1}^{k_2+k_1} dt e^{-\frac{6F}{M} t^2} t^{2(\ell+m)+1} \quad (2-8)$$

where

$$F = \frac{1}{6} \coth \frac{1}{2T} \quad (2-9)$$

and $T = T_s/\theta_E$, with θ_E being the Einstein temperature of the solid.

The Einstein model is probably valid in the very low neutron energy region (< 0.0001 eV) where multiple phonon absorption processes (Debye model) also occur. In the thermal energy region, however, the phonon absorption process given by this model probably fails since it does not allow for low frequency vibrations.

Placzek and Van Hove⁽²⁷⁾ have considered the inelastic scattering of neutrons by crystals for which the Debye model is not a good approximation. An expression

for angular and energy distribution of the inelastically scattered neutrons is derived in terms of the vibrational spectrum of the crystal which may be obtained theoretically or experimentally. Squires⁽²⁸⁾ has attempted to calculate a set of frequencies for an aluminum crystal by assuming a model of the interatomic forces in the crystal. Brockhouse and Iyengar,⁽²⁹⁾ and Pelah, Eisenhauer, Hughes and Palevsky⁽³⁰⁾ have studied lattice vibrations in Ge and ZrH by measuring the inelastically scattered slow neutrons. Singwi and Kothari⁽³¹⁾ have studied the temperature variation of the inelastic scattering cross section of graphite for neutron wave lengths below the Bragg cutoff. The model used is based on the work of Krumhansl and Brooks⁽³²⁾ in which the lattice vibrations are separated into two parts: modes normal to the layer planes characterized by a Debye temperature, θ_z , and modes in the layer planes characterized by a Debye temperature, θ_{xy} . The calculations are made for 10A neutrons and the results are in good agreement with experiments.

For monatomic gaseous moderators whose atoms have a Maxwellian velocity distribution, Wigner and Wilkins⁽³³⁾ have derived an expression for the isotropic scattering kernel $P(v, v_1)$:

$$P(v, v_1) = \frac{2\theta^2 v}{\sqrt{\pi} v_1} \exp[\beta^2(v_1^2 - v^2)] [I(\beta\theta v_1 - \beta\zeta v) \pm I(\beta\theta v_1 + \beta\zeta v)] \\ + \frac{2\theta^2 v}{\sqrt{\pi} v_1} [I(\beta\theta v - \beta\zeta v_1) \pm I(\beta\zeta v_1 + \beta\theta v)] \quad (2-10)$$

where v_1 and v are the initial and final neutron speeds: $\theta = (M + 1)/(2\sqrt{M})$; $\zeta = (M - 1)/(2\sqrt{M})$; $\beta^2 = 1/2kT$; and M is the ratio of the atom to neutron mass. The upper and lower signs refer to the processes in which $v > v_1$ and $v < v_1$, respectively. I is defined as

$$I(x) = \int_0^x \exp(-\zeta^2) d\zeta \quad (2-11)$$

Krieger and Nelkin⁽³⁴⁾ obtain Eq.(2-10) starting from Zemach and Glauber's⁽³⁵⁾ quantum mechanical scattering formulation for molecules.

2.2.2 Inelastic Scattering by U^{238}

Cranberg and Levin⁽³⁶⁾ conducted a series of experiments wherein monoenergetic neutrons in the range from 0.5 to 2 Mev were scattered from U^{238} and the spectra of the neutrons scattered at 90° were investigated by a time-of-flight method. Seven excited states of U^{238} were identified and their excitation cross sections at four incident neutron energies were obtained. These results are summarized in Table 2-4. In addition, the angular distribution of inelastic neutrons corresponding to the excitation of the 44-kev level by 550-kev incident neutrons was measured and found to be

$$d\sigma(\theta) = 0.155 [1 - (0.45 \pm 0.1) \cos^2\theta] \text{ barns/steradian}$$

where θ is the scattering angle in the laboratory system.

Table 2-4

U^{238} INELASTIC SCATTERING CROSS SECTIONS FOR
VARIOUS LEVELS OF EXCITATION

Neutron Energy, kev	Energy Level, kev	σ in barns	θ degrees	$d\sigma(\theta)$ barns/steradian
550 $\pm$ 10	0 (I = 0)	6.34 $\pm$ 0.3	145	0.29 $\pm$ 0.03
	44 $\pm$ 4 (I = 2)	1.43 $\pm$ 0.2	145	0.092 $\pm$ 0.015
	146 $\pm$ 6 (I = 4)	0.29 $\pm$ 0.06	145	0.023 $\pm$ 0.05
	309 (I = 6)	0.1	145	0.01
1000 $\pm$ 24	44 $\pm$ 4 (I = 2)	0.9 $\pm$ 0.3		
	146 $\pm$ 6 (I = 4)	0.6 $\pm$ 0.15		
	300 $\pm$ 30 (I = 6)	0.09 $\pm$ 0.05	90	0.007 $\pm$ 0.004
	730 $\pm$ 30	0.52 $\pm$ 0.07	90	0.041 $\pm$ 0.006
1500 $\pm$ 36	730 $\pm$ 30	0.26 $\pm$ 0.03	90	0.021 $\pm$ 0.025
	980 $\pm$ 20		90	0.036
	1060 $\pm$ 30	0.90 $\pm$ 0.15	90	0.036
1893 $\pm$ 30	730 $\pm$ 30	0.08 $\pm$ 0.02	90	0.007 $\pm$ 0.002
	980 $\pm$ 20		90	0.032
	1060 $\pm$ 30	0.103 $\pm$ 0.01	90	0.040
	1260 $\pm$ 40		90	0.008

The angular distribution of elastically scattered 500-kev neutrons was measured with the following results:

$$\frac{d\sigma(140^\circ)}{d\sigma(90^\circ)} = 0.85 \pm 0.04$$

$$\frac{d\sigma(130^\circ)}{d\sigma(90^\circ)} = 0.86 \pm 0.04$$

$$\frac{d\sigma(195^\circ)}{d\sigma(90^\circ)} = 0.87 \pm 0.04$$

$$\frac{d\sigma(157^\circ)}{d\sigma(90^\circ)} = 0.89 \pm 0.04$$

$$\frac{d\sigma(35^\circ)}{d\sigma(145^\circ)} = 3.57 \pm 0.1$$

2.3 SCATTERING ANGULAR DISTRIBUTION

In addition to the measurements reported in BNL-325, Supplement No. 1 angular distribution cross section measurements have been made by Langsdorf, Lane and Monahan.(37) The following is based on their work.

The experimental data are represented by an expansion in a series of Legendre Polynomials

$$\sigma_s(\mu) = (\sigma_s/4\pi) \sum_{\ell=0}^4 \omega_\ell P_\ell(\mu) \quad (2-12)$$

where μ is the cosine of the scattering angle, σ_s is the total scattering cross section (i.e., the differential cross section integrated over the solid angle) and includes the elastic plus inelastic scattering. The data is represented by the first five polynomials and are plotted as functions of the incident neutron energy. The standard deviations reported are approximately ± 0.05 for ω_1 , ± 0.10 for ω_4 and intermediate deviations for ω_2 and ω_3 ; ω_0 being normalized to the value one.

The data in this form are a more compact and practical description of angular distribution than the usual cosine plots.⁽³⁸⁾ It is particularly advantageous in reactor design calculations since it is already in a form appropriate for the Spherical Harmonics method of solution of the transport equation. Defining the transport scattering cross section as:

$$\sigma_{s, tr} = 2\pi \int_{-1}^{+1} (1-\mu) \sigma_s(\mu) d\mu = \sigma_s(1 - \frac{1}{3}\omega_1), \quad (2-13)$$

gives the average cosine of the scattering angle as a function of the energy:

$$\bar{\mu} = \frac{1}{3} \omega_1(E). \quad (2-14)$$

If the inelastic scattering is isotropic, then the transport cross section for elastic scattering is

$$\sigma_{el tr} = \sigma_{s el} \left(1 - \frac{1}{3} \omega_1 \frac{\sigma_s}{\sigma_{s el}} \right)$$

where $\sigma_{s el}$ is the elastic scattering cross section integrated over the solid angle.

The coefficients have been calculated for scattering cross sections in the Laboratory system. To express these in the Center of Mass system the following transformation matrix, derived by J. E. Monahan and G. Steiner (unpublished), may be used.

$$\omega'_\ell = \sum_k T^{-1}_{\ell k} \omega_k \quad (2-15)$$

$$T^{-1} = \begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ -\frac{2}{3}\gamma & 1 - \frac{1}{5}\gamma^2 & \frac{2}{3}\gamma & \frac{1}{5}\gamma^2 & 0 & 0 & 0 & \dots \\ \frac{3}{8}\gamma^2 & -\frac{6}{5}\gamma & 1 - \frac{9}{7}\gamma^2 & \frac{6}{5}\gamma & \frac{24}{35}\gamma^2 & 0 & 0 & \dots \\ 0 & \frac{48}{35}\gamma^2 & -\frac{12}{7}\gamma & 1 - \frac{42}{15}\gamma^2 & \frac{12}{7}\gamma & \frac{10}{7}\gamma^2 & 0 & \dots \\ 0 & 0 & \frac{50}{21}\gamma^2 & -\frac{20}{9}\gamma & 1 - \frac{370}{77}\gamma^2 & \frac{20}{9}\gamma & \frac{240}{99}\gamma^2 & \dots \\ 0 & 0 & 0 & \frac{40}{11}\gamma^2 & -\frac{30}{11}\gamma & 1 - \frac{285}{39}\gamma^2 & \frac{30}{11}\gamma & \dots \\ 0 & 0 & 0 & 0 & \frac{735}{143}\gamma^2 & -\frac{42}{13}\gamma & 1 - \frac{567}{55}\gamma^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \quad (2-16)$$

and

$$\gamma \equiv \left[\frac{m_1 m_3 (m_3 + m_4)}{m_2 m_4 (m_1 + m_2)} \frac{E_r}{E_r Q} \right]^{1/2} \quad (2-17)$$

and m_1 (incident particle), m_2 (target) are the masses of the reacting particles; m_3 , m_4 are the masses of the reaction products; E_r is the energy of the relative motion in the center of mass system, and Q is the energy of the reaction. For an elastic scattering process $Q = 0$, $m_1 = m_3$, $m_2 = m_4$, and γ then becomes

$$\gamma = \frac{m_1}{m_2} \quad (2-18)$$

The inverse transformation matrix T is

$$T = \begin{matrix} & \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \end{matrix} \\ \begin{matrix} \frac{2}{3}\gamma \\ \frac{1}{5}\gamma^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 1 - \frac{3}{5}\gamma^2 \\ \frac{6}{5}\gamma \\ \frac{24}{35}\gamma^2 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} -\frac{2}{3}\gamma \\ \frac{11}{7}\gamma^2 \\ \frac{12}{7}\gamma \\ \frac{10}{7}\gamma^2 \\ 0 \\ 0 \end{matrix} & \begin{matrix} \frac{3}{5}\gamma^2 \\ -\frac{6}{5}\gamma \\ 1 - \frac{46}{15}\gamma^2 \\ \frac{20}{9}\gamma \\ \frac{80}{33}\gamma^2 \\ 0 \end{matrix} & \begin{matrix} 0 \\ \frac{48}{35}\gamma^2 \\ -\frac{12}{7}\gamma \\ 1 - \frac{390}{77}\gamma^2 \\ \frac{30}{11}\gamma \\ \frac{1575}{429}\gamma^2 \end{matrix} & \begin{matrix} 0 \\ 0 \\ \frac{50}{21}\gamma^2 \\ -\frac{20}{9}\gamma \\ 1 - \frac{295}{39}\gamma^2 \\ \frac{42}{13}\gamma \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ \frac{40}{11}\gamma^2 \\ -\frac{30}{11}\gamma \\ 1 - \frac{581}{55}\gamma^2 \end{matrix} & \begin{matrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{matrix} \end{matrix} \quad (2-19)$$

Figures 2-15 through 2-31 give the angular distribution, and total cross sections, σ_t , for various materials of interest in reactor design. Over the energy region investigated, σ_t and σ_s are assumed equal for all materials considered.

2.4 AVERAGE CROSS SECTIONS FOR THERMAL MAXWELL DISTRIBUTION

In an attempt to standardize the nomenclature for general reactor calculations, the Tripartite Nuclear Cross Section Organization has agreed^(39,40) that effective absorption and fission cross sections be defined as suggested by Westcott:⁽⁴¹⁾

$$\hat{\sigma} = \sigma_0 (g + rs) \quad (2-20)$$

where σ_0 is the 2200 meter/sec cross section; g is a measure of the deviation from a $1/v$ behavior in a Maxwellian spectrum; s measures this deviation in the epithermal region (dE/E component of the spectrum); and r is proportional to the ratio of the neutron density in the epithermal region to the total density.

The definition (Eq. (2-20)) arises from setting a convention for the flux as measured by a $1/v$ detector, i.e., the observed reaction rate divided by the cross section at 2200 meter/sec. The reaction rate for another substance is then the product of the flux measured by a $1/v$ detector and the effective cross section, $\hat{\sigma}$, defined by Eq. (2-20).

The Maxwellian averaged cross section, $\bar{\sigma}$, for a temperature T , is obtained from the g factors by:

$$\bar{\sigma}(T) = g(T) \sigma_0 \sqrt{\pi T_0 / 4T} \quad (2-21)$$

where $T_0 = 293.6$ °K. A more detailed analysis is given in Section 3.1.

FIG. 2-15
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ${}^9\text{Be}$ AS A FUNCTION OF NEUTRON ENERGY
(REPRODUCED FROM DATA OF LANGSDORF, et al.⁽³⁷⁾)

FIG. 2-18
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ${}^{12}\text{C}$ AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-17
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{11}Na AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-18
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{12}Mg AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-19
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{13}Al AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-20
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{13}K AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-21
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{26}Fe AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-22
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{28}Ni AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-23
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{29}Cu AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-24
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{40}Zr AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-25
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{118}Cd AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-26
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{119}In AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-27
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{82}Pb AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-28
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{83}Bi AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-29
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{90}Th AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-30
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL
CROSS-SECTION OF ^{92}U AS A FUNCTION OF NEUTRON ENERGY

FIG. 2-31
COEFFICIENTS OF THE LEGENDRE POLYNOMIALS AND TOTAL CROSS-SECTION
OF STAINLESS STEEL AS A FUNCTION OF NEUTRON ENERGY

Tables 2-5 through 2-9 list the parameters needed to obtain an effective cross section for many non- $1/v$ absorbers in a Maxwellian spectrum. Since in such a spectrum $r = 0$ only g factors as a function of neutron temperature ($^{\circ}\text{K}$) are considered. For $1/v$ absorbers, $g = 1$ and these are not listed.

Table 2-5 gives the $g(T)$ factors computed and/or reported by Westcott.⁽⁴¹⁾ Included are revised data* on Cd and Pu²⁴¹.

The $g(T)$ data for Xe¹³⁵ in Table 2-6 were obtained from the cross section measurements by Smith, et al.⁽¹⁾ for the two spin factors: $g = 3/8$ and $g = 5/8$ ($J = 1$ and $J = 2$, respectively). Resonance parameters and cross section studies for Xe¹³⁵ have been summarized by Bernstein and Smith.⁽⁴²⁾

*Private communication from C. H. Westcott (Chalk River). A full compilation superseding CRRP-680 (AECL-47) is being prepared for publication in July or August 1958.

Table 2-5

g(T) FACTORS FOR MAXWELL DISTRIBUTIONS(Reproduced from data of Westcott⁽⁴¹⁾)

T, °K	In ¹¹⁵	Sm	Gd	Au	Cd	U ²³³		U ²³⁵		U ²³⁸	Pu ²³⁹		Pu ²⁴⁰	Pu ²⁴¹
	g _a	g _a	g _a	g _a	g _a	g _a	g _f	g _a	g _f	g _a	g _a	g _f	g _a	
293.60	1.017	1.620	0.893	1.005	1.321	0.998	1.000	0.975	0.978	1.002	1.074	1.053	1.028	1.042
313.60	1.021	1.697	0.882	1.006	1.388	0.996	1.000	0.970	0.973	-	1.092	1.066	-	1.051
333.60	-	1.768	0.870	-	1.457	0.995	0.000	0.965	0.969	1.003	1.113	1.081	1.039	1.062
353.60	1.028	1.832	0.858	1.009	1.528	0.994	1.000	0.960	0.965	-	1.137	1.098	-	1.074
373.60	-	1.891	0.846	-	1.601	0.993	1.001	0.956	0.961	1.004	1.163	1.118	1.051	1.088
393.60	1.036	1.942	0.834	1.011	1.674	0.992	1.001	0.951	0.957	-	1.192	1.140	-	1.102
413.60	-	1.988	0.821	-	1.747	0.991	1.002	0.948	0.954	1.005	1.225	1.164	1.063	1.118
433.60	1.043	2.029	0.809	1.013	1.821	0.991	1.002	0.944	0.950	-	1.261	1.191	-	1.135
453.60	-	2.063	0.796	-	1.893	0.990	1.003	0.941	0.947	1.006	1.299	1.220	1.076	1.152
473.60	1.050	2.093	0.784	1.015	1.965	0.990	1.003	0.938	0.944	-	1.341	1.252	-	1.171
493.60	-	2.118	0.772	-	2.034	0.990	1.004	0.935	0.942	1.007	1.386	1.286	1.089	1.190
513.60	1.058	2.139	0.760	1.017	2.102	0.990	1.005	0.932	0.939	-	1.434	1.323	-	1.211
533.60	-	2.156	0.748	-	2.168	0.990	1.005	0.930	0.936	1.008	1.485	1.362	1.102	1.231
553.60	1.066	2.169	0.736	1.020	2.232	0.990	1.006	0.928	0.934	-	1.537	1.403	-	1.252
573.60	-	2.179	0.724	-	2.293	0.990	1.007	0.925	0.931	1.009	1.593	1.445	1.116	1.273
593.60	1.074	2.186	0.713	1.022	2.352	0.990	1.008	0.924	0.929	-	1.650	1.490	-	1.295
613.60	-	2.190	0.702	-	2.408	0.991	1.009	0.922	0.927	1.009	1.709	1.536	1.130	1.316
633.60	1.082	2.192	0.691	1.024	2.462	0.991	1.009	0.920	0.925	-	1.769	1.583	-	1.338
653.60	-	2.191	0.680	-	2.512	0.991	1.010	0.919	0.923	-	1.831	1.631	-	1.359
673.60	1.091	2.188	0.669	1.027	2.561	0.992	1.011	0.917	0.922	1.011	1.894	1.681	1.152	1.381

Table 2-6

g(T) FACTORS FOR Xe¹³⁵(Reproduced from data of Westcott⁽⁴¹⁾)

T, °K	J = 1	J = 2
300.16	1.141	1.163
400.16	1.196	1.228
500.16	1.200	1.239
600.16	1.174	-
700.16	1.133	1.178

Deutsch⁽⁴³⁾ used Bernstein's⁽⁴⁴⁾ proposed parameters for the two states with an absorption measurement in a known spectrum by Petruska, et al⁽⁴⁵⁾ and found the spin factor $g = 5/8$ parameters satisfactory. Westcott reports that a re-analysis at Chalk River (unpublished) indicates the best fit is with the $g = 5/8$ curve. The g-factors in Table 2-6 were computed by ORNL (unpublished).

Table 2-7 lists the g(T) factors computed from Henderson's⁽⁴⁶⁾ computations of Maxwellian averaged cross sections using BNL-325 data. In the case of Xe¹³⁵, the resonance parameters were obtained from the best fit to the curve in BNL-325.

The g(T) factors in Table 2-8 were computed from Maxwellian averages obtained by Roberts⁽⁴⁷⁾ using data from BNL-325. The resonance parameters used for Xe¹³⁵ were the Bernstein⁽⁴⁴⁾ values for the spin factor $g = 3/8$.

The g(T) factors computed by Campbell and Freemantle⁽⁴⁸⁾ are listed in Table 2-9. The resonance parameters used to compute the g(T) factors for Xe¹³⁵ were not reported.

Table 2-7

 $g(T)$ FACTORS FOR MAXWELL DISTRIBUTIONS(Reproduced from data of Henderson⁽⁴⁶⁾)

T, °K	Li	Cd	In	Xe ¹³⁵	Sm	Eu	Gd	Dy	Hf	W
293.16	0.999	1.329	1.021	1.147	1.606	0.956	0.885	0.981	0.900	0.990
533.16	0.996	2.172	1.070	1.215	2.170	0.812	0.749	0.926	0.689	0.996
810.94	0.991	2.831	1.142	1.101	2.165	0.973	0.612	0.878	0.585	0.990
1088.72	0.985	3.063	1.230	0.957	1.957	1.031	0.509	0.841	0.545	0.999
1366.49	0.983	3.050	1.348	0.829	1.728	1.265	0.431	0.811	0.581	0.999
1644.27	0.975	2.929	1.527	0.722	1.524	1.482	0.371	0.786	0.738	1.000
1922.05	0.970	2.760	1.820	0.635	1.352	1.657	0.323	0.766	1.039	1.000

T, °K	Au	Hg	Th	U ²³³		U ²³⁵		Pu ²³⁹	
				g _a	g _f	g _a	g _f	g _a	g _f
293.16	0.999	1.000	0.936	1.001	1.006	0.981	0.980	1.053	1.083
533.16	1.000	1.030	0.937	0.994	1.001	0.946	0.944	1.414	1.360
810.94	1.003	1.036	0.939	0.997	1.001	0.933	0.926	2.121	1.917
1088.72	1.010	1.031	0.969	1.006	1.001	0.928	0.917	2.790	2.439
1366.27	1.020	1.022	0.940	1.019	1.004	0.920	0.907	3.256	2.798
1644.27	1.033	1.014	0.940	1.037	1.011	0.909	0.895	3.526	3.000
1922.05	1.048	1.006	0.939	1.061	1.024	0.896	0.883	3.649	3.006

Table 2-8

 $g(T)$ FACTORS FOR MAXWELL DISTRIBUTIONS(Reproduced from data of Roberts⁽⁴⁷⁾)

T, °K	Cd	In	Xe ¹³⁵	Sm	Eu ¹⁵¹	Gd	Er	Hf	Ir	U ²³³		U ²³⁵		Pu ²³⁹	
										g _a	g _f	g _a	g _f	g _a	g _f
232.19	1.139	0.993	1.045	1.359	1.209	0.913	0.975	0.974	0.994	0.988	0.971	0.985	0.978	1.013	1.036
293.71	1.326	1.006	1.148	1.646	1.128	0.885	1.023	1.002	0.995	0.986	0.965	0.974	0.969	1.057	1.067
464.37	1.953	1.041	1.259	2.161	0.983	0.787	1.174	1.054	0.997	0.980	0.947	0.949	0.821	1.302	1.266
696.56	2.665	1.098	1.210	2.306	0.953	0.662	1.660	1.113	1.027	0.980	0.933	0.936	0.923	1.972	1.854
928.74	3.036	1.166	1.096	2.180	1.080	0.563	2.734	1.177	1.138	0.984	0.926	0.934	0.914	2.783	2.564
1160.93	3.153	1.245	0.979	1.983	1.292	0.485	4.300	1.284	1.390	0.993	0.923	0.932	0.908	3.431	3.166
1393.11	3.124	1.336	0.874	1.783	1.524	0.423	6.067	1.481	1.785	1.004	0.924	0.926	0.899	3.907	3.593
1451.16	3.103	1.361	0.850	1.736	1.579	0.409	6.509	1.554	1.903	1.007	0.924	0.924	0.897	3.996	3.673
1509.21	3.078	1.462	0.827	1.690	1.634	0.397	6.943	1.637	2.024	1.010	0.926	0.922	0.895	4.075	3.743
1625.30	3.020	1.586	0.783	1.603	1.736	0.373	7.784	1.841	2.278	1.018	0.928	0.918	0.891	4.203	3.858
1741.39	2.952	1.752	0.743	1.521	1.835	0.352	8.576	2.096	2.598	1.026	0.932	0.913	0.866	4.297	3.942
1857.48	2.880	1.973	0.706	1.447	1.920	0.332	9.308	2.405	2.898	1.036	0.938	0.908	0.880	4.361	3.998
1973.58	2.804	2.257	0.671	1.377	1.995	0.315	9.976	2.767	3.206	1.049	0.945	0.902	0.874	4.400	4.033
2089.67	2.727	2.613	0.640	1.311	2.061	0.299	10.580	3.179	3.520	1.062	0.954	0.896	0.868	4.417	4.049
2205.76	2.650	3.050	0.610	1.250	2.117	0.284	11.118	3.637	3.836	1.077	0.964	0.889	0.863	4.418	4.049
2321.86	2.573	3.568	0.583	1.194	2.165	0.271	11.594	4.137	4.151	1.093	0.975	0.883	0.857	4.404	4.037
2554.04	2.424	4.855	0.534	1.091	2.238	0.247	12.488	5.234	4.773	1.129	1.003	0.870	0.846	4.345	3.983
2786.23	2.283	6.466	0.492	1.002	2.285	0.226	13.083	6.421	5.374	1.173	1.038	0.858	0.834	4.255	3.902
3018.45												0.846	0.823		
3250.46												0.833	0.813		
3482.83												0.822	0.802		

Table 2-9

$g(T)$ FACTORS FOR MAXWELL DISTRIBUTIONS
(Reproduced from data of Campbell and Freemantle⁽⁴⁸⁾)

T, °K	In	Xe ¹³⁵	Sm ¹⁴⁹	Au	U ²³⁵		Pu ²³⁹		Pu ²⁴⁰	Pu ²⁴¹
	g_a	g_a	g_a	g_a	g_f	g_a	g_f	g_a	g_c	g_f
300	1.020	1.262	1.624	1.000	0.952	0.952	1.044	1.064	1.021	1.044
350	1.029	1.304	1.806	1.002	0.952	0.942	1.088	1.122	1.035	1.078
400	1.039	1.332	1.949	1.005	0.934	0.933	1.139	1.191	1.051	1.122
450	1.049	1.345	2.056	1.008	0.926	0.924	1.211	1.286	1.067	1.172
500	1.060	1.348	2.131	1.012	0.919	0.918	1.296	1.399	1.084	1.226
550	1.072	1.342	2.178	1.016	0.913	0.912	1.396	1.530	1.102	1.283
600	1.084	1.329	2.204	1.020	0.908	0.907	1.508	1.668	1.121	1.343
650	1.097	1.311	2.211	1.024	0.905	0.904	1.630	1.815	1.141	1.404
700	1.110	1.291	2.205	1.029	0.902	0.902	1.755	1.968	1.163	1.465
800		1.245	2.173		0.898	0.899	2.013	2.30	1.209	1.58
900		1.189	2.110		0.894	0.896	2.263	2.63	1.267	1.68
1000		1.127	2.032		0.890	0.893	2.494	2.93	1.345	1.77
1100		1.066	1.951		0.887	0.890	2.697	3.20	1.462	1.84

Note: For U²³⁵, $g_f = g_a = 1.000$ for all temperatures listed.

2.5 RESONANCE INTEGRALS FOR INFINITE DILUTION

The resonance integral for an infinitely dilute mixture of a resonance absorber in a moderating material is defined as^(49,50,51)

$$R.I. = \int_{E_1}^{E_2} \sigma_a \frac{dE}{E} \quad (2-21)$$

In the neighborhood of a resonance, the cross section may be computed from the Breit-Wigner⁽⁵²⁾ formula

$$\sigma_a = \frac{\pi \hbar^2 g \Gamma_n \Gamma_\gamma}{2 m E_0^{1/2} E^{1/2} \left[(E - E_0)^2 + \frac{\left\{ \left(\frac{E}{E_0} \right)^{1/2} \Gamma_n + \Gamma_\gamma \right\}^2}{4} \right]} \quad (2-22)$$

where E is the energy of the incident neutron, m is the mass of the neutron, E_0 is the energy of the resonance level, Γ_n and Γ_γ are the neutron and radiative capture widths measured at E_0 ; $g = \frac{1}{2} \frac{(2J+1)}{(2I+1)}$ is the spin factor, I being the spin of the target nucleus, J is the total angular momentum of the compound nucleus and for s-wave neutrons ($\ell = 0$), $J = I \pm \frac{1}{2}$.

The upper limit of Eq. (2-21) is taken as infinite for practical purposes and the lower limit is the energy where the neutron distribution begins to deviate from a pure Maxwellian energy spectrum, ~ 0.2 ev. Since measurements are made with cadmium filtered neutrons, E_1 is usually taken as the cadmium cut-off energy, ~ 0.4 ev.

In the neighborhood of a narrow resonance the predominant energy dependent term is $(E - E_0)^2$. Neglecting all other energy dependence and extending the limits from 0 to ∞ , Eq. (2-21) with Eq. (2-22) to this approximation gives

$$\frac{\pi^2 \hbar^2 g}{m E_0^2} \frac{\Gamma_n \Gamma_\gamma}{\Gamma} \quad (2-23)$$

where

$$\Gamma = \Gamma_n + \Gamma_\gamma$$

For many such narrow resonance levels, the contribution to the resonance integral is the sum of terms such as Eq. (2-23), i.e.,

$$\frac{\pi \hbar^2}{m} \sum_i g_i \frac{\Gamma_{n_i} \Gamma_{\gamma_i}}{E_{0i}^2 \Gamma_i} = \frac{\pi}{2} (2.60 \times 10^6) \sum_i \frac{\Gamma_{n_i} \Gamma_{\gamma_i}}{E_{0i}^2 \Gamma_i} \quad (2-24)$$

To Eq. (2-24) is added the $\frac{1}{v}$ contribution above $E_1 = 0.4$ ev which is

$$2 \sigma_a (E_{th}) \left(\frac{E_{th}}{E_1} \right)^{1/2} \quad (2-25)$$

where $\sigma_a (E_{th})$ is taken as the measured capture cross section at thermal energy, 0.0253 ev.

To account for the unresolved levels in the high energy region, a small correction is applied to Eq. (2-24) above an energy E_c . Dresner⁽⁵³⁾ has estimated these corrections by assuming that the resonances are uniformly spaced and that the respective neutron and capture widths are equal and are given by the average $\bar{\Gamma}_n$ and $\bar{\Gamma}_\gamma$ per unit energy over a small energy range. With these assumptions the correction term is equal to

$$\frac{1.3 \times 10^6}{E_c} \left(\frac{2\pi \bar{\Gamma}_\gamma}{D} \right) \left[\frac{\beta - \ln(1 + \beta)}{\beta^2} \right] \text{ barns} \quad (2-26)$$

where D is the level spacing per spin state, E_c is in ev,

$$\beta = \frac{\bar{\Gamma}_\gamma}{\bar{\Gamma}_n} \sqrt{\frac{E_0}{E_c}}$$

Kuhn⁽⁵⁴⁾ has revised the Dresner correction term by accounting for the fluctuation of the reduced neutron widths using the distribution proposed by Porter and Thomas.⁽⁵⁵⁾

The Doppler-broadened cross sections are given by:⁽⁵⁶⁾

$$\sigma_a(\theta, E) = \sigma_0 \psi(\theta, x) = \frac{\sigma_0}{\sqrt{4\pi\theta}} \int_{-\infty}^{\infty} dy \frac{1}{1+y^2} e^{-\frac{(x-y)^2}{4\theta}} \quad (2-27)$$

where

$$x = \frac{E - E_0}{\Gamma/2}, \quad \sigma_0 = \frac{2\pi \hbar^2}{m E_0} \frac{\Gamma_n}{\Gamma}, \quad \theta = \frac{4 E_0 k T}{M \Gamma^2}$$

with M the mass of the target nucleus measured in units of neutron mass, T the temperature of the material, and σ_0 the peak cross section at resonance. If the thermal motion of the nucleus is neglected, $1/\theta \rightarrow \infty$; the line shape reduces to the ordinary Breit-Wigner form. Tables of the integral as a function of various parameters, i.e., θ and x , are given in Ref. 57. The resonance integral for a single resonance is independent of the Doppler effect.

Resonance integrals for dilute solution are usually determined experimentally by activation methods. The measured value is obtained from cadmium ratios of the unknown sample, x , and of a reference standard, S , by

$$R.I.(x) = R.I.(S) \frac{Cd R(s) - 1}{Cd R(x) - 1} \frac{\sigma_{th}(s)}{\sigma_{th}(x)} \quad (2-28)$$

The standards are usually indium or gold.

This method avoids absolute counting and knowledge of the thermal and epithermal neutron fluxes. For further discussion on resonance integral measurements see Ref. 58.

In Table 2-10 are listed the measured and/or computed values of the resonance integrals of various materials. Where indicated, the computations include the estimated correction for unresolved resonances given by Dresner, and the $1/v$ contribution above cadmium cut-off.

2.6 CROSS SECTION AVERAGES OVER FISSION AND OTHER SPECTRA

2.6.1 Fission Spectrum

The average cross section for the fission of U^{238} by neutrons from the fission of U^{235} has been determined by Leachman and Schmitt.⁽⁶¹⁾ The measured value in terms of ν , the average number of neutrons per fission of U^{235} , was found to be:

$$\bar{\sigma}_f(U^{238}) = (0.756 \pm 0.008)/\nu \text{ barns} \quad (2-29)$$

Using a value of $\nu = 2.47 \pm 0.03$

$$\bar{\sigma}_f(U^{238}) = 0.306 \pm 0.007 \text{ barns}$$

Activation cross sections in fission spectra for some 61 elements have been measured by Hughes, Garth, and Levin,⁽⁶²⁾ using neutrons from a converter plate.

Recent capture cross section measurements on approximately 50 nuclei have been made by Macklin, Lazar, and Lyon,⁽⁶³⁾ using Sb-Be neutrons (25 kev).

2.6.2 Cross Sections in Natural Uranium Equilibrium Spectrum

Measurements of average cross sections in a natural uranium equilibrium spectrum are generally carried out by Snell experiments. More recently, the presence of natural uranium blankets in numerous reactor systems have enabled similar measurements of spectra and detector responses at positions within the blankets approaching the

Table 2-10

RESONANCE INTEGRALS[†]

(In Barns)

Element	Measured		Calculated		Element	Measured		Calculated	
	Activation	Absorption	Resonance	1/v		Activation	Absorption	Resonance	1/v
Li		28		31	Sb ¹²¹	162			3.0
Na ²³	~0.24	0.27	0.12 th	0.25	Sb ¹²³	~138			1.1
Mg		0.9		0.027	Te		36		2.0
Al ²⁷	~0.16	0.18		0.095	I ¹²⁷	140	130 (0.5 ev)	150 ^s	2.9
Si		0.5		0.06	Ba		7.5		0.53
P ³¹	~0.092	<2		0.084	La		11		4.0
S		0.6		0.22			5.5 (0.5 ev)		
Cl		12		14.3	Pr ¹⁴¹	11.3	7.0 (0.5 ev)		4.9
K		1.1		0.89	Nd		40 (0.5 ev)		
Ca		2		2.0	Sm ^{152a}	>1750			61
Sc ⁴⁵	~10.7			10.6	Eu ^{153a}	950			1400 ^b
Ti		3.0		2.5	Lu		720 (0.5 ev)		
V ⁵¹	~2.2			2.2	Hf	1750 ^c	2800 (0.5 ev)	2778 th	51
Cr		1.9		1.3	Hf ¹⁸⁰	21.8			4.4
Mn ⁵⁵	~11.8	10.8	12 th	5.9	Ta ¹⁸¹	590		555 ^s	9.4
Fe		2.1		1.1	W ¹⁸⁶	355	170 (0.5 ev)	180 th	15
		2.3 (0.5 ev)			Re ¹⁸⁵	1160		1630 th	44
Co ⁵⁹	49.3	48	90 th	16.3	Re ¹⁸⁷	305		310 ^s	33
Ni		4		2.0	Ir ^{191a}	3500		7680 th	420
Cu	3.7	4		1.6	Ir ^{193a}	1370		976 th	57
		3.3 (0.5 ev)			Pt		69		3.7
Cu ⁶³	4.4		0.8 ^s	1.9	Au ¹⁹⁷	1558 ^d		1390 th	45
Cu ⁶⁵	2.2		0.8 ^s	0.92	Hg		31		b
Zn		2		0.48			73 (0.5 ev)		
Ga ⁶⁹	9.2		20 ^s	0.62	Tl ²⁰³	129			4.8
Ga ⁷¹	15			1.58	Tl ²⁰⁵	0.5			0.04
As ⁷⁵	36.8	33	170 th	1.8	Pb		0.1		0.08
Br ⁷⁹	147		62 ^s	4.8	Bi		0.5		0.014
Sr		16		0.53	Th ^{232†}	67 ± 6			
Y ⁸⁹	0.91			0.62		64 ± 7			
Zr		3				68 ± 3		95	
Nb ⁹³	3.87	8.3		0.47		96 ± 6			
Mo		13		1.1	U ^{233f}			650 ^g	234
Rh ¹⁰³	656	575	1080 th	6.6	U ^{234 ‡}			710 ± 60	
Pd		23		3.6	U ^{235 ‡}				
Ag		>650		27	$\int_{0.40 \text{ ev}}^{2 \text{ Mev}} \sigma_f(E) \frac{dE}{E} = 271 \pm 25$				
Ag ¹⁰⁷	74		82 ^s	13					
Ag ¹⁰⁹	1160		1206 th	37	U ^{236 ‡}	350 ± 90		340	
In ¹¹³	1050			26	U ^{238 ‡}	281 ± 20		279 ± 7	
In ^{115a}	2640		1752 th	87	Pu ^{239f}			250 ^g	b
Sn		4.3		0.26	Pu ^{240 ‡}	8700 ± 800			
Sb		115 (0.5 ev)				9000 ± 3000		8000 ± 400	
					Pu ^{242 ‡}	1275 ± 30			
					Am ^{243 ‡}	2290			

^aA near thermal resonance leads to considerable dependence on the details of the cadmium absorber that was used.^bThe capture cross section is not 1/v near thermal energies.^cCapture gamma radiation used to determine the cross section.^dAdopted as the standard using $\sigma_{th} = 98 \times 10^{-24} \text{ cm}^2$.^fFission integral.^gGraphical estimate. For Pu²³⁹, a cutoff at 2.0 ev was used and no separate 1/v contribution.^sEstimated from level parameters.thEstimated from parameters of the first large resonance and the thermal cross section.[†]Most data are reproduced from compilation of Macklin and Pomerance,⁽⁵⁹⁾ which includes detailed references.[‡]Data by Harvey and Schwartz,⁽⁶⁰⁾ and includes the Dresner correction and 1/v contribution.

equilibrium spectrum. Snell experiments are described in a series^(64,65,66) of Oak Ridge National Laboratory Reports. The initial experiments suffered for lack of sufficient natural uranium to insure an equilibrium spectrum. In subsequent experiments this difficulty was overcome.

The Snell experiments consist of measuring the responses of various detectors at positions within a large block of natural uranium. One face of the block is fed by neutrons diffusing from a neutron source such as a reactor. The neutrons diffusing into the block cause fission and give rise to a fast neutron source a short distance into the block. Knowledge of the energy responses of the detectors allows an estimate of the equilibrium natural uranium flux spectrum.

Measured relative responses of the detectors also give an average value of the detector cross sections in an equilibrium spectrum. Traverses of the detectors through regions of the block having an equilibrium spectrum gives a measure of the relaxation or diffusion length of the equilibrium spectrum neutrons. The Oak Ridge Snell experiments⁽⁶⁶⁾ give a capture to fission ratio of 11 ± 1 ; a relaxation length 9.6 cm; Pu^{239} and U^{235} gave about equal average fission cross sections; U^{235} fission to U^{238} fission ratio per atom about 336; and the average energy of the equilibrium spectrum neutrons 250 ± 150 kev.

The spectrum and response of various detectors have been measured in the natural uranium blanket of the fast reactor, Zephyr.⁽⁶⁷⁾ Figure 2-32 shows the spectrum measured at a position 37 cm from the reactor center.⁽⁶⁸⁾ From analysis of detector ratios,⁽⁶⁸⁾ the spectrum is known to contain too much high-energy flux, indicating that complete equilibrium was not attained. Table 2-11 lists the results of detector response measurements in the Zephyr envelope in a region where the spectrum most nearly approached equilibrium.⁽⁶⁹⁾

FIG. 2-32
COMPARISON OF CALCULATED EQUILIBRIUM NEUTRON SPECTRUM
IN NATURAL URANIUM WITH SPECTRUM AT A RADIUS OF 37 cm
IN THE URANIUM BLANKET OF THE ZEPHYR REACTOR

Table 2-11

COMPARISON OF CALCULATED AVERAGE CROSS
SECTIONS OF VARIOUS DETECTORS IN THE
CALCULATED EQUILIBRIUM SPECTRUM OF
NATURAL URANIUM WITH CROSS SECTIONS
MEASURED IN THE ZEPHYR REFLECTOR

Reaction	Cross Section, barns	
	Calculated	Measured (Zephyr) ^a
Pu ²³⁹ (n, f)	1.75	1.80
U ²³⁵ (n, f)	1.55	1.8 ± 0.1
U ²³⁸ (n, f)	0.0067	0.010 ± 0.002
Np ²³⁷ (n, f)	0.18	0.16 ± 0.03
U ²³⁸ (n, γ)	0.22	0.21 ± 0.01
Au ¹⁹⁷ (n, γ)	0.49	0.43 ± 0.04

^aData of Codd, Shepard, and Tait.⁽⁶⁹⁾
Measurements made in a region where
the spectrum approached as near as
possible to the equilibrium.

Additional measurements of diffusion lengths are the ANL value⁽⁷⁰⁾ of 10.0 ± 0.2 cm, and the values in the uranium envelope of Zephyr, 12 cm. In the latter case, however, it is not believed that a true uranium equilibrium spectrum was attained.

Ten-group diffusion theory calculations of equilibrium spectrum, diffusion length and detector responses in natural uranium have been made by Meneghetti, Hummel and Loewenstein at ANL.⁽⁷¹⁾ The calculated diffusion length of 9.9 cm agrees satisfactorily with results of the Oak Ridge experiment and the ANL measurements.

The calculated equilibrium spectrum⁽⁷¹⁾ and detector responses are compared with values^(68,69) measured in the blanket of the Zephyr reactor in Fig. 2-32 and Table 2-11. The calculated ratio of $\bar{\sigma}_{n,f}$ of U²³⁵ to U²³⁸ is 231, which may be compared with the value⁽⁷²⁾ 200-215 obtained at Los Alamos by Paxton and co-workers. The measured ratio of 180 ± 30 in the Zephyr blanket is probably lower because the Zephyr blanket in the region of the detector measurements may still contain too many high-energy neutrons relative to an equilibrium spectrum. Subsequent experiments by Cranberg and Levin⁽³⁶⁾ indicate that the 45-kev and 150-kev level excitations assumed in the calculations are too small. It should also be noted that the calculated Np²³⁷ to U²³⁸ fission ratio per atom is in disagreement with recent experiments at Los Alamos⁽⁷³⁾ which give a value of 15.9.

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SECTION 3

CONSTANTS FOR THERMAL HOMOGENEOUS REACTORS

3.1 THERMAL NEUTRON SPECTRUM IN AN INFINITE HOMOGENEOUS MEDIUM

Since molecular and crystalline effects in neutron thermalization are difficult to compute, the major theoretical effort, until recently, in determining the neutron spectra in the thermal energy range has been to assume the moderator to be a mixture of a monatomic gas and $1/v$ absorber.⁽¹⁻⁴⁾ The gases are characterized by various masses and are assumed to scatter neutrons elastically.

In the case of liquid moderators (H_2O and D_2O), this approach was modified to account somewhat for the scattering which occurs below the binding energy of the molecules. The modification consists essentially of assigning an "effective mass" to the molecule below this energy and then again considering elastic scattering with a monatomic gas.^(5,6) Above this energy the scatterers are considered as free atoms with normal masses.

In the following sections, the numerical calculations and experimental data on the neutron spectrum based on the various models are presented. A review of the work on neutron thermalization may be found in the papers by Cohen^(7,8) and Poole, et al.⁽⁹⁾

3.1.1 Integral Equation for Scattering by Monatomic Gases

The integral equation describing the equilibrium energy distribution of neutrons in an infinite homogeneous absorbing gaseous medium may be written:

$$\int_0^{\infty} G(v, v_1) N(v_1) dv_1 + S_0(v) = N(v) \int_0^{\infty} G(v_1, v) dv_1 + \gamma_0(v) N(v) \quad ,$$

or

$$\int_0^{\infty} G(v, v_1) N(v_1) dv_1 + S_0(v) = [V_0(v) + \gamma_0(v)] N(v) \quad (3-1)$$

where

$N(v)dv$ = the number of neutrons per unit volume with velocity in the range v to $v + dv$

$G(v, v_1)dv$ = probability that a neutron with velocity v_1 will acquire in unit time a velocity in the range v to $v + dv$ by collisions with atoms

$V_0(v)$ = probability that a neutron with velocity v will be scattered in unit time

$\gamma_0(v)$ = probability that a neutron with velocity v will be absorbed in unit time

$S_0(v)dv$ = the number of source neutrons per unit volume per unit time in the range v to $v + dv$

In the case $S_0 = \gamma_0 = 0$, a solution to Eq. (3-1) is a Maxwellian distribution.

For elastically scattered neutrons and assuming isotropic scattering in the center of mass system, the scattering kernel $G(v_1, v)$ in the collision terms, may be written:

$$G(v_1, v) = \frac{1}{2} \int_0^\infty \int_{-1}^{+1} v_r M(v_M) \sigma_s(v_r) f(v_1, v; v_M) d\mu dv_M \quad (3-2)$$

where v_r is the relative velocity of the neutron and scattering nucleus; v_M is the velocity of the scatterer; $\sigma_s(v_r)$ is the macroscopic scattering cross section; μ is the cosine of the scattering angle; $M(v_M)$ is the normalized velocity distribution of the moderator atoms; and $f(v_1, v; v_M)$ is the probability that a neutron of velocity v is scattered to the velocity v_1 and is given by

$$\begin{aligned} f(v_1, v; v_M) &= 0 && \text{for } v_1 < v_{\min} \\ &= \frac{2v_1}{v_{\max}^2 - v_{\min}^2} && \text{for } v_{\min} \leq v_1 \leq v_{\max} \\ &= 0 && \text{for } v_1 > v_{\max} \end{aligned} \quad (3-3)$$

where

$$v_{\min} = v_c - \frac{Mv_r}{M+1},$$

$$v_{\max} = v_c + \frac{Mv_r}{M+1},$$

$$v_c = \left(\frac{1}{M+1} \right) \sqrt{v^2 + M^2 v_M^2 + 2Mv_M v \mu},$$

= the velocity of the center of mass of the neutron plus atom.

3.1.2 Spectrum in Hydrogen Gas

Wigner and Wilkins,⁽¹⁾ solve Eq. (3-1) for the case where the atoms of the moderating gas have a Maxwellian velocity distribution. In addition, the nuclei scatter independently of one another and the scattering cross section is assumed constant. The absorption cross section is assumed to obey the $1/v$ law.

With the above assumptions and performing the indicated operations in Eq. (3-2) G takes the form given by Eq. (2-10) in Section 2 except that it contains the additional factor σ_s :

$$G(v, v_1) = \sigma_s P(v, v_1)$$

Wigner and Wilkins reduce Eq. (3-1) to a Riccati equation and solve analytically for small x , the velocity measured in units of the velocity corresponding to kT , up to 0.5. A numerical integration is then used to obtain the solution in the region for $x = 0.5$ to 6.0. Above this region the asymptotic form for $N(x)$ is found to be

$$N(x) = \frac{Cx}{(x + \beta\gamma)^3}$$

where $\gamma = \gamma_0/\sigma_s$ is now defined as

$$\frac{\sigma_a(v_0) v_0}{\sigma_s}, \quad \beta^2 = \frac{1}{2kT},$$

T being the moderator temperature, C is a constant, $\sigma_a(v_0)$ is the absorption cross section at velocity v_0 .

In Fig. 3-1 numerical results for hydrogen gas are presented for various values of $\beta\gamma$, the ratio of absorption to scattering at v_0 . For weak capture Eq. (3-3) shows that the neutron density per unit speed at very high speeds is proportional to $1/v^2$, a result obtained in slowing-down theory for weak capture and for stationary moderator ($T = 0$). Any deviation from this behavior is due to absorption and not from the thermal motion of the moderator. Included in Fig. 3-1 is the straight line graph of

$$\frac{1}{x^2} \ln x^2 N(x)$$

for corresponding values of $\beta\gamma$, which shows this deviation.

FIG. 3-1
NEUTRON DENSITIES AS A FUNCTION OF THERMAL
VELOCITIES FOR SEVERAL VALUES OF THE RATIO
OF ABSORPTION TO SCATTERING ($\beta\gamma$) IN
MONATOMIC HYDROGEN
(REPRODUCED FROM REF. 1)

3.1.3 Spectrum in a Heavy Monatomic Gas

Wilkins⁽²⁾ has also investigated the solution to Eq. (3-1) in the limit of very heavy moderators. In this case, the integral equation reduces to a second order differential equation. This same equation has also been derived by Hurwitz, Nelkin and Habetler⁽³⁾ (see also Cohen⁽⁴⁾) in a somewhat different manner. These authors write Eq. (3-1) for the neutron flux rather than for the neutron density. For a $1/v$ absorption, the flux

spectrum, $\phi(\epsilon)$, where $\epsilon = E/kT$ is characterized by the parameter

$$\Delta = \frac{2A \sigma_a(kT)}{\sigma_s} = \frac{4\sigma_a(kT)}{\xi \sigma_s}$$

where A is the moderator mass in units of neutron mass; $\sigma_a(kT)$ is the capture cross section at energy kT ; σ_s is a constant; and ξ is the average logarithmic energy decrement per collision; $\xi \approx 2/A$ for a heavy atom; $\xi = 1$ for H.

The neutron energy distribution, $\epsilon\phi(\epsilon)$, in a $1/v$ absorber is plotted in Fig. 3-2 for various values of Δ . The deviation from a Maxwellian distribution, $\Delta = 0$, suggests that for small Δ , the energy distribution may be written as the sum of a Maxwellian plus a non-Maxwellian component which is proportional to a slowing-down distribution, $1/E$. In this approximation the distribution function is given as

$$\epsilon\phi(\epsilon) = M(\epsilon) + \left[\frac{\Delta}{4(1-0.79887 \Delta)} \right] H(\epsilon)$$

where the functions $M(\epsilon)$ and $H(\epsilon)$ are given in Fig. 3-3.

The asymptotic form for $H(\epsilon)$ is given as

$$H(\epsilon) = \frac{\sqrt{\pi}}{2} \sum_{n=0}^{\infty} \frac{(n+1)}{\epsilon^7} + 0(\exp[-\epsilon])$$

The symbol 0 indicates that the error terms are of the order of the quantity following in parentheses.

In Fig. 3-4, the flux distribution for a heavy moderator and for monatomic hydrogen obtained from the Wigner-Wilkins solution are compared. The asymptotic behavior of the corresponding curves are matched and the absorption is chosen so that Δ equals $4\beta\gamma$ in the two cases.

3.1.4 Spectrum in Monatomic Gases - Monte Carlo Results

Coveyou, Bate, and Osborn⁽¹⁰⁾ have applied Monte Carlo techniques in calculating the thermal flux distribution in infinite capturing media of monatomic gaseous moderators.

In Figs. 3-5 through 3-16 (reproduced from Ref. 10) are summarized the results for several different values of the parameters K and A ; where

$$K = \sqrt{2/3} \sqrt{\frac{293}{T_M}} \frac{\sigma_a}{\sigma_s} \text{ (at } 293^\circ \text{ K)}$$

A = mass of moderator, in units of neutron mass

T_M = temperature of moderator

FIG. 3-2
ENERGY DISTRIBUTION OF NEUTRONS THERMALIZED
BY A HEAVY GASEOUS MODERATOR IN THE PRESENCE
OF A $1/v$ ABSORBER. C IS THE ASYMPTOTIC
VALUE OF $\varepsilon\phi(\varepsilon)$ FOR LARGE x
(REPRODUCED FROM REF. 3)

FIG. 3-3
ENERGY DISTRIBUTIONS FOR SMALL Δ
(REPRODUCED FROM REF. 3)

FIG. 3-4
ENERGY DISTRIBUTIONS FOR MODERATORS WITH $1/v$ ABSORPTION
($\sigma_a/\varepsilon\sigma_s$ IS CONSTANT)
(REPRODUCED FROM REF. 3)

FIG. 3-5
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 0.36$, $A = 1$

FIG. 3-6
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-9}$, $A = 2$

FIG. 3-7
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-7}$, $A = 2$

FIG. 3-8
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-5}$, $A = 2$

FIG. 3-9
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-5}$, $A = 9$

FIG. 3-10
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-3}$, $A = 9$

FIG. 3-11
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 1$, $A = 9$

FIG. 3-12
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-8}$, $A = 12$

FIG. 3-13
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-5}$, $A = 12$

FIG. 3-14
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-5}$, $A = 16$

FIG. 3-15
MONTE CARLO RESULTS FOR NEUTRON
SPECTRUM, $K = 2 \times 10^{-5}$, $A = 25$

FIG. 3-16
COMPARISON OF ACTUAL FLUX COMPUTATIONS BY MONTE CARLO
METHOD AND WIGNER-WILKINS METHOD FOR $K = 0.18$ AND $A = 1$

The independent variable is $x = 16v/v_T$

where

$$v_T = \sqrt{\frac{3kT_M}{m}}$$

and m is the neutron mass. From these calculations, an effective neutron temperature, T_e , was defined which is a measure of the flux distortion from a Maxwellian. The ratio T_e/T_m is also included in the figures. The effective neutron temperature is further discussed in Section 3.2.

Comparisons are shown of the computed flux (termed "actual flux" in the figures), the Maxwellian distribution ($K = 0$), and the computed neutron density distribution. The two cases for $A = 1$; $K = 0.18$, and 0.36 (Figs. 3-16 and 3-5 respectively), are duplications of calculations performed by Wigner and Wilkins ($\beta\gamma = .226$ and $.451$, respectively).

The agreement between the two methods for the case $K = 0.18$ can be seen in Fig. 3-16 where the Wigner-Wilkins and the Monte Carlo results are plotted.

3.1.5 Spectrum in H_2O and D_2O , Including Approximations to Chemical Binding and to Velocity Dependence of Scattering

Brown and St. John^(5,6) extend the theory of monatomic gases to the case of molecular scattering in liquid (D_2O and H_2O). To account for the chemical binding effects, the Sachs and Teller⁽¹¹⁾ (see also Messiah⁽¹²⁾) method is used to assign an effective scattering mass to deuterium and hydrogen in D_2O and H_2O molecules for neutron energies below an average "vibrational threshold" energy. For neutron energies above this threshold the scattering is that of free protons and deuterons.

A mass of 3.6 and 1.9, and "threshold" energies of 0.143 and 0.226 eV were assigned to D_2O and H_2O respectively in the calculations. The mass assignment for H_2O was also obtained by Kreiger and Nelkin⁽¹³⁾ using the Zemach-Glauber⁽¹⁴⁾ rigorous formalism for molecular scattering.

A further modification to the Wigner-Wilkins treatment was in taking the scattering cross section to have a velocity dependence of the form

$$\sigma_s(v_r) = A + Be^{-\alpha v_r^2}$$

where v_r is the relative velocity of neutron and moderator atom in units such that v_r^2 is twice the relative energy measured in units of eV, with $A = 20.7$ b, $B = 13.1$ b, and $\alpha = 4.5$ for H_2O ; and $A = 3.3$ b, $B = 1.6$ b, and $\alpha = 2.88$ for D_2O . The parameters were chosen so that the experimental cross section curves were fitted.

In Figs. 3-17 and 3-18, the neutron flux spectra are shown for various values of the parameter γ , which is proportional to an effective absorption cross section, for H_2O and D_2O respectively. Figure 3-19 shows the dependence of the spectrum in D_2O on moderator temperature.

FIG. 3-17
NEUTRON DISTRIBUTIONS IN H_2O AT $80^\circ C$ FOR
DIFFERENT AMOUNTS OF ABSORPTION
(REPRODUCED FROM REF. 6)

FIG. 3-18
NEUTRON DISTRIBUTIONS IN D_2O AT $80^\circ C$ SHOWING
SPECTRAL HARDENING WITH VARYING NEUTRON ABSORPTION
(REPRODUCED FROM REF. 6)

FIG. 3-19
NEUTRON DISTRIBUTIONS IN D_2O SHOWING SPECTRAL
HARDENING WITH VARYING MODERATOR TEMPERATURE
(REPRODUCED FROM REF. 6)

3.1.6 Measurement of Flux Spectrum in H₂O Poisoned with Boric Acid - Comparison with Theoretical Distribution

Poole^(9,15) has applied pulsed source time of flight techniques in measuring the thermal neutron flux in light water containing various concentrations of boric acid.

Spectra in pure water at room temperature were measured at distances of 2, 5, 10, and 12.5 cm from the neutron source. These spectra, shown in Fig. 3-20, have been normalized so that the integral of the flux in an identifiable Maxwellian flux component is unity. In all of these cases a Maxwellian component was observed with temperature, T , equal (within experimental error) to the moderator temperature. A spectrum characterized by λ/E was observed above 0.2 ev. The value of λ closely approaches an asymptotic value of ≈ 0.015 at a distance of 10 cm from the source.

FIG. 3-20
NEUTRON FLUX SPECTRUM IN WATER
(REPRODUCED FROM REF. 15)

$$\lambda = \frac{\sqrt{\pi}}{2} \sqrt{\frac{293}{T_n}} \frac{\sigma_0}{\sigma_s - 0.7 \sigma_0}$$

where σ_s is the asymptotic scattering cross section for hydrogen and T_n is the temperature of the Maxwellian component of the flux.

In Table 3-1 are listed the results of the experiments and the values obtained by the formulas of Coveyou, et al.,⁽¹⁰⁾ and Brown.⁽⁶⁾ In Fig. 3-22 the ratios of the effective neutron temperature, T_n , in the Maxwellian component to the moderator temperature, T_M , are plotted on the basis of Poole's fitting and also in terms of the formulations of Coveyou and of Brown. Also listed are the parameters T_n and λ determined by Amster⁽¹⁶⁾ by analyses of Wigner-Wilkins calculations of the neutron flux spectra in the boric acid solutions. Amster used the monatomic gaseous hydrogen model of neutron scattering and energy exchange.

In Fig. 3-23 comparisons are presented between the results of Wigner-Wilkins calculations by Amster and experimental results by Poole, the experimental results being normalized, as before, to unit area under a best-fit hardened Maxwellian

Neutron flux spectra were measured in boric acid solutions at room temperature and elevated temperatures, with poisoning ranging from $\sigma_0 = 1.5$ barns to $\sigma_0 = 7.6$ barns where σ_0 is the effective capture cross section per hydrogen atom at 2200 meters/sec. The measured spectra along with curves indicating the fitting of a hardened Maxwellian and the $1/E$ component at higher energies are included in Fig. 3-21.

In the case of boric acid poisoning, Poole finds a general formula for λ given by

FIG. 3-21
NEUTRON FLUX SPECTRA IN
BORIC ACID SOLUTION
(REPRODUCED FROM REF. 15)

Table 3-1

SUMMARIZED RESULTS ON T_n AND λ

Moderator	σ_0 , barns	T_M , °C	Position of probe, cm from source	Measured T_n , °C	Calculated T_n , °C			Measured λ , cm (All $\pm 10\%$)	Calculated λ , cm		
					Coveyou	Brown	Amster		Poole	Amster	Brown
Tap Water	0.3	18	2	35 ± 30				0.048			
Tap Water	0.3	18	5	41 ± 30				0.033			
Tap Water	0.3	18	10	10 ± 30				0.016			
Tap Water	0.3	18	12.5	24 ± 30				0.015			
Demineralized Water	0.3	18	10	31 ± 30				0.018			
Demineralized Water	0.3	18	12.5	41 ± 30				0.016			
				Average 26 ± 15				$0.016 \pm .001$			
Boric-acid Solution	1.5	18	10	33 ± 30	40	40	30	0.067	0.069	0.072	
Boric-acid Solution	2.9	18	10	37 ± 30	59	57	47	0.130	0.137	0.139	
Boric-acid Solution	3.73	18	10	67 ± 30	72	69		0.13	0.19	0.180	
Boric-acid Solution	4.55	18	10	95 ± 35	80	75	68	0.214	0.214	0.215	0.240
Boric-acid Solution	3.03	96	10	57 ± 40				0.121	0.125		
Boric-acid Solution	4.54	97	10	59 ± 40			44	0.236	0.19		
Boric-acid Solution	7.6	98	10	28 ± 50		93	79	0.328	0.356	0.34	0.35
Boric-acid Solution	9.9	98	10	42 ± 50				0.44

FIG. 3-22
 T_n/T_M AND λ AS A FUNCTION OF σ_0
 (DATA OF POOLE¹⁵)

FIG. 3-23
NEUTRON FLUX SPECTRA IN SOLUTIONS OF BORIC ACID AND WATER
(REPRODUCED FROM REF. 16)

component below the $1/E$ range. The solid lines in the graphs are Wigner-Wilkins plots normalized to give the best fit at energies in the vicinity of maximum flux; the dashed lines are extrapolations to higher energies and are proportional to $1/E$.

3.1.7 Spectrum in a Heavy Crystalline Moderator

The calculation of the neutron spectrum in an infinite homogeneous mixture of beryllium and a heavy absorber has been considered by Nelkin.⁽¹⁷⁾

The inelastic cross section used in the numerical calculation for the spectrum was obtained from the Debye model, using the inverse mass and incoherent approximations (see Section 2). The resulting kernel for scattering from E to E' is;

$$\sigma(E', E) = \sigma_B \left(\frac{3T}{A k^2 \theta^3} \right) (E'/E)^{1/2} (E' + E) \frac{\Delta E}{kT} \frac{1}{[\exp(\Delta E/kT) - 1]} \quad \text{for } \Delta E \leq k\theta$$

$$\sigma(E', E) = 0 \quad \text{for } \Delta E > k\theta$$

where

T = temperature of moderator

θ = Debye temperature of the crystal

k = Boltzmann's constant

σ_B = bound scattering cross section

A = mass of the scatterer in units of neutron mass.

In Fig. 3-24 the results of a numerical solution for the flux are compared for a Debye crystal and for a heavy gas.

FIG. 3-24
COMPARISON OF THERMAL SPECTRA FOR $1/v$
ABSORPTION IN HEAVY CRYSTALLINE MODERATOR
WITH SPECTRA IN HEAVY GASEOUS MODERATOR
(REPRODUCED FROM REF. 17)

3.2 CHARACTERIZATION OF THE SPECTRUM IN WELL-MODERATED MEDIA; EFFECTIVE CROSS SECTIONS

3.2.1 Fitting Calculated Spectra with Hardened Maxwell Distributions.

Monte Carlo computational techniques have been applied (see Section 3.1.4) to the solution of the Wigner-Wilkins problem. The calculated flux spectra have been fitted by least square analyses with Maxwell distributions corresponding to neutron temperatures, T_n . The criterion for the fit is a minimum mean square deviation of the fluxes over the range $0 \leq x \leq 35$, where

$$x = 16 \frac{v}{\sqrt{3kT_M/m}},$$

with v the neutron velocity, T_M the moderator temperature, and m the mass of the neutron. It was found that the linear relation

$$\frac{kT_n}{kT_M} = 1 + 1.11 A \frac{\Sigma_a(kT_M)}{\Sigma_s}$$

where A is the ratio of moderator to neutron mass, applied to within 5% over the range of moderator masses from 1 to 25 and for

$$0 \leq \frac{\Sigma_a(kT_M)}{\Sigma_s} \leq 0.5.$$

Brown⁽⁶⁾ has summarized the results of thermal spectra calculations in weakly absorbing H_2O and D_2O media by least squares fits of Maxwellian distributions to the computed distributions in the thermal energy range. An expression of the form

$$\frac{kT_n}{kT_M} = 1 + A \frac{\Sigma_a(kT_M)}{\Sigma_s}$$

applies well to homogeneous D_2O systems in the range

$$0 \leq \frac{\Sigma_a(kT_M)}{\Sigma_s} \leq 0.25$$

where Σ_s is the asymptotic constant free atom scattering cross section.

In Table 3-2 the parameter A is tabulated for D_2O and H_2O systems and for a variety of moderator temperatures and absorption strengths. For D_2O systems the mean value $A = 1.92$ fits the derived temperatures within 1%. In the case of H_2O systems A decreases with increasing kT_M and with increasing $\Sigma_a(kT_M)$.

Table 3-2

VALUES OF A IN EMPIRICAL FORMULA
FOR NEUTRON TEMPERATURE

$$\frac{kT_n}{kT_M} = 1 + A \frac{\Sigma_a(kT_M)}{\Sigma_s}$$

A for D₂O Systems

$\frac{\Sigma_a(.0257\text{ev})}{kT_M}$	0.0111	0.0167	0.0260	0.520
0.0257	1.80	1.96	1.92	2.00
0.0304	1.80	1.90	1.89	1.99
0.035	1.89	1.87	1.88	2.00
0.040	1.90	1.88	1.98	2.05

A for H₂O Systems

$\frac{\Sigma_a(.0257\text{ev})}{kT_M}$	0.026	0.052	0.104	0.260	0.520
0.0257	0.876	0.985	0.876	0.854	0.810
0.0304	0.805	0.805	0.805	0.745	0.685
0.035	-	0.657	0.704	0.657	0.591
0.040	-	0.614	0.614	0.562	0.509

3.2.2 Representation of Spectrum as Hardened Maxwellian Plus Epithermal Component.

A spectrum characterization appearing in the literature is the fitting of a cut-off "1/E tail" to a cut-off Maxwellian distribution of thermal neutron flux. The thermal cut-off energy is defined by the condition that at the cut-off the slowing-down density equals the absorption under the Maxwell distribution at lower energies. Continuity of the neutron flux spectrum is achieved in this model but there is an abrupt change in the shape of the flux in the vicinity of the cut-off. Presently available measurements of the neutron spectrum (see Section 3.1.6) indicate that the flux is neither Maxwellian nor 1/E in the region where the asymptotic shapes merge. Thus it is desirable to modify this model to provide a better description of the transition range.

The theoretical spectra in weakly absorbing D₂O-moderated media described by Brown and St. John⁽⁵⁾ were analyzed by Westcott,⁽¹⁸⁾ who determined the deviation of the computed flux from a Maxwell distribution by subtracting a hardened Maxwellian. In Fig. 3-25 one of these analyses is presented in graphic form. The "1/E" law applies well for energies above 10 kT_M. The transition between a hardened Maxwellian distribution and the "1/E tail" may be fitted by a step function, stepping at μkT_n . Or, as considered by Taylor,⁽¹⁹⁾ the function

$$\frac{1}{1 + \left(\frac{\mu kT_n}{E}\right)^{16}}$$

FIG. 3-25
DIFFERENCE BETWEEN COMPUTED FLUX SPECTRUM
AND HARDENED MAXWELLIAN
(REPRODUCED FROM REF. 18)

may be used in place of the step function, as a better fit to the function $E[\phi(E) - \phi(E)_{\text{Maxwell}}]$ normalized to unity at high energy E .

On the basis of these analyses, Westcott⁽¹⁸⁾ approximated the neutron flux spectrum in such media by the expression

$$\phi(E) dE = C \left[\frac{E \exp\left(-\frac{E}{kT_n}\right)}{(kT_n)^2} + \frac{\Delta\beta}{E} \right] dE ,$$

where C is a constant selected to give the desired normalization for the integrated flux; Δ is either the unit step at $E = \mu kT_n$ or the function

$$\frac{1}{1 + \left(\frac{\mu kT_n}{E}\right)^{16}}$$

and β is a measure of the magnitude of the $1/E$ flux. The two components of the spectrum are added where they overlap in energy.

Westcott has chosen the value $\mu = 5$ as a reasonable fit to the theoretical spectra computed for weakly absorbing D_2O media.

3.2.3 Effective Cross Sections for the 2200 meter/sec Flux.

Definition of Effective Cross Sections.

For experimental work, a useful concept is the "2200 meter/sec flux" which is defined by

$$nv_0 = v_0 \int_0^{\infty} n(v) dv.$$

In this expression, $v_0 = 2200$ meters/sec, and $n(v)$ is the neutron density per unit velocity interval.

An "effective cross section" for absorption

$$\hat{\sigma} = \frac{\int_0^{\infty} \sigma(v) n(v) v dv}{v_0 \int_0^{\infty} n(v) dv}$$

may be defined. It is the 2200 meters/sec value of the microscopic absorption cross section for a $1/v$ detector which would give the same reaction rate in the given spectrum as does the nucleus under consideration.

Effective Cross Section in a Pure Maxwell Distribution of Neutron Flux

For a pure Maxwellian distribution of neutrons of temperature T_n ,

$$n(v) \propto v^2 \exp \left[-\frac{mv^2}{2kT_n} \right]$$

and

$$\hat{\sigma} = g(T_n) \sigma_0,$$

where

$$g(T_n) = \frac{\int_0^{\infty} \sigma(v) v^3 \exp \left[-\frac{mv^2}{2kT_n} \right] dv}{v_0 \sigma_0 \int_0^{\infty} v^2 \exp \left[-\frac{mv^2}{2kT_n} \right] dv}.$$

In this case the average microscopic absorption cross section, $\bar{\sigma}$, defined by

$$\bar{\sigma} = \frac{\int_0^{\infty} \sigma(v) n(v) v dv}{\int_0^{\infty} n(v) v dv}$$

is related to $\hat{\sigma}$ and σ_0 through the formula

$$\bar{\sigma} = \frac{\sqrt{\pi}}{2} \sqrt{\frac{kT_0}{kT}} \hat{\sigma} = \frac{\sqrt{\pi}}{2} \sqrt{\frac{kT_0}{kT}} g \sigma_0 ;$$

$kT_0 = 0.0253$ eV is the energy of a neutron with velocity v_0 ; $T_0 = 293.6^\circ\text{K}$ (20.4°C) and σ_0 is the cross section at 2200 meters/sec.

Values of $g(T)$ for various isotopes are given in Tables 2-5 through 2-9 (see Section 2).

Effective Cross Section in a Weakly Absorbing Medium

For a flux distribution of the form

$$\phi(E) = C \left[\frac{E e^{-E/kT_n}}{(kT_n)^2} + \frac{\Delta\beta}{E} \right],$$

$$\hat{\sigma} = \sigma_0 (g + rs),$$

where

$$s = \sqrt{\frac{4}{\pi}} \sqrt{\frac{kT_n}{kT_0}} \frac{1}{\sigma_0} \int_0^\infty \left\{ \sigma(E) - g \sigma_0 \sqrt{\frac{kT_0}{E}} \right\} \frac{\Delta}{E} dE$$

and

$$r \approx \frac{\beta}{1 + 1.01\beta} \text{ for } \mu = 5.$$

s is a measure of the excess of $1/v$ absorption in the epithermal range with respect to a $1/v$ absorber whose cross section is $g\sigma_0$ at 2200 meters/sec (not the actual 2200 meters/sec cross section, σ_0) and r is a spectral index characterizing the non-Maxwellian neutron density component.

Tables 3-3 and 3-4 give values⁽¹⁸⁾ of the $s(t)$ factors for isotopes of uranium and plutonium and a number of other elements. The subscripts a and f correspond to absorption (capture plus fission) and fission respectively.

Table 3-3

$s(T)$ FACTORS FROM SOME NON - $1/v$ ABSORBERS

(Reproduced from Ref. 18)

Neutron Temperature, $T_n, ^\circ\text{C}$	Manganese	Cobalt	Rhodium	Indium ¹¹⁵	Gold	Samarium	Samarium ¹⁴⁹
20	0.415	0.927	4.36	15.31	17.32	-0.725	-1.156
40	0.429	0.958	4.49	16.05	17.90		
60						-1.004	-1.463
80	0.456	1.017	4.74	16.83	19.00		
100						-1.189	-1.675
120	0.481	1.073	4.98	17.65	20.04		
160	0.505	1.126	5.20	18.52	21.03	-1.342	-1.861
200	0.528	1.177	5.40	19.33	21.98		
220						-1.411	-1.970
240	0.550	1.226	5.60	20.10	22.89		
280	0.570	1.273	5.79	20.85	23.76	-1.444	-2.035
320	0.591	1.318	5.97	21.54	24.59		
360	0.610	1.361	6.13	22.26	25.40	-1.433	-2.066
400	0.629	1.404	6.30	22.91	26.19		

Table 3-4

s (T) FACTORS FOR URANIUM AND PLUTONIUM

(Reproduced from Ref. 18)

Neutron Temperature T, °C	U ²³³ *		U ²³⁵		U ²³⁸	Pu ²³⁹ †			Pu ²⁴⁰ ‡	Pu ²⁴¹ †
	s _a	s _f	s _a	s _f	s _a	s _a	s _f	s _{n,γ}	s _a	s _f (est.)
20	0.969	0.910	0.033	-0.046	118.6	2.325	1.792	3.684	32.3	0.956
40	1.003	0.940	0.040	-0.042		2.362	1.824	3.735	33.4	0.963
60	1.035	0.969	0.048	-0.037	126.4	2.387	1.846	3.767	34.4	0.965
80	1.067	0.997	0.056	-0.033		2.402	1.862	3.782	35.5	0.960
100	1.097	1.024	0.063	-0.028	133.8	2.413	1.873	3.789	36.4	0.946
120	1.126	1.051	0.070	-0.024		2.414	1.878	3.783	37.4	0.927
140	1.154	1.077	0.077	-0.020	140.7	2.407	1.878	3.758		0.903
160	1.182	1.102	0.083	-0.016		2.386	1.867	3.711	39.3	0.867
180	1.209	1.126	0.089	-0.012	147.4	2.351	1.844	3.644		0.827
200	1.235	1.150	0.094	-0.009		2.301	1.809	3.556	41.0	0.780
220	1.261	1.173	0.099	-0.005	153.7	2.236	1.762	3.445	41.9	0.719
240	1.285	1.196	0.104	-0.002		2.155	1.701	3.312	42.7	0.652
260	1.309	1.218	0.108	0.000	159.9	2.061	1.628	3.166		0.578
280	1.333	1.239	0.112	0.003		1.941	1.533	2.982	44.4	0.491
300	1.356	1.260	0.115	0.005	165.8	1.797	1.419	2.760		0.400
320	1.378	1.280	0.118	0.007		1.625	1.283	2.499	45.9	0.301
340	1.400	1.300	0.119	0.008	171.5	1.426	1.123	2.197		0.198
360	1.422	1.320	0.121	0.009		1.194	0.937	1.849	47.1	0.095
380	1.443	1.339	0.123	0.008		0.934	0.727	1.461		-0.007
400	1.464	1.358	0.124	0.008	179.8	0.648	0.499	1.027	48.9	-0.102

Normalizations at 2200 m/sec

$\sigma_a = 590 \text{ b}$ $\sigma_f = 532 \text{ b}$ $\sigma_a = 697.8 \text{ b}$ $\sigma_f = 581.6 \text{ b}$ $\sigma_a = 1027.3 \text{ b}$ $\sigma_f = 738.0$ $\sigma_a = 300 \text{ b}$ $\sigma_f = 971.1 \text{ b}$

*U²³³: Experimental data indicate that s_a is slightly greater than s_f (as in table).†P²³⁹ and Pu²⁴¹: In computing values of s, a sharp epithermal cutoff at 5 kT was used.‡s_a(T°C) for Pu²⁴⁰: is given by $\sqrt{\frac{273.2 + T(^{\circ}\text{C})}{293.2}} \times 32.3$

Note: The subscripts a and f denote absorption (capture plus fission), and fission, respectively.

3.3 DETERMINATION OF THERMAL GROUP PARAMETERS

3.3.1 Pulsed Source Experiments

The following description of the application of pulsed source techniques to the measurement of thermal neutron diffusion properties of moderating systems has been outlined in part on the basis of the presentation by Antonov, et al. (20)

The basis of the pulsed source method consists of the irradiation of a sample of the moderating material by very short bursts of high-energy neutrons. After a slowing-down time of some microseconds, the neutrons diffuse as essentially thermalized neutrons, and the neutron density n satisfies the time-dependent one-group diffusion equation:

$$D\nabla^2 n - \frac{n}{\ell} = \frac{\partial n}{\partial t} \quad (3-4)$$

where ℓ is the mean neutron lifetime, and D is equal to the product of the mean neutron velocity and the usual diffusion constant used in reactor calculations which gives the ratio of the current to the negative gradient of the flux; D may be considered to be a slowly varying function of the time.

A separation of variables can be achieved by setting

$$n(x, y, z, t) = S(x, y, z) e^{-t/\ell} f(t) \quad (3-5)$$

Substituting Eq. (3-5) into Eq. (3-4) yields the two equations:

$$\frac{df(t)}{dt} = -B^2 D(t) f(t) \quad (3-6)$$

and

$$\nabla^2 S(x, y, z) = -B^2 S(x, y, z) \quad (3-7)$$

Therefore, the general solution of Eq. (3-4) should be of the form:

$$n(x, y, z, t) = \sum_{\ell, m, n} A_{\ell mn} S_{\ell mn}(x, y, z) \exp - \left[t/\ell + B_{\ell mn}^2 \int_0^t D(\xi) d\xi \right], \quad (3-8)$$

where the $A_{\ell mn}$ depend upon the initial conditions. The $S_{\ell mn}$ are the eigenfunctions of Eq. (3-7) which vanish at the extrapolated boundaries of the system and $B_{\ell mn}^2$ are the corresponding eigenvalues. The neutron density is seen to be represented by the sum of a series of exponential terms.

Assuming that Fermi Age Theory provides an adequate slowing-down model, the slowing-down density is given by:

$$\nabla^2 q = \frac{\partial q}{\partial \tau} \quad (3-9)$$

where q is the slowing-down density, and τ is the age. Equation (3-9) will have a solution of the form:

$$q(\tau) = \sum_{\ell, m, n} E_{\ell mn} S_{\ell mn}(x, y, z) \exp \left[-B_{\ell mn}^2 \tau \right], \quad (3-10)$$

where the $E_{\ell mn}$ are determined by the initial distribution of fast neutrons. Association of the slowing-down density near thermal velocities with the initial diffusion distribution implies that in Eq. (3-8):

$$A_{\ell mn} = E_{\ell mn} \exp \left[-B_{\ell mn}^2 \tau \right]. \quad (3-11)$$

Since the $E_{\ell mn}$ can be evaluated, the relative magnitudes of the terms of the expansion in Eq. (3-8) can be examined and a time, t^* , may be found such that $n(x, y, z, t)$ is given approximately by a single term of the expansion for times $t > t^*$:

$$n(x, y, z, t) = S^*(x, y, z) \exp \left[-\left(t/\ell + B^2 \int_0^t D(\xi) d\xi \right) \right], \quad (3-12)$$

where B^2 is the smallest eigenvalue of the set $B_{\ell mn}^2$. With increasing time D will approach an equilibrium value for a certain set of experimental conditions, and the neutron density will decay exponentially. The equilibrium value of D has been shown to be a function of the geometrical buckling. This dependence of D on the buckling arises from the fact that the relative leakage rate of neutrons from the system increases as the velocity of the neutron increases, and therefore the mean velocity of the remaining neutrons is smaller than in an infinite medium. The phenomenon of a reduction in neutron "temperature" in finite media has been termed "diffusion cooling." A number of approaches to the diffusion cooling effect have shown⁽²⁰⁻²²⁾ that to first order,

$$D = D_0 - CB^2, \quad (3-13)$$

where D is the equilibrium value; D_0 is the value which would obtain in an infinite medium; and the constant C takes various forms depending upon the derivation. The temperature of the medium and the rate of neutron energy loss to the medium are involved in C .

Using Eq. (3-13) in Eq. (3-12), n is seen to have the exponential time dependence, $\exp(-\alpha t)$, where

$$\alpha = 1/\ell + D_0 B^2 - CB^4. \quad (3-14)$$

From a plot of the measured value of α vs. B^2 , one may obtain ℓ , D_0 , and the square of the diffusion length, $L^2 = D_0 \ell$.

Antonov⁽²⁰⁾ used a 400-kev pulsed accelerating tube to generate neutrons by the D-T reaction. The pulse frequency ranged from 50 to 1000 cycles per second, and each pulse lasted about $6 \mu\text{sec}$. In Table 3-5 the results are listed for measurements in beryllium of density 1.78 gm/cm^3 ; in graphite bricks with a mean density of 1.67 gm/cm^3 ; and in H_2O at 23°C and at 80°C .

Beckurts⁽²²⁾ has reported neutron diffusion measurements in graphite samples of various purities and densities and ranging in dimensions from $70 \times 70 \times 70 \text{ cm}$ to $200 \times 200 \times 200 \text{ cm}$. Using a 250-kev deuteron accelerator to generate neutrons from $\text{H}^3(d, n)\text{He}^4$ reactions, Beckurts has determined the values

$$\lambda_{tr} = \frac{3D_0}{v} = 2.58 \pm 0.02 \text{ cm}$$

and

$$C = (16.3 \pm 2.5) \times 10^4 \text{ cm}^4/\text{sec}$$

for graphite of density 1.6 gm/cm^3 .

Table 3-5

RESULTS OF PULSED SOURCE EXPERIMENTS

Substance and Density	Dimensions of the System	D_0 $10^5 \text{ cm}^2/\text{sec}$	C $10^5 \text{ cm}^4/\text{sec}$	ℓ $\mu \text{ sec}$	L cm
Beryllium 1.78 gm/cm^3	From $72 \times 60 \times 56 \text{ cm}$ to $20 \times 20 \times 20 \text{ cm}$	1.22 ± 0.05	3 ± 1	4000*	22.1*
Graphite 1.67 gm/cm^3	From $200 \times 180 \times 140 \text{ cm}$ to $20 \times 20 \times 40 \text{ cm}$	1.98 ± 0.03	12.0 ± 1.9	12200 ± 500	49 ± 2.0
Water at 23°C	From 0.75 liters	0.35 ± 0.01	0.04 ± 0.01	207 ± 6	2.7 ± 0.1
Water at 80°C	to 5.6 liters	0.45 ± 0.04 $- 0.02$	0.13 ± 0.05 $- 0.07$	212 ± 6	3.1 ± 0.2 $- 0.1$

*Computed from $\ell = L^2/D_0$ using $L = 22.1 \text{ cm}$ from Geraseva, et al.⁽²³⁾

The diffusion length of thermal neutrons in BeO has been determined by the method of pulsed neutron source by Ramanna, et al.⁽²⁴⁾ The neutrons were produced using a 900-kev cascade generator utilizing the D-Be reaction. The pulse width was $30 \mu \text{sec}$ and the pulsing frequency was 600 cycles/sec. For BeO of density 2.96 gm/cm^3 , and containing less than 725 ppm of iron equivalent by weight:

$$L = 28.6 \pm 0.3 \text{ cm}$$

$$D_0 = (10.69 \pm 0.06) \times 10^4 \text{ cm}^2/\text{sec}$$

$$\ell = 7661 \pm 119 \mu \text{sec}.$$

Iyengar, et al.,⁽²⁵⁾ have reported values for BeO of density 2.96 gm/cm^3 at various temperatures.

	24°C	80°C	140°C
$D_0(10^5 \text{ cm}^2/\text{sec})$	1.18 ± 0.02	1.36 ± 0.03	1.60 ± 0.03
$C(10^5 \text{ cm}^4/\text{sec})$	3.85 ± 0.08	7.5 ± 0.2	12.4 ± 0.3

Measurements have been reported by Raievski and Horowitz for heavy water and for graphite.⁽²⁶⁾ A pulsed Sb-Be source was used for the heavy water experiments. The result at 13°C was $\lambda_{tr} = 2.45 \pm 0.07 \text{ cm}$. For the measurements in graphite, the neutron source utilized a reactor beam modulated by motion of a cadmium disk. For graphite of density 1.6 gm/cm^3 , $\lambda_{tr} = 2.55 \pm 0.09 \text{ cm}$.

Von Dardel and Sjöstrand⁽²¹⁾ have reported experimental results of pulsed source measurements in light water at 22°C . Neutrons were generated in a heavy-ice target bombarded by the pulsed deuteron beam of a 150-kev accelerator. These neutrons were filtered through a cadmium shield into an aluminum vessel containing the moderator. The authors fitted the diffusion coefficient by an expansion in powers of the geometric buckling, which to first order is given by

$$D = D_0 [1 - E B^2]$$

with $D_0 = 0.363 \pm 0.008 \text{ cm}^2/\text{sec}$ and $E = 0.20 \pm 0.04 \text{ cm}^2$. The previously defined diffusion cooling constant $C \equiv D_0 E$. From the measured value of $\ell = 204.4 \pm 2.0 \text{ } \mu\text{sec}$, the derived value for the diffusion length, $L = 2.725 \pm 0.03 \text{ cm}$ was obtained.

3.3.2 Exponential Experiments in Non-Multiplying Media

Diffusion lengths of non-multiplying media can be determined from the spatial distribution of the neutron density of exponential experiments.

The work of Gerasova, et al., (23) on Be was used by Antonov (20) in his tabulation of diffusion parameters. Gerasova applied a least squares analysis to the thermal neutron distribution in a parallelepiped ($100 \times 100 \times 84 \text{ cm}$) of commercial Be of mean density 1.78 gm/cm^3 to determine $L = 22.1 \pm 1.0 \text{ cm}$.

Koechlin (27) reported a diffusion length, $L = 32.7 \pm 0.5 \text{ cm}$ for BeO of mean density 2.92 gm/cm^3 . The purity of the beryllium was not specified. The measurement was made in a block ($100 \times 101 \times 90 \text{ cm}$) of BeO separated from the thermal column of the Saclay reactor by a block ($100 \times 101 \times 60 \text{ cm}$) of graphite inserted to damp out the higher harmonics of the neutron distribution. An extrapolation distance equal to 1.7 cm was obtained empirically, in contradiction to the value $0.71 \lambda_{tr} = 1.1 \text{ cm}$ (obtained using $\lambda_{tr} = 1.65 \text{ cm}$ for BeO of density 2.80 gm/cm^3 as reported by Stephenson (28)).

DeJuren and Rosenwasser (29) determined the diffusion length in distilled water at 22°C by analyzing the results of an exponential experiment in spherical geometry. The value obtained was $L = 2.765 \pm 0.023 \text{ cm}$.

3.3.3 Thermal Diffusion Length in Graphite

A series of diffusion length measurements was carried out on graphite stacks of various geometries in order to resolve discrepancies observed between the graphite diffusion lengths measured in the Hanford reactors, and the value measured in the Hanford Standard Pile. Richey and Block (30) reported the measurements and gave the corrections necessary for the effects of absorption and scattering of neutrons by the aluminum process tubes, and for neutrons streaming in the channels of the reactors.

Lee (31) subsequently corrected the values of Richey and Block for neutron absorption in the aluminum process tubes, atmospheric nitrogen in the machined openings of the pile, nitrogen in the graphite pores, for neutron streaming in voids, and for density of the graphite. In the above corrections, Lee used a value of 0.0145 cm^{-1} for the macroscopic thermal absorption cross section of reactor grade aluminum and 4.95 barns for the microscopic transport cross section of graphite. The average of the values of the diffusion length of various Hanford reactors is 54.17 cm adjusted to a graphite density of 1.6 gm/cm^3 . The corresponding value obtained in the Standard Pile is 54.19 cm . The results are summarized in Table 3-6.

Table 3-6

CORRECTED VALUES OF DIFFUSION LENGTH FOR GRAPHITE(Results from Lee⁽³¹⁾)

Assembly	L_{avg} Measured cm	L $\sigma_t = 4,533$ cm	L $\sigma_t = 4,95$ cm	L Nitrogen in Machined Openings cm	L Nitrogen in Graphite cm
Standard					
Pile	52.5	54.4	54.4	54.4	54.19
DR	51.60	53.84	54.26	54.59	53.98
H	54.98	55.44	55.91	56.29	55.82
C	53.63	54.01	54.50	54.87	54.42
KW	51.61	53.64	54.37	54.78	54.26
KE	50.94	53.44	54.20	54.60	54.03
Average	52.02	53.73	54.35	54.72	54.19

3.3.4 Calculation of Diffusion Coefficient in Beryllium

Bhandari⁽³²⁾ has reported the result of a calculation of the diffusion coefficient D_0 , for beryllium. Assuming a Debye frequency spectrum for beryllium, and without using the incoherent approximation made by Kothari and Singwi,⁽³³⁾ Bhandari calculated the elastic and the one-phonon inelastic scattering cross sections as functions of neutron energy in the thermal range. The calculated total scattering cross section was observed to be in good agreement with the experimental results presented in BNL-325.⁽³⁴⁾

Using the calculated curve for $\sigma_{tr}(E)$, Bhandari computed $\overline{D_0}$ (= average of the diffusion coefficient, $\lambda_{tr}(E) v/3$), by averaging over a Maxwellian neutron distribution corresponding to a neutron temperature of 27°C. Bhandari obtained a value of $\overline{D_0} = 1.26 \times 10^5$ cm²/sec for beryllium of density 1.78 gm/cm³. The experimental value of $\overline{D_0}$ obtained by Antonov, et al.,⁽²⁰⁾ is $(1.22 \pm 0.05) \times 10^5$ cm²/sec for beryllium of this density.

The value of $\overline{\sigma_{tr}}$ corresponding to the value of $\overline{D_0}$ determined by Antonov is 5.74 barns. Bhandari's calculation of $\overline{D_0}$ implies a value of 5.56 barns. The experimental value quoted by Hughes⁽³⁵⁾ is 5.65 barns.

Table 3-7 summarizes results given in Sections 3.3.1 through 3.3.4.

3.3.5 Temperature Dependence of the Thermal Diffusion Length in Water

Deutsch⁽³⁶⁾ has summarized the available experimental information concerning the diffusion length, L , in light water, and has obtained a functional relationship between L and the temperature $T(^{\circ}\text{C})$ of the water. His report is reprinted below:

Table 3-7

SUMMARY RESULTS IN SECTIONS 3.3.1 THROUGH 3.3.4

Material	Density gm/cm ³	Temp., °C	Diffusion Coefficient D ₀ 10 ⁵ cm ² /sec	Diffusion Coefficient Constant, C 10 ⁵ cm ⁴ /sec	λ _{tr} cm	L; cm	Reference
H ₂ O		22	0.363 ± .008	.073 ± .015	0.420 ± 0.013	2.725 ± 0.03	21
		23	.35 ± .01	.04 ± .01		2.7 ± 0.1	20
		80	.45 ± .04	.13 ± .05		3.1 ± 0.2	20
			± .02	± .07		± 0.1	20
		22				2.765 ± 0.023	29
D ₂ O		13			2.45 ± 0.07		26
C	1.6	Room		1.63 ± .25	2.58 ± 0.02		30
	1.67	Room	1.98 ± .03	12.0 ± 1.9		49 ± 2	20
	1.6	20			2.55 ± 0.09		26
	1.6	Room				54.17	30,31
Be	1.78	Room	1.22 ± .05	3 ± 1		22.1 ± 1	20,23
	1.78	27	1.26 (Calculation)				32
BeO	2.92	Room				32.7 ± 0.5	27
	2.96	Room	1.069 ± .006	no correction for diffusion cooling		28.6 ± 0.3	24
	2.96	24	1.18 ± .02	3.85 ± .08			25
		80	1.36 ± .03	7.5 ± .2			25
		140	1.60 ± .03	12.4 ± .3			25

"The temperature dependence of the thermal diffusion length in water has been given by Fermi⁽³⁷⁾ to be, $L = 2.64 \pm 0.0061T$ (L in cm and T in °C). This expression is based on the unpublished work of Wilson, Bragdon, and Kanner, who made several measurements in the temperature range 27.2-93°C. For a Maxwellian neutron spectrum, the analysis of these results reveals that the assumption of a $1/v$ energy dependence of the hydrogen transport cross section gives an excellent fit to the data.

"The recent measurements of Von Dardel and Sjöstrand⁽²¹⁾ yield a thermal diffusion length of 2.725 ± 0.03 cm at 22°C. This result is in very good agreement with the measurement of DeJuren and Rosenwasser⁽²⁹⁾ corrected for the effect of the buckling on the neutron temperature.⁽²¹⁾ Using this value of the thermal diffusion length and assuming the $1/v$ dependence, the transport cross section of hydrogen in water at a velocity of 2200 meters/sec is 37.14 barns.

"The diffusion length at any other temperature can be found from the value at 22°C using the following expression:

$$L = \frac{2.719}{\rho} \left[\frac{T + 273.1}{295.1} \right]^{1/2} \left[\frac{1.0614}{1 + 0.0614 \left[\frac{T + 273.1}{295.1} \right]^{1/2}} \right]^{1/2}$$

where ρ is the water density and T the temperature in °C. This expression agrees very well with the results of Wilson, Bragdon, and Kanner, as well as with the results of Antónov, et al.,⁽²⁰⁾ who made measurements at 23°C and 80°C. Using the above expression for the diffusion length and approximating it by a linear temperature dependence in the 10-30°C and 30-90°C temperature intervals, the temperature coefficient in the range 30-90°C is 0.0059 cm/°C, while in the range 10-30°C the value is 0.0049 cm/°C. This latter value is in somewhat better agreement than the Fermi prescription with the value of 0.003 cm/°C reported by Von Dardel and Sjöstrand⁽²¹⁾ over a similar temperature range."

3.3.6 Calculation of Thermal Group Parameters in Hydrogenous Mixtures

This section is based upon the work of Conerty, Petrie, Storm and Zweifel.(38,39) Using computed Wigner-Wilkins spectra⁽¹⁾ for various values of the parameter

$$\beta\gamma = \frac{\Sigma_a(kT)}{\xi \Sigma_s} \quad (3-15)$$

and for various values of the moderator temperature T , the authors evaluated the average value of a number of parameters of interest in the analysis of reactors moderated by hydrogenous mixtures. In Eq. (3-15), $\Sigma_a(kT)$ is the macroscopic absorption cross section at the energy kT , ξ is the average logarithmic energy decrement per collision, and Σ_s is the free atom macroscopic scattering cross section at energies near thermal.

For any $1/v$ absorber the average absorption cross section is a function of $\beta\gamma$ and of kT :

$$\overline{\sigma_{1/v}(\beta\gamma, kT)} = \overline{\sigma_{1/v}(E_0) E_0^{1/2} E^{-1/2}(\beta\gamma, kT)},$$

where E_0 is any convenient reference energy and the averages are taken with respect to a Wigner-Wilkins spectrum cut off at $10 kT$. Figure 3-26 is a plot of the average value, $\overline{E^{-1/2}(\beta\gamma, kT)}$, as a function of $\beta\gamma$ and kT .

In Fig. 3-27 $\overline{\sigma_a(Xe^{135})}$ is shown as a function of the same variables. The resonance parameters used were determined⁽³⁸⁾ from the slow-chopper measurements by Bernstein and Smith.⁽⁴⁰⁾

$$g = 5/8; \quad \Gamma_\gamma = 0.09493 \text{ ev}; \quad \Gamma_n^0 = 0.02057 \text{ ev} \\ E_0 = 0.08415 \text{ ev.}$$

Figure 3-28 shows the average microscopic fission cross section of U^{235} , based on the energy dependence presented in BNL-325.⁽³⁴⁾ Assuming α , the capture to fission ratio, is constant for U^{235} in the thermal range, Fig. 3-28 also gives $\overline{\sigma_a(U^{235})}$ by using

$$\overline{\sigma_a(U^{235})} = (1 + \alpha) \overline{\sigma_f(U^{235})}$$

In order to obtain the average transport mean free path, the quantity which is averaged is

$$\lambda_{tr}(E) = \left[\sum_i N_i \left(\sigma_T^i - \sigma_s^i \overline{\cos \theta^i} \right) \right]^{-1} \quad (3-16)$$

where N_i , σ_T^i , σ_s^i , and $\overline{\cos \theta^i}$ are, respectively, the number of atoms per cm^3 , the microscopic total cross section, the microscopic scattering cross section, and the average cosine of the scattering angle for the i th nuclide. For non-hydrogenous materials of mass number A_i ,

$$\overline{\cos \theta^i} = \frac{2}{3A_i}$$

while for hydrogen it is assumed that

$$\overline{\cos \theta}^H = \frac{2}{3M_{\text{eff}}(E)},$$

where $M_{\text{eff}}(E)$ is given by the Radkowsky prescription⁽⁴¹⁾ as

$$\frac{M_{\text{eff}}(E)}{1 + M_{\text{eff}}(E)} = \left[\frac{\sigma_s^H(E)}{80} \right]^{1/2}$$

Water Mixtures

Figure 3-29 shows the resultant calculated $\overline{\lambda_{\text{tr}}}$ of water as a function of $\beta\gamma$ and kT and the corresponding averages for the hydrogen in water are shown in Fig. 3-30. Figure 3-31 gives the average of the hydrogen transport cross section, and Fig. 3-32 shows the average thermal diffusion length, $\overline{L}$, for water, which is taken as

$$\overline{L} = \left[\frac{\overline{\lambda_{\text{tr}}}}{3\Sigma_a} \right]^{1/2}$$

Also given in Fig. 3-32 are some experimental values of the diffusion length of water at various temperatures. Figures 3-26 through 3-32 are reproduced from Ref. 38.

In order to simplify the evaluation of $\overline{\lambda_{\text{tr}}}$ for mixtures of water and other substances, the authors have derived series expansions which utilize the values of $\overline{\lambda_{\text{tr}}}$ for water (Fig. 3-29), or hydrogen in water (Fig. 3-30), or of σ_{tr}^H (Fig. 3-31) for hydrogen in water, but which require no further integrations.

Equation(3-16) is rewritten as

$$\lambda_{\text{tr}}(E) = \left[\Sigma_{\text{tr}}^0 + \Sigma_a' + \Sigma_{\text{tr}}' \right]^{-1} \quad (3-17)$$

where

$$\Sigma_{\text{tr}}^0(E) = N_0 \sigma_{\text{tr}}^0(E)$$

$$\Sigma_a' = \sum_m N_m \sigma_a^m(E)$$

$$\Sigma_{\text{tr}}' = \sum_m \left(1 - \frac{2}{3A_m} \right) N_m \sigma_s^m$$

The indices 0 and m refer to the hydrogenous and non-hydrogenous components of the mixture; σ_{tr} and σ_a denote microscopic transport and absorption cross sections; N , A , and σ_s are as defined after Eq. (3-16). Σ_{tr}' is assumed to be energy independent.

If $\Sigma_{\text{tr}}^0 + \Sigma_a' > \Sigma_{\text{tr}}'$ and taking

$$\Sigma_{\text{tr}}^0(E) = \Sigma_{\text{tr}}^0(E_0) \left[\frac{E_0}{E} \right]^{1/2},$$

FIG. 3-26
AVERAGE VALUE OF $E^{-1/2}$ VS ($B\gamma$, kT)

FIG. 3-29
AVERAGE TRANSPORT MEAN FREE PATH OF
WATER VS ($B\gamma$, kT)

FIG. 3-27
AVERAGE VALUE OF ABSORPTION CROSS SECTION
OF Xe^{135} VS ($B\gamma$, kT)

FIG. 3-30
AVERAGE TRANSPORT MEAN FREE PATH OF
HYDROGEN IN WATER VS ($B\gamma$, kT)

FIG. 3-28
AVERAGE VALUE OF FISSION CROSS SECTION
OF U^{235} VS ($B\gamma$, kT)

FIG. 3-31
AVERAGE TRANSPORT CROSS SECTION OF
HYDROGEN IN WATER VS ($B\gamma$, kT)

(REPRODUCED FROM REF. 38)

FIG. 3-32
AVERAGE THERMAL DIFFUSION LENGTH
FOR WATER VS TEMPERATURE
(REPRODUCED FROM REF. 38)

(which is closely true in the energy range from 0.01 to 0.2 ev) then an expansion of Eq. (3-17) can be shown to yield

$$\overline{\lambda_{tr}} = \overline{R^{-1}} \overline{\Sigma_{tr}^0(\beta\gamma, kT)^{-1}} + \sum_{n=1}^{\infty} (-1)^n \overline{R^{-1}}^{n+1} \frac{\left[\frac{E}{E_0}\right]^{\frac{n+1}{2}}}{\left[\Sigma_{tr}^0(E_0)\right]^{\frac{n}{n+1}}} \frac{[\Sigma_{tr}]^n}{[\Sigma_{tr}^0(E_0)]^{\frac{n}{n+1}}}$$

where an average value of R^{-1} has been used and

$$R(E) = 1 + \frac{\Sigma_a'(E)}{\Sigma_{tr}^0(E)}$$

$R(E)$ will be a slowly varying function of energy in the range where the Wigner-Wilkins spectrum is of importance (since Σ_a' and Σ_{tr}^0 both vary with energy approximately as $E^{-1/2}$). Finally, the average of $(E/E_0)^{\frac{n+1}{2}}$ in a Maxwellian spectrum can be written in terms of the Γ function

$$\left[\frac{E}{E_0}\right]^{\frac{n+1}{2}} = \Gamma\left(\frac{n+5}{2}\right)$$

Hence,

$$\overline{\lambda_{tr}} = \overline{R^{-1}} \overline{\lambda_{tr}^0(\beta\gamma, kT)} + \sum_{n=1}^{\infty} (-1)^n \overline{R^{-1}}^{n+1} \Gamma\left(\frac{n+5}{2}\right) \frac{[\Sigma_{tr}']^n}{[\Sigma_{tr}^0(E_0)]^{\frac{n}{n+1}}}$$

The quantity $\overline{R^{-1}}$ may be obtained either as

$$\overline{R^{-1}} \approx \left[1 + \frac{\Sigma_a'(kT)}{\Sigma_{tr}^0(kT)}\right]^{-1} \quad (3-18)$$

or

$$\overline{R^{-1}} \approx \left[1 + \frac{\Sigma_a'}{\Sigma_{tr}^0}\right]^{-1} \quad (3-19)$$

Numerical examination has indicated that either Eq. (3-18) or Eq. (3-19) leads to essentially the same result for λ_{tr} .

The authors also derive a form which can be used if $\Sigma'_{tr} > \Sigma^0_{tr} + \Sigma'_a$, but they point out that this inequality is rarely if ever satisfied in practical cases.

The accuracy of these approximations was checked for 100% water using λ_{tr} for hydrogen in water as λ^0_{tr} and the agreement between a direct calculation and the series approximation was within 4%, at worst, over a wide range of values of $\beta\gamma$ and of kT .

The above method may be applied to any hydrogenous mixture for which the deviation from $1/v$ behavior of the transport cross section of the hydrogenous component is small.

Diphenyl Mixtures

Since no measurements were available for the scattering cross section of hydrogen in diphenyl, the cross section of hydrogen in 1, 3-butadiene (C_4H_6) was used in calculating $\bar{L}$ for diphenyl ($C_{12}H_{10}$). In Fig. 3-33 the total cross section of hydrogen in 1, 3-butadiene⁽⁴²⁾ and in water⁽³⁴⁾ are compared. Figure 3-34 shows the transport cross section for the two cases as calculated from $\sigma_{tr}^H = \sigma_T^H - \sigma_s^H \cos \bar{\theta}^H$, using the prescription of Ref. 41 for $\cos \bar{\theta}^H$. Figure 3-35 shows the effect of temperature on the density ρ of diphenyl. Figure 3-36 shows $\rho \lambda_{tr}$ for diphenyl as a function of $\beta\gamma$ and kT . Figure 3-37 gives $\bar{L}$ for diphenyl as a function of temperature.

FIG. 3-33
MEASURED TOTAL CROSS SECTION OF HYDROGEN IN 1,3-BUTADIENE AND IN WATER
(REPRODUCED FROM REF. 39)

FIG. 3-34
CALCULATED TRANSPORT CROSS SECTION OF
HYDROGEN IN 1,3-BUTADIENE AND IN WATER
(REPRODUCED FROM REF. 39)

FIG. 3-35
DENSITY (ρ) OF DIPHENYL ($C_{12}H_{10}$)
VS TEMPERATURE (T)
(REPRODUCED FROM REF. 39)

FIG. 3-36
VALUES OF PRODUCT OF DENSITY AND AVERAGE TRANSPORT MEAN FREE PATH
($\rho \bar{\lambda}_{tr}$) FOR DIPHENYL VS (ρ/ρ_0 , kT)
(REPRODUCED FROM REF. 39)

FIG. 3-37
THERMAL DIFFUSION LENGTH (L)
IN DIPHENYL VS TEMPERATURE (T)
(REPRODUCED FROM REF. 39)

The diffusion length of diphenyl at 185°F has been measured by Brown⁽⁴³⁾ to be 4.88 ± 0.05 cm. Calculated values of $\bar{L}$ obtained from Fig. 3-37 are 4.94 cm using σ_{tr}^H from butadiene and 5.08 cm using σ_{tr}^H from water.

For mixtures of diphenyl (D), and other materials, $\bar{\lambda}_{tr}$ may be found to good approximation from the series expansion developed above for H₂O mixtures, replacing $\lambda_{tr}^0(\beta\gamma, kT)$ by $\lambda_{tr}^D(\beta\gamma, kT)$. In order to test the accuracy of the approximation $\bar{\lambda}_{tr}$ for a diphenyl mixture was calculated using the series expansion and the results were compared with the straightforward average over the appropriate Wigner-Wilkins spectrum. The example considered was for a 700 liter volume containing diphenyl, zirconium, and 20 kg of U²³⁵. The diphenyl volume fraction was 0.50 and kT was chosen as 0.0486. $\beta\gamma$ for this mixture is 0.118. The Wigner-Wilkins averaged result was 1.75 cm while the series approximation yielded 1.71 cm. The cross section of a hydrogen atom in diphenyl was taken equal to that in water for this calculation.

3.4 THE INFINITE MULTIPLICATION FACTOR $k_{\infty} = \epsilon p (\eta f)_{eff}$

3.4.1 The Neutron Chain

The four factor formula, $k_{\infty} = \epsilon p \eta f$, for the infinite multiplication factor arises from a simplified model of the neutron fission cycle in an infinite medium. It is convenient to define the factors as they arise in following a neutron fission chain. Starting with one primary neutron produced by thermal fission, the total number of neutrons slowing down past the fission threshold of U²³⁸ (~1.4 Mev) is termed the "fast fission factor" and is denoted by ϵ . This factor includes the fraction of fission neutrons born with energies below the threshold and the neutrons arising from the cascading fission effect of the neutrons born with energy above 1.4 Mev and thus capable of producing fissions in U²³⁸. As the neutrons, ϵ in number, slow down through the resonance absorption region of uranium, a fraction of them, $1-p$, are captured, and only ϵp slow down to thermal energies. The factor p is called the "resonance escape probability." The remaining neutrons are captured while diffusing at thermal energies, and the probability, f , that they will be captured in uranium is called the "thermal utilization" of the fuel. The cycle is now complete, for η fission neutrons are produced per absorption by fuel. Therefore $\epsilon p \eta f$ fission neutrons are produced, on the average, per fission neutron; since k_{∞} is defined as the average number of fission neutrons produced after one cycle per fission neutron, $k_{\infty} = \epsilon p \eta f$.

3.4.2 The Fast Effect, ϵ

The fast effect, ϵ , is related to the ratio, F , of epi-threshold fissions to thermal fissions by

$$\epsilon = 1 + \frac{\nu_f - \left(1 + \frac{\sum_c U^{238}}{\sum_f U^{238}} \right)}{\nu_{th}} F,$$

where

ν_{th} = average number of neutrons emitted in a thermal fission, and

ν_f = average number of neutrons emitted in a fast fission.

F is given by

$$F = \frac{F_1}{1 - \frac{\nu_f}{\nu_{th}} F_1}$$

where F_1 is the number of first generation fast fissions arising from ν_{th} fission neutrons.

General Formulation

F_1 can be expressed in a general form:

$$F_1 = \int_0^{U_{\text{threshold}}} \nu_{th} \Sigma_f^{U^{238}}(u) \phi(u) du$$

where u is the lethargy $= \ln \frac{E_0}{E}$ and E_0 is some reference energy, and $\phi(u)$ is the resultant flux at lethargy u from a single fission neutron.

Clark⁽⁴⁴⁾ used the following equation to solve for $\phi(u)$ in uranium-H₂O mixtures:

$$\phi(u) \left[\Sigma_a(u) + \Sigma_{sH}(u) \right] = S(u) + \int_0^u \phi(u') \Sigma_{sH}(u') e^{-(u-u')} du' \quad (3-20)$$

where

$S(u)$ = normalized fission spectrum

$\Sigma_{sH}(u)$ = elastic scattering cross section of the hydrogen.

$\Sigma_a(u)$ = total uranium absorption cross section, including inelastic scattering.

The approximations made in deriving Eq. (3-20) include:

- (1) A single inelastic scattering in uranium degrades neutrons to sub-threshold energies.
- (2) Elastic scattering in uranium and in oxygen may be ignored.
- (3) Elastic scattering in hydrogen is isotropic in the center of mass of the scattering system.

The solution of Eq. (3-20) is given by:

$$\phi(u) = \frac{1}{\Sigma_a(u) + \Sigma_{sH}(u)} \left\{ S(u) + \exp \left[- \int_0^u \frac{\Sigma_a(v)}{\Sigma_a(v) + \Sigma_{sH}(v)} dv \right] \right. \\ \left. \int_0^u dw S(w) \frac{\Sigma_{sH}(w)}{\Sigma_a(w) + \Sigma_{sH}(w)} \exp \left[\int_0^w \frac{\Sigma_a(y)}{\Sigma_a(y) + \Sigma_{sH}(y)} dy \right] dw \right\}$$

The method and approximations used by Clark yield numerical results for ϵ which are larger than those obtained experimentally at BNL on lattices of 0.60 diameter uranium metal rods. Since the segregation of the uranium metal into rods increases the fast effect, the results based on the homogeneous approximation should be low.

Inclusion of the effect of elastic scattering in oxygen by Weil⁽⁴⁵⁾ reduced F by $\approx 10\%$ but the value was still high compared with experimental results, even when calculated for the homogeneous mixture. Figure 3-38 compares the results of Weil's calculation for the homogeneous mixture, his calculation for 0.60 diameter uranium rods and the experimental results for the rods.

FIG. 3-38
1/F AS A FUNCTION OF THE H₂O:U VOLUME RATIO
(REPRODUCED FROM REF. 45)

Mozer⁽⁴⁶⁾ formulated an expression for ϵ (now denoted by $\bar{\epsilon}$ to avoid confusion) in terms of $\epsilon(E)$, the contribution to $\bar{\epsilon}$ at each energy above the threshold:

$$\epsilon(E) = \nu P_f(E) \bar{\epsilon} + P_{in}(E) + P_{el}(E) \epsilon(E) + P_{mod} \tilde{\epsilon}_{mod}$$

In this equation P_f , P_{in} and P_{el} are the probabilities of fission, inelastic scattering and elastic scattering in the fuel, as functions of neutron energy; P_{mod} is the probability of scattering in the moderator. $\bar{\epsilon}$ is defined by:

$$\bar{\epsilon} = \int_0^{\infty} \epsilon(E) \chi(E) dE$$

where $\chi(E)$ is the normalized fission spectrum; $\tilde{\epsilon}_{mod}$ is a constant characterizing the contribution to $\epsilon(E)$ due to collision with the moderator.

Three different approximations were proposed by Mozer in determining the contribution to $\bar{\epsilon}$ from the moderator.

- (a) A collision with moderator degrades the neutron energy to subthreshold values; then $\tilde{\epsilon}_{mod} = 1$:

$$\epsilon(E) = \nu P_f(E) \bar{\epsilon} + P_{in}(E) + P_{el}(E) \epsilon(E) + P_{mod} \quad (3-21)$$

- (b) A collision with any heavy atoms present does not change the neutron energy; a collision with hydrogen leads to removal:

$$\epsilon(E) = \nu P_f(E) \bar{\epsilon} + P_{in}(E) + [P_{el} + P_{heavy}] \epsilon(E) + P_{hydrogen} \quad (3-22)$$

where P_{heavy} is the probability that a collision will result in a scattering by a heavy atom in the moderator; i.e., $P_{heavy} = P_{mod} - P_{hydrogen}$.

- (c) A collision at energy E' with a nucleus of mass A_i (measured in units of the neutron mass) of the moderator results in a uniform distribution of neutron energy E in the range $\alpha E' \leq E \leq E'$, where $\alpha_i = (A_i - 1 / A_i + 1)^2$

$$\epsilon(E) = \nu P_f(E) \bar{\epsilon} + P_{in}(E) + P_{el}(E) \epsilon(E) + \sum_i P_{mod_i} \frac{\int_{\alpha_i E}^E \epsilon(E') dE'}{(1 - \alpha_i) E} \quad (3-23)$$

The results of calculations of ϵ for homogeneous mixtures of uranium and various other materials, H_2O , D_2O , C and Pb are listed in Table 3-8.

In Table 3-8 all columns otherwise not marked are based on "old" non-elastic cross sections for uranium, as indicated in Fig. 3-39 which also includes a graph of the "new" cross sections.

Corrections for inelastic non-removal scattering by uranium using the "old" cross sections yielded the following values for $\bar{\epsilon}$.

Volume Ratio	U- H_2O	U- D_2O
0	1.235	
1:1	1.093	1.103

Table 3-8

FAST FISSION FACTOR ($\bar{\epsilon}$) IN INFINITE HOMOGENEOUS SYSTEMS

Volume Ratio (Moderator/U)	U-H ₂ O				
	U-H ₂ O	(new $\sigma_{\text{non-el}}^{\text{U}}$)	U-D ₂ O	U-C	U-Pb*
$\bar{\epsilon}$ Corresponding to Eq. (3-21)					
0	1.227	1.170		1.227	
1:4	1.135				
1:2	1.100	1.097		1.122	
1:1	1.066	1.060		1.085	
2:1	1.039	1.037		1.053	
3:1	1.028	1.027			
5:1	1.018	1.018		1.025	
10:1				1.013	

$\bar{\epsilon}$ Corresponding to Eq. (3-22)				
0	1.227	1.170	1.227	
1:4	1.151			
1:2	1.117	1.100		
1:1	1.081	1.072	1.087	
2:1	1.051	1.048		
3:1	1.037	1.036	1.040	
5:1	1.024	1.024	1.026	

($\bar{\epsilon} = 1.2211$ for the Chemical Composition UO₂)

$\bar{\epsilon}$ Corresponding to Eq. (3-23)					
0	1.227	1.170	1.227	1.227	1.227
1:4	1.162				
1:2	1.130	1.110		1.193	
1:1	1.093	1.081	1.102	1.164	1.143
2:1	1.060	1.055		1.133	1.107
3:1	1.044	1.042	1.050		
5:1	1.028	1.028	1.032	1.078	1.063
10:1				1.044	1.037

($\bar{\epsilon} = 1.2028$ for UO₂)

*Pb is assumed to slow neutrons by inelastic scattering only

$P_{\text{in}}(E)$ in the calculation, Eq. (3-23), is replaced by $P_{\text{in}}(E) \bar{\epsilon}_{\text{in}}$. The model for inelastic scattering was taken to be the decay of the compound nucleus by an evaporation process of neutron emission:

$$\bar{\epsilon}_{\text{in}} = \frac{1}{DE} \int_0^E \epsilon(E') \exp \left[-\frac{E'}{\sqrt{DE}} \right] E' dE'$$

where $D = 0.025$ Mev.

FIG. 3-39
 "NEW" AND "OLD" NON-ELASTIC CROSS
 SECTION AS A FUNCTION OF ENERGY
 (REPRODUCED FROM REF. 46)

One Group Formulation

Using a simple one group picture above the threshold

$$F_1 = \frac{\nu \chi \Sigma_f^{U^{238}}}{\Sigma_f + \Sigma_c + \Sigma_{in}}$$

and

$$\epsilon = 1 + \frac{\left[\nu^{U^{238}} - \left(1 + \frac{\Sigma_c^{U^{238}}}{\Sigma_f^{U^{238}}} \right) \chi \Sigma_f^{U^{238}} \right]}{\Sigma_c + \Sigma_{in} + \Sigma_f - \nu^{U^{238}} \chi \Sigma_f^{U^{238}}}$$

where all cross sections are the values above the threshold, and χ is the fraction of the fission spectrum above the threshold.

Hellens⁽⁴⁷⁾ represented the fast fission factor, ϵ , in a homogeneous mixture by an even simpler formula:

$$\epsilon = 1 + \frac{\left(\nu - 1 - \frac{\Sigma_c^{U^{238}}}{\Sigma_f^{U^{238}}} \right) \Sigma_f^{U^{238}}}{\Sigma_{in} + \Sigma_c - (\nu - 1) \Sigma_f^{U^{238}}}$$

where $\Sigma_f^{U^{238}}$ is an average over the full fission spectrum. Assuming $\nu = 2.48$ and $(\Sigma_c/\Sigma_f)^{U^{238}} = 0.14$, a choice of reasonable microscopic cross sections may be made to fit the computed values of $(\epsilon - 1)$ for both H_2O-U and H_2O-UO_2 systems to within 1% for non-water to water volume ratios from 0.25 to 1. These cross sections are given in Table 3-9.

Table 3-9

ONE-GROUP "CROSS SECTIONS" FOR
FAST FISSION (IN BARNS)

	σ_f	σ_c	σ_{in}
Hydrogen	0.00	0.000	1.38
Oxygen	0.00	0.024	0.171
Uranium	0.30	0.042	2.32

3.4.3 Resonance Escape Probability

Narrow Resonance (NR) Approximation for the Resonance Escape
Probability

If there is no appreciable capture in the energy range above E^* , then for pure proton moderation the resonance escape probability is given by

$$p(E) = \exp \left[- \int_E^{E^*} \frac{\Sigma_a}{\Sigma_t} \frac{dE'}{E'} \right]$$

In the case of a mixture containing non-hydrogenous moderators an approximate formula is

$$p(E) = \exp \left[- \int_E^{E^*} \frac{\Sigma_a(E')}{\xi \Sigma_t(E')} \frac{dE'}{E'} \right] \quad (3-24)$$

where

$$\xi = \frac{\sum_i \Sigma_{si} \xi_i}{\sum_i \Sigma_{si}}$$

is the average logarithmic energy decrement per collision, and the summation extends over the various constituents of the mixture.

Equation (3-24) was derived for a distribution of narrow resonances which are sufficiently separated in energy that the collision density regains its asymptotic, age-theory form between resonances. Inside the narrow resonance the collision density is essentially the same as in the asymptotic region.

Equation (3-24) may also be written in terms of the effective resonance integral, $(R.I.)_{\text{eff}}$

$$p(E) = \exp \left[- \frac{N_0}{\xi \Sigma_s} (R.I.)_{\text{eff}} \right]$$

where N_0 is number of atoms per unit volume of the resonance absorber,

$$(R.I.)_{\text{eff}} = \int \sigma_{a0 \text{ eff}}(E') \frac{dE'}{E'}$$

$$\sigma_{a0 \text{ eff}}(E) = \frac{\sigma_{a0}(E)}{1 + \frac{\sigma_{a0}(E) + \sigma_{s0}(E)}{\sigma_p(E)}}$$

σ_{a0} and σ_{s0} are the absorption and anomalous scattering cross sections of the resonance absorber.

σ_p is the potential scattering cross section per atom of resonance absorber.

$\xi \Sigma_s$ which is ordinarily taken out of the integral over energy is defined in terms of the potential scattering cross section only.

In the case of a single resonance, the resonance escape probability in the NR approximation is given by

$$p = 1 - \frac{N_0}{\xi \Sigma_s} (R.I.)_{\text{eff}}$$

It is of interest to examine the accuracy of this formula in a special case which is amenable to exact analysis; where all cross sections are energy-independent and where the moderator is monatomic of mass A . In this case the resonance escape probability in the NR approximation has the solution (48)

$$p(E) = \exp \left[- \mu \log \frac{E^*}{E} \right]$$

where μ is the solution of the transcendental equation

$$(\mu - 1) = \frac{\Sigma_s}{(\Sigma_a + \Sigma_s)(1 - \alpha^2)} \left\{ \exp \left[(\mu - 1) \log \frac{1}{\alpha^2} \right] - 1 \right\}$$

$$\text{with } \alpha = \left(\frac{A - 1}{A + 1} \right)^{1/2}$$

In Table 3-10 a comparison is made between the exact results and the results obtained from the NR approximation for a variety of moderator masses and for a range of values Σ_a/Σ_s .

Resonance Parameters in U^{238}

In Table 3-11 the parameters of the first 18 resonances in U^{238} are summarized, including the practical widths (P.W.) for several values of the moderator energy mode, kT. The practical width is defined to be the span between energies in the

Table 3-10

COMPARISON OF RESONANCE ESCAPE PROBABILITIES
COMPUTED RIGOROUSLY AND BY SIMPLE FORMULA
OF NR APPROXIMATION
(Data from Ref. 48)

Atomic Mass	$\frac{\Sigma_{ao}}{\Sigma_s}$	μ	e	e	$-\xi \frac{N_o}{\Sigma_s} \int \frac{\sigma_{ao}}{1 + \frac{N_o \sigma_{ao}}{\Sigma_s}} \frac{dE}{E}$
12	.0005	.003168	.984285		.984287
12	.005	.031588	.853902		.854121
12	.05	.306496	.215999		.221100
12	.1	.593757	5.137×10^{-2}		5.607×10^{-2}
12	.2	1.118817	3.720×10^{-3}		5.082×10^{-3}
50	.0005	.012663	.938647		.938658
50	.005	.126248	.531931		.532480
50	.05	1.225912	2.177×10^{-3}		2.401×10^{-3}
100	.0005	.025162	.881783		.881816
100	.005	.250831	.285316		.285909

Table 3-11

RESONANCE PARAMETERS FOR U^{238}

(Reproduced from Ref. 55)

Resonance Energy, E_R , ev	Total Natural Width Γ , ev	Ratio of Neutron to Total Width $\frac{\Gamma_n}{\Gamma}$	P. W. kT = 0	P. W. kT = 0.025 ev	P. W. kT = 0.05 ev	Maximum Energy Loss in U	Maximum Energy Loss in O	P.W. (0.025 ev) $E_R(1-\alpha)$
6.7 ev	0.0265 ev	0.057	1.24 ev	1.26 ev	1.27 ev	0.11 ev	1.58 ev	11.28
21.0	0.0339	0.263	1.93	1.95	1.97	0.35	4.94	5.57
36.9	0.0575	0.565	3.63	3.65	3.65	0.62	8.68	5.93
66.3	0.050	0.500	2.22	2.26	2.30	1.11	15.6	2.05
81.3	0.0271	0.078	0.43	0.76	0.98	1.36	19.1	0.56
90	0.0251	0.0036	0.081	0.13	0	1.51	21.2	0.086
103.5	0.092	0.728	3.94	3.98	4.01	1.73	24.4	2.31
117.5	0.040	0.375	1.15	1.32	1.56	1.97	27.6	0.67
146	0.0259	0.035	0.20	0.68	0.84	2.44	34.4	0.28
166	0.029	0.138	0.43	0.98	1.28	2.78	39.1	0.35
192	0.165	0.848	5.59	5.66	5.69	3.21	45.2	1.77
212	0.085	0.706	2.50	2.63	2.75	3.55	49.9	0.74
242	0.085	0.706	2.34	2.49	2.67	4.05	56.9	0.62
258	0.0263	0.049	0.18	0.75	0.89	4.32	60.7	0.17
278	0.065	0.615	1.56	1.87	2.26	4.65	65.4	0.40
297	0.065	0.615	1.51	1.85	2.29	4.97	69.9	0.37
368	0.155	0.839	3.77	3.91	4.05	6.16	86.6	0.64
418	0.105	0.762	2.23	2.56	2.96	7.00	98.4	0.37

wings of the resonance where the Doppler-broadened total resonance cross section becomes equal to the background potential scattering per atom of resonance absorber. The maximum energy losses in collisions with uranium and oxygen are also given in Table 3-11. Additional measurements by Rosen, et al.,⁽⁵⁰⁾ are given in Table 3-12.

Doppler Broadening

Owing to the thermal motion of the fuel atoms, the resulting Doppler effect must be taken into account in evaluating the effective resonance integral. It has been shown⁽⁵¹⁻⁵³⁾ that in the neighborhood of an isolated Breit-Wigner resonance, the effective microscopic capture cross section of non-stationary atoms is $\sigma_0^A \psi(\theta, x)$, where σ_0^A is the peak value of the sum of the radiative capture (n, γ) and resonance scattering (n, n) cross section of the fuel, and $\psi(\theta, x)$ is the function:

Table 3-12

ADDITIONAL RESONANCE PARAMETER
DATA FOR U^{238}

(Reproduced from Ref. 49)

Resonance Energy, E_R , ev	Natural Width, Γ , ev	Γ_n/Γ	Accuracy of Measurement of Γ_n , %
145.6	0.0264	0.053	50
165.2	.0298	.167	25
189.1	.195	.871	25
208.3	.092	.729	30
237.3	.052	.519	30
256.7	.0253	.012	75
273.6	.053	.528	20
291	.044	.432	20
309	.026	.042	50
345.6	.072	.672	25
375	.0277	.0975	50
395	.0330	.242	40
408	.0369	.322	30
431	.0339	.262	40
460	.0321	.222	75
475	.0298	.161	75
515	.0289	.135	30
531	.0308	.188	30

$$\psi(\theta, x) = \frac{1}{4\pi\theta} \int_{-\infty}^{\infty} dy \frac{1}{1+y^2} \exp \left\{ -\frac{(x-y)^2}{4\theta} \right\}$$

In this expression,

$$x = \frac{2(E-E_R)}{\Gamma}$$

$$\theta = \left(\frac{4E_R}{\Gamma^2} \right) x \left(\frac{kT\mu}{M} \right); \quad \mu = \frac{A}{A+1}; \quad M = \text{mass of absorbing nucleus}$$

kT = mode of the Maxwell distribution of velocities of the fuel atoms

E = energy in the resonance region at which $\sigma = \sigma_0^A$

$$\beta = \frac{\sigma_p}{\sigma_0^A}$$

σ_p = total potential scattering cross section per atom of fertile material.

In terms of these parameters, the effective resonance integral in the NR approximation may be written:

$$(R.I.)_{\text{eff}} = \frac{\Gamma \gamma}{E_R} \sum_p J(\xi, \beta)$$

where
$$J(\xi, \beta) = \int_0^\infty \frac{\psi(\theta, x)}{\beta + \psi(\theta, x)} dx$$

$\Sigma_p = N_0 \sigma_p$, is the macroscopic potential scattering

$$\xi = \frac{1}{\sqrt{\theta}}$$

and Γ_γ = radiative capture width of the resonance.

Dresner⁽⁵³⁾ has published a tabulation of values of $J(\xi, \beta)$, reproduced here as Table 3-13.

Table 3-13

$J(\xi, \beta)$ VS. ξ AND k , WHERE $\beta = 2^k \times 10^{-5}$

(Reproduced from Ref. 53)

k	ξ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	4.979(2)*	4.970(2)	4.969(2)	4.968(2)	4.968(2)	4.968(2)	4.967(2)	4.967(2)	4.967(2)	4.967(2)
1	3.532	3.517	3.514	3.513	3.513	3.513	3.513	3.513	3.513	3.513
2	2.514	2.491	2.487	2.485	2.485	2.484	2.484	2.484	2.484	2.484
3	1.801	1.767	1.761	1.759	1.758	1.757	1.757	1.757	1.757	1.757
4	1.307	1.257	1.248	1.245	1.244	1.243	1.243	1.243	1.242	1.242
5	9.667(1)	8.993(1)	8.872(1)	8.831(1)	8.812(1)	8.802(1)	8.796(1)	8.792(1)	8.790(1)	8.788(1)
6	7.355	6.501	6.335	6.278	6.252	6.238	6.230	6.225	6.221	6.218
7	5.773	4.777	4.562	4.485	4.450	4.430	4.419	4.412	4.407	4.403
8	4.647	3.589	3.328	3.230	3.183	3.158	3.143	3.133	3.126	3.121
9	3.781	2.759	2.471	2.354	2.297	2.265	2.245	2.232	2.223	2.217
10	3.045	2.153	1.867	1.741	1.675	1.638	1.614	1.598	1.587	1.579
11	2.367	1.676	1.423	1.301	1.235	1.194	1.168	1.151	1.138	1.129
12	1.730	1.268	1.074	9.718(0)	9.119(0)	8.739(0)	8.484(0)	8.304(0)	8.174(0)	8.077(0)
13	1.164	9.081(0)	7.815(0)	7.087	6.629	6.322	6.107	5.950	5.833	5.744
14	7.172(0)	6.014	5.342	4.914	4.624	4.419	4.268	4.154	4.066	3.997
15	4.088	3.658	3.371	3.169	3.022	2.911	2.826	2.759	2.706	2.663
16	2.204	2.067	1.966	1.889	1.829	1.781	1.743	1.712	1.687	1.666
17	1.148	1.109	1.078	1.053	1.033	1.016	1.002	9.904(-1)	9.805(-1)	9.722(-1)
18	5.862(-1)	5.757(-1)	5.671(-1)	5.599(-1)	5.539(-1)	5.488(-1)	5.445(-1)	5.408	5.376	5.348
19	2.963	2.936	2.913	2.894	2.877	2.863	2.851	2.840	2.831	2.823
20	1.490	1.483	1.477	1.472	1.468	1.464	1.461	1.458	1.455	1.453
21	7.468(-2)	7.452(-2)	7.437(-2)	7.424(-2)	7.413(-2)	7.403(-2)	7.395(-2)	7.388(-2)	7.381(-2)	7.375(-2)
22	3.739	3.735	3.732	3.728	3.726	3.723	3.721	3.719	3.718	3.716
23	1.871	1.870	1.869	1.868	1.868	1.867	1.867	1.866	1.866	1.865
24	9.358(-3)	9.356(-3)	9.355(-3)	9.352(-3)	9.350(-3)	9.349(-3)	9.348(-3)	9.346(-3)	9.345(-3)	9.344(-3)
25	4.680	4.680	4.679	4.678	4.678	4.678	4.678	4.677	4.677	4.677
26	2.340	2.340	2.340	2.340	2.340	2.340	2.340	2.340	2.340	2.340
27	1.170	1.170	1.170	1.170	1.170	1.170	1.170	1.170	1.170	1.170
28	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)	5.851(-4)
29	2.925	2.926	2.926	2.926	2.926	2.926	2.926	2.926	2.926	2.926
30	1.463	1.463	1.463	1.463	1.463	1.463	1.463	1.463	1.463	1.463
31	7.314(-5)	7.314(-5)	7.315(-5)	7.315(-5)	7.315(-5)	7.315(-5)	7.314(-5)	7.314(-5)	7.314(-5)	7.314(-5)

* Numbers in parentheses are powers of 10 which multiply the entry next to which they stand and all unmarked entries below it.

The Effective Resonance Integral of U^{238} and Th^{232} : Theoretical and Experimental Values

Using the NR approximation Dresner⁽⁵³⁾ has calculated the effective resonance integrals of U^{238} and Th^{232} . The Brookhaven cross section compilation (BNL-325) was used in determining the contributions of the resolved resonances. In calculating the contribution, I' , of the unresolved high-energy resonances, Dresner used for formula:

$$I' = \sigma_p \frac{\Gamma_\gamma}{D} \int_{E_1}^{\infty} J(\xi, \beta) \frac{dE}{E}$$

where

D = average spacing of the resonances

E₁ = energy of the highest-energy resolved resonance

ξ was taken as identically equal to 0.1 for both U²³⁸ and Th²³².

In Table 3-14 are shown results of these computations. Dresner observed that omission of Doppler broadening led to an underestimate of ≈15% in the effective resonance integral of U²³⁸ for σ_p = 20 and 2000 barns, and ≈30% for σ_p ≈ 200 barns. The corresponding underestimations for the case of Th²³² were found to be ≈25% and ≈40%, respectively.

Table 3-14

CALCULATED EFFECTIVE RESONANCE INTEGRALS OF U²³⁸ AND Th²³²
(Data from Ref. 53)

Fertile Material	Temp., °K	Potential Scattering per Atom of Fertile Material σ_p , barns	Contribution of Resolved Resonances, barns	Contribution of Unresolved Resonances, barns	Sum (not including $\frac{1}{v}$ Contribution), barns	$\frac{1}{v}$ Contribution, barns
U ²³⁸	0	20	7.9	1.7	9.6	1.4
		200	24	4.8	29	1.4
		2000	74	11	85	1.4
	300	20	8.1	3.0	11.1	1.4
		80	17	6.6	24	1.4
		200	27	9.4	36	1.4
		2000	86	12	98	1.4
		Th ²³²	0	20	3.7	2.2
200	12			6.3	18	3.5
2000	33			14	47	3.5
300	20		4.0	4.0	8.0	3.5
	80		8.9	8.6	18	3.5
	200		15	12	27	3.5
	2000		42	20	62	3.5

In Fig. 3-40 Dresner's results⁽⁵³⁾ are compared with the experimental results compiled by Macklin and Pomerance;⁽⁵⁴⁾ the 1/v contribution is included. The experimental point for σ_p ≈ 21 barns for Th²³² at 300°K is believed to be unreliable because of failure to take into account potential scattering of the material used to hold detecting foils in place.

Narrow Resonance Infinitely Heavy Absorber Approximation (NRIA)

The NR approximation is most accurate for resonances at energies where the practical width is smaller than E_R(1 - α), for in this case a single collision with a fuel atom should remove the neutron from the resonance region. (Here, E_R is the energy at the peak of the resonance; α is the maximum fractional energy loss per collision.)

FIG. 3-40
EFFECTIVE RESONANCE CAPTURE INTEGRALS OF U^{238} AND Th^{232}
DILUTED WITH OTHER MATERIALS
(REPRODUCED FROM REF. 53)

At isolated low-energy resonance levels, a single collision in the fuel may not be sufficient to remove the neutron from the resonance, and a better approximation is to then subtract the scattering by the fuel atoms.

The formula arising from this modification of the NR approximation is termed the Narrow-Resonance-Infinitely-Heavy-Absorber approximation, abbreviated to NRIA:

$$p(E) = \exp \left[- \int_E^{E^*} \frac{\Sigma_a}{\bar{\xi}(\Sigma_t - \Sigma_s^{\text{fuel}})} \frac{dE'}{E'} \right]$$

In this formula, $\bar{\xi}$ is the logarithmic energy decrement per collision, averaged with respect to the macroscopic scattering cross section of all materials except the fuel:

$$\bar{\xi} = \frac{\sum_i \Sigma_{si} \xi_i}{\sum_i \Sigma_{si}}$$

The resonance integral in this approximation is given by

$$(R.I.)_{\text{eff}} = \int \frac{\sigma_{a0}}{1 + \frac{\sigma_{a0}}{\sigma_m}} \frac{dE}{E}$$

where σ_{a0} is again the absorption cross section and σ_m is the moderator scattering (σ_p minus potential scattering of absorber) per absorber atom. The resonance escape probability in the NRIA approximation, when expressed in terms of $(R.I.)_{\text{eff}}$ is given by

$$p = \exp \left[- \frac{N_0}{\xi \Sigma_{sm}} (R.I.)_{\text{eff}} \right]$$

where $\xi \Sigma_{sm}$ refers only to the moderator scattering.

Numerical Comparison of NR, Modified NR, NRIA Approximations,
with Exact Calculations for U^{238} -H₂O Mixtures⁽⁵⁵⁾

The NR and NRIA approximations have been tested against exact numerical results by Spinney⁽⁵⁵⁾ for various resonance levels in homogeneous mixtures of uranium and H₂O. In addition, Spinney developed and tested a modified NR approximation which treats energy degradation more accurately. A modification of the NRIA approximation has also been made by Chernick and Vernon,⁽⁵⁶⁾ which appreciably increases the accuracy of that approximation. Spinney found that for a 1:1 mixture, the NR approximation was superior to the NRIA approximation for all resonances below 418 ev except for the three lowest levels. The modified NR approximation is superior to the NR approximation at every resonance below 418 ev, and with the exception of the resonance at 36.9 ev, it is superior to the NRIA approximation at every such resonance.

In the calculation of Γ_n/Γ , Γ_γ was taken to be constant (0.025 ev) for all resonances considered. In evaluating the cross sections, no modifications were made for the effects of interference between the potential scattering and resonance cross sections.

In Table 3-15 are listed the resonance absorption probabilities as computed from the various approximate formulas and by using an exact numerical solution. It was assumed that the uranium was at room temperature, and the resonance was cut off at $E_R \pm 1$ P.W. The microscopic potential scattering cross sections were taken to be $\sigma_H = 20$ b, $\sigma_p = 10$ b.

To determine the importance of Doppler broadening, the results of computations for stationary uranium atoms are listed in Table 3-16 for two mixtures of hydrogen and uranium and may be compared with the values of Table 3-15.

The integrations in Table 3-16 extended to infinity.⁽⁵⁷⁾ To ascertain the errors arising from the termination of the resonance at $E_R \pm 1$ P.W., additional computations with these limits were made for stationary fuel atoms; the results are presented in Table 3-17 and may be compared with the values in Table 3-16. It may be observed that the neglect of absorption beyond one practical width around E_R probably leads to larger errors than are inherent in the NR approximation. A measure of the accuracy of the NR approximation for a given isolated resonance in a uranium-hydrogen mixture is afforded by graphing $(1-p)_{NR}/(1-p)_{\text{exact}}$ as a function of the parameter⁽⁵⁵⁾

$$y = \frac{P.W.}{E_R(1-\alpha)} \left[\frac{\Gamma_n}{\Gamma} - A \right]$$

Table 3-15

RESONANCE ABSORPTION PROBABILITIES IN A
HOMOGENEOUS MIXTURE WITH A 1:1 ATOMIC
RATIO OF H TO U

(Reproduced from Ref. 55)

Individual resonances computed with Doppler-broadened
cross-sections and integrated over two practical widths

E_R	$\frac{\Gamma_n}{\Gamma}$	P.W.(0.025) $E_R(1 - \alpha)$	Exact	N.R.	Modified N.R.	N.R.I.A.
6.7 ev	0.057	11.28				
21.0	0.263	5.57	0.06755	0.07455 +10.4%*	0.06551 -3.0%	0.07059 +4.5%
36.9	0.565	5.93	0.05820	0.04739 -18.6%	0.08295 +42.5%	0.06110 +5.0%
66.3	0.500	2.05				
81.3	0.078	0.56	0.009596	0.010163 +5.9%	0.009632 +0.38%	0.008109 -15.5%
90	0.0036	0.086	0.001104	0.001114 +0.91%	0.001106 +0.18%	0.000998 -9.6%
103.5	0.728	2.31	0.01593	0.01182 -25.8%	0.01698 +6.6%	0.02044 +28.3%
117.5	0.375	0.67	0.009170	0.009035 -1.5%	0.009157 -0.14%	0.009500 +3.6%
146	0.035	0.28				
166	0.138	0.35				
192	0.848	1.77	0.007119	0.005068 -28.6%	0.007274 +2.2%	0.01228 +73%
212	0.706	0.74	0.005021	0.004440 -11.6%	0.004976 -0.9%	0.007689 +53%
242	0.706	0.62				
258	0.049	0.17				
278	0.615	0.40	0.003495	0.003318 -5.1%	0.003485 -0.29%	0.004936 +41%
297	0.615	0.37				
368	0.839	0.64				
418	0.762	0.37	0.001950	0.001816 -6.9%	0.001939 -0.56%	0.003855 +98%

*Percentage after the values give the deviation from the exact result.

Table 3-16

NON-DOPPLER-BROADENED RESONANCE ABSORPTION PROBABILITIES
WITH INDIVIDUAL RESONANCES INTEGRATED TO INFINITY

(Reproduced from Ref. 55)

E_R	1:1 Atomic Ratio of H to U			10:1 Atomic Ratio of H to U		
	N.R.	Modified N.R.	N.R.I.A.	N.R.	Modified N.R.	N.R.I.A.
6.7 ev	0.2104	0.07708	0.1806	0.06071	0.06215	0.06102
21.0	0.08780	0.07539	0.08401	0.02406	0.02955	0.02728
36.9	0.05625	0.11065	0.06946	0.01522	0.02654	0.02240
66.3	0.02236	0.02613	0.02595	0.005961	0.006967	0.008183
81.3	0.006521	0.006300	0.005562	0.001672	0.001675	0.001697
90	0.001137	0.001129	0.000968	0.0002019	0.0002020	0.0002004
103.5	0.01390	0.02135	0.02174	0.003691	0.004840	0.006807
117.5	0.008259	0.008361	0.008563	0.002177	0.002248	0.002670
146	0.001782	0.001763	0.001498	0.000419	0.0004189	0.000417
166	0.002976	0.002938	0.002632	0.000761	0.000762	0.000796
192	0.005957	0.009141	0.01245	0.001573	0.002006	0.003768
212	0.004684	0.005283	0.007058	0.001257	0.001362	0.002163
242	0.003842	0.004239	0.005788	0.001010	0.001077	0.001765
258	0.000900	0.000895	0.000763	0.000206	0.0002064	0.000207
278	0.002913	0.003041	0.003842	0.000763	0.000787	0.001170
297	0.002638	0.002743	0.003479	0.000690	0.000710	0.001057
368	0.002234	0.002574	0.004520	0.000585	0.000635	0.001315
418	0.001717	0.001827	0.002867	0.000446	0.000463	0.000841

Table 3-17

NON-DOPPLER-BROADENED RESONANCE ABSORPTION PROBABILITIES
WITH INDIVIDUAL RESONANCES INTEGRATED OVER TWO
PRACTICAL WIDTHS

(Reproduced from Ref. 55)

E_R	1:1 Atomic Ratio of H to U			10:1 Atomic Ratio of H to U		
	N.R.	Modified N.R.	N.R.I.A.	N.R.	Modified N.R.	N.R.I.A.
6.7 ev	0.1775	0.07416	0.1468	0.05676	0.05799	0.05706
21.0	0.07281	0.06417	0.06892	0.02242	0.02715	0.02565
36.9	0.04651	0.08037	0.05974	0.01418	0.02371	0.02137
66.3	0.01840	0.02090	0.02196	0.005548	0.006412	0.007770
81.3	0.005333	0.005185	0.004399	0.001551	0.001553	0.001577
90	0.000901	0.000896	0.000734	0.000178	0.000178	0.000176
103.5	0.01142	0.01620	0.01924	0.003435	0.004419	0.006551
117.5	0.006783	0.006851	0.007079	0.002024	0.002086	0.002518
146	0.001454	0.001442	0.001172	0.000385	0.000385	0.000383
166	0.002439	0.002414	0.002097	0.000705	0.000706	0.000741
192	0.004893	0.006928	0.01137	0.001464	0.001834	0.003658
212	0.003846	0.004244	0.006206	0.001169	0.001259	0.002077
242	0.003154	0.003419	0.005088	0.000939	0.000997	0.001694
258	0.000733	0.000730	0.000597	0.000189	0.000189	0.000190
278	0.002391	0.002477	0.003315	0.000709	0.000730	0.001116
297	0.002165	0.002235	0.002999	0.000671	0.000690	0.001008
368	0.001833	0.002058	0.004110	0.000544	0.000587	0.001274
418	0.001409	0.001482	0.002552	0.000426	0.000442	0.000832

where $A = \frac{\sigma_p - \sigma_m}{\sigma_p}$.

In Fig. 3-41 this functional relationship is depicted on the basis of the results in Table 3-18. The parameter y is tabulated in Table 3-18, together with the resonance escape probabilities obtainable from the results in Table 3-15.

FIG. 3-41
RELATIVE ERROR OF RESONANCE CAPTURE AS DETERMINED
BY NARROW RESONANCE APPROXIMATION VS THE PARAMETER, y
(REPRODUCED FROM REF. 55)

Table 3-18

RESONANCE ESCAPE PROBABILITIES IN A
HOMOGENEOUS MIXTURE WITH A 1:1
ATOMIC RATIO OF H TO U

(Reproduced from Ref. 55)

Computed with Doppler-broadened cross-sections and
integration taken over two practical widths

E_R	y	$P_{N.R.}$	$P_{N.R.I.A.}$	P_{exact}	$P_{Mod.N.R.}$
6.7 ev	-3.06	0.81429	0.84707	(0.84707)	0.84707
21.0	-0.361	0.92545	0.92941	0.93245	0.93449
36.9	1.408	0.95261	0.93890	0.94180	0.93890
66.3	0.352	0.98044	0.97643	(0.97720)	0.97759
81.3	-0.140	0.98984	0.99189	0.99040	0.99037
90	-0.0281	0.99887	0.99900	0.99890	0.99889
103.5	0.924	0.98818	0.97956	0.98407	0.98302
117.5	0.0319	0.99097	0.99050	0.99083	0.99084
146	-0.0818	0.99611	0.99678	(0.99624)	0.99620
166	-0.0672	0.99436	0.99525	(0.99451)	0.99450
192	0.920	0.99493	0.98772	0.99288	0.99273
212	0.282	0.99556	0.99231	0.99498	0.99502
242	0.234	0.99630	0.99352	(0.99590)	0.99594
258	-0.0486	0.99785	0.99814	(0.99789)	0.99787
278	0.116	0.99668	0.99506	0.99651	0.99652
297	0.107	0.99696	0.99571	(0.99681)	0.99682
368	0.326	0.99798	0.99494	(0.99767)	0.99771
418	0.160	0.99818	0.99615	0.99805	0.99806

3.4.4 The Effective Value of ηf

The thermal utilization is defined as the fraction of thermal absorptions that occur in the fuel. For a homogeneous mixture, it is given by

$$f = \frac{\int_0^{E_c} \Sigma_a^{\text{fuel}}(E) \phi(E) dE}{\int_0^{E_c} \Sigma_a^{\text{total}}(E) \phi(E) dE}$$

where Σ_a^{fuel} and Σ_a^{total} are the fuel and total macroscopic absorption cross sections and E_c is an appropriately selected cutoff energy for the thermal group.

The average value for η , the number of neutrons emitted per absorption in fuel, is specified by the requirement that

$$\eta f = \frac{\int_0^{E_c} \eta(E) \Sigma_a^{\text{fuel}}(E) \phi(E) dE}{\int_0^{E_c} \Sigma_a^{\text{total}}(E) \phi(E) dE}$$

Therefore

$$\eta = \frac{\int_0^{E_c} \phi(E) \Sigma_a^{\text{fuel}}(E) dE}{\int_0^{E_c} \Sigma_a^{\text{fuel}}(E) \phi(E) dE}$$

An approximate value of ηf may be obtained from the cross section tabulations of Westcott (Sections 2.4 and 3.2.3).

$$\eta f \approx \frac{\sum_j \nu^j N^j \sigma_{f_0}^j (g_f^j + r s_f^j)}{\sum_{j \neq \text{fertile material}} N^j \sigma_{a_0}^j (g_a^j + r s_a^j) + \sum_{k = \text{fertile material}} N^k \sigma_{a_0}^k g_a^k}$$

where the sums are taken over all materials in the reactor except those explicitly excluded in the designation of the sum. Loss of neutrons to resonance capture by fertile atoms is accounted for by the definition of the resonance escape probability, p . Since the s -factors were computed assuming constant collision density in the epithermal range, reductions in the collision density by absorption processes have been ignored. It is believed that for most thermal reactors of current interest this approximation properly accounts for most of the epithermal events not considered in the customary evaluation of p , since a large fraction of these events occur below the lowest resonances of Th^{232} and U^{238} .

3.5 SLOWING DOWN CONSTANTS

3.5.1 Fourier Transforms of Slowing Down Kernels

The critical properties of an unreflected thermal homogeneous reactor may be related to the Fourier transform of an appropriate slowing down kernel.

The slowing down kernel $P(\underline{r}, \underline{r}', E)$ represents the probability per unit volume that a neutron originating at a point $\underline{r}'$ will reach the point $\underline{r}$ with an energy E . If $\phi(\underline{r}')$ represents the thermal flux at $\underline{r}'$, the thermal slowing down density $q(\underline{r}, E_{th})$ is given by

$$q(\underline{r}, E_{th}) = \int_{\text{Reactor}} \frac{k}{p} \Sigma_a \phi(\underline{r}') P(\underline{r}, \underline{r}', E_{th}) d\underline{r}' \quad (3-25)$$

Equation (3-25) in conjunction with

$$D \nabla^2 \phi(\underline{r}) - \Sigma_a \phi(\underline{r}) + q(\underline{r}, E_{th}) = 0 \quad (3-26)$$

and the proper boundary conditions on q and ϕ completely specify the reactor. Commonly used boundary conditions are:

a) $\phi = q = 0$ at extrapolated boundary of reactor, the extrapolation distance being assumed independent of energy.

$$\begin{aligned} \text{b) } \frac{\partial \phi}{\partial Q} &= [-0.71 \lambda_t(E_{th})]^{-1} \\ \frac{\partial q}{\partial Q} &= [-0.71 \lambda_t(E)]^{-1} \end{aligned}$$

on the physical boundary of the reactor, where $\partial/\partial Q$ is the derivative with respect to the outward normal Q , and λ_t is the transport mean free path.

A solution of Eqs. (3-25) and (3-26) with given boundary conditions may be obtained by considering an infinite system of the same compositions as the finite system.

In the infinite system the slowing down kernel $P_\infty(\underline{r}, \underline{r}', E)$ is a function of $|\underline{r} - \underline{r}'|$ and E only. This is not in general the case for $P(\underline{r}, \underline{r}', E)$, but since P and P_∞ in most cases satisfy the same differential equation P may be obtained by superimposing P_∞ 's.

In the infinite medium the slowing down density $q_\infty(\underline{r}, E_{th})$ and the thermal flux $\phi(\underline{r})$ satisfy

$$q_\infty(\underline{r}, E_{th}) = \int_{\text{All space}} \frac{k}{p} \Sigma_a \phi(\underline{r}') P_\infty(|\underline{r} - \underline{r}'|, E_{th}) d\underline{r}' \quad (3-27)$$

$$D \nabla^2 \phi(\underline{r}) - \Sigma_a \phi(\underline{r}) + q_\infty(\underline{r}, E_{th}) = 0 \quad (3-28)$$

It can be shown(58) that $\phi(\underline{r})$ in Eq. (3-28) is satisfied by any solution of

$$\nabla^2 \phi(\underline{r}) + B^2 \phi(\underline{r}) = 0$$

provided B^2 is a root of

$$-DB^2 - \Sigma_a + \frac{k}{p} \Sigma_a \bar{P}_\infty(B^2, E_{th}) = 0 \quad (3-29)$$

and where $\bar{P}_\infty(B^2, E_{th})$ is the 3-dimensional Fourier transform of $P_\infty(|\underline{r}-\underline{r}'|, E_{th})$; that is

$$\bar{P}_\infty(B^2, E_{th}) = 4\pi \int_0^\infty \frac{\sin Br}{Br} P_\infty(r, E_{th}) r^2 dr \quad (3-30)$$

It also follows from Eq. (3-27) that

$$q_\infty(\underline{r}, E_{th}) = \frac{k}{p} \Sigma_a \bar{P}_\infty(B^2, E_{th}) \phi(\underline{r})$$

and consequently $\bar{P}_\infty(B^2, E_{th})$ represents the non-leakage probability for neutrons in slowing down to thermal energy, E_{th} .

For a finite reactor in which the extrapolation distance is energy independent and both P and P_∞ satisfy the same linear equation, except for points close to the boundaries of the reactor, the solution for the flux from Eq. (3-28) is the same as the solution from Eq. (3-26). Further, the slowing down densities are identical.

The criticality condition for the finite problem is obtained from Eq. (3-29)

and

$$\frac{k}{p} \frac{\bar{P}_\infty(B^2, E_{th})}{1 + L^2 B^2} = 1$$

B^2 being identified with the geometrical buckling of the reactor.

Commonly used kernels describing the slowing down of neutrons are:

$$(a) \quad P_\infty(r, E) = \frac{\exp(-r^2/4\tau(E))}{[4\pi\tau(E)]^{3/2}} \quad \text{Gaussian (Fermi) Kernel}$$

$$\bar{P}_\infty(B^2, E) = e^{-B^2 \tau(E)} \quad \text{where } \tau(E) \text{ is the Fermi age}$$

$$(b) \quad P_\infty(\underline{r}, E) = \frac{\exp(-|\underline{r}|/L_1)}{4\pi L_1^2 |\underline{r}|} \quad \text{Diffusion (Yukawa) Kernel}$$

$$\bar{P}_\infty(B^2, E) = \frac{1}{1 + L_1^2 B^2}$$

$$(c) \quad P_\infty(\underline{r}) = \frac{\exp(-\Sigma|\underline{r}|)}{4\pi|\underline{r}|^2} \quad \text{Transport Kernel}$$

$$\bar{P}_\infty(B^2) = \frac{1}{B} \tan^{-1} \frac{B}{\Sigma}$$

Convolutions of these kernels give other representations for the slowing down kernels.

Table 3-19 summarizes some measurements,⁽⁵⁹⁾ and different methods of calculating^(60,61) $\bar{P}_\infty(B^2, E)$.

A determination of $\bar{P}_\infty(B^2, E)$ from experimental data for neutrons originating from a U^{235} fission source was recently done at Oak Ridge.⁽⁶²⁾ The Fourier transforms of indium resonance and thermal neutron slowing down kernels in water for $B < 0.9 \text{ cm}^{-1}$ are shown in Fig. 3-42. Also $\bar{P}_\infty(B^2, E_{In})$ $\bar{P}_\infty(B^2, E_{th})$ as calculated from the formulas

Table 3-19

CALCULATED AND MEASURED VALUES FOR $\bar{P}_\infty (B^2, E_{th})$

B^2	Experiment ⁽⁵⁹⁾	Gauss	Yukawa ^a	Yukawa ^b	Greuling ⁽⁶⁰⁾	Selengut- Goertzel ⁽⁶¹⁾	B_3 Method ⁽⁶¹⁾
.005	0.867	.859	0.889	.868	0.887	.870	.890
.010		.738	.800	.767	.794	.770	.786
.015		.634	.727	.687	.716	.688	.725
.020		.544	.667	.622	.649	.622	.658
.030		.402	.571	.523	.543	.520	.564
.040		.296	.500	.451	.462	.440	.491
.050		.219	.444	.397	.399	.377	.431
.075		.102	.348	.305	.290	.272	.333
.100		.048	.286	.248	.220	.202	.260

*This table reproduced from "Handbook of Nuclear Engineering," Addison-Wesley, to be published.

Gauss Kernel, $\bar{P}_\infty (B^2, E_{th}) = e^{-\tau B^2}$ and $\tau = 30.4 \text{ cm}^2$ was used.

Yukawa, $\bar{P}_\infty (B^2, E_{th}) = (1 + L_1^2 B^2)^{-1}$ $L_1^2 = 25.0 \text{ cm}^2$, $L_1^2 = 30.4 \text{ cm}^2$ were used for a and b, respectively.

Greuling, $\bar{P}_\infty (B^2, E_{th}) = [(1 + \tau_1 B^2)(1 + \tau_2 B^2)(1 + \tau_3 B^2)]^{-1}$ $\tau_1 = 20.2 \text{ cm}^2$
 $\tau_2 = 4.2 \text{ cm}^2$
 $\tau_3 = 0.6 \text{ cm}^2$ were used.

FIG. 3-42
FOURIER TRANSFORMS OF INDIUM RESONANCE AND THERMAL
NEUTRON SLOWING DOWN KERNELS IN WATER FOR
 $B < 0.9 \text{ cm}^{-1}$, U^{235} FISSION SOURCE
(REPRODUCED FROM REF. 62)

$$\bar{P}_{\infty}(B^2, E_{In}) = \frac{1}{(1 + 26.27 B^2)(1 + 2.26 B^2)^2} \quad (3-31)$$

and

$$\bar{P}_{\infty}(B^2, E_{th}) = \frac{1}{(1 + 7.344 B^2)(1 + 2.625 B^2)(1 + 26.539 B^2)} \quad (3-32)$$

are shown.

The formulas of Eqs. (3-31) and (3-32) are seen to be good fits for $B \leq 0.35 \text{ cm}^{-1}$. In Fig. 3-43 the Fourier transforms of indium resonance slowing down kernels in water from various calculations, the experimental data of Hill, Roberts and Fitch⁽⁵⁹⁾ and the value using Eq. (3-31) are compared for $B < 0.5 \text{ cm}^{-1}$.

FIG. 3-43
FOURIER TRANSFORMS OF INDIUM RESONANCE SLOWING DOWN
KERNEL IN WATER FOR $B < 0.5 \text{ cm}^{-1}$. U^{235} FISSION SOURCE
(REPRODUCED FROM REF. 62)

3.5.2 Slowing Down Moments ($\overline{r^{2n}}$) - Fission Sources

In Eq. (3-30) if $\frac{\sin Br}{Br}$ is expanded in a Taylor series and integrated term by term, there results

$$\overline{P}_{\infty}(B^2, E) = 4\pi \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} B^{2n} \int_0^{\infty} r^{2n} P_{\infty}(r, E) r^2 dr$$

Normalizing $P_{\infty}(r, E)$ such that

$$4\pi \int_0^{\infty} P_{\infty}(r, E) r^2 dr = p$$

then

$$\overline{P}_{\infty}(B^2, E) = p \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} B^{2n} \overline{r^{2n}}$$

where r^{2n} is the $2n^{\text{th}}$ moment of the slowing down kernel, that is

$$\overline{r^{2n}} = \frac{\int_0^{\infty} r^{2n} P_{\infty}(r, E) r^2 dr}{\int_0^{\infty} P_{\infty}(r, E) r^2 dr}$$

The critical equations for a reactor may now be written in terms of the even moments of the slowing down kernel as

$$\frac{k}{1 + L^2 B^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} B^{2n} \overline{r^{2n}} = 1 \quad (3-33)$$

For a large thermal reactor, $B^2 \overline{r^2} < 1$, and all the terms for $n > 1$ in Eq. (3-33) may be neglected. The criticality condition may then be written in terms of the 2nd moment, $\overline{r^2}$. Under the same circumstances the criticality condition is independent of the model used for the slowing down distribution.

For the Fermi kernel

$$\overline{r^2} = 6\tau$$

and Yukawa kernel

$$\overline{r^2} = 6L_1^2.$$

The moments of the slowing down distribution have been calculated by various methods.

Various formulas have been derived to account for the slowing down in hydrogenous mixtures. Marshak⁽⁶³⁾ derives formulas for the ratio of the second moment to the 0th moment of the neutron density under varying assumptions with regard to the cross section.

Hurwitz and Zweifel⁽⁶⁴⁾ and Hellens⁽⁶⁵⁾ present various approximate methods for slowing down and age computations. The approximations used are the P_1 , B_1 and Selengut-Goertzel method associated with P_1 and B_1 .

In both the P_L and B_L methods one starts with the space Fourier transform of the Boltzman equation. The k th Fourier component of the flux $N(k, u, \mu)$ is expanded into a series of Legendre polynomials in μ , the cosine of the angle through which a neutron is scattered. Further $\sigma(\mu_0, u', U)$, the cross section for scattering a neutron of lethargy u' through an angle $\cos^{-1} \mu_0$ with a logarithmic energy reduction $U = u - u'$, is expanded in Legendre polynomial in μ_0 . Since

$$N(k, u, \mu) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} N_{\ell}(k, u) P_{\ell}(\mu)$$

$$\sigma(\mu_0, u', U) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} B_{\ell}(u', U) P_{\ell}(\mu_0)$$

using the addition theorems for spherical harmonics, the term in the Boltzman equation

$$\int d\Omega_0 \int_{u-q}^u \sigma(\mu_0, u', U) N(k, u', \mu') du'$$

becomes

$$\sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_{\ell}(\mu) \int_{u-q}^u N_{\ell}(k, u') B_{\ell}(u', U) du'$$

where $q = \ell n \left(\frac{1+\gamma}{1-\gamma} \right)^2$ and γ is the ratio of neutron mass to the mass of the scattering nucleus. Finally the Boltzman equation for the j th Legendre component of N becomes

$$N_j(k, u) = \sum_{\ell=0}^{\infty} (2\ell+1) A_{\ell j}(k, u) \left[\sum^{\gamma} \int_{u-q}^u N_{\ell}(k, u') B_{\ell}^{\gamma}(u', U) du' + S_{\ell}(k, u) \right]$$

where $\sum^{\gamma}$ indicates sum over moderators present, S_{ℓ} is the Legendre expansion of the source term, and

$$A_{j\ell} = A_{\ell j} = \frac{1}{2} \int_{-1}^1 d\mu \frac{P_j(\mu) P_{\ell}(\mu)}{\sigma_T(u) - ik\mu}$$

and σ_T is the total cross section. When the infinite set of equations for $N_j(k, u)$ is made finite by setting

$$J_{\ell}^{\gamma} = \int_{u-q}^u N_{\ell}(k, u') B_{\ell}^{\gamma}(u', U) du'$$

equal to zero, and $S_{\ell} = 0$ for $\ell > L$, one obtains the B_L approximation.

The P_L approximation is obtained when an $L+1$ point Gauss integration formula is used in the equation for $A_{\ell j}$. This is equivalent to setting $N_{\ell} = 0$ for $\ell > L$.

In either the P_L or B_L methods there are several ways of treating the system of equations for the $N_j(k, u)$. A common method is to evaluate J_{ℓ}^{γ} by a Taylor series

expansion in ω , u being near u' . In the P_1 approximation, if one retains two terms in the Taylor expansion for the N_0 component, and one term for the N_1 component of the flux, the age-diffusion approximation is obtained. If hydrogen is present the above procedure may be modified.

A modification known as the Selengut-Goertzel (S-G) method, treats all other elements besides hydrogen as outlined above. The S-G method in effect treats correctly the energy loss of a neutron colliding with a proton but neglects the correlation between loss of energy and scattering angle.

The measurement of moments is usually accomplished by measuring in a medium the distribution of neutrons corresponding to a given energy, usually at the indium resonance (1.44 ev). The activity of a foil exposed at a distance r from the source of fission neutrons, $A(r)$, can be written:

$$A(r) \propto \int_{E_{Cd \text{ cutoff}}}^{E_{fission}} \Sigma_{In}(E) \phi(E, r) dE$$

where: Σ_{In} = macroscopic absorption cross section of indium
 $\phi(E)$ = the isotropic component of the neutron flux at the energy E and the distance r .

For non-absorbing or weakly absorbing media the above relation can be written

$$A(r) \propto \int_{E_{Cd \text{ cutoff}}}^{E_{fission}} q(r, E) \frac{\Sigma_{In}(E)}{\xi \Sigma_s} \frac{dE}{E}$$

where $q(r, E)$ = slowing down density.

Since the absorption of the cadmium covered indium foil is nearly zero except at the resonance, the distribution is proportional to the slowing down density at 1.44 ev. The age is defined as:

$$\tau(E_0, E) = \frac{\bar{r}^2(E_0, E)}{6} = \frac{\int_V r^2 q(r, E) dV}{\int_V q(r, E) dV},$$

the integral being extended over all space. In terms of the activity, $A(r)$, the age becomes

$$\tau(\text{fission to 1.44 ev}) = \frac{\bar{r}^2}{6} = \frac{1}{6} \frac{\int_0^\infty r^4 A(r) dr}{\int_0^\infty r^2 A(r) dr}$$

The data on moments for various materials and mixtures follows:

H₂O

Table 3-20 (Ref. 66) compares various calculated with experimental values for fission neutrons in water at indium resonance.

Table 3-20

COMPARISON OF MOMENTS FOR FISSION NEUTRONS
IN WATER AT 1.44 ev

$\bar{r}^n$ (cm) ⁿ	n=2	n=4	n=6	n=8
Experiment	185	1.222 x 10 ^{5**}	2.270 x 10 ⁸	8.642 x 10 ¹¹
Rigorous Moments*	154	0.979	1.934	8.753
P ₁	149	0.742	0.919	1.502
B ₁	149	0.912	1.391	2.501
S-G	184	1.422	2.542	4.924
S-G Modified	184	1.694	3.470	7.091

*From Boltzman Equation with oxygen degradation neglected; calculation for 2.03 ev, reprinted from Ref. 66, p. 73

**All succeeding numbers in a given column to be multiplied by same factor.

The moments of the flux distribution in H₂O from a disc source were determined at Oak Ridge National Laboratory.⁽⁵⁹⁾

Table 3-21 (Ref. 62) shows the moments obtained in an extension of the work in Ref. 59. In order to compute the higher moments, both P_∞(r, E_{th}) and P_∞(r, E_{In}) had to be extrapolated by assuming:

$$P_{\infty}(r, E) \propto \frac{e^{-r/\lambda}}{r^2}$$

where $\lambda_{th} = 11.017$ cm for $r > 160$ cm and $\lambda_{In} = 8.786$ cm for $r > 92$ cm. The fractions of both $(\bar{r}^{2n})_{th}$ and $(\bar{r}^{2n})_{In}$ which depend on the extrapolated data are also shown in Table 3-21.

Table 3-21

SUMMARY OF THE MOMENTS OF VARIOUS SLOWING DOWN DISTRIBUTIONS IN WATER

n	$\bar{r}_{th}^n$ (a)	f.e.(c)	$\bar{r}_{In}^n$	f.e.(c)	From P _∞ (r, E _{th}), cm ⁿ
	cm ⁿ		cm ⁿ		
2	(2.348 ± 0.09) x 10 ^{2(b)}	3.08 x 10 ⁻⁶	1.847 x 10 ²	1.40 x 10 ⁻⁵	1.847 x 10 ²
4	(1.592 ± 0.13) x 10 ^{5(b)}	1.57 x 10 ⁻⁴	1.222 x 10 ⁵	7.95 x 10 ⁻⁴	1.210 x 10 ⁵
6	(2.769 ± 0.35) x 10 ^{8(b)}	2.75 x 10 ⁻³	2.27 x 10 ⁸	1.30 x 10 ⁻²	2.23 x 10 ⁸
8	(1.084 ± 0.18) x 10 ^{12(b)}	2.22 x 10 ⁻²	8.6 x 10 ¹¹	8.57 x 10 ⁻²	9.2 x 10 ¹¹
10	(8.230 ± 1.7) x 10 ^{15(b)}	9.59 x 10 ⁻²		3.33 x 10 ⁻¹	

(a) Data from experiments in Ref. 62.

(b) Uncertainty due to estimated maximum possible error in measuring the slope of the kernel. Theory indicates a much larger error.

(c) Fraction extrapolated.

Barkov, et al.⁽⁶⁷⁾ report a value of $29.4 \pm 1.5 \text{ cm}^2$ for the age to indium resonance in H_2O . A concurrent measurement⁽⁶⁸⁾ of the age from indium resonance to thermal (0.025 eV) gave a value of $1.0 \pm .5 \text{ cm}^2$.

Graphite

The age in graphite was measured at ORNL.⁽⁶⁹⁾ These results for the first three even moments are expressed in terms of the expansion of $e^{-B^2\tau}$ ($B^2\tau \ll 1$) and averaged over the fission spectrum:

As given by Hill,⁽⁶⁹⁾

$$\begin{aligned}\overline{\tau} &= 329 \text{ cm}^2 \\ \overline{\tau^2} &= 14.8 \times 10^4 \text{ cm}^4 \\ \overline{\tau^3} &= 13.3 \times 10^7 \text{ cm}^6.\end{aligned}$$

Recalculated values⁽⁷⁰⁾ of $\overline{r^2} = 1864 \text{ cm}^2$, $\overline{r^4} = 6.89 \times 10^6 \text{ cm}^4$, and $\overline{r^6} = 4.4 \times 10^{10} \text{ cm}^6$ were obtained for an adjusted density, $\rho = 1.60 \text{ gm/cm}^3$ and correcting to a point source.

Beryllium

The age in beryllium was measured at Argonne National Laboratory⁽⁷¹⁾ using a small piece of enriched uranium for a point source. These results are

$$\begin{aligned}\tau_{\text{In}} &= 80.2 \pm 2 \text{ cm}^2 \\ \tau_{\text{th}} &= 97.2 \text{ cm}^2.\end{aligned}$$

D₂O with H₂O Admixtures

A series of measurements of age from fission to indium resonance was made at the Savannah River Laboratory⁽⁷²⁾ for various D_2O - H_2O mixtures. The measurements are summarized in Table 3-22.

Table 3-22

AGE OF D_2O - H_2O MIXTURES

(Reproduced from Ref. 72)

Mole % D_2O	$\tau_{\text{In}}, \text{cm}^2$	$\lambda,^* \text{cm}$	Fraction
			Extrapolated Area
99.8	109 ± 3	8.7	.13
99.0	107 ± 5	8.7	.14
98.4	106 ± 3	9.3	.15
95.3	93 ± 3	9.0	.13
94.0	86 ± 3	7.5	.08
91.8	78 ± 3	6.7	.09

*Extrapolation beyond 30 cm was based on an assumed distribution, $e^{-r/\lambda}/r^2$, where λ has the value given.

Another measurement of τ_{In} in D_2O is reported by Martin⁽⁷³⁾ as $107 \pm 5 \text{ cm}^2$.

H₂O-Uranium

A series of such measurements using natural or slightly enriched uranium was performed at Oak Ridge National Laboratory.⁽⁷⁴⁾ The techniques of measurement were the standard ones and the mixture was in the form of a lattice with various spacings and various size rods. The experiments were directed toward maximizing k_{∞} of such a lattice. An annular air gap is included in some of the experiments to check the predicted increase in k_{∞} . The results of these experi-

ments are summarized in Table 3-23. The uranium rods were covered with cadmium to suppress the fissions. Because the suppression is not complete it is felt that the quoted

values are somewhat too high. The order of magnitude of the error may be seen in the corrected results given by Barkov, et al.⁽⁷⁵⁾ for a similar set of experiments.

Table 3-23

AGE OF H₂O-URANIUM LATTICES⁽⁷⁴⁾

Rod Diam. cm	Air Gap cm	$\frac{\text{Vol. H}_2\text{O}}{\text{Vol. Nat. U}}$	$\tau_{th}^*, \text{ cm}^2$ 25° C
3.00	none	1.14	54.7
3.00		3.27	39.9
3.00		0.273	131.0
3.00		1.78	44.0
3.00		1.36	51.7
3.00		3.17	40.1
2.79		2.06	44.9
2.79	1.143	1.57	49.2
2.79		1.42	51.0
2.79		1.92	94.5
2.79		1.56	112.0
2.00		1.80	48.4
2.00		1.50	51.5
2.00		2.10	46.5
2.00	0.900	2.01	110.0

*The measurements were actually made to indium resonance and a calculated correction made to τ_{th} .

These experiments were performed using 3.5 cm rods of natural uranium. The lattice was in the form of equilateral triangles with the spacing varied in order to change the water to metal ratio. These rods were also covered with 0.6 mm cadmium to suppress fission in the fuel. The results obtained were:

$\frac{\text{Vol. H}_2\text{O}}{\text{Vol. Nat. U}}$	$\tau, \text{ cm}^2$	$\tau, \text{ cm}^2$ (corrected for epithermal fission)
0.4	65 ± 3	58 ± 5
1.0	47 ± 2	44 ± 3
2.0	37 ± 2	35 ± 2

H₂O-Aluminum

A measurement of the age in Al-H₂O mixtures was performed at ORNL.⁽⁷⁶⁾

The results obtained were:

$\frac{\text{Vol. Al}}{\text{Vol. H}_2\text{O}}$	$\tau_{In}, \text{ cm}^2$	$\tau_{In\text{-thermal}}$ (theoretical)	$\tau_{th}, \text{ cm}^2$
0	30.8	1	31.8
0.5	49.6	2	51.6
1.0	76.8	3	79.8

H₂O-Zirconium

A measurement of the age in a zirconium-water mixture was performed at ORNL⁽⁷⁷⁾ using zirconium containing about one per cent hafnium. The thermal neutron cross section was measured to be 0.85 barns. The metal was in the form of plates 36.0 in. x 6 in. x 0.080 in. Detailed results obtained were:

$\frac{\text{Vol. Zr}}{\text{Vol. H}_2\text{O}}$	$\tau_{\text{In}}, \text{cm}^2$
0	30.03
0.25	35.7

H₂O-Iron

Some measurements due to Kendrick, on Fe-H₂O mixture are reported by Sjöstrand and Story.⁽⁷⁰⁾ These values are:

$\frac{\text{Vol. Fe}}{\text{Vol. H}_2\text{O}}$	$\tau_{\text{In}}, \text{cm}^2$
1.00	40.7
2.00	54.7

D₂O-Aluminum

In order to check a predicted anisotropy⁽⁷⁸⁾ due to neutron streaming along aluminum support structures and fuel tubes, the age was measured in two directions in an aluminum-D₂O (99.3%) lattice.⁽⁷²⁾ The metal to water ratio was not given. The results obtained were:

	$\tau_{\text{In}}, \text{cm}^2$
Parallel to rods	140 ± 3
perpendicular to rods	108 ± 5

Graphite-Bismuth

A series of measurements on a mixture of graphite and bismuth are reported by Davey, et al.⁽⁷⁹⁾ The graphite density was $\rho = 1.649 \text{ gm/cm}^3$ and bismuth density, 9.733 gm/cm^3 . The data of these experiments is summarized below:

$\frac{\text{Vol. Bi}}{\text{Vol. C}}$	$\tau_{\text{In}}, \text{cm}^2$
0	318 ± 6
0.437	498 ± 10
1.053	738 ± 14
1.626	973 ± 20

Diphenyl

Neutron age measurements in diphenyl (C₁₂H₁₀) were made at 185°F by NAA.⁽⁸⁰⁾ The value quoted was $\tau_{\text{In}} = 50.2 \text{ cm}^2$. A correction to increase this by 8% should be applied to account for the finite size of the tank yielding a corrected value $\tau_{\text{In}} = 54.2 \text{ cm}^2$.

3.5.3 Second Moments from Non-Fission Sources

The non-fission sources, in general, are effectively point sources of neutrons. Table 3-24 summarizes the available data of many sources and slowing down media. The nature of the sources is extensively covered in Section 8.

Table 3-24
SLOWING DOWN LENGTHS FOR NON-FISSION SOURCES

Source	Medium	Vol. Ratio (Room Temp.)	Detector	Age (cm ²)	Reference	Footnote
Ra (α,n) Be	H ₂ O	100%	Dy	55	81	
Rn (α,n) Be	H ₂ O	100%	Rh	54.5	82	
Ra (α,n) Be	H ₂ O	100%	In resonance	46.3	81	
Ra (α,n) Be	H ₂ O	100%	In resonance	48.5	83	
Ra (α,n) Be	H ₂ O	100%	In resonance	50.3	83	
Ra (α,n) Be	H ₂ O	100%	In resonance	45.4	84	
Ra (α,n) Be	H ₂ O	100%	In resonance	52.2	85	
Ra (α,n) Be	H ₂ O	100%	In resonance	48	86	
Ra (α,n) Be	H ₂ O	100%	ε resonance	45.5	86	
Rn (α,n) Be	H ₂ O	100%	Rh resonance	46.2	82	
Po (α,n) Be	H ₂ O	100%	In resonance	56.3	87	
Po (α,n) Be	H ₂ O	100%	BF ₃	58.5	88	
Po (α,n) Be	H ₂ O	100%	BF ₃ (B ¹⁰ 96%)	66.3	89	
Be ⁹ (d,n) B ¹⁰	H ₂ O	100%	LiI	41 ± 1.6	90	1
Be ⁹ (d,n) B ¹⁰	H ₂ O	100%	LiI	28.4	91	1
H ³ (d,n) He ⁴	H ₂ O	100%	In resonance	150 ± 6	92	2
Ra (α,n) Be	H ₂ O	100%	In resonance to thermal	1.0	84	3
Po (α,n) Be	H ₂ O	100%	In resonance to thermal	0.3	87	4
Sb ¹²⁴ (γ,n) Be	H ₂ O	100%	In resonance	5.5 ± .2	93	5
O ¹⁷ (n,p) N ^{17*} (n)N ¹⁶	H ₂ O	100%	BF ₃	15. ± 4	94	6
Sb ¹²⁴ (γ,n) Be	Graphite	100%	In resonance	147	95	7
Sb ¹²⁴ (γ,n) Be	Graphite	100%	Au	128 ± 2	96	7
Po (α,n) Be	H ₂ O:Fe	3:1	In resonance	52.3	83	
Po (α,n) Be	H ₂ O:Fe	2:1	In resonance	56.0	83	
Po (α,n) Be	H ₂ O:Fe	1:1	In resonance	57.9	83	
Ra (α,n) Be	H ₂ O:Fe	1.3:1	In resonance	53.2	81	8
Ra (α,n) Be	Be	100%	In resonance	120 ± 23	97	9
Po (α,n) Be	Be	100%	In resonance	95 ± 11	97	9
Calculated	Be	100%	1.44 ev to thermal	20	98	
D (d,n) Be	BeO	100%	LiI counter	156	91	1
Ra (α,n) Be	SiO ₂	100%	In resonance	1900 ± 90	99	10
Ra (α,n) Be	CaCO ₃	100%	In resonance	461 ± 23	99	
Ra (α,n) Be	H ₂ O:Brine	894:106	In resonance	55.3	85	11
Ra (α,n) Be	H ₂ O:Brine	894:106	I resonance	52.2	85	11
Ra (α,n) Be	Furfural					
	"H ₂ O":C	435:565	In resonance	134.5	85	12
Ra (α,n) Be	H ₂ O:Sand	388:612	In resonance	131.7	85	13
Po (α,n) Be	H ₂ O:Zr	4:1	BF ₃	77.5	89	14
Po (α,n) Be	H ₂ O:Zr	2:1	BF ₃	87.2	89	14
Po (α,n) Be	H ₂ O:Zr	1:1	BF ₃	107.9	89	14
Ra (α,n) Be	H ₂ O:Bi	1:1	Dy	93.7	81	15
Ra (α,n) Be	H ₂ O:Bi	1:1	In resonance	79.5	81	15
Ra (α,n) Be	H ₂ O:Bi	2:1	In resonance	66.3	81	15
Ra (α,n) Be	H ₂ O:Bi	1:2	In resonance	86.2	81	15
Ra (α,n) Be	H ₂ O:Pb	1:1	In resonance	72.8	81	15

1. Maximum neutron energy is 4.5 Mev and peak of distribution occurs at 3.5 Mev. Density of BeO: 2.96 gm/cm³
2. Neutron energy 14.1 Mev.
3. Based on measured $M^2 = \tau_{th} + L^2 = 54.7 \text{ cm}^2$, assumed $L^2 = 8.3 \text{ cm}^2$.
4. Based on measured $M^2 = \tau_{th} + L^2 = 64.7 \text{ cm}^2$, assumed $L^2 = 8.1 \text{ cm}^2$.
5. Source energy is approximately 30 kev with a spread of 25%. See Section 8.
6. Delayed neutrons from N¹⁷ were produced by circulating water into the reactor then in to a disc-shaped coil, which served as the source. Average energy of source neutrons was 0.9 Mev.
7. Graphite density 1.652 gm/cm³.
8. Measurements were of heterogeneous assemblies of rods. Fe rods were 1.87 x 1.87 x 32 cm.
9. Density of Be: 1.78 gm/cm³
10. Density of SiO₂: 1.61 gm/cm³
11. Approximately 900 pounds of salt were dissolved in water to give 47.4 cubic feet of solution. By chemical analysis the average NaCl content was 292 milligrams per cubic centimeter of solution, corresponding to 92% saturation. Typical Analysis of Salt Used: Sodium chloride 99.68%, Calcium sulphate 0.23%, Calcium chloride 0.09%, Magnesium chloride Trace, Iron and alumina Nil. Activation of Iodide detector is due principally to neutrons of about 40 ev.
12. Furfural:C₅H₄O₂ $\rho_{\text{equivalent water}} = \frac{\text{Mass H}_2\text{O}}{\text{Mass furfural}} \rho_{\text{furfural}} = 0.435$
13. The sand is "Flint Shot" (brand), Ottawa Silica Co., Ottawa, Illinois
14. Measurements were of heterogeneous assemblies of rods varying in diameter from 1/2" to 3/4".
15. Measurements were of heterogeneous assemblies of rods. Pb and Bi rods were 2 x 2 x 32 cm.

3.6 FAST GROUP CONSTANTS

3.6.1 Multigroup Cross Sections; 30-Group Set

The OCUSOL-A 30 group set of cross sections⁽¹⁰¹⁾ which are a modification of the ORNL "Eyewash"⁽¹⁰²⁾ cross sections are given in Table 3-25. Because the OCUSOL-A program was not directed to thermal reactors, six additional materials (H, D, Al, Mg, Zr and O) have been added.

Table 3-25
30 GROUP CROSS SECTION SET
(Microscopic cross sections in barns)

Group	Lower Energy Limit E_L	$\ln \frac{u}{E_L}$	Fission Spectrum χ	Plutonium-239				Uranium-233			
				$\xi\sigma_{tot}$	σ_{tr}	σ_a	σ_f	$\xi\sigma_{tot}$	σ_{tr}	σ_a	σ_f
*				.0667*				0.080*			
1	6.065 Mev	0.50	.02321	.0667	8	2.3	2.2	0.086	8.7	2.5	2.5
2	3.679	1.00	.10817	.0667	8	2.0	1.9	0.083	9.7	1.8	1.8
3	2.231	1.50	.21027	.065	7.8	2.0	1.9	0.085	9.7	1.85	1.85
4	1.353	2.00	.23121	.06	7.2	2.1	2.0	0.085	10	1.9	1.9
5	0.8208	2.50	.18034	.06	7.2	2.0	1.9	0.086	10	1.95	1.95
6	0.3876	3.25	.15186	.0709	8.5	1.9	1.8	0.092	10	2.05	2.05
7	0.1830	4.00	.06162	.0834	10	2.0	1.7	0.092	10	2.28	2.2
8	0.06733	5.00	.02554	.0959	11.5	2.2	1.7	0.10	11	2.8	2.5
9	0.02478	6.00	.00601	.113	13.8	2.7	2.0	0.11	11	3.2	2.8
10	0.009118	7.00	.00137	.125	15	3.2	2.2	0.12	11	4.9	4.3
11	0.003355	8.00	.00031	.142	17	4.4	2.9	0.24	21	7.9	6.9
12	0.001234	9.00	.00007	.142	17	7.0	4.5	0.27	21	12	10.5
13	454.0 ev	10.00	.00002	.108	13	12.7	7.9	0.30	21	21.2	18.6
14	319.9	10.35	0	.192	23	20.4	12.5	0.40	44.3	28.5	25
15	225.5	10.70	0	.267	32	26.8	16.2	0.50	44.3	33.1	29
16	112.0	11.40	0	.334	40	39.5	23.8	0.60	44.3	44.5	39
17	61.47	12.00	0	.642	77	67.0	38.0	0.90	82.7	66.1	58
18	33.74	12.60	0	.967	116	106.0	59.0	0.93	82.7	84.4	74
19	15.15	13.40	0	.346	41.5	73.3	31.5	0.982	111	111	94
20	10.16	13.80	0	1.38	165.8	267.0	155.8	1.13	123	124	105
21	4.564	14.60	0	.353	42.3	72.6	32.3	0.784	107	109	90
22	1.375	15.80	0	.229	27.5	17.5	14.8	2.24	216	229	189
23	0.9214	16.20	0	.420	50.3	40.3	31.3	1.26	198	209	173
24	0.6716	16.60	0	.721	86.4	76.4	57.8	1.32	147	139	126
25	0.4140	17.00	0	2.99	363.6	348.0	211.0	1.60	151	137	125
26	0.2775	17.40	0	21.5	2638	2562.0	1477.0	1.75	169	157	142
27	0.1860	17.80	0	23.8	2838	2838.0	1650.0	1.84	207	189	172
28	0.1247	18.20	0	8.24	988	978.0	600.0	2.22	247	233	212
29	0.07735	18.680	0	6.22	746.7	736.0	470.0	2.7	297	294	267
30	0.07595	18.698	0	6.08	729	719.0	470.0	2.75	333	330	300

Group	Uranium-235				Uranium-238				Thorium-232			
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	σ_f	$\xi\sigma_{tot}$	σ_{tr}	σ_a	σ_f	$\xi\sigma_{tot}$	σ_{tr}	σ_a	σ_f
*	0.060*				0.080*				0.050*			
1	0.063	7.7	1.31	1.3	0.081	9.7	0.57	0.55	0.05415	6.30	0.385	0.35
2	0.067	7.7	1.33	1.3	0.081	9.7	0.57	0.55	0.05845	6.80	0.195	0.15
3	0.068	7.7	1.34	1.30	0.081	9.7	0.58	0.55	0.06016	7.00	0.190	0.13
4	0.068	6.7	1.35	1.30	0.078	9.3	0.46	0.4	0.05673	6.60	0.180	0.10
5	0.062	7.3	1.36	1.25	0.075	9.0	0.10	0	0.05845	6.80	0.100	0
6	0.081	8.0	1.37	1.20	0.075	9.0	0.14	0	0.06532	7.60	0.163	0
7	0.081	9.3	1.44	1.35	0.078	9.3	0.19	0	0.08251	9.60	0.217	0
8	0.127	11	1.9	1.7	0.078	9.3	0.27	0	0.1031	12.0	0.306	0
9	0.127	13	2.6	2.3	0.078	9.3	0.39	0	0.1176	13.0	0.456	0
10	0.127	15	4.1	3.4	0.081	9.7	0.58	0	0.1246	14.5	0.679	0
11	0.156	17	6.8	4.7	0.084	10	1.00	0	0.1246	14.5	1.00	0
12	0.156	20	9.3	6.4	0.089	11	1.61	0	0.1246	14.5	1.163	0
13	0.156	27	14	9.6	0.101	12	3.11	0	0.1117	16	3.11	0
14	0.276	34	20	13	0.120	14	5.47	0	0.1117	19	5.47	0
15	0.276	40	25	16.5	0.159	19	10.0	0	0.1117	23	10.0	0
16	0.276	40	30	20	0.151	18	8.9	0	0.1117	28	15.1	0
17	0.441	50	39	26	0.363	43.3	35.0	0	0.1117	29	16.1	0
18	0.441	63.3	53	36	0.838	100	90.0	0	0.1117	18	5.07	0
19	0.568	70.0	59	41	0.698	83.3	75.0	0	0.1117	46.0	33.4	0
20	0.606	73.3	64	45	0.075	9.0	0.11	0	0.09884	11.5	0.315	0
21	0.801	96.7	88	42	1.42	170	162.0	0	0.1031	12.0	0.435	0
22	0.272	33.3	24	15	0.077	9.2	0.24	0	0.1071	12.5	0.735	0
23	0.744	90.0	81	62	0.080	9.5	0.41	0	0.1117	13.0	1.060	0
24	0.597	73.3	63	58	0.080	9.5	0.51	0	0.1160	13.5	1.295	0
25	0.813	103	92	85	0.080	9.5	0.63	0	0.1203	14.0	1.585	0
26	1.61	193	184	150	0.080	9.5	0.74	0	0.1246	14.5	1.945	0
27	1.96	240	231	185	0.080	9.6	0.90	0	0.1289	15.0	2.350	0
28	2.06	237	228	192	0.081	9.7	1.11	0	0.1332	15.5	2.848	0
29	2.44	300	291	250	0.083	9.9	1.39	0	0.1375	16.0	3.560	0
30	2.88	333	336	290	0.084	10	1.56	0	0.1418	16.5	3.998	0

Table 3-25 (Cont'd.)

Group	Hydrogen			Deuterium			Beryllium		
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a
*	0.47*			0.365*			0.35*		
1	1.175	1.17	0	0.943	0.8667	0	0.36	1.6	0
2	1.71	1.70	0	1.269	1.167	0	0.40	1.8	0.007
3	2.33	2.32	0	1.631	1.500	0	0.48	2.1	0.04
4	3.10	3.08	0	1.921	1.767	0	0.43	2.0	0.03
5	3.68	3.66	0.0001	2.084	1.917	0	0.66	3.0	0
6	5.85	5.82	0.0001	2.193	2.017	0	0.83	3.7	0
7	8.50	8.45	0.0001	2.248	2.067	0	0.89	4.0	0
8	12.12	12.05	0.0001	2.393	2.200	0	1.11	5.0	0
9	15.85	15.76	0.0004	2.465	2.267	0	1.24	5.6	0
10	18.40	18.30	0.0007	2.560	2.354	0	1.24	5.6	0
11	20.00	19.89	0.0012	2.750	2.529	0	1.24	5.6	0
12	20	19.89	0.0019	2.940	2.703	0	1.24	5.6	1.61
13	20	19.89	0.0027	3.129	2.877	0	1.24	5.6	0.0001
14	20	19.89	0.0032	3.257	2.995	0	1.24	5.6	0.0001
15	20	19.89	0.0042	3.324	3.057	0	1.24	5.6	0.0001
16	20	19.89	0.0052	3.452	3.175	0	1.24	5.6	0.0001
17	20	19.89	0.0062	3.547	3.261	0	1.24	5.6	0.0002
18	20	19.88	0.0111	3.661	3.3667	0	1.24	5.6	0.0002
19	20	19.88	0.0149	3.794	3.4887	0	1.24	5.6	0.0003
20	20	19.88	0.0201	3.908	3.5933	0	1.24	5.6	0.0004
21	20.2	20.09	0.0334	4.022	3.6980	0	1.24	5.6	0.0006
22	20.4	20.35	0.0401	4.212	3.8727	0	1.24	5.6	0.0010
23	20.75	20.60	0.0467	4.363	4.0120	0	1.24	5.6	0.0015
24	21.25	21.04	0.0871	4.439	4.0820	0	1.24	5.6	0.0018
25	21.9	21.78	0.0919	4.515	4.1520	0	1.24	5.6	0.0022
26	22.8	22.67	0.1074	4.591	4.2213	0	1.24	5.6	0.0027
27							1.24	5.6	0.0033
28							1.24	5.6	0.0041
29							1.24	5.6	0.0051
30							1.24	5.6	0.0058

Group	Lithium-7			Carbon			Oxygen		
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a
*	0.40*			0.181*			0.169*		
1	0.48	1.7	0	0.166	1.04	0	0.139	0.5830	0.45
2	0.57	2.0	0	0.363	1.58	0	0.242	0.4367	0.07
3	0.52	1.8	0	0.245	1.82	0	0.226	1.1220	0
4	0.44	1.6	0	0.347	1.77	0	0.363	2.550	0
5	0.38	1.3	0	0.450	2.39	0	0.665	5.50	0
6	0.34	1.2	0.0001	0.536	2.96	0	0.411	2.635	0
7	0.89	3.1	0.0004	0.663	3.60	0	0.411	3.740	0
8	0.27	.97	0.0002	0.710	4.10	0	0.423	3.360	0
9	0.27	.97	0.0002	0.726	4.30	0	0.441	3.504	0
10	0.27	.97	0.0003	0.742	4.40	0	0.459	3.6480	0
11	0.27	.97	0.0005	0.742	4.40	0	0.459	3.6480	0
12	0.27	.97	0.0007	0.742	4.40	0	0.459	3.6480	0
13	0.29	1.0	0.0011	0.742	4.40	0	0.459	3.6480	0
14	0.29	1.0	0.0014	0.742	4.40	0.0001	0.459	3.6480	0
15	0.29	1.0	0.0017	0.742	4.40	0.0001	0.459	3.6480	0
16	0.29	1.0	0.0021	0.742	4.40	0.0003	0.459	3.6480	0
17	0.29	1.0	0.0028	0.742	4.40	0.0004	0.459	3.6480	0
18	0.29	1.0	0.0036	0.742	4.40	0.0004	0.459	3.6480	0
19	0.29	1.0	0.0049	0.742	4.40	0.0005	0.459	3.6480	0
20	0.29	1.0	0.0065	0.742	4.40	0.0006	0.459	3.6480	0
21	0.29	1.0	0.0089	0.742	4.40	0.0007	0.459	3.6480	0
22	0.29	1.0	0.0140	0.742	4.40	0.0008	0.459	3.6480	0
23	0.29	1.0	0.0195	0.742	4.40	0.0009	0.453	3.6000	0
24	0.29	1.0	0.0230	0.742	4.40	0.0010	0.453	3.6000	0
25	0.29	1.0	0.0283	0.742	4.40	0.0011	0.453	3.6000	0
26	0.29	1.0	0.0354	0.742	4.40	0.0013	0.453	3.6000	0
27	0.29	1.0	0.0425	0.742	4.40	0.0017	0.459	3.6480	0
28	0.29	1.0	0.0514	0.742	4.40	0.0020	0.459	3.6480	0
29	0.29	1.0	0.0655	0.742	4.40	0.0024	0.465	3.6960	0
30	0.29	1.0	0.0744	0.742	4.40	0.0027	0.465	3.6960	0

Group	Sodium			Magnesium			Aluminum		
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a
*	0.19*			0.0696*			1.004*		
1	0.19	2.18	0.10	0.139	0.960	0	1.661	1.53	0.030
2	0.19	2.18	0.033	0.152	1.05	0	1.916	1.75	0.001
3	0.262	2.59	0.0001	0.178	1.23	0	2.19	2.00	0.0004
4	0.245	2.91	0.0002	0.209	1.44	0	2.628	2.45	0.0010
5	0.397	3.70	0.0003	0.278	2.36	0	2.92	2.80	0.0012
6	0.338	4.23	0.0004	0.474	3.997	0	2.774	3.30	0.0025
7	0.300	3.67	0.001	0.679	6.760	0	3.103	3.79	0.0075
8	0.338	3.67	0.001	0.726	8.140	0	2.92	3.65	0.0052
9	0.422	4.37	0.001	0.370	4.350	0	1.022	1.37	0.0072
10	0.558	5.63	0.001	0.335	3.953	0	1.029	1.38	0.0087
11	8.45	52.0	0.001	0.301	3.363	0.0001	1.029	1.39	0.0108
12	0.515	51.3	0.113	0.298	3.333	0.0002	1.029	1.39	0.0125
13	0.262	4.47	0.003	0.296	3.307	0.0004	1.036	1.39	0.0135
14	0.262	3.01	0.0043	0.296	3.307	0.0005	1.036	1.39	0.0150
15	0.262	3.01	0.0050	0.296	3.307	0.0006	1.036	1.39	0.0174
16	0.262	3.01	0.0065	0.296	3.307	0.0008	1.036	1.39	0.0198
17	0.262	3.01	0.0090	0.296	3.307	0.0011	1.036	1.40	0.0228
18	0.262	3.01	0.013	0.296	3.307	0.0015	1.036	1.40	0.0258
19	0.264	3.02	0.018	0.296	3.307	0.0021	1.043	1.41	0.0295
20	0.264	3.03	0.023	0.296	3.307	0.0028	1.043	1.43	0.0370
21	0.266	3.05	0.032	0.296	3.307	0.0039	1.066	1.46	0.0440
22	0.270	3.08	0.053	0.296	3.31	0.0066	1.066	1.47	0.0490
23	0.279	3.15	0.080	0.296	3.31	0.0096	1.073	1.48	0.0533
24	0.283	3.23	0.100	0.296	3.30	0.0115	1.073	1.49	0.0575
25	0.287	3.27	0.120	0.296	3.30	0.0135	1.095	1.53	0.0690
26	0.292	3.32	0.150	0.296	3.29	0.0173	1.117	1.57	0.0820
27	0.296	3.37	0.180	0.296	3.29	0.021	1.132	1.61	0.097
28	0.300	3.43	0.220	0.296	3.29	0.0255	1.146	1.65	0.116
29	0.304	3.47	0.280	0.296	3.27	0.0569	1.168	1.69	0.127
30	0.304	3.50	0.300	0.296	3.27	0.0574	1.168	1.69	0.130

Table 3-25 (Cont'd.)

Group	Chromium			Iron			Nickel			Zirconium		
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a
*	0.130*			0.100*			0.090*			0.077*		
1	0.133	3.3	0.06	0.107	2.85	0.050	0.094	2.62	0.050	0.090	2.731	0
2	0.133	3.3	0.06	0.109	2.96	0.050	0.095	2.68	0.050	0.088	2.665	0
3	0.133	3.3	0.06	0.091	2.82	0.050	0.087	2.58	0.050	0.095	2.864	0
4	0.133	3.3	0.06	0.087	2.32	0.050	0.102	2.58	0.050	0.117	3.531	0
5	0.133	3.3	0.06	0.095	2.31	0.050	0.117	3.25	0.050	0.145	4.400	0
6	0.133	3.3	0.06	0.11	2.7	0.050	0.15	3.3	0.050	0.194	6.747	0
7	0.133	3.3	0.06	0.15	3.67	0.050	0.25	5.3	0.050	0.198	8.250	0
8	0.133	3.3	0.06	0.24	4.33	0.050	0.62	12	0.050	0.198	8.933	0
9	0.133	3.3	0.06	0.26	4.67	0.050	0.62	14	0.050	0.198	8.933	0
10	0.133	3.3	0.06	0.28	5.33	0.050	0.62	16	0.050	0.176	7.941	0.0002
11	0.760	20	0.06	0.29	6.00	0.050	0.56	16	0.050	0.174	7.842	0.0004
12	0.380	10	0.06	0.30	6.67	0.050	0.56	16	0.050	0.154	6.948	0.0006
13	0.171	4.7	0.06	0.31	7.00	0.050	0.56	17	0.050	0.136	6.154	0.0010
14	0.159	4.3	0.06	0.39	9.67	0.050	0.56	17	0.09	0.132	5.956	0.0014
15	0.159	4.3	0.06	0.39	10.0	0.050	0.56	17	0.09	0.220	9.926	0.0017
16	0.159	4.3	0.06	0.39	10.3	0.050	0.56	17	0.09	0.136	6.154	0.0028
17	0.159	4.3	0.06	0.39	10.7	0.052	0.57	16	0.09	0.136	6.154	0.0030
18	0.159	4.3	0.06	0.39	11.0	0.054	0.57	16	0.09	0.136	6.154	0.0060
19	0.159	4.3	0.09	0.41	11.1	0.080	0.58	16	0.146	0.136	6.154	0.0080
20	0.163	4.3	0.12	0.40	11.0	0.107	0.57	17	0.195	0.136	6.154	0.0110
21	0.163	4.3	0.17	0.42	11.0	0.147	0.59	16	0.268	0.136	6.154	0.0180
22	0.167	4.3	0.28	0.42	11.2	0.243	0.63	17	0.450	0.136	6.154	0.0220
23	0.171	4.7	0.41	0.42	11.5	0.359	0.60	17	0.668	0.136	6.154	0.025
24	0.175	4.7	0.50	0.41	10.9	0.437	0.60	17	0.811	0.136	6.154	0.048
25	0.178	4.7	0.62	0.41	10.8	0.539	0.63	17	0.999	0.136	6.154	0.050
26	0.186	5.0	0.75	0.42	10.8	0.656	0.63	17	1.22	0.136	6.154	0.059
27	0.194	5.0	0.92	0.42	10.8	0.800	0.63	17	1.48	0.136	6.154	0.073
28	0.205	5.3	1.12	0.43	10.7	0.975	0.64	17	1.82	0.136	6.154	0.091
29	0.216	5.7	1.38	0.43	10.7	1.20	0.65	17	2.22	0.136	6.154	0.104
30	0.228	6.0	1.67	0.44	10.7	1.45	0.66	16	2.60	0.136	6.154	0.180

Group	Molybdenum			Bismuth			Two Fission Products		
	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a	$\xi\sigma_{tot}$	σ_{tr}	σ_a
*	0.07*			0.04*					
1	0.074	1.69	0	0.0611	5.50	0	0.10	6.0	0.043
2	0.077	1.82	0.015	0.0744	7.00	0	0.10	6.0	0.050
3	0.095	2.14	0.030	0.0601	7.00	0	0.10	6.0	0.060
4	0.126	2.91	0.045	0.0467	6.00	0	0.10	6.0	0.072
5	0.140	3.90	0.060	0.0506	5.00	0	0.10	6.0	0.09
6	0.186	5.73	0.075	0.0667	6.00	0	0.10	6.0	0.10
7	0.186	5.73	0.090	0.082	8.00	0	0.10	6.0	0.14
8	0.143	7.53	0.105	0.0859	9.00	0	0.10	6.0	0.19
9	0.143	7.53	0.120	0.10	10.5	0	0.10	6.0	0.29
10	0.143	7.53	0.135	0.08	10.0	0	0.10	6.0	0.38
11	0.174	7.07	0.150	0.103	10.0	0.0001	0.12	7.0	0.74
12	0.174	7.07	0.250	0.0983	10.0	0.0001	0.14	8.0	1.6
13	0.174	7.07	0.350	0.0878	10.0	0.0002	0.17	10	3.3
14	0.125	6.87	1.11	0.0878	9.2	0.0003	0.20	12	5.4
15	0.125	6.87	1.11	0.0878	9.2	0.0003	0.22	13	6.9
16	0.125	6.87	1.11	0.0878	9.2	0.0004	0.27	16	9.5
17	0.112	10.0	6.46	0.0878	9.2	0.0006	0.58	33.3	28
18	0.112	10.0	6.46	0.0878	9.2	0.0008	0.58	33.3	28
19	0.108	5.13	0.082	0.0878	9.2	0.0011	0.42	25	19
20	0.111	5.30	0.107	0.0878	9.2	0.0015	0.44	26	20
21	0.116	5.47	0.145	0.0878	9.2	0.0021	1.0	56.7	52
22	0.124	5.80	0.241	0.0878	9.2	0.0035	0.75	43.3	38
23	0.129	6.10	0.355	0.0878	9.2	0.0049	0.61	36.7	30
24	0.132	6.30	0.436	0.0878	9.2	0.0059	0.70	40.0	35
25	0.136	6.47	0.562	0.0878	9.2	0.0073	0.70	40.0	35
26	0.138	6.63	0.642	0.0878	9.2	0.009	0.49	29.0	23
27	0.140	6.80	0.800	0.0878	9.2	0.012	0.61	36.7	30
28	0.144	6.97	0.980	0.0878	9.2	0.0147	0.61	36.7	30
29	0.146	7.13	1.18	0.0878	9.2	0.0175	0.61	36.7	30
30	0.147	7.17	1.46	0.0878	9.2	0.0182	0.61	36.7	30

* $\xi\sigma_{tot}$ at 10 Mev used in obtaining average slowing down density in Group 1

A description of the methods and data employed in the OCUSOL-A cross sections appears in Ref. 101. For the six additional materials, the σ_T 's are simple arithmetic means of the data in BNL-325⁽¹⁰²⁾ and supplement; σ_{tr} was based upon σ_s and the angular distribution data from ANL-5567⁽¹⁰³⁾; σ_a , in the absence of more specific data, was taken as the $1/v$ extrapolation from the 2200 meter per second values. Cross sections of less than 0.1 mb were taken to be zero. The H and D total and transport cross sections are not given below 0.2 ev because chemical binding effects result in different values for different materials. The 0.096 ev Sm^{149} resonance remains deleted from the fission product cross section⁽¹⁰¹⁾ for the reason that it occurs in the transition region between a Maxwellian distribution and the $1/E$ region and as such its effect would be very spectrum sensitive. The fission product data are taken from Refs. 104 and 105.

This set of cross sections is not in all cases based on the latest data. Further, it has some logical inconsistencies (e.g., $\sigma_{tr} < \sigma_a + \sigma_f$ for some materials and groups). It is nevertheless given since no better set appeared available and most of the inconsistencies are not too serious for the systems to which the set should be applied.

3.6.2 Fast Group Constants in Two Group Theory

Some values of the removal cross section, Σ_F , and fast diffusion coefficient D_F are in Table 3-26. The values of Σ_F for pure moderators were based on the scheme that the neutron migrates as a fast neutron until it reaches the mean energy of the thermal group.

Table 3-26

FAST GROUP CONSTANTS IN PURE MODERATORS

(Unpublished results of M. J. Stanley, G. E., Cincinnati, Ohio)

Material	Density, gm/cm ³	D_F , cm	Σ_F , cm ⁻¹	τ , cm ²	Remarks
Be	1.85	.6873	.006796	99.6	no n-2n reaction included
Be (n, 2n)	1.85	.7002	.006968	83.67	n-2n reaction included
C	1.60	1.192	.00282	419.3	
H ₂ O	.998	1.423	.04288	32.29	
BeO	3.010	.6445	.005705	111.7	
D ₂ O	1.105	1.305	.009466	137.8	see also discussion following table.

In D₂O a special situation exists since the slowing down process is not well represented by either a Gaussian or a Yukawa kernel but rather by a convolution of the two.

In D₂O a Fermi Age to thermal, τ_f , of 125 cm² has been used for some time.⁽¹⁰⁶⁾ A very good fit to observed data is obtained by the convolution of a Yukawa and a Gaussian kernel. The Gaussian Age is $\tau_1 = 58$ cm², the Yukawa Age is $\tau_2 = 48$ cm². This is for slowing down to 1.4 ev. The slowing down density $A(r)$ is:

$$A(r) = \frac{A e^{\tau_1/\tau_2}}{8\pi r \tau_2} \left\{ \left[1 + \phi \left(\frac{r}{2\sqrt{\tau_1}} - \sqrt{\frac{\tau_1}{\tau_2}} \right) \right] e^{-\frac{r}{\sqrt{\tau_2}}} - \left[1 - \phi \left(\frac{r}{2\sqrt{\tau_1}} + \sqrt{\frac{\tau_1}{\tau_2}} \right) \right] e^{\frac{r}{\sqrt{\tau_2}}} \right\}$$

where

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy \quad (\text{Ref. 107}).$$

An age to 1.4 ev of 109 cm² is reported in Ref. 108.

In the reflector, a value of the two group age, $\tau_{2g} = 200 \text{ cm}^2$ gives good agreement with observed reflector savings.(109) In the core, having selected a value for τ_f , one can solve for τ_{2g} by

$$e^{\tau_f B^2} = 1 + \tau_{2g} B^2$$

3.6.3 Fast Group Cross Sections for Water and Water-Uranium Mixtures with the Effect of Fast Capture Included

Hellens(110) has given the results of calculation of the various moderating properties of water, and $\text{H}_2\text{O} - \text{U}$ and $\text{H}_2\text{O} - \text{UO}_2$ mixtures as a function of the amount of fast capture, fast fission, resonance capture, and buckling. Four computational schemes were used. They are the P_1 and B_1 methods with and without the Selengut-Goertzel approximations.

In addition to the four means of approximating slowing down in hydrogen, three formulations of the two group equations were considered:

$$D_F \nabla^2 \phi_F - \Sigma_F \phi_F + \nu k_F \Sigma_{fS} \phi_S = 0$$

$$D_S \nabla^2 \phi_S - \Sigma_{aS} \phi_S + k_e \Sigma_F \phi_F = 0$$

Scheme A: $k_e = 1$ $k_F = \epsilon p$

B: $k_e = p$ $k_F = \epsilon$

C: $k_e = \epsilon p$ $k_F = 1$

Figure 3-44 is the age in water as a function of buckling. Figure 3-45 is a graph of Σ_F as a function of buckling; the four approximations are much more consistent here. Figure 3-46 is a plot of the age, $\tau = D_F / \Sigma_F$, in $\text{H}_2\text{O} - \text{U}$ and $\text{H}_2\text{O} - \text{UO}_2$ mixtures. Figure 3-47 shows D_F for the various mixtures. Figures 3-48 and 3-49 give the fast fission factor and resonance escape probability, p , for the various mixtures according to Schemes A, B, and C. Figures 3-50 and 3-51 give the effects of using these constants on the age and the fast removal constant, Σ_F . Figures 3-44 through 3-51 are reproduced from Ref. 110.

FIG. 3-44
TWO-GROUP AGE (τ) IN PURE WATER

FIG. 3-45
FISSION SOURCE IN WATER. FAST
REMOVAL CROSS SECTION, Σ_F

FIG. 3-46
AGE (τ) OF FISSION NEUTRONS IN WATER MIXTURES
OF URANIUM METAL AND URANIUM OXIDE

FIG. 3-47
FAST DIFFUSION CONSTANT (D_F) FOR VARIOUS BUCKLINGS IN WATER
MIXTURES OF URANIUM METAL AND URANIUM OXIDE

FIG. 3-48
FAST FISSION FACTOR (ϵ) IN WATER MIXTURES OF URANIUM METAL AND URANIUM OXIDE
USING VARIOUS COMPUTATIONAL SCHEMES AND THE P_1 APPROXIMATION

SCHEME A

SCHEME B,C

FIG. 3-49
RESONANCE ESCAPE PROBABILITY (p) IN WATER MIXTURES OF URANIUM METAL AND URANIUM OXIDE USING VARIOUS COMPUTATIONAL SCHEMES AND THE P_1 APPROXIMATION

SCHEME A

SCHEME B

SCHEME C

FIG. 3-50
AGE (τ) OF FISSION NEUTRONS IN WATER MIXTURES OF URANIUM METAL AND URANIUM OXIDE USING VARIOUS COMPUTATIONAL SCHEMES AND THE P_1 APPROXIMATION

FIG. 3-51
FAST REMOVAL CONSTANT (Σ_F) IN WATER MIXTURES OF URANIUM METAL AND
URANIUM OXIDE USING VARIOUS COMPUTATIONAL SCHEMES AND THE P_1 APPROXIMATION

3.6.4 Fast Group Constants for Metal-Water Mixtures by the Method of Equivalence Factors

A method is presented below for obtaining the constants for the upper two groups of a three group formulation for mixtures of H_2O or D_2O and structural materials.

H_2O Mixtures⁽¹¹¹⁾

The values below are for the following group structure:

Fast group	180 kev to 10 Mev
Epithermal group	1.4 ev to 180 kev

The age in H_2O is 24 cm^2 for the fast group and 6.9 cm^2 for the epithermal group.

Assuming that within each group the energy variations of material x and H_2O are sufficiently similar, define:

$$\alpha = \frac{\xi \Sigma_s(x)}{\xi \Sigma_s(H_2O)}$$

$$\beta = \frac{\Sigma_{tr}(x)}{\Sigma_{tr}(H_2O)}$$

$V(H_2O)$ = volume fraction of H_2O

$V(x)$ = volume fraction of x

τ and D for each group may be given in terms of τ and D for H_2O :

$$\tau = \frac{\tau(H_2O)}{V(H_2O)} \frac{1}{\left[1 + \alpha \frac{V(x)}{V(H_2O)}\right] \left[1 + \beta \frac{V(x)}{V(H_2O)}\right]}; \quad D = \frac{D(H_2O)}{V(H_2O) \left[1 + \beta \frac{V(x)}{V(H_2O)}\right]}$$

Table 3-27 gives average microscopic transport cross sections for various materials in the two groups. Values for $D(H_2O)$ can be obtained from $\sigma_{tr}(H_2O)$. Values of α and β may be obtained and are given in Table 3-28.

Table 3-27

MICROSCOPIC TRANSPORT
CROSS SECTIONS

Reproduced from Ref. 111

Material	Fast Group	Epithermal Group
H ₂ O	6.03	16.44
Al	3.08	1.78
Cr	3.60	5.77
Fe	2.86	8.93
Ni	3.59	17.1
Zr	6.27	7.08
C	2.47	4.37
O	3.09	3.64
(CH) _u (Polystyrene)	3.94	10.77
(CH ₂) _u (Polyethylene)	5.41	17.17
C ₅ O ₂ H ₆ (Furfuryl Alcohol)	27.35	67.53

Table 3-28

EQUIVALENCE FACTORS IN H₂O MIXTURES

Reproduced from Ref. 111

Material	Fast Group		Epithermal Group	
	α	β	α	β
Al	0.103	0.922	0.006	0.195
Zr	0.107	1.33	0.005	0.547
Cr	0.176	1.47	0.014	0.862
Ni	0.224	1.62	0.041	2.82
Fe	0.186	1.20	0.021	1.39
C (1.6 gm/cm ³)	0.115	0.986	0.046	0.618
Polyethylene (CH ₂) _u (.91 gm/cm ³)	1.17	1.05	1.18	1.21
Polystyrene (CH) _u (1.04 gm/cm ³)	0.756	0.943	0.737	0.930
Furfuryl Alcohol (C ₅ O ₂ H ₆) (1.13 gm/cm ³)	0.664	0.943	0.640	0.850

The above numbers are based on H₂O at STP. Straightforward corrections may be made for H₂O at other densities.

D₂O Mixtures

Equivalence factors⁽¹¹²⁾ are given in Table 3-29 for various materials in D₂O mixtures. The group structure is the same as in the discussion on H₂O mixtures above. The D₂O constants are

Group	τ , cm ²	D , cm
Fast (180 kev to 10 Mev)	48	1.27
Epithermal (thermal to 180 kev)	79	1.24

Table 3-29

EQUIVALENCE FACTORS IN D₂O MIXTURES

Data from Ref. 112

Material	Fast Group		Epithermal Group	
	α	β	α	β
Mg	0.067	0.480	0.057	0.445
Al	0.058	0.479	0.021	0.172
Zr	0.033	1.11	0.028	0.799
Stainless Steel	0.028	0.473	0.051	0.887

3.7 REACTOR CONTROL

3.7.1 Linear Extrapolation Distance for Various Configurations

In the study of control rods inserted into homogeneous reactors it is often convenient to describe their effects on the behavior of the asymptotic neutron flux, at large distances from the rod. As is well known, the asymptotic transport theory and diffusion theory neutron flux of one-group theory exhibit the same spatial dependence. Thus, if the logarithmic derivative of the diffusion theory flux at the rod is set equal to that of the asymptotic transport theory flux, one would expect the two fluxes to coincide asymptotically and therefore to permit one to perform diffusion theory computations of greater accuracy. The linear extrapolation distance, λ , at the surface of a rod is defined to be the reciprocal of the logarithmic derivative of the flux evaluated at that surface.

The condition on the flux at the rod surface may be replaced by the stipulation that the flux vanish at a radius r_{eff} which is smaller than the actual radius r of the rod. This radius is denoted "effective rod radius," and it is given approximately by: $r_{\text{eff}} = r - \lambda$ where λ is the linear extrapolation distance.

In the following descriptions, λ and the rod radii are given in units of the total mean free path of the external medium, unless otherwise specified.

(a) Linear Extrapolation Distances for Black Cylinders and Black Spheres⁽¹¹³⁾

A body is said to be black to neutrons of a given velocity if the albedo of the body is zero, or equivalently, if no neutron entering the body re-enters the source medium. In the formulas presented in this section (3.7.1(a)), the black body is assumed to be placed in an isotropically and elastically scattering, non-absorbing, infinite homogeneous medium.

For a large black sphere of radius a :⁽¹¹⁴⁾

$$\lambda_a (\text{Large Black Sphere}) = 0.7104 + \frac{0.5047}{a} + \frac{0.2336}{a^2} - \left(\frac{\log a}{4} + 0.1704 \right) \frac{1}{a^3} + O\left(\frac{\log^2 a}{a^4}\right)$$

For a small black sphere of radius a :⁽¹¹⁵⁾

$$\lambda_a (\text{Small Black Sphere}) = \frac{4}{3} - \frac{5}{9} a - \left[\frac{2}{3} \left(\frac{\pi^2}{4} - 1 \right) \log a + 1.4002 \right] a^2 + O(a^3 \log^2 a)$$

For an infinitely long large black cylinder of radius a :⁽¹¹⁶⁾

$$\lambda_a (\text{Large Black Cylinder}) = 0.7104 + \frac{0.2524}{a} + \frac{0.0949}{a^2} - \left[\frac{5}{64} \log a + 0.02561 \right] \frac{1}{a^3} + O\left(\frac{\log^2 a}{a^4}\right)$$

For an infinitely long small black cylinder of radius a :⁽¹¹⁷⁾

$$\lambda_a (\text{Small Black Cylinder}) = \frac{4}{3} + \left[\left(1 - \frac{16}{3\pi^2} \right) \log a + 0.2164 \right] a + O(a^2 \log^2 a)$$

The symbol O indicates that the error terms are of the order of the quantity following in parentheses. It may be observed that the limiting cases of large spheres and large cylinders are both $\lambda = 0.7104$, the linear extrapolation distance at the surface of an infinite black slab.

These asymptotic results were combined into smooth curves, $\lambda(a)$, by application of the spherical harmonics (P_L) approximations to the neutron transport equation. In Table 3-30⁽¹¹³⁾ are given the results of P_L ($L = 1, 2, \dots, 8$) computations⁽¹¹⁸⁾ for black spheres; in Table 3-31⁽¹¹³⁾ results are supplied for black cylinders in the P_3 approximation.⁽¹¹⁹⁾

Table 3-30

LINEAR EXTRAPOLATION DISTANCE FOR BLACK SPHERES:
 P_L APPROXIMATION

Approximation								
a	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
0.00	0.5774	∞	0.6395	∞	0.6537	∞		
0.05	0.5774	16.7746	0.6894	8.7173	0.7499	6.0648		
0.10	0.5774	8.7746	0.7306	4.7543	0.8211	3.4482		
0.20	0.5774	4.7746	0.7922	2.7867	0.9116	2.1655	1.0140	1.3390
0.30	0.5774	3.4413	0.8332	2.1391	0.9585	1.7508		
0.40	0.5774	2.7746	0.8598	1.8189	0.9806	1.5471		
0.50	0.5774	2.3746	0.8764	1.6280	0.9886	1.4248	1.0291	1.1271
0.70	0.5774	1.9175	0.8909	1.4096	0.9842	1.2807		
1.00	0.5774	1.5746	0.8891	1.2417	0.9604	1.1614	0.9874	
1.50	0.5774	1.3079	0.8676	1.1008	0.9183	1.0507		
2.00	0.5774	1.1746	0.8443	1.0222	0.8847	0.9843	0.9000	
2.50	0.5774	1.0946	0.8246	0.9707	0.8589	0.9393		
3.00	0.5774	1.0413	0.8087	0.9341	0.8389	0.9069		
4.00	0.5774	0.9460	0.7853	0.8857	0.8104	0.8633		
5.00	0.5774	0.9346	0.8552	0.7914	0.8356	0.8028		
	0.5774	0.7746	0.6940	0.7297	0.7039	0.7197	0.7069	0.7159

(These Tables are reproduced from Ref. 113)

Table 3-31

LINEAR EXTRAPOLATION DISTANCE FOR
BLACK CYLINDERS: P_3 APPROXIMATION

a	P_3 approximation to λ_a	a	P_3 approximation to λ_a
0.1014	0.8354	2.0284	0.8079
0.2028	0.8905	3.0426	0.7775
0.4057	0.9171	4.0567	0.7605
0.6085	0.9074	4.8681	0.7516
1.0142	0.8709	6.0851	0.7426
1.5213	0.8335	10.142	0.7277

It was observed by Davison⁽¹¹³⁾ that the P_{2k+1} approximations underestimated λ and the P_{2k} approximations converged on λ from above. Apparently even the P_7 computations were not sufficiently accurate to permit unquestionable fits between the asymptotic cases. In Figs. 3-52 and 3-53 are graphed the various asymptotic formulae and the P_L results, as well as a "probable shape" for the transition regions.

FIG. 3-52
LINEAR EXTRAPOLATION DISTANCE (λ) AT THE SURFACE OF A BLACK SPHERE
(REPRODUCED FROM REF. 113)

FIG. 3-53
LINEAR EXTRAPOLATION DISTANCE (λ) AT THE SURFACE OF A LONG BLACK SPHERE
(REPRODUCED FROM REF. 113)

(b) Linear Extrapolation Distance at the Outer Surface of a Small Air-Gap Surrounding a Small Black Cylinder

The following formula, correct to the order indicated, applies to the linear extrapolation distance, λ_b , at the outer surface of an air gap surrounding a purely absorbing black cylinder of radius a . The radius, a , and the air-gap thickness $(b - a)$, measured in units of the total mean free path of the surrounding purely scattering medium are assumed to be small. Then:

$$\lambda_b (\text{Small Black Cylinder}) = \frac{4}{3} \frac{b}{a} + \left(1 - \frac{16}{3\pi^2}\right) b \log b - 0.0735b - 0.1429 \frac{a^2}{b} + O(ab \log^2 b),$$

which reduces to the linear extrapolation distance at the surface of the small black cylinder if $b = a$.

(c) Linear Extrapolation Distance at the Surface of a Small Black Sphere Surrounded by an Air Gap of Arbitrary Thickness (121)

For a small black sphere of radius a

$$\lambda_a = \frac{4}{3} - \frac{5}{9} a + O(a^2 \log^n a) \quad (3-34)$$

It has been shown that the effect on the asymptotic neutron density of surrounding a small black sphere of radius a with an air gap of thickness $(b - a)$ is the same as the effect induced by the replacement of this sphere by a sphere of radius a' without an air gap:

$$a' = a + \frac{a^2}{6} \left[1 - \frac{3}{2} \frac{a}{b + \sqrt{b^2 - a^2}} + \frac{1}{2} \frac{a^3}{[b + \sqrt{b^2 - a^2}]^3} \right] + O(a^3 \log^n a)$$

In the presence of the air gap, the linear extrapolation distance at the surface of the black sphere is:

$$\lambda_a = \frac{4}{3} - a \left[1 - \frac{2}{3} \frac{a}{b + \sqrt{b^2 - a^2}} + \frac{2}{9} \frac{a^3}{[b + \sqrt{b^2 - a^2}]^3} \right] + O(a^2 \log^n a)$$

By $O(a^2 \log^n a)$ it is meant that the error terms are of order $a^2 \log^n a$, $n = 0, 1, 2, \dots$. In particular there is an error term of order a^2 , as may be observed by passing to the limit $b = a$ and comparing the resultant formula with the formula for the small black sphere without an air gap.

No restriction is placed on the size of the air gap, in the case of a black sphere, as contrasted with the formula for the small black cylinder with an air gap, Eq. (3-34).

(d) Linear Extrapolation Distance at the Free Boundary of a Homogeneous Half-Space (122)

The linear extrapolation distance at a free plane boundary has been obtained for various restrictive assumptions concerning the scattering properties of the medium. It has been found that for a linear scattering law in a purely scattering medium, the emergent angular distribution, and the linear extrapolation distance in units of the transport mean free path are the same as for the isotropic case. In the case of an isotropically scattering medium with zero absorption, $\lambda = 0.7104 \ell_s$, where ℓ_s is the scattering mean free path.

If the medium is capturing and multiplying, and if scattering is linearly anisotropic, the microscopic scattering cross section, $\sigma_s(\mu)$ is given by

$$\sigma_s(\mu) = \frac{\sigma_{s0}}{2} (1 + 3b\mu)$$

where μ is the cosine of the scattering angle. Then the distance Z_0' (in scattering mean free paths) to the plane where the asymptotic solution vanishes may be obtained as a function of $(1 + f)$, the average number of neutrons emerging per collision, from Table 3-32. In that table may be found the coefficient, κ' , of the argument of the asymptotic flux, $F_0^{\text{asymptotic}}$:

$$F_0^{\text{asymptotic}}(z) \propto \sin \kappa'(z + z_0')$$

By definition of λ ,

$$\frac{1}{\lambda} = \left. \frac{d}{dz} \log F_0^{\text{as}}(z) \right]_{z=0}, \quad \text{or } \lambda = \frac{\tan \kappa' z_0'}{\kappa'}$$

Table 3-32

DIFFUSION LENGTH AND LINEAR
EXTRAPOLATION DISTANCE FOR
MILNE'S PROBLEM

(Anisotropic Scattering)

Data from Ref. 123

$1 + f$	$(1 + f)z_0'$	$(\kappa')^2$
0	$1 - 3b$	- 1
0.2	$0.7843 - 1.2197b$	$-0.99982 + 0.00350b$
0.4	$0.7300 - 0.1624b$	$-0.9714 + 0.1660b$
0.6	$0.7154 + 0.3305b$	$-0.8233 + 0.4009b$
0.8	$0.7113 + 0.5757b$	$-0.5047 + 0.3902b$
1.0	$0.7104 + 0.7104b$	0
1.2	$0.7106 + 0.7897b$	$0.6965 - 0.8201b$
1.4	$0.7117 + 0.8386b$	$1.5871 - 2.0921b$
1.6	$0.7130 + 0.8693b$	$2.6732 - 3.8270b$
1.8	$0.7142 + 0.8881b$	$3.9554 - 6.0305b$
2.0	$0.7152 + 0.8998b$	$5.4341 - 8.7063b$
2.2	$0.7162 + 0.9059b$	$7.1096 - 11.856b$
2.4	$0.7173 + 0.9084b$	$8.9819 - 15.483b$
2.6	$0.7184 + 0.9092b$	$11.051 - 19.585b$
2.8	$0.7194 + 0.9077b$	$13.318 - 24.166b$
3.0	$0.7203 + 0.9047b$	$15.781 - 29.224b$

(e) Linear Extrapolation Distance at the Surface of a Purely Absorbing Gray Cylinder⁽¹²⁰⁾

The linear extrapolation distance at the surface of a purely absorbing small gray cylinder is:

$$\lambda_a(\text{Small Gray Cylinder}) = \frac{4}{3} \left[\frac{1}{g(a^*)} - 1 \right] + \lambda_a(\text{Small Black Cylinder})$$

In this formula, a^* is the radius of the cylinder measured in capture mean free paths of the absorbing cylinder, and $g(a^*) = 2(1 - P_c)a^*$. The escape probability, P_c , is the average probability that a neutron arising from a uniform source within the cylinder will suffer a collision (in this case a capturing collision) in its first traverse of the cylinder. P_c is listed as a function of a^* in Table 3-33.⁽¹²⁴⁾ Royston⁽¹²⁵⁾ has extended the domain of λ_a to gray cylinders of larger radius by the use of the P_3 approximation. These results are given in Fig. 3-54.

The effect of a small air gap outer radius around a small gray cylindrical rod has been investigated.⁽¹²⁰⁾ Denoting by λ_b the linear extrapolation distance at the outer surface of the air gap:

$$\lambda_b(\text{Small Gray Cylinder}) = \frac{4}{3} \frac{b}{a} \left[\frac{1}{g(a^*)} - 1 \right] + \lambda_b(\text{Small Black Cylinder})$$

The linear extrapolation distance at the absorber surface is given by:

$$\lambda_a(\text{Small Gray Cylinder}) = \frac{a}{b} [\lambda_b(\text{Small Gray Cylinder})] - a \log \frac{b}{a}$$

(f) Linear Extrapolation Distance into a Purely Absorbing Gray Slab⁽¹²⁶⁾

Consider a purely absorbing gray slab of half-thickness a^* , measured in capture mean free paths, ℓ_2 , in that medium, located in an infinite homogeneous medium. The infinite medium absorbs and scatters neutrons, and it contains a uniform distribution of sources.

Let ℓ_1 denote the total mean free path in the external medium, and let α be the probability that a collision in that medium will result in a neutron capture. Then, in the P_1 (spherical harmonics) approximation to the one-group problem, the linear extrapolation distance λ at the surface of the gray slab is given by:

$$\lambda = \frac{2}{3} \frac{1 + 3E_4(2a^*)}{1 - 2E_3(2a^*)}$$

where

$$E_n(z) = \int_z^\infty dx \frac{e^{-x}}{x^n}$$

is the exponential integral of order n .

Table 3-33

COLLISION PROBABILITY P_c FOR INFINITE CYLINDER OF RADIUS a^*

(Reproduced from Ref. 125)

a^*	P_c	a^*	P_c	a^*	P_c	a^*	P_c
0.00	0.00000	1.50	0.69843	3.00	0.83714	4.50	0.88996
.02	0.02561	.52	0.70164	.02	0.83816	.52	0.89044
.04	0.04967	.54	0.70479	.04	0.83917	.54	0.89091
.06	0.07248	.56	0.70788	.06	0.84017	.56	0.89138
.08	0.09421	.58	0.71091	.08	0.84116	.58	0.89184
0.10	0.11498	1.60	0.71389	3.10	0.84214	4.60	0.89230
.12	0.13487	.62	0.71681	.12	0.84310	.62	0.89276
.14	0.15396	.64	0.71968	.14	0.84405	.64	0.89321
.16	0.17231	.66	0.72250	.16	0.84500	.66	0.89366
.18	0.18996	.68	0.72526	.18	0.84593	.68	0.89411
0.20	0.20697	1.70	0.72798	3.20	0.84685	4.70	0.89455
.22	0.22336	.72	0.73065	.22	0.84776	.72	0.89499
.24	0.23918	.74	0.73327	.24	0.84866	.74	0.89542
.26	0.25446	.76	0.73585	.26	0.84955	.76	0.89585
.28	0.26923	.78	0.73838	.28	0.85043	.78	0.89628
0.30	0.28351	1.80	0.74087	3.30	0.85130	4.80	0.89671
.32	0.29733	.82	0.74331	.32	0.85216	.82	0.89713
.34	0.31070	.84	0.74572	.34	0.85301	.84	0.89755
.36	0.32366	.86	0.74808	.36	0.85385	.86	0.89796
.38	0.33621	.88	0.75040	.38	0.85468	.88	0.89837
0.40	0.34838	1.90	0.75269	3.40	0.85550	4.90	0.89878
.42	0.36019	.92	0.75493	.42	0.85632	.92	0.89919
.44	0.37164	.94	0.75714	.44	0.85712	.94	0.89959
.46	0.38276	.96	0.75931	.46	0.85792	.96	0.89999
.48	0.39356	.98	0.76145	.48	0.85870	.98	0.90038
0.50	0.40405	2.00	0.76355	3.50	0.85948	5.00	0.90077
.52	0.41424	.02	0.76562	.52	0.86025	.02	0.90116
.54	0.42414	.04	0.76765	.54	0.86101	.04	0.90155
.56	0.43377	.06	0.76966	.56	0.86177	.06	0.90193
.58	0.44314	.08	0.77163	.58	0.86251	.08	0.90231
0.60	0.45225	2.10	0.77357	3.60	0.86325	5.10	0.90268
.62	0.46112	.12	0.77548	.62	0.86398	.12	0.90306
.64	0.46975	.14	0.77736	.64	0.86471	.14	0.90343
.66	0.47816	.16	0.77921	.66	0.86542	.16	0.90381
.68	0.48634	.18	0.78103	.68	0.86613	.18	0.90417
0.70	0.49432	2.20	0.78282	3.70	0.86683	5.20	0.90453
.72	0.50209	.22	0.78459	.72	0.86752	.22	0.90489
.74	0.50966	.24	0.78633	.74	0.86821	.24	0.90525
.76	0.51704	.26	0.78805	.76	0.86889	.26	0.90560
.78	0.52424	.28	0.78974	.78	0.86956	.28	0.90595
0.80	0.53126	2.30	0.79140	3.80	0.87023	5.30	0.90631
.82	0.53811	.32	0.79304	.82	0.87089	.32	0.90665
.84	0.54479	.34	0.79465	.84	0.87154	.34	0.90700
.86	0.55131	.36	0.79624	.86	0.87219	.36	0.90734
.88	0.55767	.38	0.79781	.88	0.87283	.38	0.90768
0.90	0.56389	2.40	0.79936	3.90	0.87346	5.40	0.90802
.92	0.56996	.42	0.80088	.92	0.87409	.42	0.90835
.94	0.57588	.44	0.80239	.94	0.87471	.44	0.90868
.96	0.58167	.46	0.80387	.96	0.87533	.46	0.90902
.98	0.58733	.48	0.80532	.98	0.87594	.48	0.90934
1.00	0.59285	2.50	0.80677	4.00	0.87654	5.50	0.90967
.02	0.59826	.52	0.80818	.02	0.87714	.52	0.90999
.04	0.60354	.54	0.80958	.04	0.87773	.54	0.91031
.06	0.60870	.56	0.81096	.06	0.87832	.56	0.91063
.08	0.61375	.58	0.81233	.08	0.87890	.58	0.91095
1.10	0.61869	2.60	0.81367	4.10	0.87947	5.60	0.91126
.12	0.62352	.62	0.81500	.12	0.88004	.62	0.91157
.14	0.62825	.64	0.81630	.14	0.88061	.64	0.91188
.16	0.63287	.66	0.81759	.16	0.88117	.66	0.91219
.18	0.63740	.68	0.81886	.18	0.88173	.68	0.91250
1.20	0.64183	2.70	0.82012	4.20	0.88228	5.70	0.91280
.22	0.64616	.72	0.82136	.22	0.88282	.72	0.91310
.24	0.65041	.74	0.82258	.24	0.88336	.74	0.91340
.26	0.65457	.76	0.82379	.26	0.88389	.76	0.91369
.28	0.65864	.78	0.82498	.28	0.88443	.78	0.91399
1.30	0.66263	2.80	0.82616	4.30	0.88495	5.80	0.91429
.32	0.66654	.82	0.82732	.32	0.88547	.82	0.91457
.34	0.67038	.84	0.82847	.34	0.88598	.84	0.91487
.36	0.67413	.86	0.82960	.36	0.88650	.86	0.91515
.38	0.67781	.88	0.83071	.38	0.88701	.88	0.91544
1.40	0.68142	2.90	0.83182	4.40	0.88751	5.90	0.91572
.42	0.68495	.92	0.83291	.42	0.88801	.92	0.91600
.44	0.68842	.94	0.83399	.44	0.88850	.94	0.91628
.46	0.69182	.96	0.83505	.46	0.88899	.96	0.91656
.48	0.69516	.98	0.83610	.48	0.88948	.98	0.91684
						6.00	0.91711

a = RADIUS OF ROD MEASURED IN EXTERNAL MEAN FREE PATH
 a^* = RADIUS OF ROD MEASURED IN ROD CAPTURE MEAN FREE PATH

FIG. 3-54
 LINEAR EXTRAPOLATION DISTANCE (λ) AT THE SURFACE OF
 A PURELY ABSORBING GRAY CYLINDER
 (REPRODUCED FROM REF. 125)

P_1 , P_3 and P_5 computations have been made for various values of α in a particular system where $\ell_1/\ell_2 = 1.2$ and $a^* = 0.2$. The results are given in Table 3-34.

Table 3-34

λ AS A FUNCTION OF α FOR A SLAB
 WITH $\ell_1/\ell_2 = 1.2$ AND $a^* = 0.2$

order of approxi- α	P_1	P_3	P_5
0.00094	2.1526	2.2890	2.2919
0.1	2.1526	2.7364	2.7575
0.1667	2.1526	3.0864	3.1602
0.3	2.1526	3.9462	4.3461

(g) Linear Extrapolation Distance at the Plane Interface of Two Homogeneous Half-Spaces⁽¹²⁷⁾

The linear extrapolation distance, λ , into an absorbing non-multiplying half-space which scatters isotropically from a non-absorbing, isotropically scattering medium may be obtained from Fig. 3-55. In that figure the abscissa is c , the ratio of scattering to total cross section in the capturing medium, and the ordinate is λ for the scattering medium. The curve marked λ_{var} was obtained by substitution of the trial function $\frac{\nu}{3(1-c)} e^{\nu x}$ for the flux in the capturing half-space, $x < 0$. Then:

$$\lambda_{\text{var}} = 0.70711 + 0.22953 c + 0.21134 c^2 + O(c^3) \quad (3-35)$$

FIG. 3-55
LINEAR EXTRAPOLATION DISTANCE AT INTERFACE OF TWO SEMI-INFINITE MEDIA:
(a) PURE SCATTERER AND (b) SCATTERER AND ABSORBER
(REPRODUCED FROM REF. 127)

For c near unity, i.e., for small ratios of capture to scattering,

$$\lambda \approx \frac{1}{\nu} \approx \sqrt{\frac{c}{3(1-c)}}$$

The function λ_1 was obtained by assuming a linear variation, $x + a$, of the neutron flux in the positive half-space (the purely scattering medium), again using the trial function $\frac{\nu}{3(1-c)} e^{\nu x}$ in the negative half-space, and applying variational techniques to estimate the value of λ_1 . For small c ,

$$\lambda_1 = 0.6667 + 0.2046 c (1 + c) + O(c^3). \quad (3-36)$$

The so-called " λ_{exact} " was found by solving the integral equation for the neutron density distribution in the positive half-space. Approximately:

$$\lambda_{\text{exact}} = 0.71045 + 0.25000 c + 0.19885 c^2 + 0.16991 c^3 + O(c^4). \quad (3-37)$$

(h) Linear Extrapolation Distance at the Surface of a Scattering and Capturing Cylinder ⁽¹²⁷⁾

Consider a homogeneous cylinder of radius a which captures neutrons and scatters isotropically and elastically. Suppose that this cylinder is surrounded by N homogeneous purely absorbing annuli, and that these $N + 1$ regions are embedded in an infinite purely scattering medium, which scatters isotropically and elastically. Let

a = radius of cylinder (all measurements are in units of the total mean free path of the infinite medium)

$$\alpha_t = \frac{\Sigma_c(\text{cylinder}) + \Sigma_s(\text{cylinder})}{\Sigma_s(\infty \text{ medium})}$$

$$c = \frac{\Sigma_s(\text{cylinder})}{\Sigma_{\text{total}}(\text{cylinder})} = \text{average number of neutrons generated per collision in the cylinder}$$

$$\alpha_i = \frac{\Sigma_c(i^{\text{th}} \text{ annulus})}{\Sigma_s(\infty \text{ medium})}$$

$$A_i = \text{cross sectional area of the } i^{\text{th}} \text{ annulus}$$

$$a_i = \text{outer radius of the } i^{\text{th}} \text{ annulus}$$

$$\tau_i = \text{thickness of the } i^{\text{th}} \text{ annulus}$$

$$R = a_N = a + \sum_{i=1}^N \tau_i$$

Then the linear extrapolation distance at the outermost surface (at radius R) is given by:

$$\lambda = \frac{4}{3} \frac{1}{P(a\alpha_t, \alpha_i, a_i)} - 4T(R) + f(R)$$

where

$$P(a\alpha_t, \alpha_i, a_i) = \frac{a}{R} P(a\alpha_t) \prod_{i=1}^N T_i + \frac{2}{\pi R} \sum_{i=1}^N \alpha_i A_i - \frac{a}{R} P(a\alpha_t) \sum_{i=1}^N (1 - T_i)$$

with

$$P(a\alpha_t) = \frac{(1-c) P_1(a, \alpha_t)}{1-c P_c(a\alpha_t, \nu)}$$

$$P_1(a\alpha_t) = \frac{4}{3} a \alpha_t \left\{ \begin{aligned} &K_1(a\alpha_t) I_1(a\alpha_t) - 1 - 2a\alpha_t \\ &+ 2(a\alpha_t)^2 \left[I_0(a\alpha_t) K_0(a\alpha_t) + I_1(a\alpha_t) K_1(a\alpha_t) + \frac{I_0(a\alpha_t) K_1(a\alpha_t)}{a\alpha_t} \right] \end{aligned} \right\}$$

$P_c(a\alpha_t, \nu)$ satisfies the relationship:

$$\frac{c}{1-c} [1 - P_c(a\alpha_t, \nu)] = 3a c B I(a, \alpha_t, c);$$

where

$$I(a, \alpha_t, c) = a \int_1^\infty \frac{du}{u(\nu^2 - u^2)} K_1(a\alpha_t u) [\nu I_1(a\alpha_t \nu) I_0(a\alpha_t u) - u I_0(a\alpha_t \nu) I_1(a\alpha_t u)];$$

and

$$B = \frac{\alpha_t \nu}{3(1-c) a I_1(a\alpha_t \nu)}$$

ν satisfying the transcendental equation $2\nu = c \log \frac{1+\nu}{1-\nu}$; and

$$T_i = 1 - 2 \frac{\alpha_i}{\pi} \left[\sqrt{a_i^2 - a^2} - \sqrt{a_{i-1}^2 - a^2} + \frac{a_i^2}{a} \arcsin \frac{a}{a_i} - \frac{a_{i-1}^2}{a} \arcsin \frac{a}{a_{i-1}} \right]$$

The functions $4T(R)$ and $P_1(a\alpha_t)$ are graphed in Fig. 3-56 and are tabulated in Table 3-35.

FIG. 3-56
FUNCTIONS $P_1(a\alpha)$ AND $4T(a)$
(REPRODUCED FROM REF. 127)

Table 3-35

VALUES OF $P_1(a\alpha)$, $4T(a)$, AND $f(a)$

$a\alpha$	(Reproduced from Ref. 127.)			$f(a)$
	$P_1(a\alpha)$	a	$4T(a)$	
0.	0.	0.	0.	0.
0.1	0.1770	0.07	0.1439	0.035
0.2	.3127	0.10	.1844	.045
0.3	.4299	0.15	.2303	.055
0.5	.5960	0.2	.2662	.062
0.7	.7080	0.3	.3206	.070
0.8	.7499	0.5	.3919	.078
1.0	.8143	0.7	.4380	.077
1.3	.8771	1.0	.4837	.074
1.6	.9155	1.5	.5297	.064
2.0	.9458	2.5	.5760	.052
2.5	.9662	5.0	.6152	.044
3.0	.9772	8.0	.6344	.044
4.0	.9877	12.0	.6420	.044
5.0	.9921			

The function $f(a)$ has been approximated for two cases of a homogeneous cylinder of radius a :

$$\alpha = 1; \text{ i.e., } \Sigma_{\text{total}}(\text{cylinder}) = \Sigma_{\text{total}}^{(\infty)}(\text{medium}), \text{ and } \alpha = \infty;$$

i.e., black cylinder. These two approximations are graphed in Fig. 3-57 as a function of a , together with a graph of $f(a)$ which Kushneriuk and McKay have suggested as an approximation of wider applicability. The suggested variation of $f(a)$ is to be used also for the case where there are absorbing annuli, in which event the graph is to be interpreted as $f(R)$ vs. R . In Table 3-35 the suggested $f(a)$ is listed for various values of the parameter a .

FIG. 3-57
VARIOUS APPROXIMATIONS FOR $f(a)$
(REPRODUCED FROM REF. 127)

The special case of a homogeneous purely absorbing cylinder in an infinite scattering medium has been examined in detail. In Fig. 3-58 the linear extrapolation distance is shown as a function of the cylinder radius for various values of α .

FIG. 3-58
LINEAR EXTRAPOLATION DISTANCE (λ) AT THE SURFACE OF
A PURELY ABSORBING CYLINDER
(REPRODUCED FROM REF. 127)

3.7.2 Effective Radii for Various Shapes of Black Bodies

There is considerable overlap between this section and Section 3.7.3 dealing with the concept of absorption area. In both sections equivalence relationships between black bodies either are directly stated or may be inferred. However, the absorption area may be used directly for reactivity analyses, whereas the correlation between the reactivity effects of a black body and of an equivalent cylindrical rod is only a step to the final result.

This section is devoted to the presentation of results taken from work of Wheeler reported in Ref. 128.

The effective radius, r_{eff} , of a black body in an infinite homogeneous medium is defined as the radius of a cylindrical rod which has the same effect on the asymptotic neutron density. In determining this asymptotic neutron density the condition is used that the density vanishes at the surface of the equivalent rod.

In Table 3-36 formulae for r_{eff} are given⁽¹²⁹⁾ for various shapes of purely absorbing black bodies in holes of various sizes. In that table, the following symbols are used:

r = actual rod radius, cm

r_{eff} = effective rod radius, cm

ℓ = transport mean free path in the external medium, cm

λ = linear extrapolation distance for a purely absorbing black cylinder in an infinite homogeneous medium, in units of the total mean free path in the external medium

r_{ℓ} = outer radius of the hole in which the black body is located, cm

s = perimeter, cm, determined by putting a measuring tape once about the bar and drawing it tight; in general, the tape will not touch the bar at all points

w = span of the thin bar, or of the cross formed from two thin bars, cm.

In Table 3-37 are listed⁽¹²⁹⁾ measured and theoretical values of relative rod effectiveness for various shapes of black bodies. Fig. 3-59 is a nomogram for the effective radius of a black rectangle.

Table 3-36

EFFECTIVE RADIUS, r_{eff} , OF BLACK CONTROL BARS

Case	r_{eff}
1) Large, snugly-embedded rod of radius r	$r \exp(-0.71 \ell / r)$
2) Snugly-embedded rod of radius r	$r \exp(-\lambda \ell / r)$
3) Small compared to ℓ , snugly-embedded bar of perimeter s	$0.46 \ell \exp(-8\pi \ell / 3s)$
4) Small compared to ℓ , snugly-embedded rod of radius r	$0.46 \exp(-4\ell / 3r)$
5) Bar in a considerably larger hole of radius r_ℓ	$r_\ell \exp(-8\pi \ell / 3s)$
6) Rod in a considerably larger hole of radius r_ℓ	$r_\ell \exp(-4\ell / 3r)$
7) Embedded strip of small thickness compared to ℓ but with w comparable with or larger than ℓ	$w/4 \exp \left\{ -[y(1-0.114y + 0.048 y^2)]^{-1} \right\}$ where $y = \frac{\sqrt{3} w}{\pi \ell}$
8) Embedded cross formed of two thin strips of width $w/\sqrt{2}$	same as preceding
9) Large, embedded, rectangular bar	See Fig. 3-57

(Data from these tables are reproduced from Ref. 129)

Table 3-37

EFFECT OF CONTROL ROD SHAPE

Shape of bar	Perimeter, cm	Observed* Effectiveness	In-hole Calculation†		Imbedded Calculation‡	
		Relative to + bar	r_0 , cm	Relative Effectiveness	r_0 , cm	Relative Effectiveness
-	18.1	0.90	1.65	0.87	1.28	0.82
+	25.6	1.00	2.38	1.00	2.24	1.00
0	28.4	1.06	2.58	1.03	3.20	1.06

*All bars had a maximum extension of 9.05 cm and were located in a square hole equivalent in area to a circle of radius $r_1 = 5.74$ cm

†Using entry 5 of Table 3-34

‡Using Table 3-34 entry 7 for -shape, entry 8 for +shape, and entry 1 (but with a value 0.58 instead of 0.71) for 0 shape.

FIG. 3-59
EFFECTIVE RADIUS OF RECTANGULAR BAR

REMOVE A LAYER OF THICKNESS 0.711 ALL AROUND THE BAR TO OBTAIN A SMALLER EFFECTIVE RECTANGLE OF WIDTH w AND THICKNESS Aw . PLOT A ON RIGHT SCALE OF GRAPH AND CONNECT TO LOWER LEFT CORNER BY A STRAIGHT LINE INTERSECTING CURVE. BELOW INTERSECTION, READ $B = w/r_0$ ON BOTTOM SCALE.

(REPRODUCED FROM REF. 129)

3.7.3 Absorption Area Method⁽¹³⁰⁾

The absorption area of a source-free black body is defined to be the ratio of the rate of neutron absorption by the body per unit height to the source intensity in the external medium. The concept of absorption area introduced by Hurwitz and Roe⁽¹³⁰⁾ may be used as a characterization of equivalence between black bodies of irregular shape. That is, two black bodies may be termed equivalent if they have equal absorption area.

Consider a black body in a homogeneous infinite medium containing a uniform distribution of neutron sources and let L^2 be the thermal diffusion area of the infinite medium. Then, for L large compared with the dimensions of the body, the absorption area C is given by

$$C \text{ (limit of large } L) \approx 2\pi L \, r_{\text{eff}} \frac{K_1\left(\frac{r_{\text{eff}}}{L}\right)}{K_0\left(\frac{r_{\text{eff}}}{L}\right)}$$

In Table 3-38 the value of r_{eff} is listed for several regular bodies.

If the boundary of the black body has a radius of curvature which is large everywhere in comparison with L , then the boundary may be approximated by a polygon whose sides are large in comparison with L , and the interaction between vertices of this polygon may be neglected. In this case, the absorption area is:

$$C(\text{limit of small } L) = L \cdot \text{perimeter} + L^2 \sum_{\text{corners}} D(\phi_i) + O(L^3)$$

where ϕ_i is the external angle at corner i .

Table 3-38

 r_{eff} FOR SIMPLE SHAPES

(Reproduced from Ref. 130)

Bar	$r_{\text{eff}} = \frac{a}{2}$ where a is the half-width of the bar
Symmetrical cross	$r_{\text{eff}} = \frac{a}{\sqrt{2}}$ where a is the distance of the end of an arm from the axis
Rectangle*	d'/d 0 0.0432 0.2219 0.3968 0.5085 0.6418 0.8030 1.000 r_{eff}/d 0.25 0.2732 0.3939 0.4038 0.4401 0.4820 0.5314 0.5902
Ellipse	$r_{\text{eff}} = \frac{a+b}{2}$ where a and b are semi-major and semi-minor axes

* d' and d are respectively the short and long sides of the rectangle. For the rectangle $r_{\text{eff}}/d = [4\kappa^2 B(\kappa^2)]^{-1}$ where κ is determined by the implicit relationship $d'/d = \frac{1-\kappa^2}{\kappa^2} \frac{B(1-\kappa^2)}{B(\kappa^2)}$ where $B(\kappa^2)$ is a complete elliptic integral

In Table 3-39 $D(\phi)$ is listed for several values of ϕ . The function $D(\phi)$ is shown in Fig. 3-60, together with the results obtained in the limit $\phi \rightarrow 0$: $D(\phi) = \frac{(4 \log 2)}{\phi}$. For $\phi \approx \pi$, $D(\phi) = \frac{(\phi - \pi)}{2}$.

For intermediate values of L , approximate absorption area formulae for a bar and for a symmetrical cross rod have been obtained by variational methods. The bar is a black slab of zero thickness and of finite width $2a$ cm.

The ratio of the absorption area to $4aL$ is listed in Table 3-40 for black bars. The absorption area is obtained in four approximations:

Column 1 In the limit of large L :

$$\frac{C}{4aL} = \frac{\pi}{4} \frac{K_1\left(\frac{a}{2L}\right)}{K_0\left(\frac{a}{2L}\right)}$$

Column 2 In the limit of small L :

$$C = L [\text{perimeter} + 2L]$$

or

$$\frac{C}{4aL} = 1 + \frac{L}{2a}$$

Column 3 A variational lower bound

Column 4 A variational upper bound

Table 3-39

VALUES OF $D(\phi)$ FOR CERTAIN ANGLES

(Reproduced from Ref. 130)

ϕ	$D(\phi)$
2π	1
$3\pi/2$	$\frac{4}{3\pi} \left(\frac{4\pi}{3\sqrt{3}} - 1 \right)$
π	0
$\pi/2$	$-4/\pi$
$\pi/3$	$-4/\sqrt{3}$

FIG. 3-60
ABSORPTION AREA CORRECTION FOR CORNERS OF EXTERIOR ANGLE ϕ
(REPRODUCED FROM REF. 130)

Table 3-40

C/4aL FOR BLACK BAR

$\frac{L}{2a}$	Large L Limit $r_{\text{eff}} = a/2$	Small L Limit $\Sigma D(\phi) = 2$	Variational Lower Bound	Variational Upper Bound
0.2	1.061	1.200	1.175	1.200
0.5	1.4073	1.500	1.468	1.496
1	1.909	2.000	1.934	1.965
2	2.786	3.000	2.782	2.819
5	5.021	6.000	4.984	5.054
10	8.2440	11.000	8.15	8.367

This table is useful in examining the range of validity of the asymptotic formulae. To facilitate such an examination, in Fig. 3-61 the ratios $\frac{C}{C(\text{limit of large } L)}$ are plotted for the various approximate formulae, together with a "Best Estimate" curve. The absolute value of $\frac{C}{4aL}$ is graphed in Fig. 3-62, together with the curve of $C(\text{limit of large } L)$.

FIG. 3-61
RATIO OF ABSORPTION AREA FOR BLACK BAR TO ABSORPTION
AREA CALCULATED FROM THE FORMULA OF THE LIMIT OF LARGE L
(REPRODUCED FROM REF. 130)

FIG. 3-62
ABSORPTION AREA FOR BLACK BAR
(REPRODUCED FROM REF. 130)

The cross rod has a full span of $2a$ cm, i.e., each of the four arms is a cm wide. Table 3-41 is analogous to Table 3-40 for the control bar. In the limit of large L ,

$$\frac{C}{8aL} = \frac{\pi\sqrt{2}}{8} \frac{K_1\left(\frac{a}{\sqrt{2}L}\right)}{K_0\left(\frac{a}{\sqrt{2}L}\right)}$$

In the limit of small L ,

$$\frac{C}{8aL} = 1 - 0.273 \frac{L}{2a}.$$

Columns 3 and 4 are respectively variational lower and upper bounds of $\frac{C}{8aL}$.

Figures 3-63 and 3-64 are analogous to those already described for the control bar.

Table 3-41

C/8aL FOR SYMMETRICAL CROSS

$\frac{L}{2a}$	Large L Limit $r_{\text{eff}} = a/\sqrt{2}$	Small L Limit $\Sigma D(\phi) = -1.093$	Variational Lower Bound	Variational Upper Bound
0.1	0.6295	0.9727		0.9732
0.2	0.6973	0.9454		0.9598
0.5	0.8802	0.8635		1.0254
1.0	1.1478	0.727	1.1850	1.2319
2.5	1.8328		1.8212	
10.0	4.5291		4.44	

Experimental confirmation of the equivalence relationship between a control bar and a symmetrical cross rod has been obtained by Bach and Kitchen.⁽¹³¹⁾ Measurements were made in a cylindrical critical assembly (Flexible Plastic Reactor), moderated with polyethylene and fueled with uranium highly enriched in U^{235} . Although the absorption area was defined earlier for black rods, the concept is meaningful for source-free gray rods as well. It is of especial significance that the one-group equivalence relationship for black crosses and bars was shown experimentally to apply to gray bodies which are strong absorbers of neutrons over a wide energy range.

In Fig. 3-65 the reactivity effects of the various absorbers are shown as a function of the span of the absorber. In order to demonstrate the geometrical dependence of the absorption area, the experimentally obtained reactivities are normalized to the reactivity effect of the 0.027-inch thick cadmium body with a span of 2 inches. The experimental data was corrected for the effect of non-zero absorber thickness on the assumption that the fractional absorption area of the edge is proportional to the fractional geometric area of the edge. This correction was made in order to provide a meaningful comparison with the zero-thickness results of Hurwitz and Roe.⁽¹³⁰⁾ The magnitude of this correction is shown in Fig. 3-66, in terms of the relative reactivity effects of 0.100-inch thick and 0.200-inch thick hafnium bars. On Fig. 3-65 are graphed the "Best Estimate" and the "small L limit" curves of Hurwitz and Roe, normalized to the 2-inch span cadmium bodies, and based on a measured diffusion length $L = 0.6$ inch.

FIG. 3-83
RATIO OF ABSORPTION AREA FOR BLACK CROSS TO ABSORPTION
AREA CALCULATED FROM THE FORMULA OF THE LIMIT OF LARGE L
(REPRODUCED FROM REF. 130)

FIG. 3-84
ABSORPTION AREA FOR BLACK CROSS
(REPRODUCED FROM REF. 130)

FIG. 3-65
RELATIVE REACTIVITY EFFECTS OF SLAB AND CROSS RODS
(SLAB AND CROSS DATA NORMALIZED TO $a = 2$ in.)
(REPRODUCED FROM REF. 131)

FIG. 3-66
THE RATIO, $R = (\Delta k/k \text{ 0.100 in. Hf SLAB}) / (\Delta k/k \text{ 0.200 in. Hf SLAB})$
SHOWING EFFECT OF EDGE CORRECTION
(REPRODUCED FROM REF. 131)

3.7.4 Reactivity Effects of a Black Cylindrical Rod

In the case of a one-group diffusion theory analysis of a small circular cylinder at the center of an unreflected cylindrical reactor, the variation of the excess effective multiplication factor, $\Delta k_{\text{eff}} = k_{\text{eff}} - 1$, with effective rod radius, r_{eff} , is given approximately by:⁽¹³²⁾

$$(\text{one-group diffusion theory}) \Delta k_{\text{eff}} = \frac{7.5 M^2}{(1 + M^2 B^2) R} \left[\frac{1}{0.116 + \log \frac{R}{2.4 r_{\text{eff}}}} \right] \quad (3-38)$$

where r_{eff} is the rod radius minus the linear extrapolation distance. In Eq. (3-38):

M^2 = neutron migration area (cm^2)

$B^2 \approx \left(\frac{2.4}{R} \right)^2 + \left(\frac{\pi}{H} \right)^2$ (cm^{-2}); R , H are the radius and height, respectively, of the core.

With the approximation that the linear extrapolation distance for thermal neutrons may be computed as if all neutrons were thermal, an approximate formula for k_{eff} may be obtained from two-group neutron diffusion theory:⁽¹³³⁾

$$k_{\text{eff}} = \frac{7.5 M^2}{(1 + L^2 B^2)(1 + \tau B^2)R^2} \left[\frac{1}{0.116 \left(1 + \frac{\tau}{L^2}\right) + \frac{\tau}{L^2} \log \frac{L\sqrt{\tau}}{M r_{\text{eff}}} + \log \frac{R}{2.4 r_{\text{eff}}}} \right]$$

where

$$M^2 = L^2 + \tau \text{ (cm}^2\text{)}$$

$$\tau = \text{age to thermal (cm}^2\text{)}$$

$$L^2 = \text{thermal diffusion area (cm}^2\text{)}$$

In the derivation of this formula, it was assumed that the rod was transparent to neutrons in the fast group.

Adler⁽¹³⁴⁾ has applied one-group diffusion theory to the analysis of the variation of effectiveness of a cylindrical rod with distance from the center of the core. Adler has shown⁽¹³⁵⁾ that the critical buckling determined by two-group diffusion theory may be obtained by a modified one-group diffusion theory. For applicability of the modified one-group method, absorbing rods must be at least one diffusion length apart and at least one diffusion length from the core-reflector interface. The variation of effectiveness of a black cylindrical rod with distance, d , from the axis of an unreflected cylinder of radius R is shown in Fig. 3-67. In that figure a comparison is presented with the curve of effectiveness predicted by the weighting of the rod worth with the square of the unperturbed flux, the result of lowest order one-group perturbation theory. It may be observed that the perturbation theory approximation overestimates rod effectiveness by 10% to 25% when the rod is off center by 25% or more of the way from the center to the edge of the core.

The effect of a radial reflector on the worth of a single rod is illustrated⁽¹³⁶⁾ in Fig. 3-68. The change in critical buckling is shown as a function of rod position for two reactors having the same buckling in the absence of the rod, one reflected and the other unreflected radially.

FIG. 3-67
EFFECTIVENESS OF ECCENTRIC CYLINDRICAL CONTROL
ROD OF RADIUS $r_{\text{eff}} = 0.01 R$
(REPRODUCED FROM REF. 129)

FIG. 3-68
CONTROL ROD EFFECTIVENESS IN
BARE VS REFLECTED REACTORS
(REPRODUCED FROM REF. 129)

FIG. 3-69
EFFECTIVENESS OF PARTIALLY WITHDRAWN AXIAL
CONTROL ROD RELATIVE TO EFFECTIVENESS WHEN COMPLETELY INSERTED
(REPRODUCED FROM REF. 129)

The preceding paragraphs have dealt with the problem of a fully inserted cylindrical rod. The relative effectiveness of a small black cylindrical rod in a bare reactor has been calculated⁽¹³⁶⁾ as a function of the fractional penetration of the rod into the reactor. In Fig. 3-69 this curve is presented for one set of rod and reactor dimensions, together with a curve based on lowest-order one-group perturbation theory weighting with the square of the unperturbed axial flux distribution. It may be observed that the perturbation theory curve lies below the more accurate curve along its entire length.

More recently,⁽¹³⁷⁾ a curve similar to Fig. 3-69 has been obtained by one-group two-dimensional calculations on a large computing machine, the IBM-704, using the CURE code.

The rod radius was varied over a range up to 20% of the bare core radius. In Fig. 3-70 is plotted the fractional reactivity change as a function of rod penetration for various rod radii. It may be observed that the curve is almost independent of rod radius over the range examined.

FIG. 3-70
FRACTIONAL REACTIVITY CHANGE AS A FUNCTION
OF ROD RADIUS AND DEPTH OF INSERTION
(REPRODUCED FROM REF. 137)

These computations⁽¹³⁸⁾ were extended to rod radii as large as 50% of the bare core radius. The fractional change in critical buckling required when the rod is inserted half way is plotted in Fig. 3-71 as a function of R/r . For $(R/r) < 5$ the drop is very sharp.

FIG. 3-71
DEPENDENCE OF FRACTIONAL CHANGE IN CRITICAL BUCKLING
ON RATIO OF RADIUS OF CORE TO RADIUS OF CENTRAL
BLACK CONTROL ROD INSERTED HALFWAY

3.7.5 Interaction Effects of Control Rods

(a) Generalizations of the Nordheim-Scalettar Method⁽¹³⁹⁾ for Cylindrical Rods

The methods described in this section have been devised to describe the effect of distributed discrete control regions in an otherwise uniform medium. Another method, based on the small-source, small-sink analysis is described in Section 4.4.5.

It is well known that the reactivity effect of a set of closely spaced black control rods fully inserted into a reactor is smaller than the sum of the worths of the individual rods. This result may be ascribed to what is generally known as the "shadowing" of each control rod by the other rods, or the "interference" between rods. That is, the flux depression in the neighborhood of a rod is enhanced by the presence of nearby rods. Beyond certain separation distances, however, the combined rod effectiveness may exceed the sum of the isolated parts. This occurs because the flux depression in the vicinity of each rod is countered partially by the rise in flux outside the region of influence of the other rods.

A commonly used method is the Nordheim-Scalettar⁽¹³⁹⁾ scheme of two-group diffusion theory. In this method the flux is written as a linear combination of regular solutions of the wave equation and of irregular terms with singularities at the positions of the control rods. Either a linear extrapolation distance or a logarithmic derivative condition is employed for the thermal flux at the surface of the thermally black rod. In various versions or modifications of the Nordheim-Scalettar method⁽¹⁴⁰⁻¹⁴⁵⁾ the two-group fast flux is assumed to be finite at the center of the rod or to vary in the

neighborhood of the rod in a manner described by a fast linear extrapolation distance. The original Nordheim-Scalettar (N-S) condition⁽¹³⁹⁾ is that the fast flux is finite at the center of the rod. For large control rods and for reactors where the ratio of fast to thermal flux is large, the N-S condition implies that the gradient of fast neutrons at the rod surface is appreciable and therefore perhaps a poor approximation to the physical situation.⁽¹⁴⁵⁾ For example, in the SPERT-I reactor the central control blade was shown experimentally to be worth 3.4% in the reflected reactor.⁽¹⁴⁴⁾ The application of the original N-S condition to the computation of an equivalent round rod in an equivalent bare reactor yielded 6.9%. This disagreement is attributed primarily to the use of the N-S condition. The use of a boundary condition reported variously in its essential import as "Codd-Rennie,"⁽¹⁴²⁻¹⁴⁶⁾ (C-R), or "Murray-Niestlie Hollow-Shell,"⁽¹⁴⁶⁻¹⁴⁸⁾ namely that the fast current into the rod is zero, led to a value of 3.2% both in the reflected core and in the equivalent bare core. On the other hand, the C-R condition of vanishing gradient of the fast flux led to a gross overestimation of the worth of an MTR-ORR type rod in the Army Package Power Reactor (APPR).⁽¹⁴⁵⁾ This rod is a thin black annulus enclosing a moderating medium of H₂O. The worth of the central rod was measured as $k_{ex} = -0.059$ to $k_{ex} = -0.063$ in comparison with an N-S calculation $k_{ex} = -0.058$ and a C-R calculation of $k_{ex} = -0.122$.⁽¹⁴⁵⁾

A further modification of the fast flux boundary condition which is applicable to moderator filled control rods has been made by Perry,⁽¹⁴⁵⁾ and Murray and Niestlie.⁽¹⁴⁷⁾ The method consists of diffusion theory calculation of a moderator-filled centrally-positioned round rod in the reactor. The regular solution $I_0(\kappa r)$ of the wave equation is used in conjunction with the usual diffusion theory boundary conditions for the fast group: continuity of flux and of current at the rod boundary, $r = a$. The fast extrapolation distance, d_1 , is defined by

$$d_1 = \frac{D_{core} I_0(\kappa a)}{D_{rod} \kappa I_1(\kappa a)}$$

where D_{core} , D_{rod} are respectively the fast diffusion coefficients outside and inside the rod, and:

$$\kappa^2 = \frac{1}{\tau} - \left(\frac{\pi}{H}\right)^2$$

τ = age to thermal in the rod material

H = extrapolated height of the reactor core

A generalization⁽¹⁴⁹⁾ of the Nordheim-Scalettar analysis to a two-group diffusion theory of a ring of uniformly spaced cylindrical rods in an otherwise homogeneous reactor may be applied to gray rods, to rods containing a fissionable material, etc.

Adler⁽¹⁵⁰⁾ has reduced the two-group Nordheim-Scalettar calculation to an equivalent one-group calculation. This one-group method was applied to the computation of the worth of rods in a bare reactor. In Fig. 3-72 a comparison is shown between the reactivity effect per rod in a four-rod ring and the effect of an isolated rod at that distance. Figure 3-73 shows the effect of a reflector on the worth of the ring of four rods; the comparison is made between a reflected and an equivalent bare reactor.

Simplifications⁽¹⁴⁰⁻¹⁴⁵⁾ of the critical determinants are possible if the control rods are distributed uniformly in rings with the center at the radial axis of a cylindrical core.

FIG. 3-72
COMPARISON BETWEEN THE EFFECT PER CONTROL ROD OF
A RING OF FOUR RODS WITH THAT OF A SINGLE ECCENTRIC
ROD AT THE SAME DISTANCE FROM THE CENTER OF A BARE SYSTEM
(REPRODUCED FROM REF. 150)

FIG. 3-73
COMPARISON OF THE EFFECT OF A RING OF FOUR RODS IN
SYSTEMS WITH AND WITHOUT REFLECTOR HAVING THE
SAME CRITICAL BUCKLING IF NO RODS ARE PRESENT
(RADIUS OF CONTROL ROD = $0.01R$)
(REPRODUCED FROM REF. 150)

Murray⁽¹⁴¹⁾ has reported results of two-group diffusion theory investigations of multiple control rod configurations in bare cylindrical reactors. For a ring of M symmetrically spaced round rods concentric with the core axis, the critical buckling, μ^2 is given by the formula:

$$\psi(\mu R) = \psi(\mu a),$$

where

$$\psi(\mu R) \approx \frac{1}{M \sum_{k=-\infty}^{\infty} \frac{J_{Mk}^2(\mu d) Y_{Mk}(\mu R)}{J_{Mk}(\mu R)} - \sum_{m=2}^M Y_0(\mu r_{1m})}$$

$$\psi(\mu a) \approx \frac{a_1 a_2 - a_3 a_4}{a_5 a_2 - a_3 a_6}$$

and

$$a_1 = \frac{J_0(\mu a)}{d_1} - J_0'(\mu a)$$

$$a_2 = S_2 \left[\frac{K_0(\nu a)}{d_1} - K_0'(\nu a) \right]$$

$$a_3 = \frac{K_0(\nu a)}{d_1} - K_0'(\nu a)$$

$$a_4 = S_1 \left[\frac{J_0(\mu a)}{d_2} - J_0'(\mu a) \right]$$

$$a_5 = \frac{Y_0(\mu a)}{d_1} - Y_0'(\mu a)$$

$$a_6 = S_1 \left[\frac{Y_0(\mu a)}{d_2} - Y_0'(\mu a) \right]$$

In these formulae:

d = radius of locus of centers of the rods (cm)

r_{1m} = distance from center of rod No. 1 to center of rod No. m

R = core radius (cm)

a = rod radius (cm)

d_1, d_2 are the fast and thermal linear extrapolation distances at the rod.

μ^2, ν^2 are the usual bucklings of the two-group diffusion theory.

S_1, S_2 are the ratios of fast to thermal flux in the two components of those fluxes (c.f. Kurath and Spinrad⁽¹⁵¹⁾)

The symbol " ' " represents differentiation with respect to r .

For small rods, $\frac{J_0(\mu R)}{Y_0(\mu R)} \approx J_0^2(\mu d) \psi(\mu a)$.

Murray reported on comparisons of theory with experiment. A control rod of radius 1.74 cm located 12.75 cm from the axis of the BSF (Bulk Shielding Facility) reactor at ORNL was found to be worth 1.37%, in comparison with ~1.38% determined by measurement. The calculation was performed on a bare equivalent core; from a calculation which took into account the effect of the reflector, the value ~1.5% was obtained. The latter type of calculation predicted that a fuel mass of 52 grams U^{235} would compensate for the reactivity loss engendered in a fully inserted 5/8-inch off-center rod in the Raleigh Research Reactor. The experimental value was 57 grams.

Using only the terms where $k = 0$ in the formula for $\psi(\mu R)$, the critical equation reduces to:

$$\frac{J_0(\mu R)}{Y_0(\mu R)} \approx \frac{M\psi(\mu a)}{1 + I(\mu a)} \cdot J_0^2(\mu d)$$

where

$$I = \sum_{m=2}^M Y_0(\mu r_{1m})$$

Using this approximate critical equation, the reactivity values of 1, 2, 3, 4, and 5 rods were computed for the equivalent bare BSF core. The results are presented in Fig. 3-74. In Fig. 3-75, the reactivity effect of a ring of two diametrically opposite rods is shown as a function of the distance of the ring of centers from the core axis. Figures 3-74 and 3-75 are from Ref. 141.

FIG. 3-74
REACTIVITY VS NUMBER OF
CYLINDRICAL CONTROL RODS

FIG. 3-75
INTERACTION OF TWO CYLINDRICAL
CONTROL RODS

(REPRODUCED FROM REF. 141)

The interaction effects of five water filled black annuli in the APPR are described⁽¹⁴⁵⁾ in Tables 3-42 and 3-43. The control system consists of a ring of four rods around a central rod. For each rod configuration criticality was attained experimentally by varying the number of boron-containing plates, and separately by varying the fuel loading. In Table 3-43 calculated results obtained by the various (N-S, C-R, M-N)

fast flux boundary conditions are compared with experiment. In the table, $W = k_{\text{eff}} - 1$, the excess k held down by the rod configuration indicated; $\sum_1 W_i$ is the sum of the values of the individual rods in the combination, taken separately.

Table 3-42

CRITICAL MASS vs. CONTROL ROD
CONFIGURATION IN APPR

(Reproduced from Ref. 145)

Configuration ^a	M, kg U ²³⁵	Configuration ^a	M, kg U ²³⁵
No rods	10.50	3X + 0	16.77
X	11.17	4X	17.31
0	12.18	4X + 0	22.82
X + 0	13.05		

^aX signifies an off-center rod position; 0 signifies the central rod position; all rods are fully inserted.

Table 3-43

COMPARISON OF CALCULATED AND EXPERIMENTAL ROD VALUES IN APPR

(Reproduced from Ref. 145)

Configuration	Calculated Worth		Experimental Worth				Calculation Method
			Uniform Poison		Fuel Addition		
	W	W/ ΣW_i	W	W/ ΣW_i	W	W/ ΣW_i	
X	0.026		0.027		0.026		Perry ⁽¹³⁸⁾ ; M-N N-S C-R
0	0.058		0.063		0.059		
0	0.122						
0	0.026						
X+0	0.099	1.18	0.089	0.99	0.085	1.00	
2X	0.068	1.26					
2X+0	0.147	1.31					
3X	0.118 ^a	1.51					
3X+0	0.188 ^a	1.38	0.166	1.15			
4X	0.171	1.64	0.173	1.60			
4X+0	0.241	1.49	0.229	1.34			

^aIn the calculation, the three off-center rods are located at the corners of an equilateral triangle.

(b) Partitioning of Reactor Core into Control Cells

A method of control analysis used frequently in reactor design is to partition the reactor core into an integral number of smaller units, one unit associated with each control rod. Each such unit is treated as a cell in an infinite lattice, and the cell boundary conditions on flux and current appropriate to the configuration are applied. In general, this method works best when the number of such control cells is large. In each cell, the approximation is made that the actual absorbing rod may be replaced by a round rod or by some other shape which is amenable to calculation.

Henry has reported⁽¹⁵²⁾ on a simplified analysis for cross rods from which one obtains the ratio $\frac{\Sigma_p}{\Sigma + \Sigma_p}$; Σ_p is the homogeneous poison cross section equivalent in control worth to the actual rod, and Σ is the average macroscopic removal cross section in the cell medium outside of the rod. Thus $\frac{\Sigma_p}{\Sigma + \Sigma_p}$ is essentially the reactivity held down by the control rod. Let the following diagram represent a quarter-section of the control cell.

Then, treating each arm of the cross as an infinite black slab of half-thickness, a , in a slab cell of half-thickness m , a cell computation yields the neutron current into the rod:

$$\frac{q}{h\Sigma + \frac{1}{L} \coth\left(\frac{m-a}{L}\right)}.$$

Then it is assumed that

$$\frac{\Sigma_p}{\Sigma + \Sigma_p} = \frac{\frac{1}{q} \text{ total exposed perimeter of rod}}{\text{total area of source region}} \frac{q}{h\Sigma + \frac{1}{L} \coth\left(\frac{m-a}{L}\right)}$$

or

$$\frac{\Sigma_p}{\Sigma + \Sigma_p} \approx \frac{2\ell}{(m-a)^2} \frac{1}{h\Sigma + \frac{1}{L} \coth\left(\frac{m-a}{L}\right)}$$

The notation used is:

q = source density, neutrons/(cm²)(sec), assumed to be constant in the region outside the rod and to be zero inside the rod;

L = diffusion length of the medium outside the rod;

h = $\frac{\text{linear extrapolation distance into the rod}}{D}$;

and the other terms are defined in the diagram. (Note that in the approximate formula, the contribution of the double cross-hatched region in the accompanying sketch has been ignored.)

L is assumed to be given by

$$L = \sqrt{\frac{D}{\Sigma}},$$

where D is the diffusion coefficient in the medium. (For a black slab, $h = 3 \times 0.7104$.)

In Table 3-44 a comparison is presented between Σ_p 's calculated by this method and also by two-dimensional calculations on a digital computer. It was assumed that the rods were black with a value $h = 2.13$.

Table 3-44

VALUES OF Σ_p CORRESPONDING TO VARIOUS CROSS-SHAPED
RODS IN VARIOUS SQUARE CELLS

(Reproduced from Ref. 152)

Cell Width (cm) 2m	20.16	20.16	20.16	21.36	21.36	21.36	21.36	19.05	19.05
Square of Thermal Diffusion Length of Medium (cm ²) L ²	1.544	1.544	1.544	2.515	2.881	4.567	5.315	6.337	11.36
Ratio of Thermal Diffusion Length to Rod Span Half Size L/Q	0.261	0.178	0.143	0.270	0.289	0.364	0.392	0.396	0.531
Values of Equivalent Poison Cross Section Σ_p (cm ⁻¹)									
Two-Dimensional Machine Code Solution	0.140	0.0219	0.0293	0.0182	0.0168	0.0192	0.0179	0.0233	0.0244
Slab Cell Approximation	0.0138	0.0216	0.0293	0.0191	0.0177	0.0208	0.0193	0.0233	0.0268
$\frac{\Sigma_p(\text{exact}) - \Sigma_p(\text{cell method})}{\Sigma_p(\text{exact})}$	0.014	0.014	0.000	-0.049	-0.054	-0.082	-0.078	-0.045	-0.098

If the cross rod fills the cell, it may be convenient to look at the cell as a square black annulus around the core medium. Stewart and Smith⁽¹⁵³⁾ have obtained an infinite series representation for the flux distribution in this square cell. The problem solved was a one-group diffusion theory cell calculation with constant source and vanishing of the thermal flux at the surface of the black boundary. The flux is given by:

$$\phi(x, y) = \sum_{\substack{n, m \geq 1 \\ \text{odd}}}^{\infty} a_{mn} \sin\left(\frac{m\pi x}{w}\right) \sin\left(\frac{n\pi y}{w}\right)$$

where

w = length of one side of the square core medium, cm

$$a_{mn} = S \left(\frac{2}{\pi}\right)^4 \left(\frac{R}{L}\right)^2 \frac{1}{mn} \frac{1}{m^2 + n^2 + \left(\frac{R}{\pi L}\right)^2}$$

L = thermal diffusion length in the core medium, cm.

S = neutron source strength per unit volume.

Defining C* to be the fraction of all neutrons absorbed by the rod,

$$C^* = 1 - \frac{64}{\pi^4} \nu^2 f(\nu)$$

where

$$\nu = \frac{\kappa w}{\pi}, \quad \kappa = \frac{1}{L}; \quad f(\nu) = \sum_{\substack{m,n \geq 1 \\ \text{odd}}}^{\infty} \frac{1}{m^2 n^2} \frac{1}{m^2 + n^2 + \nu^2}$$

For large ν :

$$f(\nu) = \frac{\pi^4}{64\nu^2} \left[1 - \frac{4}{\pi\nu} + \frac{16}{\pi^3\nu^2} \right] + O(\exp[-\pi\nu]).$$

For small ν :

$$f(\nu) = \frac{1}{\nu^2 + 2} + 0.0282 - 0.0024 \nu^2 + 0.0002 \nu^4 + O(\nu^6).$$

It is of interest to determine the size of a cylindrical cell, bounded by black material, which is equivalent to the square cell in the sense that

$$C^*(\text{circle}) = C^*(\text{square}).$$

Where

$$C^*(\text{circle}) = \frac{2I_1(\kappa r)}{\kappa r I_0(\kappa r)}$$

Applying the same boundary condition, vanishing of the flux at the outer boundary leads to the equivalence relationship described in Fig. 3-76^a between $\nu_s \equiv \frac{\kappa w}{\pi}$ and $\nu_c \equiv \frac{\kappa r}{\pi}$, where r = radius of the circle. It may be seen that the inscribed circle provides a very good approximation to equivalence even for relatively small values of ν_c . This may be compared with the assumption frequently made that the equivalent circle has approximately the same geometric area as the square. In the case of equal area,

$$\frac{\nu_s}{\nu_c} = 1.77, \text{ whereas actually } \lim_{\nu_c \rightarrow \infty} \frac{\nu_s}{\nu_c} = 2.$$

^aUnpublished correlation by D. H. Shaftman, ANL.

FIG. 3-78
ONE-GROUP DIFFUSION THEORY CALCULATIONS OF ROD WORTH

3.7.6 Multigroup Control Rod Calculations

Group diffusion techniques have been applied to control rod calculations to obtain a more realistic representation of the problem than is possible in the one group description.

Stevens⁽¹⁵⁴⁾ has extended the absorption area concept (Section 3.7.3) to control rods with epithermal as well as thermal absorption, and has combined the concept with one and one-half and two and one-half group calculations. These half-group diffusion theories assume that the slowing down age is much larger than the square of the thermal diffusion length. Except near the boundaries, the diffusion of neutrons is retained in the fast and epithermal groups but is neglected in the thermal group. Near boundaries, the thermal diffusion of neutrons is obtained by a constant source approximation derived from the slowing down of the fast neutrons.

In the one and one-half group model the absorption area of a slab is determined by the method of Hurwitz and Roe, except that a reduced diffusion length, $L' = LF$, is used. For $\tau \gg L^2$,

$$F = \frac{1}{1 + \frac{2}{3} \frac{\lambda}{L} \left(\frac{1}{f\Sigma t} - 1 \right)}$$

where

$\frac{\lambda}{3}$ = thermal diffusion coefficient in the external medium

f = self-shielding factor of the absorbing slab

Σ = macroscopic absorption cross section of the absorbing slab

t = thickness of the absorbing slab.

Similar results are obtained for cylinders and spheres; radius r . The modified absorption areas are

Cylinder:

$$C = \frac{2\pi Lr}{\frac{K_0\left(\frac{r}{L}\right)}{K_1\left(\frac{r}{L}\right)} + \frac{\Delta}{L}}$$

Sphere:

$$C = \frac{4\pi r^2 L}{\frac{r}{r+L} + \frac{\Delta}{L}}$$

where

$$\Delta = \lambda \left[\frac{\Delta'}{\lambda} + \frac{1-2f\xi}{2f\xi} \right]$$

In Fig. 5-2 (see Section 5.2) are shown values of f for various shapes of rods; ξ is defined there. Formulae for $\frac{\Delta'}{\lambda}$ are,

Cylinder: $\frac{\Delta'}{\lambda} = .7104 + .5047 \frac{\lambda}{2r}$

Sphere: $\frac{\Delta'}{\lambda} = .7104 + .5047 \frac{\lambda}{r}$

The results of one and one-half and two and one-half group computations with single cross rod (1.88 in. half-width) experiments conducted in the Flexible Plastic Reactor (FPR) are tabulated below. It can be seen that in the case of hafnium, which has significant epithermal absorption, the two and one-half group approach gives much better agreement with experiment than does the one and one-half group calculation.

Absorber Sample	Reactivity Change, Dollars		
	Experiment	1 ^{1/2} Group	2 ^{1/2} Group
B-Al (0.05 gm boron/cm ²)	0.994 ± 0.015	1.008	1.027
Cd (0.027" thick)	0.956 ± 0.015	1.076	1.023
Hf (0.100" thick)	1.396 ± 0.022	0.901	1.452
Hf (0.200" thick)	1.710 ± 0.028	1.024	1.726

A similar comparison between experiment and two and one-half group calculations for single, long, thin slabs (1.8 in. wide by $15\frac{1}{2}$ in. long) in the Advanced Test Reactor (ATR), are given below. Since the absorber worth depended upon the test location, only the relative experimental measurements have interpretative value. The calculated and measured values were arbitrarily normalized for the thick hafnium sample.

Absorber Sample	Reactivity Change, Dollars	
	Experiment	Two Group
Cd (0.125" thick)	0.326	0.311
Hf (0.050" thick)	0.285	0.271
Hf (0.103" thick)	0.380	0.369
Hf (0.205" thick)	0.470	0.470
		(normalized)
7% Boron-Stainless Steel (0.205" thick)	0.252	0.221

Pearlstein⁽¹⁵⁵⁾ combined the absorption area concept with a three-group calculation and assumed that the fast group flux and source neutrons for the epithermal group were spatially uniform. The fraction of the core which is ineffective in neutron multiplication was taken to be the ratio of the absorption area to the total fissionable area. Unity minus this fraction is called the "rod absorption factor"; it is the fraction of the slowing down density that is available for multiplication in that energy group. Curves are presented⁽¹⁵⁵⁾ which facilitate calculation of the F factors.

Voorhis and Ryan,⁽¹⁵⁶⁾ and Voorhis, et al.⁽¹⁵⁷⁾ have examined the effect of absorbing membranes of cadmium and gold^(156,157) and boron-polyethylene⁽¹⁵⁷⁾ which bisected a highly enriched slab core. A procedure is described whereby the membranes are replaced by fictitious diffusion regions whose net effect on the surrounding core is the same as the actual membrane. After the group diffusion constants for the fictitious region were determined, one dimensional, two or four group diffusion calculations were performed in the direction normal to the membrane. In the calculation, the core, reflector (water) and thin fictitious region are taken as separate diffusing media. The transverse leakage was accounted for in the calculation by increasing the group absorption by the term $D_i B_T^2$ where D_i is the i th group diffusion constant and B_T^2 is the transverse buckling.

The diffusion calculations were compared with experiment in two ways. The first determined how closely the critical size of each assembly could be predicted; the second compared experimental activation traverses through each assembly with computed neutron fluxes. Good agreement was obtained in the latter case when the activation data was arbitrarily normalized to the calculated flux plot near the core-water boundary. Table 3-45 compares the experimental and calculated values for k_{eff} .

Table 3-45

COMPARISON OF EXPERIMENTAL AND THEORETICAL CRITICALITY

(Reproduced from Ref. 157).

Absorbing Membrane	Experimental Rod-Free k_{eff}	Calculated k_{eff} With Membrane		Calculated k_{eff} Without Membrane 4-Group	Calculated Worth Δk_{eff} of Membrane 4-Group
		4-Group	2-Group		
20 mil Cd	1.002	1.013	1.022	1.157	0.144
10 mil Au	1.000	1.008	1.024	1.073	0.065
30 mil Au	1.001	1.011	1.023	1.132	0.121
50 mil Au	1.000	1.018	1.025	1.160	0.142
Boron	1.001	1.016	1.031	1.043	0.027
None	1.001	1.005	1.026		

3.8 REACTOR DYNAMICS

3.8.1 Neutron Poisoning by Xe^{135} and Sm^{149}

The poisoning effects due to formation of fission products in a reactor can be discussed in terms of Xe^{135} , Sm^{149} , and all other stable and radioactive nuclides.

Xenon¹³⁵

The fission product chain of mass number 135 is: (Section 1 - Tables 1-3, 1-5, and 1-6).

Since the Te^{135} has so short a half-life, I^{135} is considered as being formed from fission. The yield of I^{135} and direct yield of Xe^{135} from thermal fission are summarized below:

Fissionable Isotope	U^{233}	U^{235}	Pu^{239}
Yield of I^{135}	0.051	0.061	0.055
Yield of Xe^{135} (direct)	-	0.003	-

The concentrations of I^{135} and of Xe^{135} in a reactor are given by:

$$\frac{dI(t)}{dt} = \phi \Sigma_f Y^I - \lambda^I I \quad (3-39)$$

$$\frac{dX(t)}{dt} = \phi \Sigma_f Y^X + \lambda^I I - (\lambda^X + \phi \sigma_a^X) X,$$

where I, X are the concentrations (atoms per cubic centimeter of I^{135} and Xe^{135} respectively.)

$\phi \Sigma_f$ is the fission rate (fissions per cubic centimeter per second)

Y^I = fission yield of I^{135} (actually of Te^{135})

Y^X = direct fission yield of Xe^{135}

$\lambda^I = 2.89 \times 10^{-5}/\text{sec}$ is the decay constant of I^{135}

$\lambda^X = 2.09 \times 10^{-5}/\text{sec}$ is the decay constant of Xe^{135}

and time, t , is in units of seconds.

The microscopic Xe^{135} absorption cross section σ_a^X is given as 2.7×10^6 b at 2200 m/sec.⁽¹⁵⁸⁾ In addition, in Tables 2-6, 2-7, and 2-9, g -factors⁽¹⁵⁹⁾ are presented for the capture cross section of Xe^{135} for various values of the neutron temperature.

Since in general reactor applications, $\phi \Sigma_f$ and $\phi \sigma_a^X$, are slowly varying in time, the general solution of Eq. (3-39) may be simplified considerably in two special cases. Starting with fresh fuel, $I(t) = X(t) = 0$ and

Case 1

$$\phi \sigma_a^X \ll \lambda X \text{ (or } \phi \ll 10^{13} \text{ n/cm}^2 \text{ sec)}$$

$$X(t) \approx \frac{\phi \Sigma_f (Y^I + Y^X)}{\lambda X} [1 - \exp(-\lambda X t)] - \phi \Sigma_f Y^I \left[\frac{\exp(-\lambda X t) - \exp(-\lambda^I t)}{\lambda^I - \lambda X} \right]$$

Case 2

$$\phi \sigma_a^X \gg \lambda X \text{ and for } t > 1 \text{ hr}$$

$$X(t) \approx \frac{\phi \Sigma_f [Y^I + Y^X]}{\phi \sigma_a^X} - \frac{\phi \Sigma_f Y^I}{\phi \sigma_a^X} \exp(-\lambda^I t) \approx \frac{\phi \Sigma_f [Y^I + Y^X]}{\phi \sigma_a^X} [1 - \exp(-X^I t)]$$

Thus

$$\frac{X(t)}{X_{eq}} \approx [1 - \exp(-\lambda^I t)]$$

where X_{eq} is the equilibrium concentration: $X_{eq} = \frac{\phi \Sigma_f (Y^I + Y^X)}{\lambda X + \phi \sigma_a^X}$.

Therefore, for very high flux, the xenon concentration is proportional to the iodine concentration, reaching 50% of its equilibrium level in one half-life of I^{135} (6.7 hr), 95% in one day, and 99% in 2 days.

If the fractional burnup of fuel is small in time comparable to the half-life of Xe^{135} , then the iodine and xenon concentrations approach the saturation levels:

$$I_{eq} = \frac{\phi \Sigma_f Y^I}{\lambda^I}$$

$$X_{eq} = \frac{\phi \Sigma_f (Y^I + Y^X)}{\lambda X + \phi \sigma_a^X}$$

and

$$\frac{X_{eq}}{I_{eq}} = \left[\frac{Y^I + Y^X}{Y^I} \right] \cdot \frac{\lambda^I}{\lambda X + \phi \sigma_a^X}$$

It is convenient to describe the xenon poisoning effect as the ratio:

$$\frac{\phi \Sigma_a^X}{\phi \Sigma_f} = \frac{(Y^I + Y^X) \phi \sigma_a^X}{\lambda X + \phi \sigma_a^X}$$

This ratio is graphed in Fig. 3-77 for a thermal reactor where fission events occur only in U^{235} .

If, after I^{135} and Xe^{135} have attained their equilibrium values for flux ϕ , the flux is instantaneously set = 0 at $t = 0$, then

$$I(t) = I_{eq} \exp(-\lambda^I t)$$

(3-40)

$$X(t) = X_{eq} \exp(-\lambda X t) - \lambda^I I_{eq} \left\{ \frac{\exp(-\lambda^I t) - \exp(-\lambda X t)}{\lambda^I - \lambda X} \right\}$$

FIG. 3-77
RELATIVE NEUTRON ABSORPTION BY EQUILIBRIUM XENON
IN A HIGHLY ENRICHED (U^{235}) THERMAL REACTOR

The xenon concentration will increase after shutdown if $\phi \sigma_a^X > \frac{\gamma^X}{\gamma^I} \lambda^X$.

Differentiating Eq. (3-40) and solving for the time, t_{max} , at which the xenon concentration is greatest gives:

$$t_{max} = \frac{1}{\lambda^X - \lambda^I} \log \left[\frac{\lambda^X}{\lambda^I} \left(1 + \frac{\lambda^I - \lambda^X}{\lambda^I} \frac{X_{eq}}{I_{eq}} \right) \right]$$

t_{max} asymptotically approaches 11.3 hr as ϕ is increased.

Buildup of xenon after a step change in flux level from an equilibrium concentration may be obtained easily from the solution of Eq. (3-39) by using the following initial conditions:

$$\begin{aligned} I(0) &= I_{eq} \\ X(0) &= X_{eq} \\ \phi \Sigma_f &= \text{Fission rate at the new flux level.} \end{aligned}$$

Curves have been prepared by Macrea and Parr⁽¹⁶⁰⁾ for the change in equilibrium xenon poisoning following step increases or decreases in the level of neutron flux. Figures 3-78 and 3-79 show the times to reach maximum xenon concentration following a step function change in the flux from one non-zero value to a second non-zero value.

Clark and English⁽¹⁶¹⁾ have published tables of numbers from which one may obtain easily the time-dependence of xenon concentration following a step change in flux level.

FIG. 3-78
TIME TO REACH MAXIMUM XENON POISONING AFTER
FLUX INCREASE FROM ϕ_1 TO ϕ_2
(REPRODUCED FROM REF. 160)

FIG. 3-79
TIME TO REACH MAXIMUM POISONING AFTER
FLUX DECREASE FROM ϕ_1 TO ϕ_2
(REPRODUCED FROM REF. 160)

Ward's⁽¹⁶²⁾ method consists of plotting a flux independent curve of

$$\left[\frac{X}{X_{\max}} \right]^* = \lambda^X \exp(\lambda^I T) \left\{ \frac{\exp(-\lambda^X t) - \exp(-\lambda^I t)}{\lambda^I - \lambda^X} \right\}$$

for the initial condition $X(0) = 0$, and for $T = \frac{1}{\lambda^I - \lambda^X} \log \frac{\lambda^I}{\lambda^X}$, as shown in Fig. 3-80.

A translation time, t^* , is given in Fig. 3-81 such that:

$$\frac{X_{\text{eq}}}{I_{\text{eq}}} = \frac{\lambda^I}{\lambda^I - \lambda^X} [\exp([\lambda^I - \lambda^X] t^*) - 1].$$

Then the fractional buildup, $\frac{X}{X_{\max}}(t)$ for the general case $X(0) \equiv X_{\text{eq}}$ may be obtained from the universal curve by translating the time origin of the latter curve to t^* . From this same curve may be found the time to reach maximum xenon following a complete shutdown.

Samarium-149

Samarium-149 is a stable isotope arising from beta decay of promethium-149 (Pm^{149}), which is formed directly from fission and also from the beta decay of the fission product neodymium-149. The neodymium isotope has so short a half-life (2.0 hr) in comparison with that of Pm^{149} (54 hr) that the simplified chain for Sm^{149} is written:

The cumulative yields of Sm^{149} from thermal fission are:

U^{233}	U^{235}	Pu^{239}
$Y = 0.006$	0.0113	0.019

The dynamic balance equations for the simplified model are:

$$\frac{d\text{Pm}}{dt} = \phi \Sigma_f Y^{\text{Pm}} - \lambda^{\text{Pm}} \text{Pm}$$

$$\frac{d\text{Sm}}{dt} = \lambda^{\text{Pm}} \text{Pm} - \phi \sigma_a^{\text{Sm}} \text{Sm}$$

$$\lambda^{\text{Pm}} = 3.56 \times 10^{-6} / \text{sec}$$

Pm = concentration of promethium-149

Sm = concentration of samarium-149

Over intervals of constant $\phi \Sigma_f$ and $\phi \sigma_a^{\text{Sm}}$,

$$\text{Pm}(t) = \frac{\phi \Sigma_f Y^{\text{Pm}}}{\lambda^{\text{Pm}}} \left[1 - \exp(-\lambda^{\text{Pm}} t) + \text{Pm}(0) \exp(-\lambda^{\text{Pm}} t) \right]$$

$$\begin{aligned} \text{Sm}(t) = \frac{\phi \Sigma_f Y^{\text{Pm}}}{\phi \sigma_a^{\text{Sm}}} + & \left[\text{Sm}(0) - \frac{\phi \Sigma_f Y^{\text{Pm}}}{\phi \sigma_a^{\text{Sm}}} - \frac{\lambda^{\text{Pm}} \text{Pm}(0) - \phi \Sigma_f Y^{\text{Pm}}}{\phi \sigma_a^{\text{Sm}} - \lambda^{\text{Pm}}} \right] \exp(-\phi \sigma_a^{\text{Sm}} t) \\ & + \left[\frac{\lambda^{\text{Pm}} \text{Pm}(0) - \phi \Sigma_f Y^{\text{Pm}}}{\phi \sigma_a^{\text{Sm}} - \lambda^{\text{Pm}}} \right] \exp(-\lambda^{\text{Pm}} t). \end{aligned}$$

FIG. 3-80
FLUX INDEPENDENT CURVE OF XENON
CONCENTRATION AS A FUNCTION OF TIME
AFTER COMPLETE SHUTDOWN
(REPRODUCED FROM REF. 162)

FIG. 3-81
TRANSLATION TIME (t^*) AS A FUNCTION OF
RATIO OF
EQUILIBRIUM CONCENTRATION OF X_0^{135} TO I^{135}
(REPRODUCED FROM REF. 162)

Thus the equilibrium concentration of Sm^{149} is:

$$\text{Sm}_{\text{eq}} = \frac{\phi \Sigma_f Y_{\text{Sm}}}{\phi \sigma_a \text{Sm}}$$

The transient concentrations are given by Blomeke and Todd(163) in

Fig. 3-82.

FIG. 3-82
EFFECT OF IRRADIATION TIME AND NEUTRON FLUX ON
AMOUNT OF STABLE Sm^{149} FORMED IN FISSION OF U^{235}
(DATA FROM REF. 163)

3.8.2 Poisoning Due to Accumulation of Gross Fission Products

The poisoning effects of the high cross section ($\sigma_a > \sigma_a^{\text{U}^{235}}$), intermediate cross section ($\sigma_a^{\text{U}^{235}} > \sigma_a > 10b$), and low cross section ($\sigma_a < 10b$) fission products are shown in Fig. 3-83 as calculated by Deutsch.(164) The total poisoning is shown in Fig. 3-84 assuming the unknown absorption cross sections of Sm^{148} and Sm^{150} to be 250b each. The fractional fuel burnup, β , is related to the integrated flux by

$$\beta = 1 - \exp(-\sigma_a^{\text{U}^{235}} \int_0^t \phi(u) du)$$

Walker(165) has published tables and figures related to computed poisoning by important individual fission products and to total poisoning as functions of the flux levels 10^{13} , 10^{14} , and 10^{15} ($\text{n}/(\text{cm}^2)(\text{sec})$), and of the integrated flux ranging from $(0.3 \text{ to } 3.0) \times 10^{21}$ n/cm^2 . Table 3-46 gives Walker's data for known fission product yields and cross sections. The total fission product poisoning for isotopes of known cross section is presented in Figs. 3-85, 3-86, and 3-87, excluding the contributions of Xe^{135} and Sm^{149} . Poisoning is given in barns per initial fuel atom (Case I) and in barns per surviving fuel atom (Case II). The total poisoning including xenon and samarium is similarly shown in Figs. 3-88, 3-89 and 3-90. Figures 3-85 through 3-90 are reproduced from Ref. 165.

FIG. 3-83
COMPONENTS OF FISSION-PRODUCT POISONING
($\sigma_a^{U^{235}} = 687$ barns)
(DATA OF REF. 164)

FIG. 3-84
THE CONTRIBUTIONS OF Sm^{148} AND Sm^{150}
TO TOTAL FISSION PRODUCT POISONING
(DATA FROM REF. 164)

Table 3-46

NUCLEAR DATA FOR IMPORTANT FISSION PRODUCTS

Data of Walker⁽¹⁶⁵⁾

Isotope	Neutron Capture C.S. - barns -	Half-Life h-hrs. d-days y-yrs.	Isotope	Neutron Capture C.S. - barns -	Half-Life h-hrs. d-days y-yrs.
³⁴ Se ⁷⁹		6.5×10^4 y	⁵⁵ Cs ¹³³	29.0 ± 1.5	s
³⁶ Kr ⁸³	205 ± 10	s	⁵⁵ Cs ¹³⁴		2.3 y
³⁸ Sr ⁸⁹	<90	54 ± 3 d	⁵⁵ Cs ¹³⁷	<2	32 ± 4 y
³⁹ Y ⁹¹		58 ± 3 d	⁵⁶ Ba ¹⁴⁰		$1.28 \pm .05$ y
⁴⁰ Zr ⁹³	<4	11.1 ± 10^6 y	⁵⁷ La ¹³⁹	8.4 ± 1.7	
⁴⁰ Zr ⁹⁵		65 ± 2 d	⁵⁸ Ce ¹⁴¹		32 ± 1 d
⁴¹ Nb ⁹⁵		35 ± 1 d	⁵⁸ Ce ¹⁴⁴		279 ± 4 d
⁴² Mo ⁹⁵	13.4 ± 1.1	s	⁵⁹ Pr ¹⁴¹	11.2 ± 0.6	s
⁴³ Tc ⁹⁹	100 ± 25	$(2.13 \pm .02) \times 10^5$ y	⁵⁹ Br ¹⁴³		13.7 ± 0.1 d
⁴⁴ Ru ¹⁰¹	<12	s	⁶⁰ Nd ¹⁴³	300 ± 20	s
⁴⁴ Ru ¹⁰³		39.8 ± 3 d	⁶⁰ Nd ¹⁴⁵	45 ± 10	s
⁴⁴ Ru ¹⁰⁶		1.0 y	⁶⁰ Nd ¹⁴⁶	9 ± 1	s
⁴⁵ Rh ¹⁰³	150 ± 7	s	⁶⁰ Nd ¹⁴⁷		11.3 ± 0.2 d
⁴⁶ Pd ¹⁰⁴	<50	s	⁶¹ Pm ¹⁴⁷	60 ± 20	2.6 ± 0.2 y
⁴⁶ Pd ¹⁰⁵	<22	s	⁶² Sm ¹⁴⁹	$(66 \pm 2) \times 10^3$	s
⁴⁶ Pd ¹⁰⁶	<18	s	⁶² Sm ¹⁵⁰	85 ± 25	s
⁴⁶ Pd ¹⁰⁷		5×10^6 y	⁶² Sm ¹⁵¹	12000 ± 3000	73 y
⁴⁷ Ag ¹⁰⁹	84 ± 7	s	⁶² Sm ¹⁵²	220 ± 50	s
⁴⁷ Ag ^{110m}		270 d	⁶² Sm ¹⁵³		46.5 ± 0.5 h
⁴⁸ Cd ¹¹¹		s	⁶³ Eu ¹⁵³	420 ± 100	s
⁴⁸ Cd ¹¹²		s	⁶³ Eu ¹⁵⁴	1500 ± 400	16 y
⁴⁸ Cd ¹¹³	25,000	s	⁶³ Eu ¹⁵⁵	$(14 \pm 4) \times 10^3$	1.8 ± 0.2 y
⁵¹ Sb ¹²⁵		2.7 y	⁶³ Eu ¹⁵⁶		15 ± 1 d
⁵³ I ¹³¹	<145	$8.05 \pm .05$ d	⁶⁴ Gd ¹⁵⁷	160,000	s
⁵⁴ Xe ¹³¹	120 ± 15	s			
⁵⁴ Xe ¹³³	200 ± 140	5.27 ± 0.2 d			
⁵⁴ Xe ¹³⁵	3.4 ± 10^6	$9.13 \pm .05$ h

FIG. 3-85
POISONING IN U^{233} BY FISSION PRODUCTS FOR WHICH σ_c
IS KNOWN EXCEPT Xe^{135} AND Sm^{149}

FIG. 3-86
POISONING IN U^{235} BY FISSION PRODUCTS FOR WHICH σ_c
IS KNOWN EXCEPT Xe^{135} AND Sm^{149}

FIG. 3-87
 POISONING IN Pu-239 BY FISSION PRODUCTS FOR WHICH σ_c IS
 KNOWN EXCEPT Xe-135 AND Sm-149

FIG. 3-88
 POISONING IN U-233 BY FISSION PRODUCTS FOR WHICH σ_c IS KNOWN

FIG. 3-89
 POISONING IN U²³⁵ BY FISSION PRODUCTS FOR WHICH σ_c IS KNOWN

FIG. 3-90
 POISONING IN Pu²³⁹ BY FISSION PRODUCTS FOR WHICH σ_c IS KNOWN

Blomeke and Todd(163) present a vast array of fission product data for U^{235} including:

- (1) Tables of yields and cross sections.
- (2) Equilibrium concentrations of the various radioactive isotopes as a function of flux levels.
- (3) Curves of the fraction of equilibrium concentration of radioactive isotopes as a function of flux level and of time after startup.
- (4) Curves of the concentrations of radioactive isotopes as a function of time after shutdown, for various levels of flux prior to shutdown.
- (5) Curves of the buildup of individual stable fission products as a function of flux level and of time of irradiation.
- (6) Curves describing the poisoning effects of fission products as a function of flux and time.

Curves of the total poisoning effect of fission product buildup (including Xe^{135} and Sm^{149}) as a function of the flux level and of time of irradiation are shown in Fig. 3-91.

FIG. 3-91
BUILDUP OF TOTAL FISSION PRODUCT POISONING
AS A FUNCTION OF IRRADIATION TIME
AND THERMAL NEUTRON FLUX
(REPRODUCED FROM REF. 163)

Hurst(166) gives an approximate numerical correlation of the effective cross section per fission of accumulated fission products as a function of integrated flux, and also gives coefficients y_1 , y_2 , and y_3 to fit the following analytical expression for the effective neutron cross section in barns per fission:

$$\frac{\sigma_a^{fp}}{\text{fission}} = 40 y_1 e^{-40x} + 200 y_2 e^{-200x} + 1000 y_3 e^{-1000x}$$

where x is the integrated flux in n/cm^2 times 10^{-24} . These results are given in Table 3-47.

Table 3-47

CROSS SECTION PER AVERAGE FISSION AS A FUNCTION
OF IRRADIATION AND YIELDS OF PSEUDO FISSION
PRODUCTS EQUIVALENT TO FISSION PRODUCTS
WITH CROSS SECTION LESS THAN 1000 Barns

(Data of Hurst)⁽¹⁶⁶⁾

Integrated Flux($10^{21}n/\text{cm}^2$)	Fissile Nuclide	U ²³³	U ²³⁵	Pu ²³⁹	Pu ²⁴¹
	0	46.0 b	52.4 b	65.6 b	69.0 b
	1	41.0 b	47.6 b	64.1 b	68.6 b
	2	36.2 b	42.3 b	58.7 b	63.1 b
	4	28.7 b	34.1 b	48.2 b	51.6 b
	6	23.5 b	28.3 b	39.6 b	42.2 b
	8	19.8 b	23.9 b	32.9 b	35.0 b
	10	17.0 b	20.8 b	28.0 b	29.5 b
	y ₁ (40 barns)	0.506%	0.634%	0.8167%	0.8435%
	y ₂ (200 barns)	0.1288%	0.1410%	0.2270%	0.2546%
	y ₃ (300 barns)	0	0	-0.0124%	-0.0157%

In Table 3-47:

1. For fission products of unknown cross section, the assumption was made that these isotopes and the higher order isotopes in that chain have the same distribution of intermediate and low cross sections as the chains where the cross sections are known. The total cross section of the known products, therefore was multiplied by the factor,

$$2$$

total yield of products of known cross section

2. The results may be used for reactors where the flux level is less than or of the order of $10^{13} n/\text{cm}^2 \text{ sec}$.
3. Primary fission products with $\sigma_a > 1000 \text{ b}$ are omitted, but their (n, γ) daughters are included.

From these compilations the net poisoning effect of fission products may be obtained by integration over the operating time (t_0) of the reactor:

$$\text{net } \Sigma_a^{fp}(t_0) = \int_0^{t_0} dt \frac{P(t) \Sigma_a^{fp}(x)}{\int_0^{t_0} dt P(t)}$$

where

$P(t)$ is the power density at time t , and $x = \int_t^{t_0} \phi(u) du$;
 $\phi(t)$ is the neutron flux at time t .

Circulating Fuel Reactors

Lock⁽¹⁶⁷⁾ has reported on theoretical studies of fission product poisoning in circulating fuel systems. Sampson, et al.⁽¹⁶⁸⁾ have investigated the reduction in fission product poisoning obtained by removing gaseous products. The results for U^{233} and U^{235} systems are shown in Figs. 3-92 and 3-93.

FIG. 3-92
 EFFECTIVE POISONING WITH REMOVAL OF GASEOUS
 FISSION PRODUCTS (U^{233} FUEL)
 (REPRODUCED FROM SAMPSON, et al¹⁶⁸)

FIG. 3-93
 EFFECTIVE POISONING WITH REMOVAL OF GASEOUS
 FISSION PRODUCTS (U^{235} FUEL)
 (REPRODUCED FROM SAMPSON, et al¹⁶⁸)

3.8.3 Isotope Buildup in Thorium Reactors

The nuclear reactions occurring during neutron irradiation of Th^{232} are diagrammed in Fig. 3-94.

FIG. 3-94
NUCLEAR REACTIONS UNDER THERMAL NEUTRON
IRRADIATION OF Th^{232}

The thermal absorption cross sections listed in Table 3-48 are those used by Gresky and Arnold^(169,170) in theoretical studies to be described in the following paragraphs. In the table, the cross sections are compared with the values presented in BNL-325⁽¹⁷¹⁾ and its Supplement No. 1,⁽¹⁷²⁾ when significantly different. The largest important differences for $\sigma_f^{\text{U}^{233}}$, whose world value is given as 532 ± 6 b, and for $\sigma_a^{\text{Pa}^{233}}$ whose effective value in a thermal reactor has been found to be closer to 150b than to 40b. All cross sections are 2200 meter/sec values unless otherwise specified. Cross sections are in barns.

Table 3-48
CROSS SECTIONS FOR HEAVY ISOTOPES PRESENT
IN IRRADIATED THORIUM

Isotope	ORNL-1817 ⁽¹⁶⁹⁾			BNL-325 ⁽¹⁷¹⁾ and Supplement No. 1 ⁽¹⁷²⁾		
	σ_a	σ_{act}	σ_f	σ_a	σ_{act}	σ_f
Th^{232}		7.0		7.45 ± 0.15	7.34 ± 0.15	
Th^{233}		1350			1400 ± 200 (pile neutrons)	
Pa^{233}		150 (pile neutrons)			40 ± 6	
U^{233}	560	55	505	588 ± 7	56 ± 1	532 ± 6
U^{234}		70		92 ± 7	72 ± 10 (pile neutrons)	
World Consistent Set: $\eta^{\text{U}^{233}} = 2.28 \pm 0.02$						
BNL-325 Suppl. No. 1 $\nu^{\text{U}^{233}} = 2.52 \pm 0.03$						

Thorium slugs have been irradiated in the Low Intensity Test Reactor (LITR) of Oak Ridge National Laboratory and in a graphite reactor. The slugs were approximately 4 cm in diameter and 16 cm long. Using 12.5b for the resonance integral of thorium in the slugs, and 7.0b for the thermal capture cross section of thorium, a value of 8.0b was selected as an effective capture cross section of the thorium in the two reactors. (In both reactors at the site of irradiation the ratio of resonance to thermal absorption was found to be 0.13.) Using this value for thorium, the differential equations representing the reaction rates in thorium and in Pa^{233} were solved for $\sigma_{\text{ceff}}^{\text{Pa}^{233}}$; the effective capture cross section of Pa^{233} is listed in Table 3-49 for each of seven irradiated slugs. The weighted average value, 151 b, may be compared with an average of 129b, based on five slugs irradiated in the graphite reactor.

Table 3-49

U^{233} AND U^{234} YIELDS AND DERIVED
CROSS SECTIONS FOR LITR IRRADIATIONS

Time in Reactor Days	(Data from Ref. 173)		$\sigma_{\text{c}}(\text{Pa}^{233})_{\text{eff}}$ barns
	U^{233} Chain Th^{232} ($\times 10^{-6}$)	U^{234} Chain U^{233} Chain ($\times 10^{-2}$)	
62	134.0	.090 ₄	138
113	329.0	.202 ₄	149
175	488.0	.259	145
35	36.7	.032 ₈	154
42	65.8	.055 ₈	152
63	107	.078 ₁	140
133	196	.135 ₉	172

Aluminum jacketed solid thorium slugs have been irradiated in the Materials Testing Reactor (MTR)⁽¹⁷³⁾ for periods of time up to 18 months. The reactivity effect of each irradiated slug was determined by period measurements in the Reactivity Measurements Facility (RMF) at the MTR site. The results are shown in Table 3-50. A detailed mass spectrographic analysis was made⁽¹⁷⁴⁾ of a 1.33 (ID), 6.5-inch long thorium slug clad with 0.05 in. of aluminum.⁽¹⁷⁵⁾ The slug had been irradiated in the MTR for one year to an nvt of approximately 10^{21} n/cm², and had then been cooled for 3.5 years prior to analysis. The average distributions of uranium isotopes at various distances from the slug axis are shown in Table 3-51.

Detailed theoretical studies have been made by Taraba,⁽¹⁷⁶⁾ and by Gresky and Arnold of the net buildup of heavy isotopes during neutron irradiation of thorium. Figures 3-95 through 3-99 show the net concentrations of the heavy isotopes as a function of the level of thermal neutron flux, and as a function of irradiation time. The concentration of Th^{232} was held constant. In Fig. 3-100 the ratio of Pa^{233} to U^{233} concentration is given for various irradiation levels $\left(\frac{\text{grams } (\text{Pa}^{233} + \text{U}^{233})}{\text{ton } \text{Th}^{232}} \right)$ as a function of the neutron flux level. Figures 3-95 through 3-100 are reproduced from Ref. 170.

Table 3-50

SUMMARY OF DATA ON THORIUM SLUGS IN MTR IRRADIATION

Slug No.	$\Delta k/k$ per cent	Grams U^{233}	nvt ($\times 10^{20}$)
51	.107	10.6	11.6
52	.115	10.9	12.0
53	.118	10.9	10.1
54	.115	11.4	13.0
56	.056	1.5	1.3
57	.085	3.2	2.9
58	.090	4.7	4.5
59	.082	4.4	4.1
60	.042	3.1	2.8
61	.050	2.3	2.0
62	.076	4.5	4.3
63	.080	6.3	6.3
64	.062	6.1	6.0
65	.034	4.9	4.7
66	.064	4.9	4.7
67	.074	6.1	6.1
68	.065	5.2	4.9
69	.058	4.9	4.7

Table 3-51

AVERAGE RADIAL DISTRIBUTION OF URANIUM
IN IRRADIATED THORIUM SLUG

(Reproduced from Ref. 175)

Radial Interval cm	U^{233} Weight %	Isotopic Distribution % of Total Uranium		
		U^{233}	U^{234}	U^{235}
0.0 - 0.5	0.33	98.50	1.45	0.06
0.5 - 0.9	0.34	98.45	1.50	0.05
0.9 - 1.3	0.36	98.26	1.65	0.09
1.3 - 1.6	0.37	98.23	1.65	0.12
1.6 - 1.7	0.40	98.15	1.72	0.14

Computations have been made⁽¹⁶⁹⁾ of the equilibrium concentrations of the heavy isotopes in a continuously processed volume of thorium exposed to a constant neutron flux. These concentrations are shown in Fig. 3-101 for various rates of processing, and various levels of flux. In Fig. 3-102 the concentration of isotopes of mass chain 233 (Pa^{233} plus U^{233}) is presented as a function of processing rate and flux level. The ratio of Pa^{233} to U^{233} concentration is given in Fig. 3-103.

FIG. 3-95
CONCENTRATIONS OF HEAVY ISOTOPES PRODUCED
BY LONG-TERM IRRADIATION OF Th^{232} AT
FLUX OF 10^{13} neutrons/(cm²)(sec)

FIG. 3-96
CONCENTRATIONS OF HEAVY ISOTOPES PRODUCED
BY SHORT-TERM IRRADIATION OF Th^{232}
AT FLUX OF 10^{14} neutrons/(cm²)(sec)

FIG. 3-97
CONCENTRATIONS OF HEAVY ISOTOPES PRODUCED
BY LONG-TERM IRRADIATION OF Th^{232} AT
FLUX OF 10^{14} neutrons/(cm^2)(sec)

FIG. 3-98
CONCENTRATIONS OF HEAVY ISOTOPES PRODUCED
BY SHORT-TERM IRRADIATION OF Th^{232}
AT FLUX OF 10^{15} neutrons/(cm^2)(sec)

FIG. 3-99
CONCENTRATIONS OF HEAVY ISOTOPES PRODUCED BY LONG-
TERM OF Th^{232} AT FLUX OF 10^{15} neutrons/(cm^2)(sec)

FIG. 3-100
EFFECT OF NEUTRON FLUX AND EXTENT OF
IRRADIATION ON $\text{Pa}^{233}/\text{U}^{233}$ RATIO

FIG. 3-101
EFFECT OF REACTOR PROCESSING PERIOD AND NEUTRON FLUX,
ON PRODUCTS FORMED IN IRRADIATION OF Th^{232}
(REPRODUCED FROM REF. 170)

FIG. 3-102
EFFECT OF FLUX AND PROCESSING PERIOD ON gm/ton LEVEL
(REPRODUCED FROM REF. 169)

FIG. 3-103
EFFECT OF NEUTRON FLUX AND gm/ton LEVEL
ON $\text{Pa}^{233}/\text{U}^{233}$ RATIO
(REPRODUCED FROM REF. 169)

3.8.4 Studies of Isotope Concentration in Uranium-Thorium Reactors

Typical conceptual designs of U^{233} breeder reactors involve irradiation of a thorium blanket and the chemical processing of this thorium to obtain U^{233} for replenishment of the fuel in the core. Discussions of such reactor cycles have been published(177-180) presenting curves for isotope buildup, reactivity effects of irradiation, refueling requirements, etc.

Stoughton and Halperin(181) have reported studies of five different cases involving the replenishment of core fuel, initially either U^{233} or U^{235} , with U^{233} either pure or containing 5% U^{234} . Long and Lock(182) have published tables showing the amounts produced of these higher isotopes in a reactor where the U^{233} concentration is held fixed, and the various isotopes are exposed continuously to a constant flux of 10^{15} n/(cm²)(sec). These computations are summarized in Table 3-52.

Table 3-52

AMOUNTS OF HIGHER ISOTOPES PRODUCED FROM U^{233} AFTER
VARIOUS TIMES IN A FLUX OF 10^{15} n. cm.⁻² sec.⁻¹

(Reproduced from Ref. 182)

Results expressed as a fraction of original fissile material. ($\times 10^{-3}$)

Time days	1	3	10	30	100	300	1000	∞	
<u>U</u>									
N_{24}/N_{23}	4.500	13.400	43.50	123.00	334.0	611.0	721.0	721.0	$\times 10^{-3}$
N_{25}/N_{23}	0.041	0.116	1.15	7.08	30.4	62.7	74.1	75.6	"
N_{26}/N_{23}	-	-	-	0.28	12.6	96.1	421.0	900.0	"
N_{27}/N_{23}	-	-	-	0.002	0.08	0.66	3.17	6.44	"
N_{28}/N_{23}	-	-	-	-	-	0.0	1.12	21.0	"
N_{29}/N_{23}	-	-	-	-	-	-	-	0.001	"
Total									
N_U/N_{23}	4.541	13.516	44.65	130.36	377.08	770.47	1,220.4	1,724.0	"
<u>Np</u>									
N_{37}/N_{23}	-	-	-	-	0.1709	3.151	19.89	47.4	"
N_{38}/N_{23}	-	-	-	-	0.0052	0.095	0.602	1.4	"
N_{39}/N_{23}	-	-	-	-	-	-	0.004	0.07	"
N_{30}/N_{23}	-	-	-	-	-	-	0.00	0.00	"
Total									
N_{Np}/N_{23}	0.000	0.000	0.000	0.000	0.1761	3.146	20.50	48.9	"
<u>Pu</u>									
N_{48}/N_{23}	-	-	-	-	0.0459	0.847	5.35	12.74	"
N_{49}/N_{23}	-	-	-	-	0.0186	0.342	2.17	5.39	"
N_{40}/N_{23}	-	-	-	-	0.0106	0.196	1.25	3.08	"
N_{41}/N_{23}	-	-	-	-	0.0049	0.073	0.46	1.14	"
N_{42}/N_{23}	-	-	-	-	0.05	0.92	5.86	14.50	"
N_{43}/N_{23}	-	-	-	-	-	-	-	0.01	"
Total									
N_{Pu}/N_{23}	0.00	0.00	0.00	0.00	0.130	2.38	15.09	36.83	"
<u>Am</u>									
N_{53}/N_{23}	-	-	-	-	-	0.23	1.47	3.63	"
N_{54}/N_{23}	-	-	-	-	-	-	-	0.007	"
Total									
N_{Am}/N_{23}	0.00	0.00	0.00	0.00	0.00	0.23	1.47	3.64	"
<u>Cm</u>									
N_{64}/N_{23}	-	-	-	-	-	-	-	0.05	"
Total									
N_{Cm}/N_{23}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	"

Legend: $N_{2j} \equiv U^{23j}$ atoms, $j = 3$ to 9 ; $N_{3k} \equiv Np^{23k}$, $k = 7$ to 9 ; $N_{30} \equiv Np^{240}$;
 $N_{48} \equiv Pu^{238}$; $N_{49} \equiv Pu^{239}$; $N_{4n} \equiv Pu^{24n}$, $n = 0, 1, 2, 3$; $N_{5j} \equiv Am^{24j}$,
 $j = 3, 4$; $N_{64} \equiv Cm^{244}$.

Davey⁽¹⁸³⁾ has applied the g and s factors (see Section 3.2 for discussion of Westcott's notation $\hat{\sigma} = \sigma_0(g + rs)$) given by Campbell and Freemantle⁽¹⁸⁴⁾ (in Ref. 184 the related notation $\bar{S}$ and S^1 is used) to the computation of effective cross sections as a function of the ratio of epithermal to thermal absorptions in the system. Using an effective neutron temperature of 475°C, Davey computed the effective cross sections of U^{233} , U^{234} , U^{235} , and U^{236} as a function of λ , where $r = \frac{1.345\lambda}{1 + 1.345\lambda}$ is based on an epithermal flux consisting of a Maxwellian plus a $1/E$ flux beginning at ~ 2.81 kTeff. The factor λ plays the part of the factor β used by Westcott. The results are shown in Table 3-53.

Table 3-53

EFFECTIVE CROSS SECTIONS AND ETA VALUES

(Reproduced from Ref. 183)

λ	0	.05	.10	.15	.20	.25
U^{233}						
σ_a	590.0	658.2	718.3	771.7	819.4	862.2
σ_f	538.0	600.2	655.0	703.7	747.2	786.2
σ_c	52.00	58.01	63.31	68.01	72.22	76.00
U^{234}						
σ_a	92.0	151.9	204.7	251.6	293.5	331.1
U^{235}						
σ_a	626.8	632.6	637.6	642.2	646.2	649.8
σ_f	530.4	531.2	531.9	532.6	533.2	533.7
σ_c	96.4	101.4	105.7	109.6	113.0	116.1
U^{236}						
σ_a	6.0	31.25	53.54	73.30	90.97	106.8
ηU^{233}	2.289					
ηU^{235}	2.075	2.059	2.046	2.034	2.024	2.014

Carter⁽¹⁸⁵⁾ has published a collection of graphs describing the buildup of the various heavy isotopes arising from constant power operation of low enrichment fixed fuel system initially fueled with U^{235} and containing thorium as fertile material. The effects of three parameters are studied: power level (mw/ton), initial U^{235} enrichment, and the probability of escaping resonance capture by Th^{232} . Also included in this report are curves of the variation of η_{av} during such irradiation. Spinrad, et al.,⁽¹⁸⁰⁾ have also reported on such studies. In Fig. 3-104 some of their numerical results are summarized in the form $\frac{\eta_{av}}{\eta_{initial}}$ vs mwd/ton.

FIG. 3-104
VARIATION OF η FOR U^{235} - Th
SYSTEMS UNDER IRRADIATION
(REPRODUCED FROM REF. 180)

3.8.5 Heavy Isotope Buildup and Reactivity Variation of Plutonium-Uranium Systems

The nuclear reactions among the heavy isotopes under neutron irradiation of U^{238} are given in Fig. 3-105.

A large number of survey reports have appeared in the literature describing the variation of reactivity and of isotopic concentrations as a function of exposure, of the resonance escape probability, of enrichment, etc. Some of these in addition to those discussed below are referred to in reports by Graves⁽¹⁸⁶⁾; Ioffe and Okun⁽¹⁸⁷⁾; Spinrad, Carter and Egger⁽¹⁸⁸⁾ and numerous reports by Lewis⁽¹⁸⁹⁾.

The effective cross section is used in the differential equations describing the dynamics of fuel irradiation. A considerable amount of experiment and analysis has been done in order to deduce various effective cross sections and the consequences on isotope buildup and reactivity.

FIG. 3-105
NUCLEAR REACTIONS UNDER NEUTRON IRRADIATION OF U^{238}

Chalk River Irradiations of Plutonium

Effective cross sections for Pu^{240} have been obtained⁽¹⁹⁰⁻¹⁹²⁾ by theoretical analysis of measured concentrations of plutonium isotopes in irradiated samples of Pu^{239} . Fields, et al.,⁽¹⁹¹⁾ applied a method of successive approximations to the determination of effective cross sections of Pu^{240} and of Pu^{241} to be used in the irradiation dynamics equations. The concentrations of the isotopes used as points of reference in this analysis were obtained by mass spectrometric methods from three samples of plutonium irradiated at Chalk River, Canada, for periods of 6 months, 1 year, and 2 years. The experimental concentrations are given in Table 3-54, together with an isotopic description of the starting material (CW-3), one of the original samples irradiated by Fermi in 1944. The results of the theoretical analysis for the effective cross sections of Pu^{240} and Pu^{241} are shown in Table 3-55.

Table 3-54

MASS SPECTROMETER ANALYSIS (%) * FOR PLUTONIUM SAMPLES IRRADIATED IN THE CHALK RIVER PILE

(Reproduced from Ref. 191.)

Sample	CW-3	CR-1	CR-2	CR-3
Pu^{239}	92.63 ± 0.07	74.31 ± 0.22	53.3 ± 0.3	23.82 ± 0.16
Pu^{240}	6.96 ± 0.07	21.96 ± 0.20	35.6 ± 0.2	49.35 ± 0.40
Pu^{241}	0.406 ± 0.005	3.39 ± 0.07	9.17 ± 0.04	17.07 ± 0.24
Pu^{242}		0.337 ± 0.01	1.96 ± 0.03	9.76 ± 0.07
Approximate nvt	1×10^{20}	7×10^{20}	1×10^{21}	3×10^{21}

*The measured Pu^{241} concentration was corrected for decay and all the isotopic values were then readjusted to 100%.

Table 3-55

NEUTRON ABSORPTION AND CAPTURE CROSS SECTIONS
OF Pu^{240} AND Pu^{241} *

Sample	(Reproduced from Ref. 191.)			
	$\sigma_a(\text{Pu}^{240}) = \sigma_c(\text{Pu}^{240})$ (barns)	$\sigma_a(\text{Pu}^{241})$ (barns)	$\sigma_c(\text{Pu}^{241})$ (barns)	$\sigma_f(\text{Pu}^{241})$ (barns)
CR-1	560 $\pm$ 55	1450 $\pm$ 250	390 $\pm$ 80	1060 $\pm$ 210
CR-2	550 $\pm$ 55	1450 $\pm$ 250	390 $\pm$ 80	1060 $\pm$ 210
CR-3	495 $\pm$ 50	1450 $\pm$ 250	390 $\pm$ 80	1060 $\pm$ 210

*The values are based on $\sigma_a(\text{Pu}^{239}) = 1150$ barns, $\sigma_c(\text{Pu}^{239}) = 385$ barns
 $\sigma_f(\text{Pu}^{240}) = 0$, and $\sigma_a(\text{Pu}^{242}) = 30$ barns.

MTR Irradiations of Plutonium

Irradiations of Pu^{239} samples in a thermal neutron flux at $3 \times 10^{14} \text{ n}/(\text{cm}^2)(\text{sec})$ were carried out⁽¹⁹⁰⁾ for periods of up to almost 18 months in the MTR, corresponding to an integrated flux equal to $1.4 \times 10^{22} \text{ n}/\text{cm}^2$. A segment of the results of analyses of the samples are presented in Table 3-56. By measuring the concentrations of the various heavy isotopes as a function of exposure level (n/cm^2) it was possible to derive effective cross sections for the heavier plutonium isotopes and the transplutonium isotopes. The results are reported in Refs. 190 and 191. Using these effective cross sections, computations were made of the heavy isotope concentrations during irradiation in a thermal neutron flux of $3 \times 10^{14} \text{ n}/(\text{cm}^2)(\text{sec})$; the results are summarized in Fig. 3-106.

Table 3-56

PRODUCTS FROM PLUTONIUM IRRADIATIONS IN THE
CORE OF THE MATERIALS TESTING REACTOR

Sample No.	(Reproduced from Ref. 190.)		
	MTR-1	MTR-2	MTR-3
Mass of Pu^{239} inserted (mg)	350	350	170
nvt (neutrons/ cm^2)	4×10^{21}	1.1×10^{22}	1.4×10^{22}
Mass of Pu after irradiation (mg)*	50	20	6
Isotopic composition of the Pu^{**}			
Pu^{238}		0.216 ± 0.004	0.16 ± 0.02
Pu^{239}	5.33 ± 0.05	0.087 ± 0.002	0.068 ± 0.004
Pu^{240}	39.0 ± 0.4	2.02 ± 0.02	0.633 ± 0.006
Pu^{241}	21.6 ± 0.3	1.31 ± 0.01	0.308 ± 0.006
Pu^{242}	34.1 ± 0.3	96.33 ± 0.02	98.77 ± 0.03
Pu^{244}	0.0018 ± 0.0001	0.037 ± 0.002	0.052 ± 0.004

*Approximate yield. Efficiency of recovery not determined accurately.

**Mole per cent, measured by mass spectrometer.

FIG. 3-106
GROWTH AND DECAY OF THE HEAVIER PLUTONIUM
ISOTOPES, Am^{243} , Cm^{244} AS FUNCTIONS OF
TIME OF EXPOSURE OF Pu^{239} TO A THERMAL
NEUTRON FLUX OF 3×10^{14} neutrons/(cm²)(sec)
(REPRODUCED FROM REF. 191)

LITR Measurements

An effective capture cross section of Np^{239} has been derived⁽¹⁹³⁾ from theoretical analysis of isotope concentrations measured in irradiated samples of uranium metal. Aluminum-canned uranium metal cylinders (99.97% U^{238}) were irradiated in a beryllium channel adjacent to the active lattice of the LITR (Low Intensity Test Reactor) at Oak Ridge National Laboratory. Each uranium cylinder was approximately 1.8 cm in diameter and 4.0 cm long and weighed about 190 grams. The integrated flux was measured by cobalt monitors attached above and below the cylinder. After cooling for one month, the uranium was chemically processed and a purified sample of plutonium was analyzed mass-spectrographically for Pu^{240} . In Table 3-57 the ratio of concentrations, $\frac{\text{Pu}^{240}}{\text{Pu}^{239}}$, is listed as a function of irradiation time. In the same table are presented the $\sigma_{\text{ceff}}^{\text{Pu}^{239}}$ and $\sigma_{\text{ceff}}^{\text{Np}^{239}}$ obtained by two-parameter least squares fitting of the cross sections in the formula:

$$\frac{\text{Pu}^{240}/\text{Pu}^{239}}{\text{Pu}^{239}/\text{U}^{238}} = \frac{\sigma_{\text{ceff}}^{\text{U}^{239}} F_1 + \sigma_{\text{ceff}}^{\text{Np}^{239}} F_2 + \sigma_{\text{ceff}}^{\text{Pu}^{239}} F_3}{\sigma_{\text{ceff}}^{\text{U}^{238}}}$$

The factors F_1 , F_2 , F_3 are functions of the irradiation time computed from solutions of the standard differential equations representing the isotope concentrations during irradiation. The cross sections of the other isotopes were taken to be:

$$\sigma_{\text{ceff}}^{\text{U}^{238}} = 3.5 \text{ b (based on } \sigma_{\text{c}}^{\text{U}^{238}} (0.025 \text{ ev}) = 2.80 \text{ b and on an estimate of the epithermal absorption.)}$$

$$\sigma_{\text{ceff}}^{\text{U}^{239}} = 50 \text{ b (the exact value is unimportant for these analyses)}$$

Table 3-57

Pu²³⁹ AND Pu²⁴⁰ YIELDS AND DERIVED CROSS SECTIONS

(Data from Ref. 193)

Length of Irradiation (hr)	Pu ²³⁹ Production, mg		$\frac{\text{Pu}^{240}}{\text{Pu}^{239}} \%$	$\sigma_c(\text{Pu}^{239})$ (barns)	$\sigma_c(\text{Np}^{239})$ (barns)
	Computed from Co Monitor Data*	Observed**			
12.00	0.95 ₀	0.85 ₅			
12.00	0.85 ₉	0.80 ₁			
11.47	-	0.84 ₈			
12.00	0.87 ₈	0.84 ₇			
12.00	0.88 ₉	0.82 ₁			
12.00	0.87 ₇	0.86 ₄			
avg 11.9			0.0063 ± 0.0002	-	93
24.00	1.73 ₅	1.64 ₂			
24.07	1.69 ₈	1.62 ₁			
24.00	1.75 ₄	1.66 ₂			
24.00	1.71 ₄	1.62 ₄			
25.17	1.70 ₉	1.61 ₉			
24.00	1.63 ₁	1.58 ₅			
avg 24.2			0.0124 ± 0.0002	366	77
48.0	3.42 ₇	3.29 ₅			
48.0	3.46 ₃	3.33 ₄			
48.0	3.53 ₃	3.35 ₁			
48.0	3.41 ₄	3.21 ₄			
avg 48.0			0.0311 ± 0.0004	373	74
96.0	-	6.42 ₂			
95.45	6.50 ₈	6.30 ₇			
avg 95.7			0.081 ± 0.001	386	75
192.0	13.6 ₈	13.2 ₄	0.228 ± 0.003	409	-
423.0	27.3 ₉	27.0 ₅	0.590 ± 0.007	405	-

*Uncorrected for the depression in the uranium cylinder

**Corrected for Pu²⁴⁰ alphas and burnout

Agreement between the measured production of plutonium (based on $\sigma_{\text{Ceff}}^{\text{U}^{238}} = 3.5\text{b}$) and the activity of the irradiated cobalt was within 5%, well within experimental error and tending to substantiate the value selected for $\sigma_{\text{Ceff}}^{\text{U}^{238}}$. The reported mean values are $\sigma_{\text{Ceff}}^{\text{Np}^{239}} = 80 \pm 15 \text{ b}$, and $\sigma_{\text{Ceff}}^{\text{Pu}^{239}} = 400 \pm 40 \text{ b}$.

Low Level Irradiations of Uranium

Stoughton and Halperin⁽¹⁹²⁾ have reported the results of analyses for samples of uranium exposed for small values of the integrated flux. At these low exposure levels not much Pu²⁴⁰ is formed, and the reduction in Pu²⁴⁰ resonance capture because of self-shielding is not so pronounced. Accordingly, the computed effective cross section more nearly represents the effective cross section of Pu²⁴⁰ at infinite dilution in the reactor. The distributions of isotopic concentrations from two special sets of irradiations in graphite reactors and from one set at Chalk River were analyzed theoretically. Using

$\sigma_{\text{Ceff}}^{\text{U}^{238}} = 3.5 \text{ b}$ (including resonance capture), $\sigma_{\text{Ceff}}^{\text{Pu}^{239}} = 500 \text{ b}$, and using the approximation that U^{238} captures directly result in the formation of Pu^{239} , $\sigma_{\text{Ceff}}^{\text{Pu}^{240}}$ was determined for each of the irradiated samples. In Table 3-58 are shown the results of measurement and calculation for samples from one of the graphite reactors.⁽¹⁹⁴⁾ In the other graphite reactor, uranium metal slugs were irradiated to various levels of the integrated flux, covering a range of 200 to 600 grams of plutonium per ton, except for one of 57 grams per ton.

Table 3-58

EFFECTIVE CAPTURE CROSS SECTION OF Pu^{240} vs
PLUTONIUM ISOTOPIC COMPOSITION

Sample No.	Atom % Pu^{240}	Atom % Pu^{241} (Corr. for decay)	$\sigma_{\text{Ceff}}^{\text{Pu}^{240}}$ (barns)
114-42	0.49	0.008	2480
114-46	1.29	0.035	1550
114-52	2.01	0.084	1520
114-57	2.68	0.132	1340
114-61	5.77	0.488	1030

For the sample irradiated to 57 gm/ton $\sigma_{\text{Ceff}}^{\text{Pu}^{240}} = 3500 \text{ b}$ was found; the values of $\sigma_{\text{Ceff}}^{\text{Pu}^{240}}$ varied from 1000 b to 1370 b for the other ten samples. The 3500 b value was attributed to experimental errors.

Using the parameters⁽¹⁹⁵⁾ for the absorption resonance of Pu^{240} at 1 ev, a resonance integral of 8300 b was obtained.⁽¹⁹²⁾ In the two graphite reactors, the ratio of thermal to resonance flux was estimated to be 10. The effective absorption cross section of 1200 b, was in reasonably good agreement with the values derived from the experiments.

Thirteen uranium samples were irradiated⁽¹⁹²⁾ in the heavy water reactor at Chalk River. In Table 3-59 the experimental and theoretical analyses are summarized, again on the basis of $\sigma_{\text{Ceff}}^{\text{Pu}^{239}} = 500 \text{ b}$.

High Level Irradiation of Uranium at Chalk River - NRX Rod No. 683⁽¹⁹⁶⁾

A considerable effort has been allocated to experimental and theoretical investigations of irradiation effects in uranium rod No. 683 which had generated a total energy of approximately 100 mwd before its removal from NRX. This total energy output corresponds to a peak of about 3100 mwd/(metric ton U) in the central region of the rod. Eventually the rod was cut into a number of smaller rods of various lengths for reactivity measurements and for determination of radial distributions of integrated flux and isotope concentrations. An asymmetrical radial distribution of flux was observed, and in several instances determinations of concentrations were made at diametrically opposite positions in order to investigate the effects of such flux distortion. (These are designated "Radius A" and "Radius B" in the following tables.) The distribution of plutonium isotopes

was determined by radiochemical and mass spectrometric techniques as a function of penetration into the rod and at various positions along the rod. From the measured radial distribution of U^{235} burnup the integrated neutron flux was inferred based on $\sigma_{a\text{eff}}^{U^{235}} = 685 \text{ b}$, and the results are shown in Fig. 3-107.

Table 3-59

EFFECTIVE CAPTURE CROSS SECTION OF Pu^{240} IN
URANIUM METAL IRRADIATED AT CHALK RIVER

(Data from Ref. 192.)

Sample No.	% Pu^{240}	% Pu^{241}	$\frac{Pu^{241} \text{ corr.}}{Pu^{240} \text{ corr.}}$	$\frac{Pu^{240} \text{ corr.}}{Pu^{239} \text{ corr.}}$	$\sigma_{c\text{eff}}^{Pu^{240}}$ (barns)
1-RC	1.90	0.075	0.0389	0.0191	1530
2-RC	3.0	0.14	0.049	0.0297	1235
R-5	3.92	0.181	0.049	0.0381	965
3-RC	3.95	0.23	0.0617	0.0388	1190
R-4	4.10	0.238	0.0638	0.0402	1190
4-RC	4.80	0.32	0.0716	0.0468	1150
R-6	5.08	0.384	0.0813	0.0498	1220
5-RC	5.50	0.35	0.0688	0.0532	970
R-3	5.51	0.397	0.0780	0.0535	1090
6-RC	5.73	0.44	0.0833	0.0556	1125
R-1	6.83	0.535	0.0861	0.0650	995
R-2	7.15	0.581	0.0902	0.0677	1000
-	12.71	1.79	0.168	0.110	1140

FIG. 3-107
RADIAL DISTRIBUTION OF INTEGRATED NEUTRON FLUX IN
SAMPLE R FROM U^{235} DEPLETION MEASUREMENTS AT ANL
(REPRODUCED FROM REF. 196)

Using effective cross sections of 1300 b for Pu^{239} absorption and 870 b for Pu^{239} fission, and using the values 200 Mev/fission, $\epsilon = 1.035$, $\alpha (\text{U}^{235}) = 0.184$, Hanna(196) derived the irradiation level, the initial conversion ratio (ICR), and the initial plutonium production rate in two of the samples. These results are presented in Table 3-60.

Table 3-60

THEORETICAL ANALYSIS OF DATA FROM TWO SAMPLES		
(Data from Ref. 196.)		
Sample	P	G
Irradiation Level: (10^{21} n/cm ²) (mwd/metric ton U)	0.2373 ± 0.0036 970	0.6611 ± 0.0036 2740
Initial Conversion Ratio (ICR)	0.753 ± 0.008	0.769 ± 0.007
Initial Production Rate (g Pu/mwd)	0.901 ± 0.010	0.920 ± 0.008

An average value $\sigma_{\text{ceff}}^{\text{U}^{238}} = 3.78 \pm 0.04$ b may be inferred from an initial conversion ratio 0.76 ± 0.01 in conjunction with $\sigma_{\text{aeff}}^{\text{U}^{235}} = 685$ b. Thus, approximately 27% of the plutonium is produced by resonance capture of neutrons by U^{238} .

In Table 3-61 are listed values inferred from the data on the two samples based on effective cross sections also listed in that table.

Table 3-61

INFERRED EFFECTIVE CROSS SECTIONS OF PLUTONIUM ISOTOPES		
(Data from Ref. 196.)		
Sample	P	G
<u>Inferred Effective Cross Sections, barns</u>		
$\sigma_{\text{c}}^{\text{Pu}^{239}}$	431 ± 8	427 ± 3
$\sigma_{\text{a}}^{\text{Pu}^{240}}$	963 ± 45	732 ± 12
$\sigma_{\text{c}}^{\text{Pu}^{241}}$	-	440 ± 30
<u>Assumed Effective Cross Sections, barns</u>		
$\sigma_{\text{a}}^{\text{Pu}^{239}}$	1300	1300
$\sigma_{\text{a}}^{\text{Pu}^{241}}$	1500	1500
$\sigma_{\text{a}}^{\text{Pu}^{242}}$	100	100

Postulating a constant ratio

$$\frac{\sigma_{f\text{eff}}^{\text{Pu}^{239}}}{\sigma_{c\text{eff}}^{\text{Pu}^{239}}} = \frac{870 \text{ b}}{425 \text{ b}}$$

across the rod, the variation of $\sigma_{c\text{eff}}^{\text{U}^{238}}$ within two of the samples was inferred. The results are summarized in Fig. 3-108.

FIG. 3-108
EFFECTIVE CAPTURE CROSS SECTION OF U^{238} AS A FUNCTION
OF RADIAL DEPTH IN TWO SAMPLES
(REPRODUCED FROM REF. 196)

The variations of effective cross sections $\sigma_{c\text{eff}}^{\text{Pu}^{240}}$ and $\sigma_{c\text{eff}}^{\text{Pu}^{239}}$ in two of the samples have been inferred from the integrated flux level and the radial variation given by the flux distribution. The results are summarized in Fig. 3-109.

Reactivity measurements on samples from Rod No. 683 were carried out in CP-2 at Argonne National Laboratory by the danger coefficient technique. In Table 3-62 are presented results of reactivity comparisons with an unirradiated sample of natural uranium, including corrections for different masses of uranium in the samples, corrections for the effects of the aluminum jackets and corrections for changes in blackness of the samples to thermal neutrons. Further analysis is given in Ref. 197.

FIG. 3-109
EFFECTIVE CAPTURE CROSS SECTIONS OF Pu^{239} AND Pu^{240}
AS FUNCTIONS OF RADIAL DEPTH IN VARIOUS SAMPLES
(REPRODUCED FROM REF. 196)

Table 3-62

IRRADIATED SAMPLES VS NATURAL U, BOTH OF 5.6 kg U^{235} CONTENT
INCLUDING BLACKNESS CORRECTION

(Reproduced from Ref. 197.)			
Irradiation mwd/ton	δih (standard sample size, blackness corrected)	Irradiation mwd/ton	δih (standard sample size, blackness corrected)
50	-0.217	1200	-0.191
198	-0.025	1460	-0.192
390	-0.005	2000	-0.305
574	+0.015	2770	-0.840

Irradiation of Natural Uranium Rods at Windscale (198,199)

A number of rods of natural uranium were irradiated in a graphite reactor to various levels of integrated flux up to 5.16×10^{19} neutrons/cm². The results were analyzed in terms of the change in effective cross section per U^{235} initially present and were characterized by various functions of the integrated flux and parameters of the system.

3.8.6 Theoretical Studies on the Exposure Dependence of $\sigma_{\text{c eff}}^{\text{Pu}^{240}}$

Crowther and Weil⁽²⁰⁰⁾ have published results of theoretical studies showing the variation of the effective capture cross section of Pu^{240} in cylindrical fuel rods during neutron irradiation. The objective of the study was to obtain a constant value for the cross section during the entire irradiation which would yield approximately correct results. Reactor parameters investigated include lattice spacing, U^{235} enrichment and the level of the integrated thermal neutron flux. In these studies epithermal absorption in U^{235} , Pu^{239} and Pu^{241} , and epithermal $1/v$ capture in U^{238} were ignored. Resonance absorption in Pu^{240} was included by defining an effective microscopic thermal capture cross section:

$$\sigma_{\text{c eff}}^{\text{Pu}^{240}} = \bar{\sigma}_{\text{c, th}}^{\text{Pu}^{240}} \frac{\left[\sum_i \eta_i N_i \sigma_{a_i} \right] \epsilon}{1 + \tau B^2} \frac{p(1-p_0)}{M_0}$$

In this formula, $\bar{\sigma}_{\text{c, th}}^{\text{Pu}^{240}}$ is the average with respect to a Maxwellian distribution of thermal neutron flux; B^2 , τ , ϵ and p have their usual meanings; p_0 is the probability that a neutron will escape resonance capture in Pu^{240} while slowing down through resonance energies; M_0 is the atom fraction of Pu^{240} in the fuel plus fertile material. A simplified formula for p_0 was used:

$$p_0 = \exp \left[- \frac{M_0 I^{\text{Pu}^{240}}}{S} \right]$$

where

$$I^{\text{Pu}^{240}} \approx \frac{I_{\infty}^{\text{Pu}^{240}}}{1 + \frac{I_{\infty}^{\text{Pu}^{240}} M_0}{\sigma_s^{\text{fuel}}}}$$

is the effective resonance integral of Pu^{240} ; $I_{\infty}^{\text{Pu}^{240}}$ is the resonance integral at infinite dilution taken to be 9000 b. (Doppler broadening was neglected.) Finally,

$$S = \frac{\xi \Sigma_s}{N_{\text{fuel}}} F \frac{\text{vol. of moderator}}{\text{vol. of fuel}} ; \text{ where}$$

F is the resonance disadvantage factor and the other terms have their customary meanings.

To test the validity of the formula for $\sigma_{\text{c eff}}^{\text{Pu}^{240}}$, the NRX lattice was calculated.

In these and in all other analyses reported by Crowther and Weil, the non $1/v$ correction factors published by Westcott⁽²⁰¹⁾ were used for Maxwellian averages of the cross sections. The "world consistent" 2200 m/sec values of the cross sections were employed. In the case of Pu^{241} , the g -factors of Westcott were used for the fission cross section, and it was assumed that the ratio of the g factors for absorption and fission were the same as this ratio for Pu^{239} .

Figures 3-110 through 3-115 are all taken from Ref. 200.

Figure 3-110 is a comparison of the concentration-dependent Pu^{240} cross sections in NRX reported by Crowther and Weil, Stoughton and Halperin, and Hurst, et al. Two values of the uncertain initial conversion ratio were examined. The satisfactory agreement tends to substantiate the method described in the following paragraphs.

FIG. 3-110
COMPARISON OF CALCULATED EFFECTIVE THERMAL
ABSORPTION CROSS SECTIONS FOR Pu^{240}

A theoretical analysis was made of long-term irradiation effects in $\text{UO}_2\text{-H}_2\text{O}$ cylindrical fuel lattices, with enrichment of U^{235} varying from 1% to 2.5%. The effective Pu^{240} cross section is graphed in Fig. 3-111 as a function of $\text{H}_2\text{O}/\text{UO}_2$ volume ratio and of irradiation level. One parameter, S/Z , was found to lump the effects of enrichment and of lattice spacing and to permit the approximate determination of $\sigma_{\text{Ceff}}^{\text{Pu}^{240}}$ solely as a function of exposure:

$$Z = \frac{\epsilon \left[\sum_i \eta_i N_i \sigma_{ai} \right]}{1 + \tau B^2}$$

A notion of the errors involved in this lumping may be obtained from Fig. 3-112, where the effective cross section of Pu^{240} is chosen so as to match the concentration-dependent value at 8000 MWD/ton. In Fig. 3-113 are summarized the results of Fig. 3-112 and additional curves based on matching the cross section at higher irradiation levels. In Fig. 3-114 a comparison is made between the infinite multiplication factors given by concentration-dependent and a concentration-independent (1300 b) cross section for a $\text{UO}_2\text{-H}_2\text{O}$ lattice, the Dresden reactor. The concentration-independent value was chosen from Fig. 3-115, representing a single approximation for typical reactor lattices to the family of curves in Fig. 3-113.

FIG. 3-111
EFFECTIVE CONSTANT CROSS SECTION OF Pu²⁴⁰ FOR A
RANGE OF LATTICE SPACINGS AND ENRICHMENTS

FIG. 3-112
EFFECTIVE Pu²⁴⁰ CROSS SECTION AS A FUNCTION OF
IRRADIATION AND LATTICE SPACING FOR A UNIFORM
LATTICE OF 0.5 in. DIAMETER UO₂ RODS IN H₂O

FIG. 3-113
SUMMARY OF EFFECTIVE Pu^{240} CROSS SECTION SHOWING IRRADIATION DEPENDENCE

FIG. 3-114
COMPARISON BETWEEN REACTIVITY CALCULATIONS WITH
CONCENTRATION DEPENDENT Pu^{240} CROSS SECTION AND
CONSTANT CROSS SECTION

FIG. 3-115
EFFECTIVE Pu^{240} CROSS SECTION FOR
LONG-TERM REACTIVITY CALCULATIONS

3.8.7 The Effective Delayed Neutron Fraction and Prompt Neutron Lifetime

The Effective Delayed Neutron Fraction

Since delayed neutrons are born with a lower average energy than that of prompt neutrons, they will have a smaller age to thermal and, hence, a smaller probability of leaking while slowing down than will the prompt neutrons. The ratio of the effective delayed neutron fraction, β_{eff} , to the ordinary delayed fraction, β , is given formally by the ratio of the rate of creation of importance due to delayed neutrons to the total rate of creation of importance.(202) This can be written as:

$$\frac{\beta_{eff}}{\beta} = \frac{\int \chi'(v') \phi^+(r, v') \nu \Sigma_f(r, v) \phi(r, v) dr dv dv'}{\int \chi(v') \phi^+(r, v') \nu \Sigma_f(r, v) \phi(r, v) dr dv dv'} \quad (3-41)$$

where $\phi(r, v)$ is the neutron flux as a function of position and velocity

$\phi^+(r, v)$ is the function adjoint to the neutron flux (the adjoint flux)

$\nu \Sigma_f(r, v)$ is the product of the average number of neutrons emitted per fission and the macroscopic fission cross section (the product may be a function of position and of incoming neutron velocity)

$\chi(v')$ is the normalized fission spectrum

$\chi'(v')$ is the normalized spectrum of the delayed neutrons.

The integration in Eq. (3-41) is to be taken over all velocities v and v' and spatially over the whole reactor.

Gwin(203) has accounted for the increased importance of the delayed neutrons in a bare reactor by using age theory and distinguishing the prompt and delayed neutrons by means of their different ages. If τ_p and τ_d are respectively the age to thermal of the prompt and delayed neutrons,

$$k_{eff} = \frac{(1-\beta)k_{\infty} \exp(-\tau_p B^2)}{1 + L^2 B^2} + \frac{\beta k_{\infty} \exp(-\tau_d B^2)}{1 + L^2 B^2}$$

and

$$\frac{\beta_{\text{eff}}}{\beta} = \left\{ (1-\beta) \exp[-(\tau_p - \tau_d) B^2] + \beta \right\}^{-1}$$

where k_∞ is the infinite medium multiplication constant,

B^2 is the buckling, and

L^2 is the square of the diffusion length.

For an infinite system (so that $B^2 = 0$) or if τ_p is taken equal to τ_d , β_{eff} and β become identical.

For an aqueous solution of UO_2F_2 (enriched to about 90% in U^{235}) containing 0.0763 gm/cm³ of U^{235} and having a hydrogen to U^{235} ratio of 337, Gwin found⁽²⁰³⁾ for τ_p , τ_d , and β_{eff} the values 25.4 cm², 14.1 cm², and 0.0096, respectively. The analysis depended upon the experimental values of critical height and change in reactivity with height of the bare critical solution.

From analysis of an oscillator experiment, de Hoffmann⁽²⁰⁴⁾ obtained the value 0.00855 for the β_{eff} of the Los Alamos Water Boiler.

The Prompt Neutron Lifetime

An expression for the prompt neutron lifetime, ℓ , can be obtained from perturbation considerations.⁽²⁰⁵⁾ In terms of the multigroup approximation, ℓ can be written as:

$$\ell = \frac{\int \sum_i \frac{\phi_i^+(r) \phi_i(r)}{v_i} dr}{\int \sum_{i,j} \chi_j \phi_j^+(r) \nu \Sigma_{fi} \phi_i(r) dr} \quad (3-42)$$

where the indices i and j refer to the group structure, v_i is the group velocity, and the other symbols have the same meaning as in Eq. (3-41). The spatial integration is to be extended over the entire reactor.

The lifetime can be evaluated without explicit use of the adjoint functions. One inserts a uniform $1/v$ absorber of magnitude C/v into a critical reactor and evaluates the resulting change in reactivity, $\delta\rho/\rho$. In the limit as C and $\delta\rho/\rho$ approach zero,

$$\ell = \frac{\delta\rho/\rho}{C}$$

The above " $1/v$ method" may be performed either computationally or experimentally.

In the case of a bare core, Eq. (3-42) yields results in terms of the various reactor quantities such as diffusion length, buckling, etc. The lifetime in one group theory is given by

$$\ell = \frac{\ell_0}{1 + L^2 B^2}$$

where

$$\ell_0 = \frac{1}{v \Sigma_a}$$

and L is the diffusion length

B^2 is the buckling

v is the velocity

Σ_a is the macroscopic absorption cross section.

The analogous expression in two group theory is given by

$$\ell = \frac{\ell_{0F}}{1 + L_F^2 B^2} + \frac{\ell_{0S}}{1 + L_S^2 B^2}$$

where F and S refer to the fast and slow groups respectively and

$$\ell_{0F} = \frac{1}{v_F \Sigma_{aF}}$$

$$\ell_{0S} = \frac{1}{v_S \Sigma_{aS}}$$

where L_S is the slow group diffusion length and

L_F^2 corresponds to the age of the slow group neutrons.

v_F and v_S are the fast and slow group velocities, respectively,

Σ_{aS} is the slow group macroscopic absorption cross section and

Σ_{aF} is the fast group macroscopic slowing down or transfer cross section.

Experimentally, measurements of the lifetime, ℓ , only yield values for ℓ/β_{eff} . This ratio has been measured for the zero power thermal source reactor, ZPR-IV' at Argonne by three methods which are based on the statistical nature of the neutron chain. Using a special least squares analysis, (Cohn(206) obtained 69.2 ± 3.0 microseconds for ℓ based on an assumed value for β_{eff} of 0.007. The method used is the same discussed by Moore(207) who pointed out that an analysis of the power spectrum of the statistical fluctuations in neutron level of the reactor can yield the high frequency portion of the reactor transfer function. The Rossi- α (208,209) method was used by Kato and McMahon(210) to obtain the value 68.5 ± 7.0 microseconds. The method suggested by de Hoffmann(211) was used by Luckow(212) who found a range of values for ℓ between 67.5 to 72.0 microseconds. The latter two measurements were also based on a value of 0.007 for β_{eff} .

3.8.8 Boiling Reactor Stability(213,214)

Boiling natural circulation reactors can be analyzed for tendencies toward self-induced oscillations by techniques well established in the field of servomechanisms. The methods are also similar to those used in stability analysis of other reactor types. However, since the internal mechanisms of boiling reactors are more complicated and currently not completely understood, it is necessary to use rather general mathematical approaches to the problem.

The usual approach is to assume a physical model and then to empirically determine the values of the constants appearing in the model from an analysis of the behavior of the system under a wide variety of conditions. This is quite satisfactory provided the basic model is a reasonably accurate description of the system.

The differential equations whose Laplace transforms lead to the transfer function are

$$\frac{P}{P^0}(t) = g_0 k(t)$$

$$k_{fb}(t) = -h \frac{P}{P^0}(t)$$

$$k(t) = k_{fb}(t) + k_{in}(t)$$

where g_0 and h are operators whose Laplace transforms are $G(s)$ and $H(s)$ respectively, and the former is related to B_0 , the kernel of the linearized neutron kinetics equations:

$$g_0 k(t) = \int_0^\infty B_0(\sigma) k(t - \sigma) d\sigma$$

p is the deviation of the power from P^0 , k_{in} is an input reactivity, and k_{fb} is the feedback reactivity.

A fairly general description for h is

$$h = \frac{\prod_{i=1,2,\dots} \left(1 + T_{-i} \frac{d}{dt}\right) \sum_{i=1,2,\dots} \Lambda_i \delta(\tau_i, t)}{\prod_{i=1,2,\dots} \left(1 + T_i \frac{d}{dt}\right) \sum_{i=1,2,\dots} \Lambda_{-i} \delta(\tau_{-i}, t)}$$

where the operator $\delta(\tau_i, t)$ is defined

$$\delta(\tau_i, t) f(t) = f(t - \tau_i)$$

provided $f(t) = 0$ for $t < 0$. Here $\Lambda_{\pm i}$ is a constant and the T 's and τ 's are time constants. Since

$$H(s) = \frac{\prod_{i=1,2,\dots} (1 + T_{-i} s) \sum_{i=1,2,\dots} \Lambda_i \exp(-\tau_i s)}{\prod_{i=1,2,\dots} (1 + T_i s) \sum_{i=1,2,\dots} \Lambda_{-i} \exp(-\tau_{-i} s)}$$

it is seen that $\frac{\sum \Lambda_i}{\sum \Lambda_{-i}}$ is a zero frequency feedback proportionality constant between p/P^0 and $-k_{fb}$. If the reactor transfer function is measured with sufficient precision, h or H can have all its parameters evaluated. To be perfectly general these parameters can be expected to depend even on p/P^0 when $s = j\omega$, for otherwise self-sustained oscillations of finite amplitude could not be explained.

As an example, in a specific application of the above to the EBWR I it was found by simplification of many Laplace transformed hydraulic and thermodynamic relationships(214) among core variables that

$$H(s) = \frac{1}{(1 + T_1 s)(1 + T_2 s)} + \frac{2}{(1 + T_1 s)(1 + T_2 s)(1 + T_3 s)}$$

With a function of this type formulated from theoretical considerations the constants can be evaluated:

- (a) by frequency response measurements in electrically heated boiling water loops;
- (b) by transfer function measurements in the reactor;
- (c) from fundamental constants appearing in the system's theoretical equations.

For EBWR I progress is underway on (a) and (b), while for (c) it has been found from preliminary theoretical analysis that:

$$\Lambda_1 = .0096$$

$$T_1 = .392 \text{ sec, a fuel time constant}$$

$$T_2 = .122 \text{ sec, a steam transport time constant}$$

$$\Lambda_2 = .01228$$

$$T_3 = .434 \text{ sec, a fuel time constant}$$

$$T_4 = .1757 \text{ sec, a water transport time constant}$$

$$T_5 = .183 \text{ sec, a steam transport time constant}$$

where the first three and the last four constants pertain to different reactor regions.

Whatever form is used for $H(s)$, it can be used in the Laplace transform solution to give

$$G(s) = \frac{G_0(s)}{1 + G_0(s) H(s)}$$

Obtaining this reactor transfer function is then a complete quantitative solution of the stability problem.

3.8.9 Xenon Instability

A reactor that has been operating at a steady level for a sufficiently long time will contain an equilibrium amount of xenon. If there is a change in flux the loss rate of xenon follows the change immediately, but the production rate is unchanged for some time. The xenon poison will continue to decrease whenever the flux has been increased, and to increase where flux has been decreased. This will tend to emphasize the flux distortion. Eventually the increased flux will result, through the production term, in increased xenon, and thus tend to lower the flux. It is possible that such flux distortions are unstable so that the flux distribution can oscillate continually even though the mean power is kept constant by the use of control rods.

In Ref. 215 the kinetic behavior of a reactor with migration area M^2 (unpoisoned) and critical buckling κ^2 is investigated. In the analysis, steady state values are assumed to be perturbed by small quantities proportional to $e^{\omega t}$ for $t > 0$ and spatially to a function f_i , where the f_i are solutions of

$$\nabla^2 f_i + \kappa_i^2 f_i = 0$$

and

$$f_i = 0 \text{ at the outer boundary of the reactor.}$$

Retaining only the first order of small quantities and utilizing the orthogonality properties of the f_i , an equation can be found for ω . The first term in the equation, $(k - 1 - M^2 \kappa_j^2 - A \bar{X}_j)$ is the amount below critical of the j^{th} mode. $A \bar{X}$ is the local change in k due to xenon poison, A is a constant, X is the relative concentration of xenon normalized to $X \rightarrow 1$ at very high flux (i.e. neglect xenon decay rate with respect to loss through neutron capture), and

$$\bar{X}_j = \int f_j^2 X_0 dv$$

where X_0 is the unperturbed value of X and the f_j are normalized to $\int f_j^2 dv = 1$. The second term in the equation is the change in k necessary to make the perturbation grow with reciprocal period, ω . Finally, the third term is a measure of the reactivity effect of the xenon. The equation is then:

$$(k - 1 - M^2 \kappa_j^2 - A \bar{X}_j) + \left(\frac{-\omega}{\lambda} - k\beta \left\{ 1 - \sum \frac{\beta_d \lambda_d}{\omega + \lambda_d} \right\} \right) + \frac{A \phi_j \sigma_x}{\lambda_x + \omega + \bar{\phi}_j \sigma_x} \left\{ \bar{X}_j - \frac{\lambda_i}{\lambda_i + \omega} \right\} = 0$$

Here λ is the reciprocal neutron lifetime in the unpoisoned reactor; β , the total delayed neutron fraction; β_d , λ_d , the relative delayed neutron fractions and reciprocal lifetimes; λ_i , the reciprocal lifetime of iodine-135; λ_x , the same quantity for Xe^{135} ; σ_x , the Xe^{135} cross section; $\bar{\phi}_j$, the projection of ϕ_0 on f_j defined in the same fashion as $\bar{X}_j$ (ϕ is the neutron flux).

A necessary condition for $\omega \geq 0$ is $k - 1 - \kappa_j^2 M^2 \geq 0$. At very high fluxes this condition is more exact. If λ_n is defined by

$$\lambda_n = \bar{\phi}_j \sigma_x$$

then the frequency, f , of the oscillations at the onset of oscillation is given by:

$$f^2 = (2\pi)^2 \lambda_i \frac{\{\lambda_n + \lambda_x + \lambda_i (1 - \bar{X}_j)\}}{\bar{X}_j}$$

For

$$\lambda_n < \lambda_x, f \approx 2\pi \sqrt{\lambda_i \lambda_x},$$

and for

$$\lambda_n \gg \lambda_x, f \approx 2\pi \sqrt{\lambda_i \lambda_n},$$

In Ref. 216, the work presented in Ref. 215 is extended to cases where the stationary reactor power distribution is much different from the fundamental geometrical mode. The stability criterion is of the same form but somewhat more difficult to analyze since the actual flux shape as well as the fundamental mode shape is involved. Also, in Ref. 216, the effects of a power coefficient, and direct formation of Xe^{135} are included.

For a bare, one-dimensional reactor with zero power coefficient and height H , the threshold flux for onset of oscillation depends on the ratio H^2/M^2 and is 1.52×10^{13} $\text{n}/(\text{cm}^2)(\text{sec})$ for $H^2/M^2 = 2500$ and 5×10^{13} for $H^2/M^2 = 1250$. The periods in these cases are 29.4 and 22.1 hr, respectively.

3.8.10 Kinetics of Circulating Fuel Reactors

Ergen(217,218) has modified the inhour equation to account for the fact that in a circulating fuel reactor, some of the delayed neutrons are born in regions where they do not contribute to the chain reaction. The case considered is that of a zero power system in which all elements of the fuel have the same transit time around the loop. It is also assumed that the power distribution and neutron importance are spatially constant in the reactor. The derived expression involves θ and θ_1 , which are respectively the transit time of the fuel through the reactor and the transit time of the fuel through the whole loop. As θ (and hence also θ_1) becomes very large, the modified inhour equation reduces to the ordinary inhour equation since then the delayed neutrons are born inside the reacting zone before much fuel has moved outside. For high speed circulation so that θ_1 (and hence θ) is small compared to all of the delayed precursor mean lifetimes, the modified equation takes on the form (for periods $T \gg \theta$):

$$k_{\text{ex}} = \frac{\ell}{T} + \frac{\theta}{\theta_1} \sum_i \frac{\beta_i}{1 + \lambda_i T} \quad (3-43)$$

where k_{ex} is the excess multiplication constant, ℓ is the prompt neutron lifetime, and β_i and λ_i are the usual delayed neutron parameters. Equation (3-43) is seen to be just the inhour equation for stationary fuel reactors, except that the delayed neutron yields, β_i , have been decreased by the factor θ/θ_1 .

For intermediate circulating speeds, the long-delayed neutrons will be attenuated by the factor θ/θ_1 , while the neutrons from the short-lived precursors will not be appreciably attenuated. A circulating fuel reactor will, therefore, exhibit shorter asymptotic periods for a given k_{ex} than will a stationary fuel reactor and, in particular, the amount of k_{ex} needed to go prompt critical is smaller.

Studies have been made(219-223) of the stability of circulating fuel, power reactors including the case in which the delayed neutrons are ignored. Owing to the interaction of the reactivity, temperature, and power of the system, the kinetics equations are in general non-linear as contrasted with the case for zero power reactors. In addition, the equations involve a delay term even if the delayed neutrons are ignored, since owing to the circulation of the fuel, the reactor behavior at time t depends on the history during some interval preceding t . The equations which are considered for the case of no delayed neutrons are of the form,

$$\frac{d \log P}{dt} = -\frac{\alpha}{\ell} T_0 \quad (3-44)$$

$$\frac{dT_0}{dt} = \epsilon \left[P - \frac{1}{\theta} \int_0^\theta P(t-s) ds \right] \quad (3-45)$$

where

P is the reactor power

t is the time

α is the temperature coefficient of reactivity

T_0 is the temperature

ℓ is the prompt neutron lifetime

ϵ is the reciprocal heat capacity

θ is the transit time through the reactor.

The integral in Eq. (3-45) is an example of the previously mentioned delay term.

It has been shown^(219,220) by means of a plausibility argument rather than rigorous proof, that the circulation of the fuel causes a damping of the nonlinear oscillations. This is demonstrated under the assumption that there is no mechanical vibration coupled with the power oscillation, and that the shape of the hydrodynamic flow does not vary with time. The possibility of sustained oscillation exists if the transit time of the fuel through the reactor corresponds to any possible period of an oscillation. Excepting this possibility, the damping due to the circulation compensates for the loss in damping due to the fact that some of the delayed neutrons are born outside the reactor. A properly designed circulating reactor is then no worse off with respect to damping of oscillations than a reactor in which all delayed neutrons are given off in the reactor.

In case the delayed neutrons are taken into account, Eq. (3-44) is replaced by

$$\frac{dP}{dt} = -(\alpha T_0 + \beta) \frac{P}{\ell} + \frac{\beta}{\ell} \int_0^{\infty} D(s) P(t-s) ds$$

where $D(s)$ indicates the fraction of the delayed neutrons, caused by fissions at $t-s$, available at a time s later. $D(s)$ is normalized so that

$$\int_0^{\infty} D(s) ds = 1$$

The symbol β refers to the effective delayed neutron fraction in that account is taken of the loss of some of the delayed neutrons in the external loop. Again, by indirect methods, it has been shown that the nonlinear oscillations are damped.

3.9 INTEGRAL DATA

3.9.1 Critical Mass Studies of U^{235} - H_2O Solutions

Beck, Callihan, Morfitt, and Murray⁽²²⁴⁾

A series of critical mass experiments with aqueous solutions of uranyl fluoride, UO_2F_2 , were performed at Oak Ridge National Laboratory in 1949.

The isotopic concentrations of the uranium were 93.4% U^{235} , 1.1% U^{234} , and 5.5% U^{238} .

Eight interchangeable right circular cylinders made of Type 347 stainless steel and having diameters ranging from 6 to 20 inches were used as core vessels. In addition, five cylinders with diameters of $5\frac{1}{2}$ to 15 inches were made of Type 3-S aluminum. The wall thickness in all cases was $1/16$ inch.

The experiments could be performed either with or without water reflector. An iron cylinder, 24 inches in diameter and concentric with the reactors, could be filled with water to provide this reflector. In all reflected experiments at least 10 cm of water surrounded the sides and bottom of the core and this reflector was assumed to be effectively infinite. The top surface of the solution in each reactor could be covered by a thin-walled, smooth-fitting, stainless steel or aluminum water tank, 6 inches deep.

Some experiments were performed with a cadmium sheet fitted around the outside and bottom of the core, and with cadmium shot placed in the top reflector tank, thus effectively enclosing the reactor in a sheath of approximately 0.44 grams of cadmium per square centimeter.

Measurements were made to determine the maximum diameter of a core of infinite length which could be filled with fuel of a given concentration and remain subcritical. In certain cases the height of the reactor was insufficient to attain criticality, though the reciprocal multiplication curve indicated the reactor would be critical at a finite height.

Data for core dimensions and compositions are presented for this series of experiments in Tables 3-63 through 3-72, which have been reproduced from Ref. 224.

Some of the preceding data have been fitted to an empirical formula by Schuske and Morfitt.^(225,226) The formula relates the height and diameter of water reflected critical assemblies⁽²²⁷⁾ as a function of the $H:U^{235}$ ratio.

By plotting the experimentally determined critical height of a cylinder as a function of its diameter for various compositions, curves of the type shown in Fig. 3-116 are obtained. The vertical asymptote, $D = \text{constant}$, represents the diameter of a just critical infinite cylinder at the given composition. Similarly, the horizontal asymptote, $H = \text{constant}$, represents the height of a just critical infinite slab.

The data were fitted to

$$(D - a)(H - b) = c$$

Table 3-63

CRITICAL CONDITIONS FOR REACTOR CORES IN ALUMINUM VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	gm U ²³⁵ gm Sol'n.	Solution Density gm/cm ³	gm U ²³⁵ cm ³ Sol'n.	Critical Height cm	Critical Volume liters	Critical Mass kg U ²³⁵
6	52.9	0.293	1.566	0.459	70.9	12.93	5.93
	58.8	0.2745	1.51	0.415	71.8	13.10	5.44
6½	26.2	0.4179	1.98	0.827	44.5	9.56	7.91
	52.9	0.293	1.566	0.459	39.2	8.39	3.85
	56.7	0.281	1.51	0.424	40.4	8.65	3.67
	119.0	0.1685	1.26	0.212	52.6	11.26	2.39
8	26.2	0.4179	1.98	0.827	21.5	6.92	5.76
	29.9	0.394	1.926	0.759	20.7	6.71	5.09
	52.9	0.293	1.566	0.459	19.5	6.32	2.90
	58.8	0.2745	1.51	0.415	20.5	6.65	2.76
	99.5	0.1924	1.32	0.254	22.4	7.26	1.84
	192.0	0.1145	1.17	0.134	28.1	9.11	1.22
	290.0	0.0801	1.10	0.0881	40.1	13.00	1.15
10	52.9	0.293	1.566	0.459	13.4	6.79	3.12
	328.7	0.715	1.101	0.0787	22.4	11.35	0.893
	499	0.0488	1.070	0.0522	35.2	17.83	0.930
15	52.9	0.293	1.566	0.459	7.90	9.01	4.14
	56.7	0.281	1.51	0.424	8.50	9.69	4.11
	221.0	0.1014	1.14	0.116	11.30	12.88	1.49
	499.0	0.0488	1.070	0.0522	16.90	19.27	1.01
	755.0	0.033	1.04	0.0343	27.10	29.75	1.02
	999.0	0.0252	1.03	0.0260	44.30	50.50	1.31

Table 3-64

NON-CRITICAL REACTOR CORES IN ALUMINUM VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	<u>gm U²³⁵</u> <u>gm Sol'n.</u>	Solution Density <u>gm/cm³</u>	<u>gm U²³⁵</u> <u>cm³ Sol'n.</u>	Maximum Experimental Attainable		Estimated Critical	
					Height <u>cm.</u>	Mass <u>U²³⁵ kg</u>	Height <u>cm.</u>	Mass <u>U²³⁵ kg</u>
5½	56.7	0.281	1.51	0.424	74.3	4.83	subcritical	
6	26.2	0.4179	1.98	0.827	51.6	7.79	>75	>11.3
	119.0	0.1685	1.26	0.212	72.6	2.81		
6½	221	0.1014	1.14	0.116	61.6	1.47	subcritical	
8	499	0.0488	1.07	0.0522	74.5	1.26	subcritical	
10	755	0.033	1.04	0.0343	56.4	0.98	subcritical	
15	26.2	0.4179	1.98	0.827	7.5	7.07	8.60	8.10

Table 3-65

CRITICAL CONDITIONS FOR REACTOR CORES IN STAINLESS STEEL VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	gm U ²³⁵ gm Sol'n.	Sol'n. Density gm/cm ³	gm U ²³⁵ cm ³ Sol'n.	Critical Height cm	Critical Volume liters	Critical Mass kg U ²³⁵
6½	31.6	0.385	1.88	0.724	49.0	10.49	7.59
	43.9	0.326	1.65	0.538	47.1	10.08	5.42
	62.7	0.264	1.50	0.396	47.9	10.25	4.06
	86.4	0.213	1.35	0.288	53.8	11.51	3.31
7	24.4	0.430	2.02	0.869	35.4	8.79	7.63
	31.6	0.385	1.88	0.724	34.0	8.44	6.11
	43.9	0.326	1.65	0.538	32.7	8.12	4.37
	61.1	0.268	1.47	0.394	32.6	8.09	3.19
	86.4	0.213	1.35	0.288	36.0	8.94	2.57
	123.2	0.164	1.25	0.205	42.3	10.50	2.15
	174	0.124	1.19	0.148	57.3	14.22	2.10
8	24.4	0.430	2.02	0.869	23.4	7.59	6.59
	26.2	0.418	1.98	0.827	23.5	7.62	6.30
	31.6	0.385	1.88	0.724	22.6	7.33	5.31
	43.9	0.326	1.65	0.538	21.9	7.10	3.82
	56.7	0.281	1.51	0.424	22.2	7.20	3.05
	58.8	0.275	1.51	0.415	20.8	6.74	2.80
	62.7	0.264	1.50	0.396	22.9	7.42	2.94
	86.4	0.213	1.35	0.288	23.8	7.72	2.22
	99.5	0.192	1.32	0.254	24.2	7.85	1.99
	123.2	0.164	1.25	0.205	26.0	8.43	1.73
	174	0.124	1.19	0.148	30.1	9.76	1.44
	192	0.115	1.17	0.134	29.4	9.53	1.28
	226	0.0995	1.15	0.114	36.3	11.77	1.34
	320	0.0732	1.10	0.0805	60.1	19.48	1.57
9	24.4	0.430	2.02	0.869	18.4	7.55	6.56
	31.6	0.385	1.88	0.724	18.1	7.43	5.38
	43.9	0.326	1.65	0.538	17.8	7.30	3.93
	62.7	0.264	1.50	0.396	18.0	7.39	2.93
	86.4	0.213	1.35	0.288	18.7	7.67	2.21
	123.2	0.164	1.25	0.205	19.9	8.16	1.67
	174	0.124	1.19	0.148	22.2	9.11	1.35
	226	0.0995	1.15	0.114	25.3	10.38	1.18
	320	0.0732	1.10	0.0805	33.0	13.54	1.09
10	24.4	0.430	2.02	0.869	15.3	7.75	6.73
	31.6	0.385	1.88	0.724	15.3	7.75	5.61
	43.9	0.326	1.65	0.538	14.9	7.55	4.06
	62.7	0.264	1.50	0.396	15.2	7.70	3.05
	86.4	0.213	1.35	0.288	15.4	7.80	2.25
	123.2	0.164	1.25	0.205	16.8	8.51	1.74
	174	0.124	1.19	0.148	18.1	9.17	1.36
	226	0.0995	1.15	0.114	20.0	10.13	1.15
	320	0.0732	1.10	0.0805	25.0	12.67	1.02
12	62.6	0.264	1.49	0.394	12.3	8.97	3.53
	174	0.124	1.19	0.148	14.9	10.87	1.61
	226	0.0995	1.15	0.114	16.5	12.04	1.37
	320	0.0732	1.10	0.0805	18.5	13.50	1.09
	499	0.0488	1.07	0.0522	26.3	19.19	1.00
	755	0.0330	1.04	0.0343	48.7	35.53	1.22
15	56.7	0.281	1.51	0.424	10.1	11.51	4.88
	221	0.1014	1.14	0.116	13.0	14.82	1.72
	499	0.0488	1.07	0.0522	20.0	22.12	1.15
	755	0.0330	1.04	0.0343	28.8	32.83	1.13

Table 3-66

NON-CRITICAL REACTOR CORES IN STAINLESS STEEL VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	<u>gm U²³⁵</u> <u>gm Sol'n.</u>	Solution Density <u>gm/cm³</u>	<u>gm U²³⁵</u> <u>cm³ Sol'n.</u>	Maximum Experimental Attainable		Estimated Critical	
					Height cm	Mass U ²³⁵ kg	Height cm	Mass U ²³⁵ kg
6	24.4	0.4301	2.02	0.8688	56.9	9.02	subcritical	
	31.6	0.385	1.88	0.724	66.6	8.80	subcritical	
	61.1	0.268	1.47	0.394	87.6	6.30	subcritical	
	86.4	0.213	1.35	0.288	72.9	3.83	subcritical	
6½	123.2	0.164	1.25	0.205	77.2	3.39	80	3.50
	174.0	0.124	1.19	0.148	72.3	2.29	subcritical	
7	183.0	0.1194	1.18	0.141	54.4	1.90	63	2.20
	226.0	0.0995	1.15	0.114	69.9	1.98	>100	>2.80
9	499.0	0.0488	1.07	0.0522	40.5	0.87	>75	>1.60
10	499.0	0.0488	1.07	0.0522	39.4	1.04	40.7	1.08
12	999.0	0.0252	1.03	0.0260	69.0	1.31	subcritical	
15	999.0	0.0252	1.03	0.0262	50.2	1.49	52.4	1.55

Table 3-67

CRITICAL CONDITIONS FOR CADMIUM-SHIELDED

REACTOR CORES IN STAINLESS STEEL VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	<u>gm U²³⁵</u> <u>gm Sol'n.</u>	Solution Density <u>gm/cm³</u>	<u>gm U²³⁵</u> <u>cm³ Sol'n.</u>	Critical Height cm	Critical Volume liters	Critical Mass kg U ²³⁵
8	43.9	0.326	1.65	0.538	51.5	16.70	8.98
	62.7	0.264	1.50	0.396	48.4	15.69	6.21
	86.4	0.213	1.35	0.288	56.5	18.32	5.28
9	31.6	0.385	1.88	0.724	29.0	11.90	8.62
	43.9	0.326	1.65	0.538	28.8	11.82	6.36
	62.7	0.264	1.50	0.396	28.3	11.64	4.61
	86.4	0.213	1.35	0.288	29.2	11.98	3.45
	123.2	0.164	1.25	0.205	32.0	13.13	2.69
	174.0	0.124	1.19	0.148	37.3	15.30	2.26
10	31.6	0.385	1.88	0.724	21.1	10.69	7.74
	43.9	0.326	1.65	0.538	21.4	10.84	5.83
	62.7	0.264	1.50	0.396	22.0	11.15	4.42
	86.4	0.213	1.35	0.288	22.0	11.15	3.21
	221	0.1014	1.14	0.116	28.7	14.54	1.69
12	56.7	0.281	1.51	0.424	15.8	11.53	4.89
	174	0.124	1.19	0.148	19.5	14.22	2.10
	226	0.0995	1.15	0.114	20.9	15.25	1.74
	499	0.0488	1.07	0.052	32.8	23.93	1.25

Table 3-68

NON-CRITICAL CADMIUM SHIELDED REACTOR CORES
IN STAINLESS STEEL VESSELS WITH WATER REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	<u>gm U²³⁵</u> <u>gm Sol'n.</u>	Solution Density <u>gm/cm³</u>	<u>gm U²³⁵</u> <u>cm³ Sol'n.</u>	Maximum Experimental Attainable		Estimated Critical	
					Height cm	Mass U ²³⁵ kg	Height cm	Mass U ²³⁵ kg
7	31.6	0.385	1.88	0.724	50.3	9.04	subcritical	
	86.4	0.213	1.35	0.288	71.9	5.15	subcritical	
8	24.4	0.4301	2.02	0.8688	30.8	8.68	> 47	> 13.2
	31.6	0.385	1.88	0.724	35.9	8.43	subcritical	
9	24.4	0.4301	2.02	0.8688	24.7	8.80	> 32	> 11.4
	226.0	0.0995	1.15	0.114	41.2	1.93	> 46.2	> 2.16
	320	0.0732	1.10	0.0805	40.5	1.34	subcritical	
10	24.4	0.4301	2.02	0.8688	19.6	8.63	23.4	10.3
12	26.2	0.4179	1.98	0.827	13.2	7.96	15.5	9.35
	755	0.033	1.04	0.0343	58.3	1.46	subcritical	

Table 3-69

CRITICAL CONDITIONS FOR REACTOR CORES IN ALUMINUM VESSELS WITHOUT REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	<u>gm U²³⁵</u> <u>gm Sol'n.</u>	Solution Density <u>gm/cm³</u>	<u>gm U²³⁵</u> <u>cm³ Sol'n.</u>	Critical Height cm	Critical Volume liters	Critical Mass U ²³⁵ kg
10	52.9	0.293	1.566	0.459	34.0	17.22	7.90
	169.0	0.127	1.187	0.151	41.2	20.87	3.15
15	169.0	0.127	1.187	0.151	18.5	21.09	3.18
	328.7	0.0715	1.101	0.0787	21.7	24.73	1.95
	499.0	0.0488	1.070	0.0522	27.4	31.24	1.63
	755	0.033	1.04	0.0343	43.6	49.7	1.70

Table 3-70

NON-CRITICAL REACTOR CORES IN ALUMINUM VESSELS WITHOUT REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	gm U ²³⁵ gm Sol'n.	Solution Density gm/cm ³	gm U ²³⁵ cm ³ Sol'n.	Maximum Experimental Attainable		Estimated Critical	
					Height cm	Mass U ²³⁵ kg	Height cm	Mass U ²³⁵ kg
8	52.9	0.293	1.566	0.459	46.7	6.95	subcritical	
10	499	0.0488	1.07	0.0522	61.9	1.64	subcritical	

Table 3-71

CRITICAL CONDITIONS FOR REACTOR CORES IN STAINLESS STEEL VESSELS WITHOUT REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	gm U ²³⁵ gm Sol'n.	Solution Density gm/cm ³	gm U ²³⁵ cm ³ Sol'n.	Critical Height cm	Critical Volume liters	Critical Mass U ²³⁵ kg
10	43.9	0.326	1.65	0.538	32.3	16.36	8.80
	62.7	0.264	1.50	0.396	31.7	16.05	6.36
	86.4	0.213	1.35	0.288	31.9	16.16	4.65
	123.2	0.164	1.25	0.205	34.3	17.38	3.56
	174	0.124	1.19	0.148	38.7	19.61	2.90
	226	0.0995	1.15	0.114	46.6	23.61	2.69
12	174	0.124	1.19	0.148	24.7	18.02	2.67
	226	0.0995	1.15	0.114	26.2	19.11	2.18
	320	0.0732	1.10	0.0805	30.3	22.10	1.78
	499	0.0488	1.07	0.0522	48.9	35.67	1.86
15	56.7	0.281	1.51	0.424	17.1	19.49	8.26
	221	0.1014	1.14	0.116	19.5	22.23	2.58
	499	0.0488	1.07	0.0522	27.0	30.78	1.61
	755	0.0330	1.04	0.0343	41.7	47.54	1.63
20	119	0.168	1.26	0.212	14.3	28.98	6.14
	221	0.1014	1.14	0.116	15.7	31.81	3.69
	329	0.0715	1.10	0.0787	17.4	35.25	2.77
	499	0.0488	1.07	0.0522	20.5	41.45	2.17
	755	0.0330	1.04	0.0343	26.7	54.10	1.86

Table 3-72

NON-CRITICAL REACTOR CORES IN STAINLESS STEEL VESSELS WITHOUT REFLECTORS

Core Diameter Inches	H:U ²³⁵ Atomic Ratio	gm U ²³⁵ gm Sol'n.	Solution Density gm/cm ³	gm U ²³⁵ cm ³ Sol'n.	Maximum Experimental Attainable		Estimated Critical	
					Height cm	Mass U ²³⁵ kg	Height cm	Mass U ²³⁵ kg
6	24.4	0.4301	2.02	0.8688	63.1	10.0	subcritical	
7	24.4	0.4301	2.02	0.8688	43.5	9.38	subcritical	
	31.6	0.385	1.88	0.724	47.9	8.61	subcritical	
	43.9	0.326	1.65	0.538	62.0	8.28	subcritical	
	61.1	0.268	1.47	0.394	87.1	8.52	subcritical	
8	24.4	0.4301	2.02	0.8688	31.2	8.79	subcritical	
	31.6	0.385	1.88	0.724	39.3	9.22	subcritical	
	43.9	0.326	1.65	0.538	54.8	9.56	subcritical	
	62.7	0.264	1.50	0.396	71.5	9.18	subcritical	
	86.4	0.213	1.35	0.288	81.2	7.58	subcritical	
	123.2	0.164	1.25	0.205	81.7	5.43	subcritical	
	174	0.124	1.19	0.148	85.8	4.12	subcritical	
	226	0.0995	1.15	0.114	85.8	3.17	subcritical	
9	24.4	0.4301	2.02	0.8688	24.9	8.88	subcritical	
	31.6	0.385	1.88	0.724	32.0	9.51	subcritical	
	43.9	0.326	1.65	0.538	40.7	8.98	>90	>20.4
	62.7	0.264	1.50	0.396	45.3	7.36	>56	>9.1
	86.4	0.213	1.35	0.288	46.3	5.47	>57.5	>6.8
	123.2	0.164	1.25	0.205	46.0	3.87	>70	>5.9
	174.0	0.124	1.19	0.148	46.2	2.81	subcritical	
10	24.4	0.4301	2.02	0.8688	19.4	8.54	>28	>12.4
	26.2	0.4179	1.98	0.827	19.0	7.96	>25	>10.5
	31.6	0.385	1.88	0.724	27.1	9.94	>35	>12.8
	320	0.0732	1.10	0.0805	46.7	1.90	>75	>3.0
	499	0.0488	1.07	0.0522	45.9	1.21	subcritical	
12	62.6	0.2641	1.49	0.394	18.5	5.32	21.7	6.24
20	999.0	0.0252	1.03	0.0260	9.5	1.55	36	1.90

FIG. 3-116
RELATIONSHIP OF CRITICAL DIMENSIONS AS
A FUNCTION OF FUEL CONCENTRATION
(REPRODUCED FROM REF. 225)

where a and b are the respective asymptotes. The method by which the empirical equations were obtained follows a procedure described by Running.⁽²²⁸⁾ By the method there described values of a , b and c can be readily obtained for each $H:U^{235}$ ratio. A mathematical justification for approximating critical dimensions by this equation is presented by Danzker.⁽²²⁹⁾

The parameters a , b and c were determined empirically for a range of values of $H:U^{235}$ ratios from 31.6 to 400.0, and are given in Table 3-73.

Table 3-73

LIMITING DIMENSIONS OF WATER REFLECTED INFINITE SLABS AND CYLINDERS AT VARIOUS $H:U^{235}$ RATIOS (STAINLESS STEEL VESSELS)			
$H:U^{235}$ Atomic Ratio	(Reproduced from Ref. 225.)		
	Diameter of Infinite Cylinder, cm a	Height of Infinite Slab, cm b	cm^2 c
31.6	14.22	6.42	99.0
50.0	14.10	5.85	102.6
100.0	14.58	6.24	104.1
200.0	16.10	7.60	104.9
300.0	17.88	9.25	108.4
400.0	20.02	11.54	104.1
Average			104.0

A value of c which is constant is preferable to the variable c in Table 3-73. Accordingly, the average value of 104.0 cm^2 is chosen and new values of a and b are derived for this value of c , and are given in Table 3-74.

Table 3-74

LIMITING DIMENSIONS OF WATER REFLECTED INFINITE SLABS AND CYLINDERS (STAINLESS STEEL VESSELS) USING THE VALUE OF $c = 104 \text{ cm}^2$		
$H:U^{235}$ Atomic Ratio	(Reproduced from Ref. 225.)	
	Diameter of Infinite Cylinder, cm a	Height of Infinite Slab, cm b
30	14.1	6.70
50	14.0	6.12
100	14.6	6.45
150	15.4	7.00
200	16.2	7.70
250	17.0	8.55
300	17.9	9.47
350	18.9	10.50
400	20.1	11.65

Table 3-75 compares the experimental critical heights and diameters for various H:U²³⁵ ratios with values computed by means of the empirical formula using first the variable values of c from Table 3-73 and secondly the constant value of $c = 104.0 \text{ cm}^2$.

Table 3-75

COMPARISON OF CALCULATED AND EXPERIMENTAL
VALUES OF CRITICAL HEIGHT

Core Diameter inches	H:U ²³⁵ Atomic Ratio	Critical Height, cm.		
		Experimental	Using Values from Table 3-63	Using Values from Table 3-64
6½	30.0	48.6	49.4	50.0
	50.0	47.1	48.4	47.9
	100	62.6	59.8	62.6
7	30.0	34.2	34.1	35.2
	50.0	32.4	33.7	33.8
	100	37.8	38.5	37.8
	200	-	73.2	76.0
8	30.0	23.0	22.9	23.5
	50.0	22.2	22.3	22.6
	100	24.4	24.2	24.8
	200	32.8	32.9	33.3
	300	53.6	53.7	52.6
9	30.0	18.3	18.2	18.6
	50.0	17.8	17.6	17.9
	100	18.9	18.7	19.1
	200	23.4	23.4	23.5
	300	31.2	31.0	30.5
	400	-	47.8	48.9
10	30.0	15.4	15.6	15.9
	50.0	15.0	14.9	15.3
	100	15.6	15.8	16.1
	200	18.8	19.0	19.1
	300	23.7	23.7	23.4
	400	-	30.7	31.2
12	30.0	12.8	12.8	13.1
	50.0	12.6	12.1	12.4
	100	13.0	12.7	13.0
	200	15.1	14.9	15.0
	300	17.7	17.8	17.8
	400	21.5	21.4	21.7
15	50.0	10.1	10.1	10.0
	100	10.6	10.6	10.4
	200	12.5	12.4	12.5
	300	14.7	14.6	14.6
	400	17.3	17.3	17.4

Comparing the simplified calculated values in the last column of Table 3-75 with the experimental values of column 3, it is found that the average deviation from the experimental values is 1.4% with a maximum deviation of 5%, well within the expected experimental error.

In an extension of this technique, the $(D - a)(H - b) = c$ formula was found to apply equally well for reactor cores in aluminum vessels with c equal to 116 over the $H:U^{235}$ range considered. Values of a and b are given in Table 3-76.

Table 3-76

LIMITING DIMENSIONS OF WATER REFLECTED
INFINITE SLABS AND CYLINDERS (ALUMINUM
VESSELS) USING THE VALUE $c = 116$

H:U ²³⁵ Atomic Ratio	(Reproduced from Ref. 225.)	
	Height of Infinite Slab, cm b	Diameter of Infinite Cylinder, cm a
30	3.7	5.4
40	3.5	5.3
50	3.4	5.3
100	3.7	5.5
200	5.0	6.0
300	7.0	6.7
400	9.75	7.5

Fig. 3-117 relating the $H:U^{235}$ ratio to density of U^{235} facilitates the conversion from critical volume to critical mass of U^{235} .

FIG. 3-117
DENSITY OF U^{235} AS A FUNCTION OF COMPOSITION
(REPRODUCED FROM REF. 225)

Macklin has obtained an analytic expression⁽²³⁰⁾ for the functional dependence of a and b on the composition.

The critical mass measurements of Beck, Callihan, Morfitt and Murray were extended to core cylinders of 30 inch diameters, to lower H:U²³⁵ ratios, and to spherical and rectangular parallelepiped core geometries. Table 3-77 reports measurements made on water reflected and unreflected reactor cores in aluminum and stainless steel vessels, for several geometries.

Table 3-77
CRITICAL PARAMETERS OF AQUEOUS SOLUTIONS OF ENRICHED URANIUM

Fissionable material: Highly enriched aqueous solutions of UO₂F₂ (~90% U²³⁵ enrichment).
Reactor Container Material: (1) 1/16" thick 2S aluminum coated with Heresite.
(Reproduced from Ref. 231) (2) 1/16" thick stainless steel.

Solution	Container & Size	Solution Concentration		Critical Values		
		H:U ²³⁵	gm U ²³⁵ cm ³ Sol'n	Height cm	Volume liters	Mass kg of U ²³⁵
		Atomic Ratio				
<u>Aluminum</u>		<u>Totally Water-Reflected</u>				
	6" dia. cylinder	27.1	0.8288	90.0	16.38	13.58
	6.5" dia. cylinder	44.3	0.5376	39.4	8.43	4.53
	10" dia. cylinder	27.1	0.8288	13.13	6.67	5.53
	15" dia. cylinder	27.1	0.8288	8.41	9.60	7.96
	30" dia. cylinder	27.1	0.8288	5.72	26.08	21.6
		44.3	0.5376	5.46	24.9	13.39
		72.4	0.3423	6.20	28.27	9.68
	20"x20" square cross Section	27.1	0.8288	6.99	18.03	14.94
		44.3	0.5376	4.95	12.77	6.87
		72.4	0.3423	6.88	17.75	6.08
	9" dia. sphere	47.3	0.5061	--	6.32 ^a	3.20
<u>Stainless steel</u>		44.3	0.5376	118.4	21.55	11.59
	6" dia. cylinder	74.6	0.3314	∞ ^b		
<u>Aluminum</u>		<u>Partially Water-Reflected^c</u>				
	6" dia. cylinder	44.3	0.5376	76.2	13.87	7.46
	7.5" dia. cylinder	44.3	0.5376	26.4	7.54	4.05
	8" dia. cylinder	44.3	0.5376	24.3	7.89	4.24
		72.4	0.3423	24.0	7.79	2.66
	10" dia. cylinder	72.4	0.3423	17.4	8.81	3.02
		73.4	0.3370	17.5	8.90	3.00
	15" dia. cylinder	74.6	0.3314	12.7	14.44	4.79
	20" dia. cylinder	72.4	0.3423	11.3	22.86	7.83
	30" dia. cylinder	72.4	0.3423	9.9	44.96	15.39
	30"x60" cross section ^d	57.0	0.4240	8.9	103.2	43.7
<u>Aluminum</u>		<u>No Reflector</u>				
	8.76" dia. cylinder	44.3	0.5376	218.5	84.8	45.6
	9.5" dia. cylinder	44.3	0.5376	44.2	20.26	10.89
	10" dia. cylinder	27.1	0.8288	38.5	19.56	16.21
		44.3	0.5376	34.7	17.63	9.49
		73.4	0.3370	33.3	16.89	5.70
	12" dia. cylinder	44.3	0.5376	22.8	16.60	8.92
	15" dia. cylinder	27.1	0.8288	18.1	20.70	17.16
		44.3	0.5376	17.5	20.00	10.80
		74.6	0.3314	16.4	18.75	6.21
	20" dia. cylinder	27.1	0.8288	15.4	31.26	25.9
		44.3	0.5376	14.6	29.59	15.9
		73.4	0.3370	14.8	29.92	10.1
	30" dia. cylinder	44.3	0.5376	13.3	60.47	32.5
		72.4	0.3423	13.5 ^e	61.6	21.1
	20"x20" square cross Section	27.1	0.8288	16.3 ^e	42.0	34.8
		72.4	0.3423	13.9	35.84	12.3
<u>Stainless steel</u>		44.3	0.5376	f		
	8.5" dia. cylinder	74.6	0.3314	59.0	24.30	8.05

^aSphere lacked 80 cc being full; the data extrapolate to a critical mass in concentration of 3.09 kg. of U²³⁵ (0.483 g/cc, H:U²³⁵ = 49.9) in a full sphere with a capacity of 6.40 liters.

^bExtrapolated from a solution height of 143.7 cm. Reactor subcritical at infinite height.

^cNo top reflector.

^dThis vessel was coated with Unichrome rather than Heresite.

^eExtrapolated values.

^fSystem subcritical at solution height of 228.6 cm.

Brown, Noordhoff, and Bateson⁽²³²⁾

A series of critical experiments have been performed at the Atomic Power Division, Westinghouse Electric Corporation with aqueous solutions of uranyl nitrate, $\text{UO}_2(\text{NO}_3)_2$ (uranium enrichment greater than 90%). The experiments were performed in a 36-inch diameter, 72-inch high stainless steel tank with solution concentrations such that the $\text{H}:\text{U}^{235}$ atomic ratio lay between 1600 and 1800. In these experiments the reactor was always reflected at the bottom by a solid Lucite cylinder 10 inches high and it may or may not have an 8-inch radial water reflector. The top had no reflector other than the one-inch thick steel control rod support plate which lay more than 20 inches above the normal solution height.

From the critical solution heights one can calculate the critical mass of U^{235} required for each $\text{H}:\text{U}^{235}$ ratio. The relationship is tabulated in Table 3-78 for the reflected and unreflected cores operated at room temperature.

Table 3-78

$\text{H}:\text{U}^{235}$ Atomic Ratio	CRITICAL MASS OF U^{235} (Reproduced from Ref. 232.)			
	Critical Solution Height cm		Critical Mass kg	
	Reflected	Unreflected	Reflected	Unreflected
1633 ± 2	58.7 ± 0.05	61.7 ± 0.05	5.91 ± 0.04	6.21 ± 0.04
1688	69.2	74.2	6.75	7.24
1710	75.2	81.5	7.24	7.85
1721	79.1	86.1	7.57	8.24
1739	86.6	95.3	8.20	9.03
1757	95.5	106.7	8.95	10.00
1773	106.2	120.9	9.87	11.24
1776	110.0	127.0	10.21	11.78

An indication of the reactivity worth of the reflector can be obtained by multiplying the change in critical solution height with and without the reflector by the average reactivity change as determined by period measurements per centimeter of solution between those heights. The values of $\Delta\rho$ are tabulated in Table 3-79.

Table 3-79

$\text{H}:\text{U}^{235}$ Ratio	Critical Solution Height cm		Change in Height cm	$\frac{\Delta\rho}{\Delta H}$ (cm) ⁻¹	$\Delta\rho$
	Reflected	Unreflected	ΔH		
1650	61.72	65.28	3.56	12.20×10^{-4}	43.4×10^{-4}
1670	65.28	69.85	4.57	10.31	47.2
1690	69.85	74.68	4.83	8.78	42.4
1710	75.44	81.79	6.35	7.32	46.5
1730	82.80	90.93	8.13	5.94	48.3
1750	91.69	101.85	10.16	4.69	47.6
1770	104.39	117.86	13.46	3.23	43.5

Callihan, Cronin, Fox, Morfitt⁽²³³⁾

A series of critical experiments was performed in 1950 at Oak Ridge National Laboratory. The vessels used were all right circular cylinders of 2S aluminum coated with Bakelite varnish and of various radii and 1/16" thickness. The cores were set in vessels which may be filled with reflecting material. Core and reflector critical dimensions are stated in the tables. For some of the experiments hexagonal arrays of 1-inch outside diameter aluminum tubes were introduced into the core in order to contain non-fissionable materials of interest. Two such arrays were employed, one of 106 tubes and one of 36 tubes, equally spaced.

The lateral surfaces of the cores were surrounded by various materials to simulate different reflectors. The materials tested were stainless steel, phosphoric acid, uranyl nitrate of natural uranium isotopic concentration, and bismuth subcarbonate. The densities of the constituents tested and the properties of the reflectors and moderators are given in Table 3-80.

Table 3-80

PROPERTIES OF FUEL SOLUTIONS

<u>Atomic Ratios</u>			<u>Concentration</u>		<u>Density</u> gm/cm ³
<u>H:U²³⁵</u>	<u>N:U²³⁵</u>	<u>P:U²³⁵</u>	<u>gm U</u> <u>gm soln.</u>	<u>gm U²³⁵</u> <u>cm³ soln.</u>	
61.8	2.86	—	0.2485	0.359	1.55
88.0	7.48	—	0.1680	0.236	1.51
115	10.53	—	0.1339	0.187	1.50
128	6.97	15.8	0.0854	0.12	1.5
131	6.97	15.8	0.0846	0.120	1.52
157	5.37	15.8	0.0805	0.11	1.5
180	5.50	12.4	0.0812	0.11	1.5
230	7.48	—	0.0887	0.102	1.23
240	2.86	—	0.0963	0.105	1.16
316	2.90	53.1	0.0356	0.055	1.67
327	7.48	—	0.0673	0.073	1.17
352	2.86	—	0.0698	0.073	1.11
493	2.86	—	0.0524	0.053	1.08
733	2.86	—	0.0357	0.036	1.06

PROPERTIES OF REFLECTORS AND MODERATORS

<u>Material</u>	<u>Density</u>	<u>Remarks</u>
UO ₂ (NO ₃) ₂ (soln)	0.426 gm U/cm ³	Natural isotopic concentration
UO ₂ (NO ₃) ₂ (soln)	0.217 gm U/cm ³	Natural isotopic concentration
Bi ₂ O ₂ CO ₃ (dry)	0.52 gm Bi/cm	USP powder
Bi ₂ O ₂ CO ₃ (slurry)	0.85 gm Bi/cm ³ 0.82 gm H ₂ O/cm ³	Water slurry
Bi metal	9.8 gm Bi/cm ³	
H ₃ PO ₄ (soln)	0.465 gm P/cm ³	86.7% H ₃ PO ₄
Stainless Steel	7.9 gm/cm ³	AISI type 347

A limited number of experiments were done with nitric acid, phosphorus, and bismuth added (not simultaneously) to the core. These materials were contained in the hexagonal array of rods in a 15-inch core, as described above. Also, phosphoric acid was added to the core, thus retaining the characteristic of a homogeneous reactor upon introducing phosphorous. Tabulated data for these experiments are presented in Tables 3-81 through 3-89, which have been reproduced from Ref. 233.

Table 3-81

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235}
IN URANYL NITRATE SOLUTIONS

$$N:U^{235} = 2.86$$

$H:U^{235}$	Reactor Diameter, Inches									
	8.0		9.0		10.0		12.0		15.0	
	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg
Water Reflector (A minimum of 6 in. of water in all directions)										
61.8	25.5	2.97	19.7	2.90	16.6	3.02	—	—	9.4	3.85
240	36.7	1.24	25.3	1.08	20.1	1.06	15.1	1.15	12.2	1.45
352	—	—	35.0	1.04	26.2	0.96	19.3	1.02	14.4	1.20
493	—	—	—	—	37.0	1.00	24.6	0.95	17.9	1.08
733	—	—	—	—	—	—	43.2	1.11	26.7	1.07
No Reflector										
61.8	subcritical		subcritical		>40*	>7.3*	—	—	>18.0	>7.5
240	—	—	—	—	72.4	3.82	29.2	2.23	20.9	2.49
352	—	—	—	—	subcritical		36.8	1.94	24.0	1.99
493	—	—	—	—	—	—	49.5	1.91	28.3	1.71
733	—	—	—	—	—	—	subcritical		44.6	1.79

*This system may be subcritical at infinite length, however insufficient uranium precluded extension of measurement above 27 cm and 4.8 kg.

Table 3-82

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235}
IN URANYL NITRATE-NITRIC ACID SOLUTIONS

$$N:U^{235} = 7.48$$

$H:U^{235}$	Reactor Diameter, Inches							
	9.0		10.0		12.0		15.0	
	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg
Water Reflector (A minimum of 6 in. of water in all directions)								
88.0	27.7	2.69	22.4	2.67	16.3	2.81	12.7	3.42
230	—	—	22.6	1.17	16.8	1.25	13.2	1.53
327	—	—	28.2	1.05	20.0	1.07	15.1	1.26
No Reflector								
88.0	—	—	> 81.0	> 9.7	35.2	6.06	22.6	6.09
230	—	—	> 105	> 5.4	32.9	2.44	22.7	2.63
327	—	—	subcritical		40.2	2.15	25.1	2.09

Table 3-83

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235}
IN URANYL NITRATE SOLUTION
WITH STAINLESS STEEL REFLECTOR

$$N:U^{235} = 2.86$$

$H:U^{235}$	Reflector Thickness Inches	Hc cm	Mc kg
61.8	Reactor Diameter 8.0 Inches		
	0.00	subcritical	subcritical
	1.50	47.8	5.56
	2.00	39.7	4.63
	3.00	33.2	3.86
	3.50	31.5	3.66
61.8	Reactor Diameter 10.0 Inches		
	0.00	>40	>7.3
	1.00	28.0	5.09
	2.50	21.2	3.85
240	0.00	72.4	3.82
	0.25	47.3	2.51
	0.50	39.5	2.09
	1.00	32.8	1.73
	2.50	26.8	1.42
	352		
352	0.00	subcritical	subcritical
	0.50	58.7	2.15
	0.75	48.6	1.78
	1.50	37.3	1.37
	2.50	33.1	1.22
493	2.50	47.6	1.28

Table 3-84

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235} IN URANYL NITRATE SOLUTION
WITH LATERAL COMPOSITE REFLECTOR OF WATER AND STAINLESS STEEL

$$N:U^{235} = 2.86$$

Reactor Diameter Inches	$H:U^{235}$	Reflector Thickness Inches	Hc cm	Mc kg
8.0	61.8	0.00	25.5	2.97
		0.12	29.6	3.44
		0.25	32.8	3.81
		0.50	34.7	4.04
		0.75	34.0	3.96
		1.00	33.1	3.85
		1.50	29.5	3.43
		2.00	27.4	3.18
		3.50	24.2	2.82
10.0	61.8	0.00	16.6	3.02
		0.50	18.6	3.39
		1.00	18.0	3.27
		2.50	16.4	2.98
10.0	240	0.00	20.1	1.06
		0.25	23.4	1.24
		0.50	23.6	1.25
		1.00	22.8	1.20
		2.50	20.4	1.08
10.0	352	0.00	26.2	0.96
		0.50	31.3	1.15
		0.75	30.7	1.13
		1.50	28.4	1.04
		2.50	26.5	0.97
10.0	493	0.00	37.0	1.00
		2.50	38.9	1.04

Table 3-85

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235}
IN URANYL NITRATE SOLUTION WITH LATERAL WATER REFLECTOR

$$N+U^{235} = 2.86$$

Reactor Diameter Inches	$H+U^{235}$	Reflector Thickness Inches	H_c cm	M_c kg
10.0	61.8	0.00	>40	>7.3
		0.88	28.6	5.20
		1.75	24.6	4.47
		2.63	23.3	4.24
		3.50	22.9	4.16
10.0	240	0.00	72.4	3.82
		0.88	33.6	1.78
		1.75	28.4	1.51
		2.63	26.9	1.42
		3.50	26.6	1.41
10.0	352	0.00	subcritical	
		0.88	43.9	1.61
		1.75	35.3	1.30
		3.50	32.5	1.20
12.0	352	0.00	36.8	1.94
		0.88	28.6	1.52
		1.75	26.0	1.38
		4.50	24.8	1.31
12.0	493	0.00	49.5	1.91
		0.88	35.2	1.36
		4.50	30.0	1.16

Table 3-86

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235} IN URANYL NITRATE SOLUTION
WITH LATERAL REFLECTOR OF NATURAL URANYL NITRATE SOLUTION

$$N+U^{235} = 2.86$$

Reactor Diameter Inches	Reflector		$H+U^{235}$	Core	
	Concentration gm U/cm^3	Thickness Inches		H_c cm	M_c kg
10.0	0.426	0.00	61.8	>40	>7.3
		0.88		29.9	5.40
		1.75		25.3	4.60
		2.63		23.6	4.29
		3.50		23.0	4.17
10.0	0.426	0.00	240	72.4	3.82
		0.88		34.8	1.84
		1.75		29.0	1.53
		2.63		27.3	1.45
		3.50		26.4	1.40
10.0	0.426	0.00	352	subcritical	
		0.88		47.1	1.73
		1.75		36.1	1.33
		3.50		32.0	1.17
10.0	0.217	0.00	352	subcritical	
		0.88		44.7	1.64
		1.75		35.7	1.31
		3.50		32.5	1.20
12.0	0.217	0.00	493	49.5	1.91
		0.88		35.2	1.36
		4.50		29.8	1.15

Table 3-87

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235} IN URANYL NITRATE SOLUTION
WITH LATERAL REFLECTORS CONTAINING PHOSPHORUS OR BISMUTH

$$N:U^{235} = 2.86$$

<u>Reactor Diameter</u> Inches	<u>H:U²³⁵</u>	<u>Reflector Thickness</u> Inches	<u>Hc</u> cm	<u>Mc</u> kg
<u>Phosphoric Acid. 0.465 gm P/cm³</u>				
10.0	61.8	0.00	>40	>7.3
		0.88	>31.1	>5.6
		1.75	25.8	4.68
		2.63	23.6	4.28
		3.50	22.6	4.11
		3.50	16.0*	2.91*
10.0	240	0.00	72.4	3.82
		0.88	34.6	1.83
		1.75	29.2	1.55
		3.50	28.3	1.50
<u>Bismuth Sub-carbonate, dry. 0.52 gm Bi/cm³</u>				
10.0	61.8	0.00	>40	>7.3
		3.50	>36	>6.4
		3.50	21.8*	3.96*
10.0	240	0.00	72.4	3.82
		1.75	53.6	2.84
		3.50	48.8	2.58
12.0	352	0.00	36.8	1.94
		0.88	35.4	1.87
		4.50	33.5	1.77
<u>Bismuth Sub-carbonate Slurry. 0.85 gm Bi/cm³; 0.82 gm H₂O/cm³</u>				
12.0	352	0.00	36.8	1.94
		0.88	28.7	1.52
		4.50	24.7	1.31
12.0	493	0.00	49.5	1.91
		4.50	30.1	1.17

*Values obtained with core and reflector enclosed in water.

Table 3-88

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235} IN URANYL NITRATE SOLUTION
WITH ANNULAR VOID SEPARATING WATER REFLECTOR FROM REACTOR

$$N:U^{235} = 2.86$$

Reactor Diameter 10.0"

H:U ²³⁵	Thickness of Void Inches	Hc cm	Mc kg
61.8	0.0	16.6	3.02
	1.0	19.0	3.45
	2.0	21.0	3.81
	3.0	22.4	4.08
	4.0	23.6	4.28
	∞	>40	>7.3
240	0.0	20.1	1.06
	1.0	24.0	1.27
	2.0	26.5	1.40
	4.0	29.6	1.57
	∞	72.4	3.82

Table 3-89

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235}
IN URANYL NITRATE-PHOSPHORIC ACID-NITRIC ACID SOLUTIONS

Reactor Diameter Inches	N:U ²³⁵	H:U ²³⁵	P:U ²³⁵	Hc cm	Mc kg
Water Reflector (A minimum of 6 in. of water in all directions)					
10.0	5.4	157	15.8	> 27	>1.5
	5.4	157	15.8	> 27*	>1.6*
	2.9	316	53.1	>105	>2.9
12.0	2.9	316	53.1	43.5	1.75
15.0	2.9	316	53.1	26.8	1.69
	2.9	316	53.1	29.5*	1.86*
	5.5	180	12.4	18.9*	2.5*
	5.5	180	12.4	15.4	2.0
No Reflector					
10.0	5.4	157	15.8	>57	>3.2
15.0	2.9	316	53.1	43.2	2.72
15.0	5.5	180	12.4	>25	>3.3

*Without top water reflector.

The inhomogeneity in the core resulting from lumping the phosphorus and bismuth increases the critical mass only slightly in the range of concentration under investigation. The added inhomogeneity due to the aluminum tubes was evaluated by observing that when 106 tubes, open at both ends, were placed in the solution, the effect of the aluminum alone was to raise the critical mass by 4 to 5%, and is shown in Fig. 3-118. No correction has been applied to the data reported in Table 3-90.

FIG. 3-118
EFFECT OF ALUMINUM TUBES ON CRITICAL MASS
(REPRODUCED FROM REF. 233)

Table 3-90

CRITICAL HEIGHT AND CRITICAL MASS OF U^{235} IN INHOMOGENEOUS CORES
(Reproduced from Ref. 233)

Reactor Diameter 15.0"

Water Reflector (A minimum of 6 in. of water in all directions)

Fuel Composition			Bismuth Filled			Air Filled		Fuel Filled	
H:U ²³⁵	N:U ²³⁵	Tubes	Bi:U ²³⁵	Hc cm	Mc kg	Hc cm	Mc kg	Hc cm	Mc kg
88	7.48	106	—	—	—	>54	>7.7	17.3	4.39
115	10.53	106	45.3	>44	>5.0	—	—	—	—
230	7.48	106	83.5	>45	>2.8	>85	>5.2	17.4	1.90
230	7.48	36	18.5	20.2	1.96	22.3	2.17	—	—

Fuel Composition			Tubes	Average Core Composition Including Contents of Tubes			Water Filled		Phosphoric Acid Filled	
H:U ²³⁵	N:U ²³⁵	P:U ²³⁵		H:U ²³⁵	N:U ²³⁵	P:U ²³⁵	Hc cm	Mc kg	Hc cm	Mc kg
115	10.53	—	106	224	10.53	—	17.6	1.99	—	—
230	7.48	—	106	430	7.48	—	23.6	1.44	—	—
230	7.48	—	36	273	7.48	—	17.6	1.72	—	—
128	6.97	15.8	106*	291	8.16	15.8	24.1*	1.8	—	—
128**	6.97	15.8	106*	291	8.16	15.8	35.2	2.6	—	—
128	6.97	15.8	36*	163	7.22	15.8	19.6	2.3	—	—
128**	6.97	15.8	36*	163	7.22	15.8	25.9	3.0	—	—
131**	6.97	15.8	36*	155	7.22	20.7	—	—	26.9	3.1
131	6.97	15.8	36*	155	7.22	20.7	—	—	20.8	2.4
131	6.97	15.8	106*	244	8.16	39.0	—	—	29.0	2.1
131**	6.97	15.8	106*	244	8.16	39.0	—	—	>46	>3.3
316	2.90	53.1	106*	711	5.69	53.1	subcritical		—	—
316	2.90	53.1	106*	565	5.69	106.4	—	—	subcritical	
316	2.90	53.1	36*	368	3.46	63.9	—	—	35.8	1.90

*with 5% nitric acid corrosion inhibitor solution.

**without water reflector.

In the range of conditions studied, the following core and reflector effects are noted:

- (1) Nitrogen in the core acts as a weak poison. The minimum critical mass measured with uranyl nitrate was 950 gm U^{235} in a 12" reactor at H: U^{235} ratio of 493, compared to a value of 893 gm in a 10" reactor at an H: U^{235} ratio of 329 using uranyl fluoride.
- (2) Phosphorus in the core acts as a weak poison similar to nitrogen. The data are too limited for a quantitative comparison with uranyl fluoride results.
- (3) Bismuth, added to the core in the form of metal rods, gave a lower critical mass than a similar array of aluminum tubes filled with air indicating qualitatively some moderating effect. As a moderator, however, bismuth is not as effective as water or phosphoric acid.
- (4) Equal thicknesses of stainless steel, phosphoric acid and natural uranyl nitrate solutions, and a slurry of bismuth subcarbonate are comparable with water as a reflector. Dry bismuth subcarbonate powder reflector is somewhat less effective due perhaps to its low density.
- (5) Experiments with various thicknesses of stainless steel reflector and with composite reflectors of stainless steel and water show that an effectively infinite steel reflector is more efficient than one of water.
- (6) An air gap between a reactor and its water reflector raises the critical mass somewhat. In one experiment the critical mass was increased 50% by interposing a four inch wide air annulus between the reactor and the reflector. Complete removal of the water increased the mass to 3.5 times the value with a complete reflector.

Critical Mass Data for Los Alamos Water Boilers⁽²³⁴⁾

The critical masses given in Table 3-91 were determined in a series of experiments performed in LASL.

3.9.2 Thermal Flux Flattening (U^{235} - H_2O Core)

Morfitt⁽²³⁹⁾

Critical mass measurements were made at Oak Ridge National Laboratory for a cylindrical reactor loaded with fuel of uniform concentration of U^{235} , and for the same reactor with a continuously varying concentration of U^{235} so chosen that the thermal neutron flux in the core is spatially uniform in the radial direction. The fuel was an aqueous uranyl fluoride solution, UO_2F_2 , enriched to contain 93.2% U^{235} . The moderator-reflector material was light water and the reactor was operated at room temperature.

The cylindrical aluminum outer tank containing the core and reflector was 72 cm in diameter and 91.4 cm high.

Table 3-91

CRITICAL MASS DATA FOR LOS ALAMOS WATER-BOILERS

Model name and literature reference	Core		Tamper		Critical mass of U^{235} (M_C), gm	Ref.
	Material	Shape	Material	Shape		
LOPO						
AECD-3059 (1/25/51)	14.76% enriched uranium as 14.9 liters as UO_2SO_4 aqueous solution $\rho = 1.348(39^\circ C)$ 1100 gm steel including 1/32 in. stainless-steel container.	Sphere of 12 in. diameter	BeO Density 2.75 g/cc	Pseudosphere ~1 ft. thick	575	235
AECD-3063 (9/4/44)			C	Cube-4 ft. side	780	236
LA-241 (3/12/45); same as LADC-941			H ₂ O	Right cylinder diam. = 4 ft. height = 4 ft.	1200 $\pm$ 50 (Estimated)	237
HYPO						
AECD-3059 (1/25 51)	14% enriched uranium as $UO_2(NO_3)_2$ aqueous solution 3000 gm steel including cooling coil and 1/16 in. thick stainless-steel sphere container	Sphere ^a of 12 in. diameter	BeO Density 2.75 g/cc	Parallelepiped 24 x 24 x 27 in. (against core) Parallelepiped 24 x 48 x 60 in.	808 ^b	235
SUPO						
LA-1301 (2/7/52); same as AECD-3287	88.7% enriched uranium as 12.7 liters of $UO_2(NO_3)_2$ aqueous solution. Stainless-sphere container ^c	Sphere of 12 in. diameter	C	Cube 55 in. side	777	238

^aThe stainless-steel sphere was filled to within 3 or 4 cm of the top. An empty 1 in. diameter pipe passes through the core center.

^bWhen 14.5% enrichment is used, the 1 in. diameter hole is calculated to effect the critical mass by 30 gm.

^cThe core contains three 20 ft. long, 1/4 in. OD, 3/16 in. ID stainless-steel cooling coils. In addition to the 1 in. pipe in footnote a, a 1 7/8 in. ID hole runs completely through the reactor tangent to the sphere.

Since the continuous concentration gradient of the theoretical fuel distribution could not be duplicated experimentally, it was approximated, as shown in Fig. 3-119 by one which changed stepwise over the core. The division of the core into five concentric regions was found to be a good compromise between theoretical and practical considerations. The widths of the steps were chosen to be inversely proportional to the calculated concentration gradient of the idealized core having a continuously varying concentration. The predicted volume, mass, and average fuel concentration of each region was determined directly from the continuous curve in Fig. 3-119. For a critical height of 41.6 cm the critical radius was calculated to be 14.8 cm and the critical mass to be 1024 gm of U^{235} . A correction was made for the aluminum forming the partitions which increased the calculated critical radius to 15.1 cm and mass to 1038 gm U^{235} .

Table 3-92 summarizes the critical data obtained from measurement.

Table 3-92

MEASURED CRITICAL PARAMETERS

Height of core and reflector (non-uniform loading)	42.7 cm
Core volume (non-uniform loading)	30.0 l
Mass of U^{235} (non-uniform loading)	1061 gm
Mass of U^{235} (uniform loading)	1162 gm
Reduction in mass (%)	8.7

FIG. 3-119
COMPARISON OF THE FUEL LOADING FOR EXPERIMENTAL
REACTOR WITH THAT OF IDEALIZED REACTOR.
VALUES CALCULATED FOR CRITICAL HEIGHT OF 41.6 cm
(REPRODUCED FROM REF. 239)

3.9.3 Reactivity Effects Due to Special Reflectors (U^{235} - H_2O Cores)

Fox and Gilley(240)

A few critical experiments have been performed to determine the relative effect of some reflector materials on the critical parameters of cylinders containing highly enriched uranium solutions. In particular, the properties of furfural, concrete, graphite, and fire brick have been compared with those of water.

The comparison of the relative effectiveness of furfural and water as reflectors was made by using a 10" diameter aluminum cylinder and a UO_2F_2 solution which had a U^{235} concentration of 0.3370 gm/cm^3 , corresponding to an $H:U^{235}$ atomic ratio of 73.4. The critical height of the solution was measured as the thickness of the furfural or the water on the sides of the cylinder was varied in approximately 1" steps (by using up to four concentric 1" thick aluminum annular "cans" as containers for the reflector along the lateral surface of the solution container). The annular containers were 1" thick, including wall thickness, and, since each wall was $1/16$ " the reflector thickness in each container was 0.88". A correlation of the critical height as a function of the reflector thickness (Table 3-93) shows that water, which has a hydrogen density 2.4 times that of furfural (C_4H_8OCHO), is more effective as a reflector than furfural up to a thickness of approximately 3.5", after which the two materials have essentially the same effectiveness as a reflector.

The effectiveness of concrete as a neutron reflector was measured by determining the critical mass of an enriched uranium solution contained in a 9" diameter stainless steel cylinder as a function of the distance from the cylinder to a 4x4x1/2 ft thick concrete wall. The solution had a U^{235} concentration of 0.3314 g/cm³, corresponding to an H: U^{235} atomic ratio of 74.6. The concrete wall consisted of blocks of shielding concrete having a density of 2.14 g/cm³. The results (Table 3-94) show that little interaction occurs after the edge-to-edge separation is increased to about 24". Doubling the concrete wall thickness (i.e., from 6 to 12") increased the critical mass by about 2.5% when the wall was adjacent to the reactor.

A determination was made of the variation of the critical mass with the type and thickness of a reflector material placed against the bottom of a 20" diameter aluminum core cylinder. It contained an aqueous solution of UO_2F_2 at a U^{235} concentration of 0.8288 gm/cm³, corresponding to an H: U^{235} atomic ratio of 27.1. The reflectors consisted of carbon, firebrick, and water in various laminated arrangements. Table 3-95 shows the variation of the critical mass with firebrick thickness for three different thicknesses of carbon between the cylinder and the firebrick. Table 3-96 shows the variation of critical mass with the thickness of carbon (no firebrick) adjacent to the bottom of the cylinder. Table 3-97 shows the variation of critical mass with the thickness of a water reflector between the reactor and a constant thickness (5.5") of carbon which is moved back stepwise. It can be seen that, up to about 0.5", replacing carbon with water decreased the critical mass, since water is a better moderator. After about 0.5", however, the greater absorption of water more than compensates for the increased moderation, and the critical mass rises again.

Critical mass measurements described in Ref. 240 were continued.⁽²⁴¹⁾ Measurements were made for aqueous solutions of UO_2F_2 contained in aluminum cylinders and slabs, a stainless steel cylinder, and a 20 x 20 inch square aluminum vessel. In all cases the wall thicknesses were 1/16 inch and the aluminum vessels were coated on the inside with Heresite. The vessels are at various times unreflected, partially reflected, or fully reflected. The total water reflector was effectively infinite; the partial water reflector was effectively infinite on the bottom and on the lateral surface to a height equal to the critical solution height. It is to be noted that a few of the unreflected experiments were performed in a 9.5' diameter steel tank which acted as a neutron reflector. For all experiments the U^{235} enrichment of the solution was 93.2%. Results are given in Table 3-98.

Additional results in this sequence of experiments follow:

- (a) Critical height measurements were made for solutions contained in unreflected aluminum cylinders having 1/16" thick walls and bottom plates of thickness ranging from 0.5 to 1.75". From these data critical height changes relative to the 0.5" thick bottom plate critical heights were deduced. The results are given in Table 3-99.
- (b) The effect of wall thickness of an aluminum containing vessel on the critical height of a 6" thick slab of solution was also determined. In these experiments the thickness of the aluminum on the two faces was varied. The results are given in Table 3-100.

Table 3-93

CRITICAL HEIGHT AS A FUNCTION OF THICKNESS OF WATER OR FURFURAL REFLECTOR ON LATERAL SURFACE OF 10 IN. DIAMETER CYLINDER CONTAINING ENRICHED U^{235} SOLUTION WITH AN H:U²³⁵ RATIO OF 73.4

Lateral Reflector Thickness (in.)	Critical Height, Water Reflector (cm.)	Critical Height Furfural Reflector (cm.)
7/8	24.7	26.0
1 3/4	21.25	22.5
2 5/8	20.3	21.0
3 1/2	20	20

Table 3-94

CRITICAL MASS OF AN ENRICHED U^{235} SOLUTION (H:U²³⁵ = 27.1) CONTAINED IN A 9 IN. DIAMETER CYLINDER AS A FUNCTION OF THE DISTANCE BETWEEN THE CYLINDER AND A CONCRETE WALL (6 IN. THICK BY 48 IN. BY 48 IN.)

Edge-to-Edge Spacing (in.)	Critical Mass Kg. U^{235}
6	7.15
12	7.7
18	7.9
24	8
∞	8.1

Table 3-95

CRITICAL MASS OF ENRICHED U^{235} SOLUTION (H:U²³⁵ = 27.1) CONTAINED IN A 20 IN. DIAMETER CYLINDER AS A FUNCTION OF THE THICKNESS OF CARBON AND FIREBRICK ON THE BOTTOM OF THE CYLINDER

Firebrick Thickness (in.)	Critical Mass Kg. U^{235}		
	No Carbon	0.5 in. Carbon Between Firebrick and Cylinder	1.0 in. Carbon Between Firebrick and Cylinder
0	25.9	24.8	23.9
1/2	25.5	-	-
2	25.1	24.05	23.3
5	-	-	23.1
5 1/2	24.5	23.6	-

Table 3-96

CRITICAL MASS OF AN ENRICHED U^{235} SOLUTION (H:U²³⁵ = 27.1) CONTAINED IN A 20 IN. DIAMETER CYLINDER AS A FUNCTION OF THE THICKNESS OF CARBON ON THE BOTTOM OF THE CYLINDER

Carbon Thickness (in.)	Critical Mass Kg. U^{235}
0	25.9
1/2	24.8
1	23.9
2	22.5
3 1/2	21.3
5 1/2	20.5

Table 3-97

CRITICAL MASS OF AN ENRICHED U^{235} SOLUTION (H:U²³⁵ = 27.1) CONTAINED IN A 20 IN. DIAMETER CYLINDER AS A FUNCTION OF THE THICKNESS OF WATER BETWEEN THE BOTTOM OF THE CYLINDER AND 5.5 IN. THICK CARBON REFLECTOR

Water Thickness (in.)	Critical Mass Kg. U^{235}
0	18.2
1/4	18.15
1/2	18.05
3/4	18.2
1	18.5
∞	20.3

Table 3-99

CHANGE IN CRITICAL HEIGHT OF UNREFLECTED CORE
VS. THICKNESS OF BOTTOM PLATE OF THE CYLINDERS

Cylinder Diameter in.	H:U ²³⁵ Atomic Ratio	Bottom Plate Thickness in.	Change in Critical Height Relative to 0.5-in. Thick Bottom Plate Critical Height cm
30	50.1	1.5	-0.5
30	51.5	1.75	-0.6
30	51.5	1.00	-0.2
30	331.0	1.5	-0.7
30	331.0	1.0	-0.4
20	50.1	1.5	-0.6
20	50.1	1.0	-0.3
15	50.1	1.0	-0.3
10	50.1	1.5	-0.8
10	50.1	1.0	-0.5

Table 3-100

CRITICAL HEIGHT VS THICKNESS OF ALUMINUM
PLATES ON SIDES OF ALUMINUM SLAB

H:U²³⁵ Atomic Ratio = 50.1

Thickness of Aluminum Plates on Sides, in.	Critical Height, cm
0.375	30.2
0.250	31.3
0.125	32.7

- (c) A group of experiments was also performed to determine the relative neutron-reflecting properties of Plexiglass and water. A 30" diameter aluminum cylinder (1/16 in. thick walls) was placed in a 9.5-ft dia steel tank in which water could be raised to any desired height. The level of the water was adjusted to the same height as the bottom of the solution, and critical heights were measured for various thicknesses of Plexiglass replacing the water immediately beneath the cylinder. The results are given in Table 3-101.
- (d) A few rather specialized experiments were performed to determine the critical height of solutions in partially water-reflecting aluminum cylinders (1/16-in. thick walls). The critical height of an otherwise totally water-reflecting 10-in. dia cylinder was measured as a function of the thickness of a void immediately above the solution. The results are given in Table 3-102.
- (e) Experimental results were also obtained for critical heights for cylinders partially reflecting laterally (1/16-in. thick walls) and are given in Table 3-103.

Table 3-101

CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS CONTAINED
IN 30-IN. DIAMETER ALUMINUM CYLINDERS REFLECTED ON THE
BOTTOM BY PLEXIGLAS AND/OR WATER

(Reproduced from Ref. 241.)

Solution concentration: 0.470 gm of U^{235} per cm^3

H: U^{235} Atomic Ratio = 51.5

Bottom Reflector	Critical Values		
	Height, cm	Volume, liters	Mass, kg of U^{235}
None	13.3	60.0	28.2
Infinite water	9.8	44.6	21.0
0.5-in. thick Plexiglas + infinite water	9.8	44.6	21.0
1.5-in. thick Plexiglas + infinite water	9.8	44.6	21.0

Table 3-102

CRITICAL HEIGHT OF WATER-REFLECTED ENRICHED
 U^{235} SOLUTIONS CONTAINED IN 10-IN. DIAMETER
ALUMINUM CYLINDERS AS A FUNCTION OF THE
THICKNESS OF A VOID IMMEDIATELY ABOVE
THE SOLUTION

H: U^{235} Atomic Ratio = 43.2

Thickness of Void Above Solution, cm	Critical Height, cm
0	12.5
1	13.6
2	13.8
4	14.3
6	14.5

Table 3-103

CRITICAL PARAMETERS OF PARTIALLY WATER REFLECTED
ENRICHED U^{235} SOLUTIONS CONTAINED IN ALUMINUM CYLINDERS

(Reproduced from Ref. 241.)

Solution Concentration

Critical Values

Water Reflector	U^{235} , g/ cm^3	H: U^{235} Atomic Ratio	Height, cm	Volume, liters	Mass, kg of U^{235}
10-in. Diameter Cylinders					
None	0.0785	328	147.8	74.9	5.88
4-in. thick on bottom and on one-half (180 deg) of lateral surface	0.0785	328	38.0	19.3	1.52
8-in. Diameter Cylinders					
6-in. thick on one-half (180 deg) of lateral surface only	0.470	51.5	46.1	15.0	7.05
6-in. thick over entire lateral surface	0.470	51.5	25.3	8.2	3.85
Effectively infinite water reflector outside 6-in. thick void over entire lateral surface	0.470	51.5	66.4	21.6	10.2

- (f) The critical parameters of a 6-in. thick slab of solution contained in an aluminum vessel (1/8-in. thick walls) were determined for several reflector conditions and are given in Table 3-104.

Table 3-104

CRITICAL PARAMETERS OF A 6-IN. THICK SLAB OF ENRICHED U^{235}
SOLUTION UNDER VARIOUS REFLECTOR CONDITIONS

(Reproduced from Ref. 241.)

Solution concentration: 0.779 g of U^{235} per cc

H: U^{235} Atomic Ratio = 331

Reflector	Critical Values		
	Height, cm	Volume, liters	Mass, kg of U^{235}
None	121.5	221.1	17.2
3 in. of water on one face	34.5	62.8	4.89
6 in. of water on one face	33.7	61.3	4.78
6 in. of water on one face; 6 in. of Plexiglas on one end and bottom	30.5	55.5	4.32

- (g) In another group of experiments the critical height of the 6-in. aluminum slab (1/8-in. thick wall) was determined as a function of the separation distance between one edge of the slab and a 48 x 48 x 6-in. slab of water placed perpendicular to and symmetrical with the edge of the solution slab. The results are given in Table 3-105.

Table 3-105

CRITICAL PARAMETERS OF A 6-IN. THICK
SLAB OF ENRICHED U^{235} SOLUTION PER-
PENDICULAR TO AND AT VARIOUS
DISTANCES FROM A 6-IN. THICK
SLAB WATER REFLECTOR

(Reproduced from Ref. 241.)

Solution concentration:

0.0791 gm of U^{235} per cm^3

H: U^{235} atomic ratio = 325

Separation Distance, in.	Critical Values		
	Height, cm	Volume, liters	Mass, Kg. of U^{235}
1.1	100.8	183.5	14.5
5.1	105.6	192.2	15.2
15.6	109.6	199.5	15.8
42.6	111.3	202.6	16.0
∞	111.3	202.6	16.0

- (h) Still further measurements were made of critical parameters of the 6-in. aluminum slab (1/8-in. thick wall) as a function of the separation distance between the slab and a 5-ft thick concrete wall parallel to the slab. In some of these experiments a sheet of boral (60 x 48 x 1/4-in.) was placed adjacent to the concrete wall on the side facing the slab. In other experiments the boral sheet was placed adjacent to the slab on the side facing the concrete wall. The results are given in Table 3-106.

Table 3-106

Separation Distance Between Solution and Concrete Wall in.	Critical Height, cm		
	No Boral	Boral Sheet Adjacent to Wall	Boral Sheet Adjacent to Solution
0	32	42	-
5	48	56	51
12	-	69	-
18	69	77	61
27	80	86	-
42	91	97	-
90	106	107	-

3.9.4 Studies of U²³⁵-Al-H₂O Cores and Various ReflectorsMann and Martin (242)

Critical mass measurements were made at Oak Ridge National Laboratory for fuel contained in square aluminum tubes 1" x 1" x 1/16" wall thickness. The height of the tube was 70 cm and the fuel was fully enriched U²³⁵ contained in an aqueous solution of UO₂F₂. Each aluminum tube contained about 20.3 grams of U²³⁵. Such an arrangement of core material closely approximates a homogeneous mixture of water and aluminum. The aluminum to water ratio was varied by using 1/8" spacers between the tubes and small aluminum strips placed inside the tubes. Measurements were made with several reflector materials (D₂O, H₂O, and Be) located in a tank containing the reactor core. The reflectors surrounded the sides of the cores but the core ends were unreflected. The reflector thicknesses were at least 50 cm.

In stacking the tubes in the cases of D₂O and H₂O reflectors, a hexagonal geometry was used and this reasonably well approximated a cylinder. Accordingly, cylinders of cross-sectional areas equal to those of the hexagons were determined to facilitate the presentation of data.

The beryllium reflected core was a 9 x 9 fuel element array surrounded laterally by 8 inches of beryllium. This core became critical with 77 fuel tubes but 0.030" gold wires were distributed throughout the core to poison the reactor sufficiently to allow for 81 elements.

These results are contained in Table 3-107.

Table 3-107

CRITICAL MASSES IN Kg OF U²³⁵

Reflector	Vol. Ratio Al/H ₂ O	Uncorrected			Corrected†		
		Critical Mass (Kg)	No. of Fuel Rods*	Equivalent Cylinder Radius (cm)**	Critical Mass (Kg)	No. of Fuel Rods*	Equivalent Cylinder Radius (cm)**
D ₂ O	0.88	2.48	122	17.9	2.40	118	17.5
D ₂ O	0.76	2.23	110	16.9	2.15	106	16.6
H ₂ O	0.76	3.29	162	20.5	3.22	159	20.3
H ₂ O	0.65	2.82	139	19.0	2.72	134	18.7
Be	0.76	1.54	76		1.42	70	

†Correction is for void control rod channels and gold wires.

*Based upon the critical mass and the content of 20.3 grams U²³⁵ per fuel tube.

**Based upon a total area of 8.1653 cm² per fuel tube, including 1/8 in. spacers surrounding the fuel tube.

Mann, Martin, Preston, Toepfer, and McCord (243)

The critical mass measurements reported in Ref. 242 were continued (Ref. 243). The core height is now 66 cm, but the fuel element cross section is as reported in Ref. 242. The core geometry in all cases was rectangular. A measurement was made with no reflector, but with the core surrounded by cadmium sheets. A second measurement of a square array of 8 x 8 fuel tubes was made with a beryllium lateral reflector 30 cm thick, but no end reflectors. Since the assembly became critical with 61 fuel tubes, 31 pieces of 0.030" diameter gold wires were distributed throughout the core to compensate for the three tubes. Finally, a measurement was made for an assembly of 4 x 19 fuel tubes surrounded laterally by 30 cm of beryllium. There was no top or bottom reflector. About 15 lengths of gold were distributed throughout the core to compensate for excess reactivity. These data are contained in Table 3-108.

Table 3-108

CRITICAL MASSES IN Kg OF U^{235}

Reflector	Vol. Ratio Al/H ₂ O	Array of Fuel Tubes	Uncorrected Critical Mass (Kg)	Corrected Critical Mass (Kg)*
None				
(Cd Sheet)	0.65	5 x 6	5.5	5.3 ± 0.1
Be	0.65	8 x 8	1.21	1.07
Be	0.65	4 x 19	1.35	1.28

*Correction is for void control rod channels and for gold wires.

Martin and Mann(244)

Critical mass studies reported in Ref. 242 and Ref. 243 were continued. The core described in ORNL-79 was increased in steps to an ultimate size of 71 x 22.5 x 66 cm. The cores were reflected laterally by 30 cm of beryllium (not reflected on top or bottom) and in some cases this reflector was backed up by 30 cm of graphite. The effects of water coolant in the beryllium and of experimental holes in the reflector were also measured. The locations of the experimental holes is shown in Fig. 3-120. The worths of two sets of alternate holes are also cited. Measurements were made with test holes filled with air, with water, and containing poisons simulating experimental conditions.

The fuel concentration remained at 37.1 grams of U^{235} per liter of core. Hence, in order to maintain $k_{eff} = 1$ as the variations were introduced into the reactor, the aluminum strips between tubes (Mon P-357) were replaced by a sufficient number of 347 stainless steel strips of the same dimension, namely 1/8 x 5/16 x 26 inches. The absorption cross section worth of each strip was measured as $(V\Sigma_a)_{strip} = 12.3 \text{ cm}^2$. Based upon this and the volume-integrated cross section cited in Table I of ORNL-167, the number of such stainless steel strips is deduced. Also, from assembly number 2, Table 3-109, the volumes of aluminum and water are calculated, the volume ratio Al/H₂O being taken as 0.65. Thus the volume ratios of the water, aluminum, and stainless steel were computed.

Table 3-109

CRITICAL MASS AND POISON CONTENT OF U-Al-H₂O
 REACTORS WITH Be REFLECTOR Al/H₂O(Vol) = 0.65;
 Be ~ 30 cm THICK - REACTOR BARE TOP
 AND BOTTOM

Assembly Number	Dimensions L x W x H (cm)	Volume Fractions			Critical Mass, Kg
		S.S. Poison Strips	H ₂ O	Al	
1	22 x 22 x 66	0.009	0.61	0.39	1.21
2	51 x 11 x 66	0.003	0.61	0.39	1.35
3a	71 x 11 x 66	0.02	0.61	0.37	1.94
3b	71 x 11 x 66	0.02	0.61	0.37	1.94
	(30 cm graphite on one long side)				
4	71 x 14 x 66	0.04	0.61	0.35	2.44
	(30 cm graphite on one long side)				
5	71 x 17 x 66	0.06	0.61	0.33	2.94
	(30 cm graphite on one long side)				
6	71 x 22.5 x 66	0.07	0.61	0.32	3.95
	(30 cm graphite on one long side; 2% water in Be)				
7	71 x 22.5 x 66	0.06	0.61	0.33	3.95
	(30 cm graphite on all sides of Be. Seven 6 in. holes in reflector; 2% water in Be)

FIG. 3-120
 CRITICAL ASSEMBLIES AND LOCATION
 OF HOLES IN REFLECTOR
 (REPRODUCED FROM REF. 244)

In the cases of the presence of test holes, the critical mass was estimated by means of the empirical formula cited in ORNL-167:

$$\frac{\Delta M}{M} = 4.5 \frac{\Delta k}{k}$$

It is implied that the additional fuel necessary to compensate for the test holes is added at positions of average statistical weight. Table 3-110 contains data indicating the effect of test-holes on critical mass.

The effect of placing a cubical cadmium box (0.020" Cd, 10 cm along an edge) at various positions in each of the 6 inch holes is depicted in Fig. 3-121.

Two alternate sets of test holes were studied with the results contained in Table 3-111.

Johnson, Maienschein, Henry, Cochran, and Flynn⁽²⁴⁵⁾

A series of experiments designed to yield information on reactivity changes due to the presence of beam holes and partial reflectors adjacent to a swimming pool type research reactor were performed at the Bulk Shielding Facility. The beam holes consisted of $4\frac{1}{4}$ to $17\frac{3}{4}$ -inch-diameter cylindrical ducts that varied in length from 12 to $47\frac{3}{4}$ inches; the ducts were filled with either air or borated water. Reactivity changes caused by placing the ducts against the fuel of the Bulk Shielding Reactor (water-reflected Loading 22A) or against a beryllium oxide reflector of the BSR (Loading 32) were determined. Similar determinations were made for three partial reflectors: a $1\frac{1}{4}$ inch-thick boral sheet against the reflector of Loading 32; a 1 foot-thick graphite slab against the reflector of Loading 32; and a $1\frac{1}{8}$ inch-thick lead slab against the fuel of Loading 22A.

Table 3-110

EFFECT OF EXPERIMENTAL HOLES IN THE HIGH FLUX PILE CRITICAL ASSEMBLIES

(Refer to Fig. 3-120)

Assembly No. (Table 3-109)	3		5		6	
	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**
Holes extending to core - reflector interface						
6 inch hole on edge	0.0117	2.04	0.0075	3.04	0.0049	4.04
6 inch hole in center	0.0220	2.13	-	-	0.0086	4.10
8 inch hole in center	0.0480	2.36	0.0284	3.32	0.0171	4.25
Holes beginning 15 cm from core						
6 inch hole on edge	0.0017	1.95	0.0012	2.96	0.0008	3.96
6 inch hole in center	0.0034	1.97	-	-	0.0013	3.97
8 inch hole in center	0.0068	2.00	0.0040	2.99	0.0022	3.99
Holes extending to core - water filled						
6 inch hole on edge	0.0090	2.02	0.0065	3.03	0.0042	4.02
6 inch hole in center	-	-	-	-	0.0084	4.10
8 inch hole in center	0.0388	2.28	0.0245	3.26	0.0153	4.22
Holes beginning 15 cm from core - water filled						
6 inch hole on edge	-	-	0.0008	2.95	0.0006	3.96
6 inch hole in center	0.0024	1.96	-	-	0.0006	3.96
8 inch hole in center	0.0037	1.97	0.0023	2.97	0.0011	3.97

*k is the k of the pile with solid reflector.

**Critical masses estimated from $\frac{\Delta M}{M} = 4.5 \frac{\Delta k}{k}$.

FIG. 3-121
EFFECT OF ABSORPTION OF 10 x 10 x 10 cm BOX OF 20-mil CADMIUM
(REPRODUCED FROM REF. 244)

Table 3-111

SUMMARY OF TOTAL EFFECT OF ALTERNATIVE
EXPERIMENTAL HOLES

Assembly No. (Table 3-109)	3		5		6	
	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**	$\frac{\Delta k^*}{k}$	Estimated Critical Mass (kg)**
Two 8 inch holes plus five 6 inch holes	0.0933	2.75	0.0573	3.70	0.0356	4.58
Seven 6 inch holes	0.0639	2.50	0.0423	3.50	0.0262	4.42
Two 8 inch holes plus five 6 inch holes filled with water	0.0214	2.13	0.0094	3.06	0.0054	4.05
Seven 6 inch holes filled with water	0.0141	2.06	0.0066	3.03	0.0034	4.01

*k is the k of the pile with solid reflector

**Critical masses estimated from $\frac{\Delta M}{M} = 4.5 \frac{\Delta k}{k}$

The BSR is an assembly of MTR-type fuel elements comprised of fully enriched uranium contained in aluminum clad fuel plates. A full fuel element contains a total of about 140 gms of U^{235} . The reactor is cooled, moderated, shielded, and at least partially reflected by water. A beryllium oxide reflector may be used on any or all of the four vertical sides.

Loading 22A

Loading 22A consisted of 30 fuel elements in a 5 by 6 array which was completely reflected by water. The over-all active core dimensions were 15 x 18 inches x 23 $\frac{5}{8}$ inches high. This loading was composed of the original BSR fuel elements which contained an appreciable residue of lithium from the brazing flux, thus poisoning the elements.

Loading 32

Loading 32 was an arrangement of 20 fuel elements in a 4 x 5 lattice reflected on the four vertical sides by 3 inches of beryllium oxide. The over-all active core dimensions are 12 x 15 inches x 23 $\frac{5}{8}$ inches high. The fuel elements in this loading contained a minimum of lithium from the brazing flux.

The reactivity effect of the ducts may be related to their cross sectional area by the expression

$$\frac{\Delta k}{k} = CD^2$$

where D is the diameter of the duct in inches and C is a constant for a particular loading. Values of C for each duct are shown in Table 3-112

The partial reflector data are contained in Table 3-113.

Table 3-112

BSR REACTIVITY CHANGES CAUSED BY THE PRESENCE OF BEAM HOLES

(Data from Ref. 245)

Description of Duct*		Reactivity Change (% $\Delta k/k$)		$C = \frac{\% \Delta k/k}{D^2}$	
O.D.	Length in.	Loading 22A, Water-Reflected	Loading 32, BeO-Reflected	Loading 22A, Water-Reflected	Loading 32, BeO-Reflected
17 $\frac{3}{4}$	36 $\frac{3}{4}$	-1.4 ^a	-1.12 ^a	0.004 ^b	0.0035 ^b
8 $\frac{1}{8}$	47 $\frac{3}{4}$	-0.55	-0.47	0.009	0.007
6	47 $\frac{3}{4}$	-0.36 ^c	-0.25 ^c	0.010	0.007
			-0.245 ^{c,d}		
			-0.01 ^e		
6	12		-0.26		0.007
			-0.10 ^f		
4 $\frac{1}{4}$	44	-0.148		0.007	

*All ducts were filled with air unless otherwise noted.

^aThe cross-sectional area of this duct was larger than the active portion of the reactor lattice.^bThis low value was not unexpected because of the large size of the duct.^cA weld on the end of this duct held it approximately 1/8 in. away from the reactor face.^dDuct filled with borated water.^eIn this case the 1/4 in.-thick boral partial reflector was placed between the beryllium oxide reflector and the end of the duct. The reactivity change reported here is that obtained from the difference in the regulating rod position with only the boral sheet against the reflector and its position when duct was placed against the boral.^fThe end of this duct was against the reflector elements, the edge of the reactor face and at the mid-plane.^gThis duct had a 1/8 in.-thick wall and a 1/2 in.-thick end, while the other beam holes had 1/16 in.-thick walls and ends.

Table 3-113

BSR REACTIVITY CHANGES CAUSED BY THE PRESENCE OF PARTIAL REFLECTORS

(Reproduced from Ref. 245)

Description of Partial Reflector ^a	Reactivity Change, % $\Delta k/k$	
	Loading 22A	Loading 32
1 $\frac{1}{2}$ in. thick lead slab	+ 0.38	
1/4 in. thick boral slab		-1.36 ^b
1 ft thick graphite slab		+0.68

^aAll the reflectors extended well beyond the reactor face.^bThe boral sheet was held away from the bottom of the reactor face by the structure supporting the reactor itself.

Johnson, Henry, and Flynn(246)

In order to measure the effect of large voids along one face of the Bulk Shielding Reactor, an air-tight aluminum tank 26 inches high by 22.5 inches wide by 36 inches deep by 0.5 inches thick was placed against the seven element face of a 5 x 7 element core. The core dimensions were approximately 24 inches high by 21 inches wide. The fuel elements are MTR type and the core is located in a tank of water.

In addition to the void, two curtains of sufficient size to completely cover the face of the tank were provided. One of these was made of 100-mil-thick cadmium and the other of 0.25-inch thick boral. The effect of the void on reactivity was measured with and without each of these curtains inserted between reactor and tank. Three iron slabs (0.5, 1.5, and 3 inches thick) were used in conjunction with the neutron-absorbing curtains to simulate experimental equipment placed inside the tank.

Tables 3-114 and 3-115 list the experimental results.

Table 3-114

CHANGES IN REACTIVITY RESULTING
FROM FLOODING THE INITIALLY
AIR-FILLED VOID WITH WATER

(Reproduced from Ref. 246)

Experimental Condition	Changes in Reactivity,
	% $\Delta k/k$
No curtain	+2.62
Cadmium curtain	+0.60
Cadmium curtain plus 0.5 in. thick iron slab	+0.49
Cadmium curtain plus 3 in. thick iron slab	+0.15
Boral curtain	+0.42
Boral curtain plus 0.5 in. thick iron slab	+0.33

Table 3-115

NET CHANGE IN REACTIVITY PRODUCED BY ALTERING
THE REFLECTOR ON ONE SEVEN-ELEMENT FACE OF
THE SWIMMING POOL (BSF) REACTOR

Reflector Initial Condition	Reflector Final Condition			Change in Reactivity,* % $\Delta k/k$
	Tank Filled With	Curtain	Iron Slab Thickness, in.	
Reactor	Water			-0.05
Completely	Water	Cadmium		-2.31
Water -	Water	Cadmium	3	-0.89
Reflected	Water	Cadmium	0.5	-1.76
	Water	Boral		-2.59
	Water	Boral	0.5	-2.18
	Air			-2.62
	Air	Cadmium		-2.91
	Air	Cadmium	3	-1.02
	Air	Cadmium	1.5	-1.37
	Air	Cadmium	0.5	-2.17
	Air	Boral		-3.01
	Air	Boral	0.5	-2.51
Water - Filled Tank	Water	Cadmium		-2.25
	Water	Cadmium	3	-0.84
	Water	Cadmium	0.5	-1.70
	Water	Boral		-2.54
	Water	Boral	0.5	-2.12
Air - Filled Tank	Air	Cadmium		-0.29
	Air	Cadmium	3	+1.60
	Air	Cadmium	1.5	+1.24
	Air	Cadmium	0.5	+0.44
	Air	Boral		-0.40
	Air	Boral	0.5	+0.10
Air - Filled Tank plus Cadmium Curtain	Air	Cadmium	3	+1.89
	Air	Cadmium	1.5	+1.54
	Air	Cadmium	0.5	+0.74
	Water	Cadmium		+0.60
	Water	Cadmium	3	+2.02
	Water	Cadmium	0.5	+1.15
Water - Filled Tank plus Cadmium Curtain	Water	Cadmium	3	+1.41
	Water	Cadmium	0.5	+0.55
	Air	Cadmium		-0.60
	Air	Cadmium	3	+1.29
	Air	Cadmium	1.5	+0.93
	Air	Cadmium	0.5	+0.13
Air - Filled Tank plus Boral Curtain	Air	Boral	0.5	+0.50
Water - Filled Tank plus Boral Curtain	Water	Boral	0.5	+0.42

*A positive change in reactivity results when the reactivity is increased over that existing in the reference condition.

3.9.5

Critical Mass Studies of U^{235} - H_2O Slurries

Callihan, Cronin, Fox, Gallaher, Gilley, Morfitt, and Rohrer⁽²⁴⁷⁾

Critical masses were measured for aqueous slurries of $UO_3 - H_2O$ at the ORNL Critical Experiments Laboratory. The uranium was enriched to 93.2% U^{235} . Measurements were made for reflected and unreflected cores, various uranium concentrations, and two different stirrer speeds. The temperature of the slurry was about 29°C.

The critical tests were made in an aluminum cylindrical core vessel of 12" I.D., 24" height, and 1/8" thickness and having a hemispherical bottom to minimize the deposit of solids. The core was mounted in a tank about nine feet in diameter into which water could be pumped to serve as a neutron reflector on the bottom and sides of the core. The core top was uncovered; hence no reflector was provided above the slurry.

A stainless steel stirrer and aluminum baffles were designed to produce vertical circulation of the slurry with only slight surface disturbance or vortex formation. Critical mass measurements were made at stirrer speeds of 200 and 600 rpm. The critical mass is a function of the stirrer speed, and measurements indicate a maximum reactivity for a given concentration to occur at about 150 rpm. A summary of the data is given in Table 3-116.

Table 3-116
CRITICAL CONDITIONS FOR $UO_3 - H_2O$ SLURRY IN 12" DIAMETER CYLINDER CORE VESSEL
(WATER REFLECTOR ON SIDES AND BOTTOM)
(Reproduced from Ref. 247)

H:U ²³⁵ Atomic Ratio	Slurry Concentration		Stirrer Speed 600 rpm				Stirrer Speed 200 rpm			
	gm U kg	gm U* liter	Reflector Height cm	Critical Height cm	Critical Volume liter	Critical Mass U ²³⁵ kg	Reflector Height cm	Critical Height cm	Critical Volume liter	Critical Mass U ²³⁵ kg
90.2	226.1	298.0	21.1	21.1	11.5	3.20	20.2	20.1	10.9	3.03
136	164.7	199.8	26.0	22.6	12.6	2.35				
204	117.7	134.6	27.6	24.3	13.8	1.73	27.6	23.1	12.9	1.62
251	98.50	110.0	24.1	24.1	13.7	1.40	23.2	23.2	13.1	1.34
259	95.75	106.6	28.0	24.3	13.8	1.37				
366	70.14	75.81	31.5	30.3	18.2	1.29				
413	62.78	67.28	28.5	28.5	16.8	1.05	27.8	27.8	16.3	1.02
467	55.90	59.45	33.0	30.8	18.5	1.02				
511	51.41	54.39	33.8	33.8	20.5	1.04				
523	50.36	53.22	34.0	34.0	20.8	1.03	34.0	32.0	19.3	0.96
538	49.02	51.73	33.3	33.8	20.5	0.99	33.3	30.7	18.5	0.89
610	43.51	45.63	38.0	37.0	22.9	0.97				
632	42.24	44.23	35.1	35.1	21.5	0.89	32.4	32.4	19.6	0.81
668	39.90	41.67	54.0	41.3	26.0	1.01				

*The laboratory analyses were reported as mass of uranium per unit mass of slurry sample. The specific gravities of the slurries were calculated from a value of the density of the $UO_3 - H_2O$ crystal assumed to be 6 gm/cc.

3.9.6 Critical Mass Studies Involving Temperature Dependence (U^{235} - H_2O Spherical Cores)

Visner and Wood(248)

Critical concentrations of fuel as functions of core temperature, reflector temperature, reflector height, and fuel pressure have been experimentally determined for the HRE over a range of values. The fuel was a natural water solution of UO_2SO_4 , the uranium enrichment being nominally 93% U^{235} by weight.

The core vessel was a 347 stainless steel sphere of 18 in. I.D., 3/16 in. thick, and 50 liter volume. The reflector vessel had spherical sides 39 in. I.D. and a cylindrical top and bottom. The vessel thickness was 3 in. and the material was a mild steel. The reflecting material was D_2O of purity 99.1 weight per cent and thickness 10 in.

The reactor system (high pressure) was designed to operate at 250°C temperature, 1000 psig pressure, and 1000 kw reactor power. There was also a low pressure system for removal of fission gases, recombination of hydrogen and oxygen, and fuel feed.

Criticality measurements were made both with and without fuel circulation at core temperatures below 105°C and the results are given in Tables 3-117 and 3-118. The effect of delayed neutrons on reactivity due to fuel circulation may thus be studied. In the first set of measurements with circulating fuel (Table 3-117) only the high pressure system was used. A subsequent set of criticality measurements was made for elevated core temperatures, using the combined high and low pressure systems and these results are given in Table 3-119.

Table 3-117

CRITICAL FUEL CONCENTRATIONS WITH CIRCULATION (REFLECTOR FULL, RODS WITHDRAWN, HIGH PRESSURE SYSTEM ALONE)

Fuel Concentration bm U^{235} /kg H_2O	Fuel Tempera- ture, C	Reflector Tempera- ture, C	Fuel Pressure psig	Fuel Concentration bm U^{235} /kg H_2O	Fuel Tempera- ture, C	Reflector Tempera- ture, C	Fuel Pressure psig
24.73	22	25	100	26.33	79	25	100
24.82	26	25	100	26.40	80	79	100
24.90	26	25	100	26.48	82	79	120
24.89	28	25	100	26.48	83	79	120
24.85	27	25	100	26.81	92	87	260
24.92	29	25	100	26.86	93	92	100
24.95	30	25	100	26.97	96	94	120
25.02	32	25	100	27.05	99	94	100
25.05	33	25	100	27.06	100	95	300
25.08	33	25	100	27.07	100	95	100
25.10	34	25	100	27.11	100	95	100
25.11	35	25	100	27.09	101	95	300
25.13	37	25	100	27.11	101	95	100
25.32	43	25	100	27.10	102	98	600
25.47	49	25	100	27.14	102	100	100
25.44	50	25	100	27.17	102	100	300
25.93	67	25	100	27.16	103	100	910
25.98	67	25	100	27.33	104	100	120
25.98	67	64	100	27.36	105	102	120
25.94	68	60	100	27.37	105	104	120

Table 3-118

CRITICAL FUEL CONCENTRATION WITHOUT CIRCULATION,
REFLECTOR FULL, RODS WITHDRAWN, PRESSURE 100 PSIG

<u>Fuel Concentration gm U²³⁵/kg H₂O</u>	<u>Fuel Temperature C</u>	<u>Reflector Temperature C</u>
24.97	35.2	35.0
25.11	40	40
25.75	63.9	64
26.19	77.6	76
26.57	91.6	88
26.71	93.4	91
26.92	100.7	97
26.94	101.2	100
27.09	103.5	100
27.09	104.0	101

No correction in critical mass is made for the pressure.

Table 3-119

CRITICAL FUEL CONCENTRATION WITH CIRCULATION

Reflector Full, Rods Withdrawn, High and
 Low-Pressure Systems Connected Together

<u>Fuel Concentration gm U²³⁵/kg H₂O</u>	<u>Fuel Temperature C</u>	<u>Reflector Temperature C</u>	<u>Fuel Pressure psig</u>
27.11	96	89	520
27.13	98	90	440
27.15	99	92	460
27.22	100	93	600
27.24	101	93	600
27.30	103	94	420
27.39	103	95	430
27.43	105	100	520
27.45	106	101	530
28.38	120	138	460
28.44	126	123	450
29.41	144	145	380
29.46	145	147	440
30.44	164	165	440
31.64	177	184	500
32.27	198	178	1000
33.35	201	176	560
33.39	201	186	540
33.35	201	186	560
33.36	201	186	560
33.58	202	201	580
33.58	202	201	550
33.58	202	200	580
35.09	217	203	460
35.24	217	200	450
35.21	217	203	460
27.46	102	94	500
33.94	204	198	500
37.38	222	200	500
37.38	221	200	500
37.34	229	207	500
37.17	225	203	500
37.17	225	200	500
39.24	244	190	1000

The reflector was calibrated at room temperature in terms of Δk_{eff} as a function of reflector height. A reference reactor, based on a core temperature of 25°C with a full reflector is used for comparison. Measurements were made both for non-circulating fuel (Table 3-120) and for circulating fuel (Table 3-121).

Table 3-120

REFLECTOR CALIBRATION AT ROOM TEMPERATURE

Fuel Not Circulating

Data: Shim control and safety rods withdrawn,
core temperature 25°C, concentration
24.88 gm U^{235} /kg H_2O , reflector level
100% when full.

Reflector Level,* %	Δk_{eff} , %	Reflector Level,* %	Δk_{eff} , %
100.0	0.02	58.5	0.88
80.0	0.01	56.4	1.15
70.0	0.12	54.5	1.12
66.3	0.20	49.3	1.69
66.0	0.28	44.7	2.20
60.5	0.58	31.5	3.16
60.5	0.57	31.3	3.27
58.7	0.88	25.6	3.75

*Range of level indicator is 30 in. with zero
corresponding to the bottom of the core tank.

Table 3-121

REFLECTOR CALIBRATION AT ROOM TEMPERATURE

Fuel Circulating

Data: Shim inserted; control and safety
rods withdrawn; core temperature
25°C; concentration 25.48 gm
 U^{235} /kg H_2O

Reflector Level,* %	Δk_{eff} , %
73.7	-0.44
67.2	-0.27
61.2	-0.01
57.8	0.19
52.0	0.77
51.5	0.82
48.2	1.29

*Range of level indicator is 30 in. with
zero corresponding to the bottom of the
core tank.

Finally, the reflector was calibrated at 220°C, in terms of k_{eff} as a function of reflector level, and these results are given in Table 3-122.

Table 3-122

REFLECTOR CALIBRATION AT 220°C BY CONCENTRATION CHANGE

Reflector Temperature: 200°C
Shim and Safety Rods Withdrawn; Control Rod Inserted

Reflector Level, %	$\Delta k_{eff},$ %
100	Datum
92.5	-0.24
100	0.25
51	.82
40	2.64
29.5	4.36
58.5	1.24

3.9.7 Critical Mass Studies of U^{235} - D_2O Systems

Olcott(249)

Critical mass measurements have been made at Los Alamos Scientific Laboratory for ten critical assemblies of enriched uranyl-fluoride heavy water solutions. The isotopic concentration of the deuterium ranged from an initial 99.8% to about 99.0% at the end of the series. The uranium was approximately 90% U^{235} . The experiments were performed at room temperature.

Six critical measurements were made in D_2O -reflected spherical assemblies and the results are given in Table 3-123. The six cores were spheres of 0.040-inch thick stainless steel ranging in diameter from 13.5 to 18.5 inches. The reflector thickness depended upon the core diameter, the inside diameter of the stainless steel reflector vessel being 35 inches.

Table 3-123

CRITICAL DATA FOR SPHERICAL $U^{235}O_2F_2$ - D_2O CORES IN D_2O REFLECTOR

(Reproduced from Ref. 249)					
Sphere Diameter inches	Critical Volume liters	Critical Mass kg U^{235}	D/ U^{235} Atomic Ratio	U^{235} Density gm/cm ³	D_2O Reflector Thickness, inches
13.5	20.9	14.2	34.2	0.679	10.7
14.5	26.1	11.6	53.7	0.443	10.2
15.5	31.7	9.6	81.2	0.302	9.7
16.5	38.1	7.0	135.3	0.185	9.2
17.5	45.9	4.8	243	0.104	8.7
18.5	53.8	3.2	431	0.060	8.2

Four measurements were made in unreflected stainless steel cylinders of 1/8 inch thickness and then corrected for the effect of the glory hole. The critical parameters for corresponding spherical cores were then calculated. The experimental results, and the reduced data obtained by calculation are given in Table 3-124.

Table 3-124
CRITICAL DATA FOR BARE CYLINDRICAL U^{235} O_2F_2 - D_2 CORES
(CONTAINER 1/8 IN. STAINLESS STEEL)

D/ U^{235} Atomic Ratio	U^{235} Density gm/cm ³	(Reproduced from Ref. 249)		Critical Height cm	Critical Volume liters
		Cyl. inside Radius, cm	Axial Glory Hole		
230	0.1094	31.6	1-in. o.d.-s.s.	71.45	223.8
419	0.0610	31.6	1-in. o.d.-s.s.	78.74	246.6
856	0.0301	38.1	1-in. o.d.-s.s.	61.09	278.0
856	0.0301	38.1	1 1/8 in. o.d.-Al	60.83	276.8
2081	0.0124	38.1	1 1/8 in. o.d.-Al	84.75	385.8

Reduced Data					
D/ U^{235} Atomic Ratio	Cyl. critical radius, cm, glory hole removed	Cyl. critical volume, liters, glory hole removed	Critical volume of sphere in 1/8 in. s.s., ^a liters	Critical volume of ideal bare sphere, liters	Critical mass of bare sphere kg U^{235}
230	31.55	223.1	194.4	197.7	21.6
419	31.53	245.5	208.2	211.7	12.9
856	38.04	275.9	243.3	247.0	7.4
2081	38.04	384.6	335.4	340.1	4.2

^aAssumes equivalent stainless steel thickness of 0.2 cm, extrapolation distance of 2.5 cm (critical volume is increased 1.1 to 1.2% for extrapolation distance of 3.9 cm). There has been no correction for ~1 atomic per cent H^1 contamination of the D in these solutions.

It is estimated that errors of values of critical volume are less than $\pm 1\%$ for the existing conditions, and that errors of corresponding U^{235} concentrations or D/ U^{235} ratios are within $\pm 2\%$. For the unreflected systems, the standard solution and D_2O levels to which critical volumes were adjusted were selected to compensate roughly for the effect of the filling tube channel through the reflector and for asphericity of the outer reflector surface.

The critical systems were nonideal in two principal respects. First, neutron absorption by the stainless steel spheres in the reflected assemblies is estimated to increase critical volumes a maximum of ~2% above values for nonabsorbing containers. The second effect is that of light water contamination of the D_2O solutions. Other contaminants were minor and the reflector contamination by normal water is of slight significance. In the spherical cores, the hydrogen was probably between 99.6 and 99.8 atomic per cent D, and the main effect of the normal hydrogen is an increase in moderation. The correction to pure D_2O would require ~1% increase in critical volume at nominal D/ U^{235} . The large volume cylindrical solutions contained D_2O which had been incompletely protected against water vapor. Spot checks suggest that the normal hydrogen contamination averaged roughly 1 atomic per cent. In this case, the correction to pure D_2O , requires a 2-3% increase in critical volume at nominal D/ U^{235} .

Thompson(250)

A critical experiment for the Homogeneous Reactor Experiment reactor was performed in an 18-inch diameter stainless steel sphere of shell thickness 3/16 inch and 50 liter volume. The reflector was D_2O , 10 inches thick, and spherical in shape. The fuel solution was 93.4% enriched uranium in UO_2SO_4 aqueous solution at 30°C. The critical concentration was found to be 24.9 gm U^{235} /kg H_2O . The measurement was made with all rods fully withdrawn, except the regulating rod which was one-third withdrawn.

Dancoff, Fields, et al.(251)

Measurements in an exponential assembly led to the determination of critical masses of natural uranium contained in a uranyl-fluoride-heavy water solution. The assembly consisted of a sheet steel cylindrical tank which contained the fuel solution. It was located on top of the thermal column of CP-2.

Using cadmium covered foils, the relaxation distance $b_{0,1}$ was determined by activation measurements along the axis of the cylinder,

$$N(Z) = N(0) e^{-Z/b_{0,1}}$$

where Z is the axial coordinate and $N(Z)$ is the thermal neutron density at Z . End corrections were made to correct for the finite height of the assembly.

The total flux is given by

$$N(r, Z) = N(0) J_0 \left(\frac{2.4048}{R_{\text{eff}}} r \right) e^{-Z/b_{0,1}}$$

wherein only the first harmonic is used to describe the flux. Activation measurements were made at various radial distances r at heights Z and, with $b_{0,1}$ determined in the first measurements, R_{eff} is determined.

The bucklings for the assembly were then obtained from

$$B^2 = \frac{1}{b_{0,1}^2} - \frac{(2.4048)^2}{R_{\text{eff}}}$$

Using the buckling so determined as the buckling for an equivalent sphere, it is possible to deduce the critical radius of the sphere, and hence, the critical mass. Results are contained in Table 3-125.

Table 3-125

CRITICAL RADII AND QUANTITIES FOR HOMOGENEOUS REACTORS

Concentration Grams of UO_2F_2 cm ³	Critical Radius (cm)	Liters of D_2O $\cdot 10^3$	Isotopic Per Cent D_2O	kg of Uranium
.0238	452	358	99.83	6602
.0734	247	62.7	99.81	3583
.1484	208	37.3	99.81	4348
.2004	205	35.4	99.81	5598
.2493	209	37.1	99.79	7343
.2986	228	47.9	99.78	11438
.3476	244	52.2	99.77	14586

3.9.8 Critical Mass Studies of Quasi-Homogeneous U²³⁵ - Polyethylene SystemsBeck, Callihan, and Murray(252)

Cubes containing uranium enriched to approximately 35% U²³⁵ were interspersed among polyethylene blocks (essentially (CH₂)_n) in such a manner that criticality was achieved. This was done for a variety of H:U atomic ratios, reflector conditions, geometrical shapes, densities of U²³⁵, and degrees of homogeneity. Uranium bearing cubes are designated as U-cubes and the polyethylene blocks as H-cubes. The latter were made in two sizes. Characteristics of the cubes are given in Table 3-126. The lattice configurations used are represented in Fig. 3-122. Experiments were performed at room temperature.

Table 3-126

**CHARACTERISTICS OF URANIUM PLASTIC
CUBES (U CUBES)**

Dimensions	1.004 in.
Mass	79.53 gm
Total U content	51.56 gm
U ²³⁵ content	15.38 gm
Fluorine content	24.69 gm
Carbon content	2.64 gm
Oxygen content	0.08 gm
Impurity (chiefly Al)	0.56 gm

**CHARACTERISTICS OF POLYETHYLENE
BLOCKS (H CUBES)**

	"One Inch" Size	"One-half Inch" Size
Dimensions	1.005 in. cube	1.006 x 1.006 x 0.502 in.
Mass	15.11 gm	7.61 gm
Hydrogen content	2.09 gm	1.05 gm

Table 3-127 lists critical masses for various H:U ratios for the paraffin reflected assemblies and Table 3-128 for unreflected assemblies. The unreflected assemblies rest on a metallic platform.

The effect of "lumping" the U cubes was noted in a series of criticality measurements in which the ratio of H:U cubes is 1:1. Various degrees of "lumping" were achieved by using lattices B, L, K and J of Fig. 3-122. A similar set of measurements was performed for H:U cube ratios was 1:2 and 1:4. The results are given in Table 3-129.

A series of experiments was executed with the object of determining the effect of surrounding various assemblies with materials having different neutron absorbing and reflecting properties. The results for cores surrounded by boron, cadmium, paraffin and air (unreflected) are presented in Table 3-130.

Fig. 3-122
Lattice Configurations and Ratios of
H Cubes (White), U Cubes (Black), and Air Spaces (Blank)

Table 3-127

CRITICAL MASSES OF U^{235} POLYETHYLENE SYSTEMS (PARAFFIN REFLECTOR)

Experimental Conditions:
 Cubical Assembly
 Completely paraffin enclosed
 Lattice configurations refer
 to Figure 3-122

H-Cubes : U-Cubes	0:1	1:7	1:4	1:2	1:1	2:1	4:1	7:1
Lattice Configuration	A	H	S	C	B	E	G	I
Size of Assembly (Cube Units)**	17x16x16*	16x15x15*	15x14x14+	12x12x12-	10x10x10-	10x10x9-	11x11x11-	16x14x14+
Overall Density, gm/cm ³	4.8	4.3	4.0	3.5	2.8	2.2	1.7	1.4
H-Cubes	0	454	598	570	498	596	1032	2800
U-Cubes	4351***	3179*	2392	1140	498	298	258	400
Total Cubes	4351***	3633	2990	1710	996	894	1290	3200
Atoms H/atoms U^{235}	0	4.6	8	16	32	64	128	224
Critical Mass U, kg	-	215****	123.3	58.8	25.7	15.4	13.3	20.6
Critical Mass U^{235} , kg	125***	64****	36.8	17.5	7.7	4.6	4.0	6.2

*Not critical.

**These assembly dimensions are given in the number of cubes (H and U) along an edge. The actual dimensions, in inches, are slightly larger.

***This assembly was made of 1105 cubes each containing 48.76 gm U^{235} (95.3% assay) and 3246 cubes each containing 15.38 gm U^{235} (29.8% assay, which are described in this report) making a total of 223.9 kg of U and 103.8 kg U^{235} . This assembly was not critical and a very conservative estimate of the critical mass from multiplication measurements is 125 kg. The value is more probably near 160 kg.

****Extrapolated value from neutron multiplication is 49 kg U^{235} .

Table 3-128

CRITICAL MASSES OF U^{235} POLYETHYLENE SYSTEMS (UNREFLECTED)

Experimental conditions:
 Cubical assembly
 Air reflector
 Lattice Configurations
 refer to Figure 3-122

H-Cubes : U-Cubes	1:2	1:1	4:1
Lattice Configuration	C	B	G
Size of Assembly (Cube Units)	16x16x16+	14x13x13+	13x14x13-
Overall Density, gm/cm ³	3.5	2.8	1.7
H-Cubes	1405	1232	1856
U-Cubes	2810	1232	464
Total Cubes	4215	2464	2320
Atoms H/Atoms U^{235}	16	32	128
Critical Mass U, kg	144.9	63.5	23.9
Critical Mass U^{235} , kg	43.2	18.9	7.1

Table 3-129

EFFECT OF INHOMOGENEITY IN U^{235} POLYETHYLENE CRITICAL SYSTEMS

Cubical assembly.
Completely paraffin enclosed
Lattice Configuration refers
to Figure 3-122

Overall H:U cube ratio	1:1				1:2			1:4	
H to U^{235} Atomic Ratio	32				16			8	
Overall Density gm/cm ³	2.8				3.5			4.0	
Test Number	1/1	2/2	4/4	8/8	1/2 /1	1/2	2/4	1/2 /2	1/4
Lattice Configuration	B	L	K	J	F	C	R	D	S
Size of Assembly (cube units)	10x10x10-	10x10x10+	11x10x11-	11x11x12-	13x12x11+	12x12x12-	13x12x12+	15x15x14+	15x14x14+
H-Cubes	498	527	558	646	585	570	645	612	598
U-Cubes	498	527	558	646	1170	1140	1290	2450	2392
Total Cubes	996	1054	1116	1292	1755	1710	1935	3062	2990
Critical Mass U, kg	25.7	27.2	28.8	33.3	60.5	58.8	66.5	126.3	123.3
Critical Mass U^{235} , kg	7.7	8.1	8.6	9.9	18.1	17.5	19.8	37.7	36.8

Table 3-130

EFFECT OF REFLECTOR ON CRITICAL MASS

Cubical Assembly
Lattice Configuration Refers
to Figure 3-122

H-Cubes : U-Cubes	1:2			1:1				4:1		
Lattice Configuration	C			B				G		
H to U^{235} Atomic Ratio	16			32				128		
Reflector	Paraffin	Paraffin Outside cadmium 0.38 gm Cd/cm ²	Air (Untamped)	Paraffin	Paraffin outside cadmium 0.38 gm Cd/cm ²	Paraffin outside boron 0.46 gm B/cm ²	Air (Untamped)	Paraffin	Paraffin outside cadmium 0.38 gm Cd/cm ²	Air (Untamped)
Size of Assembly (cube units)	12x12x12-	15x15x15-	16x16x16+	10x10x10-	12x13x12+	13x13x13+	14x13x13+	11x11x11-	13x13x12-	13x14x13
H Cubes	570	1117	1405	498	965	1118	1232	1032	1600	1856
U Cubes	1140	2235	2810	498	965	1118	1232	258	400	464
Total Cubes	1710	3352	4215	996	1930	2236	2464	1290	2000	2320
Critical Mass U, kg	58.8	115.2	144.9	25.7	49.8	57.6	63.5	13.3	20.6	23.9
Critical Mass U^{235} , kg	17.5	34.4	43.2	7.7	14.8	17.2	18.9	4.0	6.2	7.1

Assemblies of H:U cubes in ratios of 1:2 (Table 3-131) and 2:1 (Table 3-132), completely enclosed in paraffin, were studied to determine the effect of a geometric change on critical mass. In each series of experiments the critical mass was determined with the material in cubical configuration. The critical masses were determined with the material in rectangular parallelepipeds having successively smaller square cross sectional areas, necessitating, of course, greater lengths, until a cross section was reached for which criticality apparently could not be achieved, regardless of extension in length.

Table 3-131

EFFECT OF GEOMETRY ON CRITICAL MASS

Experimental conditions:

1:2 ratio of H-cubes to U-cubes
 Lattice Configuration: C, Figure 3-122
 Atoms H/Atoms U²³⁵ = 16

Overall density = 3.5 gm/cm³
 Paraffin reflector

Size of Assembly (cube units)	12x12x12-	11x11x14+	10x10x19+	9x9x40-
H Cubes	570	578	648	1075
U Cubes	1140	1155	1295	2150
Total Cubes	1710	1733	1943	3225
Critical Mass U, kg	58.8	59.6	66.8	110.9
Critical Mass U ²³⁵ , kg	17.5	17.8	19.9	33.1

Table 3-132

EFFECT OF GEOMETRY ON CRITICAL MASS

Experimental conditions:

2:1 ratio of H-cubes to U-cubes
 Lattice Configuration: E, Figure 3-122
 Atoms H/Atoms U²³⁵ = 64

Overall density = 2.2 gm/cm³
 Paraffin reflector

Size of assembly (cube units)	10x10x9	9x9x11+	8x8x17-	8x7x26+	7x7x59*
H-cubes	596	604	712	976	1924
U-cubes	298	302	356	488	962
Total Cubes	894	906	1068	1464	2886
Critical Mass U, kg	15.4	15.6	18.4	25.2	(49.6)*
Critical Mass U ²³⁵ , kg	4.6	4.6	5.5	7.5	(14.8)*

*Not critical

A brief investigation was made of the effect of density on critical mass by stacking the H and U cubes in a cubical assembly with aluminum spacers between the cubes. These spacers were sections of aluminum tubing 7/8 in. O.D., 5/8 in. I.D., which were 1/2 in. or 1 in. long. The actual ratio of the H and U cubes in the assembly was not changed, but the interposition of aluminum spacers reduced the over-all stacking density of the assembly. Critical parameters are presented for a 4:1 ratio of H-cubes to U-cubes (Table 3-133) and for a 1:2 ratio (Table 3-134).

Table 3-133

EFFECT OF DENSITY ON CRITICAL MASS

Experimental conditions:

4:1 ratio of H-cubes to U-cubes Completely paraffin enclosed H to
Cubical assembly U²³⁵ atomic ratio = 128

Lattice configuration refers to Fig. 3-122

Volume Ratio of Materials H:U:A	4:1:0	4:1:1/2	4:1:1
Lattice Configuration	G	W	V
Size of Assembly (cube units)	11x11x11-	12x13x12	13x14x12-
Overall Density, gm/cm ³	1.7	1.6	1.4
Density, gm U ²³⁵ /cm ³	0.19	0.17	0.16
H-Cubes	1032	1364	1512
U-Cubes	258	341	378
Air Spaces (cube units)	0	170	378
Total "Cubes"	1290	1875	2268
Critical Mass U, kg	13.3	17.6	19.5
Critical Mass U ²³⁵ , kg	4.0	5.2	5.8

Snell(253)

Other critical mass measurements of assemblies of cubic shape reflected by 6 in. of paraffin were made at Oak Ridge National Laboratory. The fuel consisted of uranium, oxygen, carbon, fluorine and aluminum in the form of approximately 1 in. cubes. An array of 11 x 11 cubes measured 11 5/8 in x 11 5/8 in. The uranium was enriched 24% U²³⁵ and the atomic ratio of materials was U:O:C:F:Al = 1:2.83:5.3:3.6:0.4. By weight, the content was 55.8 U:10.6 O:15.0 C:16.1 F:2.5 Al. Additional 1 in. cubes of polyethylene were introduced into the assembly in order to vary the H:U ratio. The results are contained in Table 3-135.

Table 3-134

EFFECT OF DENSITY ON CRITICAL MASS

Experimental conditions:

1:2 ratio of H-cubes to U-cubes Completely paraffin enclosed H to
U²³⁵ atomic ratio = 16

Cubical assembly

Lattice configuration refers to Fig. 3-122

Volume Ratio of Materials, H:U:A	1:2:0	1:2:1/2	1:2:1
Lattice Configuration	C	U	T
Size of Assembly, cube units	12x12x12	14x14x13	15x15x16
Overall Density, gm/cm ³	3.5	3.0	2.7
Density, gm U ²³⁵ /cm ³	0.62	0.54	0.47
H-Cubes	570	755	903
U-Cubes	1140	1510	1807
Air spaces, cube units	0	377	904
Total "Cubes"	1710	2642	3614
Critical Mass U, kg	58.8	77.9	93.2
Critical Mass U ²³⁵ , kg	17.5	23.2	27.8

Table 3-135

CRITICAL MASSES OF FLUORINATED AND HYDROGENATED MIXTURES
CONTAINING URANIUM ENRICHED TO 24% U²³⁵

Array of Cubes	V*	H:U Atomic Ratio	Density of Core gm/cm ³	Critical Mass, kg		
				Measured (lattice)	Calculated Lattice	Homogeneous**
11 x 11 x 11.8	2	30.9	1.67	16.2	16.8	11.5
11 x 11 x 12.55	1	15.1	1.95	25.5	27.7	23.0
11 x 11 x 14.36	3:4	11.0	2.14	34.6	36.4	33.6

$$*V = \frac{\text{Number of Polyethylene Cubes}}{\text{Number of Fuel Bearing Cubes}}$$

**A correction was made for the inhomogeneity of the fuel distribution.

3.9.9 Comparisons of U²³³ and U²³⁵ Critical SystemsThomas, Fox, and Callihan(254)

In a series of experiments performed at the critical experiments facility at ORNL to determine some nuclear properties of U²³³, critical mass and buckling measurements were made over a range of temperatures, on comparable systems using both U²³³ and U²³⁵.

Two spherical core vessels having diameters of 26.4 cm and 32.0 cm were fabricated of 3S aluminum 1.27 mm thick. The vessels were mounted in cylinders of such dimensions as to provide 15 cm of natural water as an effectively infinite neutron reflector. The inside surfaces of the spheres were coated with plastic to prevent excessive corrosion. Immersion heaters placed along the periphery of the water reflector were used to effect temperature changes.

Critical mass measurements were made both for reflected and unreflected cores. Results for the latter case are listed in Table 3-136. Only in the former case was the temperature varied, as indicated in Table 3-137. Critical mass measurements for the unreflected case were made only with the 32 cm core and only at 72.0°C.

Table 3-136

CRITICAL CONDITIONS FOR UNREFLECTED
CORES IN ALUMINUM VESSELS

(Reproduced from Ref. 254) (27.0°C, Sphere 32.0 cm Diameter)		
U ²³³	gm U ²³³ /liter (at 25°C):	67.37
	H/U ²³³ atomic ratio:	381.0
	Critical mass U ²³³ :	1146.0 gm
U ²³⁵	gm U ²³⁵ /liter (at 25°C):	125.18
	H/U ²³⁵ atomic ratio:	203.5
	Critical mass U ²³⁵ :	2128.8 gm

The buckling, B^2 , was determined from the material rather than geometry, since the computation of the geometric buckling would involve the use of questionable extrapolation distances for the various temperatures. Using the equation

$$k_{\text{eff}} = \frac{\eta f}{(1 + \tau B^2)(1 + L_0^2 B^2)}$$

the U²³⁵ buckling was calculated. The results are presented in Table 3-138, together with values of L_0^2 , the thermal diffusion length in water, corrected for density changes only. The square of the fast slowing down length, τ , is taken as 27 cm². $\eta^{U^{235}}$ is taken to be 2.08 and the cross sections used in the calculation of the thermal utilization, f , are given in Table 3-139.

Table 3-137

CRITICAL CONDITIONS FOR WATER REFLECTED SPHERICAL CORES IN ALUMINUM VESSELS

(Reproduced from Ref. 254)

Fuel Concentration	Atom Ratio	Temperature	Volume	Mass of X
($\frac{\text{gm X}^a}{\text{liter}}$ (at 250C))	H/X	°C	liters	gm
UO ₂ F ₂ -H ₂ O solutions, 90% U ²³⁵				
26.4-cm diameter sphere				
61.34	418.3	32.0 ^b	9.666	591.0
61.89	414.6	39.5	9.675	596.3
64.05	400.5	65.5	9.704	611.0
65.84	389.6	83.2	9.723	623.0
67.80	378.1	96.5	9.737	638.0
32.0-cm diameter sphere ^c				
38.75	663.1	26.3	17.020	659.0
39.97	643.1	56.0	17.042	673.0
42.65	602.8	99.5	17.074	703.0
UO ₂ F ₂ - H ₂ O Solutions, 98.7% U ²³⁵				
26.4-cm diameter sphere				
95.14	268.8	27.5	9.661	918.3
97.10	263.3	39.5	9.675	935.3
104.09	245.4	74.0	9.713	989.8
106.75	239.3	85.5	9.726	1010.1
32.0-cm diameter sphere ^c				
50.29	515.1	27.0 ^b	17.020	855.8
51.50 ^d	502.6	43.0	17.032	872.0
51.44 ^d	503.4	43.0	17.032	870.8
52.13	496.5	54.0	17.042	879.2
53.07	487.6	64.5	17.049	890.3
54.27 ^d	476.8	87.5	17.065	899.6
56.28 ^d	459.6	87.5	17.065	932.0

^aX = U²³³ or U²³⁵

^bIn these experiments the vessel was critical with the order of 0.1% of its capacity unfilled and at a temperature 5°C lower than this value. Figure 1 and the capacity of the sphere calibrated over the temperature range were used to obtain this temperature at which the sphere would have been full and critical.

^cThe values of the concentrations and masses measured with this sphere are believed to be about 2% high because of a systematic error.

^dDuplicate samples.

Table 3-138

ATOMIC RATIOS AND MATERIAL BUCKLINGS
REQUIRED FOR CRITICALITY

Temperature °C	L_0^2 cm ²	26.4-cm Sphere			32.0-cm Sphere		
		H/U ²³³	H/U ²³⁵	B ² cm ⁻²	H/U ²³³	H/U ²³⁵	B ² cm ⁻²
25	8.1225	421.2	269.8	29.1 x 10 ³	663.8	516.0	22.2 x 10 ³
40	8.1998	414.5	263.0	29.2	655.2	506.8	22.4
60	8.3482	403.7	253.0	29.6	639.8	491.4	22.6
80	8.5337	390.7	242.3	29.7	620.7	473.0	22.9
100	8.7347	375.6	229.9	30.1	599.7	454.0	23.3

Table 3-139

THERMAL CROSS SECTIONS USED
IN CALCULATION OF f

Temperature °C	Energy ev	$\sigma_a^{(H)}$ barns	$\sigma_a^{U^{233}}$ barns	$\sigma_a^{U^{235}}$ barns
25	0.0257	0.328	575	662
40	0.0270	0.322	560	646
60	0.0287	0.312	545	627
80	0.0304	0.304	524	602
100	0.0321	0.297	512	587

3.9.10 References to Other Critical Mass Data

The following references contain critical mass data for a variety of geometric arrays of assemblies of homogeneous reactors. These data are particularly pertinent to hazards problems encountered in storage, handling, shipping, etc.

Fuel	Containing Vessel	Geometry	References
UO ₂ F ₂ + H ₂ O 93.2% U ²³⁵	Al, SS Cylinders with and without Cd Wrapping	Various Arrays Involving from 1 to 7 Vessels Includes Intersecting Pipes of "x" and "y" Geometry. Water or Air Reflectors	255
UO ₃ + Glycerol tri-stearate (serves as hydrogenous moderator) 5% U ²³⁵	Al Rectangular Parallelepiped	Annular Geometry with Air, Water, and/or Aluminum Separators. Water or Air Reflectors.	256
UO ₂ F ₂ + H ₂ O 93.4% U ²³⁵	5 Pipes of SS. I.D. 4.026". Length 6' Cd Wrapping.	Axes of Pipes Lie in a Plane. Center Line to Center Line Spacing 15".	257
UO ₂ F ₂ + H ₂ O 93.4% U ²³⁵	Al, SS	Pairs of Cylinders with Parallel Axes. Water or Air Reflectors.	258
UO ₂ F ₂ + H ₂ O 93.2% U ²³⁵	Al Slabs and Cylinders	Various Arrays of Slabs and Cylinders. Includes Slabs in T and L Geometry.	241

A report by Henry, Knight, and Newlon⁽²⁵⁹⁾ presents methods of calculating criticality for several vessels containing fissionable material when the vessels are arranged in different geometrical configurations. Calculations are checked with measurements.

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SECTION 4

LATTICE CONSTANTS FOR THERMAL HETEROGENEOUS SYSTEMS

4.1 THERMAL UTILIZATION

The thermal utilization, f , is the rate at which thermal neutrons are absorbed in fissile material, divided by the total rate at which thermal neutrons are absorbed. The main problem in computing thermal utilization in heterogeneous reactors arises from the non-uniform spatial distribution of the thermal flux. The thermal flux is depressed in a heavily absorbing region.

Although the term "thermal utilization" is used, the formulae presented apply to any single energy group, provided appropriate constants are used.

The thermal utilization of a lattice is usually obtained through consideration of a unit cell. Macroscopic diffusion effects are ignored so that zero current at the cell boundary is assumed. The thermal utilization of a unit cell consisting of a fuel (U) and moderator (M) region is given by:

$$f = \frac{\bar{\Phi}_U \Sigma_{a,U} V_U}{\bar{\Phi}_U \Sigma_{a,U} V_U + \bar{\Phi}_M \Sigma_{a,M} V_M} = \frac{\Sigma_{a,U} V_U}{\Sigma_{a,U} V_U + (\bar{\Phi}_M / \bar{\Phi}_U) \Sigma_{a,M} V_M}$$

where $\Sigma_{a,U}$ and $\Sigma_{a,M}$ are the macroscopic absorption cross sections; V_U and V_M are the volumes; and $\bar{\Phi}_U$ and $\bar{\Phi}_M$ are the average thermal fluxes in the fuel and moderator, respectively. The quantity $\bar{\Phi}_M / \bar{\Phi}_U$ is referred to as the disadvantage factor in the following discussion.

Various calculation methods may be applied in order to obtain the average fluxes. Usually the following two approximations are made: one, the unit cell has simple geometry, e.g., a square array of cylindrical rods would be considered to have a unit cell of cylindrically radial symmetry; two, the source for thermal neutrons in any region of the cell is constant within that region.

4.1.1 Diffusion Theory(1)

For the case of a cell containing a fuel region with zero source, surrounded by a moderator region with constant source, diffusion theory gives:

$$\frac{1}{f} = 1 + \frac{V_M \Sigma_{a,M}}{V_U \Sigma_{a,U}} G + (E-1)$$

If there is also a constant source in the fuel region the expression for f takes the form

$$\frac{1}{f} = 1 + \left\{ \frac{\left[1 + \frac{S_U V_U}{S_M V_M} \right] \left[E + G \frac{V_M \Sigma_{a,M}}{V_U \Sigma_{a,U}} \right]}{1 + \frac{S_U V_U}{S_M V_M} \left[E + (G - 1) \frac{V_M \Sigma_{a,M}}{V_U \Sigma_{a,U}} \right]} - 1 \right\}$$

where S_U and S_M are the relative source strengths in the fuel and moderator regions, respectively.

The expressions for G and E in slab, cylindrical, and spherical geometry are given in Table 4-1. In the case of no source in the fuel region, G is the ratio of the flux at the surface of the fuel region to the average flux in the fuel.

Table 4-1

DIFFUSION THEORY CALCULATIONS OF f

Cell Geometry	G	E^*
Infinite Slab	$\kappa_U t_U \coth \kappa_U t_U$	$\kappa_M t_M \coth \kappa_M t_M$
Infinite Cylinder	$\frac{\kappa_U r_U}{2} \frac{I_0(\kappa_U r_U)}{I_1(\kappa_U r_U)}$	$\frac{\kappa_M (r_M^2 - r_U^2)}{2r_U} \left[\frac{I_0(\kappa_M r_U) K_1(\kappa_M r_M) + K_0(\kappa_M r_U) I_1(\kappa_M r_M)}{I_1(\kappa_M r_M) K_1(\kappa_M r_U) - K_1(\kappa_M r_M) I_1(\kappa_M r_U)} \right]$
Sphere	$\frac{\kappa_U r_U^2}{3} \frac{\tanh \kappa_U r_U}{\kappa_U r_U - \tanh \kappa_U r_U}$	$\frac{\kappa_M (r_M^3 - r_U^3)}{3r_U} \left[\frac{1 - \kappa_M r_M \coth \kappa_M (r_M - r_U)}{1 - \kappa_M^2 r_M r_U - \kappa_M (r_M - r_U) \coth \kappa_M (r_M - r_U)} \right]$

where: t_U = half-thickness of fuel

t_M = half-thickness of moderator

r_U = radius of fuel region

r_M = outer radius of moderator = radius of cell

$\kappa^2 = \sigma_a D$ (D is the diffusion coefficient)

*For cylinders, if $\kappa_M r_M \ll 1$, then: $E - 1 \approx \frac{(\kappa_M r_M)^2}{2} \left\{ \frac{1}{1 - \left(\frac{r_U}{r_M}\right)^2} \ln \left(\frac{r_M}{r_U}\right) - \frac{3}{4} + \frac{1}{4} \left(\frac{r_U}{r_M}\right)^2 \right\}$

4.1.2 Spherical Harmonic Method

The Spherical Harmonic Method may be used to obtain results more accurate than those given by diffusion theory. In plane geometry, with isotropic scattering and constant isotropic source, the one-velocity Boltzman Equation takes the form:

$$\mu \frac{d\psi(x, \mu)}{dx} + \frac{\psi(x, \mu)}{\ell} = \frac{c}{2\ell} \int_{-1}^1 \psi(x, \mu') d\mu' + \frac{Q}{2} \quad (4-1)$$

where

μ = cosine of the angle between the neutron velocity vector and the x axis

$\psi(x, \mu)$ = angular neutron flux per unit μ interval (integrated over the azimuthal angle)

ℓ = total mean free path in the medium

$c = \frac{\Sigma_s}{\Sigma_s + \Sigma_c}$ = mean number of secondaries per collision

Q = total source strength per unit volume per second.

$\psi(x, \mu)$ may be expanded as:

$$\psi(x, \mu) = \sum_{n=0}^{\infty} \psi_n(x) \frac{2n+1}{2} P_n(\mu) \quad (4-2)$$

where the $P_n(\mu)$ are the Legendre Polynomials; $\psi_0(x)$ is the total neutron flux; and $\psi_1(x)$ the total neutron current.

Substitution of Eq. (4-2) into Eq. (4-1) yields the set of equations:

$$(n+1) \frac{d\psi_{n+1}(x)}{dx} + n \frac{d\psi_{n-1}(x)}{dx} + \frac{(2n+1)}{\ell} \psi_n(x) = \left[\frac{c \psi_0(x)}{\ell} + Q \right] \delta_{0,n}$$

where $\delta_{0,n}$ is the Kronecker delta function

$$\begin{aligned} \delta_{0,n} &= 1 \text{ if } n = 0 \\ &= 0 \text{ otherwise.} \end{aligned}$$

It is only necessary to consider the homogeneous system of equations ($Q=0$), since for each region the general solution can be obtained by adding to the solution of the homogeneous system, the particular solution

$$\psi_n(x) = \frac{\ell Q}{1-c} \delta_{0,n}$$

The P_n approximation is obtained by retaining only the first $N+1$ equations and setting

$$\frac{d\psi_{N+1}(x)}{dx} = 0$$

in the last of these equations. A solution of the form:

$$\psi_j(x) = \psi_j^0 \exp(\nu x / \ell)$$

is assumed, and a determinantal equation for ν results from the condition that the determinant of the coefficients must vanish.

The roots of the equation in the P_n approximation are the roots of

$$G_{N+1}(\nu) = 0$$

where

$$G_0(\nu) = 1$$

$$G_1(\nu) = \frac{c-1}{\nu}$$

and $G_m(\nu)$ satisfies the recursion formula

$$(m+1) G_{m+1}(\nu) + \frac{2m+1}{\nu} G_m(\nu) + m G_{m-1}(\nu) = 0$$

In general, the odd-order approximations (N odd) are used.⁽²⁾ The roots of ν are then real, and occur in pairs with opposite sign. Designating the positive roots ν_i , the general solution is given by

$$\psi_n(x) = \sum_{i=1}^{\frac{N+1}{2}} G_n(\nu_i) \left[(-1)^j A_i \exp(-\nu_i x/l) + B_i \exp(\nu_i x/l) \right]$$

where A_i and B_i are arbitrary constants.

Values of ν_i and $G_n(\nu_i)$ for various odd P_n approximations are listed in Tables 4-2 and 4-3, respectively.

Table 4-2

VALUES FOR ν_i FOR $N = 1, 3, 5, 7$
(Reproduced from Ref. 2)

Reproduced from Ref. 27											
N	1		3		5			7			
i	1		1		2		3			4	
c	1	1	2	1	2	3	1	2	3	4	
1.0	0	0	1.972027	0	1.225211	3.202945	0	1.103185	1.591779	4.458086	
0.9	0.547723	0.525560	2.055185	0.525430	1.240875	3.296688	0.525430	1.108937	1.615640	4.554851	
0.8	0.774597	0.711876	2.145773	0.710456	1.260956	3.393117	0.710413	1.116799	1.642629	4.652966	
0.7	0.948683	0.834508	2.241834	0.829085	1.286086	3.491504	0.828671	1.127655	1.672473	4.752078	
0.6	1.095445	0.922637	2.341383	0.909562	1.316073	3.591190	0.907696	1.142345	1.704602	4.851871	
0.5	1.224745	0.988751	2.442706	0.964672	1.349619	3.691603	0.959481	1.160900	1.738235	4.952060	
0.4	1.341641	1.039801	2.544478	1.002603	1.384742	3.792265	0.992327	1.181880	1.772515	5.052401	
0.3	1.449138	1.080124	2.645751	1.029086	1.419574	3.892788	1.012964	1.203058	1.806656	5.152683	
0.2	1.549193	1.112590	2.745892	1.048012	1.452833	3.992862	1.026230	1.222733	1.840027	5.252727	
0.1	1.643168	1.139170	2.844500	1.061917	1.483848	4.092252	1.035120	1.240155	1.872179	5.352384	
0	1.732051	1.161256	2.941340	1.072421	1.512380	4.190778	1.041352	1.255230	1.902832	5.451533	

(Reproduced from Ref. 2)

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For a unit cell containing two regions (denoted $\alpha = 1, 2$) with half-thicknesses t_α , mean free paths ℓ_α , secondaries per collision c_α , and sources Q_α , the solution for the flux in each region is given by

$$\psi_{\alpha,m}(x_\alpha) = \frac{Q_\alpha}{1-c} \delta_{0,m} + \sum_{i=1}^{N+1} A_{\alpha,i} G_m(\nu_{\alpha,i}) \cosh \nu_{\alpha,i} \frac{x_\alpha}{\ell_\alpha} \quad m \text{ even}$$

$$\psi_{\alpha,m}(x_\alpha) = \sum_{i=1}^{N+1} A_{\alpha,i} G_m(\nu_{\alpha,i}) \sinh \nu_{\alpha,i} \frac{x_\alpha}{\ell_\alpha} \quad m \text{ odd}$$

$$m = 0, 1, \dots, N$$

where x is measured from the midplane of each region, and the boundary condition of symmetry at the midplanes is used. Continuity of $\psi_m(x)$ at the interface may be used to determine the $A_{\alpha,i}$.

$$\sum_{i=1}^{N+1} A_{1,i} G_j(\nu_{1,i}) \cosh \frac{\nu_{1,i} t_1}{\ell_1} - \sum_{i=1}^{N+1} A_{2,i} G_j(\nu_{2,i}) \cosh \frac{\nu_{2,i} t_2}{\ell_2} = \left(\frac{Q_2}{1-c_2} - \frac{Q_1}{1-c_1} \right) \delta_{0,m} \quad m \text{ even}$$

$$\sum_{i=1}^{N+1} A_{1,i} G_j(\nu_{1,i}) \sinh \frac{\nu_{1,i} t_1}{\ell_1} + \sum_{i=1}^{N+1} A_{2,i} G_j(\nu_{2,i}) \sinh \frac{\nu_{2,i} t_2}{\ell_2} = 0 \quad m \text{ odd}$$

$$m = 1, \dots, N$$

The values ψ_n can now be determined; in particular ψ_0 (the total neutron flux), and hence, the thermal utilization and the disadvantage factor. Tables 4-4 and 4-5 give results for P_3 and P_5 calculations on thick and thin slabs, respectively, along with an indication of the rate of convergence of the method. A constant source was assumed in the non-capturing moderator and a zero source in the fuel.

The Spherical Harmonic Method may be easily generalized to include anisotropic scattering. The extension to spherical geometry is also relatively easy but the extension to cylindrical geometry⁽³⁾ is considerably more difficult.

Yvon's Method⁽⁴⁾

Sometimes called the double P_n method, this approximation is similar to the spherical harmonics method outlined above. It differs in that the angular flux is described by two functions, one for $\mu < 0$, one for $\mu > 0$, so that it is well suited to deal with problems involving thin regions and it can be expected to give better approximations at the interface than the full range approximation.

Starting with Eq. (4-1), define the solutions:

$$\begin{aligned} \psi^+ &= \psi & 0 \leq \mu \leq 1 & & \psi^- &= \psi & -1 \leq \mu \leq 0 \\ &= 0 & -1 \leq \mu \leq 0 & & &= 0 & 0 \leq \mu \leq 1. \end{aligned}$$

Table 4-4

FLUX DISADVANTAGE FACTOR
 $\bar{\Phi}_M/\bar{\Phi}_U$, FOR SLAB LATTICES*

(Thick Slabs)

1 - c ₁	t _U /ℓ _U	t _M /ℓ _M	$\bar{\Phi}_M/\bar{\Phi}_U$	
			P ₃	P ₅
0.10	2	2	1.2784	1.2897
		5	1.4200	1.4250
		10	1.6645	1.6681
0.50	2	2	2.3331	2.3905
		5	3.0499	3.0757
		10	4.2754	4.2946
0.90	2	2	3.3206	3.4268
		5	4.6226	4.6723
		10	6.8326	6.8701
0.10	5	2	1.9776	1.9926
		5	2.3045	2.3094
		10	2.9060	2.9091
0.25	5	2	3.2470	3.2919
		5	4.0816	4.0987
		10	5.5913	5.6030
0.50	5	2	5.0944	5.2015
		5	6.8010	6.8471
		10	9.8328	9.8668

*J. Chernick, "Calculation Methods for Heterogeneous Systems," Chp. IV, Addison-Wesley Handbook of Nuclear Engineering, to be published.

Table 4-5

FLUX DISADVANTAGE FACTOR
 $\bar{\Phi}_M/\bar{\Phi}_U$, FOR SLAB LATTICES*

(Thin Slabs)

1 - c ₁	t _U /ℓ _U	t _M /ℓ _M	$\bar{\Phi}_M/\bar{\Phi}_U$	
			P ₃	P ₅
0.10	0.25	0.50	1.0093	1.0133
			1.0233	1.0332
			1.0465	1.0664
0.10	0.25	1.00	1.0150	1.0201
			1.0374	1.0503
			1.0748	1.1005
0.10	0.50	0.50	1.0243	1.0334
			1.0606	1.0833
			1.1209	1.1659
0.10	0.50	1.00	1.0348	1.0445
			1.0869	1.1112
			1.1735	1.2217
0.10	1.00	0.50	1.0692	1.0885
			1.1717	1.2191
			1.3386	1.4312
0.10	1.00	1.00	1.0870	1.1032
			1.2162	1.2563
			1.4282	1.5070

*J. Chernick, "Calculation Methods for Heterogeneous Systems," Chp. IV, Addison-Wesley Handbook of Nuclear Engineering, to be published.

Then the expansion of $\psi(x, \mu)$ is

$$\psi^\pm(x, \mu) = \sum_{n=0}^{\infty} (2n+1) B_n^\pm(x) P_n^\pm(\mu),$$

$$P_n^+(\mu) = P_n(2\mu-1) \quad 0 \leq \mu \leq 1$$

$$= 0 \quad -1 \leq \mu \leq 0$$

$$P_n^-(\mu) = P_n(2\mu+1) \quad -1 \leq \mu \leq 0$$

$$= 0 \quad 0 \leq \mu \leq 1,$$

where $P_n(2\mu \pm 1)$ is the full range Legendre polynomial of argument $(2\mu \pm 1)$. It follows from these definitions that

$$\int_0^1 P_n^+(\mu) P_m^+(\mu) d\mu = \int_{-1}^0 P_n^-(\mu) P_m^-(\mu) d\mu = \frac{\delta_{n,m}}{2n+1}$$

Moreover,

$$2(2n+1) \mu P_n^{\pm}(\mu) = (n+1) P_{n+1}^{\pm}(\mu) \pm (2n+1) P_n^{\pm}(\mu) + n P_{n-1}^{\pm}(\mu).$$

The total flux, $\psi_0(x)$, is given by

$$\psi_0(x) = \int_{-1}^1 \psi(x, \mu) d\mu = B_0^+(x) + B_0^-(x),$$

and the total current, $\psi_1(x)$, by

$$\psi_1(x) = \int_{-1}^1 \mu \psi(x, \mu) d\mu = \frac{B_1^+(x) + B_1^-(x)}{2} + \frac{B_0^+(x) - B_0^-(x)}{2}$$

Making use of the orthogonality relation of the $P_n^{\pm}$ one obtains,

$$\begin{aligned} \frac{d}{dx} \left\{ n B_{n-1}^{\pm} \pm (2n+1) B_n^{\pm} + (n+1) B_{n+1}^{\pm} \right\} + \frac{2(2n+1)}{\ell} B_n^{\pm} \\ = \frac{c}{\ell} (B_0^+ + B_0^-) \delta_{n,0} + 2(2n+1) \int_{\pm} S(x, \mu) P_n^{\pm}(\mu) d\mu \end{aligned}$$

where the limits on the integral denoted by + are 0 to 1, and those denoted by - are -1 to 0.

As before, we need consider only the case $S = 0$.

Then, for $n = 0$,

$$\frac{d}{dx} \left\{ \pm B_0^{\pm} + B_1^{\pm} \right\} + \frac{2-c}{\ell} B_0^{\pm} - \frac{c}{\ell} B_0^{\mp} = 0,$$

and for $n > 0$,

$$\frac{d}{dx} \left\{ n B_{n-1}^{\pm} \pm (2n+1) B_n^{\pm} + (n+1) B_{n+1}^{\pm} \right\} + \frac{2(2n+1)}{\ell} B_n^{\pm} = 0.$$

If we insert the most general solution,

$$B_n^{\pm}(x) = \sum_i A_{n,i}^{\pm} e^{\alpha_i(x/\ell)},$$

we obtain,

$$\alpha_i \left\{ \pm A_{0,i}^{\pm} + A_{1,i}^{\pm} \right\} + (2-c) A_{0,i}^{\pm} - c A_{0,i}^{\mp} = 0$$

and

$$\alpha_i \left\{ n A_{n-1,i}^{\pm} \pm (2n+1) A_{n,i}^{\pm} + (n+1) A_{n+1,i}^{\pm} \right\} + 2(2n+1) A_{n,i}^{\pm} = 0.$$

From these equations we can determine

$$G_n^{\pm}(\alpha) = -\frac{1}{\alpha n} \frac{A_n^{\pm}}{A_{n-1}^{\pm}}$$

as

$$G_n^{\pm}(\alpha) = \frac{1}{(2n+1)(2 \pm \alpha) - \alpha^2(n+1)^2} \frac{(2n+3)(2 \pm \alpha) - \alpha^2(n+2)^2}{(2n+5)(2 \pm \alpha) - \alpha^2(n+3)^2}$$

and

$$A_{0,i}^{\pm} \left\{ 2 - c \pm \alpha_i - \alpha_i^2 G_1^{\pm}(\alpha_i) \right\} - c A_{0,i}^{\pm} = 0.$$

The eigenvalue equation for x_i is then

$$\left\{ 2 - c + \alpha - \alpha^2 G_1^+(\alpha) \right\} \left\{ 2 - c - \alpha - \alpha^2 G_1^-(\alpha) \right\} - c^2 = 0.$$

Also,

$$A_{0,i}^- = A_{0,i}^+ \frac{1}{c} \left\{ 2 - c + \alpha - \alpha^2 G_1^+(\alpha) \right\}.$$

The N^{th} order approximation is now made by requiring that all $A_n^{\pm} = 0$ for $n > N$.

In the $P_0^{\pm}$ approximation

$$\alpha_1 = 2(1-c)^{1/2}.$$

In the P_1 approximation

$$\alpha_1 = \sqrt{6} [2 - c + \sqrt{3 - 3c + c^2}]^{1/2}$$

$$\alpha_2 = \sqrt{6} [2 - c - \sqrt{3 - 3c + c^2}]^{1/2}$$

In the $P_2^{\pm}$ approximation the eigenvalues are the solutions of:

$$\alpha^6 - 12\alpha^4(7 - 2c) + 20\alpha^2(21 - \frac{43}{3}c) - 400(1 - c) = 0.$$

Further details may be obtained from Ref. 4.

4.1.3 Analytic Approximation to P_3 Method in Slab Geometry with Linearly Anisotropic Scattering⁽³⁾

The case considered is a two-region slab cell consisting of a thin fuel region and a relatively thick moderator region. It is assumed that the neutron source is zero in the fuel region, and is constant and isotropic in the moderator region. Scattering anisotropy is taken into account by including a linear anisotropic term, $P_1(\mu)$. The only space variable is the direction perpendicular to the slab plane, x ; angular dependence of the neutron velocity vector is given in terms of μ , the cosine of the angle between the neutron velocity vector and the x axis.

The following notation and symbols are used in the analysis:

V = half-thickness of region

Σ_s = macroscopic scattering cross section

Σ_a = macroscopic absorption cross section

$\Sigma_t = \Sigma_s + \Sigma_a$ = total macroscopic cross section

$$a = 1 - \frac{\Sigma_s}{\Sigma_t} = \frac{\Sigma_a}{\Sigma_t}$$

$$b = A \left(1 - \bar{\mu} \frac{\Sigma_s}{\Sigma_t} \right)$$

$\bar{\mu}$ = average cosine of scattering angle

$$\nu_1^2 = + \left(\frac{3}{2} b + \frac{14}{9} a + \frac{35}{18} \right) - \sqrt{\left(\frac{3}{2} b + \frac{14}{9} a + \frac{35}{18} \right)^2 - \frac{35}{3} b}$$

$$\nu_2^2 = + \left(\frac{3}{2} b + \frac{14}{9} a + \frac{35}{18} \right) + \sqrt{\left(\frac{3}{2} b + \frac{14}{9} a + \frac{35}{18} \right)^2 - \frac{35}{3} b}$$

$$\kappa_1 = \nu_1 \Sigma_t$$

$$\kappa_2 = \nu_2 \Sigma_t$$

$$Y = \Sigma_t V$$

All of the above symbols carry an additional subscript; U for the fuel region and M for the moderator region.

The result of the analysis for the ratio of the average fluxes in the fuel and moderator regions is given in the form:

$$\frac{\bar{\Phi}_M}{\bar{\Phi}_U} = G + (E - 1) \frac{V_U \Sigma_{a,U}}{V_M \Sigma_{a,M}} \quad (4-3)$$

If V_U is small

$$G = 1 + \frac{1}{3} \xi Y_U^2 + \dots \quad (4-4)$$

where

$$\xi = \frac{3b_U(\nu_{U,1}^2 + \nu_{U,2}^2) - \nu_{U,1}^2 \nu_{U,2}^2 + \left(\frac{1}{\nu_{U,2}} 3b_U + \frac{2}{5} \frac{a_U}{Y_M} \right) \nu_{U,1}^2 \nu_{U,2}^2 Y_U}{3b_U + \frac{1}{\nu_{M,2}} \nu_{U,1}^2 \nu_{U,2}^2 Y_U}$$

In diffusion theory (P_1 approximation) $\xi = 3b_U$. The limits of validity of Eq. (4-4) are $Y_U < 1/2$, $V_U/\Sigma_{t,U} < 1$.

In Eq. (4-3)

$$E - 1 = \left(\frac{1 - t_{M,1}}{t_{M,1}} \right) \left(1 + \psi_U \psi_M \right)$$

where

$$t_{k,i} = \frac{\tanh \kappa_{k,i} V_k}{\kappa_{k,i} V_k}$$

$$k = U, M; i = 1, 2$$

In the P_1 approximation

$$E - 1 = \frac{1 - t_{M,1}}{t_{M,1}}$$

Under the conditions of validity of Eq. (4-4), ψ_U is well approximated by:

$$\psi_U = \left(\frac{1}{a_U} \right) \left[\frac{(3b_U - \nu_{U,1}^2) (\nu_{U,2}^2 - 3b_U)}{3b_U + \frac{1}{\nu_{M,2}} \nu_{U,1} \nu_{U,2} Y_U} \right]$$

There are four conditions on $\nu_{M,2} Y_M$, $\nu_{M,1} Y_M$ in which different approximations for ψ_M hold:

$$(1) \quad \nu_{M,2} Y_M \ll 1; \quad \psi_M = \frac{1}{\sqrt{35}} \frac{a_M}{b_M} \nu_{M,2} Y_M$$

$$(2) \quad \nu_{M,2} Y_M \leq 3; \quad \nu_{M,1} Y_M \ll 1; \quad \psi_M = \frac{3}{\sqrt{35}} \frac{a_M}{b_M} (1 - t_{M,2}) \frac{1}{Y_M}$$

$$(3) \quad \nu_{M,2} Y_M \leq 3; \quad \nu_{M,1} Y_M < 1;$$

$$\psi_M = \frac{3}{\sqrt{35}} \frac{a_M}{b_M} \left(1 - \frac{27}{35} b_M \right) \left(1 + \frac{1}{15} \nu_{U,2}^2 Y_M^2 - \frac{1}{\nu_{M,2} Y_M} \right) \frac{1}{Y_M}$$

$$(4) \quad \text{Infinite Moderator } (Y_M \rightarrow \infty) \quad \psi_M = \sqrt{\frac{3}{35}} \frac{a_M}{\sqrt{b_M}}$$

A comparison of the results of this approximation with the full P_3 method is shown in Table 4-6.

Table 4-6
COMPARISON OF ANALYTIC APPROXIMATION
AND P_3 METHOD

4.1.4 Integral Transport Theory in
Cylindrical Geometry^(5,6)

	Fuel	Moderator
Σ_s	0.3924 cm ⁻¹	0.4572 cm ⁻¹
Σ_a	0.56504 cm ⁻¹	0.0002852 cm ⁻¹
$\bar{\mu}$	0.95744	0.457525
V	0.2972 cm	4.000 cm
		G
		E-1
Approximate method of Copic ⁽³⁾		1.08141
P_3 approximation		1.07973
		0.39266
		0.3950

The thermal utilization in cylindrical geometry may be approximated by the use of integral transport theory. Let the region contain a rod of fissionable or absorbing material (source = 0) of radius a , which is followed by a cooling gap of width $c - a$ (which may be 0), and finally an outer annular region of moderator (constant source), the radius of the outer boundary of the cell being denoted by b . The thermal utilization, f , is given by

$$\frac{1}{f} - 1 = \frac{b^2 - c^2}{a^2} \frac{\Sigma_{Mc}}{\Sigma_{Uc}} \left[G + a \Sigma_{Uc} \left(1 - \frac{a}{c} \right) \right] + \kappa_M^2 b^2 C + \frac{b^2 - c^2}{c} \Sigma_{Mc} \left(\frac{3}{2} \lambda - 1 \right)$$

where

Σ_{Mt} = total cross section of moderator

Σ_{Mc} = capture cross section of moderator

Σ_{Uc} = capture cross section of fissionable material

Σ_{Ut} = total cross section of fissionable material

Σ_{Us} = scattering cross section of fissionable material

$\kappa_M^2 = 3\Sigma_{Mt}\Sigma_{Mc}$

$$C = \frac{1}{2} \left[\frac{b^2/c^2}{(b^2/c^2)-1} \ln \left(\frac{b}{c} \right) - (3/4) + \frac{1}{4b^2/c^2} \right]$$

$$G = \frac{\Phi(a)}{\Phi_U}, \text{ where } \Phi_U \text{ is the average flux in the uranium.}$$

Three approximations to G in order of increasing accuracy are:

$$G = 1 + \frac{\Sigma_{Uc}}{\Sigma_{Ut}} B \quad (4-5)$$

$$G = 1 + (1 - \tau) \frac{1 - \sqrt{1 - 2\tau(1-P_c)B}}{\tau(1-P_c)} \quad (4-6)$$

$$G = 1 + \frac{\Sigma_{Uc}}{\Sigma_{Ut}} A \left[1 + \alpha \frac{\Sigma_{Us}}{\Sigma_{Ut}} + \beta \left(\frac{\Sigma_{Us}}{\Sigma_{Ut}} \right)^2 \right] \quad (4-7)$$

where

P_c is the collision probability tabulated in Ref. 7

A is defined by $P_c = \frac{A + \eta}{1 + A + \eta}$

$\eta = a\Sigma_{Ut}$

$\tau = \Sigma_{Us}/\Sigma_{Ut}$

$$B = \frac{1}{1-P_c} \left\{ (a\Sigma_{Ut})^2 \left[I_0(a\Sigma_{Ut}) K_0(a\Sigma_{Ut}) + I_1(a\Sigma_{Ut}) K_1(a\Sigma_{Ut}) \right] + a\Sigma_{Ut} I_0(a\Sigma_{Ut}) K_1(a\Sigma_{Ut}) - a\Sigma_{Ut} \right\} - 1$$

Table 4-7

VALUES OF λ

λ	$c\Sigma_{Mt}$	λ	$c\Sigma_{Mt}$
1.32	0	.815	2.5
1.04	0.5	.80	3
.94	1	.78	3.5
.88	1.5	.775	4
.846	2		

Values of λ are given in Table 4-7; A in Table 4-8; B in Table 4-9; and α, β in Fig. 4-1. A comparison of values of G calculated from Eqs. (4-5) and (4-7) with experimental results for various rods is given in Table 4-10.

Table 4-8
VALUES OF $A = (P_c/1 - P_c) - a\sum U_t$
(Reproduced from Ref. 5)

$a\sum U_t$	A	δ	$a\sum U_t$	A	δ	$a\sum U_t$	A	δ	$a\sum U_t$	A	δ	$a\sum U_t$	A	δ
0.00	0.00330	628	1.00	0.4561	131	2.00	1.2292	174	3.00	2.1402	187	4.00	3.0998	195
0.02	0.00628	599	.02	0.4692	131	.02	1.2466	172	.02	2.1589	188	.02	3.1193	193
.04	0.01227	587	.04	0.4823	133	.04	1.2638	176	.04	2.1777	189	.04	3.1386	197
.06	0.01814	587	.06	0.4956	134	.06	1.2811	175	.06	2.1966	190	.06	3.1583	193
.08	0.02401	591	.08	0.5090	135	.08	1.2989	175	.08	2.2156	191	.08	3.1776	191
0.10	0.02992	598	1.10	0.5225	137	2.10	1.3164	175	3.10	2.2347	188	4.10	3.1967	194
.12	0.03590	608	.12	0.5362	138	.12	1.3339	177	.12	2.2535	188	.12	3.2161	198
.14	0.04198	620	.14	0.5500	138	.14	1.3516	176	.14	2.2723	193	.14	3.2359	195
.16	0.04818	633	.16	0.5638	141	.16	1.3692	176	.16	2.2916	190	.16	3.2554	198
.18	0.05451	646	.18	0.5779	141	.18	1.3868	177	.18	2.3106	189	.18	3.2752	195
0.20	0.06099	661	1.20	0.5920	141	2.20	1.4045	178	3.20	2.3295	191	4.20	3.2947	192
.22	0.06760	677	.22	0.6061	144	.22	1.4223	178	.22	2.3486	190	.22	3.3139	195
.24	0.07437	694	.24	0.6205	144	.24	1.4401	180	.24	2.3676	191	.24	3.3334	191
.26	0.08131	711	.26	0.6349	146	.26	1.4581	179	.26	2.3867	191	.26	3.3525	203
.28	0.08842	727	.28	0.6495	146	.28	1.4760	178	.28	2.4058	191	.28	3.3728	191
0.30	0.09569	74	1.30	0.6641	148	2.30	1.4938	181	3.30	2.4249	192	4.30	3.3919	194
.32	0.1031	77	.32	0.6789	149	.32	1.5119	178	.32	2.4441	191	.32	3.4113	191
.34	0.1108	77	.34	0.6938	149	.34	1.5297	180	.34	2.4632	191	.34	3.4304	202
.36	0.1185	80	.36	0.7087	151	.36	1.5477	181	.36	2.4823	191	.36	3.4506	197
.38	0.1265	81	.38	0.7238	151	.38	1.5658	183	.38	2.5014	190	.38	3.4703	194
0.40	0.1346	84	1.40	0.7389	152	2.40	1.5841	180	3.40	2.5204	195	4.40	3.4897	197
.42	0.1430	84	.42	0.7541	153	.42	1.6021	184	.42	2.5399	190	.42	3.5094	192
.44	0.1514	87	.44	0.7694	155	.44	1.6205	182	.44	2.5589	194	.44	3.5286	196
.46	0.1601	89	.46	0.7849	155	.46	1.6387	179	.46	2.5783	188	.46	3.5482	199
.48	0.1690	90	.48	0.8004	156	.48	1.6566	186	.48	2.5971	193	.48	3.5681	195
0.50	0.1780	92	1.50	0.8160	156	2.50	1.6752	180	3.50	2.6164	192	4.50	3.5876	198
.52	0.1872	93	.52	0.8316	158	.52	1.6932	183	.52	2.6356	192	.52	3.6074	193
.54	0.1965	96	.54	0.8474	158	.54	1.7115	184	.54	2.6548	195	.54	3.6267	197
.56	0.2061	97	.56	0.8632	159	.56	1.7299	186	.56	2.6743	190	.56	3.6464	192
.58	0.2158	98	.58	0.8791	161	.58	1.7485	183	.58	2.6933	193	.58	3.6656	195
0.60	0.2256	101	1.60	0.8952	160	2.60	1.7668	186	3.60	2.7126	193	4.60	3.6851	198
.62	0.2357	102	.62	0.9112	161	.62	1.7854	183	.62	2.7319	196	.62	3.7049	193
.64	0.2459	104	.64	0.9273	163	.64	1.8037	185	.64	2.7515	190	.64	3.7242	196
.66	0.2563	105	.66	0.9436	162	.66	1.8222	184	.66	2.7705	194	.66	3.7438	200
.68	0.2668	107	.68	0.9598	164	.68	1.8406	187	.68	2.7899	193	.68	3.7636	194
0.70	0.2775	109	1.70	0.9762	164	2.70	1.8593	186	3.70	2.8092	191	4.70	3.7832	197
.72	0.2884	110	.72	0.9926	165	.72	1.8779	184	.72	2.8283	195	.72	3.8029	192
.74	0.2994	111	.74	1.0091	166	.74	1.8963	187	.74	2.8478	194	.74	3.8221	194
.76	0.3105	114	.76	1.0257	166	.76	1.9150	186	.76	2.8672	192	.76	3.8415	198
.78	0.3219	115	.78	1.0423	168	.78	1.9336	188	.78	2.8864	195	.78	3.8613	202
0.80	0.3334	116	1.80	1.0591	166	2.80	1.9524	187	3.80	2.9059	194	4.80	3.8815	195
.82	0.3450	118	.82	1.0757	170	.82	1.9711	188	.82	2.9253	192	.82	3.9010	199
.84	0.3568	119	.84	1.0927	168	.84	1.9899	186	.84	2.9445	196	.84	3.9209	192
.86	0.3687	120	.86	1.1095	169	.86	2.0085	185	.86	2.9641	194	.86	3.9401	195
.88	0.3807	123	.88	1.1264	171	.88	2.0270	190	.88	2.9835	191	.88	3.9596	199
0.90	0.3930	124	1.90	1.1435	170	2.90	2.0460	187	3.90	3.0026	196	4.90	3.9795	202
.92	0.4054	124	.92	1.1605	171	.92	2.0647	190	.92	3.0222	193	.92	3.9997	195
.94	0.4178	127	.94	1.1776	171	.94	2.0837	187	.94	3.0415	197	.94	4.0192	198
.96	0.4305	127	.96	1.1947	173	.96	2.1024	189	.96	3.0612	194	.96	4.0390	191
.98	0.4432	129	.98	1.2120	172	.98	2.1213	189	.98	3.0806	192	.98	4.0581	20
1.00	0.4561		2.00	1.2292		3.00	2.1402		4.00	3.0998		5.00	4.078	

TABLE 4-9

VALUES OF B AS A FUNCTION OF $a\Sigma_{Ut}$
(Reproduced from Ref. 5)

$a\Sigma_{Ut}$	B	δ	$a\Sigma_{Ut}$	B	δ	$a\Sigma_{Ut}$	B	δ
0.00	0.00000	678	1.00	0.5603	148	2.00	1.3941	181
.02	0.00678	704	.02	0.5751	148	.02	1.4122	179
.04	0.01382	726	.04	0.5899	150	.04	1.4301	183
.06	0.02108	748	.06	0.6049	151	.06	1.4484	186
.08	0.02856	770	.08	0.6200	151	.08	1.4670	180
0.10	0.03626	790	1.10	0.6351	153	2.10	1.4850	186
.12	0.04416	813	.12	0.6504	154	.12	1.5036	179
.14	0.05229	833	.14	0.6658	154	.14	1.5215	189
.16	0.06062	852	.16	0.6812	156	.16	1.5404	181
.18	0.06914	873	.18	0.6968	156	.18	1.5585	184
0.20	0.07787	891	1.20	0.7124	157	2.20	1.5769	184
.22	0.08678	910	.22	0.7281	158	.22	1.5953	188
.24	0.09588	93	.24	0.7439	159	.24	1.6141	186
.26	0.1052	95	.26	0.7598	159	.26	1.6327	180
.28	0.1147	97	.28	0.7757	161	.28	1.6507	187
0.30	0.1244	98	1.30	0.7918	161	2.30	1.6694	185
.32	0.1342	100	.32	0.8079	163	.32	1.6879	187
.34	0.1442	102	.34	0.8242	163	.34	1.7066	184
.36	0.1544	104	.36	0.8405	164	.36	1.7250	188
.38	0.1648	106	.38	0.8569	165	.38	1.7438	188
0.40	0.1754	107	1.40	0.8734	163	2.40	1.7626	186
.42	0.1861	109	.42	0.8897	166	.42	1.7812	187
.44	0.1970	110	.44	0.9063	170	.44	1.7999	191
.46	0.2080	112	.46	0.9233	164	.46	1.8190	184
.48	0.2192	114	.48	0.9397	169	.48	1.8374	191
0.50	0.2306	115	1.50	0.9566	167	2.50	1.8565	185
.52	0.2421	117	.52	0.9733	170	.52	1.8750	189
.54	0.2538	119	.54	0.9903	171	.54	1.8939	189
.56	0.2657	119	.56	1.0074	169	.56	1.9128	197
.58	0.2776	122	.58	1.0243	170	.58	1.9325	182
0.60	0.2898	122	1.60	1.0413	172	2.60	1.9507	191
.62	0.3020	125	.62	1.0585	172	.62	1.9698	187
.64	0.3145	125	.64	1.0757	174	.64	1.9885	191
.66	0.3270	127	.66	1.0931	172	.66	1.0076	186
.68	0.3397	129	.68	1.1103	174	.68	2.0262	190
0.70	0.3526	130	1.70	1.1277	175	2.70	2.0452	196
.72	0.3656	131	.72	1.1452	175	.72	2.0648	189
.74	0.3787	132	.74	1.1627	176	.74	2.0837	194
.76	0.3919	134	.76	1.1803	175	.76	2.1031	186
.78	0.4053	135	.78	1.1978	175	.78	2.1217	193
0.80	0.4188	136	1.80	1.2153	177	2.80	2.1410	188
.82	0.4324	138	.82	1.2330	177	.82	2.1598	192
.84	0.4462	138	.84	1.2507	179	.84	2.1790	193
.86	0.4600	140	.86	1.2686	182	.86	2.1983	190
.88	0.4740	141	.88	1.2868	177	.88	2.2173	189
0.90	0.4881	142	1.90	1.3045	176	2.90	2.2362	199
.92	0.5023	143	.92	1.3221	180	.92	2.2561	188
.94	0.5166	145	.94	1.3401	181	.94	2.2749	196
.96	0.5311	146	.96	1.3582	182	.96	2.2945	190
.98	0.5457	146	.98	1.3764	177	.98	2.3135	192
1.00	0.5603		2.00	1.3941		3.00	2.3327	

TABLE 4-9 (cont'd)

VALUES OF B AS A FUNCTION OF $a\Sigma_{Ut}$

$a\Sigma_{Ut}$	B	δ	$a\Sigma_{Ut}$	B	δ	$a\Sigma_{Ut}$	B	δ
3.00	2.3327	188	4.00	3.306	20	5.00	4.293	20
.02	2.3515	198	.02	3.326	20	.02	4.313	20
.04	2.3713	192	.04	3.346	20	.04	4.333	19
.06	2.3905	192	.06	3.366	19	.06	4.352	20
.08	2.4097	194	.08	3.385	20	.08	4.372	20
3.10	2.4291	192	4.10	3.405	19	5.10	4.392	20
.12	2.4483	192	.12	3.424	20	.12	4.412	19
.14	2.4675	195	.14	3.444	20	.14	4.431	21
.16	2.4870	191	.16	3.464	20	.16	4.452	20
.18	2.5061	197	.18	3.484	19	.18	4.472	19
3.20	2.5258	193	4.20	3.503	20	5.20	4.491	20
.22	2.5451	195	.22	3.523	19	.22	4.511	20
.24	2.5646	191	.24	3.542	20	.24	4.531	19
.26	2.5837	194	.26	3.562	20	.26	4.550	20
.28	2.6031	197	.28	3.582	19	.28	4.570	20
3.30	2.6228	192	4.30	3.601	20	5.30	4.590	20
.32	2.6420	198	.32	3.621	20	.32	4.610	20
.34	2.6618	190	.34	3.641	20	.34	4.630	20
.36	2.6808	190	.36	3.661	19	.36	4.650	19
.38	2.6998	200	.38	3.680	20	.38	4.669	20
3.40	2.7198	205	4.40	3.700	20	5.40	4.689	20
.42	2.7403	182	.42	3.720	20	.42	4.709	20
.44	2.7585	198	.44	3.740	19	.44	4.729	20
.46	2.7783	197	.46	3.759	20	.46	4.749	20
.48	2.7980	191	.48	3.779	20	.48	4.769	20
3.50	2.8171	185	4.50	3.799	20	5.50	4.789	19
.52	2.8356	207	.52	3.819	19	.52	4.808	20
.54	2.8563	191	.54	3.838	20	.54	4.828	20
.56	2.8754	202	.56	3.858	20	.56	4.848	20
.58	2.8956	188	.58	3.878	19	.58	4.868	20
3.60	2.9144	191	4.60	3.897	20	5.60	4.888	19
.62	2.9335	209	.62	3.917	20	.62	4.907	20
.64	2.9544	189	.64	3.937	20	.64	4.927	20
.66	2.9733	192	.66	3.957	20	.66	4.947	20
.68	2.9925	200	.68	3.977	19	.68	4.967	20
3.70	3.0125	197	4.70	3.996	20	5.70	4.987	20
.72	3.0322	190	.72	4.016	20	.72	5.007	20
.74	3.0512	195	.74	4.036	19	.74	5.027	19
.76	3.0707	192	.76	4.055	20	.76	5.046	20
.78	3.0899	201	.78	4.075	20	.78	5.066	21
3.80	3.1100	196	4.80	4.095	20	5.80	5.087	19
.82	3.1296	204	.82	4.115	20	.82	5.106	20
.84	3.1500	199	.84	4.135	19	.84	5.126	20
.86	3.1699	204	.86	4.154	20	.86	5.146	20
.88	3.1903	180	.88	4.174	20	.88	5.166	20
3.90	3.2083	196	4.90	4.194	20	5.90	5.186	19
.92	3.2279	199	.92	4.214	20	.92	5.205	20
.94	3.2478	195	.94	4.234	20	.94	5.225	20
.96	3.2673	196	.96	4.254	19	.96	5.245	20
.98	3.2869	20	.98	4.273	20	.98	5.265	20
4.00	3.306		5.00	4.293		6.00	5.285	

FIG. 4-1
VALUES OF α AND β
(REPRODUCED FROM REF. 5)

Table 4-10

COMPARISON OF CALCULATED WITH EXPERIMENTAL VALUES
OF G FOR URANIUM RODS

(Reproduced from Ref. 6)

U density, gm/cm ³	18.80		18.88		18.83	
Enrichment	0.4962 x 10 ⁻²		0.7205 x 10 ⁻²		0.9124 x 10 ⁻²	
ΣU_c , cm ⁻¹	0.2557		0.3200		0.3763	
Radius, cm	1.27	2.54	0.952	1.27	1.59	1.27
G (P ₁ approx.)	1.098	1.358	1.076	1.133	1.202	1.166
G (Eq. (4-5))	1.137	1.369	1.120	1.178	1.245	1.216
G (Eq. (4-7))	1.143	1.394	1.123	1.185	1.256	1.225
G (exptl.)	1.162 ± 0.012	1.420 ± 0.014	1.133 ± 0.011	1.190 ± 0.012	1.290 ± 0.013	1.238 ± 0.012

Note: Rods in an effectively infinite bath of D₂O

4.1.5 Empirical Fit to Radial Power Distribution in Natural Uranium
Rods in a Large Unit Cell of D₂O

A set of empirical equations has been derived which summarizes experimental data on the radial distribution of thermal neutrons in fuel rods:⁽⁸⁾

Let $\rho = a\Sigma_a$ where a is the rod radius and Σ_a is the macroscopic absorption cross section of the rod. The axis-to-surface ratio of neutron density is:

$$A(\rho) = \frac{1}{1 + 0.69713 \rho + 0.48598 \rho^2 + 0.33879 \rho^3} \quad (4-8)$$

The radial distribution of neutron density $n(r, \rho)$ normalized to unity at the surface of the rod is given by:

$$n(r, \rho) = A(\rho) [1 + 0.69713 \rho (r/a)^2 + 0.48598 \rho^2 (r/a)^4 + 0.33879 \rho^3 (r/a)^6] \quad (4-9)$$

The ratio, G , of surface to average neutron density taken over the cross section of the rod is given by

$$G(\rho) = A(\rho) [1 + 0.34856 \rho + 0.16199 \rho^2 + 0.08469 \rho^3] . \quad (4-10)$$

Equations (4-8) through (4-10) are valid in the range $0 \leq \rho \leq 2$.

Accuracy of the Fit

An analysis of the sources of errors indicates the following relative errors in the determination of the neutron density distribution in an absorbing rod as given by Eq. (4-9).

ρ	Relative Error, % (RMS Error of Fit)
0 - 0.5	1.0
0.5 - 1.0	3.0
1.0 - 2.0	5.0

4.1.6 Disadvantage Factors from Considerations of Rod Blackness and Extrapolation Length

Kushneriuk^(9,10) gives results for thermal utilization in rods similar to those obtained in Section 4.1.4, but which involve concepts of rod blackness and extrapolation length. In his formulation G has its usual definition: the ratio of surface flux to average flux in the rod, α is $\phi_{as}(a)/\phi(a)$ where $\phi(a)$ is the actual flux at the surface of the rod, $\phi_{as}(a)$ is the value at the surface of the rod of the asymptotic flux in the moderator as determined by diffusion theory with appropriate boundary conditions at the surface of the rod, and β (see also Ref. 11) is the probability that a neutron incident on the rod surface will be absorbed before escaping the rod, and is called the blackness of the rod. The extrapolation length, λ , is given by

$$\lambda = \phi_{as}(a)/\phi'_{as}(a)$$

The integral transport theory result for the blackness, β , is:

$$\beta = \frac{(1-c)P_1(a/\ell)}{1-c P_C(\nu)}$$

where

c = ratio of scattering to total cross section in the rod;

$P_C(\nu)$ = the collision probability tabulated in Ref. 7 (also see Sections 4.3.1 and 3.7.1)

$\nu = \ell/L$ is the root of $2\nu = c\ell n(1+\nu)/(1-\nu)$;

ℓ = mean free path in the rod;

L = thermal diffusion length in rod;

$$P_1\left(\frac{a}{\ell}\right) = \frac{4a}{3\ell} \left\{ I_1\left(\frac{a}{\ell}\right) K_1\left(\frac{a}{\ell}\right) - 1 - \frac{2a}{\ell} + \frac{2a^2}{\ell^2} \left[I_0\left(\frac{a}{\ell}\right) K_0\left(\frac{a}{\ell}\right) + I_1\left(\frac{a}{\ell}\right) K_1\left(\frac{a}{\ell}\right) + \frac{\ell}{a} I_0\left(\frac{a}{\ell}\right) K_1\left(\frac{a}{\ell}\right) \right] \right\}.$$

P_1 is closely related to the function B of Refs. 5 and 6, and P_C is identical in Refs. 5, 6 and 9. I_n and K_n are the modified Bessel functions of the first and second kind.

The values of G computed by the methods given in Section 4.1.4 are in good agreement with those given in Ref. 9, and can be used in the present case. For further details on the calculation of G one should see Ref. 9. One useful and simple approximation for G in the range $1.001 < G < 1.10$ is

$$G(\text{approx.}) = a [2 - \beta] / \beta \ell_a$$

where ℓ_a is the absorption mean free path in the rod.

Define

$$P_0 \left(\frac{a}{\ell} \right) = 1 + I_1 \left(\frac{a}{\ell} \right) K_1 \left(\frac{a}{\ell} \right) - \frac{3\ell}{4a} P_1 \left(\frac{a}{\ell} \right)$$

and

$$g \left(\frac{a}{\ell_a} \right) = \frac{2 - P_0 \left(\frac{a}{\ell_a} \right)}{P_1 \left(\frac{a}{\ell_a} \right)}$$

A result of integral transport theory is:

$$\alpha = \frac{2 - \frac{3}{2} P_1 \left(\frac{a}{\ell_a} \right) g \left(\frac{a}{\ell_t} \right)}{2 - P_0 \left(\frac{a}{\ell_a} \right)} \quad \left(\frac{G}{g} \right)$$

where ℓ_t is the transport mean free path in the rod.

The function $g \left(\frac{a}{\ell_t} \right)$ is given in Table 4-11.

The extrapolation length, λ , permits the determination of ϕ_{as} , and thus the amount of capture in the moderator. Values of λ or references thereto are given for a series of cases (see also Section 3.7.1).

Table 4-11
VALUES OF g
(Data of Ref. 9)

x	$g(x)$	x	$g(x)$
.07	.114	1.0	.413
.10	.139	1.5	.466
.15	.175	2.5	.524
.2	.204	5.0	.571
.3	.251	8.0	.590
.5	.314	12.0	.598
.7	.361	∞	.623

Black cylinders:

(See Refs. 12 and 13)

Gray cylinders of small radius with or without air gaps:

(See Refs. 14 and 15)

Homogeneous, purely absorbing cylinder in an infinite bath of pure, isotropic scatterer:

$$\lambda / \ell_t = [4 / (3P_1(a/\ell_a))] - g(a/\ell_t)$$

ℓ_a = absorption mean free path in the rod

ℓ_t = transport mean free path of moderator.

Non-homogeneous purely absorbing rod with radial symmetry, in an infinite bath of pure, isotropic scatterer:

$$\lambda/\ell_t = [4/(3\eta(a))] - g(a/\ell_t)$$

$$\eta(a) = 1 - \frac{4}{\pi} \int_0^{\pi/2} \text{Ki}_3(\tau) d\phi$$

$$\tau(\overline{AB}) = \int_B^A [1/(\ell_a(s))] ds$$

$\overline{AB}$ = average chord of the cylinder in units of absorption mean free paths

$$\text{Ki}_3(x) = \int_0^\infty \cosh^{-3} u \exp(-x \cosh u) du$$

and is tabulated in Ref. 16

ϕ = angle between the outward normal and the chord.

Rod with radius a , air gap of width $b-a$:

$$\frac{\lambda}{\ell_t} = \left[\frac{4b}{3aP_1(a/\ell_a)} \right] - g\left(\frac{b}{\ell_t}\right)$$

Gap is a weakly capturing sheath of thickness $b-a$, and total mean free path, ℓ_{sh} .

$$\eta(b) = \frac{a}{b} P_1\left(\frac{a}{\ell_a}\right) T + \frac{2}{b\ell_{sh}} (b^2 - a^2) - \frac{a}{b} P_1\left(\frac{a}{\ell_a}\right) (1-T)$$

$$T = 1 - \frac{2(b-a)}{\ell_{sh}}, \quad 2(b-a) \ll \pi^2 a.$$

Homogeneous rod, $c > 0$

$$\frac{\lambda}{\ell_t} = \frac{4}{3\beta} - g\left(\frac{a}{\ell_t}\right)$$

If $\frac{a}{\ell_a} \rightarrow \infty$:

$$\begin{aligned} \frac{\lambda}{\ell_t} &\rightarrow 0.7104 + 0.2046c, & c \ll 1 \\ \frac{\lambda}{\ell_t} &\rightarrow 1/\sqrt{3(1-c)}, & c \rightarrow 1 \end{aligned}$$

If there is an air gap of width $b-a$:

$$\frac{\lambda}{\ell_t} = \frac{4b}{3a\beta} - g\left(\frac{b}{\ell_t}\right)$$

N sheaths:

$$\frac{\lambda(R)}{\ell_t} = \frac{4}{3\eta(R)} - g\left(\frac{R}{\ell_t}\right)$$

R = outside radius of assembly

$\eta(R)$ = total assembly blackness

$$\eta = \frac{a}{R} \beta \prod_{i=1}^N T_i + \frac{2}{\pi R} \sum_{i=1}^N \left(\frac{A_i}{\ell_i} \right) - \frac{a}{R} \beta \left(\sum_{i=1}^N (1 - T_i) \right)$$

$$T_i = 1 - \frac{2}{\pi \ell_i} \left\{ \sqrt{a_i^2 - a^2} - \sqrt{a_{i-1}^2 - a^2} + \left(\frac{a_i^2}{a} \right) \sin^{-1} \frac{a}{a_i} - \frac{a_{i-1}^2}{a} \sin^{-1} \frac{a}{a_{i-1}} \right\}$$

$$+ (a_i^2/a) \sin^{-1} (a/a_i) - (a_{i-1}^2/a) \sin^{-1} (a/a_{i-1})$$

where:

a_i = outer radius of i^{th} sheath

$a_N = R$

a = inner rod radius

A_i = cross sectional area of i^{th} sheath

ℓ_i = absorption mean free path in i^{th} sheath; for an air gap, $\ell_i \rightarrow \infty$

Water Sheath:

$G = G(\text{approx.})$, and use two-group diffusion theory.

Rod Clusters, widely separated:

The cluster is viewed as a single homogeneous rod with $c > 0$. The effective cross section of each rod in the cluster may be determined by considering it in an infinite array of rods arranged as in the cluster.

4.1.7 Spectral Hardening

When the spectrum is hardened by capture in the rod the problem becomes more complex.⁽⁶⁾ The calculation of thermal utilization may be simplified with the assumption of no energy transfer in the rod or in the moderator. The system may then be considered to be made up of many independent energy groups. The formulation of Section 4.1.6 is then useful for the study of spectral hardening.

An equivalent set of diffusion constants is obtained by setting

$$\rho(E, x) = C(E) I_0(\kappa(E) ax)$$

where ρ is the neutron density at energy E and at a distance ax cm from the center of the rod. ($x = 1$ is the rod boundary.) These solutions are used to compute the capture in the rod at each energy and the consequent hardening.

Then

$$2\pi a \int_{\text{in}} (E) \beta(E) = 2\pi a C(E) \sum_a (E) I_1 [a\kappa(E)] / \kappa(E)$$

$$G(E) = a \sum_a (E) \left[(2 - \beta(E)) / \beta(E) \right] = \frac{a \kappa(E) I_0 [a\kappa(E)]}{2 I_1 [a\kappa(E)]}$$

can be used to find $C(E)$ and $\kappa(E)$; $\Sigma_a(E)$ is the macroscopic absorption cross section in the rod. $J_{in}(E)$ is the current into the rod. It is convenient to specify

$$J_{in}(E) = D \frac{E}{(kT)^2} \exp(-E/kT) + J_{in}^F(E) \Delta(E)$$

$$\Delta(E) = 0 \quad E < 5kT$$

$$= 1 \quad E > 5kT$$

$J_{in}^F(E)$ is the fast component usually represented by a constant divided by E . This constant is found by asserting that the slowing down density is equal to the rate of neutron absorption. This formulation can be extended to rods whose compositions vary radially; this is done in Ref. 10.

If an initially Maxwellian spectrum (temperature T °K) falls on one face of a slab of non-scattering, absorbing material, a hardened spectrum is produced. Considered here is the case of such a spectrum isotropically incident ($\cos \theta$ incident law) on one side of a plane slab of material with a $1/v$ absorption law. The thickness times the absorption cross section at 1 ev is denoted by K ; the transmission of the slab (the fraction of the neutron density remaining*) is t ; the mean velocity of the hardened distribution is given by $\bar{v} = x v_0$. For the initial spectrum

$$x = \sqrt{\frac{4T}{\pi T_0}},$$

where

$$T_0 = 293.6 \text{ °K.}$$

Values of t and x for different values of K and T are given in Table 4-12.**

The average cross section

$$\bar{\sigma} = \frac{\int \sigma(v)n(v)vdv}{\int vn(v)dv}$$

is given by $\bar{\sigma} = \frac{\hat{\sigma}}{x}$

For a neutron density distribution function $n(v)$, $\hat{\sigma}$ is defined by

$$\hat{\sigma} = \frac{\int \sigma(v)n(v)vdv}{v_0 \int n(v)dv}$$

where v_0 is equal to 2200 meters/sec.

Note that for substances like U^{238} which follow a $1/v$ -law in the thermal region (to ~1% accuracy), $\hat{\sigma}$ is invariant with respect to hardening. $\hat{\sigma}$ for non- $1/v$ substances (e.g., Pu^{239}) will vary: $\hat{\sigma}(Pu^{239})$ is also given in Table 4-12.

*The fraction of initial neutron density is also the fraction of initial reaction rate for a $1/v$ -detector, so that this quantity is the flux depression or thermal disadvantage factor as measured with a $1/v$ -law detector.

**This table was prepared by C. H. Westcott.

Table 4-12

RESULTS OF SPECTRAL HARDENING CALCULATION
(AND $\hat{\sigma}$ FOR Pu^{239})

Unhardened Values						
T = 20°C $x = 2/\sqrt{\pi} = 1.12838$ $\hat{\sigma} = 776.61$	K = 0.0059645	0.0119290	0.0178935	0.0238580	0.0357870	
	t = 0.854024	0.764130	0.693949	0.635847	0.543065	
	x = 1.15679	1.17509	1.19036	1.20385	1.22749	
	$\hat{\sigma} = 780.32$	782.86	785.07	787.09	790.78	
	K = 0.0477161	0.596451	0.0715741	0.0894676	0.1073611	
T = 100°C $x = 1.27286$ $\hat{\sigma} = 824.75$	t = 0.470986	0.412827	0.364748	0.306418	0.260197	
	x = 1.24818	1.26687	1.28406	1.30773	1.32942	
	$\hat{\sigma} = 794.17$	797.37	800.43	804.84	809.08	
	K = 0.0059645	0.0119290	0.0178935	0.238580	0.0357870	
	t = 0.866207	0.782374	0.716237	0.661007	0.571849	
T = 180°C $x = 1.40249$ $\hat{\sigma} = 900.47$	x = 1.30220	1.32124	1.33717	1.35128	1.37602	
	$\hat{\sigma} = 831.20$	835.70	839.62	843.22	849.83	
	K = 0.0477161	0.0596451	0.715741	0.0894676	0.1073611	
	t = 0.501675	0.444394	0.396534	0.337761	0.290545	
	x = 1.39773	1.41735	1.43542	1.46033	1.48319	
T = 260°C $x = 1.52092$ $\hat{\sigma} = 1005.22$	$\hat{\sigma} = 855.94$	861.74	867.31	875.38	883.20	
	K = 0.0059645	0.0119290	0.0178935	0.0238580	0.0357870	
	t = 0.875349	0.796215	0.733287	0.680392	0.594300	
	x = 1.43259	1.45225	1.46872	1.48333	1.50899	
	$\hat{\sigma} = 911.09$	918.54	925.05	931.04	942.06	
T = 260°C $x = 1.52092$ $\hat{\sigma} = 1005.22$	K = 0.0477161	0.0596451	0.0715741	0.0894676	0.1073611	
	t = 0.525870	0.469522	0.422063	0.363250	0.315511	
	x = 1.53153	1.55192	1.57073	1.59666	1.62048	
	$\hat{\sigma} = 952.26$	961.95	967.32	984.77	997.86	
	K = 0.0059645	0.0119290	0.0178935	0.0238580	0.0357870	
T = 260°C $x = 1.52092$ $\hat{\sigma} = 1005.22$	t = 0.882512	0.807182	0.746898	0.695962	0.612514	
	x = 1.55164	1.57181	1.58875	1.60377	1.63021	
	$\hat{\sigma} = 1021.10$	1032.25	1042.02	1050.99	1067.48	
	K = 0.0477161	0.0596451	0.0715741	0.0894676	0.1073611	
	t = 0.545668	0.490242	0.443263	0.384622	0.336633	
T = 260°C $x = 1.52092$ $\hat{\sigma} = 1005.22$	x = 1.65344	1.67449	1.69391	1.72070	1.74533	
	$\hat{\sigma} = 1082.74$	1097.19	1111.09	1131.16	1150.58	

4.2 RESONANCE ESCAPE PROBABILITY

In the process of slowing down, neutrons collide with nuclei and may be absorbed in resonances.

The infinite dilution resonance integral (R.I.) is given by

$$(R.I.) = \int \sigma_a(E) \frac{dE}{E}$$

where $\sigma_a(E)$ is the microscopic absorption cross section of the material under consideration and the integral extends over an energy range encompassing all the resonances. The usefulness of the resonance integral arises from the fact that in a slowing down medium the flux has a characteristic $1/E$ dependence. If a sizable number of resonance absorber atoms are present, however, there is, in general, a depletion of the flux at the resonances. The effective resonance integral, $(R.I.)_{eff}$, is then defined by

$$(R.I.)_{eff} = \int \sigma_a(E) \bar{\phi}(E) dE$$

where $\bar{\phi}(E)$ is the average over all the atoms of the sample of the resonance depleted flux normalized to $1/E$ away from the resonances. In a homogeneous mixture, if σ_a is small or σ_a has narrow, well separated resonances, then

$$\bar{\phi}(E) = \frac{\Sigma_s(E)}{\Sigma_s(E) + \Sigma_a(E)} \frac{1}{E}$$

where $\Sigma_s(E)$ and $\Sigma_a(E)$ are the macroscopic scattering and absorption cross sections of the mixture. $(R.I.)_{eff}$ is then given by

$$(R.I.)_{eff} = \int \sigma_a(E) \frac{\Sigma_s(E)}{\Sigma_s(E) + \Sigma_a(E)} \frac{dE}{E}$$

and the resonance escape probability, p , is given by

$$p = \exp \left[- \frac{N_a}{\xi \Sigma_s} (R.I.)_{eff} \right]$$

In a heterogeneous array the effective resonance integral is a complicated function of the geometry and composition of the system. It is usually given in a form that involves both the volume and surface of the fuel lump (see Refs. 17, 18) and is considered in Section 4.2.2. In the heterogeneous case, p is given by

$$p = \exp \left[- \frac{N_a V_a}{\xi \Sigma_{sm} V_m} \frac{\bar{\phi}_a}{\bar{\phi}_m} (R.I.)_{eff} \right]$$

where V_a , V_m are the volume of absorber and of moderator respectively, and $\bar{\phi}_a$ and $\bar{\phi}_m$, the average flux in the absorber and moderator. The ratio, $\bar{\phi}_m/\bar{\phi}_a$, is called the resonance disadvantage factor, and can be calculated in the same way as the corresponding thermal quantity. There are some reservations about the use of the resonance disadvantage factor; see, for example, the work of Corngold⁽¹⁹⁾ and Chernik and Vernon.⁽²⁰⁾

4.2.1 Volume Absorption Term in the Effective Resonance Integral of U^{238}

There appears in the open literature an array of values for the volume absorption term, A , of the effective resonance integral of U^{238} ,

$$(R.I.)_{\text{eff}} = A[1 + \mu(S/M)].$$

where S is the surface area of the lump, and M the mass. The various values are re-examined here in their historical order of appearance.

Creutz, Jupnik, Snyder and Wigner^(17,18) reported a value of 7.3 b for A after deducting the $1/v$ contribution. The $1/v$ contribution is:

$$\int \phi \sigma \left(\frac{1}{v} \right) \frac{dE}{E} = \int \frac{dE}{E} \frac{\sqrt{.025}}{\sqrt{E}} \sigma^{U^{238}} (.025 \text{ ev}) = 1.4 \text{ b}$$

The value $A = 7.3 \text{ b}$ was based on an infinite dilution resonance integral for U^{238} of 240 b. Using the more recent value of 292 b, (See Section 2.5)

$$A = 7.3 \frac{292}{240} \text{ b} = 8.88 \text{ b}$$

Goldstein and Hughes⁽²¹⁾ used pile neutrons to measure the U^{238} resonance integral and used the indium resonance integral as a standard. The reported value $A = 7.6 \text{ b}$ is based on an effective resonance integral for indium of 3000 b and the value is also proportional to the thermal cross section ratio of U^{238} and In^{115} . The correction factor based on the more recent data,

$$\int \frac{dE}{E} (\text{In}^{115}) = 3237 \text{ b}, \quad \sigma_{\text{th}} (U^{238}) = 2.75 \text{ b}, \quad \text{and} \quad \sigma_{\text{th}} (\text{In}^{115}) = 195 \text{ b}$$

is

$$\frac{3237}{3000} \times \frac{2.75}{2.59} \times \frac{190}{195} = 1.12.$$

This yields a corrected value of $A = 8.52 \text{ b}$.

Macklin and Pomerance⁽²²⁾ report a value of $A = 10.2 \text{ b}$ from which the $1/v$ contribution of 1.4 b should be subtracted, yielding $A = 8.8 \text{ b}$.

A value of $A = 9.3 \text{ b}$, based on a measured pile value of p may be deduced from the work of Davis.⁽²³⁾

Table 4-13 lists the various values of A discussed above and their corrected values.

Table 4-13

VALUES OF THE VOLUME ABSORPTION TERM IN U^{238}

	Uncorrected	Corrected
Creutz, Jupnik, Snyder and Wigner	7.3 b	8.88 b
Goldstein and Hughes	7.6 b	8.52 b
Macklin and Pomerance	10.2 b	8.8 b
Davis	9.3 b	9.3 b

4.2.2 Formulae for Effective Resonance Integrals in Terms of Mass and Surface Contributions

The following formulae comprise the more recent effective resonance integral formulations for uranium, thorium and thorium oxide. Except where noted, the effective resonance integral does not include the $1/v$ contribution. The mass, M , is given in grams of metal or oxide, the surface area, S , in cm^2 .

Formulae

A. Macklin and Pomerance:⁽²²⁾

1. U^{238} Metal

$$(\text{R.I.})_{\text{eff}} = 8.8 \left(1 + \frac{3.42}{M/S + 0.11} \right) \pm 15\% \text{ b}$$

2. Th^{232} Metal

$$(\text{R.I.})_{\text{eff}} = 8.37 \left(1 + \frac{0.55}{M/S + 0.08} \right) \pm 15\% \text{ b}$$

B. Sher:⁽²⁴⁾

1. U^{238} Metal

$$(\text{R.I.})_{\text{eff}} = 9.10 \left(1 + \frac{3.32}{M/S + 0.12} \right) \text{ b}$$

2. Th^{232} Metal

$$(\text{R.I.})_{\text{eff}} = 7.5 \left(1 + \frac{3.4}{M/S + 0.34} \right) \text{ b}$$

C. Dayton and Pettus:⁽²⁵⁾

1. Th^{232} Metal

$$(\text{R.I.})_{\text{eff}} = (-.3 + 24.1 \sqrt{S/M}) \text{ b}, \quad 0.1 \leq S/M \leq 1.3$$

2. ThO_2

$$(\text{R.I.})_{\text{eff}} = (-4.7 + 35.2 \sqrt{S/M}) \text{ b}, \quad 0.2 \leq S/M \leq 1.3$$

(The density of the ThO_2 is 7.5 gm/cm^3 .)

D. Davis:⁽²³⁾

Reactivity measurements on cadmium-clad cylinders of uranium, uranium oxide, thorium and thorium oxide can be interpreted to yield the following effective resonance integrals for cylinders of radius 1.73 cm and the densities listed.

Sample	Density (gm/cm^3)	Effective Resonance Integral (barns)
Uranium (U^{238})	18.9	10.9
Uranium Oxide (UO_2)	7.96	14.0 ± 0.9
Thorium	11.5	9.2 ± 0.3
Thorium Oxide (ThO_2)	9.64	12.1 ± 0.8

The measurements by Davis⁽²³⁾ for a 1.73 cm radius rod are compared with the various formulae in Table 4-14.

Table 4-14

EFFECTIVE RESONANCE INTEGRAL (BARNs)
FOR A 1.73 cm RADIUS ROD

Material	Davis	Macklin and Pomerance	Sher	Dayton and Pettus*
U ²³⁸	10.9	12.03	10.92	-
Th	9.2 ± 0.3	8.82	9.98	10.54
ThO ₂	12.1 ± 0.8		-	11.20

*Includes (1/v) contribution.

E. Hellstrand:⁽²⁶⁾

For uranium metal rods in D₂O:

$$(R.I.)_{\text{eff}} = \left(\frac{7.8}{G} + 30.2 S/M \right) b, * \quad .07 \leq S/M \leq .25$$

or

$$(R.I.)_{\text{eff}} = (2.81 + 24.7 (S/M)^{1/2}) b, \quad .07 \leq S/M \leq .53$$

where G is the ratio of surface to average flux in the rod for the resonance group. These two expressions agree within experimental error over their common range.

For uranium oxide (UO₂):

$$(R.I.)_{\text{eff}} = \left(\frac{10.5}{G} + 22.8 S/M \right) b, * \quad .2 \leq S/M \leq .7$$

or

$$(R.I.)_{\text{eff}} = (4.15 + 26.6 (S/M)^{1/2}) b, \quad .08 \leq S/M \leq .7$$

For thin rods the formula $(R.I.)_{\text{eff}} = (A/G) + (BS/M)$ is modified to read

$$(R.I.)_{\text{eff}} = (A/G) + (BKS/M) .$$

$$K = \frac{2}{1 + [1 + (\Sigma_s 2R_0)^{-1/2}]^{1/2}}$$

and is distinct from the Dancoff-Ginsburg correction factor (see Section 4.2.3); Σ_s is the potential scattering (macroscopic) cross section and R_0 the rod radius.

Theoretical Interpretation⁽²⁷⁾

The formulae displayed above are subject to several comments:

- A. The form $(R.I.)_{\text{eff}} = A + B \sqrt{S/M}$ is valid only for S/M sufficiently small that the total resonance integral is not more than $\sim 1/3$ of the resonance integral for infinite dilution. This is because a complete parametric display would take the form:

$$(R.I.)_{\text{eff}} = A(\xi \Sigma_s) \left[\frac{1 + \mu_1(\Sigma_s) S/M}{1 + \mu_2(\Sigma_s) S/M} \right]^{1/2} + \frac{B(\xi \Sigma_s) S/M}{[1 + \mu_3(\Sigma_s) S/M]^{1/2}}$$

*The formulae already include explicitly an estimate of the G term of the resonance disadvantage factor (see Section 4.2.4) and hence, when using the formulae, the disadvantage factor applied to the rod is set equal to unity.

a form which is correct in the dilute limit, but otherwise approximate and awkward to use. The variations of A and B in this formula depend on the scattering per atom, and on the logarithmic energy loss, ξ , per scatter; variations in the parameters μ_i are due to variations in total scattering only.

- B. In the formulae for $(R.I.)_{\text{eff}}$, small variations in scattering per resonance absorber atom may be approximated by varying the "A" term only in proportion to the square root of the scattering per atom. Other power laws have no theoretical significance and dubious experimental validity.
- C. The theoretical expectation is that the addition of moderating scatterer (such as oxygen) will increase the "A" term at the expense of the "B" term. Dayton and Pettus' formulae for Th and ThO₂ exhibit opposite behaviors. Moreover, the volume term in each case is negative; thus these formulae must be viewed as fitting formulae only and more limited to the range of experimental measurements than are the corresponding formulae for uranium.
- D. The experimental basis of volume integrals measured directly is very uncertain. For large lumps, a decreased volume integral for uranium, which should be observed, is masked by a flux depression in the resonance region for the calibrating material (In or Au).
- E. For clusters, or for tightly packed lattices, the value of surface area, S, is subject to the "Dancoff corrections" (see Section 4.2.3) which arise from shadowing.

4.2.3 Corrections in the Surface Term Arising from Shadowing

In a lattice of rods the presence of one rod depletes the flux at the peak of the resonance which is incident on another rod, and thus leads to a reduction of the effective surface. The factor which multiplies the surface area, S, in the formula for

the effective resonance integral is $(1 - \sum_i C_i)$, where the summation for any rod extends

over all other rods and C_i is a function of both the size of the rods and the distance to rod i. The formula for C given by Dancoff and Ginzburg⁽²⁸⁾ is

$$C = \frac{\rho d_1^2}{\lambda^3 \pi^2} \iint d\theta d\theta' \frac{Ki_3(|\vec{\rho} - \vec{\rho}'|/\lambda)}{|\vec{\rho} - \vec{\rho}'|^3/\lambda^3} \left[\cos \theta - \frac{\rho}{d_i} (1 + \cos(\theta + \theta')) \right] \left[\cos \theta' - \frac{\rho}{d_i} (1 + \cos(\theta + \theta')) \right]$$

$$|\vec{\rho} - \vec{\rho}'| = \sqrt{d^2 - 2d\rho(\cos \theta + \cos \theta') + 2\rho^2(1 + \cos(\theta + \theta'))}$$

where $\vec{\rho}$, $\vec{\rho}'$ are the radius vectors of the rod under consideration and rod i respectively, measured in each case from the center of the rod to its circumference in a plane perpendicular to the rod axis; θ and θ' , the angle these vectors make with the line of centers between the rod and rod i; and λ , the moderator mean free path; d_i is the distance between the center of the rods. Ki_3 is tabulated in Ref. 16. Limits on θ and θ' are such that a straight line joining the points of integration does not pass through the interior of either rod in the interval included between the two points. The mean value theorem of integral calculus shows that C may be written in the form

$$C = \frac{4}{\pi^2} Ki_3 \left(\delta \left(\frac{d}{\rho} \right) \frac{\rho}{\lambda} \right) f \left(\frac{\rho}{d} \right) \quad (4-11)$$

The limit of C for $\lambda \rightarrow \infty$ is $C = \frac{\sin^{-1}(\rho/d)}{\pi}$, and $Ki_3(0) = \pi/4$. Hence, $f(\rho/d) = \sin^{-1}(\rho/d)$. $\delta(d/\rho)$ may then be obtained by comparison of Eq.(4-11) with values of C obtained by numerical integration.* These results are shown in Fig. 4-2. Note that d/ρ must be greater than 2. If $d/\rho > 3.8$, the following asymptotic form may be used:

$$C = \frac{\lambda^2}{\rho d} \sqrt{\frac{\pi \lambda}{2d}} e^{-d/\lambda} \left[\frac{2(\rho/\lambda)}{\pi} \int_0^{\pi/2} d\theta \cos \theta \exp \left\{ (\rho \cos \theta)/\lambda \right\} \right]^2$$

FIG. 4-2
DANCOFF-GINSBURG CORRECTION FACTOR
(DATA FROM REF. 28)

For slabs the surface correction term, $1-C$, is $[1 - 2E_3(\Sigma_s d)]$ where d is the distance between fuel plates, Σ_s the scattering cross section (cm^{-1}) in the moderator and $E_3(x) = \int_1^\infty e^{-xu} u^{-3} du$; $E_n(x)$ is tabulated in Ref. 7.

For a hollow rod⁽²⁶⁾ of outer surface area S_0 and an inner surface area S_i with moderator in the center hole, replace S by $S_0 + \gamma S_i$. $\gamma = 2R_i \Sigma_s P_C(R_i \Sigma_s)$; R_i = inner radius, Σ_s = scattering cross section (cm^{-1}) in the moderator, and P_C is the collision probability tabulated in Ref. 7.

*The use of the mean value theorem and the graph of $\delta(d/\rho)$ was suggested by J. A. Thie, Argonne National Laboratory.

In Ref. 29 an alternative formulation is proposed for a close packed lattice of uranium rods. This formulation yields for the resonance escape probability, p

$$\ln(1/p) = [1.19 \times 10^{-12} \sqrt{\pi N \bar{\ell}}] F(q) + 5 \times 10^{-24} \lambda_s N / \omega \quad (1/\xi)$$

where

ξ = mean logarithmic energy decrement per scattering in moderator

N = number of U^{238} atoms/cm³ in the fuel

$\omega = V_{\text{mod}}/V_{\text{fuel}}$

λ_s = scattering mean free path in moderator, cm

$L = \omega \bar{\ell}$; $\bar{\ell} = 4 V_{\text{fuel}}/S_{\text{fuel}}$

$q = \lambda_s / (\lambda_s + L)$

$$F(q) = (1-q) \frac{\sqrt{\pi}}{2} \left\{ \frac{1}{[\ln(1/q)]^{3/2}} + \frac{2}{(2\pi)^{3/2}} \sum_{n=0}^{\infty} \left(\frac{\ln q}{2\pi} \right)^n \prod_{k=1}^n \left(1 + \frac{1}{2k} \right) \zeta \left(n + \frac{3}{2} \right) \cos \frac{\pi}{2} \left(n + \frac{3}{2} \right) \right\}$$

where $\zeta(x)$ = Riemann ζ function. ⁽³⁰⁾

4.2.4 Resonance Disadvantage Factor

The disadvantage factor, $\bar{\phi}_m / \bar{\phi}_a$, can be calculated as in Section 4.1.1, but there are two marked differences. One is that in the present instances the diffusion theory results are more likely to be applicable. The other is that the resonance group constants are not nearly so well known as are the thermal constants, since there is much uncertainty as to the appropriate width to be assigned to the resonance region. The transport mean free path for moderators is usually constant over the resonance band, and hence is readily calculated.

$\xi \Sigma_s / \Delta u$ is the removal cross section by elastic scattering out of the resonance group which has been assigned a logarithmic width, Δu . The appropriate value for Δu may be estimated from the resonance absorber diffusion length, $1/\kappa$, for resonance neutrons by the relation

$$\Delta u \approx \frac{N_a 3 \Sigma_s (R.I.)_{\text{eff}}}{\kappa^2}$$

where the simple approximation of diffusion theory, $\kappa^2 = 3 \Sigma_a \Sigma_s$, has been made. However, κ^2 in the resonance region has been measured only for U, and therefore estimates for other materials are extremely uncertain. Δu is sometimes taken equal to 5.6 for natural uranium. The magnitude of Δu is usually not too important, however, since the spatial distribution of the flux is quite flat. (See Ref. 1, p. 273).

In the moderator the diffusion theory parameters $\kappa_m^2 = \frac{3 \Sigma_{tr,m} \xi \Sigma_{s,m}}{\Delta u}$ and $D_m = \frac{1}{3 \Sigma_{tr,m}}$ can be applied. In the fuel the situation is more complicated. In

Ref. 1, p. 273, an experimental value of $\kappa = 0.42 \text{ cm}^{-1}$ for U^{238} is quoted.* If scattering is isotropic and Σ_t is the total cross section, κ satisfies $\frac{\Sigma_s}{2\kappa} \ln \frac{\Sigma_t + \kappa}{\Sigma_t - \kappa} = 1$, and is the positive real root. The effective absorption cross section in the fuel, $(\Sigma_a)_{\text{fuel}}^{\text{eff}} = \frac{N_a(\text{R.I.})_{\text{eff}}}{\Delta u}$. From these two quantities D for the fuel is determined. The formulae for diffusion theory constants in the resonance region are summarized in Table 4-15.

Table 4-15

FORMULAE FOR THE DIFFUSION THEORY CONSTANTS
IN RESONANCE REGION

Region	D	κ	$(\Sigma_a)_{\text{eff}}$
Moderator	$(3\Sigma_{\text{tr}})^{-1}$	$(3\Sigma_{\text{tr}} \Sigma_a)_{\text{eff}}^{+\frac{1}{2}}$	$\xi \Sigma_s / \Delta u$
Fuel	$(\Sigma_a)_{\text{eff}} \kappa^2$	positive root of: $\frac{\Sigma_s}{2\kappa} \ln \left(\frac{\Sigma + \kappa}{\Sigma - \kappa} \right) = 1$	$\frac{N_a(\text{R.I.})_{\text{eff}}}{\Delta u}$

In evaluation of the disadvantage factor it is assumed that the source in the moderator is constant, and the source in the fuel is zero. The normal diffusion theory disadvantage factor calculation (see Section 4.1.1) can be made. Further accuracy may be easily obtained by the use of the transport theory methods outlined in Section 4.1.6. If an estimate of the extrapolation length, λ , for the resonance region can be obtained using the methods outlined there, straightforward diffusion theory can be used to compute the average flux in the moderator. From the approximate formula

$$p = \exp \left[- \frac{\pi R}{2V_m} \int \frac{\beta(E)dE}{\xi \Sigma_s(E)} \right], \quad \text{Ref. 9}$$

where R is the rod radius, comparison with

$$p = \exp \left[- \frac{N_a V_a}{\xi \Sigma_{sm} V_m} \frac{\bar{\phi}_{\text{fuel}}}{\bar{\phi}_m} (\text{R.I.})_{\text{eff}} \right]$$

shows that the effective blackness, β_{eff} , may be estimated as

$$\beta_{\text{eff}} = \frac{2RN_a(\text{R.I.})_{\text{eff}}}{\Delta u}$$

This estimate for β_{eff} enables a good estimate for G , the ratio of surface to average flux in the rod, through use of the formula (see Section 4.1.6)

$$G_{\text{app}} = \frac{R(2-\beta)}{\beta \ell_a}$$

*This value is based on the assumption of flat flux in the moderator; see Ref. 31.

where ℓ_a , the total absorption mean free path in the rod may be obtained from $\ell_a^{-1} = \frac{A}{\Delta u}$; A is the volume term in the effective resonance integral. For a simpler approach see Ref. 26.

Clusters of Rods

For lattices consisting of widely separated rod clusters one may replace each cluster by an equivalent rod having as its outer boundary the circumscribed surface of the cluster. The shadowing of rods within the cluster is described in Section 4.2.3. In this way an $(R.I.)_{eff}$ is found for each rod. This yields an effective absorption cross section, $(\Sigma_a)_{eff}$, which is then homogenized with the $(\Sigma_a)_{eff}$ of the moderator within the circumscribed surface. Flux weighting, i.e., disadvantage factors, should be employed for the unit cell within the cluster if necessary. Σ_{tr} is similarly homogenized and the cluster can now be treated as one large rod. Another disadvantage factor calculation may then be applied to determine the proper flux weighting factors for the homogenized cluster and the surrounding moderator. In this calculation the source is assumed to be constant in each region and proportional to the moderator volume fraction.

4.2.5 Resonance Capture in Single Resonances by Escape Probabilities⁽¹⁹⁾

The resonance escape probability for a single resonance in the case of non-overlapping resonances in heterogeneous systems has been examined in the case of hydrogen moderator and uranium fuel.⁽³²⁾ It is assumed there is no absorption in the hydrogen. Under these conditions,

$$p = \exp \left\{ - \int \frac{dE}{E} \frac{\Sigma_{au} (1 - P_H(E))}{\Sigma_{tot u} - P_U(E) \Sigma_{su}} \right\}$$

where $P_H(E)$ is the probability that a neutron of energy E in hydrogen will suffer its next collision in hydrogen. $P_U(E)$ is the corresponding probability for a neutron in uranium for collision in the uranium. Σ_{au} is the absorption cross section in the uranium as a function of energy. Σ_{su} is the scattering cross section as a function of energy. Collision probabilities* for use here are tabulated in Ref. 7 for various cases. ξ is the logarithmic decrement per collision. The integral is over the single resonance only. In general, $P_H(E)$ is constant over a resonance. $\ln(1/p)$ has been tabulated for various U^{238} resonances in Table 4-16.

Table 4-16
VALUES OF $\ln(\ell/p)$ FOR INDIVIDUAL U RESONANCES
IN A SLAB LATTICE
(Data from Ref. 19)

$V_{H_2O}:V_U = 1:1$
Total Thickness of Fuel Slabs = .375"

Resonance Energy, ev	$\ln(\ell/p)$
6.7	0.1205
20.9	0.05222
37.0	0.04715
66.5	0.01418
81.6	0.004357
90.0	0.001079
104	0.01370
118	0.006059

*See also Tables 4-19 in Section 4.3.1 and 3-33 in Section 3.7.1

4.2.6 Monte Carlo Calculation of Resonance Escape*

Histories of neutrons are followed through the neutron energy range $E_{\max} \geq E \geq E_{\min}$, where one might choose $E_{\min}$ as the "Cd cutoff" energy and $E_{\max}$ as something of the order of .01 to .1 Mev.

Each cell consists of a number of regions. Each region is a homogeneous mixture of isotopes.

Neutron histories start with injection into the assembly at an energy $E \leq E_{\max}$ and with velocity determined by the assumption that the flux above $E_{\max}$ has a $1/E$ energy dependence and is isotropic.

Elastic scatterings are assumed to be isotropic in the center of mass system. Nuclei are considered as being at rest in the laboratory system, except for thermal motion which results in the Doppler broadening of the resonances.

Treatment of Isotopes other Than U^{238}

$$\sigma_s = \text{constant}$$

$$\sigma_a(E) = \frac{\sigma_a(1 \text{ ev})}{E}$$

where E is measured in ev.

Treatment of U^{238}

$$\sigma_{s \text{ potential}} = 10.75 \text{ barns}$$

$$\text{background } \sigma_a(E) = \frac{\text{background } \sigma_a(1 \text{ ev})}{E}$$

$$\sigma_{s \text{ res}} = 4\pi \lambda_0^2 g \left(\frac{\Gamma_n}{\Gamma} \right)^2 \psi(x, \theta) = \sqrt{10.75(4\pi) \lambda_0^2 g \left(\frac{\Gamma_n}{\Gamma} \right)^2} X(x, \theta); \text{ the resonance peak is at } E = E_0.$$

$$\sigma_{a \text{ res}} = 4\pi \lambda_0^2 g \frac{\Gamma_n \Gamma}{(\Gamma)^2} \gamma \psi(x, \theta) \sqrt{E_0/E}$$

$$x = \frac{2(E - E_0)}{\Gamma}$$

$$\theta = 4E_0 k T / (238 \Gamma^2)$$

$$\psi(x, \theta) = \frac{1}{4\pi\theta} \int_{-\infty}^{\infty} \frac{\exp \left[\frac{-(x-y)^2}{4\theta} \right]}{1 + y^2} dy$$

$$X(x, \theta) = \frac{1}{4\pi\theta} \int_{-\infty}^{\infty} \frac{2y \exp \left[\frac{-(x-y)^2}{4\theta} \right]}{1 + y^2} dy$$

*R. Van Norton: Private Communication. No numerical results are available at this time, but this work is included here because of the widespread interest in the method.

At energy E where $E_0 \leq E \leq E_0'$ the sum of the background cross section and the contributions for the resonances centered at E_0 and E_0' is used for the cross section. Thus interference is considered between the potential scattering and resonance scattering, but not between resonances.

The parameters of the 27 low-energy resonances have been measured, but for energies above 530 ev only a statistical description of the resonance structure of U^{238} is known. The description which is used is

$$z = \frac{\Gamma_n^0}{\Gamma_n^0 \text{ average}} \quad ; \quad \Gamma_n^0 = \frac{\Gamma_n}{E_0}$$

and z has the probability density

$$\frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad (\text{Porter and Thomas distribution})$$

For convenience, it is assumed that these resonances are uniformly spaced and have a constant Γ_γ . For speed of computation, a value of θ is used which is represented in the relatively sparse collection of tables of $\psi(x, \theta)$ and $X(x, \theta)$ rather than the value of θ corresponding to the selection of Γ_n , etc.

4.2.7 Measured Doppler Coefficients

When the temperature of the resonance absorber is increased there is a change in the effective resonance integral of the absorber. Assuming $(R.I.)_{\text{eff}} = A(1 + \mu S/M)$ and that all the change occurs in A , the following list of values for $(1/A)dA/dT$ has been obtained where T is the temperature in degrees Centigrade.

<u>U^{238}</u>	$\frac{1}{A} \frac{dA}{dT}$	<u>Ref.</u>
	$(1.74 \pm .2) \times 10^{-4}$	33
	$(1.1 \pm .3) \times 10^{-4}$	*
	$(1.3 \pm .3) \times 10^{-4}$	34
	$(1.56 \pm .12) \times 10^{-4}$	23
<u>Th^{232}</u>		
	2.8×10^{-4}	34

The temperature dependence of $A\mu$ (the surface term) has not been measured, and is usually taken to be zero.

*Reported by G. W. Rodeback, Reactor Information Meeting, Pittsburgh, Pa., October 1955.

4.3. FAST EFFECT

Neutrons with energy greater than ~ 1 Mev can cause fissions in U^{238} . The fast fission factor, ϵ , is defined as the number of neutrons injected into the moderator at energies below the fast fission threshold of U^{238} per neutron born in thermal fission. This definition, based on arguments given in Ref. 35 differs somewhat from that often given and includes effects associated with fast capture by U^{238} .

4.3.1 Ratio, F, of Fast Fissions to Thermal Fissions

The ratio, F, of the number of fast fissions to the number of thermal fissions is given by

$$F = \frac{\nu_{th} P\left(a, \frac{\kappa R}{a}\right) \frac{\chi \sum_f U^{238}}{\Sigma_{tot}}}{1 - \nu_f P\left(a, \frac{\kappa R}{a}\right) \frac{\chi \sum_f U^{238}}{\Sigma_{tot}} - \frac{\sum_{el}}{\Sigma_{tot}} P(a, 0)}$$

where

ν_{th} = average number of neutrons emitted in a thermal fission

ν_f = average number of neutrons emitted in a U^{238} fast fission

χ = fraction of fission neutrons born above U^{238} fission threshold

$P\left(a, \frac{\kappa R}{a}\right)$ = collision probability within fuel lump for a source distributed according to the thermal neutron flux

κ = inverse thermal neutron diffusion length, cm^{-1}

R = measure of size of fuel lump, cm.

a = measure of size of fuel lump in mean free paths at energies above U^{238} fission threshold

$P(a, 0)$ = collision probability within fuel lump for a uniform source distribution. (After one collision, the neutrons are assumed to be uniformly distributed over the lump.)

and all the macroscopic cross sections as well as the collision probabilities refer to values above the U^{238} fission threshold. Some fast effect cross sections above the threshold are given in Table 4-17. The collision probabilities⁽⁷⁾ are listed in Tables 4-18 and 4-19. (See also Table 3-33 in Section 3.7.1)

For tightly packed lattices, such as occur in H_2O moderated reactors, the formulae used in homogeneous systems can be applied since the mean free path for neutrons above the U^{238} fission threshold is of the order of several centimeters. (See Section 3.4.2)

Table 4-17

FAST EFFECT CROSS SECTIONS ABOVE U^{238} FISSION THRESHOLD

(Microscopic cross sections in barns)

	Material	U	Th ²³²	H	D	O	Al	Zr
ν_f		2.6	2.4					
σ_{el} tr.		2.0	2.0	1.7	2.7	2.0	1.5	3.0
σ_{in}		2.10	2.6				.3	.8
σ_f		.3/ χ	.076/ χ					
σ_c		.05	.06					
$\sigma_f^{U^{238}}$ above the fission threshold has an average value of .532 b; $\chi = .561$.								

Table 4-18

COLLISION PROBABILITY FOR CYLINDERS: $P(a,0)$

(Data from Ref. 7)

a	P(a,0)	a	P(a,0)	a	P(a,0)	a	P(a,0)
0.00	0.00000	0.50	0.40405	1.00	0.59285	1.50	0.69843
0.02	0.02561	0.52	0.41424	1.02	0.59826	1.52	0.70164
0.04	0.04967	0.54	0.42414	1.04	0.60354	1.54	0.70479
0.06	0.07248	0.56	0.43377	1.06	0.60870	1.56	0.70788
0.08	0.09421	0.58	0.44314	1.08	0.61375	1.58	0.71091
0.10	0.11498	0.60	0.45225	1.10	0.61869	1.60	0.71389
0.12	0.13487	0.62	0.46112	1.12	0.62352	1.62	0.71681
0.14	0.15396	0.64	0.46975	1.14	0.62825	1.64	0.71968
0.16	0.17231	0.66	0.47816	1.16	0.63287	1.66	0.72250
0.18	0.18996	0.68	0.48634	1.18	0.63740	1.68	0.72526
0.20	0.20697	0.70	0.49432	1.20	0.64183	1.70	0.72798
0.22	0.22336	0.72	0.50209	1.22	0.64616	1.72	0.73065
0.24	0.23918	0.74	0.50966	1.24	0.65041	1.74	0.73327
0.26	0.25446	0.76	0.51704	1.26	0.65457	1.76	0.73585
0.28	0.26923	0.78	0.52424	1.28	0.65864	1.78	0.73838
0.30	0.28351	0.80	0.53126	1.30	0.66263	1.80	0.74087
0.32	0.29733	0.82	0.53811	1.32	0.66654	1.82	0.74331
0.34	0.31070	0.84	0.54479	1.34	0.67038	1.84	0.74572
0.36	0.32366	0.86	0.55131	1.36	0.67413	1.86	0.74808
0.38	0.33621	0.88	0.55767	1.38	0.67781	1.88	0.75040
0.40	0.34838	0.90	0.56389	1.40	0.68142	1.90	0.75269
0.42	0.36019	0.92	0.56996	1.42	0.68495	1.92	0.75493
0.44	0.37164	0.94	0.57588	1.44	0.68842	1.94	0.75714
0.46	0.38276	0.96	0.58167	1.46	0.69182	1.96	0.75931
0.48	0.39356	0.98	0.58733	1.48	0.69516	1.98	0.76145

The ratio $P(a, \frac{\kappa R}{a})/P(a,0)$ is only a little less than 1. In particular,

$$\left. \begin{aligned} \text{at } a = 1, \frac{P(a, \kappa R/a)}{P(a,0)} &\approx 1 - .012 \frac{\kappa R}{a} \\ a = .5, \frac{P(a, \kappa R/a)}{P(a,0)} &\approx 1 - .009 \frac{\kappa R}{a} \end{aligned} \right\} \quad 0 < \frac{\kappa R}{a} < 4$$

$$\text{as } a \rightarrow 0, \frac{P(a, \kappa R/a)}{P(a,0)} \rightarrow 1$$

Table 4-19

COLLISION PROBABILITY $P(a,0)$ FOR SLABS, CYLINDERS
AND SPHERES WITH UNIFORM SOURCE

(Data from Ref. 7)

a	Slab	Cylinder	Sphere
.00	.0000	.0000	.0000
.02	.0831	.0256	.0148
.04	.1390	.0497	.0294
.06	.1849	.0725	.0436
.08	.2246	.0942	.0575
.10	.2597	.1150	.0712
.15	.3335	.1632	.1040
.20	.3932	.2070	.1352
.25	.4432	.2469	.1649
.30	.4859	.2835	.1931
.35	.5229	.3172	.2199
.40	.5554	.3484	.2454
.45	.5841	.3772	.2696
.50	.6097	.4040	.2927
.60	.6533	.4522	.3357
.70	.6890	.4943	.3748
.80	.7187	.5313	.4104
.90	.7437	.5639	.4430
1.00	.7651	.5928	.4728
1.50	.8363	.6984	.5890
2.00	.8757	.7636	.6676
2.50	.9002	.8068	.7230
3.00	.9167	.8371	.7636
4.00	.9375	.8765	.8183
5.00	.9500	.9008	.8530

a = half thickness for slabs

a = radius for cylinders or spheres in mean free paths.

4.3.2 Relation of the Fast Fission Factor ϵ to F

In Ref. 35 it is shown that the number of neutrons injected into the moderator below fast fission threshold per thermal fission, $\epsilon\nu_{th}$, is given by:

$$\epsilon\nu_{th} = \left\{ \frac{M_A \chi}{M_A + H_A + I_A + C_A} + \frac{M_B(1-\chi)}{M_B + C_B} + \frac{M_C I_A \chi}{(M_A + H_A + I_A + C_A)(M_C + C_C)} \right\} \left\{ \nu_{th} + \nu_f F \right\}$$

H = probability of causing fission on next flight;

I = probability of being inelastically scattered on next flight;

C = probability of being captured on next flight;

M = probability of being injected into moderator on next flight.

The subscript A refers to an average over the epi-threshold fission spectrum, B to the average over the subthreshold fission spectrum, C to an average over the in-elastically scattered subthreshold spectrum which, in Ref. 35, is assumed to be of the form Ee^{-E} , and χ is the fraction of fission neutrons born above fast fission threshold.

In addition it is assumed that $I_B = I_C = 0$. Also, any migration between rods without injection into the moderator is the same for both spectrum A and spectrum B.

The equation for $\epsilon\nu_{th}$ comes from the fact that per thermal fission, the sources of neutrons injected into the moderator are

$$\begin{aligned} & \chi (\nu_{th} + \nu_f F) \text{ from spectrum A} \\ & (1 - \chi) (\nu_{th} + \nu_f F) \text{ from spectrum B} \\ & \frac{I_A}{M_A + H_A + I_A + C_A} \chi (\nu_{th} + \nu_f F) \text{ from spectrum C.} \end{aligned}$$

Using the cross sections in Table 4-17, it is found that the fast fission factor, ϵ , is given by:

$$\epsilon = 1 + \frac{0.485 F (1 - 3.726 F)}{(1 - 2.431 F)} .$$

F can become as large as 0.381 for very large rods, and the above formula is adequate for all F between zero and 0.381.

Associated with fast fission is fast conversion, which may be accounted for as follows:

One thermal fission comes from $1 + \alpha_{th}$ absorptions in fissionable material, gives F fast fissions or $F(\nu_f - 1)$ extra neutrons, or $\nu_{th} + F(\nu_f - 1)$ total neutrons produced. Of these, $\nu_{th}\epsilon$ re-enter the chain reaction. Hence, $\nu_{th}(1 - \epsilon) + F(\nu_f - 1)$ neutrons are absorbed in the fast region.

The fast conversion ratio is then

$$FCR = \frac{\nu_{th}(1 - \epsilon) + F(\nu_f - 1)}{1 + \alpha_{th}}$$

which, using $\alpha_{th} = 0.183$ becomes

$$FCR = 0.268 F \frac{(1 + 2.445 F)}{(1 - 2.431 F)} .$$

Measurements of conversion ratios in NRX⁽³⁶⁾ give a production rate for Pu appreciably in excess ($\approx 7\%$) of the expected value using "best" reactivity constants. Application of the formulae in Ref. 35 reduces ϵ of NRX by about 1% from the previously calculated value (1.035), and raises the conversion ratio by 4.3%. The fast contribution to conversion thus removes a large part of the discrepancy. The reduction of ϵ by 1% is helpful since reactivity measurements for similar lattices have been best fit with $\eta\epsilon = 1.33$, which, for $\epsilon = 1.035$, gives $\eta(U) = 1.287$. This value is lower than would be derived from the cross-section tables, even assuming an appreciable resonance correction on $\sigma(U^{235})$. The 1% correction on ϵ brings $\eta(U)$ to a more reasonable value of 1.30.

Reactivity calculations on graphite lattices^(37,38) have required either larger values of resonance integrals, or smaller values of η than are generally believed to be correct. Much of the discrepancy is reduced by using the formulation of Ref. 35.

Measured conversion ratios of D₂O, natural U lattices⁽³⁹⁾ are on the high side, barely within error limits, of theory. The measurements were analyzed by correcting lattice values of conversion to those which would apply if p were decreased, at constant η , ϵ , f , to give $k_{\infty} = 1$. This correction is therefore based on an estimate of reactivity of the lattices tested. The formulae of Ref. 39 indicate that the ϵ used was 1% high, and this decreases the discrepancy between theory and experiment by a factor of two.

4.3.3 Effect of Fast Leakage

In Ref. 40 it is pointed out that leakage in the energy region of fast fission can affect the value of ϵ . Assuming the non-leakage probability, P , in this region to be $P = 1/(1 + L_f^2 B^2)$, the effect can be estimated. An estimate of L_f^2 can be obtained by equating L_f^2 to the value of $r^2/6$ at the U²³⁸ fission threshold. In Ref. 40 where a lattice of 0.6 inch rods of uranium in H₂O is studied, L_f^2 is estimated by taking the total age to indium, and subtracting from it the age from U²³⁸ fission threshold to 1.44 ev. The latter quantity can be evaluated by assuming that in pure water this age is 11 cm² and that in the lower energy group the uranium and aluminum are regarded as having infinite mass in computing the logarithmic decrement, so that

$$L_f^2 = \tau_{H_2O} - 11 (1 + V_{\text{metal}}/V_{\text{water}}).$$

In Ref. 40, $\tau_{H_2O} = 26 \text{ cm}^2$ was used. One can then show that

$$\frac{\epsilon}{\epsilon_{\infty}} = 1 - F \left[\left(\frac{1}{P} \right) - 1 \right] \frac{\nu U^{238}}{\nu U^{235}}$$

Here ϵ_{∞} is the fast fission factor for an infinite array. The chief error actually is not in the estimation of P but in $\alpha U^{238} = \text{capture to fission ratio in } U^{238} \text{ above fission threshold}$ and in νU^{238} . (See Tables 4-28 and 4-29.)

4.3.4 Interaction Fast Effect⁽³²⁾

In Section 4.3.1 it was tacitly supposed that if a neutron escapes from a rod it is degraded below fast fission threshold in the moderator. It may, however, penetrate into a neighboring rod while still above the threshold. The effect resulting from the direct interaction with neighboring rods without the neutron suffering any intervening moderator collision should be considered. This requires a redefinition of the collision probabilities to include collision in neighboring fuel elements. For a slab array it is found that:

$$P_0 = 1 - \int_0^1 t_f^{-1} \mu \frac{(1 - e^{-t_f/\mu}) (1 - e^{-t_m/\mu})}{1 - e^{-t_f/\mu} e^{-t_m/\mu}} d\mu$$

where P_0 is the collision probability in fuel for a uniform source in the fuel, t_f is the width of the fuel slab in mean free paths, t_m the width of the moderator slab in mean free paths.

Setting b = thickness of fuel slab (cm), $S = \frac{t_m}{t_f} \frac{\sigma_f}{\sigma_m}$, $(\sigma_{\text{tot mod.}}) = T (\sigma_{\text{tot U}})$, and $\sigma_{\text{nat U}} = \sigma$ (cm⁻¹), Clark has shown (Ref. 41) that $P_0 = P_0(TS, \sigma b)$. The homogeneous case is $\sigma b = 0$. Table 4-20 gives a list of values of the ratio of $P_0(TS, \sigma b)$ to $P_0(TS, 0)$ for various values of TS , σb . The corrections, $\Delta\epsilon$, to the fast effect for heterogeneous systems of .75-inch rods in a hexagonal H₂O lattice and .375-inch slabs in the H₂O lattices, compared to homogeneous systems of the same composition, are given in Table 4-21. ϵ is calculated for a homogeneous assembly. $\epsilon + \Delta\epsilon$ is the fast effect in the heterogeneous array.

Table 4-20

COLLISION PROBABILITY IN A SLAB ARRAY
(Data from Ref. 41)
 $P_0(TS, \sigma b)/P_0(TS, 0)$

$TS \backslash \sigma b$	1	2	3	∞
0	1	1	1	1
.5	.88	.85	.76	.74
1.0	1	1	1	1
2.0	1.27	1.48	1.52	1.52
∞	1	1	1	1

Table 4-21

CHANGES $\Delta\epsilon$ IN ϵ TO ACCOUNT FOR HETEROGENEITY⁽³²⁾

$V_{\text{H}_2\text{O}}/V_{\text{U}}$	ϵ	$\Delta\epsilon$.75" rods, hex. lattice	$\Delta\epsilon$.375" slabs
0	1.227	.0	0
1	1.098	.011	.010
2	1.065	.011	.010
3	1.048	.012	.011
4	1.038	.014	.012

4.4 MACROSCOPIC NEUTRON DIFFUSION IN HETEROGENEOUS LATTICES

The lattice constants f , ϵ , and p were all obtained by considering the system to consist of an infinite array of unit cells. Over-all neutron diffusion which results from the finiteness of the system must also be considered. The usual (but not the only) treatment is to reduce, on the basis of the unit cell calculations, the heterogeneous composition to an equivalent homogeneous one, and then to treat the finite system by the methods of Section 3.

Thus, a typical problem arising in the analysis of a heterogeneous reactor is that of determining the diffusion theory parameters for a homogeneous system which is equivalent to the original system in the sense of exhibiting the same reactivity and

macroscopic neutron flux shapes. In the multigroup theory, these parameters include the diffusion coefficient, $\bar{D}$, and the average cross sections, $\bar{\Sigma}$.

4.4.1 Thermal Diffusion Length with No Anisotropy Assumed

In the thermal energy region, the thermal migration area, $\bar{L}^2 = \frac{\bar{D}}{\bar{\Sigma}_a}$ is required. In a homogeneous system L^2 is obtained from the volume weighted macroscopic absorption and scattering cross sections. In reactors describable as lattice systems, consisting, for example, of periodically distributed slab, cylindrical, or spherical fuel elements in a moderating medium, it has been found reasonably accurate to obtain the average cross section by flux weighting. Thus

$$\bar{\Sigma}_{tr} = \frac{\sum_i \phi_i \Sigma_{tr_i}}{\sum_i \phi_i}, \quad \bar{\Sigma}_a = \frac{\sum_i \phi_i \Sigma_{a_i}}{\sum_i \phi_i},$$

where the index i refers to the various homogeneous areas of which the lattice cell is composed and ϕ_i is the spatially integrated flux in the i^{th} area:

$$\phi_i = \int_{\text{area } i} \phi dV$$

$\bar{L}^2$ may be obtained from the average cross section, $\bar{\Sigma}_{tr}$ and $\bar{\Sigma}_a$ in the usual way for homogeneous mixtures; $\bar{L}^2 = \frac{1}{3\bar{\Sigma}_{tr}\bar{\Sigma}_a}$. The diffusion coefficient, $\bar{D}$, is then related to $\bar{L}^2$ by the usual relation $\bar{D} = \bar{\Sigma}_a \bar{L}^2$.

In the case of reactors where $\bar{\phi}_{\text{mod}} \gg \bar{\phi}_{\text{fuel}}$, a widely used approximate formula is:

$$\bar{L} = L_1 \sqrt{1-f}$$

where L_1 is the diffusion length in the pure moderator and f is the thermal utilization of the lattice.

4.4.2 Thermal Diffusion Length with Anisotropy Present

Homogenization procedures of the preceding type (Section 4.4.1) are applicable to reactors which are essentially similar in all directions, for example, spheres of uranium distributed uniformly in a large natural uranium-graphite reactor. In most reactors of present interest, however, the fuel is fabricated into plates or into right circular cylinders. One would expect diffusion lengths to exhibit a dependence on the direction of neutron transport.

Diffusion Theory Analysis

Spinrad⁽⁴²⁾ has obtained formulae, within the structure of diffusion theory, for diffusion lengths in directions parallel and perpendicular to slab and cylindrical lattices. The assumptions involved lead to more accurate results as the degree of heterogeneity increases. The formulae are:

Slab Lattice (two-region cell)

(a) Direction parallel to plate

$$D_1 (\kappa_{\parallel}^2 - \kappa_1^2)^{1/2} \tan (\kappa_{\parallel}^2 - \kappa_1^2)^{1/2} \frac{t_1}{2} = D_2 (\kappa_2^2 - \kappa_{\parallel}^2)^{1/2} \tanh (\kappa_2^2 - \kappa_{\parallel}^2)^{1/2} \frac{t_2}{2}$$

where

D_i = diffusion coefficient of region i

κ_i^2 = reciprocal of thermal migration area in region i ; $i = 1, 2$

t_i = thickness of region i

$\kappa_{\parallel}^2$ = reciprocal of average thermal migration area in direction parallel to the plates

(b) Direction perpendicular to plate

$$\cosh \kappa_{\perp} (t_1 + t_2) = (\cosh \kappa_1 t_1) (\cosh \kappa_2 t_2) + \frac{1}{2} \left[\frac{\kappa_1 D_1}{\kappa_2 D_2} + \frac{\kappa_2 D_2}{\kappa_1 D_1} \right] (\sinh \kappa_1 t_1) (\sinh \kappa_2 t_2)$$

where $\kappa_{\perp}^2$ = reciprocal of average thermal migration area in direction perpendicular to the plate.

Cylindrical Lattice (two-region cell)

(a) Direction parallel to rods

$$D_1 (\kappa_1^2 - \kappa_{\parallel}^2) \frac{I_1([\kappa_1^2 - \kappa_{\parallel}^2]^{1/2} a)}{I_0([\kappa_1^2 - \kappa_{\parallel}^2]^{1/2} a)} = -D_2 (\kappa_2^2 - \kappa_{\parallel}^2)^{1/2} \frac{\{J_1(kA)Y_1(ka) - J_1(ka)Y_1(kA)\}}{\{J_1(kA)Y_0(ka) - J_0(ka)Y_1(kA)\}}$$

where

$\kappa_{\parallel}^2$ = reciprocal of average thermal migration area in direction parallel to the rods

$$k = [\kappa_{\parallel}^2 - \kappa_2^2]^{1/2}$$

a = radius of inner region

A = radius of outer region.

A difficult numerical problem must be solved to determine κ_1^2 in this case. For details see Ref. 42.

Transport Theory Analysis

In a more recent report, Davison⁽⁴³⁾ presented a somewhat different formulation of the problem. Davison restricts his attention to a reactor made up of parallel slabs of two different materials, a so-called "sandwich" reactor of materials 1 and 2.

(a) Direction Perpendicular to Slabs.

L_1^2 , the thermal migration area in the direction perpendicular to the slabs, is given for two cases.

Case I: The thickness of each slab is small compared to the neutron mean free path in it.

$$L_1^2 = f_1 L_1^2 \frac{\bar{\ell}^2}{\ell_1^2} + f_2 L_2^2 \frac{\bar{\ell}^2}{\ell_2^2}$$

where

L is the thermal neutron diffusion length,

t is the total thickness of a slab,

and ℓ is the total mean free path in each material and

$$\bar{\ell} = [t_1 + t_2] / [t_1/\ell_1 + t_2/\ell_2]$$

f_i is the fraction of thermal neutron captures occurring in each material,

$$f_2 = 1 - f_1$$

Case II: The thickness of each slab is large compared to the mean free path in it.

$$L_1^2 = f_1 L_1^2 + f_2 L_2^2$$

An approximation sometimes used for a case intermediate between Case I and Case II is:

$$L_1^2 = f_1 L_1^2 \frac{\bar{\ell}}{\ell_1} + f_2 L_2^2 \frac{\bar{\ell}}{\ell_2}$$

(b) Direction Parallel to the Slabs

The formula is complicated; for details see Ref. 43.

4.4.3 Slowing Down Area

The usual practice has been to calculate the slowing down area in a heterogeneous system as though it were a homogeneous mixture. Conversely, some of the quasi-empirical formulae for the slowing down area in mixtures reported in Section 3 derive their constants from measurements in heterogeneous systems. Experimental results are given in Section 3.5 and Section 4.5 (Table 4-32).

The effect of inelastic scattering by U^{238} on the age in lattices of large lumps widely spaced has sometimes been taken into account by the use of formulae of the form:*

$$\tau = \tau_0 (\tau_0 - \tau_i) \frac{\sigma_i}{\sigma} P_c$$

where τ_0 is the age to thermal in the moderator for neutrons born in fission, τ_i the average age to thermal for neutrons which have suffered one inelastic scattering collision, σ_i the inelastic scattering cross section of U^{238} , σ the total cross section of U^{238} , and P_c the probability that a fission neutron makes a collision in the lump before escaping. (See Tables 4-18, 4-19 and 3-33).

In graphite of density 1.6 gm/cm^3 $\tau_0 = 364 \text{ cm}^2$, and $\tau_i = 230 \text{ cm}^2$.

In D_2O , $\tau_0 = 120 \text{ cm}^2$ has been used together with $\tau_0 - \tau_i = 23 \text{ cm}^2$.

Anisotropy Effect

Wade⁽⁴⁴⁾ has reported that in an Al- D_2O lattice the age in the direction along the Al rods was 140 cm^2 and transverse to the rods, 108 cm^2 . The lattice consisted of 2S Al rods (1 in. dia. x 5 ft. long), suspended in clusters of four rods in contact; the clusters formed a hexagonal lattice. The clusters were housed in 63S Al tubes. The cross sectional area of the metal in the tube was 0.732 in^2 , the area of the four rods, 3.14 in^2 . The total area of a unit cell was 42.4 in^2 , so that the volume ratio of Al/ D_2O was 0.1. No other measurements have been reported which yield a similar effect.

4.4.4. Effect of Holes in a Reactor

The theoretical discussion is based on Ref. 45.

The effect of holes in a reactor, such as cooling gaps, is to increase migration areas. Let L_0^2 be the diffusion area without holes, and L^2 the diffusion area with holes of any single energy group. Assuming no anisotropy, it is found that

$$\frac{L^2}{L_0^2} = 1 + 2\phi + \left\{ \phi^2 \frac{(2r)}{(\phi\lambda)} [\exp(2r/\phi\lambda) - 1]^{-1} \right\} + Q r \phi / \lambda$$

ϕ = volume of holes/volume of material

$v\phi$ = volume of each hole

s = surface area of each hole

r = hydraulic radius = $2 v\phi/s$ for cylinders

= $3 v\phi/s$ for holes totally within the reactor volume

λ = mean free path through solid material of reactor

Q is the mean square chord length through the hole divided by the square of the mean chord length and is given in Table 4-22 for various shaped holes.

*Private communication, I. Kaplan, MIT

Table 4-22

VALUES OF Q FOR PARTICULAR SHAPES OF HOLES⁽⁴⁵⁾

Spherical hole	Q = 1.125
Regular tetrahedron hole	Q = 1.583
Infinite cylinder of circular section	Q = 1.333
Infinite cylinder of regular hexagonal section	Q = 1.397
Infinite cylinder of square section	Q = 1.487
Infinite cylinder of equilateral triangular section	Q = 1.648
Spherical shell	Q = 1.125 x f(μ)
Cylinder shell	Q = 1.333 x F(μ)

(where μ = inner-radius/outer-radius, and f and F are tabulated below).

μ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(μ)	1.000	1.001	1.005	1.016	1.04	1.07	1.11	1.18	1.27	1.43	∞
F(μ)	1.0000	1.0258	1.0536	1.0839	1.1178	1.1567	1.2023	1.2604	1.3422	1.53	∞

L^2/L_0^2 is in all cases at least as great as $(1 + \phi)^2$, the density correction.
The values of L^2/L_0^2 for a square lattice of cylindrical holes is given in Table 4-23.

Table 4-23

 L^2/L_0^2 FOR A SQUARE LATTICE OF CYLINDRICAL HOLES⁽⁴⁵⁾

Lattice spacing Hole diameter	10	5	3	2
r/λ very small	1.016 (1.000)	1.066 (1.000)	1.200 (1.000)	1.548 (1.000)
$r/\lambda = 1/3$	1.019 (1.003)	1.079 (1.012)	1.234 (1.028)	1.609 (1.039)
$r/\lambda = 1$	1.026 (1.010)	1.108 (1.039)	1.319 (1.099)	1.815 (1.172)
$r/\lambda = 3$	1.047 (1.031)	1.194 (1.066)	1.574 (1.312)	2.466 (1.593)

Note: In each case the upper figure is the value of L^2/L_0^2 .
The figure in parentheses is the ratio of this value
to the simple density correction $(1 + \phi)^2$.

In most cases, L^2 becomes an anisotropic function. Diffusion area formulae, taking into account anisotropic effects for cylindrical holes are:

$$3 L_{\parallel}^2/L_0^2 = 1 + 2\phi + \left\{ \phi^2 \left(\frac{2r}{\phi\lambda} \right) \left[\exp \left(\frac{2r}{\phi\lambda} \right) - 1 \right]^{-1} \right\} + \frac{3}{2} Qr \phi/\lambda$$

$$3 L_{\perp}^2/L_0^2 = 1 + 2\phi + \left\{ \phi^2 \left(\frac{2r}{\phi\lambda} \right) \left[\exp \left(\frac{2r}{\phi\lambda} \right) - 1 \right]^{-1} \right\} + \frac{3}{4} Qr \phi/\lambda$$

Some experimental results are given in Table 4-24 and compared with calculations by these formulae.

Table 4-24

**DIFFUSION AREAS IN GRAPHITE-URANIUM
LATTICES WITH VARIOUS SIZED HOLES**

Uranium rods are on $6\frac{3}{8}$ in. centers in a square array. Holes are interspersed equidistantly between uranium rods.

Holes		$L_{\perp}^2/L_0^2$	Reference
$1\frac{3}{4}$ in. Diameter	Experimental	$1.30 \pm .15$	46
	Calculated	1.28	
		$L_{\perp}$ cm	
$3\frac{3}{16}$ in. Square	Experimental	82.9	47
	Calculated	82.6	

4.4.5 Control Analysis in Heterogeneous Reactors

The solution of control problems in heterogeneous thermal reactors often may be obtained with sufficient accuracy by first replacing the lattice by an equivalent homogeneous mixture and then applying the techniques described in Section 3.7. Other methods have been developed to deal with the case of isolated neutron sinks in lattice reactors difficult to analyze by homogenization procedures. The "small source" theory, proposed⁽⁴⁸⁾ by Horning, and also presented⁽⁴⁹⁾ at the 1955 Geneva Conference in a somewhat different form by Feinberg and Galanin has been applied to a variety of such problems at Hanford. Lanning⁽⁵⁰⁾ has reported on a small source-small sink method applicable to radially symmetric reactors unreflected axially. The following presentation summarizes Lanning's results:

The time-independent, one-group neutron diffusion equation may be written

$$D \left(\nabla^2 - \frac{1}{L^2} \right) \phi = \sum_{i=1}^N Q_i(z) \delta(|\vec{\rho} - \vec{\rho}_i|) - \sum_{i=1}^N S_i(z) q(z, |\vec{\rho} - \vec{\rho}_i|, \tau)$$

where

N = the total number of sources plus sinks

D = diffusion coefficient of the moderator

L^2 = diffusion area of the moderator

τ = age to thermal

$Q_i(z)$ = strength of the i^{th} sink (neutrons absorbed per sec per unit length of rod)

$S_i(z)$ = strength of the i^{th} source (fast neutrons emitted per sec per unit length of rod)

$q(z, |\vec{\rho} - \vec{\rho}_i|, \tau)$ = probability that a fast neutron emitted by the i^{th} source rod will be slowed to thermal at (ρ, z) ; it is the solution of the age equation,

$$\nabla^2 q = \frac{\partial q}{\partial \tau}$$

If the reactor core is a right circular cylinder of height H , and is unreflected at the top and bottom, then the i^{th} source rod contributes

$$q_i = S_i^0 \cos(\kappa z) \exp(-\kappa^2 \tau) \exp - [(\vec{\rho} - \vec{\rho}_i)^2 / 4\tau] / 4\pi\tau.$$

Let

$$\lambda^2 = (1/L^2) + \kappa^2; \quad \kappa^2 = \pi^2/H^2;$$

$$S_i(z) = S_i^0 \cos \kappa z; \quad Q_i(z) = Q_i^0 \cos \kappa z$$

where H = effective height of reactor

One then has for the flux at any point (z, ρ)

$$\phi(z, |\vec{\rho} - \vec{\rho}_i|) = [\cos \kappa z] \left[\sum_{i=1}^N -Q_i^0 \frac{K_0(\lambda |\vec{\rho} - \vec{\rho}_i|)}{2\pi D} + \sum_{i=1}^N S_i^0 e^{-\kappa^2 \tau} G\left(\frac{\lambda}{\tau} |\vec{\rho} - \vec{\rho}_i|\right) \right]$$

where

$$G\left(\frac{\lambda}{\tau} \rho\right) = \frac{e^{\lambda^2 \tau}}{4\pi D} \left\{ 2 K_0(\lambda \rho) - E_1\left(\frac{\rho^2}{4\tau}\right) + \lambda^2 \tau E_2\left(\frac{\rho^2}{4\tau}\right) - \left(\frac{\lambda^2 \tau}{2}\right)^2 E_3\left(\frac{\rho^2}{4\tau}\right) + \dots \right\}$$

and K_0 is the modified Bessel function of the second kind, and $E_n(x) = \int_1^\infty e^{-xu} u^{-n} du$ is tabulated in Ref. 9.

One also has $S_i^0 = h_i Q_i^0$, where h_i is the number of fast neutrons produced per thermal neutron absorbed in the rod. Diffusion theory gives the following relation between Q_i^0 , ϕ_i and β_i

$$Q_i^0 = \beta_i J_{in_i} 2\pi a_i = \frac{\pi a_i \beta_i}{2 - \beta_i} \phi_i,$$

where

$\beta_i J_{in_i}$ is the net current into the i^{th} rod

β_i is the thermal blackness⁽⁵¹⁾ of the i^{th} rod

a_i is the radius of the i^{th} rod

$\phi_i = \phi(a_i)$ is the thermal neutron flux evaluated at the surface of the i^{th} rod and is a function of all the Q_i .

Having related the values Q_i^0 to ϕ_i , a homogeneous set of N linear equations in the Q_i^0 is obtained. The condition that a self-consistent solution exist gives the relation:

$$k_{\text{eff}} = \frac{\sum_{i=1}^N S_i^0 e^{-\kappa^2 \tau}}{\sum_{i=1}^N Q_i^0 + \int_{\text{reactor volume}} \phi \frac{D}{L^2} dV + \int_{\text{reactor surface}} J_{\text{out}} dS},$$

where the values of the S_i and Q_i are those obtained from the set of N linear equations.

Lanning reported on the application of this method to an annular loading of 24 fuel rods in a large graphite medium, plus eight special control rods positioned in the graphite outside this annulus. The mass of fuel required to counter the neutron losses in these control rods was calculated and also experimentally determined.

For this case one has

$$h_{\text{fuel}} = 1.96$$

$$\beta_{\text{fuel}} = 0.441$$

The moderator was graphite: $L = 52.8 \text{ cm}$ $\tau = 355 \text{ cm}^2$

For the control rods in, $h = 0.662$ $\beta = 0.897$

For the control rods out, $h = 0.055$ $\beta = 0.897$

The results for the additional fuel required to maintain criticality are:

	$\Delta M_{\text{calc.}} (\text{kg } U^{235})$	$\Delta M_{\text{obs.}} (\text{kg } U^{235})$
Rods replaced by graphite	0	0
Rods present and in	1.29	1.34
Rods present but in out position	1.14	.96

4.5 INTEGRAL DATA

Data taken from exponential and critical experiments on various lattices are presented here.

4.5.1 Light Water-Slightly Enriched Uranium Lattices

Tables 4-25 through 4-34, and Figs. 4-3 through 4-8, are taken from Kouts and Sher.⁽⁴⁰⁾ They represent a new compilation of data gleaned from experiments on lattices of .600-inch rods of uranium (slightly enriched) in normal water. The rods are clad with .028 in. of aluminum and an air gap of about .005 in. separates the U from the Al. The results on the thermal utilization are now in better agreement with the calculations of Amouyal and Benoist^(5,6) (see Section 4.1.4) than the values previously reported.⁽⁵²⁾ The tables of M^2 and k_{∞} were computed on the basis of one-group, two-group, and three-group, age, and age diffusion theory. In each case the results were internally consistent, and there is no obvious basis for a choice between the methods.

In the calculations summarized in Tables 4-31 and 4-32, the measured values of f , η , ϵ , and p were plotted against buckling to obtain the group constants. For example, in two-group theory, $V = \frac{f\eta}{(1+L_s^2 B^2)}$ is plotted against B^2 , using $L_s^2 = f L_{s,\text{fuel}}^2 + (1-f) L_{s,\text{H}_2\text{O}}^2$, where L_s^2 is the thermal diffusion area. The critical equation $k_{\infty} = \eta \epsilon p f = (1+L_s^2 B^2) (1+L_2^2 B^2)$ shows that the slope of V vs. B^2 is just L_2^2 . From the determination of L_2^2 , k_{∞} can be calculated. In three group theory, $V = \frac{f\eta}{(1+L_s^2 B^2)(1+L_1^2 B^2)}$,

and L_1^2 is assumed to equal 5, 7.5 or 10 cm², L_s^2 is defined as before and L_2^2 determined once again as the slope of V vs. B^2 . The other critical equations such as: one group, $k_\infty = 1 + M^2 B^2$; age, $k_\infty = e^{\tau B^2}$; and age diffusion, $k_\infty = e^{\tau B^2} (1 + L_s^2 B^2)$ are handled similarly.

For Table 4-32, variations in f and L_s^2 were obtained by uniform poisoning of the water; the method of obtaining L_2^2 or M^2 , or τ , is essentially the same as in the preceding example except that V is plotted against the value of the poison concentration. The column labeled enrichment variation represents results obtained by varying the enrichment with no added poison. Poison was added only to lattices with enrichments of 1.15% and 1.3%. The column called combined analysis is a weighted average of the results obtained by: (a) varying the enrichment, (b) varying the poison in 1.3% enrichment lattices, and (c) varying the poison in 1.15% enrichment lattices.

Table 4-25

INTRACELL FLUX AVERAGES

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	Enrichment (% U ²³⁵)	$\bar{\phi}_M$	$\bar{\phi}_U$	$\bar{\phi}_{Al}$
1	1.3	1.446	1.125	1.258
1.5	1.3	1.536	1.134	1.270
2	1.3	1.618	1.134	1.260
3	1.3	1.684	1.136	1.265
4	1.3	1.810	1.144	1.295
1	1.15	1.421	1.119	1.245
1.5	1.15	1.449	1.124	1.250
2	1.15	1.591	1.150	1.303
3	1.15	1.632	1.126	1.255
4	1.15	1.722	1.137	1.283
1	1.0	1.410	1.103	1.220
1.5	1.0	1.478	1.119	1.243
2	1.0	1.506	1.111	1.230
3	1.0	1.614	1.115	1.230
4	1.0	1.706	1.123	1.252

$\bar{\phi}_M$ = Average flux in moderator

$\bar{\phi}_U$ = Average flux in uranium

$\bar{\phi}_{Al}$ = Average flux in aluminum sheath

The flux is assumed flat in the air gap.

Table 4-26

THERMAL UTILIZATION, f

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	Enrichment (% U ²³⁵)		
	1.3	1.15	1.0
1	0.946	0.942	0.937
1.5	0.920	0.917	0.909
2	0.893	0.887	0.881
3	0.845	0.836	0.825
4	0.795	0.787	0.773

Table 4-27

RATIO OF U²³⁸ FISSIONS TO THERMAL FISSION

$\frac{\text{Volume Water}}{\text{Volume Uranium}}$	Enrichment (% U ²³⁵)		
	1.3	1.15	1.0
1	.1780	.1784	.1918
1.5	.1346	.1370	.1461
2	.1100	.1094	.1202
3	.0871	.0816	.0914
4	.0734	.0739	.0708

Table 4-28

FAST EFFECT FOR AN INFINITE ARRAY

$\epsilon_{\infty} - 1$

Volume Water Volume Uranium	Enrichment (% U ²³⁵)			
	1.3	1.15	1.0	Average
1	0.102	0.100	0.106	0.103
1.5	0.080	0.080	0.084	0.081
2	0.068	0.067	0.069	0.068
3	0.053	0.048	0.053	0.051
4	0.045	0.043	0.041	0.043

Table 4-29

FAST EFFECT FOR FINITE ARRAY

$\epsilon - 1$

Volume Water Volume Uranium	Enrichment (% U ²³⁵)		
	1.3	1.15	1.0
1	0.097	0.095	0.104
1.5	0.071	0.073	0.079
2	0.059	0.060	0.064
3	0.046	0.043	0.049
4	0.040	0.040	0.039

Table 4-30

EPI-Cd FISSION FRACTIONS IN U²³⁵

Volume Water Volume Uranium	Enrichment (% U ²³⁵)		
	1.3	1.15	1.0
1	.207	.174	.144
1.5	.155	.105	.116
2	.117	.082	.078
3	.062	.054	.045
4	.048	.040	.032

Table 4-31

INFINITE MULTIPLICATION FACTOR, k_{∞} , CALCULATED BY VARIOUS METHODS

Enrichment % U ²³⁵	Volume Water Volume Uranium	One Group	Two Group	Three Group			Age	Age Diffusion
				L ₁ ² 5 cm ²	L ₁ ² 7.5 cm ²	L ₁ ² 10 cm ²		
1.3	1	1.118	1.118	1.119	1.119	1.119	1.120	1.120
1.3	1.5	1.191	1.191	1.192	1.192	1.191	1.194	1.193
1.3	2	1.217	1.216	1.216	1.216	1.216	1.218	1.217
1.3	3	1.200	1.198	1.198	1.198	1.197	1.197	1.196
1.3	4	1.156	1.157	1.158	1.158	1.158	1.159	1.159
1.15	1	1.079	1.079	1.079	1.079	1.079	1.078	1.079
1.15	1.5	1.148	1.149	1.148	1.148	1.148	1.147	1.148
1.15	2	1.171	1.171	1.170	1.170	1.170	1.169	1.169
1.15	3	1.154	1.154	1.153	1.152	1.152	1.149	1.150
1.15	4	1.112	1.113	1.113	1.113	1.113	1.112	1.113
1.0	1	1.037	1.037	1.037	1.037	1.036	1.036	1.036
1.0	1.5	1.109	1.110	1.109	1.109	1.108	1.106	1.107
1.0	2	1.128	1.128	1.127	1.127	1.127	1.123	1.125
1.0	3	1.109	1.109	1.107	1.107	1.106	1.103	1.104
1.0	4	1.065	1.066	1.065	1.065	1.065	1.063	1.064

Table 4-32

RESULTS OF M^2 ANALYSES

Method				
<u>Volume Water</u> <u>Volume Uranium</u>	<u>Enrichment</u> <u>Variation</u>	<u>Poison Variation</u> <u>with 1.3%</u> <u>Enrichment</u>	<u>Poison Variation</u> <u>with 1.15%</u> <u>Enrichment</u>	<u>Combined</u> <u>Analysis</u>
M^2 , cm^2 , One Group Theory				
1	34.20	35.93	36.87	36.85 ± 0.95
1.5	38.41	38.50	36.57	36.81 ± 0.89
2	34.98	36.92	34.80	35.45 ± 0.90
3	36.05	33.10	31.91	32.76 ± 1.18
4	34.20	32.17	30.74	31.04 ± 1.17
L^2_f , cm^2 , Two Group Theory				
1	31.85	33.29	34.38	34.41
1.5	35.89	35.48	33.76	34.00
2	32.36	33.96	31.68	32.27
3	32.88	29.41	28.23	29.12
4	30.59	28.26	27.25	27.48
L^2_2 , cm^2 , Three Group Theory with $L^2_1 = 5 \text{ cm}^2$				
1	26.31	27.62	29.16	29.05
1.5	29.63	29.55	28.31	28.38
2	26.00	27.86	26.07	26.51
3	26.54	23.52	22.60	23.32
4	24.14	22.54	22.02	22.07
L^2_2 , cm^2 , Three Group Theory with $L^2_1 = 7.5 \text{ cm}^2$				
1	23.54	24.99	26.53	26.42
1.5	26.68	26.70	25.65	25.66
2	23.05	24.96	23.36	23.75
3	22.86	20.76	19.91	20.56
4	21.87	19.79	19.45	19.43
L^2_2 , cm^2 , Three Group Theory with $L^2_1 = 10 \text{ cm}^2$				
1	20.95	22.35	24.04	23.85
1.5	23.80	23.96	22.89	22.89
2	20.24	22.16	20.74	21.06
3	20.78	18.05	17.27	17.86
4	19.21	17.14	16.90	16.84
M^2 , cm^2 , Age Theory				
1	31.90	33.40	35.83	35.32
1.5	33.20	34.52	34.57	34.10
2	29.91	32.36	32.30	32.28
3	30.84	29.54	29.37	29.52
4	30.52	29.26	29.78	29.36
τ , cm , Age Diffusion Theory				
1	29.84	31.10	33.48	33.07
1.5	31.34	32.07	32.06	31.68
2	27.96	30.06	29.58	29.61
3	28.47	26.58	26.23	26.53
4	27.63	25.97	26.50	26.16

Table 4-33

BUCKLING, B^2 , AND REFLECTOR SAVINGS, λ ,
(CLEAN LATTICES)

Enrichment, % U^{235}	Volume Water Volume Uranium	B^2 , $cm^{-2} \times 10^4$	λ , cm
1.3	1	32.11 ± 0.54	7.94 ± 0.10
1.3	1.5	51.87 ± 0.50	7.44 ± 0.10
1.3	2	61.08 ± 0.32	7.04 ± 0.06
1.3	3	60.99 ± 0.26	6.70 ± 0.05
1.3	4	50.38 ± 0.27	6.64 ± 0.07
1.15	1	21.33 ± 0.41	8.10 ± 0.12
1.15	1.5	40.23 ± 0.30	7.50 ± 0.08
1.15	2	48.22 ± 0.31	7.10 ± 0.08
1.15	3	47.12 ± 0.33	6.68 ± 0.10
1.15	4	36.03 ± 0.16	6.43 ± 0.06
1.0	1	9.90 ± 0.54	8.76 ± 0.15
1.0	1.5	29.63 ± 0.42	7.47 ± 0.13
1.0	2	36.07 ± 0.39	7.04 ± 0.15
1.0	3	33.15 ± 0.31	6.51 ± 0.16
1.0	4	20.96 ± 0.26	6.46 ± 0.17

Table 4-34

BUCKLING AND REFLECTOR SAVINGS (POISONED LATTICES)

Enrichment, % U^{235}	Volume Water Volume Uranium	B_2O_3 , mg/ml	B^2 , $cm^{-2} \times 10^4$	λ , cm
1.3	1	2.587	10.67 ± 1.48	6.32 ± 0.27
1.3	1.5	1.039	38.63 ± 1.13	6.56 ± 0.26
1.3	1.5	3.452	10.84 ± 1.22	5.82 ± 0.24
1.3	2	2.587	16.38 ± 0.34	5.90 ± 0.10
1.3	3	1.724	12.87 ± 0.83	6.08 ± 0.12
1.3	4	0.500	33.21 ± 0.81	5.90 ± 0.16
1.3	4	0.855	16.93 ± 1.00	6.65 ± 0.25
1.3	4	1.059	10.97 ± 1.30	6.31 ± 0.25
1.15	1	2.98	-2.54 ± 0.58	6.16 ± 0.15
1.15	1	3.327	-6.44 ± 0.46	6.23 ± 0.10
1.15	1.5	2.022	13.17 ± 0.43	6.47 ± 0.10
1.15	1.5	3.926	-8.18 ± 0.40	5.76 ± 0.09
1.15	2	1.911	12.36 ± 0.62	6.52 ± 0.17
1.15	2	3.00	-5.96 ± 1.00	5.95 ± 0.10
1.15	3	1.308	7.70 ± 0.58	6.22 ± 0.18
1.15	4	0.344	22.16 ± 0.36	6.36 ± 0.16
1.15	4	0.746	5.58 ± 0.48	6.78 ± 0.24
1.15	4	1.300	-13.57 ± 0.46	6.53 ± 0.21

FIG. 4-3
THERMAL UTILIZATION (f) FOR URANIUM-WATER LATTICES

FIG. 4-4
RESONANCE ESCAPE PROBABILITY (P) FOR URANIUM-WATER LATTICES

FIG. 4-5
FAST EFFECT (ϵ) FOR URANIUM-WATER LATTICES

FIG. 4-6
INFINITE MULTIPLICATION FACTOR (k_{∞}) FOR URANIUM-WATER LATTICES

FIG. 4-7
BUCKLING (B^2) FOR URANIUM-WATER LATTICES

FIG. 4-8
REFLECTOR SAVING (λ) FOR URANIUM-WATER LATTICES

4.5.2 Graphite-Natural Uranium Lattices⁽³⁸⁾

The data in Tables 4-35 through 4-38 were collected during the course of designing and building the B.N.L. graphite reactor.

Table 4-35

THE MULTIPLICATION FACTOR AS A FUNCTION OF ROD RADIUS OR GRAPHITE-URANIUM VOLUME RATIO

(Data from Ref. 38)

Lattice Dimension = 8 in.; Al to U volume ratio = 0.2;
Cooling Channel Area = 36 cm².

Rod Radius cm	Volume Graphite Volume Uranium	η	ϵ	f	p	k_{∞}	f_{mod}	f_{al}
1.0	119.97	1.315	1.0224	0.8399	0.9287	1.0487	0.1529	0.0072
1.1	99.15	1.315	1.0243	0.8596	0.9168	1.0615	0.1330	0.0074
1.2	83.31	1.315	1.0261	0.8754	0.9044	1.0683	0.1169	0.0077
1.3	70.99	1.315	1.0280	0.8881	0.8915	1.0703	0.1040	0.0079
1.4	61.21	1.315	1.0299	0.8984	0.8783	1.0686	0.0935	0.0081
1.5	53.32	1.315	1.0316	0.9069	0.8647	1.0638	0.0849	0.0082
1.6	46.86	1.315	1.0333	0.9138	0.8507	1.0563	0.0778	0.0084
1.7	41.51	1.315	1.0350	0.9198	0.8368	1.0476	0.0716	0.0086

f_{mod} = fraction of thermal neutrons captured in moderator.

f_{al} = fraction of thermal neutrons captured in aluminum.

Table 4-36

THE MULTIPLICATION FACTOR AS A FUNCTION OF ROD RADIUS
OR GRAPHITE-URANIUM VOLUME RATIO

(Data from Ref. 38)

Lattice Dimension = $8\frac{1}{2}$ in.; Al to U volume ratio = 0.2;
Cooling Channel Area = 36 cm^2 .

Rod Radius cm	Volume Graphite Volume Uranium	η	ϵ	f	p	k_{∞}	f_{mod}	f_{al}
1.0	132.58	1.315	1.0224	0.8256	0.9354	1.0383	0.1673	0.0071
1.1	109.57	1.315	1.0243	0.8469	0.9247	1.0548	0.1458	0.0073
1.2	92.07	1.315	1.0261	0.8637	0.9135	1.0646	0.1287	0.0076
1.3	78.45	1.315	1.0280	0.8774	0.9108	1.0696	0.1148	0.0078
1.4	67.64	1.315	1.0299	0.8885	0.8897	1.0706	0.1035	0.0080
1.5	58.93	1.315	1.0316	0.8976	0.8774	1.0684	0.0942	0.0082
1.6	51.79	1.315	1.0333	0.9053	0.8647	1.0637	0.0864	0.0083
1.7	45.88	1.315	1.0358	0.9117	0.8521	1.0573	0.0798	0.0085

 f_{mod} = fraction of thermal neutrons captured in moderator. f_{al} = fraction of thermal neutrons captured in aluminum.

Table 4-37

THE MULTIPLICATION FACTOR AS A FUNCTION
OF COOLING CHANNEL AREA

(Data from Ref. 38)

Lattice Dimension = 8 in.; Rod Radius = 1.40 cm;
Al to U Volume Ratio = 0.2.

Radius of Channel, cm	η	ϵ	f	p	k_{∞}
2.538	1.315	1.0299	0.8908	0.8885	1.0684
2.738	1.315	1.0299	0.8922	0.8844	1.0687
2.938	1.315	1.0299	0.8941	0.8828	1.0690
3.138	1.315	1.0299	0.8956	0.8812	1.0690
3.385	1.315	1.0299	0.8978	0.8792	1.0690
3.638	1.315	1.0299	0.9001	0.8769	1.0690
4.138	1.315	1.0299	0.9049	0.8721	1.0688
4.638	1.315	1.0299	0.9091	0.8666	1.0670

Table 4-38

THE MULTIPLICATION FACTOR AS A FUNCTION
OF THE RATIO OF ALUMINUM VOLUME TO
URANIUM VOLUME

(Data from Ref. 38)

Lattice Dimension = 8 in.; Rod Radius = 1.40 cm;
Cooling Channel Area = 36 cm^2 .

Volume Aluminum Volume Uranium	f	f_{al}	f_{mod}	k_{∞}
0.	0.9057	0.0	0.0943	1.0773
0.10	0.9020	0.0041	0.0939	1.0729
0.15	0.9003	0.0061	0.0936	1.0709
0.20	0.8984	0.0081	0.0935	1.0686
0.25	0.8966	0.0101	0.0933	1.0665
0.30	0.8949	0.0120	0.0931	1.0645
0.35	0.8930	0.0140	0.0930	1.0622
0.40	0.8913	0.0160	0.0927	1.0602
0.45	0.8894	0.0179	0.0927	1.0579
0.50	0.8875	0.0199	0.0926	1.0557

 f_{mod} = fraction of thermal neutrons captured in moderator. f_{al} = fraction of thermal neutrons captured in aluminum.

4.5.3 Heavy Water-Natural Uranium Lattices⁽⁵³⁾

The results of experiments on D₂O-uranium lattices are summarized in Table 4-39. The data include: buckling, ratio of flux at the surface of the fuel to the average flux in the fuel, and the ratio of the average flux in the moderator to the average flux in the fuel.

Table 4-39

SUMMARY OF D₂O-URANIUM LATTICES

Square lattices; Al clad is 0.1 cm thick

(Data from Ref. 53)

Enrichment (% U ²³⁵)	Rod Diameter in.	Cell Side in.	Volume D ₂ O Volume Uranium	Buckling B ² , cm ⁻²	Surface Flux Avg. Rod Flux	Avg. Mod. Flux Avg. Rod Flux
0.4962	1.00	4.50	24.82	1.76 x 10 ⁻⁴	1.162	1.537
		4.90	29.64	2.10		1.563
		7.50	71.02	1.60		1.633
		12.00	183.7	0.105		1.792
	2.00	7.25	15.71	1.61	1.420	2.427
		9.00	24.80	2.25		2.594
		12.00	44.94	1.2		2.796
		3.00	19.35	7.60	1.133	1.406
	0.7205	4.50	45.09	7.48		1.474
		6.00	81.13	5.40		1.508
		1.00	3.625	15.69	1.190	1.520
		4.50	24.82	8.47		1.578
		4.90	29.64	8.36		1.598
		6.00	45.03	7.25		1.657
		7.50	71.02	5.36	1.290	1.718
		9.00	102.8	3.92		1.789
		12.00	183.7	2.11		1.901
	1.25	4.50	15.47	8.32		1.809
		6.00	28.39	8.30		1.918
		7.50	44.99	6.84		1.976
		9.00	65.29	5.35		2.064
	1.50	12.00	117.0	2.92	1.356	2.237
		6.00	19.36	8.65		2.139
		7.25	28.79	7.94		2.221
		9.00	44.97	6.29		2.341
0.9124	2.00	12.00	80.81	3.69	1.502	2.527
		7.25	15.71	8.23		2.560
		9.00	24.80	7.22		2.806
		12.00	44.94	4.76	1.238	3.174
	1.00	3.625	15.69	12.6		1.721
		4.90	29.64	12.5		1.829
		6.00	45.03	10.6		1.924
		7.50	71.02	7.81		1.968
	1.00	9.00	102.8	5.85	3.13 x 10 ⁻⁴	2.0069
		12.00	183.7	3.13 x 10 ⁻⁴		2.131

4.5.4 Light Water- UO_2 Lattices (54,55)

Experiments similar to those of Kouts and Sher (Section 4.5.1) but utilizing UO_2 as fuel, were performed by a group at the Bettis Plant. In these experiments, the enrichment was nominally 1.3% U^{235} . Some lattices of metal rods, such as used by Kouts and Sher, were also reported. The data are summarized in Tables 4-40 through 4-44. Tables 4-40, 4-41, and 4-42 are reproduced from Ref. 54 and Tables 4-43 and 4-44 from Ref. 55.

Table 4-40

THERMAL UTILIZATION OF UO_2 LATTICES

(Reproduced from Ref. 54)

Rod Dia. in.	Fuel	Fuel Density gm/cm^3	Enrichment % U^{235}	Volume Water Volume Uranium	Volume Water Volume UO_2	Disadvantage factor ϕ_m/ϕ_u	Thermal Utilization f
0.600	Metal	-	1.3	1.5	-	1.39 ± 0.04	0.918 ± 0.004
0.600	Metal	-	1.3	2.0	-	1.45 ± 0.04	0.890 ± 0.004
0.600	Metal	-	1.3	3.0	-	1.49 ± 0.04	0.843 ± 0.003
0.600	Metal	-	1.15	2.0	-	1.45 ± 0.04	0.881 ± 0.004
0.600	Metal	-	1.15	3.0	-	1.45 ± 0.04	0.835 ± 0.005
0.387	Metal	-	1.3	2.0	-	1.21 ± 0.04	0.904 ± 0.003
0.387	Metal	-	1.3	3.0	-	1.31 ± 0.04	0.857 ± 0.004
0.600	Oxide	7.53	1.3	3.0	1.07	1.09 ± 0.03	0.873 ± 0.004
0.600	Oxide	7.53	1.3	4.0	1.40	1.14 ± 0.03	0.837 ± 0.005
0.600	Oxide	7.53	1.3	5.0	1.76	1.16 ± 0.03	0.805 ± 0.005
0.384	Oxide	7.53	1.3	4.0	1.39	1.10 ± 0.01	0.836 ± 0.004
0.384	Oxide	7.53	1.3	4.9	1.73	1.10 ± 0.01	0.807 ± 0.004
0.384	Oxide	10.53	1.3	2.9	1.42	1.10 ± 0.01	0.874 ± 0.004
0.384	Oxide	10.53	1.3	3.6	1.78	1.13 ± 0.01	0.848 ± 0.004
0.384	Oxide	10.53	1.3	4.9	2.40	1.13 ± 0.01	0.809 ± 0.004

Table 4-41

RESONANCE ESCAPE PROBABILITY OF UO_2 LATTICES

(Reproduced from Ref. 54)

Rod Dia. in.	Fuel	Fuel Density gm/cm^3	Enrichment % U^{235}	Volume Water Volume Uranium	Volume Water Volume UO_2	Resonance Escape $p_{\text{U}^{238}}$
0.600	U metal	-	1.3	1.5	-	0.720 ± 0.014
0.600	U metal	-	1.3	2.0	-	0.794 ± 0.016
0.600	U metal	-	1.3	3.0	-	0.850 ± 0.017
0.600	U metal	-	1.15	2.0	-	0.785 ± 0.005
0.600	U metal	-	1.15	3.0	-	0.833 ± 0.007
0.387	U metal	-	1.3	2.0	-	0.747 ± 0.005
0.387	U metal	-	1.3	3.0	-	0.808 ± 0.005
0.600	UO_2	7.53	1.3	3.0	1.07	0.805 ± 0.006
0.600	UO_2	7.53	1.3	4.0	1.40	0.838 ± 0.003
0.600	UO_2	7.53	1.3	5.0	1.76	0.869 ± 0.003
0.384	UO_2	7.53	1.3	4.0	1.39	0.832 ± 0.007
0.384	UO_2	7.53	1.3	5.0	1.73	0.853 ± 0.004
0.384	UO_2	10.53	1.3	2.9	1.42	0.775 ± 0.004
0.384	UO_2	10.53	1.3	3.6	1.78	0.815 ± 0.004
0.384	UO_2	10.53	1.3	4.9	2.40	0.850 ± 0.004

Table 4-42

FAST EFFECT VALUES OF UO_2 LATTICES

(Reproduced from Ref. 54)

Rod Dia. in.	Enrichment % U^{235}	Fuel Composition	Fuel Density gm/cm^3	Volume Water Volume Uranium	Volume Water Volume UO_2	Fast Fission Factor, G
0.600	1.15	Metal	-	2.0	-	1.062 ± 0.006
0.600	1.15	Metal	-	3.0	-	1.048 ± 0.004
0.387	1.3	Metal	-	2.0	-	1.059 ± 0.005
0.387	1.3	Metal	-	3.0	-	1.046 ± 0.004
0.600	1.3	UO_2	7.53	3.0	1.07	1.042 ± 0.004
0.600	1.3	UO_2	7.53	4.0	1.40	1.035 ± 0.003
0.600	1.3	UO_2	7.53	5.0	1.76	1.030 ± 0.002
0.384	1.3	UO_2	7.53	4.0	1.39	1.037 ± 0.002
0.384	1.3	UO_2	7.53	5.0	1.73	1.032 ± 0.002
0.384	1.3	UO_2	10.53	2.9	1.42	1.046 ± 0.002
0.384	1.3	UO_2	10.53	3.6	1.78	1.042 ± 0.002
0.384	1.3	UO_2	10.53	4.9	2.40	1.034 ± 0.002

Table 4-43

RESULTS OF BUCKLING AND REFLECTOR SAVINGS MEASUREMENTS

(Reproduced from Ref. 55)

Lattice Designation	Critical No. of Fuel Rods at 20°C	Critical Buckling		Radial Reflector Savings		Total Axial Reflector Savings cm
		Bettis (20°C)	BNL(40)	Bettis	BNL(40)	
Fuel	Vol. H_2O Vol. U	$(\text{cm}^{-2}) \times 10^4$		cm	cm	
U Metal	2	335 $\pm$ 1	61.12**	61.08 $\pm$ 0.32	7.04 $\pm$ 0.06	
1.3%, 0.600 in.	3	264 $\pm$ 1	61.09**	60.99 $\pm$ 0.26	6.70 $\pm$ 0.05	
U Metal	2	464 $\pm$ 1	48.31**	48.22 $\pm$ 0.31	7.10 $\pm$ 0.25	
1.15%, 0.600 in.	3	377 $\pm$ 1	47.26**	47.12 $\pm$ 0.33	6.68 $\pm$ 0.10	
U Metal	2	904 $\pm$ 2	53.55 $\pm$ 0.48	52.56 $\pm$ 0.39	7.40(2)* $\pm$ 0.17	14.07(1) $\pm$ 0.71
1.3%, 0.387 in.	3	630 $\pm$ 2	58.2 $\pm$ 1.00	59.25 $\pm$ 0.33	7.22(1) $\pm$ 0.31	14.23(1) $\pm$ 0.82
UO_2 , 7.53 gm/cm ³	3	1269 $\pm$ 3	28.37 $\pm$ 0.06		8.55(5) $\pm$ 0.06	17.82(5) $\pm$ 0.23
1.3%, 0.601 in.	4	1027 $\pm$ 3	30.17 $\pm$ 0.06		8.36(4) $\pm$ 0.06	16.72(4) $\pm$ 0.19
	5	987 $\pm$ 3	29.06 $\pm$ 0.07		7.92(4) $\pm$ 0.07	15.90(5) $\pm$ 0.22
UO_2 , 7.52 gm/cm ³	4	3045 $\pm$ 3	25.28 $\pm$ 0.10		8.41(4) $\pm$ 0.12	17.06(6) $\pm$ 0.51
1.3%, 0.388 in.	5	2784 $\pm$ 3	25.21 $\pm$ 0.10		8.08(4) $\pm$ 0.12	15.37(3) $\pm$ 0.26
UO_2 , 10.53 gm/cm ³	3	2173 $\pm$ 3	32.59 $\pm$ 0.15		7.86(6) $\pm$ 0.12	15.27(11) $\pm$ 0.41
1.3%, 0.383 in.	3.6	1755 $\pm$ 3	35.47 $\pm$ 0.18		7.41(5) $\pm$ 0.13	14.69(8) $\pm$ 0.43
	5	1575 $\pm$ 3	34.22 $\pm$ 0.13		7.10(7) $\pm$ 0.10	14.52(8) $\pm$ 0.17

*Number of measurements indicated in parentheses.

**Calculated from number of fuel rods using BNL values of radial reflector savings and assuming the axial reflector savings is twice the radial value.

Table 4-44

TEMPERATURE COEFFICIENT RESULTS OF UO_2 LATTICES

(Reproduced from Ref. 55)

Lattice Designation	Fuel	Vol. H_2O /Vol. U	Temperature Coefficient $\partial\rho/\partial\theta$ ($^\circ\text{C}^{-1}$) $\times 10^4$	
			20°C	40°C
U Metal	2		-0.36(2)* $\pm$ 0.02	-0.76 $\pm$ 0.02
1.3%, 0.600 in.	3		+0.10(6) $\pm$ 0.02	-0.19 $\pm$ 0.02
U Metal	2		-0.40(12) $\pm$ 0.02	-0.74 $\pm$ 0.02
1.15%, 0.600 in.	3		+0.11(4) $\pm$ 0.02	-0.14 $\pm$ 0.02
U Metal	2		-0.46(8) $\pm$ 0.04	-0.82 $\pm$ 0.04
1.3%, 0.387 in.	3		-0.47(6) $\pm$ 0.02	-0.78 $\pm$ 0.02
UO_2 , 7.53 gm/cm ³	3		-0.71(10) $\pm$ 0.02	-0.93 $\pm$ 0.02
1.3%, 0.601 in.	4		-0.66(6) $\pm$ 0.02	-0.87 $\pm$ 0.02
	5		-0.55(6) $\pm$ 0.02	-0.83 $\pm$ 0.02
UO_2 , 7.52 gm/cm ³	4		-0.85(5) $\pm$ 0.02	-1.02 $\pm$ 0.02
1.3%, 0.388 in.	5		-0.90(4) $\pm$ 0.03	-1.01 $\pm$ 0.03
UO_2 , 10.53 gm/cm ³	3		-0.57(27) $\pm$ 0.07	-0.89 $\pm$ 0.07
1.3%, 0.383 in.	3.6		-0.69(38) $\pm$ 0.05	-0.87 $\pm$ 0.05
	5		-0.72(10) $\pm$ 0.02	-0.87 $\pm$ 0.02

*Number of measurements at all temperatures.

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SECTION 5

INTERMEDIATE REACTORS

Intermediate reactors are those reactors whose median-energy-of-fission producing neutrons lie in a range of a few electron volts to a few thousand electron volts. Physically, this range corresponds to fuel-to-moderator atom ratios of about 1:1000 to 1:20, depending upon the moderator. They are also called "resonance" reactors because the bulk of the neutron flux lies in the resonance region of the heavy element cross sections.

This section also includes the reflector moderated reactor. The core contains little or no moderator but is well reflected with a material of low mass number - commonly Be. The median energy of fission may vary from the thermal range at the reflector fuel interface, to the intermediate range in parts of the fuel region well removed from the reflector.

5.1 MULTIGROUP AGE-DIFFUSION CONSTANTS

The age-diffusion equation, for purposes of multigroup calculations, is reduced to a set of linear differential equation:^(1,2)

$$-\left(\frac{\Delta u_i}{3\xi\Sigma_{s,i}\Sigma_{tr,i}}\right)\nabla^2(q_i)_{avg} + \left(\frac{\Delta u_i\Sigma_{a,i}}{\xi\Sigma_{s,i}}\right)(q_i)_{avg} + q_i^O = q_i^I + Q_{f,i} + Q_{in,i} \quad (5-1)$$

where

Δu_i = the lethargy width of group i . $u = \ln(10/E(\text{Mev}))$

$\Sigma_{s,i}$ = scattering cross section in group i .

$\Sigma_{a,i}$ = total non-elastic cross section for group i .

$= \Sigma_{f,i} + \Sigma_{c,i} + \Sigma_{in,i}$

$\Sigma_{f,i}$ = fission cross section of group i .

$\Sigma_{c,i}$ = capture cross section of group i

$\Sigma_{in,i}$ = inelastic removal cross section for group i .

$\Sigma_{tr,i} = \Sigma_{a,i} + \Sigma_{s,i}(1 - \bar{\mu})$, where $\bar{\mu}$ is the average cosine of scattering angle.

ξ = average logarithmic energy loss per collision.

$Q_{f,i}$ = source into group i from fission.

$Q_{in,i}$ = source into group i from inelastic scattering.

q_i^I = elastic slowing down density into group i .

q_i^O = elastic slowing down density out of group i .

$q_{i,avg}$ = average elastic slowing down density in group i .

In order to solve Eq. (5-1), it is necessary to provide a relationship between q_i^O , q_i^I , and $(q_i)_{avg}$. This relationship is selected by Ehrlich and Hurwitz⁽¹⁾ to be:

$$(q_i)_{avg} = \omega_1 q_i^I + \omega_2 q_i^O \quad (5-2)$$

There remains the choice of ω_1 and ω_2 . Two different choices are in use at KAPL.

Scheme A

$\omega_1 = 0$, and ω_2 is chosen by trial and error for each region and group so that $\omega_2 \times$ (the spatial average of $(q_i^O)_{avg}$) gives a reasonable value of the spatial average of $(q_i)_{avg}$.

In addition to the more general trial and error method, there is an approximation procedure* which provides a simple formula for finding ω_2 . In the diffusion approximation model with continuous slowing down in a region of homogeneous composition and for an energy range in which elastic scattering constitutes the only source of neutrons, we have:

$$-\frac{1}{3\Sigma_a} \nabla^2 q + \frac{\Sigma_a}{\xi\Sigma_s} q = -\frac{\partial q}{\partial u} \quad (5-3)$$

If we let B_T^2 be the total buckling and approximate: $\nabla^2 q = -B_T^2 q$, then Eq. (5-3) becomes an ordinary differential equation with the solution:

$$q = q_0 \exp \left[- \int_{u_0}^u \left(\frac{\Sigma_a}{\xi\Sigma_s} + \frac{B_T^2}{3\Sigma_{tr}\xi\Sigma_s} \right) du \right] \quad (5-4)$$

For a lethargy group $u_0 \leq u \leq u_1$ chosen so that the integrand of Eq. (5-4) is approximately constant through the interval, q may be written:

$$q = q_0 \exp \left[y \frac{u-u_0}{u_1-u_0} \right]$$

where

$$y = \int_{u_0}^{u_1} du \left[\frac{\Sigma_a}{\xi\Sigma_s} + \frac{B_T^2}{3\Sigma_{tr}\xi\Sigma_s} \right]$$

The average of q in the group will be

$$\begin{aligned} \bar{q}_i &= \frac{1}{\Delta u_i} \int_{u_0}^{u_1} q du \\ &= \frac{q_0}{y} (1 - e^{-y}) \end{aligned} \quad (5-5)$$

and

$$q_i = q_0 e^{-y}$$

*R. Stark, Vallecitos Atomic Laboratory, private communication.

ω_2 is then just the ratio of $\bar{q}_i$ and q_i , so that

$$\omega_2 = \frac{(e^{-y} - 1)}{y} \quad (5-6)$$

Some modification would be necessary to extend this procedure to those energy regions containing inelastic scattering and fission sources; however, good results are obtained by using $\omega_1 = 0$ and $\omega_2 = 1$ for such regions.

Scheme B

$$\omega_1 = \omega_2 = \frac{1}{2}$$

Scheme C

Scheme C, in use at ORNL,^(1,2) assumes a linear relationship of Φ in the group. It relates ω_1 and ω_2 with the values of the reciprocal of $\xi\Sigma_s$ at the high end of the group, $(1/\xi\Sigma_s)^I$; at the low end of the group, $(1/\xi\Sigma_s)^O$; and the average over the group, $(1/\xi\Sigma_s)_{\text{avg}}$.

$$\omega_1 = \frac{(1/\xi\Sigma_s)^I}{(2/\xi\Sigma_s)_{\text{avg}}}$$

$$\omega_2 = \frac{(1/\xi\Sigma_s)^O}{(2/\xi\Sigma_s)_{\text{avg}}}$$

The criteria for choosing one particular scheme is summarized as follows:

- (1) When a small number of groups is used, $\omega_1 = 0$ and ω_2 is estimated for each group and region of the assembly with the aim of making the spatial average of $(\omega_2 q)$ approximately equal to the average of the spatial averages of q^O and q^I .
- (2) When the variation of flux with position is not too rapid, then:

$$\omega_1 = \omega_2 = 1/2 \text{ is a convenient form to assume.}$$

- (3) When Φ^I , Φ^O , and Φ_{avg} are all continuous across the interface between regions, the ORNL scheme tends to reduce oscillations at the interface between very dissimilar regions.

For greater precision, corrections to the ω 's may be needed. Once ω_1 and ω_2 are chosen, the multigroup age-diffusion equation is solved. The flux values thus obtained are used to calculate the degradation from group to group at any reactor point compared to the average degradation in the group. New values of ω_1 and ω_2 can be deduced and adjustments made as indicated. If necessary, the procedure may be repeated.

In Eq. (5-1) $\Sigma_{\text{tr},i}$, $\Sigma_{\text{a},i} = \Sigma_{\text{c},i} + \Sigma_{\text{f},i} + \Sigma_{\text{in},i}$, and $\xi\Sigma_{\text{s},i}$ are the average macroscopic cross sections for the region. The group averaged values for the microscopic cross section (σ_{tr} , σ_{a} , σ_{f} , $\xi\sigma_{\text{s}}$) of various materials⁽³⁾ are given in Table 5-1. The large number of significant figures listed are reproduced from computing machine tapes at KAPL; they are not indicative of the accuracy to which the fundamental cross sections are known. Later cross-sections for hafnium isotopes are included instead of the short file listing.⁽⁴⁾

The thermal group has been omitted since this group will always need adjustment to fit individual problems. Appropriate thermal group data are given in Section 3.1.

The multigroup program for which the cross sections were prepared could not handle inelastic scattering; therefore values of σ_{in} are not included. Inelastic effects can only be incorporated by adjustment of $\xi\sigma_{\text{s}}$.

Table 5-1
MULTIGROUP CROSS SECTIONS FOR INTERMEDIATE REACTOR CALCULATIONS
(Microscopic cross sections in barns)

				²³⁵ U			Lithium			Beryllium Larger Crystal			
Group	E _L	u = ln(10/E _L)	Δu	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	
1	6.065	Mev	0.5	0.5	1.301	1.300	0.391965	1.35193	0.0	0.269100	1.2000	0.000017	
2	3.679		1.0	0.5	1.3040	1.3000	.393315	1.35668	.0	.278379	1.2396	.000017	
3	1.353		2.0	1.0	1.3129	1.3000	.355871	1.22368	.0	.339405	1.4232	.000017	
4	0.4979		3.0	1.0	1.3815	1.3550	.303635	1.03930	.0	.637172	2.6685	.000017	
5	0.1832		4.0	1.0	1.6408	1.5575	.511299	1.31420	.000004	0.841315	3.7137	.000017	
6	0.003355	Mev	8.0	4.0	3.4311	2.8156	.303010	1.03675	.000036	1.148653	5.1183	.000018	
7	167.0	ev	11.0	3.0	15.1150	10.5242	.397021	1.36473	.000191	1.224626	5.6779	.000060	
8	32.27		12.5	1.5	41.8325	28.0583	.453437	1.56376	.000572	1.241500	5.5521	.000174	
9	10.68		13.75	1.25	64.0020	43.5700	.486909	1.67881	.001099	1.242000	5.5560	.000345	
10	3.059		15.0	1.25	71.7498	37.1550	.524911	1.81082	.002076	1.242002	5.5560	.000650	
11	1.1254		16.0	1.0	29.8088	19.4822	.556009	1.91774	.003581	1.242001	5.5560	.001137	
12	0.4140		17.0	1.0	81.2040	67.0980	.580295	2.00181	.005727	1.242001	5.5571	.001874	
13	0.1956		17.75	0.75	200.8333	161.6667	.606318	2.09219	.009236	1.233300	5.5159	.002920	
14	0.07563	ev	18.7	0.95	266.3895	222.6579	0.631480	2.17839	0.014124	1.106365	4.9326	0.004554	
Beryllium Micro Crystal				Boron			Carbon			Oxygen			
Group	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	
1	0.269100	1.2000	0.000017	0.32715	1.474	0.1513	0.195890	1.16210	0.0	0.128259	0.7523	0.000120	
2	.278379	1.2396	.000017	.30971	1.405	.1292	.233647	1.33985	.0	.184646	1.0046	.000120	
3	.339405	1.4232	.000017	.32188	1.509	.1121	.305273	1.82065	.0	.196094	1.1192	.000120	
4	.637172	2.6685	.000017	.38979	2.033	.1414	.437350	2.56138	.000001	.455593	3.0260	.000120	
5	0.841315	3.7137	.000017	.55661	3.223	0.3066	.599168	3.57074	.000001	.576000	4.0832	.000120	
6	1.148653	5.1183	.000018	.73782	4.901	1.0135	.726195	4.34246	.000004	.480000	3.8300	.000120	
7	1.224526	5.4779	.000060	.74820	8.195	4.6013	.741600	4.43900	.000020	.479950	3.8295	.000120	
8	1.241500	5.5521	.000174	.74820	16.863	13.4033	.741600	4.43900	.000059	.475190	3.7940	.000120	
9	1.242000	5.5560	.000345	.74820	30.191	27.0280	.741600	4.43900	.000116	.468593	3.7448	.000120	
10	1.242002	5.5560	.000650	.74820	51.299	49.5710	.741601	4.43905	.000216	.454509	3.6273	.000120	
11	1.242001	5.5560	.001137	.74820	89.718	87.3687	.741601	4.43904	.000377	.451823	3.6051	.000124	
12	1.242001	5.5569	.001874	.74820	143.925	142.8000	.741600	4.43904	.000621	.451313	3.6004	.000187	
13	1.240996	5.5500	.002917	.74820	219.308	218.1667	.741600	4.43900	.000959	.456000	3.6400	.000292	
14	1.221725	5.3204	0.004524	0.74820	333.577	336.3158	0.741600	4.43900	0.001472	0.467415	3.7332	0.000446	
Fluorine				Sodium			Magnesium			Aluminum			
Group	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	
1	0.15300	3.060	0.0	0.176669	1.4630	0.000290	0.163951	1.4462	0.001114	0.169162	1.6564	0.000398	
2	.15300	3.109	.0	.189482	1.5942	.000290	.170577	1.5241	.001114	.176363	1.7510	.000398	
3	.23378	5.031	.0	.204100	1.8155	.000290	.193419	1.7557	.001114	.192000	1.9977	.000398	
4	.37526	9.151	.0	.338832	2.9573	.000290	.279264	2.8089	.001114	.256000	2.9537	.000398	
5	.75021	20.883	.0	.333708	3.6194	.000290	.460165	5.1641	.001114	.288000	3.8627	.000398	
6	.53767	15.259	.00002	.373191	4.0488	.001274	.360002	4.3050	.001114	.115514	1.1837	.000429	
7	.34680	9.842	.00006	.362042	3.8335	.005863	.283178	3.4371	.001114	.115659	1.5619	.001545	
8	.34680	9.843	.00018	.280500	3.2064	.008817	.272000	3.3100	.001218	.111600	1.5191	.004415	
9	.34680	9.843	.00035	.280500	3.2264	.017693	.272000	3.3100	.002204	.111600	1.5200	.008718	
10	.34680	9.844	.00068	.280500	3.2244	.033338	.272000	3.3100	.004114	.111600	1.5284	.016144	
11	.34680	9.846	.00120	.280500	3.2545	.058180	.272000	3.3190	.007102	.111600	1.5403	.028385	
12	.34706	9.855	.00195	.280518	3.2989	.096323	.272000	3.3202	.011854	.111600	1.5579	.046912	
13	.35071	9.962	.00299	.284486	3.3790	.150214	.272000	3.3300	.018248	.111600	1.5849	.073233	
14	0.35777	10.167	0.00459	0.292695	3.5668	0.231106	0.270446	3.3153	0.022066	0.111600	1.6201	0.111029	
				Calcium			Iron			Zirconium			
Group	E _L	u = ln(10/E _L)	Δu	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	
1	6.065	Mev	0.5	0.5	0.11066	2.21	0.00002	0.102499	2.0630	0.005000	0.089705	4.11636	0.020000
2	3.679		1.0	0.5	.11311	2.26	.00003	.103373	2.1133	.005000	.085435	3.92406	.020000
3	1.353		2.0	1.0	.11803	2.36	.00005	.096261	2.0847	.005000	.100737	4.57212	.020000
4	0.4979		3.0	1.0	.12787	2.56	.00008	.093760	2.2790	.005000	.150610	6.75302	.020000
5	0.1832		4.0	1.0	.13033	2.61	.00013	.119274	3.1853	.005000	.194538	8.83524	.020000
6	0.003355	Mev	8.0	4.0	.06934	1.84	.45000	.123135	2.0118	.005040	.175744	7.82460	.296251
7	167.0	ev	11.0	3.0	.09836	1.97	.00300	.285155	7.6370	.013162	.145461	6.41565	.290793
8	32.27		12.5	1.5	.11557	2.32	.00800	.383642	10.9916	.042997	.136100	6.20000	.003261
9	10.68		13.75	1.25	.12738	2.57	.01600	.384150	11.0329	.086304	.136100	6.20000	.006439
10	3.059		15.0	1.25	.13328	2.69	.03000	.384300	11.1243	.163136	.136100	6.20001	.012014
11	1.1254		16.0	1.0	.14262	2.90	.05000	.384388	11.2625	.289152	.136100	6.20001	.020962
12	0.4140		17.0	1.0	.14852	3.05	.08000	.384700	11.4562	.476280	.136100	6.20001	.034560
13	0.1956		17.75	0.75	.15344	3.20	.13000	.384700	11.7227	.727500	.136100	6.20000	.053355
14	0.07563	ev	18.7	0.95	0.15590	3.32	0.20000	0.384700	12.1126	1.134868	0.136100	6.20000	0.081899
Hafnium - 177				Hafnium - 178			Cadmium			Stainless Steel			
Group	σ _a	σ _a		σ _a	σ _a		ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a	
1	0.475	0.0		0.450	0.1000		0.102499	2.0630	0.006096				
2	.568	.0		.515	.1000		.103373	2.1133	.006096				
3	.650	.0		.543	.1000		.096261	2.0847	.006095				
4	.826	.0		.623	.1000		.093760	2.2790	.006096				
5	1.175	.0		.758	.1000		.119274	3.1853	.006096				
6	2.619	.0		1.727	.1000		.123135	2.0118	.006146				
7	23.03	.0		12.63	.1000		.285155	7.6370	.016059				
8	67.99	.0		39.33	.1000		.383642	10.9916	.052458				
9	59.04	3.38		48.78	.1000		.384150	11.0329	.105293				
10	722.2	1446.		202.0	.3560		.384500	11.1243	.199028				
11	3708.	8.80		7.624	4.147		.384388	11.2625	.352750				
12	2577.	14.50		12.57	73.401		.384700	11.4562	.581084				
13	173.5	22.36		19.38	2142.947		.384700	11.7227	0.887500	0.174667 (10 ⁶)			
14	206.7	34.32		29.75	4811.595		0.384700	12.1126	1.384500	1.871180 (10 ⁶)			
Hydrogen in Water				Hydrogen in Polymers									
Group	ξσ _a	σ _{tr}	σ _a	ξσ _a	σ _{tr}	σ _a							
1	1.123563	0.36694	0.000018	1.123563	0.36694	0.000018							
2	1.651363	.54128	.000024	1.651363	.54128	.000024							
3	2.671317	0.86281	.000034	2.671317	.86281	.000034							
4	4.645381	1.50919	.000057	4.645381	1.50919	.000057							
5	7.663053	2.50083	.000093	7.663053	2.50083	.000093							
6	15.954918	5.07786	.000353	15.954918	5.07786	.000353							
7	19.979062	6.66219	.002087	19.979062	6.66219	.002087							
8	20.000000	6.67105	.006009	20.000000	6.67105	.006009							
9	20.000000	6.68.6											

Cross sections for hydrogen (in water and polymers) are included for convenience even though age-diffusion theory does not treat hydrogen scattering properly. Also, the prescription for chemical binding effects on hydrogen in the low-energy groups is quite arbitrary.

According to Greebler,* this set of cross sections is appropriate for reactors whose median-energy-of-fission is from a few tens to a few hundreds of electron-volts. Some modification must be considered (possibly in conjunction with Section 6) for energies of a few thousands of electron volts.

5.2 SELF-SHIELDING EFFECTS

(For a discussion of self-shielding effects in foils, see Section 8.1.2.)

Intermediate reactors usually have the fuel dispersed in a heterogeneous array. Although self-shielding effects are not as pronounced as in the conventional thermal heterogeneous reactor, nevertheless they must be considered.

There is an important difference between the treatment of thermal heterogeneous systems (see Section 4.1) and of intermediate systems. In the latter case it is usually necessary to account only for the flux depression within the fuel element compared to the surface flux. It is not necessary to account for the flux rise between fuel elements. Furthermore, the fuel elements are usually quite highly enriched; therefore they may be considered pure absorbers and scattering may be neglected.

The self-shielding factor, F , is defined as the ratio of the average flux within the fuel element, to the surface flux. An effective absorption cross section, σ_e , may be related to the true absorption cross section,^(5,6) σ_a , by:

$$\sigma_e = F\sigma_a$$

The product of σ_e , and the surface flux, Φ , then gives the reaction rate for the fuel material. The angular distribution of the incident neutron flux is assumed isotropic, i.e., the number of neutrons crossing a unit surface area per unit time and having a direction of motion in the solid angle, $d\Omega$, is proportional to $\cos \alpha d\Omega$, where α is the angle between the direction of motion and the normal to the surface. The number of neutrons striking the body is $(\Phi S/4)$ where Φ is the flux, and S is the surface area. The absorption probability, or blackness, β , for an average incident neutron is related to F by:

$$\beta = 2\xi F$$

where

$$\xi = \frac{2V}{S} \Sigma$$

V = Volume of the body

Σ = Macroscopic absorption cross section of the body and it is assumed that no scattering occurs.

*Private communication from P. Greebler of Vallecitos Atomic Laboratory.

With reference to Fig. 5-1:

α = angle between neutron velocity vector and normal to the surface.

$$\phi = -\pi/2 \text{ to } +\pi/2$$

$$\theta = -\pi/2 \text{ to } +\pi/2$$

The number of neutrons entering per unit solid angle is proportional to $\cos \alpha$; in terms of θ and ϕ , it is proportional to $\cos \theta \cos \phi$. Assuming that each collision results in an absorption, then the fraction of neutrons absorbed is:

$$1 - e^{-\ell \Sigma}$$

where ℓ is the neutron path length through the solid portion of the absorber. The general expression for the fraction of neutrons absorbed is then:

$$\beta = \frac{1}{\pi} \int \int d\theta d\phi \cos \theta \cos^2 \phi (1 - e^{-\ell \Sigma}) \quad (5-3)$$

The self-shielding factors, F , for simple geometries:

Slab

$$F = \frac{1}{2\xi} \left(1 - e^{-\xi} + \xi e^{-\xi} - \xi^2 \int_{\xi}^{\infty} \frac{e^{-u}}{u} du \right)$$

Cylinder

$$F = \frac{1}{2\xi} \left(1 - \frac{3}{16\xi^2} - \frac{15}{128\xi^4} - \frac{575}{4096\xi^6} \dots \right)$$

Sphere

$$F = \frac{2}{9\xi^3} \left[\frac{9\xi^2}{4} - \frac{1}{2} + e^{-3\xi} \left(\frac{1}{2} + \frac{3\xi}{2} \right) \right]$$

are presented graphically in Fig. 5-2. Corresponding values of β are given in Fig. 5-3. For the slab case see also Figs. 8-2 and 8-3.

For a hollow sphere:

$$F = \frac{1 - \frac{2}{m} \left\{ \frac{1}{m} - \left[\frac{1}{m} + q \right] e^{-qm} - 2 \int_q^1 x \exp \left[-m(x - \sqrt{x^2 - q^2}) \right] dx \right\}}{m \left[q^2 - \frac{2}{3} q^3 - \frac{1}{4} q^4 - \frac{1}{4 \cdot 6} q^6 - \frac{3}{4 \cdot 6 \cdot 8} q^8 - \dots \right] + \frac{2}{5} m q^3} \quad (5-4)$$

where

$$q = \left[\frac{2t}{R} - \left(\frac{t}{R} \right)^2 \right]^{1/2} = \cos^{-1} \theta_1$$

$$x = \cos \theta$$

$$m = 2 \Sigma R$$

and θ , θ_1 , R and t are defined in Fig. 5-4.

FIG. 5-1
GEOMETRICAL NOTATION FOR
NEUTRONS INCIDENT ON BODY

FIG. 5-2
SELF-SHIELDING FACTOR (F) FOR SPHERE, CYLINDER, AND SLAB
(REPRODUCED FROM REF. 7)

FIG. 5-3
ABSORPTION PROBABILITY (β) FOR SPHERE, CYLINDER, AND SLAB
(REPRODUCED FROM REF. 7)

Self-shielding factors, F , for hollow spheres are plotted in Fig. 5-5 and for various geometries of equal volume in Fig. 5-6.

FIG. 5-4
GEOMETRICAL NOTATION FOR
HOLLOW CYLINDERS AND SPHERES

FIG. 5-5
SELF-SHIELDING FACTOR (F) FOR HOLLOW SPHERES
(REPRODUCED FROM REF. 5)

FIG. 5-6
SELF-SHIELDING FACTOR (F) FOR VARIOUS
GEOMETRIES OF EQUAL VOLUME
(REPRODUCED FROM REF. 5)

For an infinite hollow cylinder:⁽⁷⁾

$$F = \frac{1 - \frac{4}{\pi} \left\{ \int_{\theta_1}^{\pi/2} \cos \theta \, \text{Ki}_3(m \cos \theta) \, d\theta + \int_0^{\theta_1} \cos \theta \, \text{Ki}_3 \left[m (\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \theta_1}) \right] d\theta \right\}}{m \cos^2 \theta_1}$$

where

$$m = 2 \Sigma R$$

and

$$\int_{\theta_1}^{\pi/2} d\theta \int_0^{\pi/2} \cos \theta \cos^2 \phi \exp(-m \cos \theta / \cos \phi) d\phi = \int_{\theta_1}^{\pi/2} \cos \theta d\theta \text{Ki}_3(m \cos \theta)$$

where θ_1 is defined in Fig. 5-4, and Ki_n are the repeated integrals⁽⁸⁾ of the Bessel function K_0 .

Self-shielding factors F for various values of ξ are given in Fig. 5-7.

The previous self-shielding factors all were based on the assumption of no scattering in the fuel. If present, the effect of scattering may be accounted for by:⁽⁹⁾

$$\sigma_e = \frac{F}{1 - \frac{\Sigma_s}{\Sigma_t}(1 - F)} \sigma_a$$

where

Σ_s = macroscopic scattering cross section

$\Sigma_t = \Sigma_s + \Sigma_a$, the total macroscopic cross section, and F is evaluated assuming no scattering, $\Sigma_a = \Sigma$.

5.3 DYNAMICS CONSIDERATIONS

5.3.1 Doppler Coefficients

Table 5-2

DOPPLER COEFFICIENTS OF UO_2 AND Hf IN INTERMEDIATE REACTORS

(Reproduced from Ref. 55)

Material	Thickness, moles/cm ²	PPA-21A		PPA-20	
		Temp. Range, °C	T dK/dT, ^c per mole	Temp. Range, °C	T dK/dT, ^c per mole
UO_2 (Highly Enriched) ^b	1.1×10^{-3a}	400-750	-0.26×10^{-4d} -0.17×10^{-4e}	325-725	-0.23×10^{-4d} -0.15×10^{-4e}
UO_2 (U^{238}) ^b	1.0×10^{-3a}	400-750	-2.07×10^{-4d}	320-700	-2.2×10^{-4d} -1.1×10^{-4e}
Hf	1.8×10^{-3} (0.86 mm wire)	400-750	-1.46×10^{-4d}	340-760	-2.1×10^{-4d}

^aThe oxide samples were distributed as an annulus enclosed in stainless steel. The center of the annulus was MgO.

^bThe coefficient was measured for two positions in the reactors.

^cThe error for all values of T dK/dT is $\pm 15\%$; T in degrees Kelvin.

^dMeasured at center of assembly.

^eMeasured in Ring No. 5.

Note: Median Energy of Fission: PPA-20 = 50 ev; PPA-21A = 125 ev.

The Doppler coefficient has been measured in two beryllium-moderated intermediate reactors at KAPL. Three materials were considered: UO_2 (highly enriched), $\text{UO}_2(\text{U}^{238})$, and hafnium. The experimental methods are described by Frost, et al.^(10,11) The theoretical aspects of Doppler coefficients are discussed in detail by Bethe⁽¹²⁾ and Feshbach.⁽¹³⁾ The results of the experiments are summarized in Table 5-2.

FIG. 5-7
 SELF-SHIELDING FACTORS (F) FOR AN INFINITE HOLLOW CYLINDER AND $\epsilon = 0.1$ TO 2.5
 (REPRODUCED FROM REF. 7)

5.3.2 Fission Product Cross Sections as a Function of Energy

An evaluation⁽¹⁴⁾ of fission product capture cross sections for use in multi-group calculations of intermediate and fast reactors is given in Table 5-3.

Table 5-3

GROUP AVERAGED CAPTURE CROSS SECTIONS OF GROSS FISSION PRODUCTS^a

(Data from Ref. 14)

Group	E (Lower)	^a barns/fission ^b	Group	E (Lower)	^a barns/fission ^b
1	6.2 Mev	0.029	8	38.0	28.0
2	3.8 Mev	0.032	9	10.8 ev	19.0
3	1.4 Mev	0.047	10	3.1 ev	52.0
4	0.5 Mev	0.073	11	1.13 ev	26.0
5	0.18 Mev	0.11	12	0.4 ev	59.0
6	0.0034 Mev	0.56	13	0.195 ev	25.0
7	170.0 ev	6.1	14	0.075 ev	1495.0
			15	(to thermal)	1390.0

^aValues not corrected for burnout or self-shielding effects.^bIncludes Sm¹⁴⁹ but not Xe¹³⁵.

A detailed list of capture cross sections by isotope and energy is given in Table 5-4. These cross sections are based on calculated values given by Greebler and Hurwitz⁽¹⁴⁾ for the product of yield and cross section. The values of the yields assumed by Greebler and Hurwitz have been divided out since more recent yield values are available. The latter values are summarized in Section 1.2.

Table 5-4

FISSION PRODUCT CAPTURE CROSS SECTIONS

(Barns)

Mass No.	Isotope	10 ² ev	10 ³ ev	10 ⁴ ev	10 ⁵ ev	Mass No.	Isotope	10 ² ev	10 ³ ev	10 ⁴ ev	10 ⁵ ev
83	Kr	5.1	1.2	0.20	0.053	127	I	21.0	4.7	0.95	0.20
84	Kr	2.7	0.36	0.04	0.008	128	Te	5.6	1.1	0.13	0.015
85	Kr	3.0	0.30	0.03	0.006	129	I	19.0	3.5	0.62	0.180
85	Rb	2.5	0.33	0.04	0.008	130	Te	4.6	0.61	0.08	0.013
86	Kr	0.9	0.09	0.014	0.003	131	Xe	15.0	2.6	0.52	0.095
87	Rb	1.1	0.11	0.014	0.002	132	Xe	7.1	1.3	0.16	0.026
88	Sr	2.2	0.42	0.053	0.009	133	Cs	24.0	4.7	0.85	0.20
89	Y	1.3	0.15	0.017	0.003	134	Xe	2.8	0.29	0.04	0.006
90	Sr	1.2	0.14	0.016	0.003	135	Cs	17.0	3.10	0.63	0.119
91	Zr	2.1	0.39	0.048	0.008	136	Xe	0.48	0.048	0.006	0.001
92	Zr	2.2	0.40	0.047	0.008	137	Cs	9.6	1.4	0.26	0.004
93	Zr	2.2	0.40	0.049	0.008	138	Ba	0.61	0.061	0.008	0.001
94	Zr	2.1	0.41	0.050	0.008	139	La	11.0	1.9	0.28	0.045
95	Mo	6.1	1.1	0.20	0.044	140	Ce	0.61	0.06	0.008	0.001
96	Zr	2.1	0.38	0.06	0.008	141	Pr	15.0	2.6	0.393	0.065
97	Mo	5.8	0.98	0.20	0.032	142	Ce	0.64	0.064	0.008	0.001
98	Mo	4.9	0.80	0.15	0.026	143	Nd	13.0	1.81	0.33	0.057
99	Tc	14.0	3.50	0.82	0.208	144	Nd	7.9	1.0	0.12	0.021
100	Mo	4.9	0.76	0.13	0.023	145	Nd	23.0	3.7	0.70	0.115
101	Ru	10.4	2.1	0.39	0.110	146	Nd	7.8	0.81	0.10	0.017
102	Ru	7.9	1.4	0.26	0.060	147	Pm	36.0	6.40	1.1	0.185
103	Rh	14.0	3.2	0.71	0.164	147	Sm	44.0	11.0	1.7	0.378
104	Ru	5.3	0.80	0.17	0.027	148	Nd	3.9	0.45	0.05	0.008
105	Pd	13.0	2.8	0.61	0.174	149	Sm	49.0	8.8	1.6	0.437
106	Pd	9.4	1.9	0.36	0.098	150	Nd	2.8	0.28	0.04	0.007
107	Pd	12.0	3.1	0.58	0.15	151	Sm	51.0	9.1	1.7	0.366
108	Pd	7.7	1.5	0.38	0.085	152	Sm	26.0	4.3	0.525	0.089
109	Ag	14.0	3.2	1.1	0.21	153	Eu	90.0	19.0	6.2	1.30
111	Cd	11.0	2.2	0.56	0.11	154	Sm	20.0	3.0	0.40	0.07
112	Cd	7.7	1.5	0.77	0.077	155	Gd	55.0	15.0	1.60	0.236
113	Cd	17.0	2.5	0.83	0.17	156	Gd	36.0	3.6	1.1	0.143
115	In	18.0	4.5	0.91	0.18	157	Gd	63.0	13.0	2.5	0.438
126	Te	5.6	1.1	0.17	0.02	236	U	20.0	3.8	0.819	0.183

5.4 INTEGRAL DATA5.4.1 Beryllium-Moderated ReactorsUnreflected

Two beryllium-moderated, unreflected reactors known as CA-1 and CA-18 are reported from ORNL.⁽¹⁵⁾ System CA-1 is an intermediate reactor, while CA-18 has a slower spectrum, approaching that of a thermal reactor. The pertinent data on composition are given in Table 5-5.

Table 5-5

COMPOSITION OF ORNL REACTORS CA-1 AND CA-18(Reproduced from Hurwitz and Ehrlich)⁽¹⁵⁾

	CA-1	CA-18
Dimensions, in.	21.0 x 21.0 x 23.2	24.0 x 28.4 x 24.1
Spacing of 0.010-inch fuel discs, in.	1	4
U ²³⁵ loading, kg	18.08	7.65
Observed reactivity	1.0054	1.0020
Be: U ²³⁵ atomic ratio	390	1560
Volume fractions		
U	0.0064	0.0016
Be	0.9060	0.9128
Al	0.0610	0.0611
Stainless Steel	0.0004	0.0003
Void	0.0262	0.0242
Observed fraction of fissions caused by neutrons below cadmium cut-off	0.46 ± 0.1	0.87 ± 0.03

Beryllium Reflected

A series of critical experiments with beryllium moderators, unreflected, and with beryllium reflectors, were performed on the KAPL "honeycomb" assembly.^{(16-20)*} In addition to the various reflector compositions, some of the assemblies were blanketed with natural uranium or lead.

The KAPL "honeycomb" is a hexagonal array of 2-inch aluminum or steel tubes.⁽²¹⁾ The core and reflector sizes are measured in terms of the number of rings of tubes filled with a particular composition. The fuel is comprised of thin (0.020 inch) discs spaced with thick (0.10 to 0.80 inch) beryllium discs. The distribution of the pieces is as homogeneous as can be obtained.

*J. R. Stehn and H. B. Stewart, KAPL, private communication.

The loading and other pertinent information on the assemblies studied is given in Table 5-6. The reactors were made critical with the exception of PPA-6, which was subcritical.

The spectrum, characterized by the medium-energy-of-fission (MEF), is shown by these experiments to be a parameter which is adjustable by changing the ratio of fuel to moderator.⁽²²⁾ When the MEF is changed a number of quantities which are energy sensitive also change. Among the more important of such quantities are the fraction of fissions below Cd-cutoff and the ratio of the effective absorption cross section per absorber atom to the effective fission cross section per fuel atom. The subcadmium fraction is easily measured by standard techniques, while the cross section ratio is derived from reactivity or "danger" coefficients. An understanding of the physical effects occurring in the reactivity coefficient measurements is obtained by normalizing the adjoint function to the iterated fission probability as developed by Hurwitz.⁽²³⁾ The iterated fission probability, $F(\underline{r}, E)$, is the average number of fissions produced per generation, an infinite number of generations after a neutron of energy E has been introduced into a critical reactor at position $\underline{r}$. The reactivity change brought about by a change in material composition is then proportional to the number of neutrons added to (or removed from) the assembly multiplied by their effectiveness represented by the iterated fission probability. The fission rate, f_T , and the fission spectrum average of $F(\underline{r}, E)$ denoted $F_H(\underline{r})$ are related as follows:

$$f_T = \int \int \phi(\underline{r}, E) \Sigma_f dE dV = \nu \int \int \phi(\underline{r}, E) \Sigma_f F_H(\underline{r}) dE dV.$$

where:

$\phi(E)$ = the neutron flux per unit energy at position $\underline{r}$ and energy E .

ν = average number of neutrons per fission

Σ_f = macroscopic fission cross section.

When a pure absorber is inserted in a void space in the reactor, the resultant reactivity change is:

$$\frac{\Delta k}{k} = \frac{-N \int \phi(\underline{r}, E) \sigma_a(E) F(\underline{r}, E) dE}{f_T}$$

where: N = number of absorber atoms

σ_a = microscopic absorption cross section of the absorber.

The reactivity coefficient, RC_a , can then be defined for one mole of absorber, and expressed in cents, c , per mole as:

$$(RC_a) (c/mole) = \frac{-N_0 F_L(\underline{r}) \int \phi \sigma_a dE}{cf_T} = \frac{\Delta a F_L}{cf_T}$$

where: c = one hundredth of the reactivity difference between delayed and prompt critical = 1 cent

$$\Delta a = N_0 \int \phi \sigma_a dE$$

N_0 = Avogadro's number

$F_L(\underline{r})$ = iterated fission probability averaged over the absorption spectrum of the sample and is given by

Table 5-6
BERYLLIUM-MODERATED INTERMEDIATE REACTORS

	FPA-1	FPA-2 ^a	FPA-3 ^b	FPA-4 ^b	FPA-5 ^b	FPA-6	FPA-7	FPA-8	FPA-9	FPA-10	FPA-11	FPA-21 ^a	FPA-25	FPA-26
CORE														
No. of rings	5	6	5	5.5 ^c	5.5 ^c	5.5 ^c	5.5 ^c	6.5 ^c	6.5 ^c	5.5 ^c	5	8	7	7
Core length, in.	16.5	20.1	17.2	18.6	18.6	17.1	17.1	17.1	18.1	18.1	17.1	27.3	27.7	27.7
Core volume, liters	-	~112	~62	85.0	85.1	78.0	78.0	111.9	118.6	86.0	62.6	273.0	208.2	208.2
Void, vol.-%	-	15.7	15.7	15.5	15.5	6.2	6.4	6.3	6.4	6.0	7.4	13.1	15.1	15
Be:U atom ratio	218	134	48	45	33	45.4	23.6	50.1	55.2	4.13	1.73	92.9	113.7	113.7
Be, kg	87.8	161.2	77.4	82.7	72.6	78.7	60.3	112.1	119.5	19.9	8.2	220.6	206.6	206.6
vol.-%	-	74.96	62.4	52.9	47.1	54.6	41.8	54.1	54.4	12.5	7.1	43.7	53.8	53
Mg, kg												98.4	-	-
vol.-%												20.7	-	-
Na, kg	0	0	6.2	19.2	24.2	12.0	17.7	18.5	18.2	32.9	24.3			
vol.-%	-	-	9.8	23.2	28.37	15.8	23.4	16.9	15.8	39.4	40.0			
Al, kg	3.4	2 ^e	2 ^e	2 ^e	2 ^e	13.9	13.9	19.9	21.1	15.3	11.1	124.2	95.4	95.4
vol.-%	-	0.74	0.74	0.01	0.72	6.5	6.5	6.5	6.4	6.4	6.4	16.5	17.3	17.3
Fe, kg	24.3	39.4	17.3	29.2	30.0	5.8	7.5	9.0	8.8	14.5	1.2			
vol.-%	-	5.2	2.87	0.293	4.35	1.0	1.4	1.0	1.0	2.2	2.2			
Stainless Steel, kg	-	-	-	-	-	82.6	102.0	113.8	130.4	191.5	130.5	93.9	211.3	211.3
vol.-%	-	-	-	-	-	12.6	15.8	12.4	13.4	25.4	25.8	4.3	13.3	16.0
U ²³⁵ , kg	10.5	31.3	42.4	47.5	58.1	45.2	66.6	58.3	56.4	125.4	123.6	61.9	47.4	47.4
vol.-%	-	1.54	3.47	2.99	3.57	3.1	4.6	2.8	2.6	7.8	10.6	1.2	2.0	2.0
U ²³⁸ , kg	-	1.6	2.2	2.6	3.1	2.4	3.6	3.1	3.0	6.7	6.5	4.38	3.30	3.3
vol.-%	-	0.1	0.21	0.18	0.22	0.2	0.2	0.2	0.1	0.4	0.6	0.1	0.1	0.1
Fuel thickness, in. (foils)	-	0.02	0.02	0.02	0.02	-	-	-	-	-	0.04			
Poison												Pyrex Rods		
END REFLECTOR														
Thickness, in.	Note: FPA-1 to FPA-5 had no end reflector					7.6	7.6	-	7.2	7.2	9.7	17.5	17.4	17.4
Volume, liters						27.4	27.4	39.3	36.9	27.0	28.1	175	130.8	130.8
% Void						5.0	5.0	5.1	5.7	6.2	5.3	13.5	12.8	12.8
Be, kg						35.8	35.8	54.3	47.4	34.1	27.6	140.5	104.7	104.7
vol.-%						70.3	70.3	70.4	69.2	68.2	52.9			
Mg, kg												117.6	-	-
Na, kg						2.8	2.8	4.0	4.0	3.0	6.8			
vol.-%						10.6	10.6	10.6	11.2	11.6	24.7			
Al, kg						5.0	5.0	7.2	6.7	4.9	5.2	-	134.7	134.7
vol.-%						6.6	6.6	6.6	6.6	6.6	6.6			
Fe, kg						1.4	1.4	2.0	2.0	1.5	3.3			
vol.-%						0.6	0.6	0.6	0.7	0.7	1.5			
Stainless Steel, kg						14.9	14.9	21.3	20.0	14.6	20.4	61.3	45.3	45.3
vol.-%						6.8	6.8	6.7	6.7	6.8	8.9			
SIDE REFLECTOR														
No. of rings	-	-	1	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	4	4	4
Volume, liters	-	-	-	-	-	52.4	52.4	61.6	65.3	52.2	49.3	518.1	463.5	463.5
% Void	-	-	21.7	21.0	21.0	5.8	5.8	5.7	5.7	5.7	5.6	13	12.8	12.8
Be, kg	76.8	-	71.3	143.4	143.4	72.7	72.7	88.0	90.6	74.2	67.2	793.4	710.3	710.3
vol.-%	-	-	-	-	-	77.0	77.0	77.0	74.9	76.7	73.4			
Na, kg	-	-	-	0.416	-	1.9	1.9	2.2	3.7	2.2	3.5			
vol.-%	-	-	-	-	-	3.7	3.7	3.7	5.8	4.2	7.4			
Al, kg	-	-	1.9	3.0	6.6	9.3	9.3	10.9	11.6	9.3	9.0			
vol.-%	-	-	-	-	-	6.6	6.6	6.6	6.4	6.4	6.4			
Fe, kg	-	-	25.4	19.0	46.9	0.91	0.91	1.08	1.79	0.99	1.7			
vol.-%	-	-	-	-	-	0.22	0.22	0.22	0.35	0.25	0.44			
Stainless Steel, kg	-	-	-	-	-	28.5	28.5	33.4	35.4	28.2	10.9	184.1	159.2	159.2
vol.-%	-	-	-	-	-	6.8	6.8	6.7	6.7	6.7	6.8			
Poison												B ₁ C Rods in curtain at interface; 4.40 kg B ₁ C		
\$ of Poison												-12		8.4
BLANKET														
Material	Pb	U	U	U		U	U	U	U	U	U			
Thickness, in.	-	12	12	12	-	-	-	-	-	-	-			
End, vol.-%	-	-	-	-	-	17.0	25.2	17.1	18.8	48.9	53.6			
Side, vol.-%	-	-	-	-	-	69.5	69.5	56.3	55.1	66.7	56.5			

- a - No reflector.
b - Weights include end reflector.
c - Tube ring contains 50% core slugs and 50% reflector slugs.
d - Includes iron in sodium containers.
e - Aluminum present for shimming.

$$F_L(\underline{r}) = \frac{\int \phi(\underline{r}, E) \sigma_a(E) F(\underline{r}, E) dE}{\int \phi(\underline{r}, E) \sigma_a(E) dE}$$

$F_L(\underline{r})$ can be evaluated in a critical assembly by observing the reactivity change associated with a resonance absorber such as gold or manganese and measuring the neutron absorption rate by the foil activation. The total fission rate, f_T , can likewise be measured. The ratio of effective absorber cross section to effective fission cross section is then given by:

$$\frac{\bar{\sigma}_a(\underline{r})}{\bar{\sigma}_f(\underline{r})} \equiv \frac{N_0 \int \phi(\underline{r}, E) \sigma_a(E) dE}{N_0 \int \phi(\underline{r}, E) \sigma_f(E) dE} = \frac{\Delta a(\underline{r})}{\Delta f(\underline{r})} = - \frac{RC_a}{\left(\frac{235}{M}\right) \left(\frac{P(\underline{r})}{P_{av}}\right) \left(\frac{F_L(\underline{r})}{C}\right)}$$

where: M = total fuel load in grams

$\frac{P(\underline{r})}{P_{av}}$ = ratio of power density at $\underline{r}$ to the average power density.

A summary of the results for typical intermediate assemblies is given in Table 5-7.

Table 5-7

SUMMARY OF SPECTRUM SENSITIVE QUANTITIES
IN PPA REACTORS⁽²²⁾

Critical Assembly	PPA-11	PPA-5	PPA-20a	PPA-12
Core Volume (liters)	62.6	85.1	293.9	272
Equivalent Core Dia. (cm)	42.8	47.9	73.4	70.8
Fuel Inventory (kg)	123.6	58.1	51.7	39.8
Be: U ²³⁵ Atom Ratio	1.73:1	33:1	137.5:1	178.5:1
Median-Energy-of-Fission (at core center)	235 kev	1500 ev	82 ev	40 ev
Fraction of Fissions below Cd-Cut-off	<0.01	0.02	0.09	0.24
Side Reflector Comp.	Be	Be	Be + B	Be + Void
σ_a/σ_f of Selected Materials				
B	.17 ^a	0.56	0.75	1.02
Cr	-	0.0010	0.0036	0.0044
Mn	-	0.071	0.031	0.022
Co	-	0.14	0.123	-
Ni	-	0.0063	0.0066	-
Nb	0.032	0.092	0.036	0.034
Mo	0.041	0.072	0.048	0.043
Rh	0.27	0.43	0.78	0.90
Pd	0.10	0.18	-	0.10
Ag	-	0.57	1.21	0.45
Cd	-	0.22	1.83	2.22
Cs	-	0.26	0.30	0.63
Sm	-	0.90	4.2	-
Ta	0.17	0.39	1.14	0.47
W	-	0.20	0.40	-
Au	0.17	0.76	1.64	-

^aThe samples used in this assembly were all sufficiently thin so that self-shielding effects were negligible.

An additional study of PPA-21 critical assemblies (Table 5-8) is of interest because of the variations in a basic loading. The boron curtain was arranged as B₄C rods placed in the interstices between the circular tubes on the periphery of the core. The remaining changes are self-evident in the table.

Table 5-8

COMPOSITION CHANGES IN PPA-21

(Data From Ref. 10)

PPA	Description	Composition, mol			Be:U ²³⁵	Approx. Reactivity Value of Boron, \$ ^a	
		Cd	Be	U ²³⁵		Core	Curtain
21A	Half of interstices filled with U-Al alloy rods (4.99 kg interstitial fuel). Boron curtain at core-reflector boundary to balance excess reactivity	-	24454	263.27	92.9	-	15.00
21B	Second half of interstices filled with beryllium (1654 mols Be added). Increase in reactivity balanced by additional boron rods at core reflector boundary	-	26108	263.27	99.2	-	19.00
21C	Beryllium (added in PPA 21B) is balanced by boron rods distributed through core. Fuel removed from interstices to make room for boron rods.	-	26108	261.10	100	4.00	15.00
21D	Beryllium (added in PPA 21B) is balanced by cadmium distributed through core (4.55 kg of Cd).	40.5	26108	261.10	100	-	15.00
21E	Interstitial beryllium removed. 4.8 kg interstitial fuel added. Loss of reactivity balanced by removal of cadmium (3.17 kg of Cd in core).	28.22	24454	280.82	87.1	-	15.00
21F	All cadmium removed from core and excess reactivity balanced by addition of boron curtain.	-	24454	280.82	87.1	-	17.50
21G	All boron removed from curtain. All interstitial fuel removed (9.78 kg). Excess reactivity balanced by cadmium distributed through core.	108.1	24454	242.04	101	-	-

^a \$1 = 0.0087 Δk/k5.4.2 Graphite-Moderated ReactorsORNL Thin Graphite Reflector Experiment

A graphite-moderated assembly was studied on a honeycomb assembly at ORNL.⁽²⁴⁾ The graphite was in the form of square blocks which varied in length. The fuel consisted of circular discs held in place by skewers.

The core size was 45 x 45 x 44.1 inch. The over-all assembly, including 3-inch side reflectors, measured 51 x 51 x 44.1 inch. The thin reflector was found equivalent to an additional fueled region of equal size.

The total uranium loading (44.449 kg fuel, or 41.515 kg U^{235}) was critical with a 10.4 x 3 x 3 inch gap in the center (control rod withdrawal). If compensation is made for the void, and extra fuel is added for the reflector, the equivalent bare reactor would be critical with $M_C = 52.48$ kg U^{235} . The details of the reactor core are given in Table 5-9.

Table 5-9

GRAPHITE MODERATED REACTOR

Material	Volume %	Density, gm/cm ³	Mass, kg
U (enriched)			44.449
U^{235}			41.515
U^{235} in equivalent Bare Core [core + 3-in. reflector]	0.162	18.7	53.234
Graphite	89.48	1.717	2889.7
Graphite impurities	.014		
Aluminum	6.08	2.71	310
Stainless Steel ^a	0.031	7.93	4.663
Void	4.26		
<hr/>			
Median energy of fission =	0.15 ev		
% Thermal Fission	= 27.7		

^aImpurities in aluminum and graphite are treated as stainless steel.

LASL Graphite Reflector Experiments

A series of multiplication experiments were performed at LASL⁽²⁵⁾ to ascertain the critical properties of some uranium-graphite systems.

The core was composed of U^{235} discs (10.5 in. dia.) of various thicknesses, interleaved with graphite discs of various thicknesses. The resulting reactors were quite heterogeneous and the spectra of some of these assemblies were nearly as fast as some all-metal fast reactors. The carbon to uranium ratio is comparable to the system PPA-11 (see Table 5-7) in which the median energy of fission is 235 kev.

The reflector was added, separately, to the top, bottom, and sides. Each loading continued until all the U was used. The critical mass was then determined by an extrapolation of the reciprocal multiplication curves.

The critical data for the series of experiments are summarized in Table 5-10.

LASL Beryllium Reflector Experiments

The honeycomb assembly at LASL was used to conduct a series of experiments on graphite-moderated, beryllium-reflected reactors.⁽²⁶⁾ The general configuration of the assemblies studied was that of a mock cylinder (Fig. 5-8).

Table 5-10

SUMMARY OF URANIUM-GRAPHITE CORE CONFIGURATIONS

U Plate Thick., in.	Graphite Plate Thick., in.	Core Densities, gm/cm ³		U (max.), kg	Mult. (max.)	1/M	M _c U ²³⁵ , kg	Crit. Core Hgt., cm
		U + Graphite	U					
Solid	0	18.82	18.82	50.5	18.08	0.055	49.8	5.08
0.315	0.5	8.33	7.29	67.4	6.44	0.155	75.2	19.84
0.315	1.0	5.79	4.52	67.4	1.55	0.645		
0.315	2.0	4.05	2.57	67.4				
0.630	0.5	11.26	10.51	67.4 ^a	28.89 ^a	0.035	58.3	10.64
0.630	1.0	8.32	7.29	67.4	7.73	0.129	73.0	19.25
0.630	2.0	5.82	4.52	67.4	2.01	0.498		
0.945	0.5	12.92	12.33	50.5	8.96	0.112	53.6	8.36
0.945	1.0	10.12	9.16	50.5	5.59	0.179	59.7	12.32
0.945	2.0	7.22	6.05	50.5	3.10	0.323	74.1	23.52

^aA complete assembly of this amount of uranium in this configuration would have become critical. The maximum multiplication was obtained with an 0.425 in. (including 0.063 in. steel support plate) separation between the two halves. A correction was made for obtaining M_cU = 93.2% U²³⁵

The series of experiments summarized in Table 5-11 establishes a relation between the critical mass and the carbon to U²³⁵ atom ratio. The studies in Table 5-12 were performed to establish the minimum core volume consistent with the material available.

The results of these surveys are shown graphically in Fig. 5-9.

Assembly No. 8, shown in Fig. 5-8, was studied extensively with respect to fission rates and cadmium ratio as a function of position within the assembly.

FIG. 5-8
CROSS SECTION OF ASSEMBLY
NO. 8, TABLE 5-11
(REPRODUCED FROM HANSEN, et al²⁶)

The cadmium ratio with U²³⁵ foils varied from ~0.5 at one core-end reflector interface, to ~0.8 at the axial center of the assembly. Radially, the minima were 0.3 and 0.5 at the side reflector-core interface, at positions 3 inches and 8 inches respectively, from the end reflector. The cadmium ratios with indium varied radially from ~1.15 at the center, to about 1.6 at the core edge, and almost independent of axial position.

UCRL Graphite and Beryllium Reflector Experiments

A series of critical experiments on graphite moderated, graphite and beryllium reflected assemblies were performed* at the University of California Radiation Laboratory (Livermore). The assemblies were stacked on a low-mass honeycombed aluminum table (density = 3 lbs/ft³). The fuel (93.5% U²³⁵) was in the form of 5.25-in-square foils, 0.001 and 0.002 inches thick. In addition, each foil was coated with ~0.92 grams of

*P. Greebler, private communication.

Table 5-11

CRITICAL CONDITIONS FOR VARIABLE C/U²³⁵ SERIES^a

(Reproduced from Hansen, et al. (26))

Assembly No.	C/U ²³⁵	Crit. Mass kg U ²³⁵	Fuel Region			Wall Reflector		East End Reflector		West End Reflector	
			Effective Graphite Density gm/cm ³	Radius, cm	Length, cm	Effect. Be Density, gm/cm ³	Thick., cm	Graphite ^b Thick. in ΔEF, cm	Be Thick., cm	Graphite ^b Thick. in ΔWF, cm	Be Thick., cm
1	108	53.6	1.42 ₁	31.3 ₀	77.7	1.65 ₉	12.4	2.5 ₄	10.1	1.0 ₂	20.2
2	108	53.6	1.42 ₁	31.3 ₀	77.7	1.65 ₉	13.7	2.5 ₄	10.1	1.0 ₂	10.1
3	343	28.7	1.42 ₁	40.5 ₆	78.7 ₄	1.66 ₆	12.9 ₆	2.5 ₄	11.4 ₇	0	8.1 ₂
4	343	33.6	1.29 ₀	46.1 ₀	78.7 ₄	1.65 ₅	14.0	2.5 ₄	7.6 ₀	0	7.6 ₀
									+3.2 ₂ ^c		+3.2 ₂ ^c
5	346	37.6	1.29 ₈	48.8 ₃	78.7 ₄	1.66 ₁	12.8	2.5 ₄	8.5 ₃	0	8.5 ₃
6	887	16.22	1.47 ₇	48.8 ₃	76.2	1.66 ₄	12.0	2.5 ₄	9.7 ₆	2.5 ₄	23.0 ₃ ^d
7	887	16.22	1.47 ₇	48.8 ₃	76.2	1.66 ₄	12.0	2.5 ₄	9.7 ₆	2.5 ₄	9.7 ₆
8	358	30.1	1.50	40.5 ₆	81.2 ₈	1.66	13.0 ₈		10.1 ₆		10.1 ₆
9	358	30.1	1.50	40.5 ₆	81.2 ₈	1.66	16.7 ₄		10.1 ₆		0
10	358	30.1	1.50	40.5 ₆	81.2 ₈	1.66	29.9 ₀ ^e		0		0

^aUnless otherwise specified, mean densities of graphite and beryllium in the east and west face reflectors are the same as listed in fuel and wall reflector regions.

^bSmall carbon regions designated ΔEF and ΔWF about the east and west faces of the core. The density and area is the same as the carbon in the core.

^cGraphite at 1.42 gm/cm³.

^dGraphite at 1.47 gm/cm³.

^eExtrapolated value. Actual critical configuration with 27.58 cm Be + 5.75 cm graphite ($\bar{\rho} = 1.56$).

Note: Assemblies 6 and 7 contained 0.002 in. foils only; assemblies 1 - 5 contained ~17.5 kg U²³⁵ as 0.002 in. foils and the remainder as 0.005 in. foils; assemblies 8-10 contained 0.005 in. foils only.

Teflon. The ATJ-graphite moderator had a density of 1.73 gm/cm^3 and the average density of the core was adjusted by varying the void fraction. The density of the beryllium in the reflector was 1.84 gm/cm^3 . The nominal carbon/ U^{235} atom ratio varied from 600 to 2340. The assemblies were controlled by poisoned rods.

Table 5-12

CRITICAL DATA FOR SMALL-CORE ASSEMBLIES

(Reproduced from Hansen, et al.⁽²⁶⁾)

Assembly No.	Critical Mass, kg U^{235}	C: U^{235} Atomic Ratio	$\bar{\rho}_C'$ gm/cm ³	$\bar{\rho}_C$ gm/cm ³	Core Length: 76.2 cm		Reflector Thickness, cm		
					Be Island Radius cm	Core Outside Radius cm	Wall	East End	West End
11	11.94	330	1.42	1.66	-	25.25	24.71	2.54 C + 10.16 Be	2.54 C + 20.32 Be
12	15.56	327	1.41	1.66	-	28.83	23.65	2.54 C ^a	2.54 C + 20.32 Be
13	8.27	332	1.38	1.66	19.38	28.83	19.99	2.54 C ^a	2.54 C + 20.32 Be

^aPeripheral beryllium extends 4 in. beyond core.

^bA safety rod fuel tube in east beryllium island was considered beryllium.

Note: These assemblies contained 0.002-in. uranium foil only.

FIG. 5-9
CRITICAL PARAMETERS OF SPHERICAL URANIUM-
GRAPHITE CORE WITH BERYLLIUM REFLECTOR
(REPRODUCED FROM HANSEN, et al.⁽²⁶⁾)

A summary of the basic assembly and the critical data is given in Table 5-13. The first part of the table shows the data for right octagonal cylinders and the second part is for parallelepipeds.

The relation between the radial buckling of an octagon and a circle of equivalent area is given by

$$B_{\text{Octag}}^2 = 1.009 B_{\text{Circle}}^2$$

The bucklings shown in Table 5-14 were obtained using this relation. The corrected value of the bucklings, when the corrections are applied by various means to eliminate the effect of the control rod channels, are also given. These are summarized as follows (corresponding to columns in Table 5-14).

a) The control channel was successively displaced from the center and the critical height measured. These results were then extrapolated to complete removal.

b) Considering the control channel void as a perturbation, δ , on the buckling and applying the results to the experiments of (a) the observed buckling is given by

$$B_{\text{Obs}}^2 = B_0^2 - \phi^2 \delta$$

where ϕ is the flux at the position of the void.

c) Usual two-group perturbation on B^2 due to reduced density of the center region.

d) Plugs were placed in half the channels and change in B^2 observed. Total effect = 2 x effect in half.

Table 5-13

SUMMARY OF CRITICAL MASS DATA FOR THE
BASIC GRAPHITE MODERATED SYSTEMS

Core Size (cm)		Reflector Thickness (cm)	C/U Absol.	C/U** Eff.	U ²³⁵ (kg)
Radial Dimensions	Height				
238 octag.*	133	0	600	675	89.3
214 octag.*	153	0	600	675	79.1
191 octag.*	167	0	600	675	76.5
167 octag.*	240	0			~112
144 octag.*	175	15-C-8 sides	600		40.5
191 x 191	152	0	600	675	82.3
191 x 191	167	0	1200	1379	43.5
191 x 144	169	15-C-2 sides	1200		33.0
144 x 144	171	15-C-4 sides	1200		25.3
191 x 191	187	0	2340	2586	25.1
191 x 144	161	15-Be-2 sides	2340		16.2
144 x 144	145	15-Be-4 sides	2340		11.0

*flat-to-flat diameter of octagon.

$$**(C/U)_{\text{Eff}} = \frac{C/U \text{ Absol.}}{\text{Disadvantage Factor}}$$

Table 5-14

SUMMARY OF BUCKLING DATA FOR THE BASIC GRAPHITE MODERATED SYSTEMS

Core Size (cm)				Correction by							
		C/U Nominal	B ² _{obs} * (x 10 ⁻⁴)	Extrapolation of ^a Channel Position		φ ² Perturbation ^b		Two-Group ^c Perturbation		Doubling ^d Void Effect	
Radial Dimensions	Height			H, cm	B ² , cm ⁻² (x 10 ⁻⁴)	H, cm	B ² , cm ⁻² (x 10 ⁻⁴)	H, cm	B ² , cm ⁻² (x 10 ⁻⁴)	H, cm	B ² , cm ⁻² (x 10 ⁻⁴)
Octag. 191	167	600	20.96								
214	153	600	20.99								
238	133	600	21.26								
Square 191	152	600	21.11	152	21.69	153.5	21.53	152.5	21.64	-	-
191	167	1200	20.17	161.2	20.69	162	20.62	161.2	20.69	158	21.03
191	187	2340	18.57	181	18.98	181	18.98	180.5	19.04	178.3	19.18

$$* \left\{ B_{\text{obs}}^2 = \left(\frac{\pi}{H + 2\delta} \right)^2 + 1.009 \left(\frac{2.048}{R + \delta} \right)^2 \right.$$

$\delta = 1.92$ cm extrapolation distance

Density of graphite for homogeneous system = 1.636 gm/cc.

a, b, c, d, - see text.

In addition to the foregoing, the disadvantage factors for fuel foils of various thicknesses were determined and are given in Fig. 5-10.

5.5 PARAMETRIC SURVEYS

Parametric surveys based solely on calculations serve to demonstrate the effect of changes in a reactor even though certain absolute quantities cannot be verified experimentally.

The results of critical mass calculations by Safanov⁽²⁷⁾ for bare, spherical, homogeneous U^{235} reactors involving several moderators are shown graphically in Fig. 5-11.

The surveys by Mandl⁽²⁸⁻³⁰⁾ and Whipple⁽³¹⁾ pertain to Pu-Be systems. The variation of critical radius with median energy of fission is shown in Fig. 5-12 for a bare homogeneous sphere, and in Fig. 5-13 for a bare homogeneous sphere with Pb coolant added. Figure 5-14 shows the variation of the ratio of moderator volume to volume of coolant and fuel, with median energy of fission.

Figure 5-14 shows the variation of the median energy of fission with the ratio of moderator volume to volume of coolant and fuel.

Several simplifying assumptions have been made for the nuclear parameters in Figs. 5-12 through 5-15. In particular, η has been assumed equal to ν , when in fact it is smaller ($\sim 0.7\nu$). Thus the figures serve only to establish trends.

FIG. 5-10
DISADVANTAGE FACTOR VS URANIUM FOIL THICKNESS

FIG. 5-11
CRITICAL MASS OF U^{235} VS CRITICAL RADIUS
FOR BARE HOMOGENEOUS SPHERICAL REACTORS
WITH VARIOUS MODERATORS
(REPRODUCED FROM SAFANOV²⁷)

FIG. 5-12
CRITICAL PARAMETERS OF Pu-Be SYSTEMS
(REPRODUCED FROM MANDL³⁰)

FIG. 5-13
CRITICAL PARAMETERS FOR Pu-Be-Pb SYSTEMS
(REPRODUCED FROM MANDL²⁸)

FIG. 5-14
VOLUME RATIO OF MODERATOR TO COOLANT AND FUEL FOR
BARE HOMOGENEOUS SPHERICAL Pu-Be-Pb CRITICAL SYSTEMS
(REPRODUCED FROM MANDL²⁸)

FIG. 5-15
CRITICAL RADII OF Pu-Be-Pb SYSTEMS WITHOUT
REFLECTOR (r_o) AND WITH INFINITE REFLECTOR (r_{∞})
(REPRODUCED FROM MANDL²⁹)

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SECTION 6

FAST REACTORS

6.1 CONSTANTS FOR FAST REACTOR CALCULATIONS

Multigroup constants useful for reactor calculations must be obtained by suitably averaging microscopic cross section information, as a function of neutron energy, over the energy interval of interest. Generally this is done by assuming a neutron spectrum as a function of energy throughout the energy interval. Whether the assumed spectrum is the correct one can be determined by analyzing spectra calculated using such multigroup constants; for example, by analyzing spectra with a finer group structure, or by examination of a smooth curve fit through the multigroup spectrum. The calculated neutron spectrum may or may not agree with that assumed originally to generate the multigroup constants. If agreement is not satisfactory, the same iterative process may be continued until such agreement is obtained.

It may be possible to arrive at a set of constants averaged over a spectrum which agrees with subsequent calculated spectra. However if the energy interval representing a group is large, then in considering a new system with different spectral properties, such constants may not be applicable. Furthermore, even though a spectrum calculation at one point may be substantially in agreement with that originally assumed to generate the multigroup constants, it may not be assumed that those same constants are always adequate to describe the spectrum throughout the system.

In order to assure that the multigroup constants are essentially independent of the spectrum within the group, the use of a large number of energy groups each representing a small energy interval suggests itself. However, many of the detailed cross sections are unknown. Therefore, there is considerable difference of opinion as to the optimum number of energy groups for analysis of various types of assemblies.

Few-group calculations have been used with reasonable success for a wide variety of criticality determinations. However, for a more complete study, multigroup sets using around ten-energy groups have been used frequently for fast reactor analysis. Generally, one, two, or three groups above the U^{238} fission threshold (~ 1.4 Mev) are adequate. Groups below the U^{238} fission threshold usually are selected to cover unit or half-unit lethargy intervals down to about 10 kev.

6.1.1 Multigroup Notation

The diffusion theory multigroup equations may be written as a set of equations of the form

$$D_j \nabla^2 \phi_j - \Sigma_{a,j} \phi_j + Q_{s,j} + Q_{f,j} = 0$$

where

$j = 1, \dots, N$ labels the energy group,

ϕ_j = the total flux in group j,

$\Sigma_{a,j} = \Sigma_{f,j} + \Sigma_{c,j} + \Sigma_{in,j} + \Sigma_{er,j}$ total removal cross section from group j,

$\Sigma_{f,j}$ = fission cross section of group j,

$\Sigma_{c,j}$ = capture cross section of group j,

$\Sigma_{in,j} = \sum_{k \neq j} \Sigma_{in,j \rightarrow k}$, inelastic removal cross section from group j,

$\Sigma_{er,j} = \sum_{k \neq j} \Sigma_{er,j \rightarrow k}$, elastic removal cross section for group j,

$Q_{s,j} = \sum_{k \neq j} (\Sigma_{in,k \rightarrow j} + \Sigma_{er,k \rightarrow j}) \phi_k$, scattering source into group j,

$Q_{f,j} = \chi_j \sum_{k=1}^N (\nu \Sigma_f)_k \phi_k$, fission source into group j,

χ_j = fraction of fission neutrons born into group j

and

D_j = diffusion coefficient of group j, related to $\Sigma_{tr,j}$ by some prescription; e.g.

$$D_j = \frac{1}{3\Sigma_{tr,j}}$$

The group macroscopic cross sections $\Sigma_{f,j}$, $\Sigma_{c,j}$, $\Sigma_{in,j \rightarrow k}$ and $\Sigma_{er,j \rightarrow k}$ may be obtained from the corresponding microscopic group cross sections for individual materials, e.g. for reaction q in material z,

$$\Sigma_{q,j} = \sum_z N^z \sigma_{q,j}^z$$

where

N^z = atoms/cm³ of material z.

The microscopic group cross sections are obtained, in principle at least, from fundamental cross sections which, using the notation of Goldstein, (1) may be written as: any type of parasitic capture

$$\sigma_{n,\gamma}(E), \quad \sigma_{n,\alpha}(E), \quad \text{etc};$$

fission

$$\sigma_{n,f}(E) \quad ;$$

elastic scattering, including moderation and angular distribution

$$\sigma_{n,n}(E), \quad \sigma_{n,n}(E \rightarrow E'), \quad \sigma_{n,n}(E \rightarrow E', \theta);$$

inelastic scattering, including angular distribution

$$\sigma_{n,n'}(E \rightarrow E'), \quad \sigma_{n,n'}(E \rightarrow E', \theta) \quad .$$

Multigroup microscopic cross sections may now be defined. One has:

group capture

$$\sigma_{c,j}^z = \frac{\int_{E_{jL}}^{E_{jH}} \left[\sigma_{n,\gamma}^z(E) + \sigma_{n,\alpha}^z(E) + \dots \right] \phi(E) dE}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE} \quad ;$$

group fission

$$\sigma_{f,j}^z = \frac{\int_{E_{jL}}^{E_{jH}} \sigma_{n,f}^z(E) \phi(E) dE}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE} \quad ;$$

scattering whether within or out of group j

$$\sigma_{\left\{ \begin{smallmatrix} \text{in} \\ \text{el} \end{smallmatrix} \right\} j \rightarrow k}^z = \frac{\int_{E' = E_{kL}}^{E_{kH}} \int_{E = E_{jL}}^{E_{jH}} \sigma_{\left\{ \begin{smallmatrix} n, n' \\ n, n \end{smallmatrix} \right\}}^z(E \rightarrow E') \phi(E) dE dE'}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE} \quad ;$$

where

E_{jL} and E_{jH} are the lower and upper limits of group j respectively,

and

$\phi(E)$ is the flux per unit energy interval at E.

The cross section for fission neutron production by group j neutrons is given by:

$$(\nu \Sigma_f)_j = \sum_z N^z (\nu \sigma_f)_j^z$$

and

$$(\nu\sigma_f)_j^z = \frac{\int_{E_{jL}}^{E_{jH}} \nu^z(E) \sigma_{n,f}^z(E) \phi(E) dE}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE}$$

The group transport cross section $\Sigma_{tr,j}$ may be obtained from:

$$\Sigma_{tr,j} = \sum_z N^z \sigma_{tr,j}^z \quad (6-1)$$

where

$$\frac{1}{\sigma_{tr,j}^z} = \frac{\int_{E_{jL}}^{E_{jH}} \phi(E) dE / \sigma_{tr}^z(E)}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE}$$

and

$$\begin{aligned} \sigma_{tr}^z(E) = & \sigma_{n,\gamma}^z(E) + \sigma_{n,\alpha}^z(E) + \dots + \sigma_{n,f}^z(E) + \int_0^{E_{jL}} \left(\sigma_{n,n'}^z(E \rightarrow E') \right. \\ & \left. + \sigma_{n,n}^z(E \rightarrow E') \right) dE' + \int_{E_{jH}}^{\infty} \left(\sigma_{n,n'}^z(E \rightarrow E') + \sigma_{n,n}^z(E \rightarrow E') \right) dE' \\ & + \int_{E_{jL}}^{E_{jH}} \int_{-1}^1 \left[\sigma_{n,n}^z(E \rightarrow E', \theta) + \sigma_{n,n'}^z(E \rightarrow E', \theta) \right] (1 - \cos \theta) d\cos \theta dE' \end{aligned}$$

In the formulation, angular distribution effects in scattering to outside the group are neglected.

An alternative prescription for $\sigma_{tr,j}$ is given by

$$\sigma_{tr,j}^z = \frac{\int_{E_{jL}}^{E_{jH}} \sigma_{tr}^z(E) \phi(E) dE}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE} \quad (6-2)$$

(Eq. 6-2 Cont'd.)

$$\begin{aligned}
&= \sigma_{c,j}^z + \sigma_{f,j}^z + \sum_{k \neq j} \sigma_{in,j \rightarrow k}^z + \sigma_{er,j \rightarrow k}^z \\
&\quad (6-2 \text{ Cont'd.}) \\
&\quad \frac{\int_{E_{jL}}^{E_{jH}} \int_{E_{jL}}^{E_{jH}} \int_{-1}^1 \phi(E) \left[\sigma_{n,n}^z(E \rightarrow E', \theta) + \sigma_{n,n'}^z(E \rightarrow E', \theta) \right] (1 - \cos \theta) d \cos \theta dE dE'}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE}
\end{aligned}$$

The last term may be denoted by

$$\sigma_{in,j \rightarrow j}^z + \sigma_{el,j \rightarrow j}^z (1 - \bar{\mu})$$

where $\bar{\mu}$ is the average cosine of the scattering angle. Although for diffusion theory an average of $1/\sigma_{tr}$, is required, it cannot be said that Eq. (6-1) is superior to Eq. (6-2) since in neither case is the really correct average (which involves the mixture) applied:

$$\frac{1}{\Sigma_{tr,j}} = \left[\frac{\int_{E_{jL}}^{E_{jH}} \left\{ (\phi(E) dE) / \sum_z N^z \sigma_{tr}^z(E) \right\}}{\int_{E_{jL}}^{E_{jH}} \phi(E) dE} \right]$$

While the multigroup constants are given in terms of a diffusion theory formalism, basically the same definitions apply in a transport theory formalism. The only major difference is that in the latter analysis Eq. (6-2) is believed to be more realistic for obtaining the transport cross section.

The rather wide application of the S_n method^(2,3,4) to the solution of fast reactor problems requires some recombination of the previously defined constants. In the case of plane geometry and the assumption of isotropic scattering in the transport approximation, the transport equation appears as:

$$\left(\mu \frac{\partial}{\partial x} + \Sigma_{tr,j} \right) N_j(x, \mu) = S_j(x)$$

where $N_j(x, \mu)$ is the angular neutron flux per unit μ interval, and μ is the direction cosine with respect to x . The source term $S_j(x)$ may be expressed in terms of the previously defined $\phi_j(x)$ where

$$\phi_j(x) = \frac{1}{2} \int_{-1}^{+1} N_j(x, \mu) d\mu$$

Then $S_j(x)$ appears as:

$$S_j(x) = \chi_j \sum_{k=1}^J \nu_k \Sigma_{f,k} \phi_k + \sum_{k=1}^J \Sigma_{k \rightarrow j} \phi_k$$

The first term on the right hand side is the $Q_{f,j}$ defined earlier. The second term differs from the previous $Q_{s,j}$ only in that the $k = j$ term is included in the sum. $\Sigma_{k \rightarrow j}$ for $k \neq j$ is the same as defined previously. $\Sigma_{k \rightarrow k}$ is given by

$$\Sigma_{k \rightarrow k} = \Sigma_{tr,k} - \Sigma_{c,k} - \Sigma_{f,k} - \Sigma_{in,k} - \Sigma_{er,k}$$

and also corresponds to

$$\sigma_{k \rightarrow k} = \sigma_{in,k \rightarrow k} + \sigma_{el,k \rightarrow k}$$

as previously defined, providing $\Sigma_{tr,k}$ is obtained by the use of Eq. (6-2).

6.1.2 Multigroup Constants

Representative 6-, 10-, and 11-group sets of fast reactor cross sections are given for materials frequently used in fast reactor calculations, and for detectors of utility in experimental measurements of reactor statics.

The first set^(5,6) (Table 6-1) consists of nine energy groups above 9.12 kev, the tenth group comprising neutrons of all lower energies. The scattering and inelastic scattering cross sections given are respective removal cross sections for the group. A maximum of six transfer groups below a given group is assumed. The elastic removal is considered to be transferred into the next lower group. The inelastic transfer matrices are based, when possible, on available data on inelastic scattering levels and upon reported measurements on gross removal cross sections. Suitable corrections for considering anisotropic scattering were based on experimental values given in LA-2099.⁽⁷⁾ Theoretical predictions were used as a guide in the absence of experimental information.

The second set of constants (Table 6-2)⁽⁸⁾ differs from that given in Table 6-1 primarily in that an additional group is introduced above the U^{238} fission threshold. This increases the range of systems that can be investigated without requiring adjustment of constants above the U^{238} fission threshold.

The third set of constants (Table 6-3)⁽⁹⁾ has been principally used for the analysis of elementary critical assemblies containing only fissile and fertile materials.

Inelastic Scattering and Elastic Moderation

The multigroup parameters given in Tables 6-1, 6-2, and 6-3 provide for inelastic scattering to various groups. Such scattering is difficult to describe because much information concerned with such processes is currently fragmentary. The lighter elements are more easily treated because inelastic thresholds are high and levels are few in the region of interest.^(10,11) Where possible, the inelastic levels are used to distribute the scattered neutrons once an assumption concerning the neutron spectrum within the energy group where scattering takes place has been decided upon.

The inelastic scattering matrix for the fissionable materials currently cannot be obtained on the basis of individual levels. However, it is known there are many levels and the threshold for inelastic scattering is on the order of a few kilovolts. It is believed there may be as many as eight levels below 200 kev. It is impossible, on theoretical grounds, to assign fractions of quite uncertain inelastic total cross sections to the excitation of these many levels. The statistical model of the nucleus is the foundation upon which scattering matrices for these materials is obtained.⁽¹²⁾

Table 6-1

MULTIGROUP CONSTANTS FOR FAST REACTOR ANALYSIS

(10 energy groups)
(microscopic cross sections in barns)

Pu ²³⁹										Fe															
j	X	E _L	u	σ _{tr}	ν	σ _f	α	σ _{er}	σ _{in}	σ _{in, j → j + k}						σ _{tr}	σ _c	σ _{er}	σ _{in}	σ _{in, j → j + k}					
										k = 123456						k = 123456									
1	0.575	1.35	2.0	4.66	3.10	1.85	0.02	0.0115	0.90	0.411	0.238	0.129	0.0702	0.0346	0.0176	2.13	0.003	0.0373	0.686	0.38	0.28	0.015	0.010	0.001	0
2	0.178	0.825	2.5	5.26	3.05	1.87	0.05	0.0436	0.70	0.348	0.179	0.0980	0.0490	0.245	0.0182	2.35	0.003	0.133	0.46	0	0.193	0.115	0.069	0.0414	0.0414
3	0.116	0.5	3.0	5.9	3.00	1.73	0.08	0.0560	0.70	0.336	0.183	0.0917	0.0466	0.0340	0.0084	2.75	0.0035	0.194	0	0	0	0	0	0	0
4	0.064	0.3	3.5	7.1	2.95	1.68	0.12	0.0809	0.40	0.200	0.100	0.0512	0.0371	0.00884	0.00248	2.90	0.004	0.204	0	0					
5	0.034	0.18	4.0	8.5	2.95	1.69	0.20	0.105	0.23	0.115	0.0591	0.0428	0.0101	0.00276	0	3.20	0.006	0.226	0	0					
6	0.017	0.11	4.5	9.5	2.95	1.72	0.33	0.121	0	0	0	0	0	0	0	3.60	0.008	0.254	0	0					
7	0.008	0.0674	5.0	10.8	2.95	1.82	0.37	0.140	0	0						4.0	0.009	0.282	0	0					
8	0.006	0.025	6.0	12.3	2.9	1.98	0.42	0.0797	0	0						6.0	0.011	0.211	0	0					
9	0.0015	0.0091	7.0	16.4	2.9	2.33	0.45	0.109	0	0						3.0	0.013	0.105	0	0					
10	0.0005	0	-	16.8	2.9	3.50	0.50	0	0	0						7.5	0.014	0	0	0					

C										U ²³⁵										Al									
j	σ _{tr}		σ _{er}	σ _{tr}	ν	σ _f	α	σ _{er}	σ _{in}	σ _{in, j → j + k}						σ _{tr}	σ _c	σ _{er}	σ _{in}	σ _{in, j → j + k}									
										k = 123456						k = 123456													
1	1.55		0.180	4.8	2.62	1.285	0.06	0.0115	1.58	0.722	0.417	0.226	0.123	0.0608	0.0308	1.7	0.001	0.0739	0.32	0.160	0.112	0.032	0.0128	0.0032	0				
2	2.17		0.690	4.8	2.55	1.230	0.08	0.0390	1.15	0.544	0.294	0.161	0.0805	0.0402	0.0299	2.45	0.0004	0.314	0.30	0	0.126	0.075	0.045	0.027	0.027				
3	2.97		0.944	5.1	2.52	1.200	0.11	0.0431	1.20	0.576	0.314	0.157	0.0798	0.0582	0.0144	3.0	0.0007	0.438	0	0	0	0	0	0	0				
4	3.24		1.030	6.3	2.50	1.290	0.14	0.0694	0.70	0.351	0.175	0.0896	0.0649	0.0155	0.00434	3.40	0.0015	0.496	0	0									
5	3.79		1.205	7.9	2.50	1.430	0.16	0.0981	0.40	0.200	0.103	0.0744	0.0176	0.0048	0	3.70	0.0017	0.540	0	0									
6	3.98		1.265	9.65	2.48	1.600	0.18	0.130	0	0	0	0	0	0	0	4.0	0.0040	0.583	0	0									
7	4.16		1.323	10.9	2.47	1.820	0.20	0.146	0	0						4.0	0.0025	0.584	0	0									
8	4.25		0.675	12.25	2.46	2.320	0.28	0.0780	0	0						3.5	0.010	0.255	0	0									
9	4.36		0.694	13.5	2.46	3.390	0.35	0.0749	0	0						1.25	0.060	0.0869	0	0									
10	4.33		0	14.3	2.46	6.000	0.42	0	0	0						1.8	0.080	0	0	0									

Np ²³⁷ Au ¹⁹⁷ U ²³⁴										U ²³⁸										Na									
j	σ _f	σ _{n,γ}	σ _f	σ _{tr}	ν	σ _f	σ _c	σ _{er}	σ _{in}	σ _{in, j → j + k}						σ _{tr}	σ _c	σ _{er}	σ _{in}	σ _{in, j → j + k}									
										k = 123456						k = 12345													
1	1.42	0.07	1.45	4.8	2.62	0.524	0.035	0.0135	2.05	0.517	0.75	0.437	0.137	0.083	0.126	2.00	0.0003	0.10	0.393	0.332	0.0259	0.0182	0.0118	0.005					
2	1.25	0.125	1.20	5.0	2.62	0.040	0.125	0.0634	1.06	0.38	0.35	0.22	0.11	0	0	2.78	0.0005	0.421	0.25	0.186	0.0641	0	0	0					
3	0.70	0.18	0.85	5.5	0	0	0.135	0.0871	0.18	0.18	0	0	0	0	0	4.14	0.0006	0.665	0.20	0.051	0.0732	0.0441	0.0268	0.005					
4	0.20	0.26	0.30	6.7	0	0	0.130	0.106	0.25	0.25	0	0	0	0	0	3.2	0.0008	0.540	0	0	0	0	0	0					
5		0.36	0.06	8.25	0	0	0.175	0.131	0.25	0.20	0.05	0	0	0	0	3.84	0.001	0.649	0	0									
6		0.55		9.65	0	0	0.225	0.154	0.25	0.25	0					3.26	0.001	0.551	0	0									
7		0.68		10.9	0	0	0.29	0.175	0.20	0.18	0.02					3.40	0.001	0.575	0	0									
8		1.05		12.25	0	0	0.38	0.0993	0.05	0.03	0.02					4.49	0.003	0.379	0	0									
9		1.95		13.5	0	0	0.50	0.109	0	0						4.47	0.001	0.378	0	0									
10		2.90		14.3	0	0	1.20	0	0	0						10.0	0.005	0	0	0									

TABLE 6-2
MULTIGROUP CONSTANTS FOR FAST REACTOR ANALYSIS
(11 ENERGY GROUPS)
(MICROSCOPIC CROSS SECTIONS IN BARNS)

				P_{239} ($N = .048 \times 10^{24}$)											
j	χ	E_L	ΔU	σ_{tr}	ν	σ_f	$\alpha(Std)$	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$					
										k=1	2	3	4	5	6
1	0.338	2.25	-	4.6	3.18	2.0	0.02	0.0142	1.35	0.513	0.382	0.221	0.120	0.0648	0.0486
2	0.236	1.35	0.5	4.6	3.09	1.95	0.04	0.0238	1.0	0.457	0.264	0.143	0.0780	0.0385	0.0195
3	0.178	0.825	0.5	5.26	3.02	1.86	0.05	0.0438	0.7	0.331	0.179	0.0980	0.0490	0.0245	0.0182
4	0.116	0.5	0.5	5.9	2.97	1.75	0.07	0.0559	0.7	0.336	0.183	0.0917	0.0446	0.0340	0.0084
5	0.066	0.3	0.5	7.1	2.95	1.70	0.10	0.0811	0.4	0.200	0.100	0.0512	0.0371	0.00884	0.00248
6	0.033	0.18	0.5	8.5	2.93	1.68	0.16	0.106	0.23	0.115	0.0591	0.0428	0.0101	0.00276	--
7	0.017	0.11	0.5	9.5	2.92	1.73	0.23	0.124	0						
8	0.008	0.067	0.5	10.8	2.91	1.80	0.30	0.142	0						
9	0.006	0.025	1.0	12.3	2.91	2.0	0.37	0.0803	0						
10	0.002	0.0091	1.0	16.4	2.91	2.25	0.47	0.110	0						
11	0	0	-	16.8	2.91	3.5	0.60	0	0						

				U^{233} ($N = .048 \times 10^{24}$)											
j	χ	E_L	ΔU	σ_{tr}	ν	σ_f	α	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$					
										k=1	2	3	4	5	6
1				4.3	2.8	1.80	0.028	0.0141	1.25	0.479	0.359	0.208	0.112	0.0625	0.030
2				4.5	2.65	1.85	0.043	0.0237	1.00	0.457	0.264	0.143	0.078	0.039	0.019
3				4.8	2.59	1.94	0.057	0.0336	0.75	0.355	0.192	0.105	0.0525	0.0262	0.0195
4				5.5	2.55	2.04	0.074	0.0497	0.35	0.168	0.0917	0.0458	0.0234	0.0168	0.0042
5				5.7	2.53	2.21	0.090	0.0536	0.10	0.0501	0.0250	0.0128	0.00927	0.00221	0.00062
6				7.0	2.52	2.27	0.10	0.0756	0						
7				9.0	2.52	2.43	0.10	0.106	0						
8				11.0	2.51	2.57	0.10	0.137	0						
9				15.0	2.51	3.0	0.10	0.0983	0						
10				20.0	2.51	4.2	0.10	0.129	0						
11				25.0	2.51	7.0	0.10	0	0						

				U^{235} ($N = .048 \times 10^{24}$)											
j	χ	E_L	ΔU	σ_{tr}	ν	σ_f	α	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$					
										k=1	2	3	4	5	6
1				4.5	2.77	1.3	0.077	0.00941	2.3	0.881	0.660	0.382	0.207	0.115	0.0552
2				4.5	2.65	1.28	0.078	0.0192	1.85	0.845	0.488	0.265	0.144	0.0712	0.0361
3				4.8	2.58	1.25	0.08	0.0366	1.15	0.544	0.294	0.161	0.0805	0.0402	0.0299
4				5.1	2.53	1.20	0.12	0.0429	1.2	0.576	0.314	0.157	0.0798	0.0582	0.0144
5				6.3	2.51	1.28	0.15	0.0694	0.7	0.351	0.175	0.0896	0.0649	0.0155	0.00434
6				7.9	2.49	1.42	0.18	0.0978	0.4	0.200	0.103	0.0744	0.0176	0.0048	0
7				9.65	2.48	1.6	0.20	0.130							
8				10.9	2.47	1.9	0.25	0.143							
9				12.25	2.47	2.3	0.30	0.0778							
10				13.5	2.47	3.4	0.35	0.0748							
11				14.3	2.47	6.0	0.42	0							

				U^{238} ($N = .048 \times 10^{24}$)											
j	χ	E_L	ΔU	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$					
										k=1	2	3	4	5	6
1				4.7	2.65	0.59	0.015	0.0144	2.87	1.06	0.46	0.66	0.39	0.12	0.18
2				4.5	2.55	0.45	0.062	0.0234	2.44	0.62	0.89	0.52	0.16	0.10	0.15
3				5.0	2.47	0.003	0.13	0.0616	1.20	0.52	0.35	0.22	0.11	0	0
4				5.5	0	0	0.143	0.0826	0.44	0.44	0				
5				6.7	0	0	0.13	0.102	0.47	0.47	0				
6				8.25	0	0	0.15	0.127	0.55	0.50	0.05				
7				9.65	0	0	0.20	0.150	0.55	0.55	0				
8				10.9	0	0	0.30	0.171	0.40	0.38	0.02				
9				12.25	0	0	0.40	0.0991	0.05	0.03	0.02				
10				13.5	0	0	0.61	0.108	0						
11				14.3	0	0	0.80	0	0						

				Th^{232} ($N = .03 \times 10^{24}$)											
j	χ	E_L	ΔU	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$					
										k=1	2	3	4	5	6
1				4.3	2.4	0.135	0.03	0.0102	3.3	1.22	0.53	0.76	0.45	0.14	0.20
2				4.5	2.3	0.10	0.04	0.0244	2.8	0.71	1.02	0.50	0.18	0.12	0.17
3				4.6	0	0	0.08	0.0569	1.25	0.54	0.37	0.23	0.11	0	0
4				5.1	0	0	0.15	0.0786	0.43	0.43	0	0	0	0	0
5				6.6	0	0	0.20	0.104	0.40	0.40	0	0	0	0	0
6				8.0	0	0	0.25	0.127	0.45	0.40	0.05				
7				9.2	0	0	0.32	0.147	0.43	0.43					
8				10.5	0	0	0.38	0.171	0.32	0.32					
9				11.6	0	0	0.50	0.0966	0						
10				13.5	0	0	0.60	0.112	0						
11				14.0	0	0	0.80	0	0						

TABLE 6-2 (CONT'D)
MULTIGROUP CONSTANTS FOR FAST REACTOR ANALYSIS
(11 ENERGY GROUPS)
(MICROSCOPIC CROSS SECTIONS IN BARNS)

C ^a									Pu ²⁴⁰ (N = .048 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1	1.4	0.311	4.6	2.65	1.6	0.02	0.0127	1.9	0.701	0.304	0.437	0.258	0.0798	0.120		
2	1.8	0.515	4.6	2.55	1.5	0.08	0.0237	1.45	0.368	0.529	0.309	0.0957	0.0594	0.0885		
3	2.17	0.690	5.3	2.47	1.35	0.18	0.0516	0.70	0.303	0.204	0.128	0.0644	0	0		
4	2.97	0.944	5.9	2.47	0.7	0.20	0.0764	0.45	0.45	0	0	0	0	0		
5	3.24	1.030	7.1	2.47	0.18	0.20	0.105	0.45	0.45	0	0	0	0	0		
6	3.79	1.204	8.5	2.47	0.03	0.20	0.131	0.45	0.410	0.040	0	0	0	0		
7	3.98	1.265	9.5	0	0	0.30	0.147	0.45	0.45	0	0	0	0	0		
8	4.16	1.322	10.8	0	0	0.45	0.170	0.25	0.238	0.012	0	0	0	0		
9	4.25	0.675	12.3	0	0	0.70	0.0974	0								
10	4.36	0.693	16.4	0	0	1.0	0.129	0								
11	4.33	0	16.8	0	0	1.5	0	0								

O ^b									Ne (N = .022 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1	1.05	0.178	1.9			0.0003	0.167	0.50	0.32	0.065	0.04	0.03	0.015	0.03		
2	1.6	0.348	2.5			0.0003	0.337	0.30	0.30	0	0	0	0	0		
3	3.6	0.871	3.3			0.0005	0.494	0.40	0.284	0.116	0	0	0	0		
4	3.0	0.726	3.2			0.0006	0.477	0.40	0.08	0.128	0.076	0.044	0.044	0.028		
5	5.7	1.379	3.3			0.0008	0.562	0								
6	3.35	0.811	3.6			0.001	0.613	0								
7	3.26	0.789	3.5			0.001	0.596	0								
8	3.26	0.789	3.3			0.001	0.562	0								
9	3.26	0.394	4.9			0.003	0.417	0								
10	3.26	0.394	4.9			0.001	0.417	0								
11	3.26	0	10.0			0.005	0	0								

Mo									(N = .0640 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1			2.5			0.010	0.0320	1.40	0.35	0.40	0.30	0.15	0.10	0.10		
2			3.2			0.017	0.0886	0.84	0.411	0.33	0.099	0	0	0		
3			4.0			0.024	0.150	0.40	0	0.108	0.117	0.068	0.042	0.065		
4			5.3			0.034	0.221	0								
5			6.4			0.040	0.267	0								
6			7.2			0.046	0.300	0								
7			7.8			0.057	0.325	0								
8			7.9			0.066	0.329	0								
9			7.4			0.090	0.154	0								
10			7.1			0.120	0.147	0								
11			7.2			0.200	0	0								

Fe									(N = .0847 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1			2.05			0.005	0.0432	1.18	0.450	0.206	0.196	0.122	0.0802	0.0448	0.0814	
2			2.0			0.005	0.0864	0.65	0	0.39	0.26	0	0	0	0	
3			1.9			0.005	0.107	0.4	0.170	0.104	0.0592	0.0332	0.0332	0	0	
4			2.2			0.005	0.157	0								
5			3.0			0.006	0.214	0								
6			3.5			0.007	0.249	0								
7			4.3			0.008	0.306	0								
8			5.3			0.008	0.378	0								
9			6.8			0.025	0.242	0								
10			4.0			0.012	0.142	0								
11			8.0			0.014	0	0								

Al									(N = .0603 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1			1.45			0.0003	0.0930	0.54	0.211	0.108	0.081	0.0648	0.0270	0.0324	0.0162	
2			1.8			0.0004	0.197	0.30	0.147	0.123	0.030	0	0	0	0	
3			2.0			0.0005	0.279	0.09	0.018	0.0279	0.0189	0.0135	0.0117	0	0	
4			2.85			0.0007	0.416	0								
5			3.1			0.0015	0.452	0								
6			4.1			0.0018	0.598	0								
7			5.6			0.0040	0.817	0								
8			4.0			0.0035	0.583	0								
9			5.5			0.0090	0.401	0								
10			1.8			0.0080	0.131	0								
11			1.6			0.0120	0	0								

Te									(N = .0553 x 10 ²⁴)							
j	σ_{tr}	σ_{er}	σ_{tr}	ν	σ_f	σ_c	σ_{er}	σ_{in}	$\sigma_{in j \rightarrow j+k}$							
									k=1	2	3	4	5	6	7	
1			3.7			0.07	0.0174	2.5	0.95	0.708	0.410	0.222	0.120	0.09		
2			4.5			0.09	0.0438	2.2	1.005	0.581	0.315	0.172	0.0847	0.0429		
3			5.0			0.10	0.0748	1.5	0.710	0.384	0.21	0.105	0.0525	0.039		
4			5.2			0.14	0.0937	0.8	0.384	0.210	0.105	0.0532	0.0388	0.0096		
5			5.6			0.20	0.119	0								
6			6.3			0.25	0.133	0								
7			7.5			0.30	0.158	0								
8			8.1			0.35	0.170	0								
9			9.9			0.40	0.104	0								
10			10.0			0.70	0.102	0								
11			17.0			1.50	0	0								

^a(N = .0838 x 10²⁴)

^b(N = .096 x 10²⁴)

Table 6-3

MULTIGROUP CONSTANTS FOR FAST REACTOR CALCULATIONS

(Data from Ref. 9)
(6 Energy Groups)
(Microscopic Cross Sections in Barns)

Pu ²³⁹													U ²³⁵												
j	χ	E_L	σ_{tr}	ν	σ_f	σ_c	σ_{in}^*	$\sigma_{in, j \rightarrow j+k}$					σ_{tr}	ν	σ_f	σ_c	σ_{in}^*	$\sigma_{in, j \rightarrow j+k}$							
								k =	1	2	3	4						5	k =	1	2	3	4	5	
1	0.204	3.0	4.25	3.48	1.90	0.03	1.27	0.20	0.27	0.45	0.31	0.04	4.25	2.94	1.21	0.05	1.79	0.27	0.37	0.65	0.44	0.06			
2	0.344	1.4	4.50	3.09	1.95	0.05	1.08	0.18	0.50	0.35	0.05		4.50	2.62	1.22	0.08	1.43	0.24	0.67	0.45	0.07				
3	0.168	0.9	4.80	2.99	1.83	0.07	0.81	0.45	0.30	0.06			4.65	2.53	1.22	0.11	1.02	0.55	0.40	0.07					
4	0.180	0.4	5.70	2.93	1.70	0.11	0.34	0.29	0.05				5.20	2.49	1.20	0.15	0.43	0.35	0.08						
5	0.090	0.1	8.40	2.88	1.67	0.17	0.05	0.05					7.90	2.46	1.43	0.23	0.08	0.08							
6	0.014	0	12.0	2.86	2.05	0.50		-					12.0	2.44	2.34	0.60		-							

U ²³³													U ²³⁸⁺												
j	σ_{tr}	ν	σ_f	σ_c	σ_{in}^*	$\sigma_{in, j \rightarrow j+k}$					σ_{tr}	ν	σ_f	σ_c	σ_{in}^*	$\sigma_{in, j \rightarrow j+k}$									
						k =	1	2	3	4						5	k =	1	2	3	4	5			
1	4.25	3.02	1.75	0.04	1.27	0.20	0.27	0.45	0.31	0.04	4.00	2.80	0.616	0.02	2.18	0.33	0.46	0.79	0.53	0.07					
2	4.50	2.70	1.83	0.06	1.08	0.18	0.50	0.35	0.05		4.40	2.50	0.485	0.05	2.04	0.35	0.96	0.64	0.09						
3	4.60	2.61	1.89	0.08	0.81	0.45	0.30	0.06			4.50	2.46	0.044	0.10	1.45	0.80	0.55	0.10							
4	5.30	2.57	1.94	0.11	0.34	0.29	0.05				5.25	0	0	0.14	0.58	0.50	0.08								
5	8.10	2.53	2.24	0.15	0.05	0.05					8.20	0	0	0.16	0.08	0.08									
6	11.80	2.51	3.23	0.40		-					11.80	0	0	0.40											

* σ_{in} and $\sigma_{in, j \rightarrow j+1}$ on Table 6-3 includes σ_{er} .† $\sigma_{n,2n} = .07$ for Group 1

More information is available⁽⁵⁾ concerning the low-lying levels of U²³⁸. However, it has been found that the more recent measurements⁽¹³⁾ imply a scattering matrix which is not in agreement with Snell-block measurements,^(14,15) assuming that $\sigma_{n,\gamma}$ for U²³⁸ is known.⁽⁸⁾ The matrix presented in Table 6-1 does predict the measured diffusion length in natural uranium while that given in Table 6-2 is in agreement with Cranberg's measurements.⁽¹³⁾

Elastic moderation is treated⁽¹⁶⁾ by assuming that the flux per unit lethargy is known throughout the energy group. An elastic transport cross section is defined for the group to describe the total number of elastic collisions which occur. A fraction of these collisions scatter the neutron out of the group. This fraction is determined by evaluating the ratio of the neutron flux per unit lethargy at the low-energy group boundary, to the total neutron flux in the group. This ratio multiplied by the logarithmic decrement of the material considered indicates which fraction of the elastically scattered neutrons leave the group.

6.1.3 Few-Group Constants

The variations of cross sections with energy are such that if only a few groups are used, the group cross sections are largely of use only in the neutron spectrum for which the cross section parameters are evaluated. Because of the resultant simplification in calculation, few-group constants are nevertheless of great utility.

Three sets of few-group constants are given in Table 6-4. These were obtained by averaging calculated spectra based on multigroup calculations.

The effective U²³⁸ fission cross section must be considered a variable with core enrichment and dilution when one-group analysis is used.

A set of two-group cross sections is quite useful because it is possible to apply them to an analytical treatment of fast reactor problems without the use of

TABLE 6-4

ONE-, TWO-, AND THREE-GROUP CONSTANTS
(Microscopic Cross Sections in barns)

Group	j	Energy Range, Mev	Fission Spectrum	Pu^{239}						U^{235}					
				ν	σ_f	σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$	ν	σ_f	σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$
One	1	0 - ∞	1.0	2.98	1.85	0.26	6.8	-	-	2.6	1.4	0.25	6.8	-	-
Two	1	1.35 - ∞	0.575	3.10	1.95	0.10	4.6	0.90	-	2.7	1.29	0.08	4.5	1.50	-
	2	0 - 1.35	0.425	2.93	1.78	0.30	7.0	-	-	2.5	1.44	0.28	7.2	-	-
Three	1	1.35 - ∞	0.575	3.10	1.95	0.10	4.6	0.45	0.45	2.7	1.29	0.08	4.5	1.00	0.50
	2	0.4 - 1.35	0.326	2.98	1.75	0.12	6.0	0.50	-	2.53	1.27	0.13	5.7	0.50	-
	3	0 - 0.4	0.099	2.91	1.80	0.35	10.0	-	-	2.47	1.77	0.49	10.0	-	-
Group	j	Energy Range, Mev	Fission Spectrum	U^{238}						Fe					
				ν	σ_f	σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$		σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$	
One	1	0 - ∞	1.0	2.6	0.095	0.16	6.9	-	-		0.006	2.7	-	-	
Two	1	1.35 - ∞	0.575	2.6	0.524	0.036	4.6	2.05	-		0.005	2.0	0.70	-	
	2	0 - 1.35	0.425	2.47	0.005	0.19	7.1	-	-		0.006	2.8	-	-	
Three	1	1.35 - ∞	0.575	2.6	0.524	0.036	4.6	1.41	0.64		0.005	2.0	0.60	0.10	
	2	0.4 - 1.35	0.326	2.47	0.01	0.13	5.8	0.25	-		0.006	2.13	0.22	-	
	3	0 - 0.4	0.099	-	-	0.26	9.6	-	-		0.006	3.24	-	-	
Group	j	Energy Range, Mev	Fission Spectrum	Au		Na				Np^{237}		Al			
				σ_c	σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$		σ_f	σ_c	σ_{tr}	$\sigma_{in,j \rightarrow j+1}$	$\sigma_{in,j \rightarrow j+2}$	
One	1	0 - ∞	1.0		0.0008	3.3	-	-			0.002	3.1	-	-	
Two	1	1.35 - ∞	0.575		0.0005	2.0	0.30	-			0.004	1.8	0.38	-	
	2	0 - 1.35	0.425		0.0008	3.5	-	-			0.002	3.5	-	-	
Three	1	1.35 - ∞	0.575	0.07	0.0005	2.0	0.24	0.06		1.42	0.004	1.8	0.33	0.05	
	2	0.4 - 1.35	0.326	0.192	0.001	3.2	0.18	-		0.68	0.0006	2.4	0.2	-	
	3	0 - 0.4	0.099	0.65	0.001	3.7	-	-		0	0.002	3.8	-	-	

NOTE: The group inelastic removal $\sigma_{in,j \rightarrow k}$ includes inelastic and elastic removal.

high-speed computing equipment. The group structure is such that group 1 represents all the neutrons above the U^{238} fission threshold (1.4 Mev), while group 2 represents all those below.

Three-group calculations are helpful in analyzing critical experiments because the threshold detectors U^{238} (n,f) and Np^{237} (n,f) combined with a detector giving an approximate integral of the flux, such as U^{235} (n,f) or Pu^{239} (n,f), facilitate, experimentally, an approximate division of the flux into three energy groups divided at about 1.35 Mev and 0.4 Mev.

6.2 INTEGRAL DATA

6.2.1 Critical Mass

There is considerable experimental information available on a variety of fast neutron systems. The results cited here are those believed amenable to calculation in spherical geometry.

Table 6-5 describes critical experiments performed with high density plutonium cores. Jezebel⁽⁹⁾ is an unreflected plutonium critical assembly at LASL. Popsy⁽⁹⁾ has essentially a Jezebel core composition but is reflected with a natural uranium blanket and has a much lower critical mass. The other assemblies represent experiments performed at UCRL.⁽¹⁷⁾

Table 6-5

OBSERVED CRITICAL MASSES - SPHERICAL PLUTONIUM CORES

Assembly	Density of Pu in Core gm/cm ³	Reflector			Critical Mass, kg Pu	Ref.
		Composition	Density, gm/cm ³	Thick., cm		
Jezebel	15.82	-	-	-	16.45 ± .05	9, 18
Popsy	15.79	Nat. U	18.8	24.1	5.79 ± .02	9, 18
	15.8	Nat. U	18.8	5.0	8.06	17
	15.8	Nat. U	18.8	2.0	10.68	17
	15.8	Beryllium	1.86	5.0	7.48	17
	15.8	Beryllium	1.86	2.0	10.35	17
	15.8	Carbon	1.63	5.0	10.05	17
	15.8	Carbon	1.63	2.0	12.75	17

Table 6-6 describes critical experiments performed with high-density uranium systems. In all assemblies, the core alloy is highly enriched uranium. The Godiva assembly⁽⁹⁾ is a spherical, unreflected assembly. Topsy⁽⁹⁾ is reflected with a natural uranium blanket, and has a much lower critical mass than Godiva.

Table 6-6

OBSERVED CRITICAL MASSES - SPHERICAL URANIUM CORES
HIGHLY ENRICHED IN U^{235} AND U^{233}

Assembly	Enrich.,* %	Density of Core Alloy	Reflector			Critical Mass kg U^{235}	Ref.
			Composition	Density, gm/cm ³	Thick., cm		
Godiva	93.9	18.75	-	-	-	48.8	9, 19
Topsy	94.1	18.75	Nat. U	18.8	20.32	16.28	9, 19
	93.5	18.75	Nat. U	18.8	5.08	23.75	19
	93.5	18.75	Nat. U	18.8	4.75	24.5	19
	93.5	18.75	Nat. U	18.8	2.84	30.3	19
	93.9	18.55	Graphite	1.69	43.2	16.9	21, 20
	93.9	18.55	Water	1.0	∞	23.2	21, 20
	93.9	18.55	Paraffin	.89	∞	22.2	21, 20
U^{233}	98.0	18.60	-	-	-	16.0	9

*All enrichments refer to U^{235} in uranium except for the single U^{233} system.

Table 6-7 summarizes the critical mass information determined from experiments with a variety of core compositions. The Jemima assemblies⁽⁹⁾ are stacks of interspersed highly enriched and natural uranium plates. The ZPR-III assemblies⁽⁶⁾ consist of small building blocks of the basic constituent materials arranged in a two-half honeycomb machine.

Table 6-7

OBSERVED CRITICAL MASSES - URANIUM CORES OF VARIOUS ENRICHMENTS AND DILUTIONS

Assembly	Sphere or Cylinder L/D	Core Composition, Volume Fraction					Reflector Composition			Density of U, gm/cm ³		Critical ^b Mass kg U ²³⁵	References	
		U ²³⁵	U ²³⁸	Fe (SST)	Al	Ca	Volume Fraction			Thick., cm	Core			Refl.
							Depleted U	Fe (SST)	Al					
Jemima-53	.58	.535	.465							0	18.8	-	86.7	9, 22
Jemima-37	1.0	.377	.623							0	18.8	-	100.7	9, 22
Jemima-29	1.18	.29	.71							0	18.8	-	123.0	9, 22
ZPR-III-6F	Sphere	.14	.159	.123	.314		.834	.073	.023	30	18.8	19	131.1 ±1	6
↑ -9A	Sphere	.117	.380	.142	.215		.834	.073		30	18.8	19	146.2	6
-2A	.97	.14	.01	.275	.314		.834	.073	.023	30	18.8	19	147.7	6
-10	1.04	.119	.579	.193			.834	.073		30	18.8	19	155.8	6
-11	.88	.095	.70	.092			.834	.073		30	18.8	19	240.6	6
-16	.92	.095	.50	.092		.182	.834	.073		30	18.8	19	204.8	6
-12	.87	.095	.355	.092		.318	.834	.073		30	18.8	19	176.8	6
-17	.93	.095	.206	.092		.454	.834	.073		30	18.8	19	156.5	6
-14	.99	.095	.007	.092		.734	.834	.073		30	18.8	19	136.0	6
↓ -15	.89	.095	.315	.117		.194	.834	.073		30	18.8	19	220.4	6
ZPR-III-5	.94	.14	.159	.123	.314	Inner Reflector			17	18.8	19	159.5	6	
						.40	.073	.25						
						Outer Reflector			17	18.8	19	159.5	6	
						.834	.073	.023						

^adensity = 1.45 gm/cm³^bCritical Masses for ZPR-III assemblies are reported as observed in Reference 6. Corrections for heterogeneity are indicated in Fig. 6-1 (See Section 6.2.3).

The designations in the first column of Table 6-7 will be used to identify the respective assemblies in subsequent sections treating spectral indices and central material replacement experiments.

Figure 6-1 compares calculated with experimentally observed critical masses when core material of the same composition as in the calculation is added at the core boundary to achieve criticality. In these predicted critical masses appropriate shape factors are applied to convert from spherical to cylindrical geometry (see Section 6.2.2). The Corrected Experimental Critical Mass (CECM) is the basis for comparing measured with predicted values. The CECM accounts for heterogeneities inherent in the experimental determinations (see Section 6.2.3).

FIG. 6-1
CRITICAL MASS AS A FUNCTION OF DILUTION

The critical assemblies referred to in Fig. 6-1 are typical of those investigated in the Argonne ZPR-III Fast Critical Facility. The assemblies are identified by the atomic ratio of U^{238} to U^{235} in the core. However, the total volume fraction of uranium was not held constant. These compositions were varied to avoid working with extremely large amounts of fissionable material. All cores were reflected by a high density depleted uranium blanket. The compositions of the various assemblies are given in Table 6-7.

The set of six-group constants (Table 6-3) has been tested against many of the high density fast neutron systems investigated at LASL with quite good agreement between theory and experiment.(9)

Paxton(21) has reported several useful empirical relationships. The effect of changing the density of an enriched uranium core which is surrounded by a thick natural uranium reflector, is given by:

$$\text{Critical Mass} \sim (\text{core density})^{-1.2}$$

The effect of changing the U^{235} enrichment in the core can be expressed:

$$(\text{Critical mass of } U^{235} + U^{238}) \sim (U^{235} \text{ enrichment})^{-1.7}$$

Critical masses of plutonium and U^{233} for small high-density cores can be roughly estimated by scaling U^{235} values downward by a factor of 3.

6.2.2 Shape Factor

The shape factor for an assembly is defined as the ratio of the spherical critical mass of a system with a similar core composition and reflector parameters, to

the critical mass of the given assembly. Figure 6-2 shows the results of experimental shape factor investigation^(6,17,19) for cylindrical assemblies of varying ratios of core height, L , to core diameter, D . The ZPR-III shape factors which represent critical masses between 130 and 220 kg U^{235} correspond very closely to the shape factors associated with high-density and highly enriched uranium cores reflected by a 2.8 cm natural uranium reflector.

FIG. 6-2
SHAPE FACTOR AS A FUNCTION OF L/D FOR VARIOUS CYLINDRICAL ASSEMBLIES

The information in Tables 6-5, 6-6, and 6-7, when used in conjunction with Fig. 6-2 affords an estimate of the critical masses of a large number of systems of varying geometries.

6.2.3 Effect of Inhomogeneity

The experimentally observed critical masses given in Section 6.2.1, in some cases, require small corrections if they are to be interpreted as corresponding to homogeneous systems. One such correction is due to the fact that the basic constituent materials of various critical facilities are of finite, but small size. Such corrections are usually small and generally not made for fast neutron systems. An experiment to estimate the magnitude of the effect of bunching of fuel was performed on ZPR-III Assemblies No. 2A, 2 (cylindrical version of 6F), and 14. These experiments introduced inhomogeneity to a considerably greater degree than ordinarily exists. In each case a reactivity increase was observed when the fuel per unit cell was shifted from the most homogeneous to the most heterogeneous arrangement. The overall compositions of a cell and the core as a whole was left unchanged. The experimental results⁽⁶⁾ are summarized below.

ZPR-III Assembly No.	Critical Mass, (kg U ²³⁵)*	
	Most Homogeneous Arrangement	Most Heterogeneous Arrangement
2A	147.7	137.8
2	136	126.8
14	136	131

*Criticality is attained by varying core radius, leaving average composition and core height unchanged.

Such reactivity changes have been, to some extent, predicted using S_n calculations for unit cells in slab geometry.⁽⁶⁾ Slab geometry is appropriate because it most nearly represents the arrangement of constituent materials in the ZPR-III assembly matrix cell.

6.2.4 Detector Ratios of Various Fast Assemblies

Currently, there is little experimental information on the characteristic neutron spectra of a fast reactor other than may be inferred from measured detector ratios in such systems. For instance, a measurement $\sigma_f^{U^{238}}/\sigma_f^{U^{235}}$ makes it possible to estimate the fraction of the neutron flux above the U^{238} fission threshold (~ 1.4 Mev). Similarly, a measurement of $\sigma_f^{U^{234}}/\sigma_f^{U^{235}}$ can be used to determine the fraction of neutrons above the fission threshold of U^{234} (~ 0.5 Mev). In addition $\sigma_c^{U^{238}}/\sigma_f^{U^{235}}$ is important toward determining the conversion ratio of a system fueled with enriched uranium.

Table 6-8 lists the experimentally determined fission ratios at the centers of various fast assemblies. Similarly, Table 6-9 lists the measured ratios of $\sigma_{n,\gamma}^{U^{238}}$ to $\sigma_{n,\gamma}^{U^{235}}$ and $\sigma_{n,\gamma}^{U^{238}}$ to $\sigma_{n,f}^{U^{238}}$. The measured values cited in both tables are principally for systems whose central spectral properties may be readily calculated by one-dimensional methods.

6.2.5 Central Material Replacement Experiments

Tables 6-10 and 6-11 list the material replacement effect on reactivity (in cents/mole) at the centers of various fast neutron systems. A cent is 1/100 of a dollar, the dollar defining the amount of reactivity required to bring a system from delayed critical to prompt critical.

The data for Jezebel, Godiva, Topsy, and ZPR-III-9A, are with respect to void. The ZPR-III-5 data are with respect to low-density (1.5 gm/cm^3) aluminum.

Effective values for the total delay fraction of the assemblies are (see Section 6.3.3):

Assembly	β_{eff}	Assembly	β_{eff}
Jezebel ⁽⁹⁾	0.0020	ZPR-III-5,11	0.0074
Godiva ⁽⁹⁾	0.0068	ZPR-III-9A	0.0073
Topsy ⁽⁹⁾	0.0073	ZPR-III-12	0.0075

Table 6-8

FISSION RATIOS MEASURED AT CENTER OF VARIOUS FAST ASSEMBLIES

Assembly	Fission Ratio ^a	Value	Method ^b	Reference	Assembly	Fission Ratio ^a	Value	Method ^b	Reference
Jezebel	25/28	5.13	F.C.	9, 18	Jemima (16) ^d	25/28	20.4	γ-C	9, 25
"	"	4.56	S.C.		"	"	17.0	R.C.	
"	37/28	4.04	F.C.		"	"	18.2	F.C.	
"	"	4.76	S.C.		"	37/28	6.27	F.C.	
Popsy	25/28	5.45	R.C.	9, 23	"	37/28	7.1	F.C.	9
"	49/25	1.41	R.C.		Jemima (09)	25/28	30	γ-C	
Godiva	25/28	6.18	R.C.		"	"	28	R.C.	
"	"	6.54	γ-C		"	49/25	1.26	γ-C	
"	"	6.17	C.C.	9, 23	Jemima (06) ^e	25/28	36	γ-C	14
"	37/28	4.49	γ-C		Jemima (05) ^e	25/28	42	γ-C	
"	"	4.82	C.C.		"	25/28	44	R.C.	
"	49/25	1.43	C.C.		"	49/25	1.23	R.C.	
"	23/25	1.67	C.C.	9, 18	Jemima (04) ^e	25/28	47	γ-C	14
"	24/25	.823	C.C.		"	"	48	R.C.	
Topsy	25/28	7.3	F.C.		"	49/25	1.11	R.C.	
"	25/28	6.63	R.C.		Natural Uranium	25/28	300-350	F.C.	15
"	"	6.83	C.C.	9, 23	Natural Uranium	25/28	220	γ-C	
"	37/28	4.56	R.C.		Exponential	"	200	R.C.	
"	"	5.02	C.C.		Experiment	"	210	F.C.	
"	49/25	1.44	C.C.	24	"	37/28	15.9	F.C.	14
"	23/25	1.63	C.C.		"	49/25	1.14	R.C.	
"	24/25	.754	C.C.		ZPR-III-10	25/28	22.7	F.C.	
EBR-I	25/28	8.70	F.C.		"	"	20.8	R.C.	
"	49/25	1.42	F.C.	9, 22	"	24/25	.331	F.C.	6
"	24/25	.625	F.C.		"	23/25	1.52	F.C.	
Jemima (53)	25/28	8.68	R.C.		"	49/25	1.22	F.C.	
"	"	9.6	γ-C	9, 22	ZPR-III-11	25/28	27.8	F.C.	6
"	"	8.9	γ-C		"	"	24.4	R.C.	
"	37/28	5.3	γ-C		"	24/25	.299	F.C.	
Jemima (37)	25/28	10.11	R.C.		"	23/25	1.51	F.C.	
"	"	10.9	γ-C	9, 22	"	49/25	1.17	F.C.	6
"	37/28	5.9	γ-C		ZPR-III-12	25/28	22.7	F.C.	
Jemima (29)	25/28	12.9-13.7	β-C		"	25/28	21.7	R.C.	
ZPR-III-2 ^c	25/28	15.6	F.C.	6	"	24/25	.293	F.C.	
"	"	17.	R.C.		"	23/25	1.46	F.C.	
"	49/25	1.16	F.C.		"	49/25	1.10	F.C.	
"	24/25	.47	F.C.	6	ZPR-III-17	25/28	20.4	F.C.	6
"	23/25	1.53	F.C.		"	24/25	.298	F.C.	
ZPR-III-2A	25/28	15.2	F.C.		"	23/25	1.46	F.C.	
"	"	11.4	R.C.		"	49/25	1.08	F.C.	
"	"	12.3	A.C.	6	ZPR-III-14	25/28	18.2	F.C.	6
"	49/25	1.28	F.C.		"	24/25	.305	F.C.	
"	24/25	0.57	F.C.		"	23/25	1.45	F.C.	
ZPR-III-5	25/28	13.5	F.C.		"	49/25	1.05	F.C.	
"	"	13.3	R.C.	6	a - $25/28 = \frac{\sigma_f^{235}}{\sigma_f^{238}} \frac{\sigma_f^{238}}{\sigma_f^{235}}$	b - F.C. = Fission counter			
"	"	12.5	A.C.		37/28 = $\frac{\sigma_f^{237}}{\sigma_f^{238}} \frac{\sigma_f^{238}}{\sigma_f^{235}}$	S.C. = Standard counter			
"	24/25	0.56	F.C.		49/25 = $\frac{\sigma_f^{239}}{\sigma_f^{238}} \frac{\sigma_f^{238}}{\sigma_f^{235}}$	R.C. = Radiochemical determination			
"	"	0.40	A.C.		23/25 = $\frac{\sigma_f^{233}}{\sigma_f^{238}} \frac{\sigma_f^{238}}{\sigma_f^{235}}$	γ-C = Gamma counting			
"	49/25	1.24	F.C.	6	24/25 = $\frac{\sigma_f^{234}}{\sigma_f^{238}} \frac{\sigma_f^{238}}{\sigma_f^{235}}$	C.C. = Comparison chamber			
"	"	1.87	R.C.			β-C = Beta counting			
"	"	1.20	A.C.			A.C. = Absolute counter			
ZPR-III-6F	25/28	14.3	A.C.	6	c - Cylinder with L/D = 1. Same core composition and reflector parameters as ZPR-III-6F.				
"	"	14.4	R.C.		d - A 16.25% enriched uranium critical assembly, very much like the higher enriched and high-density systems in Table 6-7.				
ZPR-III-9	25/28	19.6	F.C.		e - Data on Jemima (04), (05), (06) and (09) obtained from exponential experiments on high-density-uranium systems with U ²³⁵ enrichments of 4.29%, 5.14%, 6.29%, 9.14%, respectively.				
"	24/25	0.43	F.C.		*Private communication, F. Kirn, ANL.				
"	49/25	1.30	F.C.	6					
ZPR-III-15	25/28	19.4	F.C.						
"	23/25	1.54	F.C.						
"	49/25	1.22	F.C.						
"	24/25	.36	F.C.						

Table 6-9

CENTRAL VALUES OF $\sigma_{n,\gamma}^{U^{238}}/\sigma_f^{U^{235}}$ AND $\sigma_{n,\gamma}^{U^{238}}/\sigma_f^{U^{238}}$ MEASURED
BY RADIOCHEMICAL TECHNIQUE

Assembly	$\sigma_{n,\gamma}^{U^{238}}/\sigma_f^{U^{235}}$	$\sigma_{n,\gamma}^{U^{238}}/\sigma_f^{U^{238}}$	Reference
Popsy	-	0.449	18
Godiva	-	0.458	23
Topsy	-	0.529	23
Jemima (53)	-	0.816	22
ZPR-III-5	0.087	-	6
ZPR-III-6F	0.084	-	6
Jemima (09)	0.106	-	14
Jemima (05)	0.119	-	14
Jemima (04)	0.122	-	14
Natural U Exponential	0.194	-	14
ZPR-III-10	0.093	-	6
ZPR-III-12	0.10	-	6
ZPR-III-15	0.08	-	6

Table 6-10

EFFECT OF MATERIAL REPLACEMENT ON REACTIVITY AT CENTER
OF VARIOUS FAST ASSEMBLIES

(in cents/mole)

Material	Jezebel(18)	Godiva(26)	Topsy(26),h	Material	Jezebel	Godiva	Topsy
H		47.8		Sn			-4.4
D		17.8		Sb			-14.5
Li			-0.24	I			-13.5
Be	12.2 ± 0.3	7.3	9.2	La			1.8
B ¹⁰	-145.2 ± 0.7 ^a	-48.0 ^b		Ce			-4.5
B		-6.9	-12.2	Nd			-8.6
C	-5.2 ± 0.7	2.4	2.4	Ta	-75.4 ± 2.9		-17.2
Mg			1.4	W	-55.5 ± 0.9		-10.8
Al	-11.0 ± 0.6	0.5	0.7	Ir			-21.3
S			-8.9	Pt			-10.7
Ca			-6.6	Au	-66.3 ± 0.6	-8.4	-15.8
Ti	-10.5 ± 2.4		-1.9	Pb			-1.5
V	-14.2 ± 0.9		0.0	Bi	-18.3 ± 2.0	-1.7	-4.1
Cr			-1.7	Th		-1.4	-7.6
Mn			-1.0	Nat U	85.7 ± 0.8	24.7	26.6
Fe	-16.7 ± 0.4	-0.2	-2.2	U ²³³			350.5
Co	-18.7 ± 0.6	-0.6	-2.5	U ²³⁵	604 ± 3 ^c	142.1 ^d	201.1 ^e
Ni	-36.8 ± 0.6	-4.4	-7.3	Pu ²³⁹	1256 ± 4	284.1	402.5 ^f
Cu	-25.5 ± 0.7	-1.8	-3.9	Li ₂ SO ₄			-5.7
Zn		-2.5	-5.2	BeO			10.6
Ga			-4.0	CH ₂	97 ± 2.3		137.5
Ge			-4.5	CD ₂			51.7 ^g
As			-2.0	CF ₂			10.4
Zr	-22.9 ± 4.4		-2.4	MgF ₂			13.1
Nb			-7.5	Al ₂ O ₃			5.5
Mo			-3.5	Dy ₂ O ₃	-161 ± 17		
Rh			-14.1	Yb ₂ O ₃	-118 ± 16		
Pd			-10.3				

^a87% enriched in B¹⁰

^b85% enriched in B¹⁰

^c93.4% enriched in U²³⁵

^d93.7% enriched in U²³⁵

^e95.4% enriched in U²³⁵

^f98.5% Pu²³⁹

^g98.34% Deuterium

^hThick natural uranium
reflector

Table 6-11

EFFECT OF MATERIAL REPLACEMENT AT THE CENTER
OF DILUTE FAST NEUTRON SYSTEMS

(in cents/mole)

Material	ZPR-III-5	ZPR-III-9A	ZPR-III-11	ZPR-III-12
B ¹⁰		-9.7	-9.7	-15.0
C	0.33		-0.125	0.129
Al		-0.098		-0.059
Si	0.075			
S	-0.34			
Ti	0.034			
V	0.136			
Cr	-0.108			
Mn	0.070	-0.088		
Fe	-0.15	-0.223	-0.257	-0.19
Co	0.035			
Cu		-0.368		
As	-0.730			
Se	-0.095			
Nb	-0.85		-1.00	-1.30
Mo	-0.53	-0.706	-0.70	-0.905
Ru	-1.16		-1.48	-2.09
Pd	-2.00			
Ag		-3.03		
Cd	-1.11			
Sb	-1.45			
Te	-0.495			
I	-1.77			
Ce	-0.087			
Hf	-1.99			
Ta	-2.25		-2.70	-3.75
W	-1.07			
Hg	-0.59			
U ²³³			35.2	39.5
U ^{235a}	21.2 ^c		19.1	20.9
Pu ^{239b}	38.8 ^c		32.9	35.5
U ²³⁸	0.195 ^d			-0.84 ^d
B ₄ C	-7.92			
CH ₂	15.5			

^a 93.2% enriched in U²³⁵^b 94.5% Pu²³⁹^c Value is questionable^d Depleted uranium

6.2.6 Buckling of U^{235} - U^{238} and Pu^{239} - U^{238} Mixtures

The ten-group constants in Table 6-1 were used to calculate the bucklings in U^{235} - U^{238} and Pu^{239} - U^{238} mixtures. The results are plotted in Figs. 6-3 and 6-4 as a function of atomic enrichment of U^{235} or Pu^{239} , respectively, in the mixture.

FIG. 6-3
BUCKLING FOR BARE U^{235} - U^{238} SYSTEMS
ASSUMING $N^{U^{235}} + N^{U^{238}} = 0.048 \times 10^{24}$

The calculated multigroup fluxes were used to obtain the ratios of $\sigma_f^{U^{235}}/\sigma_f^{U^{238}}$, and $\sigma_f^{Np^{237}}/\sigma_f^{U^{238}}$, as a function of enrichment. The results are plotted in Figs. 6-5 and 6-6 for U^{235} and Pu^{239} . Measurements on Jezebel, (18) Godiva, and assemblies (9,14,15,23) with various enrichments of U^{235} are indicated.

Asymptotic transport theory was used to obtain the buckling. (12) The convention for the sign of the buckling value is such that the diffusion equation is written:

$$\nabla^2 \phi + \kappa^2 \phi = 0$$

Thus, $\kappa^2 > 0$ defines a multiplying medium.

For simplicity, it was assumed that the number of atoms per cubic centimeter $N = 0.048 \times 10^{24}$ throughout the analysis. The buckling of a less dense system may be readily obtained since $\kappa^2 \sim N^2$.

FIG. 6-5
FISSION RATIOS FOR BARE U^{235} - U^{238} SYSTEMS

FIG. 6-4
BUCKLING FOR BARE Pu^{239} - U^{238} SYSTEMS
ASSUMING $N Pu^{239} + N U^{238} = 0.048 \times 10^{24}$

FIG. 6-8
FISSION RATIOS FOR BARE Pu^{239} - U^{238} SYSTEMS

To emphasize the range where $\kappa^2 \approx 0$, it was found useful to have a logarithmic plot; for $\kappa^2 < 0$, $-\kappa^2$ is plotted.

6.2.7 Extrapolation Distances for High-Density Uranium Systems

The extrapolation distances for bare uranium cores with various enrichments of U^{235} in U^{238} are given in Table 6-12. The ten-group set of cross sections in Table 6-1 was used in the calculations.

Table 6-12

CALCULATED EXTRAPOLATION DISTANCES (δ) FOR HIGH-DENSITY SYSTEMS OF VARIOUS ENRICHMENTS

$$N^{\text{U}^{235}} + N^{\text{U}^{238}} = N^{\text{Pu}^{239}} + N^{\text{U}^{238}} = 0.048 \times 10^{24} \text{ atom/cm}^3$$

$N^{\text{U}^{235}} / (N^{\text{U}^{235}} + N^{\text{U}^{238}})$	δ cm	$N^{\text{Pu}^{239}} / (N^{\text{Pu}^{239}} + N^{\text{U}^{238}})$	δ cm
1.00	2.065	1.00	1.68
0.80	2.130	0.80	1.81
0.60	2.207	0.60	1.95
0.40	2.286	0.40	2.12
0.20	2.352	0.20	2.30
0.12	2.366	0.10	2.38
0.08	2.345	0.06	2.38

The critical radius, r , of a spherical bare core of given composition is obtained by the S_4 method. A multigroup fundamental mode analysis⁽¹²⁾ yields the buckling, κ^2 , for the same composition. The extrapolation distance, δ , is then obtained from the defining relation:

$$\delta = \frac{\pi}{\kappa} - r$$

6.2.8 Reflector Savings

The reflector savings is defined as the difference between the bare and reflected critical dimensions for systems of identical core composition. Direct experimental data on fast systems is sparse. The critical radius for Godiva is 8.72 cm, and for Topsy 6.35 cm, yielding a reflector savings of 2.37 cm for a thick natural uranium blanket surrounding a highly enriched uranium core. For Jezebel and Popsy, the corresponding numbers are 6.29 cm and 4.44 cm, yielding a reflector savings of 1.85 cm for a similar reflector on a plutonium core.

On larger systems for which it is not feasible to obtain a bare assembly, an experimental measure of the bare core may be obtained by extrapolation of the asymptotic flux distribution.

Table 6-13 gives the calculated reflector savings for a series of compositions similar to those investigated on the ZPR-III facility. The results are based on the ten-group set of cross sections of Table 6-1 and are given for spherical, slab, and cylindrical geometries. In the latter two cases the system is assumed infinite in the transverse directions.

Table 6-13

CALCULATED REFLECTOR SAVINGS FOR DILUTE URANIUM SYSTEMS

Note: Reflector in each system is high-density depleted uranium (83 vol-%) in a stainless steel matrix (9 vol-%).

Core Composition, Volume Fraction				Reflector Savings, cm			Critical Dimension of Reflected Core		
							Sphere Radius, cm	Cylinder Radius, cm	Slab Half- Thickness, cm
U^{235}	U^{238}	Fe	Al	Sphere	Cylinder	Slab			
0.14	0.16	0.12	0.31	11.1	11.0	9.0	21.9	13.9	5.4
0.14	0.01	0.28	0.31	11.2	11.3	9.5	23.1	14.0	5.7
0.12	0.38	0.14	0.22	7.9	8.7	7.7	25.5	16.3	7.6

The results of a two-dimensional (r, z) diffusion theory calculation on ZPR-III Assembly 2 have been compared with the results of self-consistent, one-dimensional diffusion theory calculations of the same assembly in cylindrical and slab geometries. In the self-consistent iterative procedure it is assumed that an approximate solution can be obtained from consideration of two one-dimensional problems: (a) an axially bare, radially reflected cylinder; and (b) a radially bare, axially reflected slab.

The bare axial dimension to be used in (a) is determined by the equivalent bare height in (b), and the bare radial dimension in (b) is determined from the equivalent bare radius in (a). The consistent set of bare dimensions and critical compositions are obtained by iteration.

The two-dimensional and one-dimensional calculations were performed with the same set of multigroup cross sections. The assembly in the two-dimensional calculation was found to be less reactive than in the one-dimensional calculations, indicating that the reflector savings obtained from the one-dimensional calculations are too high. However, no general conclusions should be made since similar analyses on different systems have shown an opposite effect.

A series of critical assemblies having a core composition very much like Assembly 6F (Table 6-7) were analyzed⁽⁶⁾ to determine how well the one-dimensional techniques would predict the change in critical mass effected by changing the ratio of core height, L , to diameter, D . The assemblies considered are included in the discussion of shape factor (Section 6.2.2). The ratio of observed to calculated critical mass did not vary by more than several per cent, indicating that the validity of the one-dimensional method, at least for the systems under consideration, is not too dependent on the value of L/D . Table 6-14 gives the variation of calculated reflector savings for the series of assemblies. The absolute values are believed to be too large on the basis of the above discussion.

Table 6-14

CALCULATED REFLECTOR SAVINGS FOR
CYLINDRICAL ASSEMBLIES OF VARYING L/D

L/D	Reflector Savings, cm		L/D	Reflector Savings, cm	
	Radial	Axial		Radial	Axial
0.23	16.5	14.4	1.0	15.5	15.3
0.31	16.3	14.6	1.57	15.4	16.2
0.67	15.9	15.0	2.23	15.3	17.8

6.2.9 Threshold Cross Sections by Spherical Shell Measurements

Measurements of threshold detector responses, with and without a spherical shell of the material to be studied separating the neutron source from the detector, permits the determination of effective cross sections averaged over the energy region above the detector threshold. The experimental arrangement consists of a neutron source placed some distance from the shell, and the detector placed at the center of the shell. It has been shown by Bethe;⁽²⁷⁾ on the basis of reciprocity arguments, that this arrangement is equivalent to the one with source and detector interchanged, the latter arrangement being more amenable to analysis.

Average cross sections for fission spectrum neutrons above the fission threshold of U^{238} (~ 1.4 Mev) are given in Table 6-15; corresponding values for the region above the fission threshold of Np^{237} (~ 0.5 Mev) are given in Table 6-16. The inelastic cross section includes all processes other than fission for removal from energies above the threshold.

Table 6-15

AVERAGE CROSS SECTIONS FOR FISSION SPECTRUM NEUTRONS
ABOVE THE U^{238} FISSION THRESHOLD
(in barns)

(Data from References 30, 31)

Element	σ_{in}	σ_{tr}	σ_f	Element	σ_{in}	σ_{tr}	σ_f
Aluminum	0.319	1.68		Cadmium	1.52	3.11	
Titanium	0.563	2.47		Tin	1.12	3.00	
Vanadium	0.575	2.34		Tungsten	2.23	3.82	
Iron	0.686	2.13		Gold	2.04	3.93	
Nickel	0.713	2.26		Lead	0.712	3.84	
Copper	0.900	2.35		Bismuth	0.728	4.05	
Zinc	0.955	2.32		Uranium-238	2.061	4.62	0.555
Zirconium	0.960	3.06		Uranium-235	1.58	4.77	1.285
Silver	1.66	3.07					

Table 6-16

AVERAGE CROSS SECTIONS FOR FISSION SPECTRUM
NEUTRONS ABOVE THE Np^{237} FISSION THRESHOLD
(in barns)

(Data from References 30, 31)

Element	σ_{in}	σ_{tr}	Element	σ_{in}	σ_{tr}
Aluminum	0.087	2.07	Silver	0.840	3.42
Titanium	0.187	2.24	Cadmium	0.658	3.50
Vanadium	0.224	2.35	Tin	0.369	3.51
Iron	0.276	2.01	Tungsten	1.080	3.70
Nickel	0.276	2.38	Gold	1.00	3.63
Copper	0.304	2.64	Lead	0.208	4.16
Zinc	0.305	2.58	Bismuth	0.197	4.38
Zirconium	0.302	3.99	Uranium-238	1.04	4.56

6.2.10 Integral Measurement of the Capture to Fission Ratio (α)

A series of measurement of α , the capture to fission ratio, were performed by mass spectrometric techniques for both U^{235} and Pu^{239} , as a function of position in EBR-I. (28) The results are shown in Figs. 6-7 and 6-8.

It has been found that when multigroup parameters consistent with the $\alpha(E)$ measurements summarized in Section 2.1.1 are used, the calculations for EBR-I yield α values which are lower than measured in EBR-I. The calculated results were obtained for U^{235} at ANL, using multigroup parameters similar to those listed in Tables 6-1, 6-2 and 6-3. A similar result was obtained for Pu^{239} in the analysis of Sampson and Molino. (29)

FIG. 6-7
RATIO OF CAPTURE TO FISSION (α)
FOR U^{235} IN EBR-I
(REPRODUCED FROM REF. 28)

FIG. 6-8
RATIO OF CAPTURE TO FISSION (α)
FOR Pu^{239} IN EBR-I
(REPRODUCED FROM REF. 28)

6.2.11 Coupled Fast-Thermal System

A fast-thermal system consists of a fast assembly and a thermal assembly coupled together in the sense that some of the neutrons born in one assembly cause fissions in the other.(32) Each of the assemblies is subcritical when uncoupled from the other. In particular, the system is not critical on fast neutrons.

The primary reason for interest in such systems arises from the possibility of obtaining a system, only slightly subcritical on fast neutrons, which combines the breeding ratio of a fast reactor with the long neutron lifetime of a thermal reactor.

A coupled fast-thermal reactor experiment(33) was performed at the ZPR-III facility. The composition and geometry of the radially symmetric critical system are described in Table 6-17 and Fig. 6-9, respectively. The fast core and depleted uranium blankets are similar to those in the all-fast system, ZPR-III-5 (Table 6-7). The main difference in the composition of these regions between the coupled experiment and ZPR-III-5 is that most of the stainless steel in the latter experiment was replaced by

Table 6-17

**COMPOSITION BY VOLUME FRACTION OF VARIOUS
REGIONS OF THE COUPLED EXPERIMENT**

Materials	Fast Core	Natural Uranium Inner Blanket	Depleted Uranium Low-Density Blanket	Beryllium Moderator	Depleted Uranium High-Density Blanket
U^{235}	0.140	0.003	0.0008	-	0.0019
U^{238}	0.159	0.394	0.397	-	0.833
Al	0.414	0.311	0.316	0.073	0.073
Be	-	-	-	0.835	-

FIG. 6-9
AXIAL SECTION AT CENTER OF COUPLED
FAST-THERMAL CRITICAL FACILITY

aluminum because of its much lower absorption cross section for thermal and near-thermal neutrons. It is particularly important not to have any high parasitic absorbers such as stainless steel in either the moderator or natural uranium regions.

For the configuration corresponding to Fig. 6-9 and Table 6-17 the critical mass of the fast core was found to be 137 kg of U^{235} . The prompt neutron lifetime measured by a $1/v$ method was 12.3×10^{-6} seconds, while the Rossi α method yielded 16.5×10^{-6} seconds. The effective delayed neutron fraction is estimated by methods similar to those described in Section 6.3.3 to be $\beta_{\text{eff}} = 0.0074$.

A ratio of fast fissions to thermal fissions of 7.3 was found by integrating detector responses throughout the system.

6.3 DYNAMICS CONSIDERATIONS

6.3.1 Average Group Velocities for Neutron Lifetime Calculations

The prompt neutron lifetime of a system may be evaluated by calculating the loss of reactivity, δk , resulting from the uniform introduction of a $1/v$ absorber with macroscopic cross section (cm^{-1}) of value: $[C(1/\text{sec})]/[v(\text{cm}/\text{sec})]$. The prompt neutron lifetime, ℓ , is then given by

$$\ell = \frac{\delta k}{C}$$

Average neutron velocities for the various energy groups corresponding to the ten-group set of cross sections of Table 6-1 are given in Table 6-18.

Table 6-18

GROUP AVERAGE NEUTRON VELOCITIES

Energy Interval, Mev	Neutron Velocity, 10^8 cm/sec	Energy Interval, Mev	Neutron Velocity, 10^8 cm/sec
1.35 - ∞	22.0	0.11 - 0.18	5.2
0.825 - 1.35	14.5	0.068 - 0.11	4.0
0.5 - 0.825	11.5	0.025 - 0.068	2.8
0.3 - 0.5	8.8	0.0091 - 0.025	1.7
0.18 - 0.3	6.7	0 - 0.0091	0.8

6.3.2 Rossi α Measurements

The Rossi α is the average decay constant of the population in a prompt neutron chain.^(34,35) It is determined experimentally by measurements of the statistical variation in the number of pairs of detected neutron events as a function of the time interval between them. Such measurements must be done at extremely low reactor power so that the prompt neutron chains do not overlap excessively. At delayed critical:

$$\text{Rossi } \alpha = - \frac{\beta_{\text{eff}}}{\ell}$$

where β_{eff} is the effective delayed neutron fraction and ℓ is the prompt neutron lifetime. Table 6-19 lists the Rossi α values measured on various fast systems.

Table 6-19

ROSSI α AT DELAYED CRITICAL^a

Assembly	β/ℓ	Comment	Assembly	β/ℓ	Comment
Popsy	$-20. \times 10^{-4}$	High-Density	ZPR-III-6F ^b	-9.85×10^{-4}	
Jezebel	-66.	Pu Cores	ZPR-III-5	-8.45	EBR-II
			ZPR-III-9	-9.0	
Godiva	-108.		ZPR-III-10	-10.8	
Jemima (53%)	-63.		ZPR-III-11	-10.4	
Jemima (38%)	-47.	High-Density	ZPR-III-12	-6.85	
Jemima (29%)	-37.	Uranium Cores	ZPR-III-17	-5.35	
Jemima (16.25%)	-18.		ZPR-III-14	-3.85	
			EBR-I Mock-up	-17.4	

^a ZPR-III and EBR-I data from Ref. 6. Data on all other assemblies from Ref. 9

^b With the exception of Assembly 5, cores were surrounded by a high-density, depleted uranium reflector. Assembly 5 was surrounded by a less dense reflector (~17 cm thick) which, in turn, was surrounded by a high-density reflector.

6.3.3 Effective Delayed Neutron Fractions

A typical fast reactor contains large quantities of U^{238} , both in the core and breeder blanket reflector. The neutron spectrum is such that anywhere between 15 and 30% of the fissions are in U^{238} .

The evaluation of the effective delayed neutron fraction of the system is complicated by the spatial distribution of the U^{238} fissions which differs from that of the fuel (U^{235} or Pu^{239}) fissions, and by the delayed neutron characteristics of U^{238} which differs from that of the fuel. The evaluation may be carried out by determining the fraction of total importance production rate of all fission neutrons which results from the delayed neutron.

The effective delay fractions have been determined for a variety of systems investigated in the ZPR-III facility. The latest data on delayed neutrons for fast neutron fission (Section 1.5.1) have been used in these analyses. The results are given in Table 6-20.

Table 6-20

CALCULATED EFFECTIVE DELAYED NEUTRON FRACTIONS
FOR VARIOUS ASSEMBLIES

Assembly	Calc. U^{238} Fissions per U^{235} Fission	β_{eff}	In hours/\$ of Reactivity	Volume Fractions			
				Core		Blanket	
				U^{235}	U^{238}	Uranium	Volume-%
ZPR-III-2A(6)	0.19	0.0073	300	0.14	0	Depleted	0.84
ZPR-III-6F(6)	0.24	0.0073	310	0.14	0.16	Depleted	0.84
ZPR-III-5(6) (Like EBR-II)	0.23	0.0074	307	0.14	0.16	Depleted	
						Inner Region	0.40
						Outer Region	0.84
ZPR-III-9A(6)	0.27	0.0073	318	0.12	0.38	Depleted	0.84
PBR (Conceptual 800-liter core)	0.27	0.0072	322	0.05	0.27	Depleted	0.70
EBR-I(6)	0.18	0.0068	297	0.50	0	Natural	
						Inner Region	0.48
						Outer Region	0.82
ZPR-III-10(6)	0.32	0.0074	324	0.12	0.58	Depleted	0.84
ZPR-III-11(6)	0.36	0.0074	331	0.095	0.72	Depleted	0.84
ZPR-III-12(6)	0.30	0.0075	322	0.094	0.354	Depleted	0.84
Godiva(9)	0.01	0.0068		0.938	0.62	Bare Core	
Topsy(9)	-	0.0073		0.941	0.059	Natural	~1.0
Jemima (53)(9)	0.096	0.0076		0.53	0.47	Bare Core	
Jemima (37)(9)	0.15	0.0079		0.37	0.63	Bare Core	
Jemima (29)(9)	0.19	0.0080		0.29	0.71	Bare Core	
Jemima (16)(9)	-	0.0081		0.16	0.84	Natural	~1.0
Jezebel(9)	-	0.0020		Pu Core		Bare Core	

6.3.4

Doppler Effect

A reactivity dependence on temperature in a reactor due to the Doppler effect, results from the change with temperature in the Doppler broadening of resonances to the extent that it affects the energy spectrum of the flux in the vicinity of the resonances. Such a variation is to be expected if the resonance is sufficiently strong to cause energy self-shielding (to deplete the flux in the resonance), since then the changed broadening will result in changed self-shielding.

If the flux energy dependence varies with temperature, then the integral of the product of the flux and cross section over the vicinity of the resonance will change. This is in contrast with the integral of the cross section alone, which does not change with broadening. The result is a change in the total number of absorptions which causes a change in reactivity.

In order to make theoretical estimates of the magnitude of the effect, it is necessary to know the detailed resonance structure. In the energy spectrum of fast reactors, however, level distributions and strengths may only be estimated since the resolution of present day neutron spectrometers is inadequate to obtain the parameters. At energies around 200 kev the resonance levels are broadened and so closely spaced that they tend to overlap, which decreases the possible change in the cross section due to

Doppler broadening. There is still the possibility that the distribution of level strengths and spacing is such that, in spite of the tendency of the levels to overlap on heating, there is a change in the absorption characteristics of the atom in question.

Feshbach, Goertzel and Yamauchi at NDA,⁽³⁶⁾ Lane, Lynn and Story at AERE,⁽³⁷⁾ and Bethe⁽³⁸⁾ have investigated theoretically the question of the Doppler effect for U^{235} and U^{238} for fast reactors. Their results show a strong dependence on the resonance parameters and level distribution in the hundreds of kev energy region.

A measurement^(39,40) of the uranium Doppler temperature effect was made in the fast spectrum of a mockup of the EBR-I reactor in ZPR-III. The Doppler effect was measured by thermal cycling helical samples of enriched uranium, natural uranium and plutonium and detecting the small changes in reactivity. The pile oscillation technique in conjunction with a resonant detector was utilized to measure the small oscillating component of the neutron flux. An upper limit of $2.3 \times 10^{-9} \Delta k / \Delta T$ per mol-degree C was obtained for a U^{235} sample in the spectrum of an EBR-I mockup. An upper bound of $5.1 \times 10^{-9} \Delta k / \Delta T$ per mol-degree C was obtained for the Pu sample in a plutonium-fueled assembly of the same configuration. The results for natural uranium were inconclusive. Using the formula of Goertzel and Klahr⁽⁴¹⁾ and the experimental results, an upper limit for the Doppler coefficient ($\Delta k / \Delta T$) at 20°C for U^{235} for the EBR-I type core is found to be 1.6×10^{-6} per degree C.

6.4 ACCURACY OF COMPUTATIONAL METHODS

A comparison of the accuracy of various approximations to the solution of the one-velocity Boltzmann equation in spherical geometry, prepared by Carlson^(2,3,4) is given in Table 6-21.

Table 6-21

RELATIVE ACCURACY OF VARIOUS CALCULATION METHODS FOR THE CRITICAL CORE RADIUS OF A REFLECTED SPHERE

$$f = \frac{(\nu - 1)\sigma_f - \sigma_c}{\sigma_{tr}}, \text{ excess number of neutrons emerging from a collision}$$

(Data obtained from Ref. 2)

(1)																		(2)								(3)							
Core m.f.p. = Reflector m.f.p.																		Core m.f.p. = 3/4 Reflector m.f.p.								Core m.f.p. = 3/2 Reflector m.f.p.							
1 + f Core	1 + f Reflect.	Reflect. Thick., m.f.p.	Exact ^a Crit. Core Radius, m.f.p.	% Error				Exact ^d Crit. Core Radius, m.f.p.	% Error				Exact ^d Crit. Core Radius, m.f.p.	% Error																			
				SWM ^b	MDM ^c	S ₂	S ₄		SWM ^b	MDM ^c	S ₂	S ₄		SWM ^b	MDM ^c	S ₂	S ₄																
1.3	1.00	1.5	1.896	-2.2	-1.5	-4.4	-0.9	1.825	-2.5	-2.8	-4.0	-0.8	1.999	-2.0	1.4	-4.7	-0.9																
1.3	0.95	1.5	1.973	-2.1	-1.8	-4.2	-0.7	1.917	-2.4	-3.4	-4.2	-0.8	2.057	-1.9	1.3	-4.4	-0.8																
1.3	0.80	1.5	2.132	-2.2	-2.6	-3.7	-0.6	2.103	-2.4	-4.7	-3.8	-0.7	2.184	-2.2	0.4	-4.0	-0.8																
1.7	1.00	1.5	1.044	-1.6	-4.1	-5.7	-1.2	0.997	-1.6	-6.9	-5.4	-1.0	1.103	-1.4	1.2	-5.7	-1.1																
1.7	0.95	1.5	1.075	-1.6	-4.4	-5.6	-1.1	1.035	-1.6	-7.5	-5.3	-1.0	1.125	-1.3	1.1	-5.3	-1.0																
1.7	0.80	1.5	1.148	-1.8	-5.5	-5.1	-1.0	1.122	-1.8	-9.0	-4.9	-0.9	1.180	-1.7	0.3	-5.0	-0.9																
1.3	1.00	3.0	1.752	-1.9	-1.7	-4.6	-0.9	1.651	-2.2	-3.5	-4.1	-0.8	1.893	-1.7	1.5	-4.7	-0.9																
1.3	0.95	3.0	1.886	-2.0	-2.0	-4.3	-0.9	1.811	-2.2	-4.1	-4.3	-0.8	1.996	-2.0	1.2	-4.6	-0.9																
1.3	0.80	3.0	2.108	-2.2	-2.8	-3.7	-0.7	2.074	-2.4	-5.1	-3.8	-0.7	2.165	-2.2	0.5	-4.0	-0.7																
1.7	1.00	3.0	0.992	-1.5	-4.5	-5.5	-1.0	0.934	-1.7	-7.7	-5.4	-1.1	1.068	-1.4	0.7	-5.5	-1.0																
1.7	0.95	3.0	1.043	-1.6	-4.8	-5.5	-1.1	0.995	-1.6	-8.1	-5.3	-1.0	1.104	-1.4	0.8	-5.3	-1.0																
1.7	0.80	3.0	1.137	-1.8	-5.6	-5.0	-1.1	1.109	-1.8	-9.4	-4.8	-0.9	1.174	-1.8	0.2	-4.9	-0.9																

^aThe core radius assumed to be correct; obtained by Extrapolated End Point Method (EEM) known to be in error by about -0.1%. Errors for core in column (1) are relative to these values.

^bSerber-Wilson Method.

^cModified Diffusion Method, similar to EEM but with interface correction only at the outside boundary.

^dObtained by extrapolation from S₂ and S₄, using the case of core m.f.p. = reflector m.f.p. as a guide; errors for core m.f.p. ≠ reflector m.f.p. are relative to these values.

A system of core plus reflector is used here. The modified diffusion method, MDM,⁽⁴²⁾ is similar to the extrapolated endpoint method, EEM,⁽⁴³⁾ except that the extrapolated endpoint correction is applied only at the outer boundary rather than at each interface. Both methods are applicable strictly to slab geometry, or to spherical geometry with constant mean free path. The effect of departing from constant mean free path on the modified diffusion method for spherical geometry can be seen in the table.

A comparison of diffusion theory with the S_n method in multi-group form has been carried out⁽¹²⁾ using both three-group and ten-group sets over a wide range of reactor sizes and with some variance in composition. In each calculation the radius of the core was varied until criticality was reached. The transport approximation was used in these calculations. The ratio of critical radius by diffusion theory relative to that by the S_4 method, as a function of critical radius by the S_4 method, is given in Fig. 6-10.

FIG. 6-10
RATIO OF CRITICAL RADII (DIFFUSION THEORY TO S_4)
AS A FUNCTION OF S_4 CRITICAL RADIUS

Recent comparisons⁽⁸⁾ between diffusion theory and the S_n method indicate that slight variations from Fig. 6-10 may be obtained as the properties of the reflector relative to the core are varied. In particular, as the core leakage is increased for a given size core, (possibly, by introducing a less effective reflector), then the difference between diffusion theory and the S_n method tends to increase. Similar calculations by multi-group Serber-Wilson method, SWM,^(44,45) gave an error in critical radius, relative to the S_4 method, of -0.5 to -1.0% over the range of core radii up to 22 cm.

A comparison of the improvements obtained by successively larger n values in the S_n method carried out for the case of a solid plutonium assembly by Kiehn* using 10-energy groups is given in Fig. 6-11. In the S_4 approximation the radial error is about -1%, reflected systems giving similar results.

A detailed comparison of fluxes predicted by multigroup approaches to diffusion theory, Serber-Wilson method, and S_4 for three power reactors similar to EBR-I, EBR-II and a large-scale power reactor is given in a paper by Okrent, et al.⁽¹²⁾

*Private communication, R. M. Kiehn, LASL.

FIG. 6-11
RELATIVE ERROR AS A FUNCTION OF S_N OPTION

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SECTION 7

SHIELDING CONSTANTS

Effort has been made to supply a concise selection of data on standard shielding materials for the use of the shield designer or the physicist faced with a shielding problem. The collection of data on various concretes is expected to provide a basis for determination of constants for evaluation of particular concrete mixes.

Discussion of treatment of geometry, multilayer effects, and consideration of the general shielding problem have not been included, since complete discussions are readily available. See for example: "The Reactor Shielding Design Manual," by Rockwell,⁽¹⁾ "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," by Goldstein,⁽²⁾ and "Radiation Shielding," by Price, Horton, and Spinney.⁽³⁾

An extensive shielding bibliography (TID-3032)⁽⁴⁾ has been published periodically by Technical Information Services.

7.1 RADIATIONS FROM NUCLEAR EVENTS

The spectra of gamma radiation from various sources are of primary interest in reactor shielding work. To facilitate calculations these gamma-rays may conveniently be treated as a set of monoenergetic sources, and are presented in the form of line spectra. An effective value of the gamma-ray energy, $M(E)$, and the number of photons, $N(E)$, is associated with each discrete energy E of the line spectrum. $N(E)$ and $M(E)$ are related by $M(E) = EN(E)$. All data related to fission are based on U^{235} .

7.1.1 Prompt Gamma-Rays from Fission

The energy distribution of prompt gamma-rays born directly in fission is given⁽¹⁾ in Table 7-1 in terms of several line spectra with varying degrees of detail. The coarser breakdowns are included for convenience in performing more approximate calculations.

7.1.2 Equilibrium Gamma-Rays from Fission Products

The energy distribution of equilibrium fission product gamma-rays is given⁽⁵⁾ in Table 7-2 for two different line spectra. The reactor is assumed to have been operating for a long time so that fission product activities are essentially at their saturation values.

Table 7-1

PROMPT FISSION GAMMA-RAY SPECTRA

Spectrum A*			Spectrum B		Spectrum C	
E, Mev	N(E), γ/fission	M(E), Mev/fission	E, Mev	M(E), Mev/fission	E, Mev	M(E), Mev/fission
0.5	3.1	1.55				
1.0	1.9	1.90	1.0	3.450	1.0	3.45
1.5	0.84	1.26				
2.0	0.55	1.10	2.0	2.360	2.0	3.085
2.5	0.29	0.725				
3.0	0.15	0.450	3.0	1.175		
3.5	0.062	0.217				
4.0	0.065	0.260	4.0	0.477	4.0	1.035
4.5	0.024	0.108				
5.0	0.019	0.095	5.0	0.203		
5.5	0.017	0.094				
6.0	0.007	0.042	6.0	0.136	6.0	0.256
6.5	0.004	0.026	7.0	0.026		
	7.028	7.827		7.827		7.827

*Reproduced from Reactor Shielding Design Manual, Table 3.2⁽¹⁾
from data of R. L. Gamble.⁽⁸⁾

Note: In addition, Bertini, et al.,⁽⁹⁾ estimate the effective prompt capture gamma-ray source from U^{235} in a thermal reactor to be 1.18 Mev per fission, based on a value of 0.184 for α of U^{235} .

Table 7-2

EQUILIBRIUM FISSION PRODUCT GAMMA-RAY SPECTRA

Energy Group	Spectrum A				Spectrum B		
	Energy Range, Mev	Effective Energy, Mev	M(E)		Effective Energy, Mev	M(E)	
			Mev per Fission†	Mev/(sec)(watt)* of Reactor Power		Mev per Fission†	Mev/(sec)(watt) of Reactor Power
1	0.1 - 0.4	0.4	0.645	2.0×10^{10}	1	5.16	1.6×10^{11}
2	0.4 - 0.9	0.8	3.87	1.2×10^{11}			
3	0.9 - 1.35	1.3	0.645	2.0×10^{10}	2	1.737	5.38×10^{10}
4	1.35 - 1.8	1.7	1.06	3.3×10^{10}			
5	1.8 - 2.2	2.18	0.677	2.1×10^{10}	3	0.322	1.0×10^{10}
6	2.2 - 2.6	2.5	0.290	9.0×10^9			
7	> 2.6	2.8	0.032	1.0×10^9			
Total			7.219	2.24×10^{11}			

*Data from Reactor Shielding Design Manual, Table 3.5⁽¹⁾

†3.1 fissions/(watt)(sec) assumed

7.1.3 Capture Gamma-Rays

The prompt capture gamma-ray spectra resulting from thermal neutron captures in various elements were summarized by Mittelman.⁽⁶⁾ His summary groups the gamma-rays into energy intervals 2 Mev in width. A compilation by Deloume⁽⁷⁾ breaks the gamma-ray spectrum into ten groups of 1 Mev energy width, and is presented in

Table 7-3. References for these data are included adjacent to Table 7-3, although an appreciable portion of the data was obtained by Deloume through private communication with Dr. G. A. Bartholomew of Chalk River, Ontario, Canada.

7.1.4 Inelastic Scattering Gamma-Rays

Cross sections for inelastic scattering are not well known. A summary of available information on the energies of gamma-rays produced by inelastic scattering of neutrons is given in Table 7-4.⁽²⁾

7.1.5 Decay of Fission Product Gamma-Ray Activity

The activity of fission products after reactor shutdown is discussed extensively in the literature. Moteff⁽¹⁰⁾ gives curves for gamma activity after shutdown for reactor operating times of 10 minutes and 1, 10, 100, and 1000 hours, over the time period from 4 hours to 10 days after shutdown. Clark⁽¹¹⁾ extended these curves to show gamma activity from 30 minutes to 100 days after reactor shutdown and showed the activity due to all the important individual nuclides. Blomeke⁽¹²⁾ presents an extensive tabulation of fission product data including gross and individual activities as functions of operation time, time after shutdown, reactor flux level, etc.

Figures 7-1, 2, 3, and 4 show the rates of beta and gamma energy release, and a seven-group gamma-ray energy spectrum for fission products due to thermal neutron fission of U^{235} as calculated by Perkins and King.⁽¹³⁾ The curves apply to reactor operating times of 1, 10, 100, and 1000 hours respectively. Figure 7-5 shows the data of Fig. 7-4 replotted on a semilogarithmic scale for short times (up to 250 minutes) after shutdown. The energy group structure for the fission product gamma-ray spectrum is as indicated in Table 7-2, spectrum A.

The activity decay rate after the time of fission is discussed in Sec. 1 (see Fig. 1-6).⁽¹³⁾ An experimental determination of gamma-rays from fission products between 125 and 1600 seconds after fission is reported by Zobel and Love.⁽¹⁴⁾

7.1.6 Fission Product Decay Power and Integrated Energy Release

Figure 7-6 shows the total fission product power (beta and gamma) generated in a U^{235} -fueled reactor as a function of time after shutdown following infinite operation, based on the curves of Untermeyer and Weills⁽¹⁵⁾ renormalized to be consistent with the data of Perkins and King.⁽¹³⁾ The corresponding integrated fission product energy generated from time of shutdown is plotted in Fig. 7-7. Conversion factors are included for convenience in converting the energy into units of Btu per watt of reactor power.

7.1.7 Activation Gamma-Ray Data

Table 7-5 gives the induced activities of some reactor coolant materials. Pure beta emitters are neglected. The activation cross sections for O^{16} and O^{17} are averaged over the fission neutron spectrum since the (n,p) reactions involved have threshold energies in the Mev range.⁽¹⁶⁾

Table 7-3

PROMPT GAMMA-RAYS RESULTING FROM NEUTRON CAPTURE

Data from Deloume⁽⁷⁾

Capture Gamma-Ray Energy, in Mev per Neutron Capture

Element	Atomic Number	Energy of Emitted Gamma-Rays, Mev										References
		0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	
Aluminum	13				1.29	1.29	.51	.42	1.90			B,P
Antimony	51				.78	.64	.45	.28				K
Arsenic	33				1.01	.93	.65	.64	.17			K
Barium	56				1.89	1.08	.69	.10	.01	.01	.01	L
Beryllium	4				.85			5.12				C,G,T
Bismuth	83					4.17						A
Boron (B ¹⁰)*	5					3.51		.19		.05		
Cadmium	48	.46	.34	.40			.52	.13	.11	.03	.02	L,N,O,P
Calcium	20	.07	.94	.41	1.08	1.96	1.28	2.32	.07			D,P,S
Carbon	6				1.10	3.47						C,G
Chlorine	17	.40		1.57	1.07	1.57	1.45	1.89	1.06	.12		D,P,T
Chromium	24				.26	.29	.62	.75	1.87	3.93	1.11	E,P
Cobalt	27				.77	.87	1.57	1.70	.57			F
Copper	29					.61	.62	.94	3.13	.016		F
Fluorine	9					1.83	3.20	2.82				B
Gadolinium	64				.64	.30	.15	.06				L
Gold	79				1.01	1.53	1.31	1.10	.01			K
Hafnium	72					.50	.51	.17	.026			
Hydrogen	1			2.23								
Indium	49				.76	.55	.26					K
Iron	26	.05	.30	.20	.37	.45	.84	.67	2.66	.25	.20	E,Q
Lead	82								7.40			A,N
Lithium (Li ⁶)**	3							.51	.94			
Magnesium	12			.62	2.68	.25	.31	.27	.01	.26	.05	B,I,P
Manganese	25					1.24	.98	1.11	1.78			F
Mercury	80	.21	.50	.56	1.35	1.94	1.91	.63	.04			L,N
Molybdenum	42				1.93	1.41	.90	.66	.16	.05		L
Nickel	28	.07	.06	.04	.20	.51	.81	1.32	1.35	4.57	.98	E,P,S
Niobium	41				1.22	.88	.68	.20	.04			
Nitrogen***	7				1.15	.72	5.06	1.07	.66	.35		C
Phosphorous	15				2.17	1.51	.66	1.04	.58			D,P
Platinum	78				1.07	.71	.75	.08	.06			L
Potassium	19	.24	.61	.83	1.80	1.86	1.99	.24	.39	.01		D,J,P,S
Praseodymium	59				.76	.59	.45					K
Rhodium	45				.82	.68	.45	.17				K
Samarium	62	.53	.21	.77	1.21	.55	.28	.08	.07			L,N
Silicon	14		.37	1.61	2.14	3.89	.47	.92	.62	.14		B,P,Q
Silver	47			.24	1.52	1.28	.81	.23	.04			K
Sodium	11	.80	.39	1.44	1.05		.33	.83				B,O,P,T
Strontium	38				1.46	1.03	.91	2.18	.79	.29	.02	L
Sulphur	16	.40		1.77	1.62	1.92	4.64	.29	.22	.09		D,P,T
Tantalum	73					.05	.10	.04				K
Thallium	81				.94	2.15	2.68	.96				K
Tin	50				3.64	1.89	1.24	.65	.25		.04	L
Titanium	22	.14	1.59	.08	.37	.72	.09	6.58	.11	.02	.01	E,P,Q
Tungsten	74				1.28	.77	.58	.33	.03			L
Uranium (U ²³⁸)†	92					1.89	.43		.27			R
Vanadium	23				.39	.50	1.33	2.16	1.07	.008		F,P
Zirconium	40				2.52	1.71	1.07	1.15	.15	.11		L

* $\sigma_{n,\gamma} B^{10} = 0.5 \pm .2b$ ** $\sigma_{n,\gamma} Li^6 = 0.023 \pm .008b$ *** $\sigma_{n,\gamma} N = 0.080 \pm .020b$

†Added to original Deloume compilation

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Table 7-4

ENERGIES OF GAMMA-RAYS EMITTED IN INELASTIC SCATTERING

(Reproduced from data of Goldstein,² with additions)*

Element	Threshold for Inelastic Scat., Mev	Energies, Mev	Reference [†]	Element	Threshold for Inelastic Scat., Mev	Energies, Mev	Reference [†]
B ¹⁰	0.717	0.717 2.20	7, 12	Cu	0.90	0.37 0.66 0.96 1.14 1.32 1.47 1.55 1.9 2.07 2.42 2.58	2, 4, 8, 9, 7
C ¹²	4.42	4.42	7				
N ¹⁴	2.30	2.30	7				
O ¹⁶	6.09	6.09	7				
F	0.11	0.11 0.20 1.23 1.36 1.46 1.56	7, 12	Mn	0.90	0.58 0.67 0.83 1.16 1.5 1.86 2.2	9, 4
Mg		1.37 1.62 1.82	7	Zr	0.92 (Zr ^{92,94}) 2.17 (Zr ⁹⁰)	0.69 0.92 1.14 1.5 2.17 3.23	4, 12
Al	0.85	0.17 0.85 1.02 1.20 1.70 2.3 3.10	1, 2, 4, 7, 12	Ta		0.137 0.164 0.350 0.485	7
Ca	1.16 (Ca ⁴⁴) 3.35 (Ca ⁴⁰)	0.51 1.16 3.74 3.90	7	Pb	0.85 (Pb ^{206,207})	0.35 0.53 0.57 0.88 1.1 1.4 1.7 2.6 3.0	1, 3, 10, 12, 7
Cr	1.44	1.43 0.97 0.75	1, 3, 4, 5	Bi	0.90	0.49 0.90 1.60 2.6 3.35	1, 3, 4, 5, 7
Fe	0.85	0.12 0.46 0.63 0.85 1.25 1.44 1.67 2.10 2.60 3.52	1, 2, 3, 4, 5, 7, 12	U ²³⁸ *	0.044	0.044 0.15 0.30 0.98 1.06 1.26	13
Co		0.60 1.15 1.49 1.7 2.5	4				
Ni	1.33	0.59 (?) 0.85 1.33 1.47 2.18 2.66	1, 3, 4, 7				

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FIG. 7-1
DECAY RATES AFTER REACTOR OPERATION FOR 1 HOUR

FIG. 7-2
DECAY RATES AFTER REACTOR OPERATION FOR 10 HOURS

FIG. 7-3
DECAY RATES AFTER REACTOR OPERATION FOR 100 HOURS

FIG. 7-4
DECAY RATES AFTER REACTOR OPERATION FOR 1000 HOURS

(Reproduced from Ref. 13)

FIG. 7-5
DECAY RATES AFTER REACTOR OPERATION FOR 1000 HOURS
(UP TO 250 MINUTES AFTER SHUTDOWN)

FIG. 7-6
FISSION PRODUCT POWER VS TIME AFTER
SHUTDOWN FOLLOWING INFINITE OPERATION

FIG. 7-7
INTEGRATED FISSION PRODUCT ENERGY GENERATED VS
TIME AFTER SHUTDOWN FOLLOWING INFINITE OPERATION

Table 7-5

INDUCED ACTIVITIES FROM SOME REACTOR COOLANTS(Reproduced from Reactor Shielding Design Manual, Table 4.1⁽¹⁶⁾)

Target Isotope	Isotopic %	Cross Section at 2200 meter/sec, barn	Radioactive Product of Reaction	Half-Life	Energy of Radiation, Mev (γ)	Gammas per Disintegration
Na ²³	100	0.53	Na ²⁴	14.9 hr	2.76; 1.38	1 each
K ⁴¹	6.8	1.15	K ⁴²	12.4 hr	1.51	0.25
O ¹⁸	0.204	0.00021	O ¹⁹	29.4 sec	1.6	0.7
O ¹⁶	99.8	$\sim 0.019 \times 10^{-3} a$	N ¹⁶	7.4 sec	6.13; 7.10	0.76; 0.06
O ¹⁷	0.039	$\sim 0.0052 \times 10^{-3} a$	N ¹⁷	4.1 sec	1(neutron)	1(neutron)

^aFast (n, p) reactions; cross sections averaged over fission spectrum. Data quoted is from Roys and Shure.⁽²¹⁾ A previous determination by Henderson and Tunnicliffe⁽²²⁾ gives cross sections for N¹⁶ of 0.0185×10^{-3} barn and for N¹⁷ of 0.0093×10^{-3} barn.

Table 7-6 gives activation data for isotopes produced due to capture of thermal neutrons. Gamma-ray activity data is given for both primary and possible secondary products of neutron capture. For shielding applications pure beta emitters are not important, hence they are not included.

Activation cross sections are compiled from BNL-325 and Supplement No. 1.⁽¹⁷⁾ The comments given in the introduction to the section, "Thermal Cross Sections" in BNL-325 will apply.

In Table 7-6 the metastable state is indicated by an asterisk. The activation cross sections refer to the direct formation of each individual state. The half-life of the parent isotope is given when the emitter is a daughter of a primary activation product.

The value of the half-life of each radioisotope is taken from the Trilinear Chart of Nuclides.⁽¹⁸⁾ Energies of the photons emitted are given together with their respective percentage yields per disintegration.^(18,19) The superscript a indicates that gamma-ray energies and per cents are effective values obtained by summing yields of individual gammas and assigning to them the highest energy represented within the group. If the yield of a gamma-ray is less than 1% per disintegration and is not the highest energy gamma-ray given off, it is omitted. If a gamma-ray is the highest energy photon given off and appreciably higher than the next lower one, it is included even if its yield is less than 1%.

7.1.8 Cross Sections for Photoneutron Production

The curves shown in Fig. 7-8 represent a best fit to experimental data for the photoneutron production cross section of deuterium and beryllium.⁽²⁰⁾

Table 7-6
GAMMA-RAY ACTIVITY DUE TO THERMAL NEUTRON CAPTURE

s = second
m = minute
h = hour
d = day
y = year
* = metastable

Radioactive Isotope	Half Life	E _γ , Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns	Radioactive Isotope	Half Life	E _γ , Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns
B ¹²	.027s	~4.5	~ 4	B ¹¹	81.2	<50 mb	Zn ^{71*}	3 h	0.38 0.49 0.61	100 100 100	Zn ⁷⁰	0.62	?
N ¹⁶	7.4 s	6.0 6.1 7.1	20 20 80	N ¹⁵	0.37	24 ± 8 mb	Zn ⁷¹	2.2 m	0.51	100	Zn ⁷⁰	0.62	85 ± 20 mb
O ¹⁹	29.4 s	1.4	70	O ¹⁸	0.20	0.21 ± 0.04mb	Ga ⁷⁰	21 m	1.04	0.8	Ga ⁶⁹	60.2	1.4 ± 0.3
F ²⁰	10.7 s	1.63	100	F ¹⁹	100	9 ± 2 mb	Ga ⁷²	14.2 h	0.69 ^a 0.89 1.86 2.51	28.2 105 30.3 53.1	Ga ⁷¹	39.8	5.0 ± 1.0
Ne ²³	40.2 s	0.44 1.65	29 1	Ne ²²	8.8	36 ± 15 mb	Ge ^{75*}	48 s	0.139	100	Ge ⁷⁴	36.7	?
Na ²⁴	15.1 h	2.76 1.38 4.14	99.96 99.96 0.04	Na ²³	100	0.53 ± 0.02	Ge ⁷⁵	82 m	0.63 ^a 0.264	0.80 11.15	Ge ⁷⁴ Ge ^{75*}	36.7 48 s	.45 ± 0.08
Mg ²⁷	9.45 m	1.015 0.181 0.834	29 1 70	Mg ²⁶	11.3	26 ± 2 mb	Ge ^{77*}	52 s	0.159	14.2	Ge ⁷⁶	7.7	30 ± 20 mb
Al ²⁸	2.3 m	1.78	100	Al ²⁷	100	0.21 ± 0.02	Ge ⁷⁷	12 h	0.56 ^a 0.79	21.6 4.3	Ge ⁷⁶ Ge ^{77*}	7.7 52 s	0.2 ± 0.1
Si ³¹	2.65 h	1.26	0.07	Si ³⁰	3.12	0.11 ± 0.01			1.36 2.0	4 1		(14.2%)	
S ³⁷	5.04 m	3.12	90	S ³⁶	0.016	0.14 ± 0.04	As ⁷⁶	26.8 h	2.05	1	As ⁷⁵	100	4.2 ± 0.8
Cl ^{38*}	1.0 s	0.66	100.0	Cl ³⁷	24.5	~5 mb			1.40 1.20 0.64 0.55	2 9.22 8.78 39.78			
Cl ³⁸	37.5 m	2.15 1.65	47 31	Cl ³⁷	24.5	0.56 ± 0.12							
A ⁴¹	1.83 h	1.37	99.3	A ⁴⁰	99.6	0.53 ± 0.02	Se ^{81*}	56.8 m	0.103	100	Se ⁸⁰	49.8	30 ± 10 mb
K ⁴²	12.46 h	1.53	18	K ⁴¹	6.91	1.15 ± 0.11	Se ⁸³	25.0 m	0.176 0.950	100 100	Se ⁸²	9.2	4 ± 2 mb
Ca ⁴⁷	4.9 d	1.31 0.83 0.48	92.9 7.14 7.14	Ca ⁴⁶	0.0033	0.25 ± 0.10	Br ⁸³	2.33h	0.046	20	Se ^{83*} Se ⁸³	67 s 25 m	
Ca ⁴⁹	8.8 m	3.10 4.05	89.78 9.88	Ca ⁴⁸	0.185	1.1 ± 0.1	Kr ^{83*}	1.88h	0.032 0.009	100 100	Br ⁸³	2.33 h	
Sc ^{46*}	19.5 s	0.14	100	Sc ⁴⁵	100	10 ± 4	Br ^{80*}	4.5 h	0.048 0.036	100 100	Br ⁷⁹	50.54	2.9 ± 0.5
Sc ⁴⁶	85 d	1.12 0.89	99.9 100	Sc ⁴⁵ Sc ^{46*}	100 19.5 s	12 ± 4	Br ⁸⁰	18.5 m	0.62	14	Br ⁷⁹ Br ^{80*}	50.54 4.5 h	8.5 ± 1.4
Sc ⁴⁷	3.43 d	0.16	74	Ca ⁴⁷	4.9 d		Br ⁸²	35.9 h	0.688 ^a 0.817 1.469	152.4 108 69.8	Br ⁸¹	49.5	3.5 ± 0.5
Ti ⁵¹	5.79 m	0.605 0.928 0.323	1.4 4.2 95.8	Ti ⁵⁰	5.25	0.14 ± 0.03	Kr ⁷⁹	34.5 h	0.217 ^a 0.398 0.833	34.4 12.7 5.84	Kr ⁷⁸	0.35	2.0 ± 0.5
V ⁵²	3.76 m	1.44	100	V ⁵¹	99.8	4.5 ± 0.9	Kr ^{85*}	4.4 h	0.305 ^a 0.150	22 78	Kr ⁸⁴	57	0.10 ± 0.03
Cr ⁵¹	27.8 d	0.32	10	Cr ⁵⁰	4.31	13.5 ± 1.4	Kr ⁸⁷	78.0 m	2.57 2.05 0.85 0.403	21.9 3.57 13.57 86.9	Kr ⁸⁶	17.4	60 ± 20 mb
Mn ⁵⁶	2.58 h	2.98 2.13 2.65 1.81 0.845	0.4 15 1.8 24 99	Mn ⁵⁵	100	13.4 ± 0.3	Rb ^{86*}	1.02m	0.56	100	Rb ⁸⁵	72.15	?
Fe ⁵⁹	45.1 d	0.191 1.29 1.10	3 43 57	Fe ⁵⁸	0.31	0.9 ± 0.2	Rb ⁸⁶	18.6 d	1.08	10	Rb ⁸⁵ Rb ^{86*}	72.15 1.02 m	0.72 ± 0.15
Co ^{60*}	10.5 m	0.058	99.7	Co ⁵⁹	100	16 ± 3	Rb ⁸⁶	17.8 m	2.68 0.91 1.85 4.89	6.1 13.6 24.0 small	Rb ⁸⁷	27.85	0.12 ± 0.03
Co ⁶⁰	5.3 y	1.17 1.33	99.9 100	Co ⁵⁹ Co ^{60*}	100 10.5 m (99.7%)	20 ± 3	Sr ^{85*}	70.0 m	0.233 0.007 0.225 0.150	1.3 84.7 84.7 14	Sr ⁸⁴	0.55	?
Co ⁶¹	1.65 h	0.07	100	Co ⁶⁰	5.28y	6 ± 2	Sr ⁸⁵	64 d	0.513	100	Sr ⁸⁴ Sr ^{85*}	0.55 70.0 m (86%)	1.0 ± 0.3
Ni ⁶⁵	2.56 h	0.37 1.49 1.12	4.1 24.9 18.1	Ni ⁶⁴	1.16	1.6 ± 0.2	Sr ^{87*}	2.8 h	0.388	100	Sr ⁸⁶	9.87	1.3 ± 0.4
Cu ⁶⁴	12.87 h	1.34	0.43	Cu ⁶³	69	3.9 ± 0.8	Zr ⁹⁵	63.3 d	0.754 0.722	54 43	Zr ⁹⁴	17.4	0.09 ± 0.03
Cu ⁶⁶	5.15 m	1.05	9	Cu ⁶⁵	31	1.8 ± 0.4	Zr ⁹⁷	17 h	1.15 1.72 0.58	80 20 80	Zr ⁹⁶	2.8	0.10 ± 0.05
Zn ⁶⁵	245 d	1.11	45.5	Zn ⁶⁴	48.9	0.5 ± 0.1							
Zn ^{69*}	13.8 h	0.439	100	Zn ⁶⁸	18.6	97 ± 10 mb							
As ⁷⁷	38.8 h	0.524	0.52	Ge ^{77*}	52 s (85.8%)								
		0.246	1.51	Ge ⁷⁷	12 h								
Se ⁷⁵	119.9 d	0.121 ^a 0.265 0.280 0.136 0.402	24.7 58.7 17.8 59.9 17.8	Se ⁷⁴	0.87	26 ± 6							
Se ^{77*}	17.5 s	0.162	100	Se ⁷⁶	9	7 ± 3							
Se ^{79*}	3.9 m	0.096	100	Se ⁷⁸	23.5	0.4							

a = effective values

Table 7-6 (Cont'd.)

Radioactive Isotope	Half Life	Ey, Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns	Radioactive Isotope	Half Life	Ey, Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns
Nb ^{94*}	6.6 m	0.0414 0.90	99.9 ~0.1	Nb ⁹³	100	1.0 ± 0.5	Cd ¹¹⁷	50 m	0.425 0.281	100 100	Cd ¹¹⁶	7.58	?
Nb ^{95*}	84 h	0.235	100	Zr ⁹⁵	63.3 d (3%)		Sn ^{117*}	14 d	0.562 0.160	100 100	In ¹¹⁷	66 m	
Nb ⁹⁵	35 d	0.770	99	Nb ⁹⁵ Zr ⁹⁵	84 h 63.3 d (97%)		In ^{118*}	50 d	0.192 0.722 0.556	96.5 3.5 3.5	In ¹¹³	4.2	56 ± 12
Nb ^{97*}	60 m	0.747	100	Zr ⁹⁷	17 h		In ¹¹⁸	72 m	1.300	0.09	In ¹¹³	4.2	2.0 ± 0.6
Nb ⁹⁷	74 m	0.665	100	Nb ^{97*}	60 m		In ^{116*}	54.2 m	0.137 ^a 1.49 0.40 1.27 2.09	3 21 25 129 25	In ¹¹⁵	95.8	145 ± 15
Mo ^{93*}	6.8 h	0.26 0.68 1.48	100 100 100	Mo ⁹²	15.86	<6 mb	Sn ¹¹³	112 d	0.400	100	Sn ¹¹²	0.95	1.3 ± 0.3
Mo ⁹⁹	67 h	0.780 ^a 0.740 0.377 0.041 0.181	8 6 1 5 6	Mo ⁹⁸	23.75	0.45 ± 0.10	In ^{113*}	1.75h	0.258 0.393 0.135	11 89 11	Sn ¹¹³	112 d	
Mo ¹⁰¹	14.6 m	0.515 ^a 0.083 0.896 2.08	96 93.5 3.5 8.5	Mo ¹⁰⁰	9.62	0.20 ± 0.05	Sn ^{117*}	14 d	0.162 0.320 0.159	99 1 99	Sn ¹¹⁶	14.24	6 ± 2 mb
Tc ^{99*}	6.04h	0.142 0.002 0.140	1.0 99.0 99.0	Mo ⁹⁹	67 h (94%)		Sn ^{119*}	275 d	0.065 0.024	100 100	Sn ¹¹⁸	24.01	10 ± 6 mb
Tc ¹⁰¹	14 m	0.307 ^a 0.939 0.545 0.130	87 3 10.4 3.5	Mo ¹⁰¹	14.6 m		Sn ¹²³	40 m	0.153	100	Sn ¹²²	4.71	0.16 ± 0.04
Ru ⁹⁷	2.8 d	0.57 0.325 0.216	1.0 3.5 95.5	Ru ⁹⁶	5.5	10 ± 4 mb	Sn ¹²⁵	9.5 m	1.39 0.326	1.9 99.7	Sn ¹²⁴	5.98	0.2 ± 0.1
Tc ^{97*}	92 d	0.099 0.090	100 100	Ru ⁹⁷	2.8 d		Sb ¹²⁵	2.4 y	0.637 ^a 0.175 0.465 0.0354	22 11 60 11	Sn ¹²⁵	9.5 m 9.5 d	
Ru ¹⁰³	39.7 d	0.610 0.495 0.055	6.45 90.5 1.09	Ru ¹⁰²	31.5	1.2 ± 0.3	Te ¹²⁵	58 d	0.110 0.0354	100 100	Sb ¹²⁵	2.4 y	
Ru ¹⁰⁵	4.5 h	0.730	100	Ru ¹⁰⁴	18.7	0.7 ± 0.2	Sn ¹²⁵	9.5 d	1.97 0.81 1.07	1.2 1.0 3.75	Sn ¹²⁴	5.98	4 ± 2 mb
Rh ^{103*}	54.0 m	0.040	100	Ru ¹⁰³ Pd ¹⁰³	41.0 d 17.0 d (94%)		Sb ^{122*}	3.5 m	0.0753 0.0607	100 100	Sb ¹²¹	57.25	?
Rh ^{105*}	40 m	0.130	100	Ru ¹⁰⁵	4.5 h		Sb ¹²²	2.8 d	1.137 0.686 1.258 0.566	0.73 3.4 0.66 66.3	Sb ¹²¹ Sb ^{122*}	57.25 3.5 m	6.8 ± 1.5
Rh ¹⁰⁵	36.5 h	0.310	30	Rh ^{105*}	40 m		Sb ^{124*}	21 m	0.0185	100	Sb ¹²³	42.75	30 ± 15 mb
Rh ^{104*}	4.4 m	0.051 1.53	100 0.0065	Rh ¹⁰³	100	12 ± 2	Sb ^{124*}	1.3 m	0.012	100	Sb ¹²³	42.75	30 ± 15 mb
Rh ¹⁰⁴	42 m	1.24 0.556	0.11 1.98	Rh ^{104*} Rh ¹⁰³	4.4 m 100	140 ± 30	Sb ¹²⁴	60.9 d	1.37 ^a 2.09 0.967 0.644 0.725	3.75 49.2 2.16 108.6 16	Sb ¹²³ Sb ¹²³ Sb ¹²³ Sb ^{124*} Sb ^{124*}	42.75 21.0 m 1.3 m	2.5 ± 0.5
Pd ¹⁰³	17 d	0.498 0.362 0.298	4 2.84 2.05	Pd ¹⁰²	0.96	4.8 ± 1.5	Te ^{121*}	154 d	0.0818 0.214	100 100	Te ¹²⁰	0.089	?
Pd ^{109*}	4.8 m	0.17	100	Pd ¹⁰⁸	26.71	?	Te ¹²¹	17 d	0.575 0.506	87 13	Te ^{121*}	154 d	
Ag ^{109*}	39.2 s	0.0875	100	Pd ¹⁰⁹ Cd ¹⁰⁹	13.6 h 1.30y		Te ^{123*}	104 d	0.0887 0.159	100 100	Te ¹²²	2.46	1.1 ± 0.5
Pd ¹¹¹	22 m	0.73 0.65 0.56	<1 <1 <1	Pd ¹¹⁰	11.81	0.3 ± 0.1	Te ^{125*}	58 d	0.110 0.0354	100 100	Te ¹²⁴	4.61	5 ± 3
Ag ¹⁰⁸	2.3 m	0.60 0.62 0.43	0.22 0.8 0.28	Ag ¹⁰⁷	51.35	44 ± 9	Te ^{127*}	105 d	0.089 0.0585	98 1.5	Te ¹²⁶	18.71	90 ± 20 mb
Ag ^{110*}	270 d	0.883 ^a 0.945 1.382 1.519	203 48 40 8	Ag ¹⁰⁹	48.65	2.8 ± 0.5	Te ¹²⁷	9.35h	0.418 0.370 0.170	Small Small Small	Te ¹²⁶ Te ^{127*}	18.71 105 d (98%)	0.8 ± 0.2
Ag ¹¹¹	7.5 d	0.340 0.243	8 1	Pd ¹¹¹	22 m		Te ^{129*}	33 d	0.106	100	Te ¹²⁸	31.8	15 ± 5
Cd ¹⁰⁷	6.7 h	0.846	<1	Cd ¹⁰⁶	1.21	1.0 ± 0.5	Te ¹²⁹	74 m	1.12 0.21 0.72 0.475 0.027	10.4 1.7 2.0 17.1 98	Te ¹²⁸ Te ^{129*}	31.8 33 d	0.13 ± 0.03
Ag ^{107*}	44.3 s	0.094	100	Cd ¹⁰⁷	6.7 h		Te ^{131*}	30 h	0.180 ^a 0.239 1.12 0.099 0.446	21.7 70 90 84 50	Te ¹³⁰	34.5	<8 mb
Cd ^{111*}	48.6 m	0.148 0.247	100 100	Cd ¹¹⁰	12.4	0.2 ± 0.1	Te ¹³¹	24.8 m	0.773 0.446 0.147 0.455 0.990	5 45 17.3 0.2	Te ¹³⁰ Te ^{131*}	34.5 30.0 h (21.7%)	0.22 ± 0.05
Cd ^{113*}	5.1 y	0.265	0.1	Cd ¹¹²	24.1	30 ± 15 mb	I ¹²⁸	24.98m	0.540 0.455 0.990	1.8 17.3 0.2	I ¹²⁷	100	5.5 ± 0.5
Cd ^{115*}	43 d	1.30 0.935	1 2	Cd ¹¹⁴	28.86	0.14 ± 0.03	Xe ¹²⁵	18 h	0.243 0.056 0.187	95 5 5	Xe ¹²⁴	0.096	?
Cd ¹¹⁵	53 h	0.033 0.523 0.49 0.26	1.0 23.8 12.4 2.32	Cd ¹¹⁴	28.86	1.1 ± 0.3	I ¹²⁵	60 d	0.035	100	Xe ¹²⁵	18 h	
In ^{115*}	4.5 h	0.335	95	Cd ¹¹⁵	53.0 h								
Cd ^{117*}	2.9 h	0.84 1.27 1.55 0.425 0.281	61 30 9 61 91	Cd ¹¹⁶	7.58	1.5 ± 0.3							
In ^{117*}	1.9 h	0.312 0.160	22 23	Cd ^{117*}	2.9 h 50 m								
In ¹¹⁷	66 m	0.562 0.712 0.160	91 9 91	In ^{117*}	1.9 h (22%)								

a = effective values

Table 7-6 (Cont'd.)

Radio- active isotope	Half Life	Ey. Mev	Yield-% per Disinte- gration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns	Radio- active isotope	Half Life	Ey. Mev	Yield-% per Disinte- gration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns
Xe ¹²⁷	36.4 d	0.368 0.170 0.200 0.145 0.056	10 85 85 2 4	Xe ¹²⁶	0.09	?	Gd ¹⁵⁹	18 h	0.364 0.23 0.136 0.079 0.056	10 6 4 2 12	Gd ¹⁵⁸	24.87	4 ± 2
Xe ¹²⁹ *	8 d	0.0400 0.196	100 100	Xe ¹²⁸	1.919	?	Gd ¹⁶¹	3.63m	0.360 0.102 0.316 0.060	10 90 90 10	Gd ¹⁶⁰	21.90	0.8 ± 0.3
Xe ¹³³ *	2.3 d	0.232	100	Xe ¹³²	26.89	?	Tb ¹⁶¹	7 d	0.106 ^a 0.0573 0.0277 0.0783	2 18 27 5	Gd ¹⁶¹	3.63m	
Xe ¹³³	5.27d	0.081	100	Xe ¹³² Xe ¹³³ *	26.89 2.3 d	0.2 ± 0.1	Tb ¹⁶⁰	73 d	0.093 ^a 0.976 1.45 0.466 0.297	80 63 26 39 63	Tb ¹⁵⁹	100	>22
Xe ¹³⁵ *	15.6 m	0.52	100	Xe ¹³⁴	10.44	?	Dy ¹⁶⁵ *	1.25m	0.108 0.515 0.361	90 6 4	Dy ¹⁶⁴	28.18	510 ± 20
Xe ¹³⁵	9.2 h	0.370 0.620 0.250	2 3 97	Xe ¹³⁴ Xe ¹³⁵ *	10.44 15.6 m	0.2 ± 0.1	Dy ¹⁶⁵	139 m	0.279 ^a 0.361 1.02 0.71 0.094	1 40 8 2 10	Dy ¹⁶⁴ Dy ¹⁶⁵ *	28.18 1.25m (90%)	2100 ± 300 ^b
Ca ¹³⁸ *	3.2 h	0.137 0.0105 0.127	0.8 98.2 98.2	Ca ¹³³	100	17 ± 4 mb	Ho ¹⁶⁶	27.2 h	1.378 0.080	1 48	Ho ¹⁶⁵	100	60 ± 12
Ca ¹³⁴	2.3 y	0.200 ^a 0.801 1.17 1.37	13 210 4.02 6.70	Ca ¹³³ Ca ¹³⁴ *	100 3.2 h	26 ± 5	Er ¹⁶⁹	9.4 d	0.0084	15	Er ¹⁶⁸	27.1	2 ± 0.4
Ba ¹³¹	11.6 d	1.03 ^a 0.82 0.495 0.245 0.122	2.9 5.6 79.7 17.2 61	Ba ¹³⁰	0.101	10.1 ± 1.0	Er ¹⁷¹	7.5 h	0.308 ^a 0.013 0.126 0.118 0.005	95 3 40 55 65	Er ¹⁷⁰	14.9	9 ± 2
Ba ¹³³ *	38.8 h	0.276 0.012	100 100	Ba ¹³²	0.097	?	Tm ¹⁷⁰	127 d	0.084	24	Tm ¹⁶⁹	100	118 ± 30
Ba ¹³³	7.2 y	0.360 0.070 0.292 0.081	96.5 3.5 3.5 100	Ba ¹³² Ba ¹³³ *	0.097 38.8 h	7 ± 2	Yb ¹⁶⁹	32 d	0.0084 ^a 0.094 0.118 0.20 0.31	60 60 60 50 40	Yb ¹⁶⁸	0.14	11000 ± 3000 ^b
Ba ¹³⁵ *	28.7 h	0.268	100	Ba ¹³⁴	2.42	?	Yb ¹⁷⁵	4.1 d	0.396 ^a 0.144 0.251 0.138 0.114	10 5 2 3 13	Yb ¹⁷⁴	31.84	60 ± 40
Ba ¹³⁷ *	2.60m	0.661	100	Ba ¹³⁶	7.81	?	Yb ¹⁷⁷	1.8 h	1.24 ^a 0.140 0.147 0.118	4 1 9 3	Yb ¹⁷⁶	12.73	5.5 ± 1.0
Ba ¹³⁹	85 m	1.43 0.163	19 85	Ba ¹³⁸	71.66	0.5 ± 0.1	Lu ¹⁷⁶ *	3.7 h	0.089	90	Lu ¹⁷⁵	97.4	35 ± 15
La ¹⁴⁰	40.2 h	2.57 ^a 0.920 0.486 1.60 0.130	20 40 73 76 26	La ¹³⁹	99.9	8.4 ± 1.7	Lu ¹⁷⁷	6.7 d	0.321 0.208 0.072 0.250 0.113	2 2 3 3 5	Lu ¹⁷⁶ Yb ¹⁷⁷	2.6 1.8 h	4000 ± 800
Ce ¹³⁷ *	35 h	0.255	100	Ce ¹³⁶	0.193	0.6 ± 0.2	Hf ¹⁷⁵	70 d	0.430 0.089 0.342	1.5 13.7 97.5	Hf ¹⁷⁴	0.19	?
Ce ¹³⁷	8.7 h	0.445	3	Ce ¹³⁶ Ce ¹³⁷ *	0.193 35 h	6.3 ± 1.5	Hf ¹⁸⁰ *	5.5 h	0.444 0.333 0.216 0.093 0.0576	80 20 20 80 100	Hf ¹⁷⁹	13.75	?
Ce ¹³⁹ *	55.0 a	0.740	100	Ce ¹³⁸	0.250	7 ± 5 mb ^b	Hf ¹⁸¹	44.6 d	0.482 ^a 0.137 0.0039 0.346	89.3 110 1 13.2	Hf ¹⁸⁰	35.25	10 ± 3
Ce ¹³⁹	140 d	0.1665	100	Ce ¹³⁸ Ce ¹³⁹ *	0.25 55 a	0.6 ± 0.3	Ta ¹⁸² *	16.5 m	0.180	100	Ta ¹⁸¹	100	30 ± 10 mb
Ce ¹⁴¹	32 d	0.1449	75	Ce ¹⁴⁰	88.48	0.31 ± 0.10	Ta ¹⁸²	115 d	0.229 ^a 0.100 0.084 1.289	45 65 55 94	Ta ¹⁸¹ Ta ¹⁸² *	100 16.5 m	19 ± 7
Ce ¹⁴³	32 h	1.10 ^a 0.861 0.351 0.294 0.057	6 17 24 21 36	Ce ¹⁴²	11.07	0.95 ± 0.05	W ¹⁸¹	140 d	0.152 0.136	0.176 0.114	W ¹⁸⁰	0.135	10 ± 10
Pr ¹⁴²	19.3 h	1.61	2.8	Pr ¹⁴¹	100	10 ± 3	W ¹⁸⁵ *	1.7 m	1.30 1.65	100 100	W ¹⁸⁴	30.6	2.1 ± 10.6
Nd ¹⁴⁷	11.06d	0.69 0.318 0.165 0.532 0.092	3 20 3 25 60	Nd ¹⁴⁶	17.26	1.8 ± 0.6	W ¹⁸⁷	24 h	0.686 0.480 0.206 0.072 0.132	12 8 5 3 3	W ¹⁸⁶	28.4	34 ± 7
Nd ¹⁴⁹	2.0 h	0.65 ^a 0.124 0.285	15 81 68	Nd ¹⁴⁸	5.74	3.7 ± 1.2	Re ¹⁸⁶	91 h	0.768 0.123 0.137	0.05 2 22	Re ¹⁸⁵	37.07	100 ± 20
Pm ¹⁴⁹	52 h	0.285 1.0	100 100	Nd ¹⁴⁹	2 h		Re ¹⁸⁸ *	18.7 m	0.060 0.105	100 100	Re ¹⁸⁷	62.93	?
Nd ¹⁵¹	12 m	0.73 ^a 0.085 0.117 0.421 1.14	40 60 120 60 100	Nd ¹⁵⁰	5.63	?	Re ¹⁸⁸	17 h	1.96 ^a 0.633 0.155	0.20 1 9	Re ¹⁸⁷ Re ¹⁸⁸ *	62.93 18.17 m	75 ± 15
Pm ¹⁵¹	27.5 h	0.715 ^a 0.340 0.177 0.275 0.069	10 42 115 70 105	Nd ¹⁵¹	12 m								
Sm ¹⁴⁵	400 d	0.0613	100	Sm ¹⁴⁴	3.16	<2							
Sm ¹⁵³	47 h	0.17 0.07 0.1	25 15 53	Sm ¹⁵²	26.63	140 ± 40							
Sm ¹⁵⁵	24 m	1.05 0.246	100 100	Sm ¹⁵⁴	22.53	5.5 ± 1.1							
Eu ¹⁵⁵	1.7 y	0.102 0.084 0.018	50 27 27	Sm ¹⁵⁵	24 m								
Eu ¹⁵²	9.2 h	1.39 ^a 0.983 0.344 0.122	1.54 21.4 2.8 15.2	Eu ¹⁵¹	47.8	1400 ± 300 ^b							
Gd ¹⁵³	236 d	0.1	100	Gd ¹⁵²	0.2	<125							

a = effective values

b = pile neutrons

Table 7-6 (Cont'd.)

Radioactive Isotope	Half Life	E _γ , Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns	Radioactive Isotope	Half Life	E _γ , Mev	Yield-% per Disintegration	Parent Isotope	Isotopic % or Half-Life	Activation Cross Section barns
Os ¹⁸⁵	95 d	0.879 ^a 0.233 0.160 0.646	15 2 2 85	Os ¹⁸⁴	0.018	<200	Au ¹⁹⁹	3.15d	0.209 0.050 0.159	3.7 20.6 89.9	Pt ¹⁹⁹	31 m	
Os ^{191*}	14 h	0.074	100	Os ¹⁹⁰	26.4	?	Au ¹⁹⁸	2.7 d	1.089 0.4118	0.16 99.018	Au ¹⁹⁷	100	96 ± 10
Ir ^{191*}	4.9 m	0.042 0.129	100 100	Os ¹⁹¹	16 d		Hg ^{197*}	24 h	0.164 0.133	96.6 96.6	Hg ¹⁹⁶	0.146	?
Os ¹⁹³	31.5 h	0.387 ^a 0.460 0.281 0.139 0.073	5 5 12 7 36	Os ¹⁹²	41	1.6 ± 0.4	Hg ¹⁹⁷	65 h	0.191 0.077	1.2 100	Hg ^{197*}	24 h (96.6%)	
Ir ^{192*}	1.45m	0.056	100	Ir ¹⁹¹	38.5	260 ± 100	Au ^{197*}	7.4 m	0.407 0.130 0.277	2 1.4 1.4	Hg ^{197*}	24 h (3.4%)	
Ir ¹⁹²	74.5 d	0.485 ^a 0.613 1.06	208 23 0.04	Ir ¹⁹¹ Ir ^{192*}	38.5 1.45m	700 ± 200	Hg ²⁰³	45.4 d	0.279	100	Hg ²⁰²	29.8	3.8 ± 0.8
Ir ¹⁹⁴	19 h	2.05 ^a 1.66 1.18 0.643 0.328	0.14 8.9 9.4 18.5 19.3	Ir ¹⁹³	61.5	130 ± 30	Hg ²⁰⁵	5.5 m	0.203	Small	Hg ²⁰⁴	6.85	0.43 ± 0.10
Pt ^{193*}	3.5 d	0.130	100	Pt ¹⁹²	0.78	90 ± 40	Tl ²⁰⁴	3 y	0.38	0.008	Tl ²⁰³	29.5	8 ± 3
Pt ^{195*}	~6 d	0.130 0.031 0.099	60 40 40	Pt ¹⁹⁴	32.8	?	Th ²³³	23.5 m	0.662 0.448 0.350 0.172 0.098	0.05 0.10 0.004 0.03 0.25	Th ²³²	100	7.34 ± 0.15
Pt ¹⁹⁷	18 h	0.279 0.191 0.077	0.9 10.6 99.1	Pt ¹⁹⁶	25.4	0.80 ± 0.10	Pa ²³³	26.95d	0.417 ^a 0.104 0.313 0.058 0.028	20 35 80 111 39	Th ²³³	23.5 m	
Pt ¹⁹⁹	31 m	0.96 ^a 0.54 0.197 0.246 0.316	3 41 81 42 41	Pt ¹⁹⁸	7.2	3.9 ± 0.8	U ²³⁹	23.5 m	0.075	100	U ²³⁸	99.274	2.76 ± 0.09
							Np ²³⁹	2.33d	0.067 ^a 0.285 0.105 0.228	62 87 30 4	U ²³⁹	23.5 m	

a = effective values

FIG. 7-8
PHOTONEUTRON CROSS SECTIONS FOR Be^9 AND D^2
(REPRODUCED FROM AEC-8645, FIG. 2.1.2²⁰)

7.2 NEUTRON CONSTANTS

Section 7.2 is limited to effective constants useful for hand calculations in shield design. Multigroup fast constants are discussed in Sections 5 and 6 and thermal neutron constants are tabulated in Section 9.

7.2.1 Attenuation of Fission Neutrons in Water

Figures 7-9 and 7-10 are plots of centerline attenuation data for water measured in the Bulk Shielding Facility at ORNL.* In addition to the fast neutron tissue dose curve the thermal flux and gamma-ray dose curves are presented. The effect of photoneutrons on the thermal flux at large distances from the reactor is clearly demonstrated in Fig. 7-9. Figure 7-10 shows the same data through first 200 centimeters of water from the reactor face.

Figure 7-11 is a similar set of data from the Lid Tank Shield Facility at ORNL.⁽²³⁾ The fission source for these radiations is a 28 inch diameter plate of enriched uranium (20.8% U^{235}).⁽²⁴⁾ These data have been corrected as suggested in the reference cited to account for effects due to the measuring foils, the geometry of the containing structure used, and a 5% correction to the effective source power normalization, although a $\pm 5\%$ uncertainty in source plate power is quoted. Similar data for transformer oil and slightly borated water (1.34 wt. % natural boron) are also presented in Ref. 23.

7.2.2 Removal Cross Section

The removal cross section, originally defined by Albert and Welton,⁽²⁵⁾ is a parameter widely used in fast neutron attenuation calculations. It is a measure of the ability of a material to remove a fast neutron from further consideration as a fast

*Furnished by E. P. Blizard, ORNL, June, 1958.

FIG. 7-9
NEUTRON AND GAMMA RAY ATTENUATION IN WATER AT
CENTERLINE OF BULK SHIELDING FACILITY

FIG. 7-10
NEUTRON AND GAMMA RAY ATTENUATION IN WATER AT CENTERLINE
OF BULK SHIELDING FACILITY (EXPANDED SCALE)

neutron for shielding attenuation purposes by interaction of the neutron with the material and subsequent interaction with a hydrogenous material. Neutrons which are either degraded in energy by inelastic scatterings, or deflected by wide angle elastic scatterings in the material are unlikely to penetrate large distances into a medium such as water, or a concrete which contains several atomic per cent of hydrogen.

Table 7-7 lists the removal cross sections determined in the Lid Tank by analysis of neutron attenuation measurements in water beyond slabs of various materials.⁽²⁶⁾ Figure 7-12 is a plot of the removal cross section in cm^2 per gram of various elements as a function of atomic weight. The smooth variation with atomic weight indicates that removal cross sections may be estimated with reasonable accuracy.

Table 7-7

REMOVAL CROSS SECTIONS FOR VARIOUS MATERIALS

(Reproduced from data of Chapman and Storrs⁽²⁶⁾)

Material	σ_R , barns	N_0 at 20°C (atom/ cm^3)	Σ_R cm^{-1}
Hydrogen	1.00 ± 0.05		
Lithium	1.01 ± 0.04	0.0460×10^{24}	0.046
Beryllium	1.07 ± 0.06	0.120	0.128
Boron	0.97 ± 0.10	0.139	0.135
Graphite	0.72 ± 0.05	0.113	0.092
Oxygen	0.92 ± 0.05		
Fluorine	1.29 ± 0.06		
Aluminum	1.31 ± 0.05	0.0603	0.079
Chlorine	1.2 ± 0.8		
Iron	1.98 ± 0.08	0.0848	0.168
Nickel	1.89 ± 0.10	0.0913	0.173
Copper	2.04 ± 0.11	0.0846	0.173
Tungsten	2.51 ± 0.55	0.0631	0.198 ^a
Lead	3.53 ± 0.30	0.0330	0.116
Bismuth	3.49 ± 0.35	0.0282	0.098
Uranium	3.6 ± 0.4	0.0473	0.17
Boric Oxide (B_2O_3)	4.30 ± 0.41		
Boron Carbide (B_4C)	5.1 ± 0.4		
Fluorothene ($\text{C}_2\text{F}_3\text{Cl}$)	6.66 ± 0.8		
Heavy Water (D_2O)	2.76 ± 0.11		
Lithium Fluoride (LiF)	2.43 ± 0.34		
Oil (CH_2)	2.84 ± 0.11		
Paraffin ($\text{C}_{30}\text{H}_{62}$)	80.5 ± 5.2		
Perfluoroheptane (C_7F_{16})	26.3 ± 0.8		

^aValue given by Miller⁽²⁷⁾

7.2.3

Conversion Factors: Neutron Flux to Dose

In Table 7-8 the first collision dose rate (dose imparted to a small volume of tissue) is given in rems as a function of neutron energy.⁽²⁸⁾ It is obtained by multiplying the first collision dose, which includes all of the energy imparted by the neutron to heavy ionizing particles, by the appropriate RBE (Relative Biological Effectiveness). The RBE given for thermal and epi-thermal neutrons in NBS63⁽²⁸⁾ is of the order of 10, as compared to 2 or 3 as given previously by Goldstein.⁽²⁹⁾ Depth dose curves have been computed by Snyder using a Monte Carlo method⁽³⁰⁾ and are also reported in NBS Handbook 63.⁽³¹⁾ The depth dose refers to the energy actually imparted to tissue as a function of depth beneath the surface, due to all primary and secondary ionizing events throughout the tissue body.

FIG. 7-12
EXPERIMENTAL REMOVAL CROSS SECTION AS A
FUNCTION OF ATOMIC WEIGHT FOR VARIOUS ELEMENTS
(DATA FROM CHAPMAN AND STORRS²⁶)

Table 7-8

NEUTRON FLUX - TO - DOSE CONVERSION FACTORS
FIRST COLLISION RBE DOSE

(Data from NBS Handbook 63(28))

For Neutrons at Energy E, Multiply Neutron Flux, $\phi(E)$,
neutrons/cm²-sec by Conversion Factor, F,
To Obtain Dose in mrem/hr

Conversion Factor, F			Conversion Factor, F		
Neutron Energy, E		$\frac{\text{mrem/hr}}{\text{neut/cm}^2\text{-sec}}$	Neutron Energy, E		$\frac{\text{mrem/hr}}{\text{neut/cm}^2\text{-sec}}$
10	Mev	0.13	0.025	Mev	0.010
5	"	0.11	0.01	"	0.0036
2.5	"	0.094	0.001	"	0.00026
1.0	"	0.097	100	ev	0.000054
0.5	"	0.068	10	"	0.000072
0.25	"	0.047	1.0	"	0.00016
0.1	"	0.030	0.1	"	0.00043
0.05	"	0.019	0.025	"	0.00086

7.3 GAMMA-RAY CONSTANTS

The theory of gamma ray attenuation has been treated extensively in the literature, see for example, Goldstein,⁽²⁾ Goldstein and Wilkins,⁽³²⁾ Fano,⁽³³⁾ Taylor,⁽³⁴⁾ and Grodstein.⁽³⁵⁾ The data presented in this section are tabulated for discrete gamma-ray energies so that they may be applied in calculations using line spectra.

7.3.1 Total Gamma-Ray Attenuation Coefficient

Table 7-9 gives total gamma-ray attenuation coefficient for some common materials.⁽³⁶⁾ The product of these numbers and the density of the material gives the familiar total cross section, μ_t , Table 7-10.

7.3.2 Energy Absorption Coefficient

Only a fraction of the events represented by the total attenuation cross section actually remove the gamma-ray. In particular, Compton scattering can cause a change in the direction and the energy of a photon without absorbing it. The energy absorption coefficient is a measure of the fraction of the gamma-ray energy which is converted from radiant energy into heat:

$$\phi(E) (\text{Mev}/(\text{cm}^2)(\text{sec})) (\mu_e(E) (\text{cm}^{-1})) = \text{Heat Generation Rate} (\text{Mev}/(\text{cm}^3)(\text{sec}))$$

Table 7-11 lists energy absorption coefficients. The product of these coefficients and the density of the material gives the energy absorption cross section, μ_e , Table 7-12.

7.3.3 Buildup Factors

Since not all gamma interactions are absorptive, and some scattered radiation may ultimately penetrate a shield, a numerical correction is commonly used in attenuation calculations. The uncollided flux is calculated using total cross sections, and then multiplied by the appropriate buildup factor to give a more realistic value of the transmitted flux. Buildup factors have been determined theoretically⁽³⁷⁾ by Monte Carlo calculations and by the method of moments. The buildup factor (B) is a function of the distance (x) into the shield, measured in mean free paths, ($\mu_t x$), as well as of the particular material and gamma-ray energy.

Table 7-13 gives a set of buildup factors for a point isotropic source, applicable for shields at least one mean free path in thickness.⁽³⁸⁾ Table 7-14 gives coefficients A_1 , a_1 , and a_2 , for an analytical fit to the buildup factor.⁽³⁴⁾

$$B(\mu_t x) = A_1 \exp(-a_1 \mu_t x) + A_2 \exp(-a_2 \mu_t x)$$

where

$$A_2 = 1 - A_1$$

Values have been calculated⁽³⁷⁾ for energy and energy absorption buildup factors, and for plane monodirectional sources.

Table 7-9

TOTAL MASS ATTENUATION COEFFICIENTS (μ/ρ)(in cm^2/gm)(Data of Grodstein)⁽³⁵⁾

Gamma-Ray Energy Mev

Material	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.5	2	3	4	5	6	8	10.0
H	.295	.265	.243	.212	.189	.173	.160	.140	.126	.113	.103	.0876	.0691	.0579	.0502	.0446	.0371	.0321
Be	.132	.119	.109	.0945	.0847	.0773	.0715	.0628	.0565	.0504	.0459	.0394	.0313	.0266	.0234	.0211	.0180	.0161
C	.149	.134	.122	.106	.0953	.0870	.0805	.0707	.0636	.0568	.0518	.0444	.0356	.0304	.0270	.0245	.0213	.0194
N	.150	.134	.123	.106	.0955	.0869	.0805	.0707	.0636	.0568	.0517	.0445	.0357	.0306	.0273	.0249	.0218	.0200
O	.151	.134	.123	.107	.0953	.0870	.0806	.0708	.0636	.0568	.0518	.0445	.0359	.0309	.0276	.0254	.0224	.0206
Na	.151	.130	.118	.102	.0912	.0833	.0770	.0676	.0608	.0546	.0496	.0427	.0348	.0303	.0274	.0254	.0229	.0215
Mg	.160	.135	.122	.106	.0944	.0860	.0795	.0699	.0627	.0560	.0512	.0442	.0360	.0315	.0286	.0266	.0242	.0228
Al	.161	.134	.120	.103	.0922	.0840	.0777	.0683	.0614	.0548	.0500	.0432	.0353	.0310	.0282	.0264	.0241	.0229
Si	.172	.139	.125	.107	.0954	.0869	.0802	.0706	.0635	.0567	.0517	.0447	.0367	.0323	.0296	.0277	.0254	.0243
P	.174	.137	.122	.104	.0928	.0846	.0780	.0685	.0617	.0551	.0502	.0436	.0358	.0316	.0290	.0273	.0252	.0242
S	.188	.144	.127	.108	.0958	.0874	.0806	.0707	.0635	.0568	.0519	.0448	.0371	.0328	.0302	.0284	.0266	.0255
A	.188	.135	.117	.0977	.0867	.0790	.0730	.0638	.0573	.0512	.0468	.0407	.0338	.0301	.0279	.0266	.0248	.0241
K	.215	.149	.127	.106	.0938	.0852	.0786	.0689	.0618	.0552	.0505	.0438	.0365	.0327	.0305	.0289	.0274	.0267
Ca	.238	.158	.132	.109	.0965	.0876	.0809	.0708	.0634	.0566	.0518	.0451	.0376	.0338	.0316	.0302	.0285	.0280
Fe	.344	.183	.138	.106	.0919	.0828	.0762	.0664	.0595	.0531	.0485	.0424	.0361	.0330	.0313	.0304	.0295	.0294
Cu	.427	.206	.147	.108	.0916	.0820	.0751	.0654	.0585	.0521	.0476	.0418	.0357	.0330	.0316	.0309	.0303	.0305
Mo	1.03	.389	.225	.130	.0998	.0851	.0761	.0648	.0575	.0510	.0467	.0414	.0365	.0349	.0344	.0344	.0349	.0359
Sn	1.58	.563	.303	.153	.109	.0886	.0776	.0647	.0568	.0501	.0459	.0408	.0367	.0355	.0355	.0358	.0368	.0383
I	1.83	.648	.339	.165	.114	.0913	.0792	.0653	.0571	.0502	.0460	.0409	.0370	.0360	.0361	.0365	.0377	.0394
W	4.21	1.44	.708	.293	.174	.125	.101	.0763	.0640	.0544	.0492	.0437	.0405	.0402	.0409	.0418	.0438	.0465
Pt	4.75	1.64	.795	.324	.191	.135	.107	.0800	.0659	.0554	.0501	.0445	.0414	.0411	.0418	.0427	.0448	.0477
Tl	5.16	1.80	.866	.346	.204	.143	.112	.0824	.0675	.0563	.0508	.0452	.0420	.0416	.0423	.0433	.0454	.0484
Pb	5.29	1.84	.896	.356	.208	.145	.114	.0836	.0684	.0569	.0512	.0457	.0421	.0420	.0426	.0436	.0459	.0489
U	1.06	2.42	1.17	.452	.259	.176	.136	.0952	.0757	.0615	.0548	.0484	.0445	.0440	.0446	.0455	.0479	.0511
Air	.151	.134	.123	.106	.0953	.0868	.0804	.0706	.0655	.0567	.0517	.0445	.0357	.0307	.0274	.0250	.0220	.0202
NaI	1.57	.568	.305	.155	.111	.0901	.0789	.0657	.0577	.0508	.0465	.0412	.0367	.0351	.0347	.0347	.0354	.0366
H ₂ O	.167	.149	.136	.118	.106	.0966	.0896	.0786	.0706	.0630	.0575	.0493	.0396	.0339	.0301	.0275	.0240	.0219
Concrete	.169	.139	.124	.107	.0954	.0870	.0804	.0706	.0635	.0567	.0517	.0445	.0363	.0317	.0287	.0268	.0243	.0229
Tissue	.163	.144	.132	.115	.100	.0936	.0867	.0761	.0683	.0600	.0556	.0478	.0384	.0329	.0292	.0267	.0233	.0212

Table 7-10
TOTAL GAMMA-RAY ATTENUATION CROSS SECTIONS, (μ_T)
(in cm^{-1})

		Gamma-Ray Energy, Mev																	
Material	Density	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.5	2	3	4	5	6	8	10.0
Be	1.85	.244	.220	.202	.1748	.1567	.1430	.1323	.1162	.1045	.0932	.0849	.0729	.0579	.0492	.0433	.0390	.0333	.0298
C	2.25	.335	.302	.275	.239	.2144	.1958	.1811	.1591	.1431	.1278	.1166	.0999	.0801	.0684	.0608	.0551	.0479	.0437
Na	.9712	.147	.126	.115	.099	.0886	.0809	.0748	.0657	.0590	.0530	.0482	.0415	.0338	.0294	.0266	.0247	.0222	.0209
Mg	1.741	.279	.235	.212	.185	.1643	.1497	.1384	.1217	.1092	.1975	.0891	.0770	.0627	.0548	.0498	.0463	.0421	.0397
Al	2.70	.435	.362	.324	.278	.2489	.2268	.2098	.1844	.1658	.1480	.1350	.1166	.0953	.0837	.0761	.0713	.0651	.0618
Si	2.42	.416	.336	.303	.259	.2309	.2103	.1941	.1709	.1537	.1372	.1251	.1082	.0888	.0782	.0716	.0670	.0615	.0588
P	1.83	.318	.251	.223	.190	.1698	.1548	.1427	.1254	.1129	.1008	.0919	.0798	.0655	.0578	.0531	.0500	.0461	.0443
S	2.07	.389	.298	.263	.224	.1983	.1809	.1668	.1463	.1314	.1176	.1074	.0927	.0768	.0679	.0625	.0588	.0551	.0328
K	0.87	.187	.130	.110	.092	.0816	.0741	.0684	.0599	.0538	.0480	.0439	.0381	.0318	.0284	.0265	.0251	.0238	.0232
Ca	1.55	.369	.245	.205	.169	.1496	.1358	.1254	.1097	.0983	.0877	.0803	.0699	.0583	.0524	.0490	.0468	.0442	.0434
Fe	7.86	2.704	1.438	1.085	.833	.7223	.6508	.5989	.5219	.4677	.4174	.3812	.3333	.2837	.2594	.2460	.2389	.2319	.2311
Cu	8.933	3.814	1.840	1.313	.965	.8183	.7325	.6709	.5842	.5226	.4654	.4252	.3734	.3189	.2948	.2823	.2760	.2707	.2725
Mo	9.01	9.280	3.505	2.027	1.171	.8991	.7668	.6857	.5838	.5181	.4595	.4208	.3730	.3289	.3144	.3099	.3099	.3144	.3190
Sn	7.298	11.53	4.109	2.211	1.117	.795	.6466	.5663	.4722	.4145	.3656	.3350	.2978	.2678	.2591	.2591	.2613	.2686	.2795
I	4.94	9.040	3.201	1.675	.815	.563	.4510	.3912	.3226	.2821	.2480	.2272	.2020	.1828	.1778	.1783	.1803	.1862	.1946
W	19.3	81.25	27.79	13.66	5.655	3.358	2.413	1.949	1.473	1.235	1.050	.9496	.8434	.7817	.7759	.7894	.8067	.8453	.8975
Pt	21.37	101.51	35.05	16.99	6.924	4.082	2.885	2.287	1.710	1.408	1.184	1.071	.9510	.8847	.8783	.8933	.9125	.9574	1.019
Tl	11.86	61.20	21.35	10.27	4.104	2.419	1.696	1.328	.9773	.8005	.6677	.6025	.5361	.4981	.4934	.5017	.5135	.5384	.5740
Pb	11.34	59.99	20.87	10.16	4.037	2.359	1.644	1.293	.9480	.7757	.6452	.5806	.5182	.4774	.4763	.4831	.4944	.5205	.5545
U	18.7	19.82	45.25	21.88	8.452	4.843	3.291	2.543	1.780	1.416	1.150	1.025	.9051	.8322	.8228	.8340	.8509	.8957	.9556
NaI	3.667	5.757	2.083	1.118	.568	.407	.3304	.2893	.2409	.2116	.1863	.1705	.1511	.1346	.1287	.1272	.1272	.1298	.1342
H ₂ O	1.00	.167	.149	.136	.118	.106	.0966	.0896	.0786	.0706	.0630	.0575	.0493	.0396	.0339	.0301	.0275	.0240	.0219
Concrete	2.35	.397	.327	.291	.251	.2242	.2045	.1889	.1659	.1492	.1332	.1215	.1046	.0853	.0745	.0674	.0630	.0571	.0538

Table 7-11

ENERGY ABSORPTION MASS ATTENUATION COEFFICIENTS, (μ_e/ρ)(in cm^2/gm)(Calculated from Data of Grodstein⁽³⁵⁾ by Blizard, et al. ORNL)

Material	Gamma-Ray Energy, Mev																	
	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.50	2	3	4	5	6	8	10.0
H	.0411	.0487	.0531	.0575	.0589	.0591	.0590	.0575	.0557	.0533	.0509	.0467	.0401	.0354	.0318	.0291	.0252	.0255
Be	.0183	.0217	.0237	.0256	.0263	.0264	.0263	.0256	.0248	.0237	.0227	.0210	.0183	.0164	.0151	.0141	.0127	.0118
C	.0215	.0246	.0267	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0237	.0209	.0190	.0177	.0166	.0153	.0145
N	.0224	.0249	.0267	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0238	.0211	.0193	.0180	.0171	.0158	.0151
O	.0233	.0252	.0271	.0289	.0296	.0297	.0296	.0289	.0280	.0268	.0257	.0238	.0212	.0195	.0183	.0175	.0163	.0157
Na	.0289	.0258	.0266	.0279	.0283	.0284	.0284	.0276	.0268	.0257	.0246	.0229	.0207	.0194	.0185	.0179	.0171	.0168
Mg	.0335	.0276	.0278	.0290	.0294	.0293	.0292	.0285	.0276	.0265	.0254	.0237	.0215	.0203	.0194	.0188	.0182	.0180
Al	.0373	.0283	.0275	.0283	.0287	.0286	.0286	.0278	.0270	.0259	.0248	.0232	.0212	.0200	.0192	.0188	.0183	.0182
Si	.0435	.0300	.0286	.0291	.0293	.0290	.0290	.0282	.0274	.0263	.0252	.0236	.0217	.0206	.0198	.0194	.0190	.0189
P	.0501	.0315	.0292	.0289	.0290	.0290	.0287	.0280	.0271	.0260	.0250	.0234	.0216	.0206	.0200	.0197	.0194	.0195
S	.0601	.0351	.0310	.0301	.0301	.0300	.0298	.0288	.0279	.0268	.0258	.0242	.0224	.0215	.0209	.0206	.0206	.0206
A	.0729	.0368	.0302	.0278	.0274	.0272	.0270	.0260	.0252	.0242	.0233	.0220	.0206	.0199	.0195	.0195	.0194	.0197
K	.0909	.0433	.0340	.0304	.0298	.0295	.0291	.0282	.0272	.0261	.0251	.0237	.0222	.0217	.0214	.0212	.0215	.0219
Ca	.111	.0489	.0367	.0318	.0309	.0304	.0300	.0290	.0279	.0268	.0258	.0244	.0230	.0225	.0222	.0223	.0225	.0231
Fe	.225	.0810	.0489	.0340	.0307	.0294	.0287	.0274	.0261	.0250	.0242	.0231	.0224	.0224	.0227	.0231	.0239	.0250
Cu	.310	.107	.0594	.0368	.0316	.0296	.0286	.0271	.0260	.0247	.0237	.0229	.0223	.0227	.0231	.0237	.0248	.0261
Mo	.922	.294	.141	.0617	.0422	.0348	.0315	.0281	.0263	.0248	.0239	.0233	.0237	.0250	.0262	.0274	.0296	.0316
Sn	1.469	.471	.222	.0873	.0534	.0403	.0346	.0294	.0268	.0248	.0239	.0233	.0243	.0259	.0276	.0291	.0316	.0339
I	1.726	.557	.260	.100	.0589	.0433	.0366	.0303	.0274	.0252	.0241	.0236	.0247	.0265	.0283	.0299	.0327	.0353
W	4.112	1.356	.631	.230	.121	.0786	.0599	.0426	.0353	.0302	.0281	.0271	.0287	.0311	.0335	.0355	.0390	.0426
Pt	4.645	1.556	.719	.262	.138	.0892	.0666	.0465	.0375	.0315	.0293	.0280	.0296	.0320	.0343	.0365	.0400	.0438
Tl	5.057	1.717	.791	.285	.152	.0972	.0718	.0491	.0393	.0326	.0301	.0288	.0304	.0326	.0349	.0354	.0406	.0446
Pb	5.193	1.753	.821	.294	.156	.0994	.0738	.0505	.0402	.0332	.0306	.0293	.0305	.0330	.0352	.0373	.0412	.0450
U	9.63	2.337	1.096	.392	.208	.132	.0968	.0628	.0482	.0383	.0346	.0324	.0332	.0352	.0374	.0394	.0443	.0474
Air	.0233	.0251	.0268	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0238	.0211	.0194	.0181	.0172	.0160	.0153
NaI	1.466	.476	.224	.0889	.0542	.0410	.0354	.0299	.0273	.0253	.0242	.0235	.0241	.0254	.0268	.0281	.0303	.0325
H ₂ O	.0253	.0278	.0300	.0321	.0328	.0330	.0329	.0321	.0311	.0298	.0285	.0264	.0233	.0213	.0198	.0188	.0173	.0165
Concrete	.0416	.0300	.0289	.0294	.0297	.0296	.0295	.0287	.0278	.0272	.0256	.0239	.0216	.0203	.0194	.0188	.0180	.0177
Tissue	.0271	.0282	.0293	.0312	.0317	.0320	.0319	.0311	.0300	.0288	.0276	.0256	.0220	.0206	.0192	.0182	.0168	.0160

Table 7-12

GAMMA-RAY ENERGY ABSORPTION CROSS SECTION (μ_e)(in cm^{-1})

Gamma-Ray Energy, Mev

Material	Density	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.50	2	3	4	5	6	8	10.0
Be	1.85	.0339	.0401	.0438	.0474	.0487	.0488	.0487	.0474	.0459	.0438	.0420	.0389	.0339	.0303	.0279	.0261	.0235	.0218
C	2.25	.0484	.0554	.0601	.0648	.0666	.0668	.0666	.0650	.0630	.0603	.0576	.0533	.0470	.0427	.0398	.0374	.0344	.0326
Na	.9712	.0281	.0251	.0258	.0271	.0275	.0276	.0276	.0268	.0260	.0250	.0239	.0222	.0201	.0188	.0180	.0174	.0166	.0163
Mg	1.741	.0583	.0481	.0484	.0505	.0512	.0510	.0508	.0496	.0481	.0461	.0442	.0413	.0374	.0353	.0338	.0327	.0317	.0313
Al	2.70	.1007	.0764	.0743	.0764	.0775	.0772	.0772	.0751	.0729	.0699	.0670	.0626	.0572	.0540	.0518	.0508	.0494	.0491
Si	2.42	.1053	.0726	.0692	.0704	.0709	.0702	.0702	.0682	.0663	.0636	.0610	.0571	.0525	.0499	.0479	.0469	.0460	.0457
P	1.83	.0917	.0576	.0534	.0529	.0531	.0531	.0525	.0512	.0496	.0476	.0458	.0428	.0395	.0377	.0366	.0361	.0355	.0357
S	2.07	.1244	.0727	.0642	.0623	.0623	.0621	.0617	.0596	.0578	.0555	.0534	.0501	.0464	.0445	.0433	.0426	.0426	.0426
K	0.87	.0791	.0377	.0296	.0264	.0259	.0257	.0253	.0245	.0237	.0227	.0218	.0206	.0193	.0189	.0186	.0184	.0187	.0191
Ca	1.55	.172	.0758	.0569	.0493	.0479	.0471	.0465	.0450	.0432	.0415	.0400	.0378	.0357	.0349	.0344	.0346	.0349	.0358
Fe	7.86	1.769	.6367	.3844	.2672	.2413	.2311	.2256	.2154	.2051	.1965	.1902	.1816	.1761	.1761	.1784	.1816	.1879	.1965
Cu	8.933	2.769	.956	.5306	.3287	.2823	.2644	.2555	.2421	.2323	.2206	.2117	.2046	.1992	.2028	.2064	.2117	.2215	.2332
Mo	9.01	8.307	2.649	1.270	.5559	.3802	.3135	.2831	.2532	.2370	.2234	.2153	.2099	.2135	.2253	.2361	.2469	.2667	.2847
Sn	7.298	10.721	3.437	1.620	.6371	.3897	.2941	.2525	.2146	.1956	.1810	.1744	.1700	.1773	.1890	.2014	.2124	.2306	.2474
I	4.94	8.704	2.752	1.284	.494	.2910	.2139	.1808	.1497	.1354	.1245	.1191	.1166	.1220	.1309	.1398	.1477	.1615	.1744
W	19.3	79.362	26.171	12.178	4.439	2.335	1.517	1.156	.8222	.6813	.5829	.5423	.5320	.5539	.6002	.6466	.6852	.7527	.8222
Pt	21.37	99.264	33.252	15.365	5.599	2.949	1.906	1.423	.9937	.8014	.6732	.6261	.5984	.6326	.6838	.7330	.7800	.8548	.9360
Tl	11.86	59.976	20.364	9.381	3.380	1.803	1.153	.8515	.5823	.4661	.3866	.3570	.3416	.3605	.3866	.4139	.4198	.4815	.5290
Pb	11.34	58.839	19.879	9.310	3.334	1.769	1.127	.8369	.5727	.4559	.3765	.3470	.3323	.3459	.3742	.3992	.4230	.4672	.5103
U	18.7	180.08	43.702	20.495	7.330	3.890	2.468	1.810	1.174	.9013	.7162	.6470	.6059	.6208	.6582	.6994	.7368	.8284	.8864
NaI	3.667	5.376	1.745	.8214	.3260	.1988	.1503	.1298	.1096	.1001	.0928	.0887	.0862	.0884	.0931	.0983	.1030	.1111	.1192
H ₂ O	1.00	.0253	.0278	.0300	.0321	.0328	.0330	.0329	.0321	.0311	.0298	.0285	.0264	.0233	.0213	.0198	.0188	.0173	.0165
Concrete	2.35	.0978	.0705	.0679	.0691	.0698	.0697	.0693	.0674	.0653	.0639	.0602	.0562	.0508	.0477	.0456	.0442	.0423	.0416

Table 7-13

DOSE BUILDUP FACTOR $B(\mu_{tx})$ FOR A POINT ISOTROPIC SOURCE

(Reproduced from data of Goldstein and Wilkins(38))

		Gamma-Ray Energy, Mev							
	μ_t^x	0.5	1.0	2.0	3.0	4.0	6.0	8.0	10.0
Water	1	2.52	2.13	1.83	1.69	1.58	1.46	1.38	1.33
	2	5.14	3.71	2.77	2.42	2.17	1.91	1.74	1.63
	4	14.3	7.68	4.88	3.91	3.34	2.76	2.40	2.19
	7	38.8	16.2	8.46	6.23	5.13	3.99	3.34	2.97
	10	77.6	27.1	12.4	8.63	6.94	5.18	4.25	3.72
	15	178	50.4	19.5	12.8	9.97	7.09	5.66	4.90
	20	334	82.2	27.7	17.0	12.9	8.85	6.95	5.98
Aluminum	1	2.37	2.02	1.75	1.64	1.53	1.42	1.34	1.28
	2	4.24	3.31	2.61	2.32	2.08	1.85	1.68	1.55
	4	9.47	6.57	4.62	3.78	3.22	2.70	2.37	2.12
	7	21.5	13.1	8.05	6.14	5.01	4.06	3.45	3.01
	10	38.9	21.2	11.9	8.65	6.88	5.49	4.58	3.96
	15	80.8	37.9	18.7	13.0	10.1	7.97	6.56	5.63
	20	141	58.5	26.3	17.7	13.4	10.4	8.52	7.32
Iron	1	1.98	1.87	1.76	1.55	1.45	1.34	1.27	1.20
	2	3.09	2.89	2.43	2.15	1.94	1.72	1.56	1.42
	4	5.98	5.39	4.13	3.51	3.03	2.58	2.23	1.95
	7	11.7	10.2	7.25	5.85	4.91	4.14	3.49	2.99
	10	19.2	16.2	10.9	8.51	7.11	6.02	5.07	4.35
	15	35.4	28.3	17.6	13.5	11.2	9.89	8.50	7.54
	20	55.6	42.7	25.1	19.1	16.0	14.7	13.0	12.4
Tungsten	1	1.28	1.44	1.42	1.36	1.29	1.20	1.14	1.11
	2	1.50	1.83	1.85	1.74	1.62	1.43	1.32	1.25
	4	1.84	2.57	2.72	2.59	2.41	2.07	1.81	1.64
	7	2.24	3.62	4.09	4.00	4.03	3.60	3.05	2.62
	10	2.61	4.64	5.27	5.92	6.27	6.29	5.40	4.65
	15	3.12	6.25	8.07	9.66	12.0	15.7	15.2	14.0
	20		7.35	10.6	14.1	20.9	36.3	41.9	39.3
Lead	1	1.24	1.37	1.39	1.34	1.27	1.18	1.14	1.11
	2	1.42	1.69	1.76	1.68	1.56	1.40	1.30	1.23
	4	1.69	2.26	2.51	2.43	2.25	1.97	1.74	1.58
	7	2.00	3.02	3.66	3.75	3.61	3.34	2.89	2.52
	10	2.27	3.74	4.84	5.30	5.44	5.69	5.07	4.34
	15	2.65	4.81	6.87	8.44	9.80	13.8	14.1	12.5
	20	2.73	5.86	9.00	12.3	16.3	32.7	44.6	39.2
Uranium	1	1.17	1.31	1.33	1.29	1.24	1.16	1.12	1.09
	2	1.30	1.56	1.64	1.58	1.50	1.36	1.27	1.20
	4	1.48	1.98	2.23	2.21	2.09	1.85	1.66	1.51
	7	1.67	2.50	3.09	3.27	3.21	2.96	2.61	2.26
	10	1.85	2.97	3.95	4.51	4.66	4.80	4.36	3.78
	15	2.08	3.67	5.36	6.97	8.01	10.8	11.2	10.5
	20			6.48	9.88	12.7	23.0	28.0	28.5

Table 7-14

COEFFICIENTS FOR AN ANALYTIC FIT TO BUILDUP FACTOR EQUATION(Reproduced in part from data of Taylor⁽³⁴⁾)*

$$B(\mu_t x) = A_1 \exp(-a_1 \mu_t x) + A_2 \exp(-a_2 \mu_t x)$$

$$A_2 = 1 - A_1$$

Material		Gamma-Ray Energy, Mev							
		0.5	1	2	3	4	6	8	10
Water	A ₁		13.5	8.1	5.6	4.5	3.4	2.8	2.5
	-a ₁		0.100	0.068	0.059	0.0555	0.0525	0.05	0.0473
	+a ₂		0.010	0.0405	0.073	0.11	0.156	0.17	0.1719
Aluminum	A ₁		8.0	5.5	4.5	3.8	3.1	2.3	2.25
	-a ₁		0.11	0.082	0.074	0.066	0.064	0.062	0.060
	+a ₂		0.044	0.093	0.116	0.130	0.152	0.150	0.128
Iron	A ₁	10.0	8.0	5.5		3.75	2.9	2.35	2.0
	-a ₁	0.0948	0.0895	0.0788		0.075	0.0825	0.0833	0.095
	+a ₂	0.012	0.04	0.07		0.082	0.075	0.0546	0.0116
Tungsten	A ₁		3.3	2.9	2.7	2.05	1.2	0.7	0.6
	-a ₁		0.043	0.069	0.086	0.118	0.171	0.205	0.212
	+a ₂		0.148	0.188	0.134	0.070	0.00	0.052	0.144
Lead	A ₁	1.65	2.45	2.60	2.15	1.65	0.96	0.67	0.50
	-a ₁	0.032	0.045	0.071	0.097	0.123	0.175	0.204	0.214
	+a ₂	0.296	0.178	0.103	0.077	0.064	0.059	0.067	0.08
Concrete	A ₁	12.5	10.0	6.3	4.7	3.9	3.1	2.7	2.6
	-a ₁	0.11	0.088	0.069	0.062	0.059	0.059	0.056	0.050
	+a ₂	0.01	0.029	0.058	0.073	0.079	0.083	0.086	0.084

*The referenced document includes a discussion of the errors involved in these analytical fits. In general, the errors are less than a few per cent and, in no case greater than 8%.

7.3.4 Conversion Factors: Gamma-Ray Flux to Dose

Gamma-ray flux-to-dose conversion factors are given in Table 7-15.⁽³⁹⁾

7.4 SHIELDING PROPERTIES OF CONCRETES

The compositions of a number of different shielding concretes are described and their nuclear properties tabulated. One of the uncertainties in shield design work has been the variation in content of the various concretes. This is particularly true of the water content, which has the vital role of a moderator in the concrete. In cases where water retention is not known, two or three values are assumed. The resulting constants then indicate a range of numbers with the parameter being water content.

Table 7-15

GAMMA-RAY FLUX - TO - DOSE CONVERSION FACTORS(Data Taken from Reactor Shielding Design Manual,⁽³⁹⁾ Fig. 2.2)

For gamma-rays of energy E: Multiply gamma-ray flux, $\phi(E)$ in photons/cm²-sec by conversion factor N, or multiply gamma-ray flux $\phi(E)$ in Mev/cm²-sec by conversion factor M, to obtain dose in mr/hr.

Gamma-Ray Energy, E Mev	Conversion Factor	
	N $\gamma/\text{cm}^2\text{-sec}$ mr/hr	M Mev/cm ² -sec mr/hr
10	10.8×10^{-3}	1.08×10^{-3}
9	9.8	1.09
8	8.8	1.1
7	8.0	1.14
6	7.1	1.18
5	6.2	1.24
4	5.3	1.32
3	4.3	1.43
2	3.25	1.62
1	1.93	1.93
0.5	1.02	2.06

7.4.1 Physical Compositions

The various concretes considered, their densities and mix proportions are listed in Table 7-16.

Table 7-17 gives a detailed breakdown of the elemental compositions of the concretes, upon which the calculations of the cross sections are based.

7.4.2 Nuclear Constants

Neutron constants for the concretes are presented in Table 7-18. The values for the age to thermal energy (τ), and the two-group fast neutron diffusion coefficient (D_f) are approximate, but form a consistent set for comparison purposes. Measured removal cross-sections and fast neutron relaxation lengths are included.

Table 7-19 gives the effective thermal neutron absorption cross section, and the effective number of gamma-ray photons emitted per neutron capture for a four-line gamma energy spectrum.

Gamma-ray attenuation coefficients for ordinary concrete (Type 04) are included in Tables 7-9 and 7-10.

Table 7-16

COMPOSITION AND DENSITY OF VARIOUS CONCRETES

Concrete	*Symbol	Density		Water	Composition, lb/yd ³			Steel Punchings	Total	Ref.
		gm/cm ³	lb/ft ³		Portland Cement	Aggregate				
Ordinary										
1	01	2.33	145							40
2a	02-a	2.3	144	260	318	3300	-	3878		40
2b	02-b	2.22	139			(Sand and Gravel)				
3	03	2.39	149							41
4	04	2.35	147							42
Ferrophosphorous										
a	FP-a	4.68	292	384	730	6870	-	7894		†
b	FP-b	4.57	285			(Ferrophosphorus Slag)				
Barytes										
a	BA-a	3.46	216	370	550	4980	-	5900	40,43	
b	BA-b	3.35	212			(BaSO ₄ Ore)				
Magnetite										
a (EBWR Shield)	M-a	3.62	226	330	875	4900	-	6105	44,45	
b	M-b	3.52	220			(Magnetite Ore)				
Brookhaven (Limonite)	BR	4.26	266							40
Ilmenite										
1 (New York State)	I-1	3.68								46†
2 (Swedish)	I-2	3.62								47†
Magnetite and Steel Punchings										
1a (MIT Reactor Shield)	MS1-a	4.5	280	333	550	3310	3360	7553	44	
1b	MS1-b	4.4	274			(Magnetite Ore)				
2a (EBWR Shield)	MS2-a	4.74	295	340	940	1900	4800	7980	44,45	
2b	MS2-b	4.64	290			(Magnetite Ore)				
Limonite and Steel Punchings										
a (CP-5 Shield)	LS-a	4.64	289	347	980	1825	4680	7832	††	
b	LS-b	4.54	283			(Limonite Ore)				

*Where appearing, "a" and "b" indicate 100% and 50% water retention, respectively.

†Ilmenite Ore substituted for Magnetite in M-a.

‡Victor Chemical Works, Chicago, Illinois.

††Spectrochemical Analysis by Hanford, unpublished.

Table 7-17

ELEMENTAL COMPOSITION OF CONCRETES

(in gm element / cm³ concrete)

Ordinary Concretes						Heavy Concretes										Heavy Concretes with Steel Punchings						
Element	O1	O2-a	O2-b	O3	O4	Element	FP-a	FP-b	BA-a	BA-b	M-a	M-b	BR	I-1	I-2	Element	MS1-a	MS1-b	MS2-a	MS2-b	IS-a	IS-b
H	0.00484	0.023	0.0144	0.020	0.013	H in water	0.0253	0.0127	0.0243	0.0122	0.0217	0.0109	0.0211	0.0217	0.0217	H in water	0.0221	0.0110	0.0224	0.0112	0.0228	0.0114
O in water	0.0384	0.183	0.114	0.159	0.103	O in water	0.202	0.101	0.194	0.0972	0.174	0.087	0.763	0.174	0.174	in ore						0.172
in dry mix	1.1106	1.037	1.036	0.957	1.062	in ore	-		0.828		0.894			1.037	0.916	O in water	0.176	0.0882	0.0180	0.0898	0.182	0.0911
C	0.130	0.0023	0.118	-	-	in cement	0.156		0.118		0.187			0.187	0.187	in ore	0.725		0.347		0.408	
Mg	0.00486	0.005	0.057	0.006	-	Mg in ore	-		-		0.0263		0.008	0.280	0.0139	in cement			0.201		0.209	
Al	0.0119	0.078	0.085	0.107	-	in cement	0.00522		0.00396		0.0063			0.0063	0.0063	Mg in ore	0.0179		0.0102		-	
Si	0.438	0.775	0.342	0.737	-	Al in ore	-		-		0.0644		0.0213	-	-	in cement	0.0040		0.0067		0.007	
S	0.00192	-	0.007	0.003	-	in cement	0.0183		0.0139		0.0220			0.022	0.022	Al in ore	0.0417		0.0239		0.0043	
K	-	0.0299	0.004	0.045	-	Si in ore	-		-		0.0339		0.0589	0.047	-	in cement	0.0139		0.0236		0.0245	
Ca	0.581	0.100	0.582	0.194	-	in cement	0.0465		0.0352		0.0567		-	0.0557	0.0557	Si in ore	0.0230		0.0132		0.0043	
Fe	0.00726	0.032	0.026	0.029	-	P	1.206		-		0.0004		-	-	-	in cement	0.0350		0.0599		0.0623	
Na	-	0.0368	-	0.400	-	S	-		0.361		0.0052		0.0035	-	-	S	-		-		-	
ρ , gm/cm ³ :	2.33	2.30	2.22	2.39	2.35	Ca in ore	-		0.0203		0.0166		0.259	-	0.0012	K	-		-		0.0043	
						in cement	0.195		0.148		0.234		-	0.234	0.234	Ca in ore	0.0113		0.00446		-	
						Ti	-		-		0.192		-	0.998	0.914	in cement	0.1480		0.251		0.261	
						V	-		-		0.0113		-	-	-	Ti	0.130		0.074		-	
						Cr	-		-		0.00575		-	-	-	P	0.0003		0.00017		-	
						Mn	-		-		0.0067		0.067	0.0247	0.0305	V	0.0046		0.0026		0.0043	
						Fe in ore	2.814		0.150		1.661		3.055	0.583	1.001	Fe in ore	1.127		0.445		0.633	
						in cement	0.0121		0.00917		0.0145		-	0.0145	0.0145	in cement	0.009		0.0156		0.0162	
						Ba	-		1.551		-		-	-	-	in punchings	2.002		2.851		2.772	
ρ , gm/cm ³ :	4.68	4.57	3.46	3.35	3.62	3.52	4.26	3.62	3.62							Zn	-		-		0.0043	
																ρ , gm/cm ³ :	4.5	4.4	4.74	4.64	4.64	4.54

Table 7-18

NEUTRON CONSTANTS FOR CONCRETES

Concrete*	Density, gm/cm ³	κ_{th} , cm ⁻¹	D _{th} , cm	Σ_R , cm ⁻¹		λ , cm Exptl.	τ cm ²	D _f , cm
				Calc.	Exptl.			
01-a	2.33	0.0744	1.00	0.0776	0.083**		349	1.40
b				0.0715			581	
c							1370	
02-a	2.30	0.130	0.531	0.0882			101	1.14
b	2.22	0.102	0.707	0.0806			205	1.51
c							1530	2.04
03-a	2.39	0.127	0.558	0.0866	0.086		119	1.23
b							222	1.39
c							1250	1.65
04-a	2.35	0.108	0.715	0.088			179	1.31
b							316	1.57
c							1450	1.73
FP-a	4.68	0.474	0.348	0.129	0.148**		51.6	
b	4.57	0.405	0.463	0.117			92.4	
c							275	
BA-a	3.46	0.200	0.484	0.0947		12.5	86.9	1.10
b	3.35	0.154	0.722	0.084			179	1.29
c							1650	1.62
M-a	3.62	0.379	0.418	0.112			75.4	
b	3.52	0.322	0.559	0.102			141	
c							708	
M	3.29					9.52 [†]		
BR-a	4.26	0.477	0.387	0.116		6.3	79.1	.889
b	4.3			0.135			153	
c							925	
I-1	3.62	0.581	0.315	0.127				
2	3.62	0.454	0.442	0.111				
MS1-a	4.5	0.504	0.365	0.125			69.0	
b	4.4	0.436	0.472	0.115			131	
c							713	
MS2-a	4.74	0.504	0.356	0.127			56.0	0.814
b	4.64	0.436	0.458	0.117			113	0.932
c							958	1.00
LS-a	4.64	0.586	0.269	0.136	0.114**			
b	4.54	0.528	0.324	0.126				
LS	4.24					7.7 [†]		
LS	4.15					7.9 [†]		

*Assumed Water Retention:
a = 100%; b = 50%; c = 0%.

**ANL Shield Facility⁽⁴⁸⁾.

[†]Hanford Facility⁽⁴⁹⁾.

[†]ORNL Core Hole Facility⁽⁵⁰⁾.

ρ = Density.

κ_{th} = Reciprocal diffusion length for thermal neutrons.

D_{th} = Diffusion coefficient for thermal neutrons.

Σ_R = Removal cross section for fast neutrons.

τ = Age down to thermal energy.

D_f = Diffusion coefficient for fast neutrons.

λ = Measured relaxation length for fast neutrons.

Table 7-19

CAPTURE GAMMA-RAY SPECTRA OF CONCRETES

Concrete*	Σ_a^{**} cm ⁻¹	N(E) Effective Number of Capture Gamma Rays of Energy E Emitted per Neutron Absorption			
		2 Mev	4 Mev	6 Mev	8 Mev
01	0.00553	0.655	0.832	0.699	0.066
02-a	0.0089	0.830	0.678	0.211	0.168
02-b	0.0074	0.788	0.953	0.296	0.236
03	0.00895	0.693	0.512	0.454	0.077
04	0.00836	0.745	0.834	0.335	0.129
FP-a	0.0781	0.154	0.281	0.230	0.444
b	0.0759	0.129	0.289	0.237	0.457
BA-a	0.0193	0.663	0.555	0.301	0.120
b	0.0171	0.620	0.624	0.339	0.135
M-a	0.0600	0.384	0.252	0.374	0.349
b	0.0581	0.365	0.260	0.386	0.360
BR	0.0879	0.137	0.243	0.232	0.448
I-1a	0.0888	0.807	0.299	0.758	0.163
2a	0.0941	0.710	0.287	0.686	0.204
MS1-a	0.0939	0.225	0.247	0.289	0.435
b	0.0897	0.208	0.251	0.295	0.445
MS2-a	0.0962	0.182	0.248	0.265	0.453
b	0.0942	0.215	0.243	0.260	0.444
LS-a	0.0923	0.176	0.232	0.216	0.451
b	0.0903	0.158	0.237	0.221	0.461

*Assumed water retention: a = 100%; b = 50%

** Σ_a = Thermal neutron absorption cross section at 20°C corrected (0.8862) for 1/v absorption in a Maxwell distribution.

7.4.3

Temperature Dependence of Removal Cross Section

The shielding effectiveness of a concrete depends heavily on the water content. The quantity of contained water decreases as the concrete is heated, changing the value of the removal cross section. Measured fast neutron relaxation lengths for four types of concrete as a function of temperature are presented in Table 7-20.

Table 7-20

MEASURED RELAXATION LENGTHS FOR SEVERAL CONCRETES
AS A FUNCTION OF CONCRETE TEMPERATURE

(Sulfur Foil Activation Method)

Concrete Type	Density	Relaxation Length, cm Temperature				Reference
		Unheated	100°C	185°C	320°C	
Iron-Limonite	4.231	7.4	7.9	9.0	10.0	51
Magnetite-Limonite	3.39	8.9	9.7	10.5	10.5	51
Magnetite	3.59(?)	8.75	9.52	9.75	10.1**	49, 52
Ordinary	2.47	11.8*	-	-	-	53

* $\lambda = 11.8$ cm for first 2 ft; $\lambda \sim 12.4$ cm for next 2 ft. Measurements at higher temperatures in progress.** $\lambda = 10.1$ cm for first 28 in.; $\lambda \sim 11.1$ after 28 in.

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SECTION 8

CONSTANTS RELATED TO INTERPRETATION
OF EXPERIMENTAL DATA

This section is devoted to those quantities necessary to the proper interpretation of experiments and, insofar as possible, specific techniques are not explored. The data presented are therefore more in the character of corrections to be applied to data derived from a given technique of measurement. It is difficult to draw a line between proper technique and corrections, and in this sense the selection of material is rather arbitrary. Generous use has been made of summaries such as "Interpretation of Counting Data"⁽¹⁾ and bibliographies such as "A Directory to Nuclear Data Tabulations."⁽²⁾

8.1 FLUX PERTURBATION BY FOILS

The activation observed by counting a foil which has been exposed at a specified point P in a reactor represents an approximate measure of the flux existing at P before the foil was inserted. The corrections necessary to translate the observed activation reading to a number which is proportional to the unperturbed flux include:

- (1) Flux depression in the vicinity of the foil.
- (2) Shadowing of the center portion of the foil by the outer layers of the foil.
- (3) Mutual shadowing of neighboring foils.
- (4) Shadowing of a foil due to a covering foil of a different material.

8.1.1 Thermal Flux Depression

The presence of a foil will result in the depression of the flux in its vicinity. This is due to the absorption by the foil of neutrons which otherwise would have had a chance to be scattered back by the surrounding medium into that vicinity. The magnitude and extent of the depression depend on the size and blackness of the foil, on the angular distribution of neutrons in its vicinity, and on the mean free paths of neutrons in the surrounding medium.

The magnitude of the thermal flux depression may be approximated by the diffusion theory development of Bothe⁽³⁾ as modified by Tittle⁽⁴⁾ or by the transport theory development of Skyrme.⁽⁵⁾ The factor F_{SP} , the ratio of the unperturbed flux to the flux at the foil surface is, according to Tittle:

$$F_{SP} = 1 + \frac{\beta}{2} \left[\frac{3R}{2\lambda_{tr}} \cdot \frac{L}{R+L} - 1 \right] \quad \text{if } R > 2\lambda_{tr}$$

or

$$F_{SP} = 1 + \frac{0.34 \beta R}{\lambda_{tr}} \quad \text{if } R \ll \lambda_{tr}$$

where

R = foil radius.

λ_{tr} = transport mean free path in medium around foil.

β = probability that a neutron will be absorbed in one transversal through the foil (See Section 8.1.2).

$$L^2 = \frac{\lambda_{tr}\lambda_a}{3 \left(1 - \frac{2\lambda_{tr}}{5\lambda_a}\right)^2}$$

and

$\lambda_a = (N_a\sigma_a)^{-1}$, or absorption mean free path in medium around the foil.

σ_a = Maxwell averaged microscopic absorption cross section (corresponding for a $1/v$ absorber to the value at $E = 4/\pi kT$).

The magnitude of the calculated flux perturbation in graphite is given in Fig. 8-1, along with data reported by various experimenters.⁽⁶⁾

FIG. 8-1
MEASURED AND THEORETICAL FLUX DEPRESSION
BY INDIUM FOILS IN GRAPHITE MEDIUM
(REPRODUCED FROM REF. 6)

8.1.2 Self-Shielding for Foils

(For a discussion of other self-shielding effects see Section 5.2.)

In addition to depressing the external flux, the foil materials may attenuate flux quite abruptly internally, thus producing less activation density internally than at the surface. Consequently, the ratio of the gross activation of the foil to the quantity calculated by multiplying the macroscopic cross section for activation by the surface flux on the foil is always less than unity. This ratio is referred to as the self-shielding factor of the foil.

A quantity that enters into the evaluation of the self-shielding factor is the transmission probability, T , of a neutron traversing the foil without being absorbed. T is related to the blackness, β , by $T = 1 - \beta$. For the case of monoenergetic isotropic flux and a non-scattering foil,

$$T_{iso} = e^{-x} (1 - x) + x^2 [E_1(x)]$$

and for a collimated flux,

$$T_{coll} = e^{-x}$$

where

x = foil thickness measured in mean free paths.

$E_1(x)$ = the exponential integral

$$= \int_x^\infty \frac{e^{-t}}{t} dt$$

FIG. 8-2
TRANSMISSION PROBABILITY (T) VS FOIL THICKNESS (x)
(REPRODUCED FROM REF. 8)

FIG. 8-3
SELF-SHIELDING FACTOR (F) VS FOIL THICKNESS (x)
(REPRODUCED FROM REF. 8)

Plots of $T_{iso}(x)$ and $T_{coll}(x)$ as a function of foil thickness are shown in Fig. 8-2.(8) (See also Fig. 5-3.)

The self-shielding factor, F , for an isotropic flux, is given by:

$$F_{iso} = \frac{1 - T_{iso}}{2x}$$

and for a collimated flux by

$$F_{coll} = \frac{1 - T_{coll}}{x}$$

Plots of $F_{iso}(x)$ and $F_{coll}(x)$ as a function of foil thickness are shown in Fig. 8-3.(8) (See also Fig. 5-2.)

The case of sandwiched foils has also been investigated.(8) A foil of thickness $(y_2 - y_1)$ is considered to be sandwiched between two foils of equal thickness y_1 . All foils are considered to be of the same material, and have macroscopic absorption cross section Σ . The absorption rate per unit area, P , is then calculated in the separate foils, assuming that the outside faces of the outer foils are subjected to an isotropic flux whose incoming component is $\phi/4$. It is convenient to separate the resonance and non-resonance contributions. The non-resonance contri-

bution is then broken up energy-wise into sub-intervals in which an approximate analytic expression can usually be found for the cross section and the integration performed

analytically. The additional assumption is made that the flux, ϕ , is a slowly varying function of energy in the non-resonance sub-interval and may therefore be removed from the integral as an average flux, $\bar{\phi}_i$.

The absorption rate, P , for the case of $\Sigma y_i \ll 1$ is given for the outer and inner foils by:

$$P_{\text{outer}} = \frac{\bar{\phi}_{\text{reson}}}{4} \int_{E_{\text{reson}}} (1 - T_1 + T_2 - T_{1+2}) dE + y_1 \sum_i \bar{\phi}_i \int_{E_i} \Sigma(E) dE$$

and

$$P_{\text{inner}} = \frac{\bar{\phi}_{\text{reson}}}{2} \int_{E_{\text{reson}}} (T_1 - T_2) dE + (y_2 - y_1) \sum_i \bar{\phi}_i \int_{E_i} \Sigma(E) dE,$$

where

$$T_j = e^{-\Sigma y_j} (1 - \Sigma y_j) + (\Sigma y_j)^2 \{ E_1(\Sigma y_j) \}$$

8.1.3 Foil-Induced Resonance Flux Depression

FIG. 8-4
RESONANCE ACTIVATION LOSS DUE TO NEARBY FOIL

neutrons between foils. Approximate values of C for parallel discs of the same size, on the same axis, and black in the resonance region, are given in Fig. 8-4.*

*K. Plumlee, ANL, private communication.

Interaction between neighboring resonance absorbers is given by the development of Dancoff and Ginsburg.⁽⁹⁾ An individual foil is not greatly affected by its own depressing effect on the surrounding resonance energy flux since most resonances are narrow so that neutrons at exactly the resonance energy have only one free path before removal by slowing down or capture. It is further assumed that resonance energy neutrons are uniformly produced in the moderator and will surely be captured upon striking the foil. Then the resonance flux on one object will be multiplied by a factor $(1-C)$ for each additional resonance absorber installed in the system. C is given by

$$C = \frac{-1}{\pi \Sigma} \int_{\Sigma} d\Sigma \int_e \ell - S/\lambda \cos \alpha d\Omega$$

where Σ is the surface of one object; Ω is the solid angle subtended by the other on an increment $d\Sigma$ of the first; α is the angle between the line-of-sight distance S (within $d\Omega$) and the normal to $d\Sigma$; and λ is the mean-free-path length of resonance energy neu-

8.1.4 Foil Covers

Some typical foil covers employed and their functions are:

1. Cadmium covers to shield the foil from flux at energies below ~ 0.4 ev, to permit determination of thermal and "epi-cadmium" flux components. Two quantities are usually defined:

(a) The cadmium ratio, C. R.

$$C.R. = \frac{\text{uncovered activity}}{\text{covered activity}} = \frac{(\text{thermal} + \text{epithermal}) \text{ activity}}{\text{epithermal activity}}$$

(b) The sub-cadmium fraction, C. F.

$$C. F. = \frac{(\text{uncovered} - \text{covered}) \text{ activity}}{\text{uncovered activity}} = \frac{\text{thermal activity}}{(\text{thermal} + \text{epithermal}) \text{ activity}}$$

2. Boron covers to shield the foil from flux below a rather broad cutoff determined by the thickness of boron used.

3. Resonance absorbers to shield the foil from a particular energy band of the flux. Since resonance absorption by gold is a surface effect (primarily), a gold cover will shield enclosed gold foils from resonance activation. The reduction in activation effected by the cover may be used to measure the resonance flux.

The effective cutoff energy of a cadmium filter is defined to equal the cutoff energy of a step function filter permitting the same activation in a sandwiched absorber.

A step function filter is one whose transmission is defined as:

$$\text{transmission} = \begin{cases} 0, & 0 < E < E_{\text{cutoff}} \\ 1, & E_{\text{cutoff}} < E < \infty \end{cases}$$

The thermal activation of the absorber foil by neutrons which penetrate the cadmium shield, and the loss of activation due to epithermal neutrons absorbed in the cadmium, represent a measure of the deviation of the cadmium filter from a step function filter.

FIG. 8-5
ATTENUATION OF EPICADMIUM NEUTRONS
BY CADMIUM COVERING
(REPRODUCED FROM REF. 10)

Typical empirical data showing the activation of indium and gold foils under Cd covers of varying thickness are presented in Fig. 8-5.⁽¹⁰⁾ The indium activation varies slowly as the cadmium cover thickness is increased beyond 30 mils. It is assumed that this decrease in activation with increasing cover thickness represents the loss of epithermal neutrons. Extrapolation of the straight line curve above a cadmium cover thickness of 30 mils, back to a cover thickness of zero yields an intercept. This intercept is interpreted to be the epithermal activation observed when there is no loss of epithermal flux in the cadmium. The ratio

of the intercept to the actual value of the observed activation at varying cadmium thicknesses is the correction factor F_{Cd} which is plotted in Fig. 8-6 as a function of the indium foil thickness.⁽⁴⁾

Up to a cadmium cover thickness of 20 mils the indium foil activation decreases with a markedly steeper slope. It is assumed the rapid loss of activation with cadmium thickness is a measure of the effectiveness of the cadmium as absorber of thermal neutrons. The difference between the extrapolated straight line as discussed in the previous paragraph, and the observed curve below 20 mil-thickness is the activation from thermal neutrons. This is shown in Fig. 8-7.⁽¹¹⁾

Calculated values of the effective cutoff energy of a cadmium filter as a function of cadmium thickness for a $1/v$ -absorber sandwiched between cadmium layers are shown in Fig. 8-8 for various absorber thicknesses.⁽¹¹⁾

The amount of $1/v$ contribution to the resonance integral as a function of cadmium thickness has been calculated by Jervis,⁽¹²⁾ and is shown in Fig. 8-9. The cadmium thickness corresponding to a given cutoff energy is based on an independent calculation by Jervis.

In addition to cadmium, neutron shields have been designed to provide several different effective "cutoff" energies. Kanne, et al.⁽¹³⁾ used such shields in the measurement of capture to fission ratios in U^{235} and Pu^{239} . The characteristics of the shields are summarized in Table 8-1. The resulting spectrum of neutrons producing fissions in Pu^{239} (the cross section of Pu^{239} weighted by the transmission of the shield and the dE/E incident

FIG. 8-8
MULTIPLIER F_{Cd} TO CORRECT ACTIVATION
OF CADMIUM-COVERED INDIUM FOILS FOR
EPICADMIUM ABSORPTION BY CADMIUM COVER
(REPRODUCED FROM REF. 4)

FIG. 8-7
THERMAL NEUTRON ACTIVATION OF
CADMIUM-COVERED INDIUM FOILS
(REPRODUCED FROM REF. 10)

FIG. 8-8
EFFECTIVE CUTOFF ENERGY AS A FUNCTION OF (A) THICKNESS
OF CADMIUM-COVERED $1/v$ ABSORBERS AND (B) CADMIUM THICKNESS
(DATA FROM REF. 11)

FIG. 8-9
FACTOR TO OBTAIN $1/v$ COMPONENT OF RESONANCE INTEGRAL

flux), is shown in Fig. 8-10. The construction of the shields, the materials, and the obliquity correction factors are discussed by Kanne, et al. in Ref. 14.

Table 8-1

NEUTRON SHIELDS
(Reproduced from Ref. 13)

Material	Thickness gm/cm ²	Mean fission energy in Pu ²³⁹ , (ev) ($\phi = dE/E$)	Cutoff energy (ev) (33% transmission)
Cd (0.176 ev resonance)	5	30	5
In (1.44 ev resonance)	3		
Ir (0.635 and 1.35 ev resonances)	0.5		
Ordinary Boron	0.8	100	30
B ¹⁰ (enriched isotope used to reduce scattering and size)	0.5	1,200	200
	1.7	15,000	5000

FIG. 8-10
SPECTRUM OF NEUTRONS PRODUCING FISSIONS IN
Pu²³⁹ ENCLOSED IN VARIOUS SHIELDS (SEE TABLE 8-1)
(REPRODUCED FROM REF. 13)

Similar shields were used by Hummel, et al.,⁽¹⁵⁾ in the analysis of blanket transmission in a coupled slow-fast critical system. The calculated transmission differences for the foil covers are shown in Fig. 8-11 and the resulting product of difference of transmission and $\sigma_f^{U^{235}}$ is shown in Fig. 8-12.

8.1.5 Intercalibration of Foils

Although two foils of similar material but of different thickness may be intercalibrated in a specified flux, in general the intercalibration will not be valid in another spectrum. However a factor, R , can be defined which must be invariant if the intercalibration is to hold in a different spectrum. If the ratio of counting rates observed from two foils of thicknesses t and $t + \Delta t$ exposed to the same flux, Φ_i , for the same

time is C_i , then the ratio of C_i observed in a spectrum Φ_i to the ratio C_j observed in a flux spectrum Φ_j is:

$$\frac{C_i}{C_j} = \frac{1 + \frac{d}{dt} \left(\frac{\bar{\Phi}_i}{\Phi_i} \right) \left(\frac{\Delta t}{\bar{\Phi}_i / \Phi_i} \right)}{1 + \frac{d}{dt} \left(\frac{\bar{\Phi}_j}{\Phi_j} \right) \left(\frac{\Delta t}{\bar{\Phi}_j / \Phi_j} \right)}$$

FIG. 8-11
CALCULATED DIFFERENCE OF TRANSMISSION, ΔT , BETWEEN SHIELDS
CONTAINING 5 AND 10 mg/cm^2 OF B^{10} WITH AND WITHOUT Cd COVERS AND
BETWEEN SHIELDS CONTAINING 100 AND 200 mg/cm^2 OF B^{10}
(REPRODUCED FROM REF. 15)

FIG. 8-12
TRANSMISSION DIFFERENCE WEIGHTED FISSION CROSS SECTION
OF U^{235} FOR BORON FILTERS WITH Cd COVERS
(REPRODUCED FROM REF. 15)

where

$\bar{\Phi}$ = average flux in foil
 Φ = unperturbed flux

Provided

$$R \equiv \frac{(d/dt) \left(\frac{\bar{\Phi}}{\Phi} \right)}{\bar{\Phi}/\Phi}$$

does not change from spectrum Φ_i to spectrum Φ_j , the intercalibration of the foils remains valid. The factor R is shown as a function of indium thickness in Fig. 8-13. It is evident that for indium foil thicknesses greater than 150 mg/cm², R changes slowly from a spectrum characterized by a cadmium ratio of 1.9, to a completely thermalized spectrum.⁽¹⁶⁾

FIG. 8-13
 VARIATION OF R WITH CADMIUM RATIO
 (REPRODUCED FROM REF. 16)

8.2 CALIBRATION OF GAMMA, BETA, AND ALPHA DETECTORS

8.2.1 Gamma Counter Calibration Data

The activation of foils is measured conveniently by counting either the beta or the gamma emission. When it is preferable to observe the emitted beta-particles because of ease of shielding against background radiation, comparative measurements may be made easily in which the over-all counting geometry and the characteristics of activated foils and counting equipment are preserved. If an attempt is made to interpret the data in terms of the absolute activation of any one foil, a number of correction factors must be considered. These factors pertain to detector sensitivity, geometry, absorption and scattering suffered by the beta radiation both inside the activated foil and en route to the detecting equipment.

Corresponding correction factors pertaining to activation measurements through the counting of gamma radiation from foils refer mainly to the detector sensitivity to gamma-rays as a function of the material of the detector, its dimension, and the energy of the incident gamma-ray. The absorption and scattering of the gamma-ray as it penetrates the activated foil and travels into the detector are considerably less important than in the case of beta-particle counting. Since a most convenient and sensitive gamma detector is the thallium-activated NaI crystal, optically coupled to the front of a commercial photomultiplier tube, detection efficiencies are given for this type of crystal.

Figure 8-14 shows the linear absorption coefficient for NaI as a function of energy of incident gamma radiation. The assumed density of the NaI is⁽¹⁷⁾ 3.667 gm/cm³.

The photopeak efficiency, i.e., the ratio of the integrated photopeak (counts due to events which result in the gamma-ray giving up its full energy to the crystal) to the total gamma counts, is given in Fig. 8-15 for a NaI crystal, 1½-inch diameter by 1 inch high.⁽¹⁾

Figure 8-16 shows the calculated photopeak efficiency for a NaI crystal (1½-inch diameter by 1 inch high) as a function of energy and distance between the source and crystal, with the source centrally located below the crystal.⁽¹⁾

The results of recent computations,⁽¹⁸⁾ by the Monte Carlo method, of the total efficiencies (fraction of gamma rays incident on the crystal experiencing any interaction in the crystal), and of the photofractions (fraction of gamma-rays incident on the crystal which are totally absorbed in the crystal), for NaI crystals, are given in Tables 8-2 and 8-3.

FIG. 8-16
CALCULATED PHOTOPEAK EFFICIENCY AS A FUNCTION OF ENERGY
AND DISTANCE (L) FROM SOURCE TO NaI CRYSTAL
(CRYSTAL SIZE: 1-1/2 in. dia. x 1 in. high)
(REPRODUCED FROM REF. 21)

Table 8-2

TOTAL EFFICIENCIES OF NaI CRYSTALS FOR A BROAD PARALLEL BEAM

(Reproduced from the data of Miller, et al.,⁽¹⁸⁾)

Height of Crystal	1 in.		2 in.		4 in.		8 in.	
Energy, Mev	Monte Carlo ^a	Anal.	Monte Carlo ^a	Anal.	Monte Carlo ^a	Anal.	Monte Carlo ^a	Anal.
0.279	0.795 ± 0.004	0.798	0.957 ± 0.004	0.959	0.997 ± 0.003	0.998	1.000 ± 0.003	1.000
0.661	0.496 ± 0.005	0.498	0.749 ± 0.005	0.748	0.937 ± 0.004	0.936	0.995 ± 0.004	0.996
1.33	0.374 ± 0.004	0.370	0.613 ± 0.005	0.604	0.847 ± 0.005	0.843	0.976 ± 0.005	0.975
2.62	0.297 ± 0.004	0.298	0.509 ± 0.003	0.507	0.761 ± 0.004	0.757	0.942 ± 0.004	0.941
4.45	0.276 ± 0.001	0.278	0.484 ± 0.002	0.478	0.734 ± 0.001	0.728	0.931 ± 0.002	0.926

^aFor 5000 histories.

Table 8-3

PHOTOFRACTIONS FOR NaI CRYSTALS FOR A BROAD PARALLEL BEAM
AS CALCULATED BY THE MONTE CARLO METHOD^a

(Reproduced from Ref. 18)

Crystal Size		Energy, Mev				
Radius in.	Height, in.	0.279	0.661	1.33	2.62	4.45
1	1	0.816 ± 0.004	0.399 ± 0.007	0.233 ± 0.007	0.147 ± 0.006	0.103 ± 0.006
1	2	0.855 ± 0.003	0.481 ± 0.005	0.286 ± 0.006	0.199 ± 0.005	0.141 ± 0.005
1	4	0.869 ± 0.003	0.530 ± 0.005	0.334 ± 0.005	0.232 ± 0.005	0.172 ± 0.004
1	8	0.869 ± 0.003	0.548 ± 0.005	0.367 ± 0.005	0.258 ± 0.004	0.192 ± 0.004
2	1	0.845 ± 0.004	0.450 ± 0.007	0.286 ± 0.007	0.191 ± 0.007	0.139 ± 0.006
2	2	0.894 ± 0.003	0.576 ± 0.005	0.385 ± 0.006	0.282 ± 0.006	0.213 ± 0.006
2	4	0.913 ± 0.003	0.663 ± 0.005	0.477 ± 0.005	0.361 ± 0.005	0.295 ± 0.005
2	1	0.915 ± 0.003	0.701 ± 0.004	0.543 ± 0.005	0.419 ± 0.005	0.335 ± 0.005
4	1	0.864 ± 0.004	0.485 ± 0.007	0.335 ± 0.007	0.208 ± 0.007	0.184 ± 0.007
4	2	0.919 ± 0.003	0.631 ± 0.005	0.452 ± 0.006	0.346 ± 0.006	0.282 ± 0.006
4	4	0.943 ± 0.002	0.752 ± 0.004	0.592 ± 0.005	0.474 ± 0.005	0.409 ± 0.006
4	8	0.944 ± 0.002	0.811 ± 0.004	0.694 ± 0.005	0.582 ± 0.005	0.519 ± 0.005
8	1	0.869 ± 0.004	0.502 ± 0.007	0.353 ± 0.007	0.222 ± 0.007	0.176 ± 0.007
8	2	0.927 ± 0.003	0.664 ± 0.005	0.490 ± 0.006	0.383 ± 0.007	0.309 ± 0.006
8	4	0.953 ± 0.002	0.802 ± 0.004	0.653 ± 0.005	0.547 ± 0.005	0.479 ± 0.006
8	8	0.954 ± 0.002	0.878 ± 0.003	0.792 ± 0.004	0.693 ± 0.005	0.630 ± 0.005
12	1	0.872 ± 0.004	0.510 ± 0.007	0.358 ± 0.008	0.231 ± 0.007	0.198 ± 0.007
12	2	0.931 ± 0.003	0.674 ± 0.005	0.503 ± 0.006	0.401 ± 0.007	0.319 ± 0.006
12	4	0.958 ± 0.003	0.823 ± 0.004	0.679 ± 0.005	0.569 ± 0.005	0.502 ± 0.006
12	8	0.960 ± 0.002	0.902 ± 0.003	0.828 ± 0.004	0.733 ± 0.004	0.667 ± 0.005
16	1	0.872 ± 0.004	0.511 ± 0.007	0.363 ± 0.008	0.246 ± 0.008	0.202 ± 0.007
16	2	0.934 ± 0.002	0.679 ± 0.005	0.510 ± 0.006	0.407 ± 0.007	0.333 ± 0.007
16	4	0.960 ± 0.002	0.831 ± 0.005	0.692 ± 0.005	0.576 ± 0.005	0.518 ± 0.006
16	8	0.962 ± 0.002	0.912 ± 0.003	0.848 ± 0.004	0.752 ± 0.004	0.687 ± 0.005

^aFor 5000 histories⁽¹⁸⁾.

8.2.2 Absolute Beta-Counting of Indium Foils

It is frequently desirable to determine absolute neutron fluxes. One method is to determine the absolute activity of an indium foil exposed in the flux. Determination of absolute beta activities requires the knowledge of the effects due to the sample's self-absorption and self-scattering, absorption in the air and window of the detector, the geometry and efficiency of the detector, back-scattering of the sample support, and contributions from radiations other than betas resulting from the decay of the nucleus. These effects are discussed at length in Refs. 22 through 25, but since most are sensitive functions of the experimental situation; only those factors dependent upon the properties of the foils will be given here and they are: self-absorption, self-scattering and back-scattering.

The absolute beta activity of a foil may be related to the counting rate per milligram (specific activity) as follows:

$$A_{th}(x) = Q f_{bs} f_s f_{eff} f_{geom} f_w f_{\gamma}$$

where:

$A(x)$ = a computed counting rate per milligram at the end of an irradiation.

x = foil thickness in milligrams/cm².

Q = the absolute disintegration rate per milligram (arising from thermal neutrons only).

f_{bs} = back-scattering correction factor = ratio of the counting rate with back-scattering to that without back-scattering.

f_s = self-absorption and self-scattering correction factor = ratio of the counting rate of the sample to the counting rate of a sample of the same activity but of zero thickness.

f_{eff} = efficiency factor of the detector system = the ratio of the number of betas actually counted to the number incident on the counting volume.

f_{geom} = geometry factor = the ratio of the solid angle subtended by the detector window at the source to 4π .

f_w = air and window absorption correction factor.

f_γ = photon counting correction factor = the ratio of the observed counting rate due to photons plus betas to the counting rate due to betas alone.

The quantity $A_{th}(x)$ is obtained from the measured activity, $A(x)$, by applying the corrections for flux depression and self shielding discussed in Section 8.1, and is given by:

$$A_{th}(x) = A(x) \frac{F_{sp}(x)}{F(x)}$$

where: F_{sp} and F isotropic or collimated are defined in Sections 8.1.1 and 8.1.2 respectively.

Because of the short half life, 54.3 min, the observed count must be corrected in various ways in order to obtain $A(x)$. If M is the observed counting rate per milligram in a time interval, t_c , counted at a time, t_w , after the end of the irradiation, then $A(x)$ is given by:

$$A(x) = \frac{\lambda M e^{\lambda t_w}}{(1 - e^{-\lambda t_c})}$$

where: λ = decay constant of indium.

The above expressions may be used to solve for the self-absorption and self-scattering correction factors

$$f_s = \frac{1}{Q f_{geom}} \frac{1}{f_{eff} f_w} \frac{A(x) \frac{F_{sp}(x)}{F(x)}}{f_\gamma f_{bs}}$$

For a windowless 2π counter f_{eff} and f_w are unity. It was assumed that for small increments of thickness the backscattering was proportional to the thickness and for each thickness of In foil and mounting material used, a count was taken with and without an added increment of mounting material. f_{bs} , the backscatter factor, is the ratio of the count with added mounting material to that without added material.

The self-absorption correction factor for the case in which no backscattering material additional to the supporting film was used is given in Fig. 8-17 and the factor ($f_s f_{bs} f_\gamma$) for the thick silver backing is given in Fig. 8-18. These factors were obtained with a 2π windowless counter. The same factors for an end-window GM counter are given in Figs. 8-19 and 8-20 respectively.

The estimated errors in the data from which the curves are taken are:

- a) Foil weights: $\leq 2\%$
 - b) Counting error: $\leq 3\%$
 - c) Errors in estimating counts due to back-scattering of supporting films, γ -rays, and neutron self-shielding: $< 3\%$
 - d) Error in Cd ratio: $\leq 1.5\%$
- Estimated total error (r.m.s. value of above errors): $\cong 5\%$

FIG. 8-17
SELF-ABSORPTION CORRECTION FACTOR (f_s) FOR BACKLESS
INDIUM FOILS IN A 2π WINDOWLESS COUNTER
(REPRODUCED FROM REF. 22)

FIG. 8-18
SELF-ABSORPTION, BACK SCATTERING, AND PHOTON CONTRIBUTION
CORRECTION FACTOR ($f_s f_{bs} f_\gamma$) FOR INDIUM FOILS COUNTED ON
THICK SILVER BACKING IN A 2π WINDOWLESS COUNTER
(REPRODUCED FROM REF. 22)

FIG. 8-19
SELF-ABSORPTION CORRECTION FACTOR (f_s) FOR INDIUM
FOILS COUNTED WITH AN END WINDOW GM COUNTER AT 2.2 cm
(REPRODUCED FROM REF. 22)

FIG. 8-20
SELF-ABSORPTION, BACK SCATTERING, AND PHOTON CONTRIBUTION
CORRECTION FACTOR ($f_s f_{bs} f_\gamma$) FOR INDIUM FOILS COUNTED
WITH AN END-WINDOW GM COUNTER AT 2.2 cm
(REPRODUCED FROM REF. 22)

8.2.3 Alpha-Particle Counting

The corrections to α -counting techniques are very similar to those applied to β -counting techniques. Because the range of the α -particle is so much shorter than that of β -particles, more care must be taken to achieve the same degree of accuracy. The corrections considered here are the same kind as those considered for β -counting: backscattering and self-absorption in the samples.

The nature of the scattering of α -particles is such that these losses will be negligible in "low geometry" counters, i.e., counters whose apertures are approximately 1% of the total solid angle.

Backscattering - The backscattering of α -particles from various mounting materials was considered by Crawford.⁽²⁶⁾ The sample is considered to be spread evenly in a thin layer on a backing material. The results, given in terms of the ratio of the backscattering in selected materials to that of gold taken as a standard, are given in Table 8-4.

Table 8-4

**BACKSCATTERING OF Pu^{239}
ALPHA PARTICLES**
(Reproduced from Ref. 26)
(Range in air = 3.68 cm)

Sample Mount Material	Backscattering
Au	1.00
Pt	0.99
U_3O_8	.894
SiO_2 quartz	.19
Al	.23
Be	.10

The absolute value of the ratio of the backscattering in gold (ratio of counts observed to true counts without backscattering) is found to be 1.032. This value is in close agreement with the value of approximately 1.04 for Pt obtained by Cunningham, et al.⁽²⁷⁾ The relative values for U_3O_8 and quartz (Table 8-4) were also found to agree with experiment.⁽²⁷⁾

The correction of these quantities for different ranges of α -particles is accomplished by the quantity:

$$\frac{\Phi}{\Phi_\mu} = \frac{S_\mu}{S} \frac{\rho}{\rho_\mu} = \frac{\text{backscatter correction for the sample}}{\text{backscatter correction for standard}}$$

where:

$$\rho = \left(\frac{S}{S_1}\right)^2 \frac{\int_0^{S_1} \left(\frac{S_1 - t}{S - t}\right)^2 dt}{S} = 2 \left(\frac{S}{S_1}\right)^2 \left\{ \left(1 - \frac{S_1}{S}\right) \ln \left(1 - \frac{S_1}{S}\right) + \frac{2S_1}{S} - \left(\frac{S_1}{S}\right)^2 \right\}$$

ρ_μ is identical to ρ except that S and S_1 are replaced by S_μ and $S_{1\mu}$, respectively.

S and S_μ = ranges in air of the α -particles from the sample and standard respectively.

S_1 and $S_{1\mu}$ = the maximum distance in the backing material that the α -particles from the sample and standard, respectively, can travel in the foil and still produce a detectable count (usually ~90% of S in a 2π windowless counter).

Self-Absorption - It has been calculated⁽¹⁾ that the fraction of α -counts lost in a 2π counter due to absorption within a smoothly spaced sample is:

$$\text{Fraction lost} = \frac{1}{2} \frac{t}{R}$$

where:

t = sample thickness

R = effective range of the α -particles in the source material (maximum distance the α -particle can travel and still produce a detectable event).

8.3 NEUTRON SOURCES

8.3.1 Spectra of (α, n) Sources

Hess,⁽²⁸⁾ in a recent survey of the neutron spectra measurements and calculations, as well as neutron yields per alpha particle, has summarized the important information on neutron sources available in the first half of 1957.

FIG. 8-21.
NEUTRON SPECTRUM OF A Po- α -B SOURCE
(REPRODUCED FROM REF. 28)

The spectra for various neutron sources are given in Figs. 8-21 to 8-26.⁽²⁸⁾ In addition, Fig. 8-27 shows the results of Canadian work on Ac-Be sources,⁽²⁹⁾ while Fig. 8-28 gives a redetermination of the Po-Be spectrum.⁽³⁰⁾

The spectrum calculation⁽²⁸⁾ in Fig. 8-21 was performed for three conditions of isotropy of the neutron emission, with respect to the incoming alpha particle, in the center-of-mass system; corresponding to isotropic, $(\cos)^2$, and $(\sin)^2$ distributions. The effect is seen to be small.

The measured spectra shown were determined mainly from photographic plate scanning. The more recent measurements have been made with direct counting equipment, using recoil or scintillation spectrometers.

FIG. 8-22
NEUTRON SPECTRUM OF A Po- α -BF₃ SOURCE
(REPRODUCED FROM REF. 28)

FIG. 8-23
NEUTRON SPECTRUM OF A Po- α -Li SOURCE
(REPRODUCED FROM REF. 28)

FIG. 8-24
NEUTRON SPECTRUM OF A Po- α -Be SOURCE
(REPRODUCED FROM REF. 37)

FIG. 8-25
NEUTRON SPECTRUM OF A Pu- α -Be SOURCE
(REPRODUCED FROM REF. 37)

FIG. 8-26
NEUTRON SPECTRUM OF A Ra- α -Be SOURCE
(REPRODUCED FROM REF. 37)

FIG. 8-27
NEUTRON SPECTRUM OF AN Ac²²⁷- α -Be SOURCE
(REPRODUCED FROM REF. 29)

FIG. 8-28
NEUTRON SPECTRUM OF A Po-Be SOURCE
(REPRODUCED FROM REF. 30)

8.3.2 Yields of (α, n) Sources

The experimental, and calculated yields for neutron sources are given in Table 8-5.(28)

Table 8-5

YIELDS OF (α, n) NEUTRONS SOURCES

Source	Measured Yield, neut/10 ⁶ alphas	Reported by	Calculated Yield, neut/10 ⁶ alphas	Reported by
Po-Li	2.4 4.7	Roberts(42) Segre and Wiegand (1944)(43)	2.9 2.6	Roberts(42) Hess (1957)(28)
Po-Be	72 73	Roberts(42) Segre and Wiegand (1944)(43)	80 58 67	Roberts(42) Hess (1957)(28) Runnalls (1956)(44)
Po-B	21 19	Roberts(42) Segre and Wiegand (1944)(43)	24 24	Roberts(42) Hess (1957)(28)
Po-BF ₃	13.5	Richards (1944)(34)	15.4	Hess (1957)(28)
Po-F	10.4	Roberts(42)	12	Roberts(42)
Pu-Be	42 60 $\pm$ 2	Stewart (1955)(40) Runnalls (1956)(44)	35	Hess (1957)(28)
Ra-Be	1.4 x 10 ⁷ neut/sec per gm Ra (new source) (For source contain- ing 4.5 gm Be and 1 gm Ra)	Anderson (1948)(45)	1.35 x 10 ⁷ neut/sec per gm Ra (new Source) 1.56 x 10 ⁷ neut/sec per gm Ra (old source)	Hess (1957)(28) Hess (1957)(28)
Am ²⁴¹ -Be	74 $\pm$ 2	Runnalls (1956)(44)		
Cm ²⁴² -Be	112 $\pm$ 3	Runnalls (1956)(44)		
Ac-Be			709	Runnalls (1956)(44)

8.3.3 Photo-Neutron Sources

A useful source of neutrons for calibration and other purposes is the photo-disintegration of the deuteron and the beryllium nucleus. An extensive survey of sources of photo-neutrons is given by Wattenberg.(46) The necessary information for assembling such a source is summarized in Table 8-6.

Because the reactions are endothermic they give rise to monoenergetic neutrons when the photons are monoenergetic. From the conservation of energy and momentum one can derive the neutron energy in terms of the photon energy as:

$$E_n = \frac{A-1}{A} \left[E_\gamma - Q - \frac{E_\gamma^2}{1862(A-1)} \right] + \delta$$

where E_n = neutron energy in Mev
 E_γ = photon energy in Mev
 A = mass number of the target nucleus
 Q = threshold energy of the reaction

Table 8-6

PHOTO-NEUTRON SOURCES

(From Data of Ref. 46.)

Data for Gamma Emitter Target	Half Life	E_{γ}^* , Mev	E_n , Mev	Yield** (10^4 n/sec-curie)
F ²⁰ + Be	12 s	2.2	.50	
Na ²⁴ + Be	14.8 h	2.76	1.0	14
Na ²⁴ + D			.29	27
Al ²⁸ + Be	2.4 m	1.8	.15	
Cl ³⁸ + Be	37 m	2.15	.46	
Sc ⁴⁴ + Be	4.1 h	1.8	.15	
Mn ⁵⁶ + Be	2.59 h	(1.8, 2.13, 2.7)	(.15, .45, .95)	2.9
Mn ⁵⁶ + D			.26	.31
Ga ⁷² + Be	14.1 h	1.87, 2.21, 2.51	.78	4.8
Ga ⁷² + D			.16	5.8
As ⁷⁶ + Be	26.8 h	2.2, 1.8	.28, 1.44	
As ⁷⁶ + D			.50	
Y ⁸⁸ + Be	105 d	1.9, 2.8	.24, 1.04	10
Y ⁸⁸ + D			.31	0.3
In ¹¹⁶ + Be	54 m	1.8, 2.1	.15, .46	0.82
Sb ¹²⁴ + Be	60 d	1.7 (1.67)	.06 (.035)	19
La ¹⁴⁰ + Be	40 h	2.50	.76	.28
La ¹⁴⁰ + D			.16	.83
Ra ²²⁸ + Be	6.7 y	1.8, 2.2, 2.62	.15, .88	3.5
Ra ²²⁸ + D			.22	9.5

*Only those γ s whose energy exceeds the threshold of the (γ -n) reaction are listed.

**When the target material is thin, the yield is for 1 gram of material at 1 cm distance. Mean values of data are presented.

δ = a small spread in energy which depends upon the angle between the incident photon and the emitted neutron and is given by:

$$\delta_{\max} \cong E_{\gamma} \left[\frac{2(A-1)(E_{\gamma} - Q)}{931 A^3} \right]^{1/2}$$

The corrections for thick sources may be computed from the γ -attenuation tables in Section 7 or from a summary of such linear attenuation coefficients given by Davison and Evans.⁽⁴⁷⁾ For non-slab geometries the corrections for absorption can be computed from the tables given by Case, et al.⁽⁴⁸⁾

8.4 CONSTANTS RELATED TO REACTOR KINETICS EXPERIMENTS8.4.1 Deviation of Observed from Asymptotic Period*

The worth of a step change in reactivity in a reactor system may be obtained by observation of the kinetics behavior of the system. This is usually done by use of the inhour equation which relates the reactivity and the asymptotic period.

The introduction of a step change in reactivity into a critical reactor results in an observed rate of reactor power change characteristic of the amount and sign of the reactivity step, and of the general kinetic characteristics of the reactor. A premature observation of the change in reactor power with time may show a period which is in considerable error owing to the presence of transient terms. The waiting time following a step change in reactivity to observe a period within 1%, 5%, or 10% of the asymptotic value is shown in Figs. 8-29 and 8-30 for a reactor without a source. The delayed neutron data used are those of Keepin given in Section 1.5.1.

The error in reactivity as determined from the inhour equation associated with an error in asymptotic period, is shown in Fig. 8-31.

FIG. 8-29
TIME AFTER POSITIVE STEP CHANGE IN REACTIVITY TO OBSERVE
A PERIOD WITHIN 1%, 5%, AND 10% OF THE ASYMPTOTIC VALUE
AS A FUNCTION OF POSITIVE ASYMPTOTIC PERIOD.

*B. J. Toppel, submitted for publication to Nuclear Science and Engineering.

FIG. 8-30
TIME AFTER NEGATIVE STEP CHANGE IN REACTIVITY TO OBSERVE
A PERIOD WITHIN 1%, 5%, AND 10% OF THE ASYMPTOTIC VALUE
AS A FUNCTION OF NEGATIVE ASYMPTOTIC PERIOD

FIG. 8-31
RELATIVE ERROR IN REACTIVITY AND ASYMPTOTIC PERIOD AS A
FUNCTION OF ASYMPTOTIC PERIOD FOR BOTH POSITIVE AND NEGATIVE PERIODS

8.4.2 Period Measurements with Source in Reactor*

The reactor kinetics equations(49) are:

$$\frac{dn}{dt} = \left[k(1 - \beta) - 1 \right] \frac{n}{\ell} + \sum_i \lambda_i C_i + S$$

$$\frac{dC_i}{dt} = \frac{k\beta_i n}{\ell} - \lambda_i C_i$$

where S is the source. The other symbols are defined in Section 1.5.3. The complete solution for the experimentally interesting case of $k \neq 1$ is:

$$\frac{n(t)}{n_0} = \sum_{i=1}^{N+1} (A_j + B_j) e^{s_j t} + C$$

where the s_j are the $N+1$ roots of the inhour equation

$$\rho = \frac{k-1}{k} = \frac{\ell s + s \sum_{i=1}^N \frac{\beta_i}{s + \lambda_i}}{1 + \ell s}$$

and

$$A_j = (1 - \rho) \frac{\ell + k_{\text{eff}_0} \sum_{i=1}^N \frac{\beta_i}{s_j + \lambda_i}}{\ell(1 - \rho) + \sum_{i=1}^N \frac{\beta_i \lambda_i}{(s_j + \lambda_i)^2}}$$

$$B_j = (1 - \rho) \frac{1 - k_{\text{eff}_0}}{s_j \left[\ell(1 - \rho) + \sum_{i=1}^N \frac{\beta_i \lambda_i}{(s_j + \lambda_i)^2} \right]}$$

$$C = \frac{\rho - 1}{\rho} (1 - k_{\text{eff}_0})$$

The subscript 0 refers to initial values which obtain until $t = 0$ at which time the effective multiplication constant is changed abruptly from k_{eff_0} to the final k . Owing to the normalization used, the results do not depend explicitly upon S but rather upon k_{eff_0} .

The deviation from the critical state of an assembly is reflected sensitively in the accuracy of measurements of periods subsequent to the introduction of the step change in reactivity. If the initial value of k has actually been unity, i.e., if there was no detectable effect from built-in neutron sources, then the only inaccuracy in a period measurement would be occasioned by a premature observation of the period. If a source is present, the error in period is considerably more severe, even after a long time has elapsed.

*B. J. Toppel, Submitted for publication to Nuclear Science and Engineering.

The influence of the extraneous source is illustrated for various degrees of initial subcriticality in Figs. 8-32 to 8-36.

DEVIATION OF OBSERVED PERIOD FROM ASYMPTOTIC PERIOD AS A FUNCTION OF TIME AFTER POSITIVE STEP CHANGE IN REACTIVITY TO $\rho = 5.0 \times 10^{-4}$ FOR VARIOUS INITIAL VALUES OF k

WAITING TIME AFTER POSITIVE STEP CHANGE IN REACTIVITY TO OBSERVE A PERIOD WITHIN 10% OF THE ASYMPTOTIC VALUE AS A FUNCTION OF POSITIVE ASYMPTOTIC PERIOD FOR VARIOUS INITIAL VALUES OF k

FIG. 8-34
WAITING TIME AFTER POSITIVE STEP CHANGE IN REACTIVITY TO
OBSERVE A PERIOD WITHIN 5% OF THE ASYMPTOTIC VALUE AS A FUNCTION
OF POSITIVE ASYMPTOTIC PERIOD FOR VARIOUS INITIAL VALUES OF k

FIG. 8-35
WAITING TIME AFTER POSITIVE STEP CHANGE IN REACTIVITY TO
OBSERVE A PERIOD WITHIN 1% OF THE ASYMPTOTIC VALUE AS A FUNCTION
OF POSITIVE ASYMPTOTIC PERIOD FOR VARIOUS INITIAL VALUES OF k

FIG. 8-38
DEVIATION OF OBSERVED PERIOD FROM THE RECIPROCAL OF THE ALGEBRAICALLY
LARGEST ROOT OF THE INHOUR EQUATION AS A FUNCTION OF TIME AFTER
A STEP CHANGE IN REACTIVITY TO $\rho = -1.0 \times 10^{-3}$ FOR VARIOUS INITIAL VALUES OF k

The effect of varying the prompt neutron lifetime is illustrated in Table 8-7 where the deviation of the observed period from the asymptotic period is given at various times after a reactivity step.

Table 8-7

INFLUENCE OF VARYING PROMPT NEUTRON LIFETIME^a

ρ	l , sec	Deviation %	t , sec
1×10^{-4}	1×10^{-3}	1.167	3000
"	4×10^{-5}	1.113	"
"	1×10^{-7}	1.111	"
"	1×10^{-3}	43.510	100
"	4×10^{-5}	43.399	"
"	1×10^{-7}	43.394	"
1×10^{-3}	1×10^{-3}	1.222	"
"	4×10^{-5}	1.149	"
"	1×10^{-7}	1.146	"
"	1×10^{-3}	43.882	4
"	4×10^{-5}	42.886	"
"	1×10^{-7}	42.844	"

$$^a k_{\text{eff}_0} = 0.9999$$

The effect of varying the delayed neutron parameters is illustrated in Table 8-9. In this calculation, use was made of the data of Keepin, et al. (see Section 1.5.1), and the early data of Hughes,⁽⁵⁰⁾ and a set of parameters (Table 8-8) representative of a D_2O -moderated reactor in which account is taken of the delayed photo-neutrons.

Table 8-8

DELAYED NEUTRON PARAMETERS

i	$\lambda_i, \text{sec}^{-1}$	β_i	i	$\lambda_i, \text{sec}^{-1}$	β_i
1	6.26×10^{-7}	2.90×10^{-7}	8	1.24×10^{-2}	2.30×10^{-4}
2	3.63×10^{-6}	7.10×10^{-7}	9	1.69×10^{-2}	1.42×10^{-4}
3	4.37×10^{-5}	2.24×10^{-6}	10	3.05×10^{-2}	1.53×10^{-3}
4	1.17×10^{-4}	1.61×10^{-5}	11	1.11×10^{-1}	1.37×10^{-3}
5	4.28×10^{-4}	1.43×10^{-5}	12	2.77×10^{-1}	4.50×10^{-4}
6	1.50×10^{-3}	2.33×10^{-5}	13	3.01×10^{-1}	2.75×10^{-3}
7	4.81×10^{-3}	4.83×10^{-5}	14	1.14×10^0	8.00×10^{-4}
			15	3.01×10^0	2.90×10^{-4}

Table 8-9

INFLUENCE OF DELAYED NEUTRON PARAMETERS*

Asymptotic Period, sec	$k_{\text{eff}0}$	Delayed** Parameters	Waiting Time in Seconds to Achieve Accuracy Indicated		
			1%	5%	10%
8289	1.0	a	152.9	58.0	36.5
"	"	b	15390.	6980.	5160.
"	"	c	156.5	57.1	35.8
1075	"	a	141.7	55.2	35.0
"	"	b	2525.	1164.	640.
"	"	c	145.2	54.3	34.6
134	"	a	88.4	41.8	27.8
"	"	b	237.	83.6	44.9
"	"	c	89.4	41.0	27.9
10.5	"	a	22.3	11.68	7.44
"	"	b	23.8	12.59	8.08
"	"	c	22.7	11.90	7.72
8289	0.9999	a	37370.	24000.	18245.
"	"	b	34090.	21000.	15440.
"	"	c	37250.	23900.	18175.
1075	"	a	4309.	2580.	1837.
"	"	b	3990.	2332.	1630.
"	"	c	4264.	2531.	1785.
134	"	a	350.7	142.7	67.5
"	"	b	357.0	157.8	83.1
"	"	c	338.8	134.3	60.9
10.5	"	a	23.7	12.36	7.91
"	"	b	24.9	13.13	8.43
"	"	c	24.0	12.60	8.13

* $\ell = 4.0 \times 10^{-5}$ sec. for all cases**^adata from Sec. 1.5.1^bdata from Table (8-8)^cdata of Hughes, et al.⁽⁵⁰⁾

8.4.3 Effect of Source on Reactor Transfer Function*

The effect of various degrees of subcriticality in a reactor with a source upon the reactor transfer function is plotted in Figs. 8-37 to 8-40. The delayed neutron data used are those of Keepin given in Section 1.5.1. Figures 8-37 to 8-40 are from the data of Toppel.*

FIG. 8-37
MAGNITUDE (M) OF TRANSFER FUNCTION VS FREQUENCY (ω) FOR
VARIOUS DEGREES OF SUBCRITICALITY IN REACTOR WITH SOURCE

FIG. 8-38
PHASE (ϕ) OF THE TRANSFER FUNCTION VS FREQUENCY (ω) FOR
VARIOUS DEGREES OF SUBCRITICALITY IN REACTOR WITH SOURCE

*B. J. Toppel, private communication.

FIG. 8-39
MAGNITUDE OF TRANSFER FUNCTION (M) IN
REACTOR WITH SOURCE AS A FUNCTION
OF $1-k$ FOR $\omega = 1$ RADIAN/SECOND

FIG. 8-40
PHASE OF THE TRANSFER FUNCTION (ϕ)
VS $1-k$ FOR $\omega = 1$ RADIAN/SECOND

8.5 STATISTICS AND ERRORS IN DATA REPORTING(51)

The interpretation of data may depend upon the accuracy of counting data and the criterion for judging accuracy may be one of several kinds of error. When a large number of readings of a single quantity have been taken, the average value, $\bar{n}$, of the readings is found and the root-mean-square, "standard" deviation, σ , is determined. The probability, $P(|\bar{n} - n| > k\sigma)$, that a single reading, n , deviates from the mean, $\bar{n}$, by more than a fractional amount, k , of σ is called by various names depending upon the size of k . Table 8-10 summarizes these names and the associated values of k and the probability, $P(k\sigma)$.

Table 8-10

PROBABILITY OF EXCEEDING A GIVEN DEVIATION

Terminology	-	(Data from Ref. 51)			
		Probable error	Standard error	Nine-tenths error	Ninety-Five Hundredth error
$k\sigma$	0	0.6745σ	1.000σ	1.6449σ	1.9600σ
$P(\bar{n} - n > k\sigma)$	1.000	0.5000	0.3173	0.1000	0.0500

$\bar{n}$ = average of a number of observations

n = a single observation

σ = root-mean-square of "standard" deviation $[= (\bar{n} e^{-\lambda t})^{\frac{1}{2}}]$
for a radioactive species with decay constant, λ , and
counted for a time, t . When $\lambda t \ll 1$, $\sigma \approx (\bar{n})^{\frac{1}{2}}]$

$P(|\bar{n} - n| > k\sigma)$ = the probability of obtaining a deviation $|\bar{n} - n|$,
greater than $k\sigma$.

The most common application of these errors is in the counting of radioactive foils which have a decay constant λ . Then applying the binomial distribution to probabilities determined by the radioactive decay law one shows that the true average number decaying in time t is:

$$\bar{n} = N_0(1 - e^{-\lambda t}) \quad \text{where } N_0 = \text{number of parent atoms}$$

and the standard deviation, σ , is:

$$\sigma = (\bar{n} e^{-\lambda t})^{\frac{1}{2}}$$

Thus, when

$$\lambda t \ll 1$$

$$\sigma = (\bar{n})^{\frac{1}{2}}$$

These results are directly applicable when a large number of intervals have been separately counted. When only one count is taken (and assuming $\lambda t \ll 1$) the true average count ($\bar{n}$) is not known but it is assumed that n and $\bar{n}$ are not widely different. In these circumstances σ^2 is identified with the count, n , observed and

$$n \approx \bar{n} = \sigma^2$$

Use can now be made of Table 8-10 to express the precision of the result; the usual scheme is to report the quantity $(n \pm n^{1/2})$ and referring to the table this would mean that the probability that $|n - \bar{n}| > n^{1/2}$ is 0.3173.

Any desired accuracy is then achieved by making n suitably large. Table 8-11 gives the total counts necessary for a given size error.

Table 8-11
TOTAL COUNTS REQUIRED
FOR A GIVEN ERROR

(Reproduced from Ref. 51)

% Error	Total Counts Required		
	Standard Error	Probable Error	Nine-Tenths Error
0.1	10^6	4.5×10^5	2.7×10^6
0.3	1.1×10^5	5.1×10^4	3×10^5
1.0	10^4	4.5×10^3	2.7×10^4
3.0	1.1×10^3	506	3×10^3
10	10^2	45	271

A counting rate is determined by $n = (r/t)$ and the standard deviation in the counting rate, σ_r , is given by:

$$\sigma_r = \frac{n^{1/2}}{t} = \left(\frac{r}{t}\right)^{1/2}$$

In making observations with counters or other pulse recording instruments having a "dead time" or a time after each event in which the system is inoperative, a correction to the observed count is necessary to find the true counting rate. If R is the true counting rate, R' the observed counting rate, and t is the "dead time" then the true counting rate is given by:

$$R = \frac{R'}{1 - R' t}$$

and the system is inoperative for a fractional time, $R't$ during the counting period so that $RR't$ counts were lost.

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SECTION 9

PROPERTIES OF ELEMENTS AND REACTOR MATERIALS

9.1 NUCLEAR CONSTANTS

The text, tabulated data (Table 9-1), and references in this section are reproduced in their entirety by special permission of NUCLEONICS.⁽¹⁾

"The widespread acceptance of an earlier thermal-neutron-data by Kroeger (Ref. 1, Table 9-1) has stimulated this updated compilation. Intended primarily for calculations related to reactor studies, it should also prove useful in neutron-source and shielding studies.

"The material, for the most part, is found in the references or calculated using standard techniques, such as are presented by Glasstone and Edlund (Ref. 2, Table 9-1). The cross sections are the 2,200-m/sec cross sections. Non-1/v cross sections are corrected to yield a result equivalent to a 1/v cross section in a Maxwellian thermal-neutron flux.

"The average value for the cosine of the scattering angle, $\bar{\mu}_0$, is calculated from $\bar{\mu}_0 = 2/3A$, where A is the mass of the scattering nucleus.

"The average logarithmic energy decrement, ξ , is found from the approximation $\xi = 2/(A + 2/3)$ for the elements with mass larger than 10. For the lighter elements,

$$\xi = 1 + \left[(A-1)^2 / 2A \right] \left[\ln \left\{ (A-1) / (A+1) \right\} \right]$$

"For the fissionable elements, for which the moderation area has little meaning, that column contains the microscopic fission cross section.

"The diffusion coefficient is

$$D = L^2 \Sigma_a \text{ or } D = \lambda_s / 3(1 - \bar{\mu}_0),$$

where

$$\lambda_s / (1 - \mu_0)$$

is the transport mean free path. The transport cross sections and mean free path were deleted to allow space for the resonance integral, the moderation area and the diffusion coefficient.

"The cross sections given in the table are the absorption (a), scattering (s) and total (t) cross sections."

TABLE 9-1

THERMAL-NEUTRON DATA FOR THE ELEMENTS

Atomic no.	Element or compound	Atomic wt.	Density (g/cm ³)	Nuclear spin (h ² /4π ²)	Microscopic cross section (barns)			Macroscopic cross section (cm ⁻¹)			Mean free path (cm)			Resonance integral (barns)	Diffusion coefficient (cm ² /sec)	Diffusion length (cm)	Atomic no.		
					1-μ ₀	ε	σ _a	Σ _a	Σ _s	Σ _t	λ _a	λ _s	λ _t						
1	H	1.008	0.83	5.33	0.3386	1.000	0.33	38	38	1.7	0.002	0.002	0.6	525	521	1.610	518	1	
2	D	2.014	0.00335	0.00335	0.676	0.948	0.66	103	103	0.032	3.45	3.45	0.5	0.290	0.290	2.731	0.164	2	
3	He	4.003	0.00178	2.63	0.0033	0.884	0.570	0.007	0.8	0.007	0.003	0.449	0.449	0.43	0.293	0.223	116	0.62	3
4	Li	6.940	0.534	0.0031	0.0031	0.854	0.425	0.007	0.8	0.007	0.003	2.13	2.13	50	4.70	4.70	2.8	0.16	4
5	Be	9.012	1.85	0.0126	0.0126	0.929	0.709	0.010	7.0	7.0	0.053	0.865	0.865	0.304	1.54	0.299	28	2.605	5
6	B	10.81	2.025	0.0178	0.0178	0.939	0.713	0.010	6.8	6.8	0.051	0.501	0.501	1.370	2.00	2.00	32	0.75	6
7	C	12.011	2.26	0.0234	0.0234	0.944	0.718	0.003	4.8	4.8	0.385	0.385	3.845	2.60	2.60	1.83	0.112	1.78	7
8	O	16.000	0.0013	5.33	0.9524	1.136	1.88	10	11.9	9.9	60	60	0.1	2.000	1.667	2.710	661	7	8
9	F	19.000	0.0014	5.33	0.9583	1.120	20	4.2	4.2	0.002	21	21	—	4.760	4.760	4.1	166	8	
10	Ne	20.183	0.0009	2.63	0.9649	1.021	1	7.4	7.4	0.016	20	20	—	5.000	5.000	1.9	179	9	10
11	Na	22.991	0.971	0.0054	0.9710	0.6845	0.515	4.0	4.0	0.012	0.015	0.015	0.3	9.1	9.1	0.27	15.8	11	11
12	Mg	24.32	1.74	0.041	0.9722	0.0811	0.063	3.6	3.6	0.003	0.155	0.158	368	6.45	6.45	0.9	28.5	12	12
13	Al	26.98	2.699	0.0602	0.9754	0.0723	0.230	1.4	1.63	0.004	0.084	0.086	72.5	11.9	10.2	0.18	20.0	5.52	13
14	Si	28.09	2.42	0.0522	0.9762	0.0688	0.13	1.7	1.83	0.007	0.099	0.096	147	11.3	10.6	0.5	23.7	3.81	14
15	P	30.975	1.82	0.0354	0.9785	0.0632	0.20	5	5.20	0.007	0.177	0.184	141	8.55	5.43	2	16.4	1.90	15
16	S	32.064	2.07	0.0389	0.9792	0.0612	0.49	1.1	1.59	0.019	0.043	0.062	524	23.3	16.2	0.6	19.5	1.72	16
17	Cl	35.457	0.0032	5.33	0.9810	0.0561	32.6	16	48.6	0.002	0.80	0.002	625	1.248	1.248	12	429	294	17
18	Ar	39.944	0.0018	2.63	0.9833	0.052	1.8	21.2	0.652	0.39	5.3	0.5	0.2	9.8	9.8	0.2	13.1	18	18
19	K	39.100	0.87	0.0134	0.9829	0.0604	19.7	1.5	34.7	0.026	0.020	0.047	37.9	4.8	21.5	1.1	223	131	19
20	Ca	40.08	1.55	0.0233	0.9833	0.0492	0.43	3.0	34.3	0.010	0.070	0.080	100	14.3	12.5	2	21.7	4.70	20
21	Sc	44.96	2.5	0.0235	0.9852	0.0384	23.5	24	17.5	0.787	0.804	15.9	1.29	1.24	0.628	10.7	0.666	0.349	21
22	Ti	47.88	4.5	0.0566	0.9861	0.0411	5.8	4	9.8	0.328	0.226	0.555	3.05	4.43	1.80	3.0	1.86	1.73	22
23	V	50.95	5.96	0.0704	0.9869	0.0387	5.1	5	10.1	0.359	0.252	0.711	2.79	2.84	1.41	2.21	1.48	0.808	23
24	Cr	52.01	7.1	0.0822	0.9872	0.0385	2.9	3.0	5.9	0.238	0.247	0.485	4.20	4.06	2.06	1.9	2.18	1.13	24
25	Mn	54.94	7.2	0.0789	0.9878	0.0359	13.2	2.3	15.5	0.104	0.181	0.22	0.962	5.52	0.808	10.8	0.898	0.839	25
26	Fe	55.85	7.86	0.0801	0.9881	0.0353	7.53	11	13.5	0.031	0.115	0.115	4.65	3.00	1.79	2.3	1.27	0.245	26
27	Co	58.94	8.9	0.0910	0.9887	0.0335	37.0	7	44.0	0.070	0.637	4.00	0.297	1.57	0.250	48	0.273	0.251	27
28	Ni	58.71	8.90	0.0913	0.9887	0.0335	4.6	17.5	22.1	0.420	1.60	2.02	2.38	0.63	0.49	4	0.688	0.199	28
29	Cu	63.54	8.94	0.0848	0.9886	0.0309	3.69	7.2	10.9	0.313	0.611	0.924	3.19	1.64	1.08	4	1.26	0.497	29
30	Zn	65.38	7.14	0.0658	0.9897	0.0304	1.06	3.6	4.66	0.207	0.237	0.307	1.43	4.22	0.326	2	4.38	1.34	30
31	Ga	69.72	5.91	0.0611	0.9905	0.0283	2.77	4	6.77	0.142	0.204	0.346	7.00	5.40	2.89	9.21	3.18	1.44	31
32	Ge	72.60	5.36	0.0445	0.9909	0.0271	2.35	3	3.55	0.105	0.134	0.239	9.52	7.46	4.18	12	4.53	2.15	32
33	As	74.91	5.73	0.0461	0.9911	0.0264	4.1	6	10.1	0.189	0.277	0.466	5.29	3.61	2.15	33	2.37	1.06	33
34	Se	78.96	4.8	0.0516	0.9916	0.0247	11.8	11	22.8	0.432	0.403	0.885	2.31	2.48	1.29	48	1.26	0.686	34
35	Br	79.916	3.12	0.0235	0.9925	0.0219	6.6	12.6	15.5	0.815	0.141	0.296	6.45	7.05	3.38	147	3.54	1.94	35
36	Kr	83.80	0.0037	2.63	0.9921	0.0236	28	7.2	35.2	73	70	91	1.40	3.300	1.200	1.180	1.010	36	36
37	Rb	85.48	1.53	0.0108	0.9922	0.0233	0.70	12	12.2	0.008	0.130	0.138	3.28	7.69	7.69	1.83	2.55	37	37
38	Sr	87.62	2.54	0.0175	0.9925	0.0226	1.16	10	11.2	0.020	0.175	0.195	49.3	5.71	5.13	16	9.62	1.88	38
39	Y	88.92	5.01	0.0373	0.9925	0.0223	1.3	3	4	0.048	0.112	0.160	20.8	8.93	6.25	0.91	7.57	2.75	39
40	Zr	91.22	6.4	0.0423	0.9927	0.0218	0.180	8	8.2	0.008	0.338	0.346	132	2.96	2.83	3	11.4	0.99	40
41	Nb	92.91	8.4	0.0454	0.9928	0.0214	1.1	5	6.1	0.060	0.272	0.333	16.7	3.70	3.0	8.3	4.43	1.18	41
42	Mo	95.95	10.2	0.0640	0.9931	0.0207	2.5	7	9.5	0.100	0.448	0.608	6.25	2.23	1.64	13	2.09	0.699	42
43	Tc	98		0.0000	0.9932	0.0203	100												43
44	Ru	101.1	12.2	0.0727	0.9934	0.0197	2.46	6	8.5	0.179	0.436	0.615	5.59	2.30	1.63		1.99	0.709	44
45	Rh	102.91	12.5	0.0732	0.9935	0.0193	150	5	155	0.110	0.350	0.14	0.091	2.78	0.88	575	0.107	0.126	45
46	Pd	106.4	12.16	0.0689	0.9937	0.0187	8	3.6	11.6	0.551	0.248	0.799	1.81	0.03	1.25	23	1.30	0.931	46
47	Ag	107.88	10.5	0.0586	0.9938	0.0184	62	6	68	3.63	0.352	3.98	0.275	2.84	0.251	600	0.290	0.305	47
48	Cd	112.41	8.65	0.0464	0.9940	0.0178	3.215	7	3.222	154	0.325	154	0.007	3.08	0.007		0.0083	0.011	48
49	In	114.82	7.28	0.0382	0.9942	0.0173	190	2.2	19.2	0.786	0.084	7.34	0.138	11.9	0.136	2,570	0.171	0.212	49
50	Sn	118.70	6.5	0.0330	0.9944	0.0167	6.6	4	4.6	0.132	0.132	0.152	50.0	15.8	5.59	4.3	11.0	1.25	50
51	Sb	121.76	5.95	0.0330	0.9940	0.0163	5.5	4.3	9.8	0.182	0.142	0.324	5.94	7.08	3.53	11.5	3.21	1.87	51
52	Te	127.61	6.24	0.0295	0.9944	0.0155	4.5	5	9.5	0.133	0.148	0.281	7.52	6.76	3.56	36	3.79	1.91	52
53	I	126.91	4.93	0.0234	0.9948	0.0157	6.7	3.6	10.3	0.157	0.084	0.241	6.37	11.9	4.15	130	4.29	2.89	53
54	Xe	131.30	0.0069	2.63	0.9949	0.0150	35	4.3	39.3	0.001	0.001	0.001	1,053	9.991	909	1,012	972	54	
55	Cs	132.91	1.873	0.0085	0.9950	0.0152	29	20	49	0.246	0.170	0.416	0.47	0.88	2.40	2	2.49	1.52	55
56	Ba	137.36	3.5	0.0154	0.9951	0.0145	1.17	8	9.2	0.018	0.123	0.141	55.6	8.1	7.09	7.5	12.1	2.64	56
57	La	138.92	6.19	0.028	0.9952	0.0143	8.9	15	24	0.239	0.043	0.642	4.18	2.47	1.57	11	1.77	0.746	57
58	Ce	140.13	6.78	0.0292	0.9952	0.0142	0.7	9	9.7	0.021	0.262	0.283	4.89	3.83	3.83	5	7.82	1.58	58
59	Pr	140.92	6.78	0.0296	0.9953	0.0141	11.2	4	15.2	0.024	0.116	0.440	3.30	6.52	2.27	7	2.38	1.85	59
60																			

References for Table 9-1

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9.2 PHYSICAL PROPERTIES OF THE ELEMENTS

The physical properties of the elements⁽²⁻⁶⁾ are listed in Table 9-2.

9.3 ISOTOPIC RATIOS

The isotopic abundance of U^{235} in natural uranium has been determined by several workers as follows:

Greene, et al. ⁽⁷⁾	$0.71057 \pm 0.00059\%$
Lounsbury ⁽⁸⁾	0.7204%
White, et al. ⁽⁹⁾	0.718%

Commercially prepared D_2O admixture of H_2O is usually 99.8% pure D_2O .⁽¹⁰⁾
The natural isotopic abundance of D_2O in H_2O is 149 ppm.

9.4 COMPOSITION OF REACTOR MATERIALS

The composition of various grades and forms of graphite⁽¹¹⁾ and beryllium⁽¹²⁾ are given in Tables 9-3 and 9-4, respectively.

Table 9-2

PHYSICAL PROPERTIES OF THE ELEMENTS

Element	Symbol	Atomic Number	Coefficient of Linear Thermal Expansion Near 20°C Per °C x 10 ⁻⁶	Thermal Conductivity Near 20°C cal/sq cm/cm per °C/sec	Specific Heat Near 20°C cal/(gm)(°C)	Heat of Fusion cal/gm.	Melting Point °C	Boiling Point °C
Actinium	Ac	89			0.029 ^a	15.0 ^a	1600	3300 ^a
Aluminum	Al	13	23.9	0.53	0.215	94.6	660.2 ± 0.1	2060
Americium	Am	95						
Antimony	Sb	51	8.5 to 10.8	0.045	0.049	38.3	630.5 ± 0.1	1440
Argon	A	18		0.406 x 10 ⁻⁴	0.125	6.7	-189.4 ± 0.2	-185.8
Arsenic	As	33	4.7		0.082	88.4 ^a	814	610 (Sublimes)
Astatine	At	85			0.033 ^a	13.6 ^a	300 ^a	380 ^a
Barium	Ba	56	18.1 to 21.0 ^c		0.068	13.3 ^a	704 ± 20	1640
Beryllium	Be	4	12.4	0.38	0.52	260	1280 ± 40	2770
Bismuth	Bi	83	13.3	0.020	0.034	12.5	271.3 ± 0.1	1420
Boron	B	5	8.3		0.309	490 ^a	2300 ± 300	
Bromine	Br	35			0.070	16.2	-7.2 ± 0.2	58
Cadmium	Cd	48	29.8	0.22	0.055	13.2	320.9 ± 0.1	765
Calcium	Ca	20	22	0.3	0.149	51.6 ^a	850 ± 20	1440
Carbon	C	6	0.6 to 4.3	0.057	0.165		3700 ± 100	4830
Cerium	Ce	58			0.042	15.6 ^a	600 ± 50	1400 ^b
Cesium	Cs	55	97		0.052	3.8	28 ± 2	690
Chlorine	Cl	17		0.172 x 10 ⁻⁴	0.116	21.6	-101 ± 2	-34.7
Chromium	Cr	24	6.2	0.16	0.11	75.6	1890 ± 10	2500
Cobalt	Co	27	12.3	0.165	0.099	58.4	1495 ± 1	2900
Copper	Cu	29	16.5	0.94	0.092	50.6	1083.0 ± 0.1	2600
Curium	Cm	96						
Dysprosium	Dy	66			0.04 ^a	25.2 ^a	1500 ^a	2300 ^a
Erbium	Er	68			0.04 ^a	24.5 ^a	1500 ^a	2600 ^a
Europium	Eu	63			0.04 ^a	16.4 ^a	1150 ± 50 ^b	1400 ^a
Fluorine	F	9			0.18	10.1	-223 ± 10	-188.2
Francium	Fr	87			0.034 ^a	2.2 ^a	30 ^a	700 ^a
Gadolinium	Gd	64			0.055 ^a	23.5 ^a	1300 ^a	2700 ^a
Gallium	Ga	31	18		0.079	19.2	29.78 ± 0.02	2070
Germanium	Ge	32	6.0 ^c		0.073	104.6 ^a	958 ± 10	2700 ^b
Gold	Au	79	14.2	0.71	0.031	16.1	1063.0 ± 0.0	2970
Hafnium	Hf	72			0.034 ^a	29.1 ^a	1700	>3200 ^b
Helium	He	2		3.32 x 10 ⁻⁴	1.25		-271.4 ± 0.2	-268.9
Holmium	Ho	67			0.039 ^a	24.8 ^a	1500 ^a	2300 ^a
Hydrogen	H	1		4.06 x 10 ⁻⁴	3.45	15.0	-259.4 ± 0.1	-252.7
Indium	In	49	33	0.057	0.057	6.8 ^a	156.4 ± 0.1	2000 ± 10 ^b
Iodine	I	53	93	10.4 x 10 ⁻⁴	0.052	14.2	114 ± 1	183
Iridium	Ir	77	6.8	0.14	0.031	32.8 ^a	2454 ± 3	5300
Iron	Fe	26	11.7	0.18	0.11	65	1539 ± 3	2740
Krypton	Kr	36		0.21 x 10 ⁻⁴	0.059 ^a	4.7 ^a	-157 ± 0.5	-152
Lanthanum	La	57			0.045	19.4 ^a	826 ± 5	3350 ^a
Lead	Pb	82	29.3	0.083	0.031	6.3	327.4 ± 0.1	1740
Lithium	Li	3	56	0.17	0.79	159	186 ± 5	1370
Lutecium	Lu	71			0.028 ^a	26.3 ^a	1700 ^a	1900 ^a
Magnesium	Mg	12	26	0.38	0.25	89	650 ± 2	1110
Manganese	Mn	25	22		0.115	64	1245 ± 10	2150
Mercury	Hg	80		0.0201	0.033	2.7	-38.87 ± 0.02	357
Molybdenum	Mo	42	4.9	0.35	0.061	70	2625 ± 50	4800
Neodymium	Nd	60			0.045	18 ^a	840 ± 40	3090 ^a
Neon	Ne	10		0.00011	0.25 ^a	4.0 ^a	-248 ± 0.3	-246.0
Neptunium	Np	93						

Table 9-2 (Cont'd.)

Element	Symbol	Atomic Number	Coefficient of Linear Thermal Expansion Near 20°C Per °C x 10 ⁻⁶	Thermal Conductivity Near 20°C cal/(gm)(°C)	Specific Heat Near 20°C cal/(gm)(°C)	Heat of Fusion cal/gm.	Melting Point °C	Boiling Point °C
Nickel	Ni	28	13.3	0.22	0.105	74	1455 ± 1	2730
Niobium	Nb	41	7.1		0.065	68.9 ^c	2415 ± 15	3700 ^a
Nitrogen	N	7		0.000060	0.247	6.2	-210.0 ± 0.1	-195.8
Osmium	Os	76	4.6		0.031	36.8 ^a	2700 ± 200	5500
Oxygen	O	8		0.000059	0.218	3.3	-218.8 ± 0.1	-183.0
Palladium	Pd	46	11.8	0.17	0.058	34.2	1554 ± 1	4000
Phosphorus	P	15	125		0.177	5.0	44.1 ± 0.1	280
Platinum	Pt	78	8.9	0.17	0.032	27	1773.5 ± 1	4410
Plutonium	Pu	94	50.3 ^d				632 ^d	
Polonium	Po	84			0.03 ^a	14.3 ^a	600	960 ^a
Potassium	K	19	83	0.24	0.177	14.5	63 ± 1	770
Praseodymium	Pr	59			0.048	17 ^a	940 ± 50	3000 ^a
Promethium	Pm	61			0.045 ^a	20.7 ^a	1000 ^a	2700 ^a
Protoactinium	Pa	91			0.03 ^a	15.2 ^a	3000	4000 ^a
Radium	Ra	88			0.03 ^a	8.8 ^a	700	1140 ^a
Radon	Rn	86			0.02 ^a	3.1 ^a	-71	-61.8
Rhenium	Re	75			0.033	42.4 ^a	3170 ± 60	5600 ^a
Rhodium	Rh	45	8.3	0.21	0.059	50.5 ^a	1966 ± 3	4500
Rubidium	Rb	37	90		0.080	6.1	39 ± 1	680
Ruthenium	Ru	44	9.1		0.057	60.3 ^a	2500 ± 100	4900
Samarium	Sm	62			0.04 ^a	17.6 ^a	>1300	1590 ^a
Scandium	Sc	21			0.134 ^a	85.6 ^a	1200	2400 ^b
Selenium	Se	34	37		0.084	6.6	220 ± 5	680
Silicon	Si	14	2.8 to 7.3	0.20	0.162	337	1430 ± 20	2300
Silver	Ag	47	19.7	1.0	0.056	25	960.5 ± 0.0	2210
Sodium	Na	11	71	0.32	0.295	27.5	97.7 ± 0.2	892
Strontium	Sr	38			0.176	25	770 ± 10	1380
Sulfur	S	16	64	6.31 x 10 ⁻⁴	0.175	9.3	119.0 ± 0.5	444.6
Tantalum	Ta	73	6.5	0.13	0.036	41.4 ^a	2996 ± 50	4100 ^b
Technetium	Tc	43			0.058 ^a	55.6 ^a	2700	4600 ^a
Tellurium	Te	52	16.8	0.014	0.047	7.3	450 ± 10	1390
Terbium	Tb	65			0.041 ^a	24.5 ^a	327 ± 5	2500 ^a
Thallium	Tl	81	28	0.093	0.031	7.2	300 ± 3	1460
Thorium	Th	90	11.1		0.034		1800 ± 150	4500 ^b
Thulium	Tm	69			0.038 ^a	26 ^a	1600 ^a	2100 ^a
Tin	Sn	50	23	0.16	0.054	14.5	231.9 ± 0.1	2270
Titanium	Ti	22	8.5		0.126	77.2 ^a	1820 ± 100	>3000 ^b
Tungsten	W	74	4.3	0.48	0.032	44	3410 ± 20	5930
Uranium	U	92		0.064	0.028	15.5 ^a	1130	3900 ^a
Vanadium	V	23	7.8		0.120	82.4 ^a	1735 ± 50	3400
Xenon	Xe	54		1.24 x 10 ⁻⁴	0.038 ^a	4.2 ^a	-112 ± 1	-108.0
Ytterbium	Yb	70			0.035 ^a	12.7 ^a	824 ^a	1500 ^a
Yttrium	Y	39			0.068 ^a	46.1 ^a	1490 ± 200	2500 ^b
Zinc	Zn	30	39.7	0.27	0.0915	24.09	419.46	906
Zirconium	Zr	40	5		0.066	43.8 ^a	1750 ± 700	>2900 ^b

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^dAECD-3647 The Reactor Handbook, Vol. 3, Sec. 1.

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Table 9-3

COMPOSITION OF COMMERCIAL-AND REACTOR-GRADE GRAPHITE

(in ppm)

Element	Commercial Grade		Reactor Grade		
	AGX	C-18	AGHT	AGOT	CS
Ash	1500	1200-2000	700-1700	350-700	350-700
B	1.3	1.0	-	-	-
Al	20	10	-	-	-
Ca	2000	1200	500	100-200	-
Ti	50	35	100	< 100	-
Fe	40	100	< 100	< 100	-
V	70	200	< 100	< 100	< 100

Table 9-4

COMPOSITION OF VARIOUS GRADES AND SHAPES OF BERYLLIUM METAL

(in %)

Plates, Pebbles, Billets

Grade	A		B	C	D
	Plates	Pebbles	Plates	Pebbles	Billets
BeF ₂ reduced to Be, %	70	70 55			
Assay	96 - 99	96 - 99 99+	99+	99+	99+
BeO	0.1 - 0.25	0.1 - 0.25 0.1 - 0.25	0.1 - 0.25	0.1 - 0.25	0.1 - 0.25
Mg	0.5 - 1.0	0.5 - 0.8 0.15 - 0.40	0.05 - 0.09	0.08 - 0.25	0.01 - 0.05
Ca	0.001 - 0.005	0.001 - 0.005 0.001 - 0.005	0.1 - 0.2	0.001 - 0.005	0.001 - 0.005
Be ₂ C	0.1 - 0.3	0.1 - 0.3 0.1 - 0.3	0.1 - 0.3	0.1 - 0.3	0.1 - 0.3

Powder - Sintered Shapes

Grade	QT	QPT	QM	QRM	QMV
Assay	98.0+	98.5+	98.5+	98.7+	98.7+
BeO	1.0 - 1.8	1.0 - 1.8	1.0 - 1.8	1.0 - 1.8	1.0 - 1.8
Mg	0.35 - 0.60	0.15 - 0.35	0.05 - 0.09	0.08 - 0.25	0.01 - 0.05
Ca	0.001 - 0.005	0.001 - 0.005	0.1 - 0.2	0.001 - 0.005	0.001 - 0.005
Be ₂ C	0.1 - 0.3	0.1 - 0.3	0.1 - 0.3	0.1 - 0.3	0.1 - 0.3
Density	1.84 - 1.86	1.85 - 1.86	1.85 - 1.86	1.85 - 1.86	1.85 - 1.86

Grade Legend

A Primary (or technical)	QT Open furnace - primary
B Remelted in CaF ₂ -BeF ₂ Flux-Premium	QPT Pressure furnace - primary
C Remelted in MgF ₂ -BeF ₂ slag in pressure furnace	QM Remelted under CaF ₂ -BeF ₂ Flux-open furnace
D Vacuum melted	QRM Remelted under MgF ₂ -BeF ₂ slag in pressure furnace
	QMV Vacuum cast-powdered

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