UNCLASSIFIED

OAK RIDGE NATIONAL LABORATORY

Operated By

UNION CARBIDE NUCLEAR COMPANY

UCC

POST OFFICE BOX P
OAK RIDGE, TENNESSEE

ORNL

CENTRAL FILES NUMBER

55-9-4

DATE: September 1, 1955

SUBJECT: Breeding Ratio and Chemical Processing Method

TO: R. B. Briggs

FROM: Paul R. Kasten

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

Photostat Price \$ 3.30

Microfilm Price \$ 2.40

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

UNCLASSIFIED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

CLASSIFICATION CANCELLED

DATE FEB 16 1957

For The Atomic Energy Commission

H. F. Canale
Chief, Declassification Branch *fm*

Chief, Declassification Branch

UNCLASSIFIED

Date: September 14, 1955

Subject: ERRATUM TO ORNL-CF-55-9-4

Copy No. 43A

From: Paul R. Kasten

Replace Equation (1) on page 4 with

$$\Sigma_p(t) = \int_{-\infty}^t S(t-x) e^{-\frac{t-x}{T}} F(x) dx = \int_0^{\infty} S(v) e^{-\frac{v}{T}} F(t-v) dv \quad (1)$$

vkx

DECLASSIFIED

CONFIDENTIAL

2.

SUMMARY

This study is preliminary in nature, the purpose being to indicate the influence of processing method (continuous or batch) upon breeding ratio for one- and two-region reactors. The results obtained for batch fuel reprocessing are compared with those for continuous fuel reprocessing, using the same average cycle time for each case. Two specific reactors were considered. For both the one- and two-region reactors the breeding ratio was higher ($\sim .01 - .03$) if the fuel fluid was reprocessed by the batch process.

INTRODUCTION

Companies operating future nuclear power stations may not wish to become involved in the large scale chemical operations associated with continuous reprocessing of homogeneous fuel systems. Reprocessing on a batch cycle would allow the power producer to divorce himself from the major fuel processing itself. However, the processing method ultimately employed will be based upon economic factors. If large central processing facilities serve many reactor stations, the cost of chemical processing itself should be comparable, whether a batch or a continuous fuel reprocessing cycle is used by a particular reactor station. There will be some cost associated with fuel shipment to a central facility, but such costs are estimated to be small ($\sim .18$ mills/kwhr)¹.

CONFIDENTIAL

DECLASSIFIED

183 003

Another economic factor is the breeding ratio of the reactor system. The study reported here is concerned with the effect of the processing method upon the breeding ratio, assuming that the average cycle time is the same for both batch and continuous cycles. In either processing method it is assumed that fission gases and precipitated fission products (in the case of fuel solutions) may be removed continuously with equal facility.

One-region and two-region homogeneous reactors will be investigated. The particular reactors chosen are based upon economic studies², and represent minimum cost electrical power producing reactor systems in which the fuel is continuously reprocessed. The one-region reactor has a 12-foot spherical core region, operates at a power density of ~ 20 kw/liter, contains about 6 gm fissionable material/liter of fluid and about 250 gm Th/liter, has a continuous fuel reprocessing cycle time of 400 days, contains 5.5% poisons, and has a breeding ratio of 1.012. The two-region reactor consists of a 5-foot spherical core surrounded by a 2-1/4-foot blanket containing 1000 gm/liter. The core region generates 400 Mw of thermal energy; contains 6% poisons, about 2.3 gm fissionable material/liter of fluid, and about 6.6 gm U/liter; and operates on a reprocessing cycle time of 340 days. The blanket region generates 80 Mw of thermal energy, contains 3 gm U^{233} /kg Th and about 3% poisons, and operates on a reprocessing cycle time of 140 days.

CONFIDENTIAL

DECLASSIFIED

183 004

AVERAGE POISON CROSS SECTION

An indication of the effect on breeding ratio is to compare the average poison cross section of reactors operated on batch and continuous processing cycles. As pointed out by Young³, the average poison cross section in a reactor operated on a batch cycle is smaller than the poison cross section in a continuously processed reactor if the chemical processing time is the same in the two cases. To illustrate this, consider the average history of the products of a single average fission and let $S(t)$ describe the variation of their total neutron absorption cross section subsequent to their fission birth at $t = 0$. Actually, S is a function of time and neutron flux; however, studies of $S(t, \phi)$ and $S(t, \phi_0)$, where ϕ is variable and ϕ_0 is constant, indicate that little difference exists up to about 20% burnup^{4,5}. Here, S will be considered a function of time only.

Continuous Processing

The probability that a particle remains in a continuously purified system is $e^{-t/T}$, where T is the fluid residence time associated with fuel reprocessing. Let $\Sigma_p(t)$ be the poison cross section at time t , and let x be a time previous to t . Letting $F(x)$ be the rate of production of fission fragments, then the total cross section of all the fission products in the system at time t is given by

$$\Sigma_p(t) = \int_t^{\infty} S(t-x) e^{-\frac{-(t-x)}{T}} F(x) dx = \int_0^{\infty} S(x) e^{-\frac{x}{T}} F(x+t) dx \quad (1)$$

CONFIDENTIAL

DECLASSIFIED

183 005

Assuming the power is constant, then F is a constant and the average poison cross section is independent of time and is given by

$$\bar{\Sigma}_p^c = F \int_0^{\infty} S(x) e^{-\frac{x}{T}} dx \quad (2)$$

where the superscript c represents continuous processing and the bar signifies the average value.

Batch Processing

Consider now batch processing in which the system is entirely purified after time T . If the poison cross section is initially zero, at time t the fission product cross section is

$$\Sigma_p^b(t) = \int_0^t S(x) F(t-x) dx \quad (3)$$

where the superscript b represents batch processing. The average value of the poison cross section over one processing cycle time is

$$\begin{aligned} \bar{\Sigma}_p^b &= \frac{1}{T} \int_0^T \Sigma_p^b(t) dt = \frac{1}{T} \int_0^T dt \int_0^t S(x) F(t-x) dx \\ &= \int_0^T S(x) \left(1 - \frac{x}{T}\right) \frac{\int_x^T F(t-x) dt}{T-x} \end{aligned} \quad (4)$$

Again considering the power level to be constant, the average poison cross section reduces to

$$\bar{\Sigma}_p^b = F \int_0^T S(x) \left(1 - \frac{x}{T}\right) dx \quad (5)$$

Comparing Equations 5 and 2 it is seen that $\bar{\Sigma}_p^c > \bar{\Sigma}_p^b$ since $e^{-\frac{x}{T}} > 1 - \frac{x}{T}$; also, the integral extends from $0 \rightarrow \infty$ in Equation 2 rather than from $0 \rightarrow T$.

The above indicates that a reactor operating on a batch processing cycle may have a greater breeding ratio than the corresponding reactor operating on a continuous purification cycle. Rather than the average poison cross section itself, however, the breeding ratio is influenced more by the average value of the poison cross section times the flux. Since the flux varies with time in a reactor operating on a batch process and the specific variation is dependent upon the particular reactor system, an a priori statement cannot be made as to the desirability of batchwise over continuous processing. To indicate that the batchwise operation is indeed better the following calculations are made for one- and two-region reactor systems.

ONE-REGION REACTOR

A large, bare, homogeneous reactor is considered here, and any fuel additions as a function of time will consist of fissionable-type fuel only. Using the modified one-group criticality relation, the macroscopic fuel absorption cross section is given by

$$\Sigma_f = a + b \Sigma_p \quad (6)$$

where

$$a = \frac{1 + B^2(L_o^2 + T)\Sigma_{mod} + (1 + B^2 T)}{(1 + \alpha)(\eta\epsilon_p - 1 - B^2 T)} \Sigma_x$$

$$b = \frac{1 + B^2}{(1 + \alpha)(\eta\epsilon_p - 1 - B^2 T)}$$

CONFIDENTIAL

DECLASSIFIED

183 007

Σ_x = macroscopic cross section of fertile material, and the other symbols have their usual meaning.

The previous assumption of constant power appears logical for a power-producing one-region reactor. Thus, the same number of neutrons will be available for absorption in fertile material and poisons for either batch or continuous processing, since any change in leakage losses would be negligible. A comparison of neutron absorptions for the two cases will therefore show which processing method results in the higher breeding ratio. The absorption by the poisons during time T are given by

$$A = \int_0^T \Sigma_p \phi V dt \quad (7)$$

where A = total absorptions by poisons in cycle time T

V = volume of reactor.

The flux variation with time is given by

$$\phi(t) = \frac{\gamma P}{\Sigma_f} = \frac{\gamma P}{a+b \Sigma_p(t)} \quad (8)$$

where γ = proportionality constant

P = reactor power.

The ratio of absorptions in poisons for the batch-to-continuous cases is thus given by

$$\frac{A^b}{A^c} = \frac{\int_0^T \phi^b dt \int_0^t S(x) dx}{\phi^c T \int_0^\infty e^{-\frac{x}{T}} S(x) dx} = \frac{\int_0^T S(x) \left(1 - \frac{x}{T}\right) \frac{\phi^b(T-x)}{\phi^c} dx}{\int_0^\infty e^{-\frac{x}{T}} S(x) dx} \quad (9)$$

where $\bar{\phi}(y) = \frac{1}{y} \int_0^y \phi \, dt$

The ratio of ϕ^b to ϕ^c will always be equal to or less than the value of the ratio at time zero, since

$$\frac{\phi^b(t)}{\phi^c(t)} = \frac{a + b F \int_0^\infty e^{-\frac{x}{T}} S(x) \, dx}{a + b F \int_0^t S(x) \, dx} \leq 1 + \frac{b \sum_p^c}{a} \quad (10)$$

Taking values for a 12-foot spherical reactor (power density ~ 20 kw/liter, about 6 gm fissionable material/liter solution, continuous processing cycle time of 400 days, 5.5% poisons, breeding ratio = 1.012) obtained in economic studies², the ratio $b \frac{\sum_p^c}{a}$ is .09. Equation 9 now becomes

$$\frac{A^b}{A^c} < 1.09 \frac{\int_0^T S(x) \left[1 - \frac{x}{T} \right] \, dx}{\int_0^\infty S(x) e^{-\frac{x}{T}} \, dx} \quad (11)$$

The breeding ratio thus depends upon the value of $S(t)$ as a function of time. If S is independent of time, then Equation 11 gives

$$\frac{A^b}{A^c} < \frac{1.09}{2} \quad (12)$$

If S decays exponentially with time in the manner $S(x) = S_0 e^{-mt}$, then Equation 11 becomes

$$\frac{A^b}{A^c} = 1.09 \left[1 - \frac{1}{mT} + \frac{e^{-mT}}{mT} \right] \left[1 + \frac{1}{mT} \right] \quad (13)$$

If mT is zero, then the result is the same as Equation 12. If mT is 1, then $\frac{A^b}{A^c} < 1.09 \frac{2}{e} \approx .8$. If mT is 10, then $\frac{A^b}{A^c} < 1.08$. In the reactor considered, 6 gm of fissionable material/liter at 20 kw/liter corresponds to 100% burnup after 300 days, or 133% burnup after 400 days exposure. Sampson et al.⁵ present data on the value of $S(t)$, and for the above conditions,

$$\begin{aligned} S(t) &\approx S_0 e^{-mt} & 0 \leq t < 160 \text{ days} \\ &\approx S_0 c_1 & 160 < t \leq 400 \text{ days} \end{aligned} \quad (14)$$

where $c_1 = .365$ for continuous processing
 $= .455$ for batch processing
 $m = \frac{3}{T}$
 $T = 400$ days

(The above relation was obtained for U^{235} fuel with all fission gases removed; for the batch case poison absorptions are slightly overestimated. If U^{233} were the fuel, the relation would change so as to decrease

$\frac{A^b}{A^c}$.)

Inserting the above expression for S into Equation 11 gives $\frac{A^b}{A^c} \approx .7$, and so the breeding ratio will be slightly greater for the batch process.

Specifically, the breeding ratio will increase from 1.012 for continuous reprocessing to about 1.03 for batch reprocessing. The important point is that no breeding ratio penalty is incurred because of batch reprocessing of the fuel.

TWO-REGION REACTOR

The two-region reactor considered will be the most economic power reactor as determined in ORNL-1810. This reactor consists of a 5-foot spherical core surrounded by a 2-1/4-foot blanket containing 1000 gm Th/liter. The operating power is 480 Mw, of which 80 Mw is generated in the blanket region. The blanket chemical processing time is 140 days, and there are 3 gm U^{233} /kg Th in the blanket.

Core Region

For the core region the problem is similar to the one-region reactor, except that the core power changes with time. To simplify the analysis, it will be assumed initially that the core power is constant. Even so, the ratio $\frac{\phi^b}{\phi^c}$ will change with time due to buildup of fission products, and will have its maximum value initially. The ratio $\frac{A^b}{A^c}$ is then

$$\frac{A^b}{A^c} = \frac{\int_0^T \phi^b v \sum_p^b dt}{\int_0^T \phi^c v \sum_p^c dt} < \left(\frac{\phi^b}{\phi^c} \right)_0 \frac{\sum_p^b}{\sum_p^c} \quad (15)$$

where the subscript zero signifies evaluation at time zero.

CONFIDENTIAL

In the continuous processing case there are 6% poisons present in the core, so that $\left(\frac{\phi^b}{\phi^c}\right) < 1.06$. However, the core power will not remain constant in the batch case if it is assumed that the total reactor power is constant at 480 Mw. The initial (and maximum) value of $\frac{P^b}{P^c}$ will be $\frac{480}{400} = 1.2$,

where P^b = core power with batch reprocessing

P^c = core power with continuous reprocessing

Thus, considering the core power to change with time, Equation 15 becomes

$$\frac{A^b}{A^c} < 1.2(1.06) \frac{\sum_p^b}{\sum_p^c} \quad (16)$$

Extrapolating the poison cross section data presented by Sampson et al⁵ for U^{233} ($\sim 3000\%$ burnup in the core region) it is found that

$$\frac{\sum_p^b}{\sum_p^c} < .7, \text{ so that}$$

$$\frac{A^b}{A^c} < .9 \quad (17)$$

Since relatively fewer neutrons are absorbed by fission products by using the batch reprocessing method, more neutrons will leak into the blanket and be absorbed by fertile material, leading to a higher breeding ratio.

CONFIDENTIAL

Blanket Region

If batch reprocessing is used, the blanket power will vary from zero to an appreciable magnitude. The problem here will be to determine the effect of the blanket reprocessing method upon the breeding ratio.

The power in the blanket will be a function of time if batch processing is used, so that

$$\bar{\Sigma}_p^b = \frac{1}{T} \int_0^T S(x)(T-x) \bar{P}(T-x) dx \quad (18)$$

Assuming that the blanket power follows the equation

$$P(t) = c_2 t \quad (19)$$

where c_2 is a constant

Equation 18 now becomes

$$\bar{\Sigma}_p^b = \frac{c_2 T}{2} \int_0^T S(x) \left(1 - \frac{x}{T}\right)^2 dx \quad (20)$$

For the continuous processing case, the blanket power will be constant, so that

$$\bar{\Sigma}_p^c = P^c \int_0^\infty S(x) e^{-\frac{x}{T}} dx \quad (21)$$

and

$$\frac{\bar{\Sigma}_p^b}{\bar{\Sigma}_p^c} = \frac{\frac{c_1 T}{2} \int_0^T S(x) \left(1 - \frac{x}{T}\right)^2 dx}{P^c \int_0^\infty S(x) e^{-\frac{x}{T}} dx} < \frac{\bar{P}^b}{P^c} \quad (22)$$

CONFIDENTIAL

DECLASSIFIED

183 013

where

P^c = blanket power for continuous processing

$$\bar{P}^b = \frac{c_2 T}{2} = \text{average blanket power for batch processing}$$

The blanket cycle time for the continuously reprocessed reactor considered is 140 days; the blanket contains 3 gm U^{233} /kg Th; and the blanket power is 80 Mw, with the total reactor power about 480 Mw. Since the breeding ratio is ~ 1 , about 480 gm of fuel will be bred per day. For the continuous reprocessing cycle the blanket will contain $\sim 5 \times 10^4$ gm U^{233} . It requires $\frac{5 \times 10^4}{480} \approx 100$ days to build up this much fuel on a batch cycle. Therefore, on a batch processing cycle of 140 days

$$\frac{\sum_p^b}{\sum_p^c} < \frac{\bar{P}_b}{\bar{P}_c} < \frac{140}{2(100)} = .7 \quad (23)$$

To evaluate the breeding ratio, the flux should be included in the above calculation. ORSORT studies⁶ indicate that the average thermal blanket flux rises about 10% above its initial value as U^{233} concentration increases from zero to 6 gm U^{233} /kg Th in the blanket. Thus,

$$\frac{A^b}{A^c} < \sim .77 \quad (24)$$

and so the average breeding ratio over time T would be greater for the batch fuel reprocessing method, considering the blanket alone.

CONFIDENTIAL

DISCUSSION

For both the one- and two-region reactors studied, the batch fuel reprocessing cycle results in a higher breeding ratio than if continuous reprocessing were used. The increase in the breeding ratio appears to be in the range .01 - .03. The results of this study, however, are preliminary in nature due to the assumption of $S(t, \emptyset) = S(t)$. For the high fuel burnups associated with the core of the two-region reactor such an assumption may not be realistic.

A detailed investigation is now proceeding pertaining to the time behavior of critical reactor systems, which will eliminate many of the approximations made in the above analysis. A variety of reactors will be studied to determine the economic features associated with batch chemical processing.

Paul R. Kasten
Paul R. Kasten

vkh

CONFIDENTIAL

183 015

DECLASSIFIED

REFERENCES

1. J.E. Vivian, Processing of Spent Power-Reactor Fuels, MIT-5005, December, 1954.
2. H.C. Claiborne and M. Tobias, Some Economic Aspects of Thermal Breeder Reactors, ORNL-1810, to be issued.
3. G. Young, On the Formulation of the Fission Product Poisoning Problem, CL-GY-1, December 4, 1946.
4. J.W. Webster, The Low Cross Section Fission Products Poisons, IDO-16100, June 8, 1953.
5. J.B. Sampson, W.C. Robb, J.R. Stehn and J.K. Davidson, Poisoning in Thermal Reactors Due to Stable Fission Products, KAPL-1226, October 4, 1954.
6. H.C. McCurdy, Group Leader, et al, ORSORT Summer Problem, Thorium Rod Blanket Study for Thermal Breeder Reactors, to be issued.

0-13

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

0377281030