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ANALOG TECHNIQUE FOR OPTIMIZING DYNAMIC SYSTEMS

R. A. HARVEY

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By

R. A. Harvey

Engineering Development Department
Engineering Analysis Section

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R. A. Harvey

ABSTRACT

An iterative analog program has been developed for optimizing a sampled quadratic-like function of an independent parameter. A minimum-seeking sampled dynamic system is discussed with results that show the effects of changing the control loop gain. The program and results for a digital computer implementation using the MIMIC program is presented also.

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ANALOG TECHNIQUE FOR OPTIMIZING DYNAMIC SYSTEMS

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INTRODUCTION

During the development of a model for system optimization, an analog computer technique for optimizing a performance function of a sampled dynamic system was developed. The purpose of this report is to document this technique so that others can use it on the analog computer with iterative capability and on the digital computer with the MIMIC program. The availability of iterative analog computing equipment permits the handling of more sophisticated problems that can be done easily without the iterative capability. Among these problems are automatic optimization of dynamic systems, adaptive control, design optimization, automatic curve fitting, and others.

Some analog techniques for automatically optimizing various types of systems were presented in an earlier report.⁽¹⁾ Random and perturbation methods were used to optimize quadratic-like functions of parameters in dynamic processes. This report describes a method which converges more rapidly and absolutely to the optimum value than do the methods presented in the earlier report.

DISCUSSION

TYPE OF SYSTEM TO BE OPTIMIZED

The described technique can be used for optimizing a function of a single parameter as well as for optimizing a function of multiple interacting parameters. However, consideration of parameter interactions is the subject of another report and is not pertinent to an understanding of the optimizing analog techniques; hence, only the single parameter system is considered.

(1) R. A. Harvey, G. R. Taylor and R. D. Benham. "Analog Computer Methods for Parameter Optimization," Simulation, vol. 6, p. 181. March 1966.

In this report systems are classified according to characteristics pertinent to closing an automatic optimizing loop. Figure 1 shows a generalized system with an index of performance (I) to be optimized, and with parameters (X) which are manipulated to result in optimum performance. The inputs U represent disturbances to the system.

FIGURE 1. The General System to be Optimized

The easiest type of system to optimize is the algebraic system in which the sign of dI/dX does not change (Curve A in Figure 2). A simple feedback control system can be used to maintain I at any desired value. A dynamic system whose steady-state characteristic is represented by Curve A is easy to optimize also; however, the time required for convergence might be longer than for the purely algebraic system.

FIGURE 2. Examples of Functions for $I = F(X)$

A more difficult system to optimize automatically with the analog computer is one whose steady-state characteristic is represented by Curve B in Figures 2 where the sign of dI/dX changes. This is a common type of function in optimization problems. The approach is to feed back the derivative, dI/dX , and control X so that dI/dX is equal to zero. The derivative is easily obtained with digital programs for algebraic systems, but not so easily obtained with analog computer programs.

A still more difficult system to optimize is the time-sampled system in which dI/dX changes sign. Dynamic problems where predictions are made over some computing interval are often of this type. Examples are: the minimum expenditure of fuel for a space vehicle to accomplish a given maneuver or mission, the minimization of energy losses in a power system over some prediction period, and curve or function matching using an integral-error-squared criterion. In each of these examples, it is necessary that a computing cycle be completed before the total fuel consumption, energy loss, or integral-error-squared values can be determined. This is the type of computation for which the described technique was developed.

ANALOG COMPUTER IMPLEMENTATION

The analog computer diagram in Figure 3 was developed for an EASE model 2133 iterative analog computer. However, the techniques are applicable to any computer with track and store capabilities. The block labeled "process" represents the system to be optimized, where X is the parameter to be adjusted and I_p is the function to be minimized. Symbols on the integrators indicate that control of the compute and initial condition modes from the master timer is direct (M), or complementary ($\overline{M}$). The instantaneous losses are represented by f so that I represents the time integrated losses. Integrator 1 stores the peak value of the integrated losses at the end of a computing interval.

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The analog program was developed for the case where I_p (peak integrated losses) is a quadratic-like function of the parameter X and where it is desired to find the value of X that results in minimum I_p . The system solves for dI_p/dX , and then adjusts X until dI_p/dX is equal to zero.

The derivative dI_p/dX is determined by

$$\frac{dI_p}{dX} = \frac{dI_p}{dt} \div \frac{dX}{dt} \quad (1)$$

so that it is necessary to solve for both dI_p/dt and dX/dt . Integrators 3 and 4 solve (approximately) for dI_p/dt , and integrators 7 and 9 solve for dX/dt . Integrators 6 and 8 serve only to equalize the X time-delay with the I_p time-delay. The divider indicated by MO performs the operation indicated by Equation (1).

The divisors for analog dividers cannot go through zero or change sign; hence, amplifier 12 and comparator relay KO are used to maintain the MO divisor always as a negative voltage. Relay contacts B and amplifier 14 at the output of the divider operate to maintain the correct sign of dI_p/dX when the comparator relay operates (as dX/dt goes through zero). A bias voltage of one-volt is applied from potentiometer 13 so that the divisor never becomes more positive than minus one-volt. Switching of KO generally occurs when dI_p/dt is in the vicinity of zero; thus, the switching transients are usually negligible.

The output dI_p/dX from amplifier 15 is an error voltage which is applied through gain potentiometer 10 to accumulator integrators 10 and 5. The output of the accumulator is the parameter X .

Convergence of dI_p/dX to zero requires that negative feedback be used. Feedback with the circuit as shown might be positive or negative, depending on the effects of X , and on whether optimum is at maximum or at minimum I_p . The phase of feedback can be changed by reversing the B contacts of the comparator or by adding an inverting amplifier in series with amplifier 15. Each system must be considered individually for phase and polarity of voltages.

Figure 4 shows the results of varying the loop gain in an automatic minimum-seeking sampled analog system. In A (Figure 4) the parameter value was scanned (loop open) and the peak loss, I_p , was recorded. It can be seen in A that as the parameter value increases, the losses go through a minimum. The automatic minimum-seeking loop was closed for B in Figure 4, and it can be seen that the losses soon went to the minimum value automatically. Traces C and D in Figure 4 show the effect of reducing and increasing, respectively, the loop gain. With low gain the response is slow, and with high gain there is a tendency to oscillate. The one-second timing markers can be seen at the bottom of the chart.

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FIGURE 4. Strip Chart Recordings of the Performance Index (losses) and the Parameter Value for an Open Loop Scan and for Several Values of Closed Loop Controller Gains

DIGITAL COMPUTER IMPLEMENTATION

The same optimizing technique was implemented with a digital computer by using the MIMIC program. A simple representation of a sampled quadratic function was obtained by integrating a quadratic function over a fixed time interval.

$$F = \int_0^T AX^2 dt \quad (2)$$

where X is the parameter input, and F is the index of performance to be optimized by adjusting X .

Figure 5 shows the block diagram used for the example. Timing generators TG1 and TG2 with the complementary functions TG1C and TG2C were used to control the iteration cycles. All of the timing functions were derived from the same timing angle (TANGLE); therefore, they are synchronous. Since the function switch (FSW) represents a three-position switch, it was not necessary to invert dX/dt ; therefore, it was not required to invert dF/dX as in the analog computer program. A control value of zero is shown on Figure 5 to indicate that dF/dX is forced to zero; however, the input function is not actually needed since the input is always zero.

The MIMIC digital program and the results are shown in Figures 6 and 7. A recording time interval of $DT = 1$ was chosen arbitrarily, but the $DTMAX$ must be chosen such that the track-and-store (TAS) functions can be activated a sufficient number of times during each cycle of the timing angle (TANGLE). The one-to-ten relationship between $DTMAX$ and one cycle of TANGLE is not mandatory, but it is a reasonable guide.

The output data from the digital computer show the parameter, X , the peak function being minimized, $TAS2$, the time, T , and the derivative, $DFDX$, of the peak function with respect to X . After about

FIGURE 5. Digital Optimization of a Sampled Quadratic Function with the MIMIC Program

TEST OF MIMIC LOGICAL FUNCTIONS FOR OPTIMIZATION		
	CON(F1,X1,2PI)	1
	CON(DT,DTMIN,A,DTMAX)	1.1
X	INT(-.3*DFDX,-5.)	2
F	INT(A*X*X,FI,TRUE,TG1C)	3
TANGLE	2PI*T	4
TG1	FSW(SIN(TANGLE+2PI/8.)-.707,FALSE,TRUE,TRUE)	5
TG1C	NOT(TG1)	6
TAS1	TAS(F,TG2C,0.)	7
TG2	FSW(SIN(TANGLE+2PI/8.)+.707,FALSE,TRUE,TRUE)	7.1
TG2C	NOT(TG2)	7.2
TAS2	TAS(TAS1,TG1,0.)	8
TAS3	TAS(TAS2,TG2C,0.)	9
M1	TAS2-TAS3	10
DFPDT	TAS(M1,TG1,0.)	11
TAS4	TAS(X,TG2C,0.)	11.1
TAS5	TAS(TAS4,TG1,0.)	11.2
TAS6	TAS(TAS5,TG2C,0.)	11.3
M2	TAS5-TAS6	11.4
DXDT	TAS(M2,TG1,0.)	11.5
DXFSW	FSW(DXDT,TRUE,FALSE,TRUE)	11.6
DXFSWC	NOT(DXFSW)	11.7
DXFSW MDXDT	EQL(DXDT)	11.8
DXFSWC MDXDT	EQL(.01)	11.9
DFDX	DFPDT/MDXDT	11.9.1
	FIN(T,30.)	12
	HDR(T,X,TAS2,DFDX)	13
	HUR	15
	CUT(T,X,TAS2,DFDX)	14
	END	19
SORT DIAGNOSTICS FOLLOW		

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FIGURE 6. MIMIC Program for Optimizing a Sampled Quadratic Function

IFN	LCV	RESULT	FTN	A	B	C	D	E	F
1			CON	FI	XI	2PI			
2			CON	DT	DTMIN	A	DTMAX		
3		01	NEG	5.					
4		005	MPY	A	X	X			
5		TANGLE	MPY	2PI	T				
6		008	DIV	2PI	8.				
7		009	ADD	TANGLE	008				
8		010	SIN	009					
9		011	SUB	010	.707				
10		TG1	FSW	011	FALSE	TRUE	TRUE		
11		TG1C	NOT	TG1					
12		015	DIV	2PI	8.				
13		016	ADD	TANGLE	015				
14		017	SIN	016					
15		018	ADD	017	.707				
16		TG2	FSW	018	FALSE	TRUE	TRUE		
17		TG2C	NOT	TG2					
18		TAS2	TAS	TAS1	TG1	0.			
19		TAS3	TAS	TAS2	TG2C	0.			
20		M1	SUB	TAS2	TAS3				
21		DFPDT	TAS	M1	TG1	0.			
22		TAS4	TAS	X	TG2C	0.			
23		TAS5	TAS	TAS4	TG1	0.			
24		TAS6	TAS	TAS5	TG2C	0.			
25		M2	SUB	TAS5	TAS6				
26		DXDT	TAS	M2	TG1	0.			
27		DXFSW	FSW	DXDT	TRUE	FALSE	TRUE		
28		DXFSWC	NOT	DXFSW					
29	DXFSW	MDXDT	EQL	DXDT					
30	DXFSWC	MDXDT	EQL	.01					
31		DFDX	DIV	DFPDT	MDXDT				
32			FIN	T	30.				
33		002	MPY	.3	DFLX				
34		00	NEG	002					
35	X		INT	00	01				
36	F		INT	005	FI	TRUE	TG1C		
37		TAS1	TAS	F	TG2C	0.			
38			HDR	T	X	TAS2	DFDX		
39			HDR						
40			OUT	T	X	TAS2	DFDX		
41			END						

FURTHER DIAGNOSTICS AND EXECUTION FOLLOW

FI	XI	2PI	
0.00000	-5.00000+00	6.28319+00	
DT	DTMIN	A	DTMAX
1.00000+00	1.00000-01	1.00000+00	1.00000-01

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FIGURE 7. Program Generated Function Language and Constants

TAS2

10 readout intervals, ~~is~~ is within 1% of the final value of zero. It was found also, as one would expect, that lower loop gain results in slower convergence and higher loop gain results in the tendency to oscillate.

T	X	TAS2	DFDX
0.00000	-5.00000+00	0.00000	0.00000
1.00000+00	-4.92833+00	1.16667+01	-2.33333+00
2.00000+00	-4.28433+00	9.98488+00	-3.13377+00
3.00000+00	-3.34147+00	6.73168+00	-3.67967+00
4.00000+00	-2.24121+00	3.52800+00	-2.95179+00
5.00000+00	-1.35923+00	1.46734+00	-2.24098+00
6.00000+00	-6.91161-01	4.79971-01	-1.39515+00
7.00000+00	-2.75881-01	1.06269-01	-7.82207-01
8.00000+00	-4.32233-02	1.15228-02	-3.41343-01
9.00000+00	5.79000-02	1.40370-04	-8.54129-02
1.00000+01	8.28404-02	2.37262-03	5.12703-02
1.10000+01	6.79254-02	2.61647-03	-4.19464-02
1.20000+01	8.03222-02	2.58907-03	-4.52222-03
1.30000+01	8.10593-02	3.06387-03	1.19407-01
1.40000+01	4.58073-02	1.83117-03	4.54087-02
1.50000+01	3.19094-02	8.94397-04	6.04585-02
1.60000+01	1.39408-02	2.38264-04	2.66642-02
1.70000+01	5.98150-03	4.52455-05	1.86803-02
1.80000+01	4.37889-04	4.67740-06	6.58256-03
1.90000+01	-1.51191-03	1.96356-07	1.58810-03
2.00000+01	-1.97203-03	1.44672-06	-1.51365-03
2.10000+01	-1.52552-03	1.40963-06	-1.56519-04
2.20000+01	-1.46674-03	1.05159-06	-2.52225-03
2.30000+01	-7.15371-04	8.42945-07	-8.60542-04
2.40000+01	-4.59396-04	1.59275-07	-1.02293-03
2.50000+01	-1.53650-04	4.22222-08	-3.96099-04
2.60000+01	-3.75151-05	4.24109-09	-2.37119-04
2.70000+01	3.25892-05	4.54610-11	-5.10019-05
2.80000+01	4.75077-05	7.89082-10	2.55437-05
2.90000+01	4.02229-05	8.38015-10	-5.01241-05
3.00000+01	5.49098-05	1.07882-09	1.99437-05

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FIGURE 8. Results of MIMIC Program for Optimizing a Sampled Quadratic Function

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