STRESS INDICES AT LUG SUPPORTS
ON PIPING SYSTEMS

E. C. Rodabaugh
W. G. Dodge
S. E. Moore

OAK RIDGE NATIONAL LABORATORY

OPERATED BY UNION CARBIDE CORPORATION • FOR THE U.S. ATOMIC ENERGY COMMISSION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22151
Price: Printed Copy \$5.45; Microfiche \$0.95

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Contract No. W-7405-eng-26

Reactor Division

STRESS INDICES AT LUG SUPPORTS ON PIPING SYSTEMS

E. C. Rodabaugh
Battelle-Columbus Laboratories

W. G. Dodge
Oak Ridge National Laboratory

S. E. Moore
Oak Ridge National Laboratory

MAY 1974

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

GG

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

CONTENTS

	<u>Page</u>
FOREWORD	v
NOMENCLATURE	ix
ABSTRACT	1
1. INTRODUCTION AND SCOPE	1
2. THRUST AND BENDING MOMENT LOADS	7
Maximum Stress Intensities	7
Comparisons with ASME Boiler Code, Section I, Paragraph PW-43	13
Comparisons with Welding Research Council Bulletin 107	19
Local Stresses	32
3. SHEAR AND TORSIONAL LOADS	34
4. INTERNAL PRESSURE	37
5. THERMAL GRADIENTS	41
6. CODE USE OF STRESS INDICES	42
Satisfaction of Primary-Stress-Intensity Limit	43
Satisfaction of Primary-Plus-Secondary Stress- Intensity Range	46
Satisfaction of Peak-Stress-Intensity Range	46
Simplified Elastic-Plastic Discontinuity Analysis	48
7. SUMMARY AND CONCLUSIONS	49
ACKNOWLEDGMENTS	51
REFERENCES	51
APPENDIX A: RECOMMENDED ADDITION TO ASME CODE SECTION III TO COVER STRAIGHT PIPE WITH INTEGRAL STRUCTURAL ATTACHMENTS ..	53
APPENDIX B: DIRECTIONS AND LOCATIONS OF MAXIMUM STRESSES FOR THRUST AND MOMENT LOADINGS	58
APPENDIX C: EXAMPLE ANALYSIS OF A LUG SUPPORT	70

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

FOREWORD

The work reported here was performed at Oak Ridge National Laboratory (ORNL) and at Battelle-Columbus Laboratories under Union Carbide Corporation Nuclear Division subcontract No. 2913, as part of the ORNL Piping Program - Design Criteria for Piping, Pumps, and Valves - under the direction of W. L. Greenstreet, Associate Head, Solid Mechanics Department, and S. E. Moore, Program Coordinator. The program is funded by the U.S. Atomic Energy Commission (USAEC) as the AEC-supported portion of an AEC-industry cooperative effort for the development of design criteria for nuclear-power-plant piping components. This joint effort is coordinated by the Pressure Vessel Research Committee (PVRC) of the Welding Research Council, under the Subcommittee to Develop Stress Indices for Piping, Pumps, and Valves.

Prior reports and open literature publications under the ORNL piping program are listed below.

1. W. L. Greenstreet, S. E. Moore, and E. C. Rodabaugh, "Investigations of Piping Components, Valves, and Pumps to Provide Information for Code Writing Bodies," ASME Paper 68-WA/PTC-6, American Society of Mechanical Engineers, New York, Dec. 2, 1968.
2. W. L. Greenstreet, S. E. Moore, and R. C. Gwaltney, Progress Report on Studies in Applied Solid Mechanics, ORNL-4576 (August 1970).
3. E. C. Rodabaugh, Phase Report No. 115-1 on Stress Indices for Small Branch Connections with External Loadings, ORNL-TM-3014 (August 1970).
4. E. C. Rodabaugh and A. G. Pickett, Survey Report on Structural Design of Piping Systems and Components, TID-25553 (December 1970).
5. E. C. Rodabaugh, Phase Report No. 115-8, Stresses in Out-of-Round Pipe Due to Internal Pressure, ORNL-TM-3244 (January 1971).
6. S. E. Bolt and W. L. Greenstreet, "Experimental Determination of Plastic Collapse Loads for Pipe Elbows," ASME Paper 71-PVP-37, American Society of Mechanical Engineers, New York, May 1971.
7. G. H. Powell, R. W. Clough, and A. N. Gantayat, "Stress Analysis of B16.9 Tees by the Finite Element Method," ASME Paper 71-PVP-40, American Society of Mechanical Engineers, New York, May 1971.
8. J. K. Hayes and B. Roberts, "Experimental Stress Analysis of 24-Inch Tees," ASME Paper 71-PVP-28, American Society of Mechanical Engineers, New York, May 1971.

9. J. M. Corum et al., "Experimental and Finite Element Stress Analysis of a Thin-Shell Cylinder-to-Cylinder Model," ASME Paper 71-PVP-36, American Society of Mechanical Engineers, New York, May 1971.
10. W. L. Greenstreet, S. E. Moore, and J. P. Callahan, Second Annual Progress Report on Studies in Applied Solid Mechanics (Nuclear Safety), ORNL-4693 (July 1971).
11. J. M. Corum and W. L. Greenstreet, "Experimental Elastic Stress Analysis of Cylinder-to-Cylinder Shell Models and Comparisons with Theoretical Predictions," Paper No. G2/5, First International Conference on Structural Mechanics in Reactor Technology, Berlin, Germany, Sept. 20-24, 1971.
12. R. W. Clough, G. H. Powell, and A. N. Gantayat, "Stress Analysis of B16.9 Tees by the Finite Element Method," Paper No. F4/7, First International Conference on Structural Mechanics in Reactor Technology, Berlin, Germany, Sept. 20-24, 1971.
13. R. L. Johnson, Photoelastic Determination of Stresses in ASA B16.9 Tees, Research Report 71-9E7-PHOTO-R2, Westinghouse Research Laboratory (November 1971).
14. E. C. Rodabaugh and S. E. Moore, Phase Report No. 115-10 on Comparisons of Test Data with Code Methods for Fatigue Evaluations, ORNL-TM-3520 (November 1971).
15. W. G. Dodge and S. E. Moore, Stress Indices and Flexibility Factors for Moment Loadings on Elbows and Curved Pipe, ORNL-TM-3658 (March 1972).
16. J. E. Brock, Elastic Buckling of Heated, Straight-Line Piping Configurations, ORNL-TM-3607 (March 1972).
17. J. M. Corum et al., Theoretical and Experimental Stress Analysis of ORNL Thin-Shell Cylinder-to-Cylinder Model No. 1, ORNL-4553 (October 1972).
18. W. G. Dodge and J. E. Smith, A Diagnostic Procedure for the Evaluation of Strain Data from a Linear Elastic Test, ORNL-TM-3415 (November 1972).
19. W. G. Dodge and S. E. Moore, "Stress Indices and Flexibility Factors for Moment Loadings on Elbows and Curved Pipe," Welding Research Council Bulletin 179, December 1972.
20. W. L. Greenstreet, S. E. Moore, and J. P. Callahan, Third Annual Progress Report on Studies in Applied Solid Mechanics (Nuclear Safety), ORNL-4821 (December 1972).

21. W. G. Dodge and S. E. Moore, ELBOW: A Fortran Program for the Calculation of Stresses, Stress Indices, and Flexibility Factors for Elbows and Curved Pipe, ORNL-TM-4098 (April 1973).
22. W. G. Dodge, Secondary Stress Indices for Integral Structural Attachments to Straight Pipe, ORNL-TM-3476 (June 1973).

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

NOMENCLATURE

- $A_\ell = 4L_1L_2$ = loaded area (base of lug)
 $A_p = 2\pi r t$ = cross-sectional area of pipe
 B = stress index for sustained loads
 B_T, B_L, B_C = stress indices for radial thrust, longitudinal moment, and circumferential moment loadings respectively
 B_1, B_2 = stress indices as defined by the Code*
 C = stress index for primary-plus-secondary stresses
 C_T, C_L, C_C = stress indices for radial thrust, longitudinal moment, and circumferential moment loadings respectively; the normalizing stress is the nominal stress at base of the lug
 C_T^*, C_L^*, C_C^* = analogous to C_T , etc., except that the normalizing stress is the nominal stress in the pipe
 C_1, C_2, C_3, C_3' = stress indices as defined by the Code
 D_o = outside diameter of the pipe
 E = modulus of elasticity as defined by the Code
 K = stress index for local stresses
 K_e = fatigue evaluation factor defined by the Code in Paragraph NB-3653.6
 K_ℓ = stress index for local stresses at lug fillet welds
 K_1, K_2, K_3 = peak stress indices as defined by the Code
 L_1 = one-half the width of the loaded area in the circumferential direction
 L_2 = one-half the length of the loaded area in the longitudinal direction
 L = one-half the length of a simply supported cylindrical shell
 L_a, L_b, L_c, L_d = lug dimensions used in defining indices for shear and torsional loadings
 M_i, M_i^* = magnitudes of moment-vector ranges as used in the Code's design analysis
 M_L, M_C, M_T = longitudinal, circumferential, and torsional moments respectively
 $\bar{M}_T$ = torsional-moment-load term
 M_L^*, M_C^*, M_T^* = range of longitudinal, circumferential, and torsional moments respectively
 $\bar{M}_T^*$ = range of $\bar{M}_T$

*Code, as used in this report, refers to ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components; see Ref. 1.

P = design pressure as specified by the Code

P_o = range of operating pressure according to the Code

p = internal pressure

Q_1, Q_2 = circumferential and longitudinal shear forces respectively

Q_1^*, Q_2^* = range of circumferential and longitudinal shear forces respectively

r = mean pipe radius

S_{alt} = alternating stress intensity as defined by the Code

S_e = expansion stress intensity

S_{ℓ} = nominal stress at the base of the lug

= $W/4L_1L_2$ for radial thrust load

= $6M_L/8L_1L_2^2$ for longitudinal moment load

= $6M_C/8L_1^2L_2$ for circumferential moment load

S_n = primary-plus-secondary stress-intensity range

S_p = peak-stress-intensity range

t = pipe wall thickness

T_{ℓ} = average temperature of lug

T_w = average temperature of pipe

W = radial thrust load

W^* = range of radial load

x = axial coordinate

Z = section modulus

$Z_p = \pi r^2 t$, section modulus of the pipe

$Z_{\ell L} = 8L_1L_2^2/6$, lug section modulus for longitudinal moment loading M_L

$Z_{\ell C} = 8L_1^2L_2/6$, lug section modulus for circumferential moment loading M_C

α = coefficient of thermal expansion

$\beta_1 = L_1/r$, nondimensional parameter perpendicular to the pipe axis

$\beta_2 = L_2/r$, nondimensional parameter parallel with the pipe axis

$\gamma = r/t$, radius-to-thickness ratio of cylindrical shell

$\emptyset$ = circumferential coordinate

σ = normal stress

τ = shear stress

STRESS INDICES AT LUG SUPPORTS ON PIPING SYSTEMS

E. C. Rodabaugh W. G. Dodge
S. E. Moore

ABSTRACT

Stress indices and the appropriate simplified design formulas are developed for analyzing integral lug attachments on straight pipe according to the philosophy of Section III of the ASME Boiler and Pressure Vessel Code for Class I Piping Systems. Indices are developed for the evaluation of primary stresses, primary-plus-secondary stresses, and peak stresses due to internal pressure in the pipe; for radial thrust and transverse shear forces and torsional and bending moment loads acting on the lug; and for a thermal gradient between the pipe and the lug. The indices for thrust and bending moment loads are based on an extensive parameter study and are represented by simple formulas that may be used directly by designers and/or incorporated into codes and standards. From comparisons with other methods of analysis these formulas are considered to be more accurate and easier to use. Indices for the other loadings are based in part on strength-of-materials theory and information in the literature. Specific recommendations are made for incorporating the stress indices and design formulas into the ASME Code. As an example, a simple pipe support design is analyzed using the recommended formulas.

Key words: stress indices, stress analysis, pipe supports, pipe attachments, straight pipe, piping code, ASME BPV Section III, lugs, lug supports, nuclear piping, pressure vessel code, ORNL piping program.

1. INTRODUCTION AND SCOPE

The ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components¹ (hereinafter referred to as the Code) includes a "simplified" method of design analysis for piping components (Subsubarticle NB-3650)* which makes use of stress indices. The method uses three types

*References to articles, subarticles, paragraphs, tables, or figures from the Code are identified by number (e.g., NB-xxxx), as appropriate.

of indices (B, C, and K), which are related to the three categories of Code-allowable stresses: primary, primary-plus-secondary, and peak stresses, respectively, in conjunction with five rather simple equations. In the Code the design formulas are given as Eqs. (9) to (14) in Paragraphs NB-3652 and NB-3653, and stress indices for many commonly used piping components are given in Table NB-3683.2-1. The Code equations are reproduced here in Table 1.* Reference to these equations as Eqs. (C9)–(C14) is made throughout this report when referring to Code equations in order to avoid confusion with numbered equations of the text.

To the extent that stress indices are provided for the particular piping component and loading, the Code-analysis method is a relatively simple way to check a piping design for compliance with Code requirements. At present, however, stress indices are not provided for lugs or other types of pipe supports. The purpose of this report is to develop stress indices for points on piping systems that are supported or restrained by lugs which are integral with the pipe, that is, attached with full-penetration welds.

Three types of loadings are involved in the analysis: (1) mechanical loads, such as thrust, shear, and bending moments; (2) internal pressure; and (3) thermal gradients. Conceptually B, C, and K stress indices could be developed for each of the loadings and added to Table NB-3683.2-1 for direct use in Eqs. (C9)–(C14). However, since this table and its footnotes are already quite lengthy and involved and since the evaluation of stresses at lugs involves an almost independent set of definitions, the approach taken here is to modify the existing Code equations by adding new terms, specifically for integral lugs. The modified equations take the generic form shown in Table 2, where the new terms are identified by the subscript "lug." Definitions for these additions in terms of appropriate B_l , C_l , and K_l stress indices for each of the three types of loadings are developed in Sections 2 to 5. Their application, as a supplement to the present Code analysis, is discussed in Section 6, with specific recommendations for Code use given in Appendix A.

*Table 1 is presented with appropriate definitions given in the nomenclature, but without the accompanying footnotes, qualifications, or cross references given in the Code.

Table 1. Equations^a for the simplified design-analysis procedures of the Code

Design stress formula	Code equation number
$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M_i \leq 1.5S_m$	(C9)
$S_n = C_1 \frac{PD_o}{2t} + C_2 \frac{D_o}{2I} M_i$ $+ \frac{1}{2(1-\nu)} E\alpha \Delta T_1 + C_3 E_{ab} \alpha_a T_a - \alpha_b T_b \leq 3S_m$	(C10)
$S_p = K_1 C_1 \frac{PD_o}{2t} + K_2 C_2 \frac{D_o}{2I} M_i$ $+ \frac{1}{2(1-\nu)} K_3 E\alpha \Delta T_1 + K_3 C_3 E_{ab} \alpha_a T_a - \alpha_b T_b $ $+ \frac{1}{1-\nu} E\alpha \Delta T_2 $	(C11)
$S_e = C_2 \frac{D_o}{2I} M_i^* \leq 3S_m$	(C12)
$C_1 \frac{PD_o}{2t} + C_2 \frac{D_o M_i}{2I} + C_3 E_{ab} \alpha_a T_a - \alpha_b T_b \leq 3S_m$	(C13)
$S_{alt} = \frac{K_e}{2} S_p$	(C14)

^aAbstracted from Paragraph NB-3650 of the Code; see Nomenclature and the Code for symbol definitions.

Table 2. Modified stress index equations for the design analysis of pipe with integral attachments remote from other discontinuities^a

Primary stress-intensity limit:

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M_i + (P_L + P_b)_{lug} \leq 1.5S_m$$

Primary-plus-secondary stress-intensity-range limit:

$$S_n = C_1 \frac{PD_o}{2t} + C_2 \frac{D_o}{2I} M_i + \frac{1}{2(1-\nu)} E\alpha|\Delta T_1| + (S_n)_{lug} \leq 3S_m$$

Peak stress-intensity range:

$$S_p = K_1 C_1 \frac{PD_o}{2t} + K_2 C_2 \frac{D_o}{2I} M_i + \frac{1}{2(1-\nu)} K_3 E\alpha|\Delta T_1| + \frac{1}{1-\nu} E\alpha|\Delta T_2| + (S_p)_{lug}$$

Nominal expansion-stress limit:

$$S_e = C_2 \frac{D_o}{2I} M_i^* \leq 3S_m$$

Nominal primary-plus-secondary membrane-plus-bending stress-intensity-range limit (excluding thermal-bending and thermal-expansion stresses):

$$C_1 \frac{PD_o}{2t} + C_2 \frac{D_o}{2I} M_i \leq 3S_m$$

Alternating stress intensity:

$$S_{alt} = K_e \frac{S_p}{2}$$

^aSince the intended application is for lugs remote from other discontinuities, the term $|\alpha_a T_a - \alpha_b T_b|$ in Eqs. (C10), (C11), and (C13) is zero.

Maximum stress intensities induced in the pipe-lug component by the different types of loading are discussed in Sections 2 to 5. Stress indices for thrust and bending moment loads applied to the lug are discussed in Section 2 based on the analysis and numerical results given by Dodge² for the maximum surface stresses (membrane plus bending) in the pipe. The primary-plus-secondary stress indices (C-type indices) developed in Ref. 2 are renormalized to conform with the equations of Table 1. The maximum membrane and surface stresses from the numerical parameter study are tabulated in Appendix B since they were not reported in Ref. 2. Local stresses and the corresponding K-type stress indices for thrust and bending moment loads are also discussed in Section 2.

In addition to the stress-index analysis method for integral lug attachments given in this report, there are two methods that are widely used in design: (1) the design equations prescribed in Paragraph PW-43 of the ASME Boiler and Pressure Vessel Code, Section I, Power Boilers,³ referred to herein as the PW-43 method, and (2) the analysis presented in Welding Research Council Bulletin 107 (WRC-107 method).⁴ Since both of these methods are concerned only with thrust and bending moment loads, comparative results with the indices developed in Section 2 are also given in this section. Conclusions drawn from these comparisons indicate that the PW-43 method gives maximum allowable loadings which are comparable with the results from the stress-index analysis, although the applicable range for the PW-43 method is limited. Comparisons with the WRC-107 method show that the results are similar for square lugs but that for long rectangular lugs the WRC-107 method may be incorrect.

Stresses due to transverse shear and torsional moment loads on the lug are discussed in Section 3. The analyses are based on rather simple strength-of-materials assumptions that are believed to be conservative and therefore adequate for defining stress indices for Code use. An example calculation for the design of shear lug supports for a vertical pipe section is given in Appendix C.

Calculated stresses due to internal pressure, discussed in Section 4, are based on an analysis of a cylindrical shell with a solid circular insert. For square or rectangular lugs, approximations are introduced

which should give conservative results, although further study is needed to verify this point.

Stresses due to temperature differences between the lug and the pipe are discussed in Section 5. Here again the analytical model is only an approximation, and a more rigorous analysis would be desirable although the results given here are believed to be conservative.

All the basic analyses used in this report are concerned with the stresses in the pipe and not in the lug itself. A portion of the lug, however, is included within the jurisdiction of the Code rules given in Subarticle NB-3600 Piping Design, whereas the remainder of the lug would be covered by Subsection NF, Component Supports.* For the type of lugs considered in this report, the lug-pipe interface and the portion of the lug within a distance of $0.353r$ from the surface of the pipe, where r is one-half the maximum cross-sectional dimension of the lug, are within the jurisdiction of NB-3600. With relatively simple modifications and/or restrictions, the equations developed in this report for calculating stresses in the pipe wall due to thrust, shear, and moment loads on the lug can also be used to evaluate the adequacy of the lug-pipe interface. These modifications and restrictions are therefore included as appropriate. The development of stress indices given here applies basically to rectangular integral lugs on straight pipe remote from other discontinuities.

In piping systems, it may also be desirable to place lugs on elbows or tees. While the indices developed here may (with appropriate modifications) be applicable to such components, we believe that such extensions should not be attempted until after the present approach to the integral-lug problem has been studied by potential users.

*According to the December 1972 draft of Subsection NF, this boundary is established in Subarticle NF-1000, Paragraph NF-1512.

2. THRUST AND BENDING MOMENT LOADS

Maximum Stress Intensities

Stress indices developed by Dodge² for calculating the maximum surface stresses (membrane plus bending) in straight circular pipe due to thrust and moment loadings at lug attachments as indicated in Fig. 1 are

ORNL-DWG 73-5841R

Fig. 1. Coordinate system and positive directions for loadings.

given by

$$C_T^* = 12.0(r/t)^{0.64} \eta^{1.54} , \quad (1a)$$

$$C_L^* = 1.2(r/t)^{0.74} \eta^{4.74} , \quad (1b)$$

$$C_C^* = 1.8(r/t)^{0.90} \eta^{3.40} , \quad (1c)$$

where C_T^* is for a radial thrust load, C_L^* is for a longitudinal moment load, and C_C^* is for a circumferential moment load acting on the lug. The nondimensional parameter η is given by

$$\eta = -(X_1 \cos \theta + Y_1 \sin \theta) - \frac{1}{A_0} (X_1 \sin \theta - Y_1 \cos \theta)^2 , \quad (1d)$$

where θ is a constant and X_1 and Y_1 are given in terms of the constants X_0 and Y_0 and the nondimensional width and length parameters β_1 and β_2 :

$$X_1 = X_0 + \log_{10} \beta_1 , \quad (1e)$$

$$Y_1 = Y_0 + \log_{10} \beta_2 . \quad (1f)$$

Here $\beta_1 = L_1/r$ is one-half the width of the loaded area in the circumferential direction divided by the mean pipe radius r and $\beta_2 = L_2/r$ is one-half the length of the loaded area in the longitudinal direction divided by r .

Appropriate numerical values for the constants A_0 , θ , X_0 , and Y_0 are tabulated below for each of the three different types of load.

Load	A_0	θ	X_0	Y_0
Thrust	2.2	40°	0	0.05
Longitudinal moment	2.0	50°	-0.45	-0.55
Circumferential moment	1.8	40°	-0.75	-0.60

Maximum stress intensities in the pipe $\bar{\sigma}_T$, $\bar{\sigma}_L$, and $\bar{\sigma}_C$ corresponding to the thrust and moment loadings W , M_L , and M_C on the lug are then given

by the following stress-index equations in terms of the pipe dimensions:

$$\bar{\sigma}_T = C_T^* W / A_p , \quad (2a)$$

$$\bar{\sigma}_L = C_L^* M_L / Z_p , \quad (2b)$$

$$\bar{\sigma}_C = C_C^* M_C / Z_p , \quad (2c)$$

where A_p is the cross-sectional area of the pipe and Z_p is the section modulus of the pipe. This form of stress index has certain advantages in that, for moment loading, the nominal stresses (M_L/Z_p) and (M_C/Z_p) are the same as used in the Code for moments acting on the pipe cross section. However, as indicated in the Introduction, it appears desirable to consider the additional stresses in the pipe caused by the lug separately from the stresses presently covered by the Code. With this approach we may change the normalizing base of Dodge's C^* -type indices to the nominal stresses in the lug. Thus C_T , C_L , and C_C may be defined as

$$C_T = \bar{\sigma}_T / (W/A_\ell) , \quad (3a)$$

$$C_L = \bar{\sigma}_L / (M_L/Z_{\ell L}) , \quad (3b)$$

$$C_C = \bar{\sigma}_C / (M_C/Z_{\ell C}) , \quad (3c)$$

where $A_\ell = 4L_1L_2$ is the cross-sectional area of the lug at the pipe surface, $Z_{\ell L} = (4/3)L_1L_2^2$ is the section modulus of the lug at the pipe surface for an applied moment M_L , and $Z_{\ell C} = (4/3)L_1^2L_2$ is the section modulus of the lug at the pipe surface for an applied moment M_C . Conversion from the C^* indices to the C indices is obtained by the following relationships:

$$C_T = C_T^* (A_\ell / A_p) = C_T^* [4L_1L_2 / (2\pi r t)] = C_T^* (2\beta_1\beta_2\gamma/\pi) , \quad (4a)$$

$$C_L = C_L^* (Z_{\ell L} / Z_p) = C_L^* [8L_1L_2^2 / (6\pi r^2 t)] = C_L^* [4\beta_1\beta_2^2\gamma / (3\pi)] , \quad (4b)$$

$$C_C = C_C^* (Z_{\ell C} / Z_p) = C_C^* [8L_1^2L_2 / (6\pi r^2 t)] = C_C^* [4\beta_1^2\beta_2\gamma / (3\pi)] , \quad (4c)$$

where $\gamma = r/t$ is the radius-to-thickness ratio of the pipe.

Numerical values of C_T , C_L , and C_C are given in Tables 3 to 5 respectively. Table 3, for thrust load, gives the ratios of the maximum primary-plus-secondary stress intensity in the pipe to the nominal compressive (or tensile) stress in the lug at the pipe surface. (See Fig. 1 for assumed positive load directions.) Reversing the direction of the load reverses the sign of all stresses. Tables 4 and 5, for longitudinal and circumferential moment loadings M_L and M_C , respectively, give the ratios of the maximum primary-plus-secondary stress in the pipe to the nominal

Table 3. Stress indices C_T for thrust loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.66	2.05	6.30	19.9	62.1
	0.1	1.11	3.44	10.7	33.5	104.0
	0.2	1.72	5.36	16.7	52.1	162.0
	0.3	2.12	6.61	20.6	64.2	200.0
	0.4	2.38	7.43	23.2	72.2	225.0
	0.5	2.55	7.95	24.8	77.2	241.0
0.1	0.05	1.02	3.19	9.95	31.0	96.7
	0.1	1.74	5.44	16.9	52.8	165.0
	0.2	2.74	8.53	26.6	82.9	258.0
	0.3	3.38	10.5	32.8	102.0	319.0
	0.4	3.80	11.8	36.9	115.0	358.0
0.2	0.05	1.46	4.54	14.1	44.1	137.0
	0.1	2.52	7.84	24.4	76.2	237.0
	0.2	3.96	12.4	38.5	120.0	374.0
	0.3	4.87	15.2	47.3	147.0	460.0
0.3	0.05	1.68	5.22	16.3	50.8	158.0
	0.1	2.93	9.12	28.4	88.6	276.0
	0.2	4.60	14.4	44.7	139.0	434.0
0.4	0.05	1.77	5.52	17.2	53.6	167.0
	0.1	3.12	9.72	30.3	94.4	294.0
	0.2	4.89	15.3	47.5	148.0	462.0
0.5	0.05	1.78	5.56	17.3	54.0	168.0
	0.1	3.17	9.87	30.8	95.9	299.0

Table 4. Stress indices C_L for longitudinal moment loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.086	0.286	0.955	3.19	10.7
	0.1	0.197	0.658	2.20	7.34	24.5
	0.2	0.385	1.29	4.30	14.4	47.9
	0.3	0.514	1.72	5.73	19.1	64.0
	0.4	0.593	1.98	6.61	22.1	73.8
	0.5	0.634	2.12	7.07	23.6	78.9
0.1	0.05	0.119	0.399	1.33	4.45	14.9
	0.1	0.290	0.970	3.24	10.8	36.2
	0.2	0.605	2.02	6.75	22.6	75.4
	0.3	0.846	2.82	9.44	31.5	105.0
	0.4	1.01	3.39	11.3	37.8	126.0
0.2	0.05	0.144	0.480	1.60	5.36	17.9
	0.1	0.368	1.23	4.11	13.7	45.8
	0.2	0.811	2.71	9.05	30.2	101.0
	0.3	1.18	3.93	13.1	43.8	146.0
0.3	0.05	0.147	0.491	1.64	5.48	18.3
	0.1	0.388	1.30	4.33	14.5	48.3
	0.2	0.880	2.94	9.82	32.8	110.0
0.4	0.05	0.142	0.475	1.59	5.30	17.7
	0.1	0.383	1.28	4.28	14.3	47.7
	0.2	0.888	2.97	9.91	33.1	111.0
0.5	0.05	0.134	0.447	1.49	4.99	16.7
	0.10	0.368	1.23	4.11	13.7	45.8

bending stress in the lug at the pipe surface. It may be noted in these tables that there are several combinations of β_1 , β_2 , and γ for which C_T , C_L , and C_C are less than unity. For these dimensional parameters, the maximum stress is less than the nominal stress in the lug at the lug-pipe interface. Since the analysis procedure is intended to cover this interface as well as the pipe, a lower limit of 1.0 must be imposed on the values of the C_T , C_L , and C_C indices. These indices are used in

Table 5. Stress indices C_C for circumferential moment loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.066	0.246	0.918	3.43	12.8
	0.1	0.104	0.388	1.45	5.41	20.2
	0.2	0.148	0.553	2.06	7.70	28.7
	0.3	0.172	0.640	2.39	8.92	33.3
	0.4	0.184	0.688	2.57	9.58	35.7
	0.5	0.190	0.711	2.65	9.90	37.0
0.10	0.05	0.184	0.685	2.56	9.55	35.6
	0.1	0.303	1.13	4.22	15.7	58.7
	0.2	0.450	1.68	6.27	23.4	87.4
	0.3	0.536	2.00	7.46	27.8	104.0
	0.4	0.587	2.19	8.18	30.5	114.0
0.2	0.05	0.459	1.71	6.40	23.9	89.1
	0.1	0.798	2.98	11.1	41.5	155.0
	0.2	1.24	4.64	17.3	64.7	241.0
	0.3	1.52	5.67	21.2	79.0	295.0
0.3	0.05	0.735	2.74	10.2	38.2	143.0
	0.1	1.32	4.94	18.4	68.8	257.0
	0.2	2.13	7.95	29.7	111.0	413.0
0.4	0.05	0.986	3.68	13.7	51.3	191.0
	0.1	1.83	6.84	25.5	95.3	356.0
	0.2	3.02	11.3	42.1	157.0	586.0
0.5	0.05	1.21	4.50	16.8	62.7	234.0
	0.1	2.30	8.59	32.1	120.0	447.0

Section 6 for developing the additaments $(S_n)_{lug}$ for primary-plus-secondary stresses and $(S_p)_{lug}$ for peak stresses.

The parametric study which formed the basis for developing Eqs. (1a), (1b), and (1c) included only the values of β_1 and β_2 listed in Tables 3 to 5; thus the equations should not be used for β_1 or β_2 values higher than 0.5. In addition, the equations should not be used for $\beta_1\beta_2$ greater than 0.075. These limitations permit a small extrapolation beyond the

$\beta_1\beta_2$ products included in the study but prevent large extrapolations. While the study included parameter values for γ up to 80, the general limitation on stress indices of $D_o/t = 100$ is recommended for the indices for lugs. These limitations are included in Appendix A, Recommended Addition to ASME Code Section III to Cover Straight Pipe with Integral Structural Attachments.

Ratios of the maximum calculated membrane stress to the maximum calculated secondary stress intensity are given in Tables 6 to 8 for radial thrust, longitudinal moment, and circumferential moment loadings respectively. For all three cases the tabulated values show that the membrane stresses are small with respect to the bending stresses. This fact is used in Section 6 as a part of the basis for establishing the B_ℓ indices used in the new term $(P_L + P_b)_{\text{lug}}$ for primary stresses. The directions and locations of the maximum stresses are included in Appendix B for reference.

Comparisons with ASME Boiler Code,
Section I, Paragraph PW-43

Paragraph PW-43 in Section I of the ASME Boiler Code³ gives a design method for loading on structural attachments which considers only radial thrust and longitudinal moment loadings; circumferential moment loadings are not considered. According to this code, the design is acceptable if the following inequality is satisfied:

$$\frac{W}{2L_2} + \frac{6M_L}{4L_2^2} \leq Y, \quad (5)$$

where Y is the maximum allowable load per unit length of the loaded area (lb/in.), W is the radial thrust load (lb), M_L is the longitudinal moment load (in.-lb), and L_2 is one-half the length of the loaded area (in.). The maximum load Y is given graphically in Fig. PW-43.1 of Ref. 3 (the load being identified there as P). The value of Y depends on whether the direction of loading is tensile or compressive and also on the outside diameter of the pipe for tensile loads. For arbitrary loading directions

Table 6. Ratios of maximum membrane stress
to maximum secondary stress intensity^a
for thrust loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.081	0.098	0.120	0.145	0.168
	0.1	0.093	0.112	0.131	0.148	0.187
	0.2	0.109	0.122	0.131	0.170	0.210
	0.3	0.118	0.122	0.144	0.185	0.224
	0.4	0.121	0.116	0.155	0.196	0.235
	0.5	0.123	0.117	0.165	0.207	0.248
0.1	0.05	0.103	0.122	0.144	0.164	0.202
	0.1	0.116	0.136	0.156	0.169	0.208
	0.2	0.135	0.149	0.156	0.190	0.222
	0.3	0.145	0.149	0.169	0.210	0.244
	0.4	0.151	0.143	0.182	0.227	0.264
0.2	0.05	0.139	0.164	0.187	0.203	0.229
	0.1	0.150	0.167	0.178	0.208	0.244
	0.2	0.176	0.186	0.183	0.219	0.255
	0.3	0.192	0.190	0.181	0.226	0.265
0.3	0.05	0.177	0.203	0.225	0.232	0.229
	0.1	0.187	0.202	0.204	0.228	0.250
	0.2	0.209	0.210	0.210	0.251	0.280
0.4	0.05	0.217	0.242	0.257	0.252	0.223
	0.1	0.227	0.241	0.235	0.233	0.242
0.5	0.05	0.261	0.284	0.290	0.270	0.203

^aMaximum secondary stress intensity for a nominal thrust load of $W/A_\ell = 1.0$ is the same as the stress index $C_T = C_T^*(2\beta_1\beta_2\gamma/\pi)$, as defined by Eq. (4a).

Table 7. Ratios of maximum membrane stress to maximum secondary stress intensity^a for longitudinal moment loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.022+ ^b	0.038+	0.063	0.105	0.166
	0.1	0.038+	0.064	0.103	0.157	0.214
	0.2	0.075	0.115	0.157	0.192	0.191
	0.3	0.112	0.149	0.188	0.198	0.179
	0.4	0.145	0.166	0.209	0.202	0.159
	0.5	0.171	0.174	0.219	0.199	0.194
0.1	0.05	0.031+	0.053+	0.087	0.140	0.215
	0.1	0.052+	0.086	0.136	0.202	0.269
	0.2	0.094	0.144	0.193	0.219	0.216
	0.3	0.136	0.180	0.215	0.218	0.186
	0.4	0.168	0.193	0.230	0.217	0.168
0.2	0.05	0.050+	0.085+	0.131	0.195	0.271
	0.1	0.080+	0.128	0.195	0.272	0.339
	0.2	0.137	0.204	0.270	0.295	0.274
	0.3	0.191	0.249	0.272	0.239	0.180
0.3	0.05	0.069+	0.114+	0.169	0.233	0.298
	0.1	0.110+	0.168	0.244	0.322	0.369
	0.2	0.182	0.263	0.338	0.353	0.304
0.4	0.05	0.089+	0.143	0.203	0.264	
	0.1	0.139+	0.350	0.287	0.360	0.386
0.5	0.05	0.109+	0.172	0.234	0.291	

^aMaximum secondary stress intensity for a nominal longitudinal moment load of $M_L/Z_{\theta L} = 1.0$ is the same as the stress index $C_L = C_L^*[4\beta_1\beta_2^2\gamma/(3\pi)]$, as defined by Eq. (4b).

^bThe "+" sign indicates that the maximum value may actually be larger, since the numerical parameter study did not extend far enough to locate the true maximum.

Table 8. Ratios of maximum membrane stress to maximum secondary stress intensity^a for circumferential moment loading

Shape parameter		Radius-to-thickness (γ) ratio				
β_1	β_2	5	10	20	40	80
0.05	0.05	0.010+ ^b	0.017+	0.028+	0.042+	0.063
	0.1	0.013+	0.021+	0.034+	0.051+	0.071
	0.2	0.016+	0.026+	0.040+	0.058+	0.079+
	0.3	0.018+	0.028+	0.042+	0.059+	0.076+
	0.4	0.020+	0.029+	0.042+	0.057+	0.073+
	0.5	0.021+	0.030+	0.042+	0.055+	0.069+
0.1	0.05	0.017+	0.027+	0.041+	0.060	0.085
	0.1	0.020+	0.032+	0.049+	0.069	0.097
	0.2	0.025+	0.039+	0.059+	0.080	0.100
	0.3	0.029+	0.043+	0.062+	0.083+	0.101
	0.4	0.031+	0.045+	0.063+	0.082+	0.100+
0.2	0.05	0.034+	0.046	0.064	0.087	0.108
	0.1	0.039+	0.053	0.072	0.095	0.118
	0.2	0.048+	0.065	0.086	0.107	0.126
	0.3	0.056+	0.076+	0.095	0.113	0.128
0.3	0.05	0.048	0.064	0.085	0.102	0.113
	0.1	0.053	0.070	0.092	0.110	0.115
	0.2	0.064	0.084	0.106	0.125	0.129
0.4	0.05	0.063	0.083	0.101	0.114	0.111
	0.1	0.068	0.087	0.107	0.113	0.112
0.5	0.05	0.080	0.100	0.118	0.122	0.108

^aMaximum secondary stress intensity for a nominal circumferential moment load of $M_C/Z_{\ell C} = 1.0$ is the same as the stress index $C_C = C_C^*[4\beta_1^2\beta_2\gamma/(3\pi)]$, as defined by Eq. (4c).

^bThe "+" sign indicates that the maximum value may actually be larger, since the numerical parameter study did not extend far enough to locate the true maximum.

and all pipe diameters, an approximate value for Y is given by

$$Y = 30,000 \frac{t^2}{D}, \quad (6)$$

where t is the pipe wall thickness and D is the outside diameter. Substituting Eq. (6) into Eq. (5), the maximum allowable loads must satisfy the following:

$$\frac{W}{2L_2} + \frac{6M_L}{4L_2^2} = 30,000 \frac{t^2}{D}. \quad (7)$$

The basis for the design method is not known to the authors. However, in discussing the background of PW-43, Melworm and Berman⁵ state

"It was assumed that the tube material behaved elastically. Circumferential tube stress determinations were made for pressure and external load. Thick wall (Lamé) theory was used to determine the pressure stresses and thin-ring combined with curved beam theory were used to obtain the stresses caused by external load. The maximum circumferential tube stress was limited to an empirical value of 20,000 psi."

Apparently the 20,000-psi limit was placed on the sum of the circumferential stresses in the pipe due to both internal pressure and external loads. If that were so, and assuming that the circumferential stress due to pressure was equal to a typical value of 15,000 psi, the allowable stress which would be available for the external loads would be 5000 psi. We can thus write an equation for the maximum allowable stress in the pipe due to W and M_L in terms of the lug dimensions and unknown dimensionless coefficients C'_T and C'_L :

$$C'_T \frac{W}{A_\ell} + C'_L \frac{M_L}{Z_{\ell C}} = 5000 \text{ psi}, \quad (8)$$

where the cross-sectional area of the lug $A_\ell = 4L_1L_2$ and the section modulus $Z_{\ell C} = (4/3)L_1L_2^2$. Simply rearranging the terms in Eq. (7) gives

$$\frac{2L_1}{6t} \frac{D}{t} \left[\frac{W}{4L_1L_2} + \frac{M_L}{(4/3)L_1L_2^2} \right] = 5000 \text{ psi}. \quad (9)$$

From Eqs. (8) and (9),

$$C'_T = C'_L = L_1 D / 3t^2 ,$$

or, in terms of nondimensional parameters $\beta_1 = L_1/r$ and $\gamma = r/t$,

$$C'_T = C'_L = (2/3)\beta_1\gamma^2 .$$

It is thus apparent that the PW-43 analysis method implies equal stress indices C'_T and C'_L for both thrust and moment loadings, although the width of the lug ($2L_1$) does not enter into the PW-43 criteria [Eq. (5)]. Typically the lug width would be about 0.2 times the pipe diameter, in which case $\beta_1 = 0.1$. Using this value for β_1 , a direct comparison can be made between the PW-43 design method and the stress indices C_T and C_L developed in this report.

Table 9 gives some comparisons of $(2/3)\beta_1\gamma^2$ with C_T and C_L . As judged by the stress index, the PW-43 method tends to be unconservative for thrust load (except for $\gamma = 40$) and conservative for moment load.

Table 9. Comparison of PW-43 method with C indices for thrust and longitudinal moment loading

β_1	β_2	Index source	Radius-to-thickness (γ) ratio			
			5	10	20	40
0.1	Any	PW-43 ^a	1.67	6.67	26.7	107.0
0.1	0.2	C_T	2.74	8.53	26.6	82.9
		C_L	0.60	2.02	6.75	22.6
0.1	0.3	C_T	3.38	10.5	32.8	102.0
		C_L	0.85	2.82	9.44	31.5
0.1	0.4	C_T	3.80	11.8	36.9	115.0
		C_L	1.01	3.39	11.3	37.8

^aIndex = $(2/3)\beta_1\gamma^2$ for both thrust and longitudinal moment.

In general, however, the PW-43 method gives permissible loadings in the same range as the stress-index method when the latter method is used with $(S_n)_{\text{lug}} = 5000$ psi. The reason for the conservatism with respect to moment load is that PW-43 treats the maximum nominal stress due to a moment as equivalent to a thrust load acting along the entire length of the lug; that is, $6M/2L_2 = W$.

Comparisons with Welding Research
Council Bulletin 107

Welding Research Council Bulletin 107, Local Stresses in Spherical and Cylindrical Shells Due to External Loadings,⁴ is a widely used design guide based on theoretical and numerical results developed by Bijlaard^{6,7} for distributed loads on cylindrical shells as shown in Fig. 2. WRC Bulletin 107 also includes empirical corrections to Bijlaard's results that were based principally on results from tests of two models of cylindrical shells with nozzles. For lack of a more directly applicable analysis, Bijlaard's results have been used to estimate stresses due to thrust or moment loads applied to a nozzle in a cylindrical shell and to loads applied through a lug or other relatively rigid attachment.

The data given in WRC Bulletin 107 for thrust load are based on calculations for cylinders with length-to-radius ratios $L/r = 8$. The value of L/r used for moment loadings is not clearly stated; however, the stresses are only slightly dependent upon this ratio. The stresses for thrust load were calculated for the center of the loaded area, while those for moment loads are for the edge of the loaded area.

Dodge² calculated stresses based on Bijlaard's theory with the following refinements: (1) certain terms in the solution were retained in the numerical analysis as recommended by Weil, Laupa, and Murphy in their discussion at the end of Ref. 6; (2) the angularity of the surface loads was taken into consideration; and (3) the stresses due to "beam bending" were subtracted out. For sufficiently large values of L/r , the stresses become independent of L/r . (In using the stress-index method, as given in Section 6, the stresses due to beam bending are included in the existing Code equations.)

ORNL-DWG 72-12146R

BIJLAARD THEORY

LOADS ARE EQUILIBRATED BY DISTRIBUTED SHEARS AT END OF CYLINDER, $x = \pm L$

Fig. 2. Illustration of Bijlaard's distributed-load equivalence with thrust and moment loads.

The stress indices C_T , C_L , and C_C are based on calculated stresses at the edge of or outside* the loaded area. The major difference between Bulletin 107 and the stress indices is for thrust loading; Bulletin 107 gives stresses at the center of the loaded area, whereas the stress indices are based on stresses at the edge of the loaded area. Detailed comparisons for each of the three loadings are discussed below.

Thrust loading

Table 10 shows comparisons for thrust loadings on square loaded areas. Dodge** gives maximum stresses at the edge of the loaded area, whereas Bulletin 107 is based on stresses at the center of the loaded area. Stresses calculated using WRC 107 are higher than these reported in Ref. 2, and a check of the detailed calculations indicates the discrepancy is almost entirely due to the difference between the stresses at the center of the loaded area and those at the edge of the loaded area. Indices obtained by the approximate formula, Eq. (4a), however, are higher than or of the same order as the maximum stress obtained by Bulletin 107. This is primarily due to the conservatism introduced in developing approximate formulas with the required simplicity; for more details see Ref. 2.

Table 11 shows comparisons for thrust loadings on rectangular loaded areas for β_1/β_2 ratios of 1/4, 1/2, 2, and 4. The equations included in WRC 107 are approximations developed by Bijlaard⁷ based on a relatively few calculated results for these ratios. To avoid additional interpolations or extrapolations here, the same ratios were used in making the comparisons. The WRC 107 equations give, in effect, an "equivalent" square area. As shown in the table, the approximate equations of Bulletin 107 appear to be adequate, since, with only a few minor exceptions, the stresses obtained with these equations are not less than the more accurate results obtained by Dodge. For thrust loads on rectangular areas,

*For most parameters included in the study, the maximum surface stresses were at the edge of the loaded area. See Appendix B.

**Reference to Dodge in these comparisons is meant to indicate the detailed stresses presented in Ref. 2 and used as the basis for developing the approximate formulas, Eqs. (4a), (4b), and (4c).

Table 10. Comparisons for thrust loadings
on square loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\phi o}/S_L$	$\sigma_{\phi i}/S_L$	σ_{xo}/S_L	σ_{xi}/S_L
0.05	0.05	5	WRC 107	-0.49	0.39	-0.44	0.34
			Dodge	-0.42	0.32	-0.33	0.28
			Eq. (4a)	0.66			
		10	WRC 107	-1.71	1.35	-1.52	1.12
			Dodge	-1.54	1.15	-1.13	0.87
			Eq. (4a)	2.05			
		20	WRC 107	-6.08	4.48	-5.10	3.54
			Dodge	-5.39	3.96	-3.83	2.62
			Eq. (4a)	6.39			
		40	WRC 107	-21.44	15.04	-16.80	11.04
			Dodge	-18.46	13.37	-12.50	7.34
			Eq. (4a)	19.9			
		80	WRC 107	-73.44	49.44	-57.68	34.48
			Dodge	-60.95	43.93	-39.38	18.47
			Eq. (4a)	62.1			
0.10	0.10	5	WRC 107	-1.51	1.11	-1.35	0.94
			Dodge	-1.26	0.88	-0.89	0.70
			Eq. (4a)	1.74			
		10	WRC 107	-5.02	3.66	-4.37	2.82
			Dodge	-4.14	3.11	-2.93	1.96
			Eq. (4a)	5.44			
		20	WRC 107	-17.87	12.27	-14.48	8.56
			Dodge	-14.83	10.51	-9.34	4.96
			Eq. (4a)	16.9			
		40	WRC 107	-58.88	37.12	-45.76	23.36
			Dodge	-47.82	34.73	-28.75	10.89
			Eq. (4a)	52.8			
		80	WRC 107	-179.5	115.5	-136.8	53.63
			Dodge	-147.7	112.5	-87.45	18.99
			Eq. (4a)	165.0			
0.20	0.20	5	WRC 107	-4.23	2.78	-3.59	2.07
			Dodge	-3.27	2.12	-2.04	1.38
			Eq. (4a)	3.96			
		10	WRC 107	-13.15	8.35	-10.83	5.30
			Dodge	-10.74	7.33	-6.31	3.08
			Eq. (4a)	12.4			
		20	WRC 107	-41.92	27.20	-31.74	11.26
			Dodge	-34.03	24.99	-19.53	5.76
			Eq. (4a)	38.5			
		40	WRC 107	-125.7	77.06	-89.60	17.92
			Dodge	-103.4	61.91	-60.96	8.23
			Eq. (4a)	120.0			
		80	WRC 107	-351.5	226.0	-254.2	16.13
			Dodge	-306.2	257.5	-193.0	2.00
			Eq. (4a)	374.0			

^aFor thrust load (W of Fig. 2), $S_L = W/4L_1L_2$. Subscripts ϕ and x refer to the circumferential and axial directions respectively; i is inside surface and o is outside surface.

Table 11. Comparisons for thrust loadings on rectangular loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\phi o}/S_\ell$	$\sigma_{\phi i}/S_\ell$	σ_{xo}/S_ℓ	σ_{xi}/S_ℓ
0.05	0.20	5	WRC 107	-1.58	1.20	-1.22	0.81
			Dodge	-1.25	0.87	-0.89	0.70
			Eq. (4a)	1.72			
		10	WRC 107	-5.42	3.94	-3.76	2.24
			Dodge	-4.38	3.07	-2.91	1.95
			Eq. (4a)	5.36			
		20	WRC 107	-18.72	13.92	-12.00	6.24
			Dodge	-14.74	10.43	-9.33	4.99
			Eq. (4a)	16.7			
		40	WRC 107	-56.00	40.0	-32.3	11.5
			Dodge	-47.5	34.4	-28.8	11.0
			Eq. (4a)	52.1			
		80	WRC 107	-189.9	141.9	-107.5	30.7
			Dodge	-146.2	111.3	-85.9	17.6
			Eq. (4a)	162.0			
0.05	0.10	5	WRC 107	-0.89	0.69	-0.73	0.53
			Dodge	-0.73	0.52	-0.59	0.50
			Eq. (4a)	1.11			
		10	WRC 107	-3.16	2.32	-2.41	1.64
			Dodge	-2.60	1.83	-2.04	1.53
			Eq. (4a)	3.44			
		20	WRC 107	-10.88	8.32	-8.24	5.20
			Dodge	-8.97	6.16	-6.78	4.41
			Eq. (4a)	10.7			
		40	WRC 107	-37.12	26.24	-22.88	11.68
			Dodge	-29.72	19.84	-21.42	11.57
			Eq. (4a)	33.5			
		80	WRC 107	-114.4	77.60	-80.00	35.20
			Dodge	-93.79	61.37	-64.78	25.77
			Eq. (4a)	104.0			
0.20	0.10	5	WRC 107	-2.28	1.51	-2.16	1.39
			Dodge	-2.03	1.29	-1.55	1.19
			Eq. (4a)	2.52			

Table 11 (continued)

β_1	β_2	γ	Method	$\sigma_{\phi o}/S_\ell$	$\sigma_{\phi i}/S_\ell$	$\sigma_{x o}/S_\ell$	$\sigma_{x i}/S_\ell$
0.40	0.10	10	WRC 107	-7.52	4.48	-6.61	3.76
			Dodge	-6.90	4.35	-4.97	3.14
			Eq. (4a)	7.84			
		20	WRC 107	-23.84	14.56	-21.44	11.20
			Dodge	-21.14	13.86	-15.17	7.09
			Eq. (4a)	24.4			
		40	WRC 107	-75.20	40.00	-61.57	24.45
			Dodge	-66.77	42.39	-44.00	12.25
			Eq. (4a)	76.2			
		80	WRC 107	-220.2	117.8	-170.4	38.53
			Dodge	-188.3	125.3	-124.4	9.04
			Eq. (4a)	237.0			
		5	WRC 107	-2.85	1.38	-3.08	1.72
			Dodge	-2.84	1.42	-2.36	1.74
			Eq. (4a)	3.12			
		10	WRC 107	-8.77	3.33	-9.02	4.22
			Dodge	-9.07	4.39	-7.04	4.03
			Eq. (4a)	9.72			
		20	WRC 107	-25.28	9.28	-24.90	8.90
			Dodge	-26.51	12.20	-19.46	7.08
			Eq. (4a)	30.3			
		40	WRC 107	-71.42	17.66	-64.00	12.80
			Dodge	-70.57	31.94	-49.47	5.53
			Eq. (4a)	94.4			
		80	WRC 107	-177.4	31.49	-165.4	6.66
			Dodge	-172.5	83.69	-121.6	-20.11
			Eq. (4a)	294.0			

^aFor thrust loads $S_\ell = W/4L_1L_2$; for subscript notation, see footnote a of Table 10.

the stress-index equation, Eq. (4a), tends to be more conservative than either the WRC 107 or Dodge results for β_1/β_2 greater than 1; it is also more conservative than Dodge but less conservative than WRC 107 for β_1/β_2 less than 1.

Moment loadings

Tables 12 and 13 show comparisons for longitudinal and circumferential moment loadings, respectively, on square areas. For the range of parameters covered in these comparisons, the WRC 107 results and those obtained by Dodge are essentially the same — within the accuracy with which the graphs in Bulletin 107 can be read and interpolated. This indicates that the modifications made in Ref. 2 do not produce significant changes within this parameter range. The parameters for which large empirical corrections were made in Bulletin 107 are not included. For both types of moment loadings, Eqs. (4b) and (4c) tend to be conservative with respect to the other two calculations. For the case where $\beta_1 = \beta_2 = 0.2$, for example, Eqs. (4b) and (4c) give results which range from about 1.1 to 1.7 times the results of the other two.

Comparisons for longitudinal and circumferential moment loadings on rectangular areas are shown in Tables 14 and 15 respectively. For longitudinal moment loadings (Table 14), the approximate equations of Bulletin 107 apparently failed to locate the maximum stress and for some parameters tended to underestimate the magnitude. See, for example, the case for $\beta_1 = 0.05$, $\beta_2 = 0.20$, and $\gamma = 80$, where WRC 107 gives a maximum stress of $-25.4S_\ell$ on the outside surface in the axial direction (σ_{x0}), whereas Dodge's calculations (based on Bijlaard's complete equations) gives a maximum stress of $-39.3S_\ell$ on the outside surface in the circumferential direction ($\sigma_{\phi 0}$). On the other hand, the stress-index equation [for this case, Eq. (4b)] always tends to be conservative with respect to the maximum.

The comparisons for a circumferential moment loading on rectangular areas given in Table 15 show that the approximate equations of WRC 107 give locations and directions for the maximum stresses that generally agree with the more detailed calculations of Dodge. However, for cases

Table 12. Comparisons for longitudinal moment loadings on square loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\phi 0}/S_\ell$	$\sigma_{\phi 1}/S_\ell$	σ_{x0}/S_ℓ	σ_{x1}/S_ℓ
0.05	0.05	5	WRC 107	-0.04	0.03	-0.06	0.05
			Dodge	-0.03	0.03	-0.05	0.05
			Eq. (4b)			0.09	
		10	WRC 107	-0.14	0.12	-0.21	0.21
			Dodge	-0.13	0.12	-0.21	0.21
			Eq. (4b)			0.29	
		20	WRC 107	-0.54	0.45	-0.81	0.79
			Dodge	-0.52	0.42	-0.80	0.78
			Eq. (4b)			0.95	
		40	WRC 107	-2.16	1.62	-3.21	3.07
			Dodge	-2.11	1.51	-3.15	2.91
			Eq. (4b)			3.19	
		80	WRC 107	-8.51	5.31	-11.95	11.09
			Dodge	-8.29	4.93	-11.08	10.21
			Eq. (4b)			10.7	
0.10	0.10	5	WRC 107	-0.14	0.12	-0.21	0.21
			Dodge	-0.14	0.12	-0.21	0.21
			Eq. (4b)			0.29	
		10	WRC 107	-0.55	0.40	-0.80	0.76
			Dodge	-0.57	0.42	-0.81	0.78
			Eq. (4b)			0.97	
		20	WRC 107	-2.16	1.30	-2.93	2.71
			Dodge	-2.26	1.40	-2.98	2.76
			Eq. (4b)			3.24	
		40	WRC 107	-7.92	3.86	-10.06	8.89
			Dodge	-8.59	4.21	-10.22	8.98
			Eq. (4b)			10.8	
		80	WRC 107	-27.18	9.69	-29.72	23.53
			Dodge	-30.13	10.58	-31.58	25.25
			Eq. (4b)			36.2	
0.20	0.20	5	WRC 107	-0.56	0.34	-0.75	0.69
			Dodge	-0.55	0.33	-0.74	0.69
			Eq. (4b)			0.81	
		10	WRC 107	-2.06	0.88	-2.53	2.20
			Dodge	-2.03	0.92	-2.50	2.21
			Eq. (4b)			2.71	
		20	WRC 107	-7.19	2.28	-8.45	6.91
			Dodge	-7.25	2.38	-7.52	5.96
			Eq. (4b)			9.05	
		40	WRC 107	-21.63	4.99	-22.87	16.01
			Dodge	-22.15	4.29	-19.76	12.77
			Eq. (4b)			30.2	
		80	WRC 107	-58.88	2.56	-52.56	25.26
			Dodge	-50.65	4.25	-48.48	21.75
			Eq. (4b)			101.0	

^aFor longitudinal moment loading, $S_\ell = 6M_L/(8L_1L_2^2)$; for sub-script notation see footnote a of Table 10.

Table 13. Comparisons for circumferential moment loadings on square loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\theta\theta}/S_\ell$	$\sigma_{\theta i}/S_\ell$	$\sigma_{x\theta}/S_\ell$	σ_{xi}/S_ℓ
0.05	0.05	5	WRC 107	-0.05	0.05	-0.03	0.03
			Dodge	-0.05	0.05	-0.03	0.03
			Eq. (4c)	0.066			
		10	WRC 107	-0.21	0.21	-0.13	0.13
			Dodge	-0.20	0.20	-0.13	0.12
			Eq. (4c)	0.25			
		20	WRC 107	-0.85	0.83	-0.51	0.50
			Dodge	-0.78	0.75	-0.50	0.47
			Eq. (4c)	0.92			
		40	WRC 107	-3.43	3.29	-2.08	1.89
			Dodge	-3.09	2.91	-1.99	1.78
			Eq. (4c)	3.43			
		80	WRC 107	-13.37	12.49	-8.01	6.84
			Dodge	-12.12	11.05	-7.80	6.45
			Eq. (4c)	12.8			
0.10	0.10	5	WRC 107	-0.21	0.21	-0.14	0.13
			Dodge	-0.22	0.21	-0.13	0.12
			Eq. (4c)	0.30			
		10	WRC 107	-0.85	0.81	-0.53	0.48
			Dodge	-0.87	0.83	-0.52	0.47
			Eq. (4c)	1.13			
		20	WRC 107	-3.32	3.08	-2.02	1.70
			Dodge	-3.42	3.19	-2.04	1.73
			Eq. (4c)	4.22			
		40	WRC 107	-12.93	11.65	-7.82	6.01
			Dodge	-13.21	11.92	-7.86	6.04
			Eq. (4c)	15.7			
		80	WRC 107	-47.23	40.83	-29.08	19.05
			Dodge	-49.37	42.88	-29.17	19.04
			Eq. (4c)	58.7			
0.20	0.20	5	WRC 107	-0.84	0.78	-0.53	0.43
			Dodge	-0.86	0.80	-0.53	0.44
			Eq. (4c)	1.24			
		10	WRC 107	-3.15	2.93	-2.02	1.50
			Dodge	-3.39	3.05	-2.06	1.56
			Eq. (4c)	4.64			
		20	WRC 107	-12.06	10.46	-7.53	4.76
			Dodge	-12.45	10.78	-7.58	4.87
			Eq. (4c)	17.3			
		40	WRC 107	-41.30	34.47	-25.34	12.54
			Dodge	-43.87	36.70	-26.54	13.00
			Eq. (4c)	64.7			
		80	WRC 107	-137.7	112.1	-86.02	24.58
			Dodge	-142.5	117.4	-85.72	24.96
			Eq. (4c)	241.0			

^aFor circumferential moment loading, $S_\ell = 6M_C/(8L_1^2L_2)$; for subscript notation see footnote a of Table 10.

Table 14. Comparisons for longitudinal moment loadings on rectangular loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\phi 0}/S_\ell$	$\sigma_{\phi 1}/S_\ell$	σ_{x0}/S_ℓ	σ_{x1}/S_ℓ
0.05	0.20	5	WRC 107	-0.107	0.069	-0.239	0.233
			Dodge	-0.240	0.183	-0.242	0.229
			Eq. (4b)			0.385	
		10	WRC 107	-0.416	0.180	-0.863	0.827
			Dodge	-0.955	0.661	-0.896	0.806
			Eq. (4b)	1.29			
		20	WRC 107	-1.57	0.368	-3.02	2.84
			Dodge	-3.43	2.103	-2.82	2.31
			Eq. (4b)	4.30			
		40	WRC 107	-5.54	0.0107	-9.85	9.05
			Dodge	-11.85	6.77	-8.59	6.06
			Eq. (4b)	14.4			
		80	WRC 107	-19.64	-4.44	-25.37	22.56
			Dodge	-39.34	22.27	-26.79	15.33
			Eq. (4b)	47.9			
0.05	0.10	5	WRC 107	-0.081	0.073	-0.130	0.128
			Dodge	-0.090	0.085	-0.124	0.122
			Eq. (4b)			0.197	
		10	WRC 107	-0.302	0.256	-0.489	0.475
			Dodge	-0.392	0.314	-0.469	0.450
			Eq. (4b)			0.658	
		20	WRC 107	-1.174	0.910	-1.839	1.751
			Dodge	-1.595	1.152	-1.814	1.692
			Eq. (4b)			2.20	
		40	WRC 107	-4.301	2.992	-6.63	6.17
			Dodge	-6.199	3.894	-6.43	5.70
			Eq. (4b)			7.34	
		80	WRC 107	-15.18	8.91	-21.45	19.0
			Dodge	-22.63	12.11	-20.97	17.08
			Eq. (4b)	24.5			
0.20	0.10	5	WRC 107	-0.238	0.192	-0.285	0.268
			Dodge	-0.155	0.108	-0.293	0.287
			Eq. (4b)			0.368	

Table 14 (continued)

β_1	β_2	γ	Method	$\sigma_{\phi o}/S_\ell$	$\sigma_{\phi i}/S_\ell$	$\sigma_{x o}/S_\ell$	$\sigma_{x i}/S_\ell$
0.40	0.10	10	WRC 107	-0.897	0.621	-1.05	0.94
			Dodge	-0.632	0.357	-1.12	1.07
			Eq. (4b)			1.23	
		20	WRC 107	-3.46	2.07	-3.69	3.12
			Dodge	-2.48	0.96	-3.97	3.66
			Eq. (4b)			4.11	
		40	WRC 107	-12.28	6.09	-12.12	9.24
			Dodge	-9.00	1.54	-12.67	11.11
			Eq. (4b)			13.7	
		80	WRC 107	-41.57	16.43	-32.80	19.91
			Dodge	-28.55	-25.43	-34.06	27.40
			Eq. (4b)			45.8	
		5	WRC 107	-0.380	0.304	-0.379	0.345
			Dodge	-0.168	0.090	-0.365	0.360
			Eq. (4b)			0.383	
		10	WRC 107	-1.42	0.980	-1.29	1.09
			Dodge	-0.679	2.38	-1.32	1.27
			Eq. (4b)			1.28	
		20	WRC 107	-5.23	3.25	-4.44	3.40
			Dodge	-2.57	3.07	-4.38	4.12
			Eq. (4b)			4.28	
		40	WRC 107	-17.65	10.54	-13.90	9.15
			Dodge	-8.65	-1.405	-12.45	11.50
			Eq. (4b)			14.3	
		80	WRC 107	-53.72	30.06	-34.22	15.03
			Dodge	-25.91	-11.11	-28.52	26.40
			Eq. (4b)			47.7	

^aFor longitudinal moment loading, $S_\ell = 6M_L / (8L_1 L_2^2)$; for subscript notation see footnote a of Table 10.

Table 15. Comparisons for circumferential moment loadings on rectangular loaded areas^a

β_1	β_2	γ	Method	$\sigma_{\theta o}/S_\ell$	$\sigma_{\theta i}/S_\ell$	σ_{xo}/S_ℓ	σ_{xi}/S_ℓ
0.05	0.20	5	WRC 107	-0.100	0.098	-0.044	0.042
			Dodge	-0.101	0.100	-0.042	0.039
			Eq. (4c)	0.148			
		10	WRC 107	-0.399	0.393	-0.167	0.156
			Dodge	-0.403	0.395	-0.161	0.147
			Eq. (4c)	0.553			
		20	WRC 107	-1.57	1.54	0.637	0.563
			Dodge	-1.59	1.55	-0.654	0.568
			Eq. (4c)	2.06			
		40	WRC 107	-6.34	6.20	-2.42	2.01
			Dodge	-6.22	6.01	-2.56	2.06
			Eq. (4c)	7.70			
		80	WRC 107	-23.7	23.12	-9.68	7.38
			Dodge	-22.9	2.19	-9.68	6.55
			Eq. (4c)	28.7			
0.05	0.10	5	WRC 107	-0.077	0.076	-0.039	0.037
			Dodge	-0.077	0.076	-0.038	0.037
			Eq. (4c)	0.104			
		10	WRC 107	-0.310	0.305	-0.150	0.143
			Dodge	-0.312	0.306	-0.157	0.148
			Eq. (4c)	0.388			
		20	WRC 107	-1.20	1.17	-0.569	0.523
			Dodge	-1.23	1.19	-0.608	0.549
			Eq. (4c)	1.45			
		40	WRC 107	-4.98	4.81	-2.29	2.02
			Dodge	-4.95	4.76	-2.29	1.99
			Eq. (4c)	5.41			
		80	WRC 107	-18.62	17.68	-9.05	7.27
			Dodge	-19.32	18.28	-8.95	7.09
			Eq. (4c)	20.2			
0.20	0.10	5	WRC 107	-0.447	0.395	-0.366	0.310
			Dodge	-0.525	0.486	-0.392	0.344
			Eq. (4c)	0.798			

Table 15 (continued)

β_1	β_2	γ	Method	$\sigma_{\phi 0}/S_\ell$	$\sigma_{\phi 1}/S_\ell$	σ_{x0}/S_ℓ	σ_{x1}/S_ℓ
0.40	0.10	10	WRC 107	-1.72	1.45	-1.41	1.09
			Dodge	-2.05	1.82	-1.51	1.24
			Eq. (4c)	2.98			
		20	WRC 107	-6.89	5.31	-5.27	3.55
			Dodge	-7.75	6.50	-5.66	4.13
			Eq. (4c)	11.1			
		40	WRC 107	-26.6	15.75	-18.49	1.03
			Dodge	-27.9	21.74	-20.03	1.21
			Eq. (4c)	41.5			
		80	WRC 107	-86.45	60.82	-63.63	25.89
			Dodge	-92.32	66.77	-64.69	28.41
			Eq. (4c)	155.0			
		5	WRC 107	-0.90	0.676	-0.815	0.550
			Dodge	-1.17	1.011	-0.830	0.617
			Eq. (4c)	1.83			
		10	WRC 107	-3.45	2.40	-3.05	1.66
			Dodge	-4.35	3.56	-3.00	1.91
			Eq. (4c)	6.84			
		20	WRC 107	-12.9	7.92	-12.91	2.15
			Dodge	-15.0	11.66	-10.11	4.96
			Eq. (4c)	25.5			
		40	WRC 107	-44.0	25.93	-35.72	9.79
			Dodge	-48.2	36.67	-31.48	10.76
			Eq. (4c)	95.3			
		80	WRC 107	-149.2	96.75	-113.6	21.39
			Dodge	-142.5	108.1	-92.52	12.84
			Eq. (4c)	356.0			

^aFor circumferential moment loading, $S_\ell = 6M_C/(8L_1^2L_2)$; for subscript notation see footnote a of Table 10.

where $\beta_1 > \beta_2$, the WRC 107 method tends to underestimate the magnitude of the maximum stress. For circumferential moment loadings, Eq. (4c) tends to be conservative over the entire parameter range.

In summary, for both thrust and moment loadings, comparisons among the WRC 107 method, Dodge's direct calculations² of Bijlaard's analysis,⁶ and the stress-index equations developed in Ref. 2 [Eqs. (4a), (4b), and (4c) herein] show that the stress-index method tends always to be conservative. For square loaded areas, the WRC 107 method gives about the same results as the stress-index method. For loaded areas which are not square (i.e., $\beta_1 \neq \beta_2$), Bulletin 107 uses an approximate method (based on Ref. 7) which appears to be adequate for thrust loadings over the parameter range considered and fair for moment loadings within the parameter range $1/4 \leq \beta_1/\beta_2 \leq 4$; within this range the magnitude is conservative, but the location and direction may be in error. On the other hand, the WRC 107 method may be misleading for moment loadings on integral lugs which are long and narrow (i.e., for cases where $\beta_1/\beta_2 < 1/4$ or $\beta_1/\beta_2 > 4$).

Local Stresses

The C indices represent a combination of membrane stresses and linear through-the-wall-thickness bending stresses. They do not, however, provide any allowance for the highly localized stresses that would be present at the toe of the fillet weld joining the lug to the pipe surface. In the Code's analysis procedure of NB-3650, these stresses are evaluated by means of K indices. Different K-index values are provided in Table NB-3683.2-1 for butt welds which have been ground smooth and those which have been left as welded. Thus, in keeping with the philosophy of NB-3650, K_ℓ indices are proposed for lugs with either as-welded fillet welds or with ground fillet welds as shown in Fig. 3. The K_ℓ indices will be used in defining the added term $(S_p)_{\text{lug}}$ in Section 6.

An appropriate K_ℓ index for an as-welded lug is deemed to be that presently given by K_2 in Table NB-3683.2-1 for girth fillet welds for socket-weld fittings, slip-on flanges, and socket-welding flanges. The background and justification for this case, where K_2 is set equal to 2.0, is given in Ref. 8. Thus for as-welded lugs, $K_\ell = 2.0$ is proposed.

Fig. 3. As-welded and ground welds between lug and pipe.

If the fillet weld joining the lug to the pipe is ground to a smooth radius r_f as shown in Fig. 3 and if $r_f \geq t$ (the wall thickness of the pipe), then $K_t = 1.3$ would be appropriate. This value is based on the bending stress-concentration factors shown in Fig. 4 for $r/t \geq 1.0$. These curves were presented by Wichman, Hopper, and Merston⁴ in WRC Bulletin 107. Use of the bending curve was chosen since the major part of the surface stress at the toe of the weld is a bending stress (see, for example, Tables 5 to 7).

Fig. 4. Stress concentration factors from Ref. 4.

It may be noted in Appendix B that for moment loadings there are a number of cases where the maximum surface stress is well outside the loaded area. For these particular cases the local stress at the toe of the fillet weld would not be at the same position as the maximum surface stress. For conservatism, however, we will assume that the surface stress at the toe of the fillet weld is equal to the maximum surface stress, wherever that maximum occurs.

3. SHEAR AND TORSIONAL LOADS

A complete set of external loads which may act on a lug consists of three moments and three forces. Two of the moments, M_L and M_C , and one of the forces, radial thrust W , were discussed in the previous section. This section covers the torsional moment M_T and the two shear forces Q_1 and Q_2 illustrated in Fig. 5. Equations for calculating the shear stresses τ_1 and τ_2 in the pipe wall (also shown in Fig. 5) due to the shear loads

(a) SHEAR FORCE LOAD Q_1 (b) SHEAR FORCE LOAD Q_2 (c) TORSIONAL MOMENT LOAD M_T

Fig. 5. Representation of the nomenclature for shear-force and torsional-moment loadings on a lug.

Q_1 and Q_2 are given in Ref. 4:

$$\tau_1 = \frac{Q_1}{4L_1 t}, \quad (10)$$

$$\tau_2 = \frac{Q_2}{4L_2 t}. \quad (11)$$

The concept behind these equations is that the shear load is balanced by membrane shear stresses in the pipe wall parallel to the edges of the lug.

A similar equation can be derived for the shear stress τ_3 in the pipe wall due to the torsional moment M_T by writing the force balance equation for the stresses τ_3 and t' shown in Fig. 5c. Assume that τ_3 and t' are constant along the short edges ($2L_c$) and the long edges ($2L_d$) of the lug, respectively, and that the shear forces acting on the short edges are equal to the shear forces acting on the long edges. That is, $2L_d t' \tau' = 2L_c t \tau_3$; $L_c \leq L_d$. Then, summing moments gives

$$M_T = (2L_c t \tau_3)(2L_d) + (2L_d t)(\tau_3 L_c/L_d)(2L_c) .$$

Solving this equation for τ_3 and multiplying the result by 2.0 to account for the fact that the shear stress is not uniformly distributed gives an estimate for the maximum shear stress in the pipe wall due to a torsional moment M_T :

$$\tau_3 = \frac{M_T}{2L_c L_d (1 + L_c/L_d) t} , \quad (12)$$

where $L_c = \min (L_1, L_2)$ and $L_d = \max (L_1, L_2)$.

It should be noted that Eqs. (10) to (12) should give conservative estimates for the maximum shear stresses in the pipe wall. They are obviously inadequate, however, for the maximum shear stress in the lug at the interface when either L_1 or L_2 is less than the pipe wall thickness t . In order to cover both the pipe and the pipe-lug interface, Eqs. (10) and (11) should be replaced by

$$\tau_1 = \frac{Q_1}{4L_1 L_a} , \quad (13)$$

where L_a is taken as the smaller of L_2 and t , and

$$\tau_2 = \frac{Q_2}{4L_2 L_b} , \quad (14)$$

where L_b is taken as the smaller of L_1 and t .

The maximum shear stress in a rectangular bar under torsional loading is given by Timoshenko and Goodier⁹ as

$$\tau_{3\max} = \frac{M_T}{8k_2 L_c^2 L_d},$$

where the factor k_2 is tabulated as a function of L_d/L_c over the range $1.0 \leq L_d/L_c \leq \infty$. An approximate fit within the range $1.0 \leq L_d/L_c \leq 10$ is given by

$$8k_2 = 1.57 + 0.093 (L_d/L_c).$$

The approximation agrees with Ref. 9 at $L_d/L_c = 1.0$ and 10 and is up to 16% conservative for intermediate values. Thus, the maximum shear stress in the lug due to a torsional moment M_T is approximately

$$\tau_3 = \frac{M_T}{(1.57 + 0.093 L_d/L_c) L_c^2 L_d}. \quad (15)$$

In application, τ_3 should be taken as the greater of the values calculated by Eqs. (12) and (15) in order to cover both the pipe wall and the lug-pipe interface.

These expressions for τ_1 , τ_2 , and τ_3 , multiplied by 2.0 to convert from maximum shear stress to stress intensities, will be used in Section 6 in defining the terms $(P_L + P_b)_{\text{lug}}$, $(S_n)_{\text{lug}}$, and $(S_p)_{\text{lug}}$.

4. INTERNAL PRESSURE

Internal pressure acting in a pipe with an integral attachment will produce local stresses in the pipe because of the resistance to expansion of the lug. A rough estimate of these stresses can be obtained from the data given by Van Dyke¹⁰ for a rigid circular insert in a cylindrical shell with internal pressure loading. The stresses are a function of the parameter δ , defined as:

$$\delta^2 = \frac{a^2 \sqrt{12(1 - \nu^2)}}{8rt}, \quad (16)$$

where

a = radius of rigid circular insert,

ν = Poisson's ratio,

r = cylindrical shell radius,

t = cylindrical shell wall thickness.

For Poisson's ratio of 0.3, $\delta = 0.643(a/r) \sqrt{r/t}$.

Membrane stresses at $\theta = 0$ and 90° are plotted in Fig. 6. Membrane stresses at other values of θ lie between those shown. It may be noted that for small values of δ the membrane stresses are the same as those for a rigid insert in a flat plate with a 2-to-1 tensile stress field.

Except for values of δ less than about 0.4, the maximum surface stress (membrane plus bending) occurs at $\theta = 0$. This stress is also shown in Fig. 6. The maximum surface stress for values of δ up to 2.0 is reasonably well represented by the equation:

$$\begin{aligned} \frac{\sigma_m}{pD_o/2t} &= 1.5 + 1.3\delta^2 \\ &= 1.5 + 0.537(a/r)^2 (r/t), \end{aligned} \quad (17)$$

which is also shown in Fig. 6.

Application of the Van Dyke analysis for a rigid circular insert to lugs requires some engineering judgment as to what is to be used as the equivalent of (a/r) for lugs with dimensions β_1 and β_2 . For lugs with $\beta_1 = \beta_2$, it would appear appropriate to let $\beta_1 = \beta_2 = (a/r)$ in Eq. (17); however, for $\beta_1 \neq \beta_2$, an appropriate value for (a/r) is not obvious. Unfortunately, neither a directly applicable analysis nor relevant test data are available. Either the analysis or the test data should be developed. In the meantime, it is considered adequately conservative to let (a/r) in Eq. (17) be equal to $(\beta_1\beta_2)^{1/2}$.

With the limitation of $\beta_1\beta_2 \leq 0.075$ and $D_o/t \leq 100$ (recommended in Section 2, Thrust and Moment Loads), the largest value of δ will be $0.643 \times \sqrt{0.075} \times \sqrt{50} = 1.245$ and the maximum stress, as determined by Eq. (17), is $3.516pD_o/2t$.

The stresses defined by Eq. (17) damp out within a short distance from the insert and also decrease rapidly as θ varies from 90 to 0° . Thus even if the maximum values exceed the yield strength of the material,

Fig. 6. Stresses at the edge of a rigid circular insert in a cylindrical shell with pressure loading.

the maximum strains will be limited by the surrounding elastic material. For this reason, it seems appropriate to consider the stresses defined by Eq. (17) as peak stresses.

At the toe of the fillet weld, the maximum stress may be intensified if the weld is not ground smooth. Therefore, different values will be assigned for the stress index $K_{\ell p}$ (local stress at the lug due to internal pressure), depending upon whether the fillet weld is ground smooth or not, in the same manner as is done for K_{ℓ} for moment loadings. The peak-stress term to be included in S_p for internal pressure is therefore

$$\sigma_p = K_{\ell p} \frac{pD_o}{2t} .$$

For as-welded fillet welds the index $K_{\ell p}$ is thus

$$\begin{aligned} K_{\ell p} &= K_{\ell}(1.5 + 1.3U^2) \\ &= 2.0(1.5 + 0.537\beta_1\beta_2\gamma) , \end{aligned}$$

where the value of $K_{\ell} = 2.0$ is taken from the previous section and the remainder is taken from Eq. (17). Similarly for ground fillet welds

$$K_{\ell p} = 1.3(1.5 + 0.537\beta_1\beta_2\gamma) ,$$

where $K_{\ell} = 1.3$ is the value assigned in the previous section for ground welds.

So far as gross plastic yielding is concerned, the presence of a lug attachment will not adversely affect the behavior of the component; hence, a primary-stress index $B_{\ell p}$ for lugs is taken as zero. Recall that B-type indices are used in Eq. (C9) for primary-stress evaluation; B_{ℓ} -type indices are used to define the added term $(P_L + P_b)_{\text{lug}}$ discussed in Section 6. Similarly, additional primary-plus-secondary stresses due to internal pressure will be negligible; thus an appropriate value for $C_{\ell p}$ is also zero.

5. THERMAL GRADIENTS

If the temperature of the lug is different from that of the pipe, the difference in thermal expansions will induce stresses in the pipe wall around the lug. Since these local stresses will damp out within a short distance from the lug, they can be classified as peak stresses and represented by K-type indices to be used in defining the term $(S_p)_{\text{lug}}$. Thermal stress contributions to the terms $(P_L + P_b)_{\text{lug}}$ and $(S_n)_{\text{lug}}$ will be taken as zero, as was done in the previous section for local-pressure stresses.

In general, the temperature gradients in the vicinity of the lug will form a rather complex pattern. However, in keeping with current Code practice, it will be assumed that each part is at some average temperature and the thermal stresses will be estimated accordingly. The average temperature of the portion of the lug within $2t$ of the surface of the pipe is defined as T_ℓ , and the average temperature of the pipe under the lug and within a distance of $\sqrt{rt}$ from the edge of the lug is defined as T_w . An approximate estimate of the maximum thermal stresses can be obtained by assuming that the lug is an elliptical insert at temperature T_ℓ in an infinite flat plate at temperature T_w and that the lug and weld metal have the same modulus of elasticity E and coefficient of thermal expansion α as the pipe. Lugs that are long compared with their width will be covered by assuming that the major axis of the elliptical area is large compared with the minor axis. For this case Goodier^{11,12} gives the magnitude of the maximum thermal stress as

$$\sigma_T = E\alpha |T_\ell - T_w| \quad (18)$$

The thermal stresses may also be intensified at the toe of the fillet weld for the same reasons given earlier for thrust and/or moment loads and internal pressure. It is therefore recommended that the same values be used for the peak thermal stress index $K_{\ell T}$ as were recommended for the other loadings (i.e., $K_{\ell T} = K_\ell = 2.0$ for as-welded fillet welds, and $K_{\ell T} = K_\ell = 1.3$ for ground fillet welds). The additament $(S_p)_{\text{lug}}$ will thus

incorporate the thermal-stress term

$$\sigma_T = K_\ell E \alpha |T_\ell - T_w| \quad (19)$$

6. CODE USE OF STRESS INDICES

The Code gives stress indices for a number of piping components in Table NB-3683.2-1. Since this table and its footnotes are already quite lengthy and involved and since the evaluation of stresses at lugs involves an almost independent set of loadings, it is deemed appropriate to add a new paragraph, NB-3683.3, Indices for Integral Structural Attachments. The proposed wording for this new paragraph is included herein as Appendix A. The general intent is to "add on" appropriate terms identified earlier as $(P_L + P_b)_{\text{lug}}$, $(S_n)_{\text{lug}}$, and $(S_p)_{\text{lug}}$ to Code equations (C9), (C10), and (C11) respectively. In addition, the thermal-stress terms at a structural discontinuity $C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ in Eq. (C10), $K_3 C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ in Eq. (C11), and $C'_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ in Eq. (C13) will be dropped because the stress indices for lugs developed herein are not intended to apply if the lug is at a "gross discontinuity."

One other departure from current Code practice is also made by establishing criteria for evaluating the adequacy of the lug-pipe interface. This is accomplished by using the lower limit of 1.0 on C_T , C_L , and C_C introduced in Section 2 and the alternate equations for stresses in the lug-pipe interface introduced in Section 3 for shear and torsional moment loads. Since failure of the interface alone does not constitute failure of the pressure boundary, an aspect is introduced which is not specifically covered in NB-3600, analysis of a zone which is not a pressure boundary but which is within the jurisdiction of NB-3600. Including these special limits for the lug-pipe interface in the manner described below effectively sets the stress limits for the interface just as if it were part of the pressure boundary, including addition of the stresses due to internal pressure, moment loads, and thermal gradient with those due to loads applied to the lug.

Satisfaction of Primary-Stress-Intensity Limit

Primary-stress intensities in Class I piping systems are limited to $1.5S_m$ by Eq. (C9) (NB-3652). For points in the vicinity of integral lug attachments, modification of this equation is proposed by adding the term $(P_L + P_b)_{lug}$, which includes additional primary local membrane-stress intensities P_L and additional primary bending-stress intensities P_b caused by the presence of the lug. Thus

$$B_1 \frac{PD}{2t} + B_2 \frac{D}{2I} M_i + (P_L + P_b)_{lug} \leq 1.5S_m, \quad (20)$$

where the indices B_1 and B_2 are those currently given in Table NB-3683.2-1, and the definition of $(P_L + P_b)_{lug}$ in terms of appropriate B_l indices is the subject of this section. It should be noted that the magnitude of the moment vector M_i , as determined by an analysis of the piping system, would not be the same on the two sides of the lug, but in most cases the difference would be small. However, for cases where the difference is significant, the intent is to use the absolute mean value of the two. A footnote in the Code will be needed to cover this point.

Equation (C9) is now included as part of Paragraph NB-3652, "Consideration of Design Conditions." In USAS B31.7 (Ref. 13), the same equation was included in Paragraph 1-705.1, "Satisfaction of Primary Stress Intensity." The change in design concept implied by this change in the title of the heading is not trivial. If the force loads on lugs are not specified as design conditions in the Design Specification, these loads would not be included in the evaluation of Eq. (C9). If Eq. (C9) is to continue to be a valid check of primary-stress intensity, the design specification writer must be careful to include all sustained mechanical loads as design conditions. The present definition of M_i given in NB-3652.2,

" M_i = Resultant moment loading due to loads caused by (1) weight, (2) earthquake, considering only one-half the range of the earthquake and excluding the effects of anchor displacement due to earthquake, and (3) other sustained design mechanical loads ...,"

is inadequate in this respect. It is recommended that the definition be changed to read as follows:

" M_i = Magnitude of the resultant moment due to loads caused by
 (1) weight, (2) earthquake, considering only one-half the
 range of the earthquake and excluding the effects of
 anchor displacement due to an earthquake, and (3) all
 other sustained mechanical loads"

It appears appropriate to assume that most force loads such as weight or earthquake inertial forces on lugs will be "sustained" forces. The intent of Eq. (C9) is to limit the stresses resulting from such forces so that gross plastic deformations do not occur. A lower bound to this limit can be obtained by simply limiting the calculated elastic-membrane-stress intensity to the yield strength of the pipe material S_y , or $1.5S_m$, since S_m is essentially limited to $2/3S_y$.^{*} For the six external loadings on lugs, this lower bound will actually be quite conservative, since in application the loadings causing the maximum stresses will be thrust W and longitudinal and circumferential bending moments M_L and M_C respectively. These loadings produce mainly bending stresses, and for collapse to occur from bending stresses plastic hinges must be formed. A limit-load analysis could be used to establish a less-conservative lower bound. However, at present, no such analysis is available, and after formulating the analysis method a rather extensive parameter study would be required to establish appropriate B indices.

Tables 6 to 8 of Section 2 show that the maximum membrane stresses caused by the loadings W , M_L , and M_C are always substantially less than $2/3$ the maximum surface stresses (membrane-plus-bending stresses represented by the stress indices C_T , C_L , and $C_C \geq 1.0$). Limiting these stresses to $1.5S_y$ (where S_y is the yield stress) thus automatically limits the maximum membrane stress intensities to less than S_y , or $1.5S_m$. Since the limit on Eq. (20) is $1.5S_m$ [the same as Eq. (C9)], an appropriate set

^{*}For some austenitic steels and nonferritic materials, the value of S_m may be up to 0.9 of the nominal yield strength. However, the strain-hardening properties of these materials are generally assumed to be such that the value of S_m is not greater than $2/3$ the effective yield strength.

of B_ℓ indices for use in the term $(P_L + P_b)_{\text{lug}}$ is simply

$$B_T = (2/3)C_T \geq 2/3 ,$$

$$B_L = (2/3)C_L \geq 2/3 ,$$

$$B_C = (2/3)C_C \geq 2/3 .$$

Membrane shear stresses were derived in Section 3 for the transverse shear loads Q_1 and Q_2 and the torsional moment load M_T . Since the membrane stress intensity from these loads is twice the maximum shear stress, S_y (or $1.5S_m$) is an appropriate primary stress limit. Thus 2.0 times the right-hand side of Eqs. (13) and (14) and twice the greater of Eqs. (12) and (15) can be used directly in defining $(P_L + P_b)_{\text{lug}}$.

The quantity $(P_L + P_b)_{\text{lug}}$ to be added to the left side of Eq. (C9), including all six external loadings, is therefore

$$\begin{aligned} (P_L + P_b)_{\text{lug}} = & B_T \frac{W}{A_\ell} + B_L \frac{M_L}{Z_{\ell L}} \\ & + B_C \frac{M_C}{Z_{\ell C}} + \frac{Q_1}{2L_1L_a} + \frac{Q_2}{2L_2L_b} + \bar{M}_T , \end{aligned} \quad (21)$$

where, in order to cover the lug-pipe interface as well as the pipe wall, L_a is taken as the smaller of L_2 and t , L_b is the smaller of L_1 and t , and $\bar{M}_T$ is the greater of

$$\bar{M}_{Tp} = \frac{M_T}{L_c L_d [1 + (L_c/L_d)] t} ,$$

and

$$\bar{M}_{T\ell} = \frac{2M_T}{[1.57 + 0.093(L_d/L_c)] L_c^2 L_d} ,$$

where $L_c = \min(L_1, L_2)$ and $L_d = \max(L_1, L_2)$. In Eq. (21), the loading terms W , M_L , M_C , Q_1 , Q_2 , and $\bar{M}_T$ represent the absolute value of the sustained loading amplitudes and not the loading ranges.

Satisfaction of Primary-Plus-Secondary
Stress-Intensity Range

Primary-plus-secondary stress intensities in Class I piping systems are limited to $S_n \leq 3S_m$ by Eq. (C10)(NB-3653.1). As discussed earlier, we propose to modify this equation by dropping the term $C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ and adding the term $(S_n)_{lug}$; thus

$$S_n = C_1 \frac{P_o D_o}{2t} + C_2 \frac{D_o}{2I} M_1 + \frac{1}{2(1-\nu)} E \alpha |\Delta T_1| + (S_n)_{lug} \leq 3S_m \quad (22)$$

The stress indices C_T , C_L , and C_C discussed in Section 2 for thrust W and moment loadings M_L and M_C and twice the shear stresses τ_1 , τ_2 , and τ_3 discussed in Section 3 are directly applicable in defining $(S_n)_{lug}$. However, even though τ_1 , τ_2 , and τ_3 are the same as used in defining $(P_L + P_b)_{lug}$ above, the resulting stresses will not necessarily be the same because Eq. (C9) is in terms of sustained loading amplitudes whereas Eq. (C10) is in terms of loading ranges. Additional internal-pressure stresses and thermal-gradient stresses due to the presence of the lug were classified as peak stresses in Sections 4 and 5, and thus $(S_n)_{lug}$ does not contain either a pressure term or a temperature term. Thus, for evaluating the stresses in a piping system at an integral lug, the quantity $(S_n)_{lug}$ to be added to S_n is

$$(S_n)_{lug} = C_T \frac{W^*}{A_\ell} + C_L \frac{M_L^*}{Z_{\ell L}} + C_C \frac{M_C^*}{Z_{\ell C}} + \frac{Q_1^*}{2L_1 L_a} + \frac{Q_2^*}{2L_2 L_b} + \bar{M}_T^* \quad (23)$$

where W^* , M_C^* , M_L^* , Q_1^* , and Q_2^* are the ranges of the loads and $\bar{M}_T^*$ is calculated using the range of $\bar{M}_T$.

Satisfaction of Peak-Stress-Intensity Range

Peak stress intensities in Class I piping systems are evaluated by Eq. (C11) given in NB-3653.2. For the integral-lug problem this equation is to be modified by dropping the term $K_3 C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ and adding

$(S_p)_{\text{lug}}$:

$$S_p = K_1 C_1 \frac{P_o D_o}{2t} + K_2 C_2 \frac{D_o}{2I} M_i + \frac{1}{2(1-\nu)} K_3 E \alpha |\Delta T_1| + \frac{1}{(1-\nu)} E \alpha |\Delta T_2| + (S_p)_{\text{lug}}, \quad (24)$$

where the additament $(S_p)_{\text{lug}}$ will define the additional peak stresses caused by the presence of the lug for internal pressure in the pipe; thrust, shear, and/or moment loads on the lug; and a temperature gradient between the pipe and the lug.

The peak stress due to internal pressure was given in Section 4 as $K_{\ell p} P_o D_o / 2t$, with $K_{\ell p} = K_{\ell} (1.5 + 0.537 \beta_1 \beta_2 \gamma)$. However, since Eq. (C11) already has the internal pressure term $K_1 C_1 (P_o D_o / 2t)$, which is retained in Eq. (24), and since $K_1 = C_1 = 1.0$ for straight pipe remote from gross discontinuities, the internal pressure term for $(S_p)_{\text{lug}}$ becomes

$$[(K_{\ell} - 1)(1.5 + 0.537 \beta_1 \beta_2 \gamma)] P_o D_o / 2t.$$

Peak stresses due to thrust and moment loads on the lug are treated in the same manner as for the primary-plus-secondary stresses except that each of the indices C_T , C_L , and C_C is to be multiplied by the index K_{ℓ} . The thermal-stress term was defined by Eq. (19) in Section 5 as $K_{\ell} E \alpha |T_{\ell} - T_w|$, and the local-peak-stress index K_{ℓ} was defined in Section 2 under the discussion on local stresses as 2.0 for as-welded fillet welds or 1.3 for ground fillet welds.

The quantity $(S_p)_{\text{lug}}$ is thus defined as the sum of the individual loading terms:

$$(S_p)_{\text{lug}} = [(K_{\ell} - 1)(1.5 + 0.537 \beta_1 \beta_2 \gamma)] \frac{P_o D_o}{2t} + K_{\ell} \left[C_T \frac{W^*}{A_{\ell}} + C_L \frac{M_L^*}{Z_{\ell L}} + C_C \frac{M_C^*}{Z_{\ell C}} + \frac{Q_1^*}{2L_1 L_a} + \frac{Q_2^*}{2L_2 L_b} + \bar{M}_T^* \right] + K_{\ell} E \alpha |T_{\ell} - T_w|. \quad (25)$$

Simplified Elastic-Plastic Discontinuity Analysis

If the piping system component fails to meet the $3S_m$ limit of Eq. (C10) [corresponding to Eq. (22) in the present context] for all pairs of load sets, the Code provides an alternate procedure for qualification under NB-3653.6 provided that the nominal value of the expansion stress intensity S_e defined by Eq. (C12) is less than $3S_m$ and that the range of stress intensity defined by Eq. (C13) is also less than $3S_m$. The procedure is to increase the alternating stress intensity S_{alt} for fatigue evaluation by the factor $K_e \geq 1.0$ prescribed by the Code under Eq. (C14).

Equation (C12) establishes a limit on thermal-expansion and anchor-movement stresses with the intent of limiting the stress range to about the same magnitude as permitted in predecessor piping codes [e.g., ANSI B31.1.0].¹⁴ Since these predecessor codes do not limit expansion stresses for pipe with lug attachments to a lower value than for pipe without lug attachments, there is no reason to change Eq. (C12) for a pipe with a lug. Equation (C13) is intended to limit the possibility of ratchetting when S_n exceeds $3S_m$. The relatively conservative values recommended for the B indices should give more than adequate protection against such a possibility. Accordingly, no addition to Eq. (C13) is considered necessary for lugs.

Equation (C14) does not require any changes, although, obviously, the numerical results will be influenced by the additional terms included in the evaluation of S_p [Eqs. (24) and (25)]. For the analysis of lug attachments, however, it seems advisable to impose an additional constraint on the use of Eqs. (C14), etc., when the component fails to meet the $3S_m$ limit of Eq. (22). This is because loadings on the lugs caused by expansion of the piping system may add to the primary weight loads to give a total loading which is greater than the capacity of the lug for a single load application. For other piping components the stresses corresponding to the "constraint of free-end displacement" loads are categorized as S_n stresses, which under the rules of NB-3653.6 have a very high absolute upper limit for small numbers of cycles. For example, the S_n limit for type 304 stainless steel can be as high as 400,000 psi. While the fatigue

limitation based on the peak stresses S_p would probably prevent the majority of such conditions from occurring in practice, it is considered desirable to impose the additional limit for lugs:

$$(S_n)_{\text{lug}}^{**} = C_T \frac{W^{**}}{A_\ell} + C_L \frac{M_L^{**}}{Z_{\ell L}} + C_C \frac{M_C^{**}}{Z_{\ell C}} + \frac{Q_1^{**}}{2L_1L_a} + \frac{Q_2^{**}}{2L_2L_b} + \bar{M}_T^{**} \leq 3S_m, \quad (26)$$

where the loads W^{**} , M_L^{**} , ..., $\bar{M}_T^{**}$ are defined as the maximum loads from any source.

7. SUMMARY AND CONCLUSIONS

A simplified design analysis procedure using stress indices has been developed for analyzing points in a Class I piping system where an integral lug attachment is welded to straight pipe. The procedure follows the general method outlined in Subsubarticle NB-3650 of the Code and is recommended for addition to the Code in a new paragraph, NB-3683.3, Indices for Integral Structural Attachments. The proposed wording for this paragraph is given as Appendix A. New terms (additaments) for addition to the Code equations when considering integral structural attachments have been developed. These additaments, defined herein as $(P_L + P_b)_{\text{lug}}$, $(S_n)_{\text{lug}}$, and $(S_p)_{\text{lug}}$, are expressed in terms of internal pressure in the pipe, applied mechanical loads on the lug, the average temperatures of the lug and the pipe, and appropriate stress indices.

The stress indices developed in this report for the various loadings are based on available analytical solutions, some of which are more accurate than others. In each case, however, the results are believed to be conservative within the parameter ranges defined by the limits placed on the stress-index equations. For the cases of thrust and/or bending moments applied to the lugs, rather extensive comparisons between the stress-index analysis method developed here and two existing and widely used design analysis methods were made. These are the PW-43 method³ and the WRC-107 method.⁴

A three-way comparison for thrust and moment loads among the stress-index analysis method, the WRC-107 method, and Ref. 2 calculations using

Bijlaard's theoretical analysis indicates that the stress-index method is always conservative. The WRC-107 method tends to be conservative for loaded areas that are square and gives about the same results as the stress-index method. For loaded areas that are not square, the WRC-107 method is generally conservative for thrust loadings but less conservative than the stress-index method. For moment loadings, however, the WRC-107 method tends to break down as the length-to-width (or width-to-length) ratio increases and apparently should not be used when this ratio exceeds 4 (or is less than $1/4$).

Comparisons made between the stress-index method and the PW-43 method, which limits the permissible thrust and longitudinal moment loads rather than the maximum stresses, indicate that the PW-43 method would result in higher stresses for thrust loads and lower stresses for longitudinal moment loads. For typical design conditions, indications are that the two methods would give comparable values for the permissible loads. The PW-43 method does not consider circumferential moment loads.

The indices presented here for transverse shear and torsional moment loadings on the lug and for a temperature difference between the lug and the pipe are based on approximate analytical models which perhaps could be improved with further study. The indices, however, are believed to be conservative and adequate for piping-system design; and unless user experience indicates the need for more rigorous analyses, no further study is planned.

The indices developed for internal pressure loading are based on an analysis of a cylindrical shell with a solid circular insert. For lugs that are square or rectangular, approximations are introduced which should give conservative results; however, further study is needed to verify this point.

Finally, it should be pointed out that the stress-index-analysis method presented here for lugs is both simple to use and at least as accurate as any other known method. For those loadings where the accuracy can be questioned, the conservative practice of adding maximum stress intensities, even though these maximums may not occur at the same point or be oriented in the same direction, should provide adequate protection to the design.

ACKNOWLEDGMENTS

The authors wish to thank the members of Task Group I of the Pressure Vessel Research Committee, Subcommittee for Piping, Pumps, and Valves, and the members of the ASME Boiler and Pressure Vessel Code Committee, Subgroup on Piping (SGD)(SC III), for their review and valuable suggestions.

REFERENCES

1. ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components, American Society of Mechanical Engineers, New York, 1971.
2. W. G. Dodge, Secondary Stress Indices for Integral Structural Attachments to Straight Pipe, ORNL-TM-3476 (June 1973).
3. ASME Boiler and Pressure Vessel Code, Section I, Power Boilers, American Society of Mechanical Engineers, New York, 1970.
4. K. R. Wichman, A. D. Hopper, and J. L. Mershon, Local Stresses in Spherical and Cylindrical Shells Due to External Loadings, Welding Research Council Bulletin No. 107, August 1965.
5. R. F. Melworm and I. Berman, "Welded Attachments to Tubes - Experimentation, Analysis, and Design," Welding J. Res. Suppl. 45(10), 454-64 (1966).
6. P. P. Bijlaard, "Stresses from Local Loadings in Cylindrical Pressure Vessels," Trans. ASME 77(6), 805-16 (1955).
7. P. P. Bijlaard, "Stresses from Radial Loads and External Moments in Cylindrical Pressure Vessels," Welding J. Res. Suppl. 34(12), 608-17 (1955).
8. E. C. Rodabaugh, W. G. Dodge, and S. E. Moore, Stress Indices for Girth-Welded Joints, Including the Effects of Radial Weld Shrinkage, Mismatch, and Tapered-Wall Transitions, ORNL-TM-3643 (to be published).
9. S. Timoshenko and J. N. Goodier, Theory of Elasticity, 2d Ed., pp. 275-78, McGraw-Hill, New York, 1951.
10. Peter Van Dyke, "Stresses in a Cylindrical Shell with a Rigid Inclusion," AIAA J. 3(1), 125-37 (January 1967).
11. J. N. Goodier, "Thermal Stresses and Deformation," J. Appl. Mech. 24, 467-74 (September 1957).

12. R. J. Roark (Ed.), Formulas for Stress and Strain, 4th Ed., p. 375, McGraw-Hill, New York, 1967.
13. USA Standard Code for Pressure Piping, Nuclear Power Piping, USAS B31.7-1969, American Society of Mechanical Engineers, New York, 1969.
14. Power Piping, USAS B31.1.0, Sect. 1 of the American Standard Code for Pressure Piping, American Society of Mechanical Engineers, New York, 1967.

Appendix A

RECOMMENDED ADDITION TO ASME CODE SECTION III
TO COVER STRAIGHT PIPE WITH INTEGRAL
STRUCTURAL ATTACHMENTS

The recommended addition to the ASME Boiler Code, Section III, consists of a new paragraph, NB-3683.3, to read as follows:

NB-3683.3 STRESS INDICES FOR INTEGRAL STRUCTURAL ATTACHMENTS

(a) Limitations to Applicability of Indices

- (1) The attachment is welded to the pipe with a complete-penetration weld.
- (2) The attachment material, weld material, and pipe material have essentially the same moduli of elasticity and coefficients of thermal expansion.
- (3) $\beta_1 \leq 0.5$, $\beta_2 \leq 0.5$, and the product $\beta_1 \times \beta_2 \leq 0.075$, where β_1 and β_2 are defined in (c).
- (4) The attachment is made on straight pipe, with the nearest edge of the attachment weld located at a minimum distance of $\sqrt{rt}$ from any other weld or other discontinuity; r and t are defined in (c).
- (5) $D_o/t \leq 100$.

(b) Equations To Be Satisfied

For points on a piping system with integral structural attachments, the term $C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b|$ shall be deleted from Eqs. (10), (11), and (13). Equations (9), (10), and (11) shall use the B, C, and K indices given in Table NB-3683.2-1 for "straight pipe, remote from welds or other discontinuities" and shall include the following additional terms [see also Footnote (1)]. Symbols are defined in (c).

¹The value of M_l , under equilibrium conditions, is not identical on the two sides of the attachment when M_L or M_C is applied to the lug. Usually the change in M_l will be negligible; if it is significant, however, the mean value is to be used in Eqs. (9) to (11).

Equation (9), add to left-hand side:

$$(P_L + P_b)_\ell = B_T W/A_\ell + B_{L_L} M/Z_{\ell L} + B_{C_C} M/Z_{\ell C} + Q_1/2L_1 L_a + Q_2/2L_2 L_b + \bar{M}_T .$$

Equation (10), add to S_n :

$$S_{n\ell} = C_T W^*/A_\ell + C_{L_L} M^*/Z_{\ell L} + C_{C_C} M^*/Z_{\ell C} + Q_1^*/2L_1 L_a + Q_2^*/2L_2 L_b + \bar{M}_T^* .$$

Equation (11), add to S_p :

$$S_{p\ell} = [(K_\ell - 1)(1.5 + 0.537\beta_1\beta_2\gamma)] \frac{P_D}{2t} + K_\ell [S_{n\ell}] + K_\ell E\alpha |T_\ell - T_w| .$$

The following equation must also be satisfied:

$$S_{n\ell}^{**} = C_T W^{**}/A_\ell + C_{L_L} M^{**}/Z_{\ell L} + C_{C_C} M^{**}/Z_{\ell C} + Q_1^{**}/2L_1 L_a + Q_2^{**}/2L_2 L_b + \bar{M}_T^{**} \leq 3S_m .$$

The value of S_m shall be the lesser of that for the pipe material or of that for the lug material.

(c) Definitions (see also Fig. NB-3683.3; Fig. A.1 herein)

r = mean pipe radius (in.)

t = nominal pipe-wall thickness (in.)

$\gamma = r/t$

$\beta_1 = L_1/r$

$\beta_2 = L_2/r$

L_1 and L_2 are defined in Fig. NB-3683.3

L_a = lesser of L_2 and t

L_b = lesser of L_1 and t

L_c = lesser of L_1 and L_2

L_d = greater of L_1 and L_2

$C_T = 7.64(\gamma)^{1.64}\beta_1\beta_2\eta^{1.54} \geq 1.0$

$C_L = 0.51(\gamma)^{1.74}\beta_1\beta_2^2\eta^{4.74} \geq 1.0$

$C_C = 0.76(\gamma)^{1.90}\beta_1^2\beta_2\eta^{3.40} \geq 1.0$

$\eta = -(X_1 \cos \theta + Y_1 \sin \theta) - \frac{1}{A_o} (X_1 \sin \theta - Y_1 \cos \theta)^2$

ORNL - DWG 73 - 5839

- (a) Graphic representation of L_1 , L_2 , W , M_L , Q_1 , Q_2 , and M_T ; L_1 and L_2 are to be measured along the surface of the run pipe. Welds or fillet radii between attachment and pipe are not to be included.

ORNL - DWG 73 - 6629

- (b) Ground weld or integrally cast attachment; $K_\ell = 1.3$. For as-welded or fillet radii $r_f \geq t$, use $K_\ell = 2.0$.

Fig. A.1. Proposed additional Code illustration entitled "Figure NB-3683.3: Nomenclature Illustration."

$$X_1 = X_0 + \log_{10} \beta_1$$

$$Y_1 = Y_0 + \log_{10} \beta_2$$

Load	A ₀	θ	X ₀	Y ₀
Thrust	2.2	40°	0	0.05
Longitudinal moment	2.0	50°	-0.45	-0.55
Circumferential moment	1.8	40°	-0.75	-0.60

$$R_T = (2/3)C_T, R_L = (2/3)C_L, R_C = (2/3)C_C$$

$$A_\ell = 4L_1L_2 \text{ (in.}^2\text{)}$$

$$Z_{\ell L} = 4L_1L_2^2/3 \text{ (in.}^3\text{)}$$

$$Z_{\ell C} = 4L_1^2L_2/3 \text{ (in.}^3\text{)}$$

$$K_\ell = 2.0 \text{ for "as-welded" fillet welds}$$

$$K_\ell = 1.3 \text{ for "ground" fillet welds per Fig. NB-3683.3}$$

W, M_L, M_C, Q₁, Q₂, and M_T are absolute values of the sustained load amplitudes as shown in Fig. NB-3683.3. These are loads caused by (1) weight, (2) earthquake, considering only one-half the range of the earthquake and excluding the effects of anchor displacement due to earthquake, and (3) all other sustained mechanical loads. Units: W, Q₁, and Q₂: lb; M_L, M_C, and M_T: in.-lb

W*, M_L*, M_C*, Q₁*, Q₂*, and M_T* are ranges of loadings due to (1) thermal expansion, (2) anchor movements from any cause, (3) earthquake effects, and (4) all other mechanical loads which go through a range of magnitude

W**, M_L**, M_C**, Q₁**, Q₂**, and M_T** are absolute values of maximum loads from any cause

$$\bar{M}_T = \text{greater of } M_T / \{L_C L_d [1 + (L_C / L_d)]\} \text{ and } M_T / [0.8 + 0.05(L_d / L_C)] L_C^2 L_d$$

$$\bar{M}_T^* = \text{same as } \bar{M}_T, \text{ except use the loading range } M_T^*$$

$$T_\ell = \text{average temperature of that portion of the lug within a distance of } 2t \text{ from the surface of the pipe (°F)}$$

$$T_w = \text{average temperature of the portion of the pipe under the lug and within a distance of } \sqrt{rt} \text{ from the edge of the lug (°F)}$$

$E\alpha$ = modulus of elasticity (E) times the mean coefficient of thermal expansion (α), both at room temperature (psi/°F)

P_o = range of operating pressure (psi)

D_o = outside diameter of pipe (in.)

Appendix B

DIRECTIONS AND LOCATIONS OF MAXIMUM STRESSES
FOR THRUST AND MOMENT LOADINGS

The stress indices developed in Section 2 for thrust and bending moment loads W , M_L , and M_C were based on the maximum surface stresses (membrane plus bending) calculated for the region of the pipe near the pipe-lug interface. Figure B.1 shows the region of the pipe-lug intersection and the points at which stresses were calculated. Positions (1) and (2) on the edges of the lug are the locations at which the maximum stress occurred most often. Stresses were calculated using the computer program LUGS (Ref. 2) at each point in the rectangular array, starting at the center of the lug and extending a distance along the pipe of $X = x/L_1 = 2$ or 3 and a distance around the pipe of $\theta = \phi/\beta_1 = 2$ or 3, depending on the width and length parameters β_1 and β_2 of the particular lug as shown below:

For $\beta_1 \leq 0.05$;	$0 \leq \theta \leq 3.0$;	$\Delta\theta = 0.25$
For $\beta_1 \geq 0.1$;	$0 \leq \theta \leq 2.0$;	$\Delta\theta = 0.1667$
For $\beta_2 \leq 0.1$;	$0 \leq X \leq 3.0$;	$\Delta X = 0.25$
For $\beta_2 \geq 0.2$;	$0 \leq X \leq 2.0$;	$\Delta X = 0.1667$

Stresses inside the loaded area (underneath the lug) were calculated but not used in establishing the stress indices. Because of symmetry, the stresses were evaluated for only one quadrant.

ORNL-DWG 73-5838

Fig. B.1. Region of the pipe-lug intersection used in calculating stresses.

The maximum stresses reported here are the result of a rather extensive parameter study for lug-pipe intersection dimensions within the parameter range $0.05 \leq \beta_1 \leq 0.5$; $0.05 \leq \beta_2 \leq 0.5$; and $\gamma = r/t = 5, 10, 20, 40$, and 80 . The locations and directions of the maximum surface stresses (membrane plus bending) for the three loadings W , M_L , and M_C are tabulated as functions of β_1 and β_2 in Tables B.1 to B.5 for γ values of $5, 10, 20, 40$, and 80 respectively. The direction is indicated as either "C" or "L" for circumferential or longitudinal under the columns labeled D. The coordinates of the maximum stress location are indicated by the pair of numbers under the columns "X" and " θ ," except when the maximum occurred at positions (1) and (2) shown in Fig. B.1, in which case a (1) or (2) appears under the column labeled "X." In all but the five cases marked, the maximum stress was a compressive stress on the outside surface; and for all but three cases, the maximum surface stress was the maximum stress intensity.

Tables B.6 to B.10 give the directions and locations of the maximum membrane stresses for $\gamma = r/t = 5, 10, 20, 40$, and 80 , respectively, using the same format as used for the surface stresses. For moment loadings M_L and M_C , there were a number of cases where the maximum membrane stress was apparently located outside the area used in the calculations. These are indicated by "3+" in the appropriate columns.

Table B.1. Direction and location of maximum surface stresses for pipe radius-to-thickness ratio of 5

β_1	β_2	W			M_L			M_C		
		η^a	x^b	θ	D^a	x^b	θ	D^a	x^b	θ
0.05	0.05	C	(1)		L	(1)		C	0	1.25
	0.10		(2)							1.25
	0.20									1.25
	0.30									1.25
	0.40									1.50
	0.50				C	0.83	1.00		0.17	1.50
0.10	0.05		(1)		L	(1)			(2)	
	0.10		(1)							
	0.20		(2)							
	0.30				C					
	0.40				C					
0.2	0.05		(1)		L	1.25	0		1.00	0.83
	0.10					(1)			(2)	
	0.20									
	0.30									
0.3	0.05									
	0.10									
	0.20									
0.4	0.05					1.25	0.17		1.00	0.83
	0.10					(1)				
0.5	0.05				L ^c	1.25	0			

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

^cInside surface, positive sign.

Table B.2. Direction and location of maximum surface stresses for pipe radius-to-thickness ratio of 10

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(1)		L	(1)		C	0	1.25
	0.10		(2)		L					
	0.20				C					
	0.30									
	0.40									
	0.50					0.83	1.00		0.17	1.75
0.10	0.05		(1)		L	(1)			(2)	
	0.10		(1)		L					
	0.20		(2)		L					
	0.30				C					
	0.40				C					
0.2	0.05		(1)		L	1.25	0		1.00	0.83
	0.10				L	(1)			(2)	
	0.20				L				(2)	
	0.30				C				(2)	
0.3	0.05				L				1.00	0.83
	0.10								(2)	
	0.20								(2)	
0.4	0.05								1.00	0.67
	0.10								1.00	0.83
0.5	0.05								1.00	0.83

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.3. Direction and location of maximum surface stresses for pipe radius-to-thickness ratio of 20

β_1	β_2	W			M_L			M_C		
		D^a	X^b	θ	D^a	X^b	θ	D^a	X^b	θ
0.05	0.05	C	(1)		L	(1)		C	0	1.25
	0.10		(2)		L					
	0.20				C					
	0.30									
	0.40					0.67	1.00			1.50
	0.50					0.67	1.00		0.17	1.50
0.10	0.05		(1)		L	(1)			(2)	
	0.10		(1)		L					
	0.20		(2)		C					
	0.30		(2)		C					
	0.40		(2)		C					
0.2	0.05		(1)		L				1.00	0.83
	0.10				L				(2)	
	0.20				L				(2)	
	0.30				C				(2)	
0.3	0.05				L				1.00	0.83
	0.10								(2)	
	0.20								(2)	
0.4	0.05					1.25	0		1.00	0.83
	0.10					(1)			1.00	0.83
0.5	0.05					1.25	0.17		1.00	0.83

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.4. Direction and location of maximum surface stresses for pipe radius-to-thickness ratio of 40

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(1)		L	(1)		C	0	1.25
	0.10		(2)		L	(1)			(2)	
	0.20				C	(1)			0	1.25
	0.30				C	0.83	1.00		0	1.75
	0.40				C	0.67	1.00		0	1.25
	0.50				C	0.83	1.00		0	1.25
0.10	0.05		(1)		L	(1)			(2)	
	0.10		(1)		L				(2)	
	0.20		(1)		C				(2)	
	0.30		(2)		C				0	1.17
	0.40		(2)		C				(2)	
0.2	0.05		(1)		L				1.00	0.67
	0.10				L				(2)	
	0.20				C				(2)	
	0.30				C				(2)	
0.3	0.05				L	1.00	0.33		1.00	0.67
	0.10				L	(1)			1.00	0.83
	0.20				C	(1)			(2)	
0.4	0.05				L	1.00	0.50		1.00	0.67
	0.10				c	1.00	0.17		1.00	0.67
0.5	0.05				L	1.00	0.05		1.00	0.83

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

^cMaximum stress intensity occurs on inside surface, $S_L = 169.4$, $S_C = -20.69$.

Table B.5. Direction and location of maximum surface stresses for pipe radius-to-thickness ratio of 80

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(1)		L	(1)		C	0	1.25
	0.10		(2)		C				(2)	
	0.20								0.17	1.50
	0.30								0	1.75
	0.40					0.67	1.00		0	1.75
	0.50					0.67	1.00		0	1.75
0.10	0.05		(1)		L	(1)			(2)	
	0.10				L					
	0.20				C					
	0.30				C					
	0.40				C				0	1.17
0.2	0.05				L	1.00	0.33		1.00	0.67
	0.10				L	(1)			(2)	
	0.20				C	(1)			(2)	
	0.30				C	(1)			(2)	
0.3	0.05				L	1.00	0.50		1.00	0.67
	0.10				c	(1)			1.00	0.67
	0.20				C	1.00	0.50		(2)	
0.4	0.05				C	3+	0		1.00	0.67
	0.10				d	(1)				
0.5	0.05		1.00	0.33	C ^e	3+	0			

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

^cMaximum stress intensity occurs on inside surface, $S_L = 265.1$, $S_C = -81.46$.

^dMaximum stress intensity occurs on inside surface, $S_L = 194.4$, $S_C = -81.78$.

^eOn inside surface.

Table B.6. Direction and location of maximum membrane stresses for pipe radius-to-thickness ratio of 5

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(2)		C	3+	0	L	0	3+
	0.10					3+	0			
	0.20					1.17	0			
	0.30					(1)				
	0.40					(1)				
	0.50					(1)				
0.10	0.05					3+	0			
	0.10					3+	0			
	0.20					1.17	0			
	0.30					(1)				
	0.40					(1)				
0.2	0.05		(1)			3+	0			
	0.10		(2)			3+	0			
	0.20		(2)			1.17	0			
	0.30		(2)			(1)				
0.3	0.05		(1)			3+	0			1.67
	0.10		(1)			3+	0			1.67
	0.20		(2)			1.33	0			1.67
0.4	0.05		(1)			3+	0			1.33
	0.10		(1)			3+	0			1.33
0.5	0.05		(1)			3+	0			1.17

^aDirection of the maximum surface stress; C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.7. Direction and location of maximum membrane stress for pipe radius-to-thickness ratio of 10

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(2)		C	3+	0	L	0	3+
	0.10	C				1.5	0			
	0.20	C				(1)				
	0.30	C				(1)				
	0.40	C				(1)				
	0.50	L				(1)				
0.10	0.05	C				3+	0			
	0.10					1.5	0			
	0.20					(1)				
	0.30					(1)				
	0.40					(1)				
0.2	0.05		(1)			3+	0			1.67
	0.10		(2)			1.67	0			1.67
	0.20		(2)			(1)				1.83
	0.30		(1)			(1)				3+
0.3	0.05		(1)			3+	0			1.17
	0.10		(1)			1.67	0			1.17
	0.20		(2)			(1)				1.33
0.4	0.05		(1)			3+	0		(2)	
	0.10		(1)			1.83	0		(2)	
0.5	0.05		(1)			3+	0		(2)	

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.8. Direction and location of maximum membrane stress for pipe radius-to-thickness ratio of 20

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	C	(2)	↓	C	2.00	0	L	0	3+
	0.10	C				1.17	0			
	0.20	L				1.00	0.83			
	0.30	L				0.83	1.25			
	0.40	L				0.67	1.00			
	0.50	L				0.67	1.00			
0.10	0.05	C		↓		2.00	0			
	0.10	C				1.17	0			
	0.20	C				(1)				
	0.30	L				0.67	1.00			
	0.40	L				0.67	1.00			
0.2	0.05	C	(1)	↓		2.25	0			1.33
	0.10	C	(2)			1.33	0			1.33
	0.20	C	(2)			(1)				1.50
	0.30	L	(2)			(1)				1.67
0.3	0.05	C	(1)	↓		2.50	0	(2)		
	0.10	C	(1)			1.33	0	(2)		
	0.20	L	(1)			(1)		0		1.17
0.4	0.05	C	(1)	↓		2.75	0	(2)		
	0.10	C	(1)			1.50	0	(2)		
0.5	0.05	C	(1)	↓	↓	2.75	0	↓	1.00	0.83

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.9. Direction and location of maximum membrane stresses for pipe radius-to-thickness ratio of 40

β_1	β_2	W			M_L			M_C		
		D^a	X^b	θ	D^a	X^b	θ	D^a	X^b	θ
0.05	0.05	C	(2)		C	1.50	0	L	0	3+
	0.10	C				(1)				
	0.20	L				0.67	1.00			
	0.30	L								
	0.40	L								
	0.50	L								
0.10	0.05	C				1.50	0			1.67
	0.10	C				(1)				1.83
	0.20	L				(1)				2+
	0.30	L				0.67	1.00			2+
	0.40	L				0.67	1.00			2+
0.2	0.05	C	(1)			1.50	0		(2)	
	0.10	L				1.17	0		(2)	
	0.20	L				(1)			0	1.17
	0.30	L				(1)			0	1.13
0.3	0.05	C				2.00	0		(2)	
	0.10	L				1.17	0		(2)	
	0.20	L				(1)			(2)	
0.4	0.05	C				2.00	0		1.00	0.83
	0.10	L				1.17	0		(2)	
0.5	0.05	C				2.00	0		1.00	0.83

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Table B.10. Direction and location of maximum membrane stresses for pipe radius-to-thickness ratio of 80

β_1	β_2	W			M_L			M_C		
		D ^a	X ^b	θ	D ^a	X ^b	θ	D ^a	X ^b	θ
0.05	0.05	L	(1)		C	1.25	0	L	0	2.25
	0.10		(2)		C	(1)				2.50
	0.20				C	0.67	1.00			3+
	0.30				C					
	0.40				L					
	0.50				L					
0.10	0.05		(1)		C	1.25	0			1.33
	0.10		(1)		C	(1)				1.33
	0.20		(2)		C	(1)				1.67
	0.30		(2)		C	0.67	1.00			2.00
	0.40		(2)		L	(1)				2+
0.2	0.05		(1)		C	1.50	0		(2)	
	0.10					(1)			(2)	
	0.20					(1)			(2)	
	0.30					(1)			0	1.17
0.3	0.05					1.50	0		1.00	0.83
	0.10					(1)			(2)	
	0.20					(1)			(2)	
0.4	0.05					3+	0		1.00	0.67
	0.10					(1)			1.00	0.67
0.5	0.05					3+	0		1.00	0.67

^aDirection of the maximum surface stress: C indicates circumferential and L indicates longitudinal.

^bA (1) or (2) under this column indicates that the location of the maximum stress corresponds to the positions shown in Fig. B.1.

Appendix C

EXAMPLE ANALYSIS OF A LUG SUPPORT

Loads and Primary-Stress-Intensity Limit

The determination of the loads applied to the lugs is not a subject of this report. However, the engineering judgment involved in determining the loads can be as significant as the accuracy of the equations relating loads to stresses developed herein. This aspect can be illustrated by analyzing a 24-in. sched-160 vertical pipe ($D_o = 24$ in., $t = 2.343$ in.) supported by lugs as shown in Fig. C.1, with the size of the

ORNL-DWG 73-5837

TOTAL LOAD = 13,550 lb (25 ft OF 24-in. SCHED-160 PIPE)

$t = 2.343$ in.

$W = 542$ lb/ft

Fig. C.1. Example of lugs used to support vertical pipe.

lug to be controlled by the modified primary-stress-intensity limit equation given in Table 2 of the text:*

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M_i + (P_L + P_b)_{lug} \leq 1.5S_m, \quad (C.1)$$

where, from Eq. (21) of the text,

$$(P_L + P_b)_{lug} = \frac{B_T W}{A_\ell} + \frac{B_L M_L}{Z_{\ell L}} + \frac{B_C M_C}{Z_{\ell C}} + \frac{Q_1}{2L_1 L_a} + \frac{Q_2}{2L_2 L_b} + \bar{M}_T. \quad (C.2)$$

We will assume that the primary stress intensity in the pipe without considering the lug supports is 20,000 psi. Specifically, $PD_o/2t$ is assumed to be 20,000 psi, and the magnitude $D_o M_i/2I$ is assumed to be 10,000 psi.** The stress indices $B_1 = 0.5$ and $B_2 = 1.0$ are the values given in Table NB-3683.2-1 of the Code for straight pipe remote from welds or other discontinuities. Thus,

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M_i = 20,000 \text{ psi}. \quad (C.3)$$

It is also assumed that the four support-reaction forces Q_2 act at some offset distance X from the pipe surface as shown in Fig. C.1. The longitudinal moment M_L acting on the pipe is then

$$M_L = Q_2 X. \quad (C.4)$$

Assume further that

$$\begin{aligned} 1.5S_m &= 30,000 \text{ psi}, \\ W &= M_C = Q_1 = \bar{M}_T = 0, \\ L_1 &= L_2/2. \end{aligned} \quad (C.5)$$

*Equations numbered C.1, C.2, etc., in this appendix are not to be confused with equations abstracted from the Code and given similar numbers in the main text.

**It should be noted that M_i used in Code Eqs. (9) and (13) is a magnitude of a moment vector, whereas M_i used in Code Eqs. (10), (11), and (12) is a range of a moment vector. These do not necessarily have the same numerical value.

The section modulus of the lug is therefore

$$Z_{\ell L} = 2/3 L_2^3 . \quad (C.6)$$

The equation for determining the lug dimension L_2 from Eqs. (C.1), (C.2), and (C.3) is then

$$(P_L + P_b)_{\text{lug}} = \frac{3B_L Q_2 X}{2L_2^3} + \frac{Q_2}{2L_2 L_b} = 10,000 \text{ psi} . \quad (C.7)$$

As will be apparent from the results below, the dimensionless parameters involved in this example (i.e., $\gamma = 4.62$, $\beta_1 \leq 0.093$, $\beta_2 < 0.186$) are such that C_L is controlled by the lower limit of 1.0, $B_L = 2/3$ (see Table 4 in the text), and $L_b = L_1$ (because L_1 is less than t). Hence, Eq. (C.7) for determining L_2 becomes

$$(P_L + P_b)_{\text{lug}} = \frac{Q_2 X}{L_2^3} + \frac{Q_2}{L_2^2} = 10,000 \text{ psi} . \quad (C.8)$$

The question arises as to what should be used for Q_2 and X . If an ideal situation is assumed in which each lug takes one-quarter of the total load and the load is introduced adjacent to the pipe surface, then $Q_2 = 13,550/4$, $X = 0$, $L_2 = 0.582$ in., and the minimum size of the lug is 0.582 by 1.164 in. However, unless some carefully designed spring supports are used, it is more likely that, at some time, the entire load will be taken by one lug. It might be possible to design supports under the lug that would introduce the load quite close to the pipe wall, perhaps making X as little as 0.5 in. Table C.1 shows various lug sizes required for various values of X if all the load (13,550 lb) is supported by one lug.

However, if the analyst had assumed the ideal conditions of $Q_2 = 13,550/4$ and $X = 0$, if he had specified a lug with $2L_2 = 1.164$ and $2L_1 = 0.582$, if all of the load were then inadvertently taken by one of the four lugs, and if the supports were such that X was effectively 2 in., then

$$(P_L + P_b)_{\text{lug}} = \frac{(13,550)(2)}{(0.582)^3} + \frac{(13,550)}{(0.582)^2} = 144,470 \text{ psi} .$$

In this case the support might fail.

Table C.1. Minimum lug dimensions for support
of the 24-in. sched-160 pipe^a

X (in.)	L ₂ (in.)	Stress ^b due to bending (psi)	Stress ^c due to shearing (psi)	Lug size (in.)		β_2	β_1
				2L ₂	2L ₁		
0	1.16	0	10,000	2.32	1.16	0.107	0.054
0.5	1.36	2693	7,325	2.72	1.36	0.126	0.063
1.0	1.50	4014	6,022	3.00	1.50	0.139	0.069
2.0	1.71	5419	4,633	3.42	1.71	0.158	0.079
4.0	2.01	6674	3,354	4.02	2.01	0.186	0.093

^aTotal load = 13,550 lb (25 ft at 542 lb/ft).

^bBending stress = $Q_2 X / L_2^3$.

^cShearing stress = Q_2 / L_2^2 .

If the pipe used in the above example had been a 24-in. sched-40 pipe ($t = 0.687$ in., $r = 11.656$ in., $W = 542$ lb/ft) rather than a sched-160 pipe, it would not have been possible to establish the value of the stress index B_L without evaluating C_L for specific lug dimensions as prescribed in Appendix A; that is,

$$B_L = (2/3)C_L - (2/3)0.517^{1.74}\beta_1\beta_2^{2.74} \geq 2/3. \quad (C.9)$$

This makes determining the minimum lug size more complex, and a trial-and-error solution is indicated. For this purpose a rather simple time-sharing computer program in the BASIC language was used.* The results are given in Table C.2. It is interesting to note that for the smaller lug sizes, when $X \leq 0.5$, the stresses in the lug control the design (i.e., $B_L = 2/3$), whereas for the larger sizes the stresses in the pipe are controlling.

*The program listing is given at the end of this appendix.

Table C.2. Minimum lug dimensions for support
of the 24-in. sched-40 pipe^a

X	Stress ^b due to bending	Stress ^c due to shearing	2L ₁	2L ₂	B _L
0	0	10,000	0.65	1.31	2/3
0.5	3765	6,235	0.83	1.66	2/3
1.0	5532	4,467	0.98	1.96	0.810
2.0	7569	2,431	1.33	2.66	1.385
4.0	8482	1,518	2.02	4.04	2.731

^aD_n = 24 in., t = 0.687 in., r = 11.656 in.;
total load = 4278 lb (25 ft at 171.1 lb/ft).

^bBending stress = $Q_2 X / L_2^3$.

^cShearing stress = Q_2 / L_2^2 .

Assumed Operating Conditions and Pipe Properties

The example analysis for the sched-160 pipe is continued below to illustrate the use of the remainder of the equations given in Appendix A.

A very simple set of operating conditions will be assumed, consisting of 50 cycles during which:

$$\frac{P_D}{2t} \text{ varies from 0 to 16,000 psi to 0,}$$

$$\frac{D}{2I} M_1 \text{ varies from 0 to 8000 psi to 0,}$$

$$\frac{1}{2(1-\nu)} E\alpha |\Delta T_1| \text{ varies from 0 to 18,000 psi to 0,}$$

$$\frac{1}{1-\nu} E\alpha |\Delta T_2| \text{ varies from 0 to 9000 psi to 0,}$$

$$E\alpha |T_\ell - T_w| \text{ varies from 0 to 15,000 psi to 0,}$$

the thermal expansion of the piping system is such that the tensile stress in the pipe below the lugs varies from 0 to 500 psi to 0.

The conservative assumptions are made that the entire load will be taken by one lug and that $X = 2.0$ in. From Table C.1 the minimum lug dimensions required to satisfy the primary-stress-intensity limit are $2L_2 = 3.42$ in. and $2L_1 = 1.71$ in.

The pipe cross section is assumed to be circular within a tolerance of $0.08t$, so that $K_1 = 1.00$. The pipe material is assumed to be an austenitic stainless steel with an allowable stress intensity S_m of 20,000 psi at the maximum operating temperature, and the weld between the lug and pipe is assumed to be ground to a radius as required by Fig. A.1. Hence $K_\ell = 1.3$.

Primary-Plus-Secondary Stress-Intensity Range Limit

Following the procedure outlined in Appendix A [NB-3683.3(b)], the limiting equation for satisfaction of the allowable primary-plus-secondary stress-intensity range is

$$S_n = C_1 \frac{PD_o}{2t} + C_2 \frac{D_o}{2I} M_i + \frac{1}{2(1-\nu)} E\alpha |\Delta T_1| + S_{nl} \leq 3S_m \quad (C.10)$$

With the assumed operating conditions and with $C_1 = C_2 = 1.0$, the numerical value of S_n is

$$S_n = 16,000 + 8,000 + 18,000 + S_{nl} = 42,000 + S_{nl} ,$$

where S_{nl} is calculated using the range of loads M_L^* and Q_2^* operating on the lugs:

$$S_{nl} = C_{T_1} \frac{M_L^*}{Z_{\ell L}} + \frac{Q_2^*}{2T_2 T_{1b}} = C_{T_1} \frac{Q_2^* X}{Z_{\ell L}} + \frac{Q_2^*}{2T_2 T_{1b}} \quad (C.11)$$

The range Q_2^* is obtained by multiplying the axial stress in the pipe, assumed here to be 500 psi, by the cross-sectional metal area of the pipe (159.4 in.^2), or $Q_2^* = 79,700$ lb. The numerical value of S_{nl} is thus

$$S_{nl} = (1.0) \frac{(79,700)(2)}{(1.333)(0.855)(1.71)^2} + \frac{79,700}{(2)(1.71)(0.855)} = 75,090 \text{ psi} ,$$

and

$$S_n = 42,000 + 75,090 = 117,090 \text{ psi} , \quad (C.12)$$

which is well above the $3S_m$ limit of 60,000 psi.

The lug design might still be acceptable provided that the restriction placed on the maximum load stresses $S_{nl}^{**} \leq 3S_m$ [see Appendix A, NB-3683.2(b); also Eq. (26) of the text] is met and the design is qualified under the simplified elastic-plastic discontinuity analysis procedures of NB-3653.6 as modified in Appendix A. For this particular design the maximum load stress condition is

$$S_{nl}^{**} = C_L \frac{M_L^{**}}{Z_{\ell L}} + \frac{Q_2^{**}}{2L_2L_b} \leq 3S_m , \quad (C.13)$$

where $M_L^{**} = Q_2^{**}X$, and the maximum load Q_2^{**} is equal to the range Q_2^* plus the deadweight W ; that is,

$$Q_2^{**} = 79,700 + 13,550 = 93,250 .$$

Thus,

$$S_{nl}^{**} = (1) \frac{(93,250)(2)}{(1.333)(0.855)(1.71)^2} + \frac{93,250}{(2)(1.71)(0.855)} = 87,850 \text{ psi} , \quad (C.14)$$

which is greater than $3S_m = 60,000$ psi. Therefore the design is unacceptable. In order to pass this restriction, the tensile stress in the pipe would need to be equal to or less than 315 psi, rather than 500 psi as assumed.

Simplified Elastic-Plastic Discontinuity Analysis [NB-3653.6]

Even though the design failed to pass the maximum-load stress condition $S_{nl}^{**} \leq 3S_m$, the example is continued to show that the potentially overloaded condition is not necessarily prevented by the fatigue analysis.

In order to qualify the design under Paragraph NB-3653.6 of the Code, two conditional equations must first be satisfied; that is, Code Eqs. (12) and (13). (Paragraph NB-3653.2, Satisfaction of Peak Stress Intensity Range, was ruled out since $S_n = 117,090 > 3S_m$.) Code Eq. (12) for the expansion stress S_e is (see Table 2 of the text):

$$S_e = C_2 \frac{D_o}{2I} M_1^* \leq 3S_m, \quad (C.15)$$

where the magnitude of $D_o/2I M_1^*$ was postulated to be 10,000 psi. With $C_2 = 1.0$, the expansion stress limit Eq. (C.15) is obviously met.

The other conditional equation [Code Eq. (13), Table 1 of the text] was modified for the analysis of lug supports in that the term involving $|\alpha_a T_a - \alpha_b T_b|$ was dropped. Thus the appropriate limit is

$$C_1 \frac{P_o D_o}{2t} + C_2 \frac{D_o M_1}{2I} \leq 3S_m, \quad (C.16)$$

where P_o is the range of operating pressure and M_1 is the magnitude of the moment vector used in the primary-stress-intensity limit check. Accordingly, with $C_1 = 1.0$, $P_o D_o/2t = 16,000$ psi, and $D_o M_1/2I = 10,000$ psi the left-hand side of the above equation is 26,000 psi. This value is less than $3S_m = 60,000$ psi; hence, Code Eq. (13) is also satisfied.

We may thus proceed to evaluate the alternating stress

$$S_{alt} = K_e \frac{S_p}{2} \quad (C.17)$$

using the modified equation for S_p given in Appendix A [NB-3683.3(b)] and K_e calculated according to the rules of NB-3653.6(c). The peak-stress intensity range is

$$S_p = K_1 C_1 \frac{P_o D_o}{2t} + K_2 C_2 \frac{D_o}{2I} M_1 + \frac{1}{2(1-\nu)} K_3 E \alpha |\Delta T_1| + \frac{1}{1-\nu} E \alpha |\Delta T_2| + S_{pl}, \quad (C.18)$$

and, with $K_1 = K_2 = K_3 = 1.0$, we get

$$S_p = 16,000 + 8,000 + 18,000 + 9,000 + S_{pl} = 51,000 + S_{pl} .$$

Since

$$\begin{aligned} S_{pl} &= [(K_\ell - 1)(1.5 + 0.537\beta_1\beta_2\gamma)] \frac{P_o D_o}{2t} + K_\ell(S_{nl}) + K_\ell E \alpha |T_\ell - T_w| \\ &= [(1.3 - 1)(1.5 + 0.031)] \times 16,000 + 1.3[75,090] + 1.3 \times 15,000 \\ &= 7,350 + 97,620 + 19,500 = 124,470 , \end{aligned}$$

therefore

$$S_p = 51,000 + 124,470 = 175,470 . \quad (C.19)$$

For austenitic stainless steel with material parameters $m = 1.7$ and $n = 0.3$ from Paragraph NB-3228.3(b) and with $S_n \geq 3mS_m$, that is,

$$S_n = 117,090 > (3)(1.7)(20,000) = 102,000 ,$$

the factor K_e is given by

$$K_e = \frac{1}{n} = \frac{1}{0.3} .$$

The alternating stress S_{alt} from Eq. (C.17) is thus

$$S_{alt} = \frac{(175,470)}{(2)(0.3)} = 292,450 \text{ psi} .$$

From Fig. 1-9.2 of the Code, the allowable number of fatigue cycles N at $S_a = S_{alt} = 295,000$ psi is 60. Because 50 cycles at operating conditions were postulated, the lug design is adequate except for not meeting the overload limitation. For this particular design the new requirement $S_{nl}^{**} \leq 3S_m$ [Eq. (26) of the text] is thus necessary to prevent a potentially dangerous condition.

LISTING OF THE COMPUTER PROGRAM LUGS (BASIC)

```

00010 DIM C(3),Z(6),L$(6)
00020 PRINT
00030 PRINT
00040 PRINT"      NB-3683.3 STRESS INDICES FOR INTEGRAL STRUCTURAL";
00050 PRINT" ATTACHMENTS"
00060 PRINT
00070 PRINT
00080 PRINT"      INPUT DATA"
00090 PRINT
00100 PRINT"ATTACHMENT LENGTH = ";
00110 INPUT L2
00120 PRINT" ATTACHMENT WIDTH = ";
00130 INPUT L1
00140 PRINT" MEAN PIPE RADIUS = ";
00150 INPUT R
00160 PRINT"      PIPE THICKNESS = ";
00170 INPUT T
00180 PRINT
00190 FOR I = 1 TO 6
00200 LET L$(I) = "PIPE"
00210 NEXT I
00220 LET L1 = 0.5*L1
00230 LET L2 = 0.5*L2
00240 LET B1 = L1/R
00250 LET B2 = L2/R
00260 LET D1 = T
00270 IF L2>T THEN 300
00280 LET D1 = L2
00290 LET L$(5) = "LUG"
00300 LET D2 = T
00310 IF L1>T THEN 340
00320 LET D2 = L1
00330 LET L$(4) = "LUG"
00340 LET D3 = L1
00350 LET D4 = L2
00360 IF L1<L2 THEN 390
00370 LET D3 = L2
00380 LET D4 = L1
00390 PRINT
00400 IF B1<0.5 THEN 430
00410 PRINT"*** WARNING ***      BETA-1 EXCEEDS THE LIMIT OF 0.5"
00420 PRINT
00430 IF B2<0.5 THEN 460
00440 PRINT"*** WARNING ***      BETA-2 EXCEEDS THE LIMIT OF 0.5"
00450 PRINT
00460 IF B1*B2<0.075 THEN 500
00470 PRINT"*** WARNING ***      THE PRODUCT OF BETA-1 AND BETA-2 EXCEEDS"
00480 PRINT"                        THE LIMIT OF 0.075"
00490 PRINT
00500 LET Z(1) = 4.0*L1*L2
00510 LET Z(2) = Z(1)*L2/3.0
00520 LET Z(3) = Z(1)*L1/3.0
00530 LET Z(4) = 2.0*L1*D1
00540 LET Z(5) = 2.0*L2*D2
00550 LET Z(6) = D3*D4*(1.0 + D3/D4)*T
00560 LET A = D3*D3*D4*(0.8 + 0.05*D4/D3)

```

```

00570 IF Z(6) < A THEN 600
00580 LET Z(6) = A
00590 LET L$(6) = "LUG"
00600 FOR I = 1 TO 6
00610 LET Z(I) = 1.0/Z(I)
00620 NEXT I
00630 LET G = R/T
00640 LET X = 0.4343*LOG(B1)
00650 LET Y = 0.05 + 0.4343*LOG(B2)
00660 LET A = 0.6428*X - 0.7660*Y
00670 LET A = -0.4545*A*A
00680 LET A = A - (0.7660*X + 0.6428*Y)
00690 LET C(1) = 7.64*B1*B2*(G1.64)*(A1.54)
00700 IF C(1) > 1.0 THEN 730
00710 LET C(1) = 1.0
00720 LET L$(1) = "LUG"
00730 LET X = -0.45 + 0.4343*LOG(B1)
00740 LET Y = -0.55 + 0.4343*LOG(B2)
00750 LET A = 0.7660*X - 0.6428*Y
00760 LET A = -0.5*A*A
00770 LET A = A - (0.6428*X + 0.7660*Y)
00780 LET C(2) = 0.51*B1*B2*B2*(G1.74)*(A4.74)
00790 IF C(2) > 1.0 THEN 820
00800 LET C(2) = 1.0
00810 LET L$(2) = "LUG"
00820 LET X = -0.75 + 0.4343*LOG(B1)
00830 LET Y = -0.60 + 0.4343*LOG(B2)
00840 LET A = 0.6428*X - 0.7660*Y
00850 LET A = -0.5555*A*A
00860 LET A = A - (0.7660*X + 0.6428*Y)
00870 LET C(3) = 0.76*B1*B1*B2*(G1.90)*(A3.40)
00880 IF C(3) > 1.0 THEN 910
00890 LET C(3) = 1.0
00900 LET L$(3) = "LUG"
00910 LET K = 1
00920 PRINT"(P + P ) = "C(1)*Z(1)" * THRUST @ "L$(1)
00930 PRINT" L B"
00940 PRINT" + "C(2)*Z(2)" * LONG. MOMENT @ "L$(2)
00950 PRINT
00960 PRINT" + "C(3)*Z(3)" * CIR. MOMENT @ "L$(3)
00970 PRINT
00980 PRINT" + "Z(4)" * CIR. SHEAR @ "L$(4)
00990 PRINT
01000 PRINT" + "Z(5)" * LONG. SHEAR @ "L$(5)
01010 PRINT
01020 PRINT" + "Z(6)" * TORQUE @ "L$(6)
01030 PRINT
01040 PRINT
01050 IF K=2 THEN 1130
01060 FOR I = 1 TO 3
01070 LET C(I) = 2.0*C(I)/3.0
01080 NEXT I
01090 PRINT" S = "C(1)*Z(1)" * THRUST @ "L$(1)
01100 PRINT" N"
01110 LET K = K + 1
01120 GO TO 940
01130 PRINT"DO YOU HAVE ANOTHER CASE? ";
01140 INPUT Q$
01150 IF Q$="YES" THEN 60
01160 END

```

INTERNAL DISTRIBUTION

1. S. E. Beall	32. W. J. McAfee
2. M. Bender	33. H. C. McCurdy
3. S. E. Bolt	34. J. G. Merkle
4. J. W. Bryson	35. A. J. Miller
5. J. R. Buchanan	36-60. S. E. Moore
6. J. P. Callahan	61. A. M. Perry
7. C. E. Childress	62. T. W. Pickel
8. J. A. Clinard	63. H. Postma
9. J. M. Corum	64. C. E. Pugh
10. W. B. Cottrell	65. R. A. Schmidt
11. F. L. Culler	66. M. R. Sheldon
12-16. W. G. Dodge	67. J. E. Smith
17. M. H. Fontana	68. I. Spiewak
18. A. P. Fraas	69. D. A. Sundberg
19. W. R. Gall	70. D. B. Trauger
20-24. W. L. Greenstreet	71. G. D. Whitman
25. R. C. Gwaltney	72. G. T. Yahr
26. W. O. Harms	73. ORNL Patent Office
27. H. W. Hoffman	74-75. Central Research Library
28. P. R. Kasten	76. Document Reference Section
29. K. C. Liu	77-82. Laboratory Records Department
30. R. N. Lyon	83. Laboratory Records - Record Copy
31. R. E. MacPherson	

SUBCONTRACTORS AND CONSULTANTS

84. A. J. Edmondson, University of Tennessee, Knoxville, TN
85. R. H. Gallagher, Cornell University, Ithaca, NY
86. J. K. Hayes, Combustion Engineering, Chattanooga, TN
87. R. W. Holland, University of Tennessee, Knoxville, TN
88. A. T. Hopper, Battelle Memorial Institute, Columbus, OH
89. L. E. Hulbert, Battelle Memorial Institute, Columbus, OH
90. N. Krishnamurthy, Auburn University, Auburn, AL
91. R. L. Maxwell, University of Tennessee, Knoxville, TN
92. F. M. O'Hara, Oak Ridge, TN
93. E. T. Onat, Yale University, New Haven, CT
94. G. H. Powell, University of California, Berkeley, CA
95. Y. R. Rashid, General Electric Co., San Jose, CA
- 96-100. E. C. Rodabaugh, Battelle Memorial Institute, Columbus, OH
101. W. F. Swinson, Auburn University, Auburn, AL
102. S. Y. Zamrik, Pennsylvania State University, University Park, PA
103. Z. Zudans, Franklin Institute, Philadelphia, PA

- 104-105. Director, Division of Reactor Research and Development, U.S Atomic Energy Commission, Washington, D.C.
- 106. RRD Senior Site Representative, ORNL
- 107. Research and Technical Support Division, AEC-ORO, P. O. Box E, Oak Ridge, TN.
- 108. Patent Office, AEC, ORO
- 109-185. Nozzle Analysis and Nuclear Design Criteria - Piping-Pumps-Valves Programs Distribution
- 186-454. Given distribution as shown in TID-4500 under UC-79, UC-79h, and UC-79k