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Reactor Fuel Elements and Coolant Channel

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Abstract

An analysis of the temperature structure in the GCR-2 fuel elements and coolant stream at the position where the maximum fuel element surface temperature exists is presented. Results were obtained by numerical methods on the IBM 704 digital computer. The effect of variation in channel size is shown, and a method of data correlation is suggested. Preliminary conclusions are presented as to the effect of temperature structure on the design and testing of gas cooled reactor fuel element configurations.

In view of the rate at which gas cooled reactor work is proceeding results of the calculations are being published in their current incomplete form. The study is continuing in an effort to refine the calculations, and experimental data will eventually be available with which to verify the analytical conclusions.

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Nomenclature

C_p	specific heat
f	friction factor (as defined in McAdams' Heat Transmission)
g_c	conversion factor
k	static thermal conductivity
K	"total thermal conductivity" including the effects of eddy diffusivity
q	rate of heat flux
r_w	radius of a straight tube
T	temperature
u	local axial velocity
V	average velocity
w	rate of volume heat generation
y	distance from nearest solid boundary (except where used to denote coordinate system)
ϵ_h	eddy diffusivity for heat exchange
ϵ_m	eddy diffusivity for momentum exchange
μ	dynamic viscosity
ρ	density
τ	fluid shear stress
τ_w	fluid shear stress at a solid boundary

Introduction

Early in the GCR-2 design study a fuel element configuration as shown in Fig. I which consisted of seven cylindrical capsules, with one centrally located and the remaining six arranged around it centered on the vertices of a regular hexagon was chosen. This configuration has sufficient surface to volume ratio to permit removal of heat from the fuel without fins, and the internal thermal path is short enough that temperatures inside the fuel can be kept within desired limits. There is also the advantage that both the capsule and the enclosed uranium oxide fuel slugs are

Figure I. GCR-2 Fuel Element Configuration

easy to fabricate. This basic geometry was held throughout the design study. Variations were limited to channel diameter and the diameter of the central hole in the fuel slugs. The latter does not constitute a heat transfer surface and was neglected in this analysis.

From the known axial power density schedule and coolant flow rate it is possible to calculate the mixed mean coolant temperature at any axial position by a simple heat balance. Using the Dittus-Boelter equation, $Nu = 0.023 Re^{0.8} Pr^{0.4}$, or an equivalent expression, yields the "average" capsule surface temperature, where "average" refers to the single surface temperature which would exist if the heat flux from each surface element of every fuel rod at a given point in the reactor

were the same and the film coefficient were likewise everywhere constant. The two temperature schedules so calculated can be plotted against the axial dimension to give the familiar plot which is commonly used to show temperature in gas cooled reactors.

It is often assumed that in heat transfer to a turbulent fluid such a large part of the temperature drop occurs in the boundary layer that the remainder taking place in the main stream is negligible. The results of these calculations show that this is not a safe assumption to make arbitrarily. It is obvious from Fig. I that the heat flux from each surface element of every capsule is not the same but is, in fact, dependent on the peculiarities of the coolant flow distribution associated with the arrangement of the capsules in the channel. Thus there will evidently be a temperature variation along the peripheries of the several rods which will be dependent in general on the heat sink each surface element has at its disposal and the resistance to heat flow that each surface element encounters.

At the time of the GCR-2 report was written, the existence of such a temperature variation was recognized, and the maximum surface temperature was arbitrarily assumed to exceed the average surface temperature by 300°F . With a 1500°F metallurgical limit imposed on the stainless steel, the average surface temperature became 1200°F . If the 300°F margin were insufficient the 1500°F metallurgical limit would be exceeded at design conditions, and if it were excessive the calculated performance of the GCR-2 would be less than its full potential. Since the importance of the effect of temperature variation was realized, the Reactor Projects Division initiated analytical and experimental studies to more closely determine the necessary margin between the average temperature and the metallurgical limit. The results of the analytical approach are presented in this paper.

Method of Analysis

It will be apparent as the method of analysis is developed that the objects of the investigation are some quick, approximate answers rather than a precise mathematical exercise. Where simplifying assumptions are made for the sake of expediency, they will be pointed out and their effects discussed. No attempt is made at this time to determine the effect of slots in the channel wall used for hanging the fuel elements although it is known they will increase the temperature differences on some of the capsules.

If a heat balance is written for the coolant on the differential volume element shown in Fig. II in which there is heat conduction in the x and y directions and heat transport in the z direction, the equation which results is the general heat conduction equation with variable thermal conductivity and variable sinks.

Figure II

$$\frac{\partial}{\partial x} \left[K(x,y) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[K(x,y) \frac{\partial T}{\partial y} \right] - \rho C_p u(x,y) \frac{\partial T}{\partial z}(x,y,z) = 0$$

In the fuel region Poisson's equation applies

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{w}{k_{UO_2}} = 0$$

It is these equations which must be solved together with appropriate boundary conditions in order to determine the temperature structure at a given axial location.

Before a solution can be obtained it is necessary to determine K , u , and $\partial T / \partial z$ in the first equation as functions of position.

The "total thermal conductivity", K , which includes the effects of diffusivity was determined as described in Appendix A. At the time the problem was being solved, the machine code for numerical relaxation problems did not include in its routine considerations for variable conductivity. Therefore a hand calculation of K was performed, yielding K as a function of position, and the results averaged. This has the effect of depressing the temperature gradients in the region between a solid boundary and the point of occurrence of the "average" conductivity and steepening them for points beyond this regime. This effect is shown in Fig. III. Note that "solid boundary" as used in the above discussion and portrayed in Fig. III is in reality the surface where the laminar sublayer meets the turbulent core (i.e. the laminar sublayer-turbulent core interface).

Figure III

One of the refinements which are being made in the continuing work is a revision to the code to permit K to vary in x and y .

The axial velocity, u , was determined as outlined in Appendix B. The velocity was assumed to depend on the distance a fluid particle was from the nearest wall in the same manner that the universal velocity profile expresses the velocity field in a pipe for the case of established flow.

The axial temperature gradient, $\partial T / \partial z$, was assumed constant in x and y and was evaluated at the value of z where the maximum capsule surface temperature in the reactor exists. The justification for assuming the axial temperature gradient a constant over the flow area is presented in Appendix C. The axial section for which the calculation was performed was on the order of $300 L/D$'s from the inlet, and it is quite reasonable to expect that the temperature field, as well as the velocity field, was the "established" or "equilibrium" temperature profile.

At the present time the most which can be said is that the equilibrium temperature profile represents a maximum temperature variation which will be approached in the reactor. A calculation based on an infinite slab indicates that the temperature variations existing at the outlet end of a 40 in. fuel element would be 70 to 80 percent of those calculated by assuming equilibrium.

Discussion

The results of the computations are illustrated on the three temperature contour plots for the three channel sizes considered (Figs. IV through VI). Also, in Fig. VII is shown the variation of capsule surface temperature with position on the surface. The short horizontal bars on the curves in Fig. VIII indicate the average surface temperature actually necessary to transfer the amount of heat postulated in this calculation ($18.0 \text{ Btu/ft}^2 \text{ sec}$). As mentioned previously, the average surface temperature at this location in the reactor was calculated as 1200°F from the given heat flux, a mixed mean gas temperature of 858°F , and a heat transfer coefficient as obtained from the Dittus-Boelter equation for the heating of fluid in parallel flow on the outside of tubes.

It was expected that the local fuel element surface temperatures would vary above and below 1200°F , but the results show that the temperatures of the elements of surface fluctuate around a consistently higher average surface temperature. Furthermore, it is seen that the larger the channel diameter, the higher is the average surface temperature about which the temperatures of the surface elements fluctuate.

This leads to an interesting hypothesis.....that a correction must be made to the heat transfer coefficient, as obtained from such relations as the Dittus-Boelter equation, which takes into account the effects of the detailed shape of the fluid passage. The phenomenon is analogous to "fin efficiency" corrections for extended surfaces, except that in this instance the factor is a result of the shape of the fluid passage rather than the shape of the solid conductor. One can thus define a "passage shape efficiency", η_{ps} , as the ratio of the ΔT required to transfer a given amount of heat across a film (as calculated by the Dittus-Boelter or similar equations) in a certain configuration to the ΔT actually required to transfer that amount of heat. Using this definition it was found that η_{ps} for the fuel cluster varied from 85% to 52% as the channel diameter was increased from 3.05 in. to 3.45 in.

If this hypothesis is valid it offers an explanation for some of the failures of longitudinal finned surfaces to perform as predicted in channels.

It is planned to direct further investigations, both analytical and experimental, toward developing relationships between passage shapes and "passage shape efficiency".

In order to better correlate the numerical data obtained, resort was made to a semi-empirical analysis. Here again, in view of the complexity of the factors which control the temperature structure, no attempt was made to be rigorous. Instead, a simple relationship for estimating the temperature difference between a point on an outer rod closest to the channel wall and a point diametrically opposite on the same rod was formulated. While this expression is by no means generally accurate, it is at least indicative of the magnitude of this temperature difference across a fuel capsule over the range of channel diameters investigated. Let ΔT represent the aforementioned temperature difference.

FIGURE IV. TEMPERATURE DISTRIBUTION IN GCR-2 FUEL ELEMENTS AND COOLANT STREAM
HEAT FLUX.....18.0 BTU/FT² SEC.
AVERAGE COOLANT TEMP.....858° F.
CHANNEL DIAMETER.....3.05"
FUEL ELEMENT DIAMETER.....0.75"

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FIGURE V. TEMPERATURE DISTRIBUTION IN GCR-2 FUEL ELEMENTS AND COOLANT STREAM
HEAT FLUX.....18.0 BTU/FT² SEC.
AVERAGE COOLANT TEMP.....358° F.
CHANNEL DIAMETER.....3.25"
FUEL ELEMENT DIAMETER.....0.75"

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FIGURE VI. TEMPERATURE DISTRIBUTION IN GCR-2 FUEL ELEMENTS AND COOLANT STREAM
HEAT FLUX.....18.0 BTU/FT² SEC.
AVERAGE COOLANT TEMP.....858° F.
CHANNEL DIAMETER.....3.45"
FUEL ELEMENT DIAMETER.....0.75"

Let r = rod radius

ρ = radius of circle on which outer rods are centered

R = radius of channel

ϕ = heat flux from rod surface

K = "total thermal conductivity" of coolant

Assuming that the five variables above are the only ones affecting ΔT , dimensional analysis yields the dimensionless groups $\Delta TK/\phi r$, ρ/r , and R/r . A study of the ΔT across an infinite slab in an analogous configuration and in an analogous heat transfer situation suggests that the form of the relationship between these three groups is

$$\frac{\Delta TK}{\phi r} = \left[\frac{\rho - r}{\rho - 2r} \right]^a - \left[\frac{\frac{2R - \rho - r}{r}}{\frac{R - \rho - r}{\rho + r}} \right]^b$$

The exponents a and b were determined from the results of the machine calculations to be 2.0 and 0.375 respectively.

Thus

$$\frac{\Delta TK}{\phi r} = \left[\frac{\rho - r}{\rho - 2r} \right]^{2.0} - \left[\frac{\frac{2R - \rho - r}{r}}{\frac{R - \rho - r}{\rho + r}} \right]^{0.375}$$

In the above relation a positive sign for ΔT means that the temperature of the inward facing surface of an outer rod is higher than the temperature of the outward facing surface.

The temperature of the surface of the inner rod is practically constant at a value close to the temperature of the inward facing surface of the outer rods. The equation therefore also gives an indication of the difference between the surface temperature of the central rod and the temperature of the surface of the outer rods facing the channel wall. Note also that when $\Delta TK/\phi r$ is zero it does not follow that the temperature is constant around the surface of the outer rods; only that the central rod surface temperature and the inward and outward facing surface temperatures of the outer rods are all nearly equal. There will still exist a temperature variation around the surface of the outer rods which is symmetrical about a rod diameter such as A-B in Fig. VII.

An interesting result can be obtained from the last equation. It is seen from this expression that $\Delta TK/\phi r$ is a function of geometry alone. Therefore, for a given geometry and heat flux the quantity ΔTK is constant. Hence the relative temperature differences resulting from use of two different cooling fluids in a given geometry with a given heat flux (still assuming, of course, that r , ρ , R , ϕ , and K are the only variables affecting ΔT) can be expressed as $\Delta T_2/\Delta T_1 = K_1/K_2$. As shown in Appendix A, Eq. (g), $K = k + 0.4 C_p \sqrt{f/2} \rho V y (1 - \frac{y}{r_w}) - C_p \mu$ for $0 \leq y \leq \frac{r_w}{2}$.

In the turbulent region both k and $C_p \mu$ are small compared to $0.4 C_p \sqrt{f/2} \rho V y (1 - \frac{y}{r_w})$.

Therefore $\frac{\Delta T_2}{\Delta T_1} = \frac{K_1}{K_2} \approx \frac{(C_p \sqrt{f/2} V)_1}{(C_p \sqrt{f/2} V)_2}$ at corresponding positions in

a given geometry. For a given amount of heat removal rate and geometry

$$(C_p \rho V)_1 = (C_p \rho V)_2 \quad \text{and hence} \quad \frac{\Delta T_2}{\Delta T_1} \approx \sqrt{\frac{f_1}{f_2}}$$

On this basis, the ΔT occurring with CO_2 , for example, will be related to that occurring with He by the factor $\sqrt{0.006/0.004} = 1.22$.

Conclusions

1. Surface temperature differences existing on a seven capsule cluster fuel element arrangement in a cylindrical channel as shown in Fig. I can be expressed by the dimensionless group $\Delta T K / \phi r$, which is, in turn, a function of capsule radius, capsule spacing, and channel size. If the capsule and channel diameters are fixed the capsule spacing can be adjusted to minimize surface temperature variation. For GCR-2 conditions the minimum temperature variation will be approximately $\pm 100^\circ F$ about the average surface temperature (see dashed curve in Fig. VII).

2. The temperature structure of the seven capsule cluster was found to be quite sensitive to capsule location relative to the channel wall. A movement of 10 mils from the point of minimum temperature difference can be expected to change the temperature difference across a capsule by approximately $20^\circ F$. Greater displacements will produce much more than proportionally greater temperature differences.

It is also expected that the shift of a single capsule as a result of dimensional tolerances or imperfect alignment with the channel can produce similar temperature differences. Therefore it is unlikely that the same fuel element cluster can be used in three different channel diameters as was assumed in the original GCR-2 design.

3. The present GCR-2 fuel element concept is probably unstable from a standpoint of thermal deflection. A $20^\circ F$ temperature difference resulting from a 10 mil displacement can, if maintained over the length of the capsule, result in a maximum deflection at the mid-length of approximately 50 mils toward the hot side. This will produce further temperature difference, more deflection, etc. until the affected capsule touches an adjacent one or the channel wall.

Although, admittedly, the temperature difference along the surface of the rods is not expected to exist along the entire capsule length, the "feed-back" is so great that instability of the fuel element is expected.

As mentioned in ORNL-2500*, part 3, section 6.2, shortening the fuel element will improve it by decreasing the deflection resulting from a given temperature difference and also by decreasing the amount of temperature difference likely to occur. However, the sensitivity of the seven capsule configuration to asymmetry appears to be such that decreasing the length by a factor of two may not completely remove the danger of a diverging progression of events with temperature difference along the surface of the capsules causing bowing which in turn magnifies the temperature difference, etc.

4. It is concluded that the ideal fuel element configuration must not only satisfy the obvious requirements of sufficient surface area and short internal thermal path, it must also be as generally symmetrical as possible in either polar or rectangular geometry. Examples are single rods or annuli in individual channels or suitably spaced slabs.

The symmetry of the seven rod cluster can be improved if it is envisioned as seven separate rods in individual channels located at the center and corners of a hexagon. If the hexagon is decreased in size until the channels begin to merge a configuration will result which is shown in Fig. VIII.

Figure VIII

5. The results are based on an average heat flux of $18 \text{ Btu/ft}^2 \text{ sec}$ and a total conductivity corresponding to that of helium at a Reynolds number of 6×10^4 . Changing the heat flux will affect the temperature structure in direct proportion. Changing the coolant conditions will change the temperature structure approximately as the inverse ratio of the square roots of friction factors. Carbon-dioxide for instance under comparable conditions would give temperature differences approximately 1.2 times those of helium.

Recommendations

Any hot spots existing on the surface of the fuel rods for any appreciable distance along them which result from anomalies in the flow, wakes and eddies from support structures, etc. will similarly make possible such instabilities as noted above. Such effects are intractable analytically and must be investigated in the experimental program. The experimental program should endeavor to ascertain the following:

- a. The temperature on the surfaces of the several rods as a function of angular position on the rods and position along the channel.
- b. The temperature on the surfaces of the rods as a function of displacement of the rods from their intended location.
- c. Whether or not the bowing of the rods leads to a divergent progression of events, with bowing causing greater temperature difference across the rods which in turn causes still more bowing, etc.
- d. The length along the rods in the axial direction through which hot spots persist for various angular and axial positions.
- e. Effects of flow separation, flow irregularities, and variations of geometry from the idealized, simple configuration used in this analysis on the surface temperature of the capsules.

Appendix A

Determination of "total thermal conductivity"

The eddy diffusivity for heat flow ϵ_h is defined by the equation

$$q = - (k + \rho c_p \epsilon_h) \frac{dT}{dy} \quad (a)$$

while the eddy diffusivity of momentum ϵ_m is defined analogously by

$$\tau_{g_c} = (\mu + \rho \epsilon_m) \frac{du}{dy} \quad (b)$$

For flow in a long straight tube the shear stress is proportional to radius and equation (b) becomes

$$\tau_{w g_c} (1 - \frac{y}{r_w}) = (\mu + \rho \epsilon_m) \frac{du}{dy} \quad (c)$$

Noting that $\tau_w = f \frac{\rho V^2}{2 g_c}$ and solving (c) for ϵ_m yields

$$\epsilon_m = \frac{f/2 V^2}{du/dy} (1 - \frac{y}{r_w}) - \frac{\mu}{\rho} \quad (d)$$

If one assumes that the eddy diffusivity for heat transfer is equal to the eddy diffusivity for momentum transfer, the total thermal conductivity (term in parenthesis in equation (a)) is

$$K = k + c_p \frac{f/2 \rho V^2}{du/dy} (1 - \frac{y}{r_w}) - c_p \mu \quad (e)$$

In the laminar sub-layer the diffusivity is zero and $K = k$. For long straight tubes the velocity in the turbulent core is given as a function of distance from the wall by the universal velocity profile as

$$u = 5.5 V \sqrt{f/2} + 2.5 V \sqrt{f/2} \ln \frac{V \rho \sqrt{f/2}}{\mu} y \quad (f)$$

Substituting du/dy as obtained from equation (f) into (e) yields

$$K = k + 0.4 c_p \sqrt{f/2} \rho V y (1 - \frac{y}{r_w}) - c_p \mu \quad (g)$$

It will be noticed that K from equation (g) maximizes at $y = \frac{r_w}{2}$ and then decreases with increasing y. This is contrary to reality, and so the value for K for values of y greater than $r_w/2$ is commonly assumed to be the same as at $r_w/2$. This assumption was made in this analysis in the process of determining the average "total thermal conductivity".

It was assumed that the flow between solid boundaries behaved as though the region between the boundaries was a long straight tube with the boundaries acting as the walls of the tube.

Appendix B

Determination of the Velocity Field

As indicated in Fig. IX the only region that need cause concern is the turbulent regime, for the buffer layer has been apportioned among the laminar sublayer and turbulent core, and subsequently the laminar sublayer is treated as a thin film at the surface of the rods and at the channel wall. The manner in which the buffer layer was distributed between the other two hydrodynamic regions is illustrated in the figure. From the known shape of the velocity—distance-from-wall relations for the laminar and turbulent regions on a log-log plot as obtained from the "universal velocity distribution", an extrapolation of the two curves yielded the intersection "a". This point corresponds to a dimensionless distance from the wall which was taken as the interface between the laminar sublayer and the turbulent core.

The velocity—distance-from-wall correlation as obtained from the universal velocity distribution was assumed to hold in the peculiar geometry of the septafoil with the "distance" in this case being the perpendicular distance from the nearest solid boundary.

FIGURE IX UNIVERSAL VELOCITY PROFILE (From R.C. Martinelli, Trans. ASME, 69, 947-959 (1947).)

Appendix C

The axial temperature gradient, $\partial T / \partial z$, was assumed constant in the x and y dimensions. This assumption is commonly made in similar calculations and is justified mainly by the simplification which it effects. The axial temperature gradient would be a constant in x and y if a condition of aerodynamic and thermal "equilibrium" profiles existed. Such a situation is approached asymptotically and some discussion is in order as to how close an approach is to be expected in the GCR-2 type fuel element.

Since the interest here is in finding the maximum fuel element temperature which may exist in the reactor the calculations are performed to determine the temperature variation at the point where the maximum "average" temperature exists. This point is approximately 200 diameters from the inlet to the active core and 50 diameters from the nearest upstream gap between fuel elements. It is apparent that turbulence will be well established at this point. The Reynolds number is of the order 6×10^4 , and the point of interest is sufficiently removed from the inlet to be far from the transition zone. In addition, Jakob and Dow find that for axial flow outside isolated heated cylinders transition to turbulent flow occurred almost immediately if the cylinders had hemispherical nose pieces (as do the GCR-2 fuel capsules). It seems safe to assume, therefore, that an equilibrium aerodynamic profile will exist at the point of the calculation. The degree of attainment of thermal equilibrium is less certain. Eckert** presents data which indicates that for water ($Pr = 4$) there is no appreciable change in heat transfer coefficient in tubes at Reynolds numbers above approximately 1×10^4 and beyond 50 diameters from the inlet which would imply that entrance effects are no longer appreciable and that the hydrodynamic and thermal profiles are close to equilibrium. Since for helium $Pr = 0.7$ and since low Prandtl numbers are associated with greater variation of temperature in the turbulent zone it may be argued that helium would take more diameters to reach a given degree of thermal equilibrium than water. However the 50 diameters shown by Eckert's data includes the distance required for transition from laminar to turbulent flow which, as discussed previously, is not expected on the fuel element. The point of interest is in a region of decreasing power density with z. It is believed this will decrease the length required to attain thermal equilibrium, all other things being equal.

At the present time a numerical investigation is under way to study the buildup of the temperature field in a fluid flowing with the velocity distribution described in Appendix B, having thermal conductivities calculated by the method described in Appendix A, and entering the geometry

* Jakob, Max and Dow, W. M., Heat Transfer from a Cylindrical Surface to Air in Parallel Flow with and without Unheated Starting Sections -- ASME Transactions 1946.

** Eckert, "Introduction to Transfer of Heat and Mass", p 116.

(infinite parallel plates) with a constant temperature. The results of this problem will not be available for some time and so a brief analysis is presented for the following simplified case.

Figure X

Assumptions:

- (a) constant velocity everywhere
- (b) constant thermal conductivity everywhere
- (c) no conduction in x direction.

The differential equation describing the temperature structure is:

$$\frac{\partial^2 T}{\partial y^2} - \frac{\rho_p^c u}{K} \frac{\partial T}{\partial x} = 0$$

where T is the fluid temperature above inlet conditions.

The boundary conditions are

$$(1) \quad T \Big|_{x=0} = 0$$

from the statement of the problem.

$$(2) \left. \frac{\partial T}{\partial y} \right|_{y=0} = 0$$

because there is no heat flow across the centerline.

$$(3) \left. \frac{\partial T}{\partial y} \right|_{y=y_1} = \frac{q}{K}$$

where q is the heat flux per unit area per unit time entering from one side

$$(4) \left. \frac{\partial T}{\partial x} \right|_{x \rightarrow \infty} = \frac{q}{y_1 \rho C_p u}$$

as obtained by a simple heat balance assuming that at $x \rightarrow \infty$ each streamline will rise in temperature at the same rate

The solution to this boundary value problem is:

$$T = \frac{2 y_1 q}{\pi^2 K} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos \frac{n\pi}{y_1} y e^{-\frac{n^2 \pi^2 K}{y_1^2 \rho C_p u} x} + \frac{q}{K y_1} \left(\frac{y^2}{2} + \frac{x K}{\rho C_p u} \right) - \frac{q y_1}{6 K}$$

This can be made dimensionless by defining

$$\alpha \equiv \frac{x K}{y_1^2 \rho C_p u} \quad \beta \equiv \frac{y}{y_1} \quad \text{to obtain}$$

$$\frac{TK}{q y_1} = \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos n\pi \beta e^{-n^2 \pi^2 \alpha} + \frac{\beta^2}{2} + \alpha - \frac{1}{6}$$

If one defines the heat transfer coefficient, h , by the equation $h \equiv \frac{q}{T_w - \bar{T}_f}$

where T_w is the wall temperature and $\bar{T}_f$ is the average fluid temperature integrated over y at any x one gets

$$\frac{1}{h} = \frac{2y_1}{\pi^2 K} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos \frac{n\pi}{y_1} y}{n^2} e^{-\frac{n^2 \pi^2 K}{y_1^2 \rho c_p u} x} + \frac{1}{3} \frac{y_1}{K} + \frac{\delta}{k}$$

where δ is the laminar sublayer thickness at x , and k is the static thermal conductivity of the fluid.

We now investigate the behavior of the temperature field for flow similar to the conditions in the GCR-2.

The case of the 3.25 in. diameter channel represents a typical example and its parameters are as follows:

$$\begin{aligned} \rho &= 0.106 \text{ lb/ft}^3 \\ u &= 192 \text{ ft/sec} \\ \mu &= 0.0871 \text{ lb/ft hr} \\ k &= 0.158 \text{ Btu/hr ft } ^\circ\text{F} \\ K &= 3.15 \text{ Btu/hr ft } ^\circ\text{F} \\ c_p &= 1.25 \text{ Btu/lb } ^\circ\text{F} \\ y_1 &= 0.01623 \text{ ft} \end{aligned}$$

The last parameter is obtained by making the equivalent diameter between infinite parallel plates the same as the equivalent diameter for the 3.25 in. diameter channel (0.0649 ft). It is seen by differentiating the equation which describes the temperature as a function of x and y that the temperature gradient in the x direction is constant only when the infinite series becomes small compared to the other term (i.e. when $x \rightarrow \infty$). Let us see how far down the channel one must travel before the infinite series in the expression for $\partial T / \partial x$ becomes less than 5% of the total contribution to the gradient.

We have

$$\frac{\partial T}{\partial x} = \frac{2q}{y_1 \rho c_p u} \sum_{n=1}^{\infty} (-1)^n \cos \frac{n\pi}{y_1} y e^{-\frac{n^2 \pi^2 K}{y_1^2 \rho c_p u} x} + \frac{q}{y_1 \rho c_p u}$$

The infinite series will be 5% of the total contribution to $\frac{\partial T}{\partial x}$ when

$$2 \sum_{n=1}^{\infty} (-1)^n \cos \frac{n\pi}{y_1} y e^{-\frac{n^2 \pi^2 K}{y_1^2 \rho c_p u} x} = \frac{1}{19}$$

Substituting the typical numerical values yields $x \approx 2.8$ ft at $y = 0$, the last streamline to reach a state in which the gradient is constant with x . The equivalent diameter of 2 parallel infinite plates spaced 0.03246 ft apart is 0.0649 ft and so thermal equilibrium is reached approximately 43 L/D 's from the inlet. On the basis of this simplified model it seems reasonable to presume that thermal equilibrium has been reached in the channel at the point under investigation.

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- 36. W. F. Banks - ACF Industries
- 37. A. W. Larson, Jr. - AEC, Oak Ridge
- 38. W. I. Thompson - General Dynamics Corporation
- 39. General Nuclear Engineering Corporation
- 40-54. TISE, AEC

Robert H. Smith