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PERFORMANCE STATISTICS OF THE FORTRAN IV (H)
LIBRARY FOR THE IBM SYSTEM/360

by

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	5
I. INTRODUCTION	5
II. TESTING PROCEDURES	5
III. RESULTS.	7
APPENDICES	
A: Summary of Results.	8
B: Decimal Equivalents of Selected Inverse Powers of 2	33
C: Typical Test Program.	34
ACKNOWLEDGEMENT.	37

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ABSTRACT

Testing procedures are described and results tabulated for accuracy and timing tests of the basic FORTRAN IV (H) Subroutine Library provided by the manufacturer for the IBM System/360 computer.

I. INTRODUCTION

The software provided by the manufacturer as part of the IBM System/360 Model 50 Computer delivered to Argonne National Laboratory in 1966 included an extensive library of subroutines for the elementary functions. For the most part, the library was written (in assembly language) by numerical analysts of The University of Chicago under contract to IBM. The resulting product carried documentation¹ indicating a quality of testing unusual in such subroutine packages.

Partly to verify the performance claims made for this package and partly as a prelude to providing some replacement routines of improved performance, the authors undertook an independent certification of the library. The techniques used, differing in detail and in emphasis from those used at The University of Chicago, are extensions of similar techniques originally used by the authors to test the subroutine package for the Control Data 3600 computer. Although the overall picture presented in the original library documentation has been verified, the new certifications present a much more detailed statistical picture of the library and spotlight both its strengths and weaknesses.

II. TESTING PROCEDURES

The library tested was the FORTRAN IV (H) Library contained in Releases 6 and 7 of OS/360 (the latter being implemented in mid-February at Argonne). All certification runs were made under the standard operating system, OS/360, on a Model 50 computer.

The tests themselves can be divided into three categories — accuracy, timing, and error returns. By far the greatest effort was involved in the accuracy tests. Consider the testing of a single-precision function subroutine.

¹IBM System/360 Operating System FORTRAN IV Library Subprograms,
IBM Systems Reference Library, C28-6596-1 (November 1966).

The first step was to divide the domain of the routine into several subdomains, the subdomains usually corresponding in some manner to subdomains used internally in the routine under test. Next, several thousand uniformly distributed pseudo-random, single-precision, floating-point numbers, together with their identically valued double-precision counterparts, were generated for each subdomain. The single-precision numbers were used as arguments for the routine being tested; the double-precision numbers were used as arguments for the corresponding double-precision subroutine. The results of the double-precision computation were considered as "correct." The single-precision results were then converted to double-precision, and the maximum relative error,

$$M = \max \left| \frac{F(x) - f(x)}{f(x)} \right| ,$$

where $F(x)$ is the test value and $f(x)$ is the "correct" value, and the root-mean-square of the relative error,

$$\sigma = \sqrt{\frac{1}{n} \sum_i \left(\frac{F(x_i) - f(x_i)}{f(x_i)} \right)^2} ,$$

were computed in double precision. In addition, the single-precision results were compared against the results of the double-precision computation correctly rounded to single precision, and the frequency of the errors in units of the last bit of the mantissa was tabulated.

Since the System/360 machines do not conveniently allow computation in higher than double precision, the Control Data 3600 computer with its longer word length was used to generate the standard values for testing of the double-precision routines. The procedure here was to generate pseudo-random, double-precision, floating-point numbers in the usual CDC 3600 format (an 84-bit mantissa) and to convert them to a mantissa length (56 bits, including possible leading zeros due to normalization in base 16 arithmetic) compatible with the System/360 double-precision arithmetic by rounding and zeroing out the extra bits. These converted arguments were then used to generate the corresponding function values on the CDC 3600, and the results were rounded to correct System/360 format. The pairings of arguments and function values were then transmitted to the System/360 machine via magnetic tape, and the testing carried on there in a manner analogous to the single-precision testing. All computations on the System/360 were carried out in double precision. Since the maximum relative error and the root mean square of the relative error were not computed in double-plus precision, these results are not as accurate as those for the single-precision subroutines.

Timing tests were performed by invoking the routine under test up to 10,000 times, with either a fixed argument or random arguments, in a loop. The "overhead" of the looping instructions was timed in each case by repeating the same loop with a dummy subroutine whose only executable instruction was a jump back to the calling program. Because of the coarseness of the internal clock (measuring in increments of 0.01 sec), the indicated timings varied by as much as 5 μ sec from run to run on a given test, and they should not be considered as precise values.

The tests of error returns consisted in supplying arguments outside the domain of the subroutine under test. Without exception, this resulted in termination of the job.

III. RESULTS

Appendix A contains tabular summaries of the tests described above, arranged in alphabetical order according to the entry points of the routines under test. For the most part, the table headings are self-descriptive, but the headings for the frequency of error tabulations may require some explanation. A typical heading might be

0	1	2	3	3BT	4BT	OTHER
---	---	---	---	-----	-----	-------

The column headed 0 contains the number of times that the result from the routine under test agreed exactly with the "correct" result. The column headed 1 contains the number of times the two results disagreed by one unit in the last bit of the mantissa. The column headed 4BT contains the number of times the results disagreed in the fourth (from the end) bit; i.e., the magnitude of the error was between 8 and 15 units in the last bit.

In a few cases, the error patterns were skewed one way or the other, as in ALOG, for example. The errors were then tabulated according to the sign of the error. In these cases, an error of N units indicates -N units have to be added to the "test" result to obtain the "correct" result.

At times there appears to be a discrepancy in the error tables when a test with perhaps four-bit errors results in a smaller maximum relative error than a test with perhaps two-bit errors. This is due to the wide variation in the number of significant bits in the mantissa of a normalized floating-point number in base 16 representation. In single-precision, a number may contain as many as 24 significant bits in the mantissa (no leading bits set to zero) or as few as 21 (three leading bits set to zero). The tabulated frequency of errors measures the error in units of the last bit; the relative error measures in terms of the number of correct significant bits. A 24-bit number with the last four bits in error can contain more correct significant bits than a 21-bit number with only the last two bits in error; hence the 24-bit number would have a smaller relative error.

The same number of random arguments was used in each subinterval for testing a particular function, allowing a direct comparison of the performance of a given routine for various intervals.

To aid in estimating the number of correct significant bits in a maximum relative error, Appendix B lists the decimal equivalents of selected inverse powers of 2.

Appendix C lists a typical test program.

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ALGAMA LOGARITHM OF THE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0.0 , 1.0)	409	359	137	141	327	443	16	168	.245E-02	.617E-04
(1.0 , 2.0)	587	773	163	43	153	215	44	22	.705E-03	.167E-04
(2.0 , 8.0)	462	967	170	46	171	145	0	39	.539E-02	.121E-03
(8.0 , 12.0)	340	530	364	315	451	0	0	0	.125E-05	.303E-06
(12.0 , -)	714	1041	201	42	2	0	0	0	.106E-05	.322E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0., 2.0)	874.0
RANDOM IN (2.0, 8.0)	1000.0
RANDOM IN (8.0, -)	620.0
.5	876.0
1.5	877.0
10.0	618.0

ERROR RETURN IHC291 TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ALOG NATURAL LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT											MAX REL ERROR	ROOT MEAN SQUARE
	OTHER	-4	-3	-2	-1	0	1	2	3	4	OTHER		
(0.0 , .0625)	0	0	0	0	28	1681	291	0	0	0	0	.274E-06	.959E-07
(.0625 , .125)	0	0	0	0	0	1653	347	0	0	0	0	.306E-06	.136E-06
(.125 , .25)	0	0	0	0	0	1164	836	0	0	0	0	.609E-06	.294E-06
(.25 , .5)	1	190	473	347	27	647	315	0	0	0	0	.804E-06	.291E-06
(.5 , .6065)	114	185	276	324	351	307	279	130	34	0	0	.592E-06	.232E-06
(.6065 , .779)	0	11	73	230	417	558	515	179	17	0	0	.625E-06	.211E-06
(.779 , .8825)	0	0	0	2	130	642	828	380	18	0	0	.129E-05	.384E-06
(.8825 , .9395)	0	0	0	1	60	1483	456	0	0	0	0	.952E-06	.285E-06
(.9395 , 1.0)	68	72	137	228	286	521	497	182	9	0	0	.121E-05	.224E-06
(1.0 , 1.065)	0	0	114	822	889	175	0	0	0	0	0	.134E-05	.291E-06
(1.065 , 1.134)	0	0	3	6	1147	844	0	0	0	0	0	.130E-05	.434E-06
(1.134 , 1.285)	0	0	143	934	817	106	0	0	0	0	0	.123E-05	.555E-06
(1.285 , 1.65)	0	0	158	894	841	107	0	0	0	0	0	.697E-06	.278E-06
(1.65 , 2.0)	0	2	360	904	694	40	0	0	0	0	0	.371E-06	.193E-06
(2.0 , 4.0)	0	0	0	0	465	817	23	237	320	137	1	.797E-06	.313E-06
(4.0 , 8.0)	0	0	0	0	848	1152	0	0	0	0	0	.596E-06	.285E-06
(8.0 , 16.0)	0	0	0	0	554	1446	0	0	0	0	0	.496E-06	.162E-06
(16.0 , 256.0)	0	0	0	0	207	1762	31	0	0	0	0	.256E-06	.694E-07

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, FOR ARGUMENTS RANDOM IN
(2.8 , 5.6) IS 363.0 MICROSECONDS

ERROR RETURN IHC2531 TESTED

NC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ARCCOS ARCSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3 BITS	OTHER		
(0.0, 1.0)	3419	2942	1406	1321	912	0	.155E-05	.349E-06
(-1.0, -.5)	1699	301	0	0	0	0	.668E-06	.173E-06
(-.5, 0.0)	549	1451	0	0	0	0	.771E-06	.416E-06
(0.0, .5)	1213	787	0	0	0	0	.798E-06	.363E-06
(.5, .84)	142	382	506	524	446	0	.108E-05	.306E-06
(.84, 1.0)	102	466	734	485	213	0	.158E-05	.425E-06

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0,1)	449.0
-0.8	606.0
-0.3	346.0
.3	319.0
.7	586.0
.9	587.0

ERROR RETURN IHC257I TESTED

WC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ARSIN ARCSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3 BITS	OTHER		
(0.0, 1.0)	3509	3261	573	1012	1645	0	.897E-06	.258E-06
(-1.0, -.5)	461	202	244	412	681	0	.687E-06	.312E-06
(-.5, 0.0)	975	1025	0	0	0	0	.918E-06	.190E-06
(0.0, .5)	948	1052	0	0	0	0	.848E-06	.183E-06
(.5, .84)	2	92	334	605	967	0	.634E-06	.307E-06
(.84, 1.0)	1496	504	0	0	0	0	.737E-06	.314E-06

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0,1)	440.0
-0.8	605.0
-0.3	318.0
.3	296.0
.7	591.0
.9	591.0

ERROR RETURN IHC257I TESTED

WC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ATAN ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-5	-4	-3	-2	-1	0	1	2	3	4		
(0.0 , .26795)	0	0	0	1	2056	6620	905	312	100	6	.836E-06	.187E-06
(.26795, .4142)	0	0	0	106	1632	5071	3001	190	0	0	.527E-06	.132E-06
(.4142 , 1.0)	0	0	0	21	2396	5656	1817	110	0	0	.288E-06	.662E-07
(1.0 , 2.4142)	24	524	2160	1174	4124	1994	0	0	0	0	.108E-05	.511E-06
(2.4142, 3.732)	0	0	0	0	6503	3497	0	0	0	0	.994E-06	.547E-06
(3.732, 10.0)	0	0	0	0	7518	2482	0	0	0	0	.949E-06	.546E-06
(10.0, 3076.0)	0	0	0	0	7612	2388	0	0	0	0	.785E-06	.498E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0.,1.0)	279.0
0.134	280.0
0.7071	348.0
3.073	385.0
6.866	313.0

NO ERROR RETURN

WC
3/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CABS COMPLEX ABSOLUTE VALUE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	OTHER	-48T	-38T	-3	-2	-1	0	1	OTHER		
THETA(0,2PI)											
R(0,1)	56	518	734	248	256	178	10	0	0	.193E-05	.775E-06
THETA(0,2PI)											
R(0,-)	32	283	390	223	467	549	56	0	0	.200E-05	.881E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 400 MICROSECONDS

NO ERROR RETURN

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CCDS COMPLEX COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
X(0 , .35)	2064	2084	359	271	210	11	1	0	.190E-05	.324E-06
Y(0 , -)	452	1649	886	652	1161	186	14	0	.222E-05	.607E-06
X(.35, -)	1542	2138	521	300	403	77	8	11	.102E-04	.561E-06
Y(0 , -)	1583	2107	501	312	421	54	14	8	.428E-04	.901E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
X(0, .35) Y(0, -)	1282.0
X(.35, -) Y(0, -)	1328.0

ERROR RETURN IHC275I TESTED

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2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDABS COMPLEX DOUBLE ABSOLUTE VALUE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	OTHER	-4BT	-3BT	-3	-2	-1	0	1	OTHER		
THETA(0,2PI)											
R(0,1)	331	936	449	75	78	73	34	13	11	.360E-14	.510E-15
THETA(0,2PI)											
R(0,-)	136	870	456	110	103	224	98	1	2	.349E-14	.109E-14

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 657 MICROSECONDS

NO ERROR RETURN

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDCOS COMPLEX DOUBLE COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

	LE-6	-5B	-4B	-3B	-3	-2	-1	0	1	2	3	3BT	4BT	5BT	6E+6	MAX REL ERROR	ROOT MEAN SQUARE
X(0,.35)	370	281	330	216	55	98	118	97	55	58	41	116	87	58	20	.253E-14	.961E-15
Y(0,-)	98	60	56	92	28	34	45	41	57	51	71	188	306	368	505	.260E-11	.741E-13
X(.35,-)	452	166	127	101	27	39	45	45	37	37	29	94	145	156	500	.103E-11	.461E-13
Y(0,-)	436	158	151	84	37	43	43	34	36	38	29	121	153	181	456	.689E-11	.188E-12

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
X(0,.35) Y(0,-)	2445.0
X(.35,-) Y(0,-)	2428.0

ERROR RETURN IHC285I TESTED

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1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDEXP COMPLEX DOUBLE EXPONENTIAL

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER	MAX REL ERROR	ROOT MEAN SQUARE
X(0,-)	74	128	224	317	432	392	266	129	20	18	.465E-12	.113E-13
Y(0,PI)	74	169	230	365	431	339	279	97	14	2	.818E-13	.213E-14
X(0,-)	73	125	197	313	389	372	285	156	43	47	.118E-11	.293E-13
Y(PI,-)	63	110	173	330	387	389	291	142	54	61	.351E-11	.818E-13

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 2265 MICROSECONDS

ERROR RETURN IHC281I TESTED

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDLOG COMPLEX DOUBLE LOGARITHM

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT											MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER			
X(-, +)	110	227	317	247	309	251	119	225	150	45		.374E-12	.133E-13
Y(-1, +1)	620	1283	56	40	1	0	0	0	0	0		.222E-15	.818E-16
X(-, +)	148	320	440	224	57	79	173	375	161	23		.234E-12	.936E-14
Y(-, +)	548	1352	59	41	0	0	0	0	0	0		.234E-15	.103E-15

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 2510 MICROSECONDS

ERROR RETURN IHC283I TESTED

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDSIN COMPLEX DOUBLE SINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT															MAX REL ERROR	RCOT MEAN SQUARE
	LE-6	-5B	-4B	-3B	-3	-2	-1	0	1	2	3	3BT	4BT	5BT	GE+6		
X(0, .35)	448	390	392	225	60	67	86	61	39	44	20	67	51	42	8	.454E-14	.117E-14
Y(0, -)	367	280	324	224	49	79	109	117	68	53	44	122	89	55	20	.238E-14	.956E-15
X(.35, -)	398	187	163	117	31	45	49	47	38	47	35	142	150	154	397	.378E-11	.110E-12
Y(0, -)	400	183	134	136	34	31	45	47	50	38	27	110	143	178	444	.832E-12	.340E-13

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
X(0, .35) Y(0, -)	2428.0
X(.35, 0) Y(0, -)	2449.0

ERROR RETURN IHC285I TESTED

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CDSQRT COMPLEX DOUBLE SQUARE ROOT

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT											MAX REL ERROR	ROOT MEAN SQUARE
	OTHER	4BT	3BT	2BT	-1	0	1	2BT	3BT	4BT	OTHER		
X(-1,+1)	6	330	434	197	96	206	246	259	189	35	2	.193E-14	.197E-15
Y(-1,+1)	10	189	342	212	155	187	149	251	297	198	10	.198E-14	.197E-15
X(-, +)	105	97	120	366	321	235	135	169	165	117	170	.173E-14	.370E-15
Y(-, +)	104	116	132	283	218	223	221	289	152	122	140	.173E-14	.376E-15

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 1335 MICROSECONDS

NO ERROR RETURN

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CEXP COMPLEX EXPONENTIAL

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT											MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER			
X(0,-)	930	785	192	78	9	4	1	1	0	0		.171E-04	.516E-06
Y(0,PI)	867	835	195	82	16	1	3	0	1	0		.298E-04	.850E-06
X(0,-)	917	775	197	95	10	3	1	0	2	0		.659E-04	.164E-05
Y(PI,-)	842	813	223	102	15	2	2	0	1	0		.575E-04	.134E-05

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 1205 MICROSECONDS

ERROR RETURN IHC2711 TESTED

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CLOG COMPLEX LOGARITHM

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER		
X(-, +)	139	444	150	155	488	491	9	16	63	45	.146E-02	.453E-04
Y(-1,+1)	1028	839	72	43	18	0	0	0	0	0	.139E-05	.354E-06
X(-, +)	119	800	270	77	317	390	3	1	4	19	.211E-03	.695E-05
Y(-, +)	654	1175	89	58	24	0	0	0	0	0	.124E-05	.485E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 1282 MICROSECONDS

ERROR RETURN IHC273I TESTED

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

COS COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	OTHER			
(0. ,PI/2)	1880	2304	667	19	44	71	1	1	13		.187E-03	.292E-05
(PI/2, PI)	1275	3257	422	3	16	27	0	0	0		.333E-05	.214E-06
(PI , 2PI)	1583	2812	512	6	38	46	0	0	3		.468E-04	.746E-06
(2PI, 8PI)	1581	2826	489	9	44	46	1	0	4		.491E-03	.696E-05
(8PI,64PI)	1647	2740	513	6	47	43	0	3	1		.146E-04	.381E-06

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0,PI/2)	308.0
.7854	280.0
5.0	323.0
20.0	329.0

ERROR RETURN IHC254I TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

COSH HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	4	5	6	OTHER		
(0.0 , 0.35)	1542	1458	0	0	0	0	0	0	.997E-06	.535E-06
(0.35,12.88)	1486	1060	129	113	80	35	29	68	.113E-05	.272E-06
(12.88, -)	1954	557	185	201	103	0	0	0	.675E-06	.182E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0., .35)	564.0
RANDOM IN (.35,12.9)	563.0
RANDOM IN (12.88, -)	559.0
0.2	562.0
8.94	563.0
100.0	556.0

ERROR RETURN IHC256I TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

COT COTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0 ,PI/4)	2321	1541	541	311	222	56	8	0	.137E-05	.382E-06
(PI/4,8PI)	1515	1783	859	397	324	95	15	12	.195E-02	.276E-04

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0,PI/4)	367.0
.1	345.0
.5	345.0
2.0	345.0
10.0	350.0

ERROR RETURN IHC258I TESTED

ET
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CSIN COMPLEX SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BTS	5BTS	OTHER		
X(0 , .35)	445	1638	880	632	1179	210	16	0	.222E-05	.620E-06
Y(0 , -)	2101	2094	305	277	214	9	0	0	.142E-05	.305E-06
X(.35, -)	1543	2111	520	307	426	70	15	8	.428E-04	.907E-06
Y(0 , -)	1574	2126	517	303	393	69	7	11	.102E-04	.552E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
X(0,.35) Y(0, -)	1269.0
X(.35,-) Y(0, -)	1317.0

ERROR RETURN IHC275I TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

CSQRT COMPLEX SQUARE ROOT

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	OTHER	-3BT	-2BT	-1	0	1	2BT	3BT	OTHER		
X(-1,+1)	23	361	515	193	343	289	216	59	1	.108E-05	.342E-06
Y(-1,+1)	16	232	353	218	383	245	345	193	15	.118E-05	.348E-06
X(-, +)	164	251	288	379	221	251	214	136	96	.164E-05	.588E-06
Y(-, +)	128	226	239	304	216	301	240	205	141	.181E-05	.584E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMP 815 MICROSECONDS

NO ERROR RETURN

NC
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DARCOS DOUBLE ARCCOSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT												MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1	2	3	3BT	4BT	5BT	GE+6		
(-.5,0.5)	0	0	0	440	681	879	0	0	0	0	0	0	.211E-15	.162E-15
(0.5,1.0)	28	133	220	259	273	989	51	12	27	6	0	2	.106E-13	.399E-15

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG. MICROSECS.
 RANDOM IN (-.5,.5) 958.0
 RANDOM IN (.5,1.0) 1354.0

ERROR RETURN IHC267I TESTED

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DARSIN DOUBLE ARCSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1	2	3	3BT			
(-.5 , 0.5)	0	5	119	686	382	667	139	2	0		.222E-15	.934E-16
(0.5 , 1.0)	39	95	187	412	609	362	190	83	23		.301E-15	.110E-15

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG. MICROSECS.
 RANDOM IN (-.5,.5) 931.0
 RANDOM IN (.5,1.0) 1362.0

ERROR RETURN IHC267I TESTED

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DATAN DOUBLE ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2	3		
(0.0 , .26795)	0	0	2316	2684	0	0	.222E-15	.714E-16
(.26795, .4142)	0	65	1599	2681	652	3	.148E-15	.449E-16
(.4142 , 1.0)	0	3	1343	3033	619	2	.706E-16	.253E-16
(1.0 , 2.4142)	3	1737	2821	438	1	0	.222E-15	.108E-15
(2.4142, 3.732)	0	2246	2754	0	0	0	.188E-15	.119E-15
(3.732, 10.0)	0	2459	2541	0	0	0	.170E-15	.110E-15
(10.0, 3076.0)	0	2298	2702	0	0	0	.149E-15	.959E-16

AVERAGE EXECUTION TIMES ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0.0,.26795)	636.0
RANDOM IN (.26795,1.0)	790.0
RANDOM IN (1.0, 3.732)	882.5
RANDOM IN (3.732,10.0)	727.0
RANDOM IN (10.0,3076.)	744.0

NO ERROR RETURN

KH
3/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DCDS DOUBLE COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(-PI/2, PI/2)	78	332	183	64	176	135	1	31	.173E-13	.107E-14
(PI/2, -)	93	191	123	86	275	124	23	85	.224E-12	.846E-14

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (-PI/2,PI/2)	662.0
RANDOM IN (PI/2, -)	665.0

ERROR RETURN IHC264I TESTED

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DCOSH DOUBLE HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS														MAX REL ERROR	ROOT MEAN SQUARE
	ERROR IN THE LAST HEXADECIMAL DIGIT															
	LE-5	-4B	-3B	-3	-2	-1	0	1	2	3	3BT	4BT	GE+5			
(0.0 , 0.35)	0	0	0	0	0	554	446	0	0	0	0	0	0	.222E-15	.162E-15	
(0.35, 19.3)	422	27	11	36	57	194	161	51	22	14	3	2	0	.244E-14	.884E-15	
(19.3, -)	475	29	23	34	31	28	37	30	29	25	32	23	204	.226E-14	.881E-15	

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0.,.35)	978.0
RANDOM IN (.35, -)	965.0
0.2	972.0
10.0	970.0

ERROR RETURN IHC2661 TESTED

KH
3/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DCOTAN DCUBLE COTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0. , PI/4)	2557	1813	185	91	235	100	19	0	.389E-15	.931E-16
(PI/4, 8PI)	311	530	370	241	634	762	438	1714	.863E-11	.164E-12

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (PI/4,8P)	790.0
0.1	752.0
0.5	722.0
2.0	767.0
10.0	757.0

ERROR RETURN IHC2681 TESTED

ET
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DERF DOUBLE ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BITS	OTHER		
(0.0 , 1.317)	938	754	277	29	2	0	.217E-15	.315E-16
(1.317, 2.04)	856	1098	46	0	0	0	.295E-16	.114E-16
(2.04 , 6.092)	1050	950	0	0	0	0	.139E-16	.958E-17

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
.8	840.0
1.6	1186.0
3.2	2267.0
6.4	178.0

NO ERROR RETURN

WC
4/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DERFC DOUBLE COMPLEMENTARY ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BITS	4BITS	5BITS	6BITS	7BITS	OTHER		
(-6.0 , -1.317)	1047	953	0	0	0	0	0	0	0	0	.115E-15	.768E-16
(-1.317, 0.0)	1036	964	0	0	0	0	0	0	0	0	.221E-15	.101E-15
(0.0 , 1.317)	927	765	277	29	2	0	0	0	0	0	.413E-15	.817E-16
(1.317, 2.04)	489	657	234	164	262	153	41	0	0	0	.442E-15	.127E-15
(2.04 , 3.9192)	45	71	91	68	308	418	461	329	181	28	.386E-14	.124E-14
(3.9192,13.306)	37	86	71	83	316	430	435	308	192	42	.441E-14	.133E-14

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
- .8	901.0
.8	863.0
1.6	1176.0
3.2	2254.0
14.3	193.0

NO ERROR RETURN

WC
4/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DEXP DOUBLE EXPONENTIAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	7BTS	8BTS		
(0.0 , 0.332)	1125	875	0	0	0	0	0	0	0	0	.222E-15	.126E-15
(0.332, 0.664)	1247	753	0	0	0	0	0	0	0	0	.159E-15	.835E-16
(0.664, 1.328)	442	1041	495	22	0	0	0	0	0	0	.221E-15	.104E-15
(1.328, 2.656)	353	550	463	314	320	0	0	0	0	0	.175E-15	.706E-16
(2.656, 5.312)	571	941	306	116	64	2	0	0	0	0	.337E-15	.850E-16
(5.312, 10.625)	797	711	223	113	151	5	0	0	0	0	.222E-15	.712E-16
(10.625, 21.250)	62	101	104	80	324	465	400	295	164	5	.236E-14	.123E-14
(21.250, 42.500)	54	123	117	95	291	439	411	296	166	8	.242E-14	.109E-14
(42.500, 85.000)	56	101	106	133	402	475	397	264	66	0	.193E-14	.848E-15
(85.000, 170.00)	60	131	90	123	386	431	376	298	100	5	.223E-14	.906E-15
(-0.0 , -0.332)	912	1023	65	0	0	0	0	0	0	0	.372E-16	.129E-16
(-0.332, -0.664)	1231	760	9	0	0	0	0	0	0	0	.486E-16	.142E-16
(-0.664, -1.328)	520	530	433	337	180	0	0	0	0	0	.180E-15	.738E-16
(-1.328, -2.656)	1164	743	93	0	0	0	0	0	0	0	.197E-15	.697E-16
(-2.656, -5.312)	723	819	206	100	141	11	0	0	0	0	.222E-15	.650E-16
(-5.312, -10.625)	538	865	338	135	124	0	0	0	0	0	.275E-15	.850E-16
(-10.625, -21.25)	66	176	88	72	243	393	425	327	187	23	.232E-14	.120E-14
(-21.25, -42.50)	74	119	128	114	334	413	370	267	171	10	.218E-14	.103E-14
(-42.50, -85.00)	51	130	112	136	416	475	370	224	86	0	.185E-14	.798E-15
(-85.00, -170.0)	68	110	105	86	394	462	377	284	111	3	.227E-14	.927E-15

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, FOR ARGUMENTS RANDOM IN
(-85.0, -170.0) IS 655.0 MICROSECS

ERROR RETURN IHC262I TESTED

NC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DGAMMA DOUBLE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	7BTS	OTHER		
(0.0 , 1.0)	1104	896	0	0	0	0	0	0	0	0	.222E-15	.980E-16
(1.0 , 2.0)	1383	556	61	0	0	0	0	0	0	0	.313E-16	.966E-17
(2.0 , 4.0)	518	547	62	78	270	514	11	0	0	0	.177E-14	.474E-15
(4.0 , 8.0)	2	38	105	98	240	428	721	353	15	0	.294E-14	.957E-15
(8.0 , 16.0)	31	63	69	70	179	346	450	362	310	120	.667E-14	.210E-14
(16.0, 57.574)	0	4	1	1	13	24	61	103	215	1578	.623E-13	.296E-13

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
.06	1476.0
1.3	1430.0
7.9	1829.0
13.7	2097.0

ERROR RETURN IHC300I TESTED

WC
4/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DLGAMA DOUBLE LOGARITHM OF THE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0.0 , 1.0)	360	283	61	30	189	762	172	143	.287E-12	.734E-14
(1.0 , 2.0)	727	650	173	70	177	156	20	27	.105E-12	.315E-14
(2.0 , 8.0)	73	219	345	351	589	229	62	132	.324E-13	.112E-14
(8.0 , 12.0)	265	341	194	217	627	346	10	0	.441E-15	.119E-15
(12.0 , -)	500	319	123	99	350	570	39	0	.203E-14	.342E-15

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0., 1.0)	2298.0
RANDOM IN (1.0, 2.0)	2209.0
RANDOM IN (2.0, 8.0)	2460.0
RANDOM IN (8.0, -)	1414.0
.7	2293.0
1.7	2215.0
5.0	2394.0
10.0	1413.0

ERROR RETURN IHC301I TESTED

KH
3/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DLOG DOUBLE LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	OTHER			
(0.177, 0.354)	1098	902	0	0	0	0	0		.213E-15	.107E-15
(0.354, 0.707)	0	79	7	0	20	1894	0		.519E-15	.247E-15
(0.707, 1.000)	798	832	287	46	37	0	0		.312E-15	.776E-16
(1.000, 1.414)	585	856	406	88	65	0	0		.327E-15	.860E-16
(1.414, 2.828)	2	116	5	0	54	1823	0		.444E-15	.205E-15
(2.828, 5.657)	936	1064	0	0	0	0	0		.213E-15	.111E-15

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, FOR ARGUMENTS RANDOM IN
(2.8, 5.6) IS 777.0 MICROSECONDS

ERROR RETURN IHC263I TESTED

NC
12/66

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DPOWER DOUBLE EXPONENTIATION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT												MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	9BT	ABT	OTHER		
X (2**-4, 2**4)	100	239	147	448	383	251	211	147	74	0	0	0	.455E-14	.147E-14
Y (-4.0 , 4.0)	34	83	55	221	295	420	347	295	176	54	10	10	.341E-13	.453E-14
X (2**-16, 2**16)	32	63	43	182	189	240	282	265	235	171	147	151	.876E-13	.207E-13
Y (-8.0 , 8.0)	28	48	40	136	139	184	241	247	230	245	217	245	.761E-13	.244E-13
X (2**-64, 2**64)	12	43	33	135	161	221	287	278	277	217	157	179	.629E-13	.210E-13
Y (-32.0, 32.0)	25	31	31	87	117	163	228	285	298	288	217	230	.689E-13	.233E-13
X (2**-4, 2**4)														
Y (-64.0, 64.0)														

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, FOR X, Y BOTH RANDOM IN
(0.0, 1.0) IS 1605.0 MICROSECONDS

ERROR RETURN IHC253I (IN DLOG) TESTED

NC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DSIN DOUBLE SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER			
(-PI/2, PI/2)	367	420	93	58	48	5	9	0		.448E-15	.564E-16
(PI/2, -)	85	205	154	91	255	115	15	80		.117E-12	.629E-14

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (-PI/2,PI/2)	666.0
RANDOM IN (PI/2, -)	668.0

ERROR RETURN IHC264I TESTED

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DSINH DOUBLE HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT														MAX REL ERROR	ROOT MEAN SQUARE
	LE-5	-4B	-3B	-3	-2	-1	0	1	2	3	3BT	4BT	GE+5			
0.0 , 0.35)	0	0	0	0	0	579	421	0	0	0	0	0	0	.221E-15	.659E-16	
0.35, 19.3)	345	22	16	26	49	116	213	98	36	28	21	8	22	.227E-14	.839E-15	
19.3, -)	475	29	23	34	31	28	37	29	29	26	32	23	204	.224E-14	.880E-15	

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0.,.35)	448.0
RANDOM IN (.35, -)	972.0
0.2	450.0
10.0	976.0

ERROR RETURN IHC266I TESTED

KH
3/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DSORT DOUBLE SQUARE ROOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	-1	0	1		
(0.0 , -)	3	1996	1	.708E-16	.180E-17
(0.06, 1.0)	8	1991	1	.299E-16	.128E-17
(1.0 , 16.0)	10	1990	0	.164E-15	.591E-17

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS 355.0 MICROSECONDS

ERROR RETURN IHC261I TESTED

NC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DTAN DOUBLE TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0.0 , PI/4)	842	2763	1280	56	15	24	20	0	.629E-15	.890E-16
(PI/4 , 8PI)	306	546	368	241	641	714	398	1786	.421E-11	.956E-13

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (PI/4,8PI)	784.0
.1	717.0
.5	692.0
2.0	772.0
10.0	732.0

ERROR RETURN IHC268I TESTED

ET
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

DTANH DOUBLE HYPERBOLIC TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	-4BT	-3BT	-3	-2	-1	0	1	2		
(0.0, 0.55)	0	0	0	0	0	873	1123	4	.221E-15	.535E-16
(0.55, 20.1)	1	16	10	7	29	982	955	0	.222E-15	.337E-16
(20.1, -)	0	0	0	0	0	2000	0	0	.000	.000

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0.0, .55)	553.0
RANDOM IN (.55,20.1)	897.0
RANDOM IN (20.1, -)	72.0

NO ERROR RETURN

KH
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ERF ERROR INTEGRAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BITS	OTHER		
(-3.9192, -2.04)	965	1035	0	0	0	0	.597E-07	.346E-07
(-2.04 , -1.317)	1065	934	1	0	0	0	.104E-06	.343E-07
(-1.317, 0.0)	815	956	153	58	18	0	.784E-06	.141E-06
(0.0 , 1.317)	805	939	172	65	19	0	.891E-06	.140E-06
(1.317 , 2.04)	1060	939	1	0	0	0	.102E-06	.342E-07
(2.04 , 3.9192)	953	1047	0	0	0	0	.604E-07	.349E-07

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
-1.1	376.0
1.1	353.0
1.8	395.0
2.6	840.0
5.1	167.0

NO ERROR RETURN

WC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

ERFC COMPLIMENTARY ERROR INTEGRAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(-13.306, -1.317)	1804	196	0	0	0	0	0	0	.453E-06	.119E-06
(-1.317, 0.0)	1086	914	0	0	0	0	0	0	.916E-06	.356E-06
(0.0 , 1.317)	830	939	147	65	19	0	0	0	.394E-05	.595E-06
(1.317 , 2.04)	1138	458	146	89	129	40	0	0	.937E-06	.204E-06
(2.04 , 3.9192)	562	671	326	181	236	24	0	0	.149E-05	.362E-06
(3.9192, 13.306)	40	81	55	57	201	333	419	814	.150E-04	.822E-05

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
-5.1	891.0
-1.1	399.0
1.1	372.0
1.8	389.0
3.8	844.0

NO ERROR RETURN

WC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

EXP EXPONENTIAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT		MAX REL ERROR	ROOT MEAN SQUARE
	0	+,-1		
(0.0 , .3466)	1990	10	.473E-06	.233E-06
(.3466 , .6931)	1987	13	.341E-06	.166E-06
(.6931 , 1.3863)	1980	20	.228E-06	.102E-06
(1.3863 , 2.7726)	1933	67	.125E-06	.402E-07
(2.7726 , 5.5452)	1949	51	.439E-06	.120E-06
(5.5452 , 11.0904)	1950	50	.458E-06	.115E-06
(11.0904 , 22.1807)	1935	65	.455E-06	.118E-06
(22.1807 , 44.3614)	1904	96	.427E-06	.117E-06
(44.3614 , 88.7229)	1797	203	.475E-06	.121E-06
(-88.7229 , 174.673)	1648	352	.483E-06	.121E-06
(-174.673 , -100.0)	1725	275	.470E-06	.117E-06
(-100.0 , -10.0)	1882	118	.460E-06	.115E-06
(-10.0 , -1.0)	1964	36	.458E-06	.116E-06
(-1.0 , 0.0)	1926	74	.790E-07	.308E-07

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, FOR ARGUMENTS RANDOM IN (0.0, 1.0) IS 374.0 MICROSECONDS

ERROR RETURN IHC252I TESTED

NC
12/66

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

GAMMA GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0.0 , 1.0)	1038	957	5	0	0	0	0	0	.985E-06	.344E-06
(1.0 , 2.0)	1223	776	1	0	0	0	0	0	.101E-06	.364E-07
(2.0 , 4.0)	669	1236	95	0	0	0	0	0	.922E-06	.360E-06
(4.0 , 8.0)	40	488	491	232	548	189	12	0	.224E-05	.822E-06
(8.0 , 16.0)	41	84	91	84	277	373	400	650	.226E-04	.783E-05
(16.0 , 57.0)	15	44	36	34	110	264	411	1086	.381E-04	.145E-04

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
.4	486.0
1.1	490.0
2.8	521.0
5.9	634.0
13.7	1004.0

ERROR RETURN IHC290I TESTED

WC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

POWER EXPONENTIATION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER		
X (2**-4, 2**4)												
Y (-4.0 , 4.0)	501	443	432	297	202	107	18	0	0	0	.433E-05	.108E-05
X (2**-16, 2**16)												
Y (-16.0, 16.0)	129	171	189	238	312	371	294	219	75	2	.268E-04	.760E-05
X (2**-32, 2**32)												
Y (-8.0 , 8.0)	145	184	190	218	247	295	254	204	125	138	.115E-03	.217E-04
X (2**-64, 2**64)												
Y (-4.0 , 4.0)	111	165	181	166	197	258	285	298	210	129	.601E-04	.174E-04
X (2**-8, 2**8)												
Y (-32.0, 32.0)	100	123	197	210	255	364	282	277	156	36	.392E-04	.111E-04
X (2**-4, 2**4)												
Y (-64.0, 64.0)	109	136	184	204	200	303	328	230	184	122	.644E-04	.170E-04

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS FOR ARGUMENTS RANDOM IN
(0.0, 1.0) IS 865.0 MICROSECONDS

ERROR RETURN IHC253I (FOR LOG) TESTED

NC
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

SIN SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	OTHER		
(0.0,PI/2)	1221	3369	372	5	16	17	0	0	0	.138E-05	.195E-06
(PI/2, PI)	1835	2322	667	17	76	70	1	2	10	.426E-03	.641E-05
(PI , 2PI)	1598	2775	530	11	35	47	0	1	3	.139E-04	.384E-06
(2PI, 8PI)	1619	2826	458	11	30	48	0	0	8	.311E-03	.452E-05
(8PI,64PI)	1499	2819	563	15	51	49	0	2	2	.235E-04	.543E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0,PI/2)	309.0
.7854	303.0
5.0	302.0
20.0	306.0

ERROR RETURN IHC254I TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

SINH HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	4	5	6	OTHER		
(0.0 , 0.35)	1345	1655	0	0	0	0	0	0	.929E-06	.229E-06
(0.35, 12.88)	1427	1019	159	151	83	40	37	84	.107E-05	.275E-06
(12.88, -)	1954	557	185	201	103	0	0	0	.675E-06	.182E-06

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0., .35)	197.0
RANDOM IN (.35, 12.9)	563.0
RANDOM IN (12.88, -)	562.0
0.2	191.0
8.94	563.0
100.0	559.0

ERROR RETURN IHC256I TESTED

KH
2/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

SQRT SQUARE ROOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	-1	0	1		
(0.0 , -)	2340	7211	449	.849E-06	.991E-07
(0.06, 1.0)	911	7794	1295	.224E-06	.452E-07
(1.0 , 16.0)	4897	5103	0	.913E-06	.234E-06

AVERAGE EXECUTION TIME ON IBM360-50, EXCLUDING JUMPS, 200.0 MICROSECONDS

ERROR RETURN IHC251I TESTED

NC
12/66

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

TAN TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS, ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(0.0 , $\pi/4$)	630	1721	1846	615	147	39	2	0	.207E-05	.328E-06
($\pi/4$, 8π)	1529	1777	847	424	287	99	21	16	.194E-02	.275E-04

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN ($\pi/4$, 8π)	345.0
0.1	320.0
0.5	315.0
2.0	345.0
10.0	340.0

ERROR RETURN IHC258I TESTED

ET
1/67

TEST OF IBM 360 FORTRAN SUBROUTINE LIBRARY

TANH HYPERBOLIC TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1	2			
(0.0 , 0.55)	0	0	5	2352	2643	0	0		.863E-06	.171E-06
(0.55 , 9.01)	28	48	60	72	2336	2456	0		.667E-06	.610E-07
(9.01 , -)	0	0	0	0	5000	0	0		.296E-07	.112E-08

AVERAGE EXECUTION TIME ON IBM360-50 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDOM IN (0. , .55)	176.0
RANDOM IN (.55, 9.01)	538.0
RANDOM IN (9.01, -)	80.0

NO ERROR RETURN

KH
2/67

APPENDIX B

Decimal Equivalents of Selected
Inverse Powers of 2

<u>n</u>	<u>2^{-n}</u>	<u>n</u>	<u>2^{-n}</u>
18	3.815 E-06	50	8.882 E-16
19	1.907 E-06	51	4.441 E-16
20	9.537 E-07	52	2.220 E-16
21	4.768 E-07	53	1.110 E-16
22	2.384 E-07	54	5.551 E-17
23	1.192 E-07	55	2.776 E-17
24	5.960 E-08	56	1.388 E-17
25	2.980 E-08	57	6.939 E-18

SAMPLE LIBRARY SUBROUTINE TEST PROGRAM

```

C      TEST PROGRAM FOR SINGLE PRECISION -ALOG- FUNCTION
C      NANCY CLARK
      DIMENSION X(2000), K(19), A(25), IZ(2)
      EQUIVALENCE (Y,IY), (DZ,IZ)
      DOUBLE PRECISION DX,DY,DZ
      COMMON K
C      --- THE DATA A(I) ARRAY DEFINES TEST INTERVALS ---
      DATA A(1)/0.0/, A(2)/.0625/, A(3)/.125/, A(4)/.25/
      X, A(5)/.5/, A(6)/.6065/, A(7)/.779/, A(8)/.8825/
      X, A(9)/.9395/, A(10)/1.0/, A(11)/1.065/, A(12)
      X /1.134/, A(13)/1.285/, A(14)/1.65/, A(15)/2./,
      X A(16)/4./, A(17)/8./, A(18)/16./, A(19)/256./
C      --- FOLLOWING LOOP NECESSARY FOR COMPATIBILITY OF
C      RESULTS WITH EARLIER TEST ON THIS FUNCTION ---
      DO 6210 I=1,18000
6210  X(I)=RANF(-1)
C      --- READ IN TITLE AND PRINT TITLE AND HEADINGS ---
      READ 1
      PRINT 1
      PRINT 66
      PRINT 63
      PRINT 64
C      -- FORM THE ARRAY OF RANDOM NUMBERS TO BE USED ---
      DO 6220 I=1,2000
6220  X(I)=RANF(-1)
C      --- SET UP FOR TEST OF EACH INTERVAL SPECIFIED ---
      DO 6250 L=1,18
      CON1=A(L)
      CON2=A(L+1)-A(L)
      RMS=0
      REM=0
      DO 6230 I=1,11
6230  K(I)=0
C      --- PERFORM ACCURACY TEST FOR A TEST INTERVAL ---
      DO 6240 I=1,2000
      Y=CON1+CON2*X(I)
C      --- COMPUTE DOUBLE AND SINGLE PRECISION LOGS ---
      DY=Y
      DZ=DLOG(DY)
      Y=LOG (Y)
      DY=Y
C      --- COMPUTE AND TABULATE ERRORS IN LOGARITHM ---
      RE=DABS((DY-DZ)/DZ)
      IF(RE.GT.REM) REM=RE
      RMS=RMS+RE*RE

```

```

J=IY-IZ(1)
IF(IZ(2).LT.0) J=J-1
IF(IZ(1).LT.0) J=-J
IF(J.GT.5) J=5
IF(J.LT.-5) J=-5
6240 K(J+6)=K(J+6)+1
RMS=SQRT(RMS/2000.)
C --- PRINT RESULTS OF ERROR TEST ON AN INTERVAL ---
6250 PRINT 65, A(L),A(L+1),(K(I),I=1,11),REM,RMS
C --- FORM RANDOM ARGUMENT ARRAY FOR TIMING TEST ---
DO 6260 I=1,2000
6260 X(I)=X(I)*9.0+1.0
C --- TIMING TEST OF ALOG WITH RANDOM ARGUMENTS ---
PRINT 6
CALL CLOCK (T0)
C --- TIMING OF THE LOOPING AND LINKAGE OVERHEAD ---
DO 6270 I=1,2000
DO 6270 J=1,5
6270 Y=DUMMY(X(I))
CALL CLOCK (T1)
C --- TIMING OF ALOG WITH THE OVERHEAD INCLUDED ---
DO 6280 I=1,2000
DO 6280 J=1,5
6280 Y=LOG(X(I))
CALL CLOCK (T2)
C --- SUBTRACT OVERHEAD TO GET A TIMING FOR ALOG ---
T1=T2-T1-T1+T0
PRINT 8, T1
C --- TEST ERROR RETURN WITH AN ILLEGAL ARGUMENT ---
PRINT 12
Y=LOG (0.)
Y=LOG (-1.)
RETURN
1 FORMAT( 72H
X
)
63 FORMAT('OACCURACY FOR RANDOM ARGUMENTS'//10X
X 'INTERVAL'19X 'ERROR IN LAST HEXADECIMAL DIGIT'/)
64 FORMAT(23X 'OTHER -4 -3 -2 -1 0 1 2'
X,' 3 4 OTHER MRE RMS'/)
65 FORMAT(1X,F9.4,' TO'F9.4,1X,3I4,2I5,I6,2I5,3I4,
X 2X,2E10.3)
6 FORMAT(///' AVE TIME IN MICROSECS. EXCLUDING JUMP'
X //8X'ARG'8X ' TIME ')
8 FORMAT(' RANDOM IN (1.,10.) ',F7.0)
12 FORMAT(///' TEST OF ERROR RETURN ')
66 FORMAT(/49X 'ALOG' 6X,A8)
END

```

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