A STUDY OF CORE FUEL SYSTEMS FOR
A FAST BREEDER POWER REACTOR

to

ATOMIC POWER DEVELOPMENT ASSOCIATES

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SUMMARY AND RECOMMENDATIONS

This report covers the first phase of a program aimed toward the development of materials and a core-subassembly design which would demonstrate economic power production in a fast power breeder reactor of the PRDC type. This core design was to be developed in time for the second loading in the PRDC plant.

The first phase objective was to survey and evaluate potential fuel materials and subassembly designs leading to the recommendation of a fuel-subassembly design, fuel and structural materials, and a basic fabrication technique for further development in the next phase.

The ground rules established by APDA for the study were based upon the performance requirements of the PRDC reactor plant with some modification to permit higher power generation. Also, they were based upon a value of the plutonium produced of about \$44 per g.

The scheduled need for the completion of this program is such that a period of about 3 years could be devoted to the second-phase development of the selected fuel system.

These factors dictated that the study consider fuels and designs based upon present technology which had a reasonable chance of being developed in this period. Also, the achievable burnup fractions of the fuel materials studied, one of the most important factors in the economic evaluation, were assumed on a similar basis.

Evaluation of potential fuel systems under these conditions led to the following materials and designs as showing the most promise:

- (1) Alloy fuels - uranium-10 w/o molybdenum or gamma-phase-type alloy clad in zirconium in a flat-plate subassembly design
- (2) Ceramic fuel - 90 per cent dense UO_2 pellets in sodium-filled stainless steel pins. A parallel-plate design utilizing a stainless steel radiator-type assembly was economically attractive but would require release of fission gases to the main coolant stream.
- (3) Dispersion or cermet fuels - A 30 volume per cent dispersion of UO_2 in uranium-10 w/o molybdenum or gamma-phase-type alloy, zirconium clad, in a flat-plate-subassembly design.

The plant economics for subassembly designs of these fuel systems were evaluated at a core power level of 616 megawatts. This power was chosen as about the minimum level for economic operation under the ground rules. The plant economics indicated no significant economic advantage of one system over the others.

The technical evaluation of these fuel systems was based upon over-all performance capability, expected ability to achieve high-uranium-burnup fractions, reactor safety, likelihood of low-cost fabrication production, and long-range potential. Consideration of these factors led to the selection of the dispersion or cermet fuel as having the most potential for the Core II application. The wrought-alloy fuel in a parallel-plate design held the best hope of achieving the program objectives with the least extrapolation from present technology.

The following recommendations are submitted concerning the future direction of this program:

- (1) That the initial work in Phase II of the program be devoted to the development of the uranium-10 w/o molybdenum or gamma-phase-type alloy system including both UO_2 dispersions in a uranium-10 w/o molybdenum matrix and uranium-10 w/o molybdenum in wrought form. The initial objective should be the selection of the specific material showing the most satisfactory radiation stability for high-burnup fractions.
- (2) That experimental evaluation studies be undertaken to establish the feasibility of pressure-bonded zirconium-clad flat-plate-type sub-assemblies of the reference uranium-10 w/o molybdenum fuels.

Once basic feasibility was established and the most promising specific fuel material selected, the emphasis should shift to the development of fabrication techniques applicable to large-scale low-cost production.

It is important to recognize that the engineering designs and the economic
 evaluation of the designs were strongly influenced by the ground rules of the [redacted] in large sense the reactor power level and the subassembly design were optimized to a core size of 139 subassemblies and a plutonium price of \$44 per g.

During the course of the study the AEC announced its intention to reduce the buy-back price of plutonium to \$30 per g. However, it was decided that the present study should be completed with the existing APDA ground rules. Thus, while outside the scope of the study reported here, it is believed that economic power production with a reduced value for plutonium of \$30 per g or less would probably require either a larger core size and a higher power level or a plant concept offering significantly reduced fixed costs.

It is believed that the fuel materials and subassembly designs proposed here will contribute a substantial improvement in economic performance even at lower power levels and plutonium prices. An example of this is shown for a reactor power level of 430 megawatts. At this level the economic analysis of appropriate pin and plate designs indicates that the proposed pressure-bonded plate design has a revenue deficiency of about \$2.7 million per year less than the comparable pin design.

INTRODUCTION

Background and Objectives of the Study

The Atomic Power Development Associates, Inc., is undertaking the development of a fast breeder power reactor. To demonstrate this concept of power generation, the Power Reactor Development Corporation is constructing the Enrico Fermi Reactor at Monroe, Michigan.

The first core for this PRDC reactor is being developed primarily from the standpoint of engineering feasibility. However, it was recognized by APDA that the ultimate success of this reactor concept depended upon a core fuel system producing economic power. In support of this goal Battelle was requested to undertake a research and development program with the following objectives:

- (1) To determine a core subassembly design and materials that will permit operation of a PRDC-type reactor under operating conditions and a burnup fraction which will yield a reasonable profit from the power produced.
- (2) To determine fabrication methods of subassembly designs which are compatible with the operation and cost requirements.
- (3) To construct one experimental core loading or partial core loading for verification of the design and fabrication techniques under reactor operating conditions.

The objectives of this program were to be pursued in three phases over a 5-year period. This report constitutes a summary of the findings of Phase I of the program covering the first of the above objectives.

Summary of Ground Rules

This Phase I study was performed on the basis of certain ground rules set forth by APDA. The complete ground rules are given in Appendix A. A brief summary of the ground rules is as follows:

- (1) Uranium-235 is the fissionable material.
- (2) The core subassemblies are to operate under the conditions of the PRDC reactor plant design with one major exception. The maximum coolant-flow rate allowable is that which may be attained with a 65-psi pressure drop through the core section of a subassembly. Maximum power generation is unlimited. Appropriate increases in plant cost factors are to be applied, however.
- (3) The maximum number of core fuel subassemblies is 139.

- (4) The fuel-cycle cost rates except fabrication cost, the reactor operating costs, and plant capital costs were supplied by APDA. These costs included a return on plant investment of 6 per cent after taxes, and an annual fuel-inventory charge of 12.56 per cent.
- (5) The steam-revenue value was based on steam temperature and ranged from \$0.45 to \$0.55 per 10^6 Btu for 800 to 1000 F steam, respectively.
- (6) The uranium-235 cost and plutonium revenue were calculated from the AEC price schedule dated January 5, 1955. Thus, the plutonium revenue was based upon a value of about \$44 per g within the range of plutonium quality found.

During the course of the study the AEC announced its intention to reduce the buy-back price of plutonium to \$30 per g. While it was recognized that this would have a significant effect on the absolute value of the revenue figures developed in the study, APDA indicated that the existing plutonium price ground rule should be maintained for the following reasons:

- (1) Since the AEC price schedule is subject to change annually, there was no assurance that the price of \$30 per g would remain firm.
- (2) The evaluation and selection of fuel materials and subassembly-design configurations for further development, one of the primary objectives of the Phase I study, would probably be independent of the absolute value of revenue.

Accordingly, the plutonium price was retained at the \$44 per g level. However, the general effect of a reduced plutonium value was investigated and is discussed in the section on reactor economics.

Method of Attack

The ultimate fuel system to be recommended has to meet two basic criteria:

- (1) It must be feasible from an engineering and materials standpoint based upon present technology or on future developments which could be reasonably expected to be achieved within the next few years.
- (2) It must show potential capability for producing economic power under the ground rules.

Within these limits the primary variables are the design configuration of the fuel elements within the assembly and the materials of construction. From these, the operating performance, the method of fabrication, and the allowable burnup may be estimated, and the ultimate revenue surplus or deficiency per day can be calculated.

Approximately the first half of the program was devoted to screening a variety of fuel materials and designs, to determining methods of deriving designs compatible with

the ground rules, and to setting up a system of analysis for evaluating the economics of any fuel system. The last half of the program was spent in detailed studies of the most promising fuel systems.

In general, the study of each fuel system began with a proposed fuel material which appeared to have applicability. Evaluations of present technology were made to try to arrive at a set of performance and fabrication characteristics and to set an upper value to allowable burnup. With this information a reference design and reference fabrication procedure were established. Finally, the performance of the reference design and the fabrication cost estimate were entered into the economic analysis and the revenue deficiency or surplus determined.

In the several fuel systems which survived to the final analysis, the design, fabrication, and economics analyses were refined to optimize the design and the economics at each of several power levels.

The final economic evaluation was made by considering each fuel system at the same power level. This power level was chosen arbitrarily at approximately the minimum power level for which economic power could be achieved.

The ultimate fuel system to be recommended for continued study in Phase II of the program was chosen on the combined basis of its economic potential, a qualitative estimate of the regions of uncertainty in performance, and the directness with which the development problems appeared to be solvable.

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GENERAL METHOD OF ANALYSIS OF REACTOR ECONOMICS

In screening potential fuel materials and designs for the Core II application it was desirable to know the parameters which would affect the economic performance most significantly. Further it was desirable to have a relatively rapid procedure to feed back the economic effects of design or materials changes. This need was fulfilled by the derivation of an "Economics Equation".

The Economics Equation

The purpose of this analysis was to provide a relatively straightforward system to evaluate the expenses and revenue of Core II operation in terms of the core design and materials parameters. The analysis which was used for all but the final designs is derived in Appendix B. The closed-form economic equation that is developed there is:

$$\begin{aligned}
 R = D + PX f \left[1 - \frac{L_f}{JBmF} (1 - JBmF) \right] + \frac{PX}{JBmF} \left[C_f(1 - JBmF) + C_u(U - JBmF) + NC_n \right] \\
 + P \times b \left[L_f(I - C_p) + C_p \right] + \frac{C_b N_b}{L} \\
 + \frac{0.1256}{365} fM \left[\frac{270 PX}{M} \left(\frac{1}{JBmF} - 1 \right) + 1 - \frac{JBmF}{2} \right] \\
 - P \times I(b + g) - C_s P_t
 \end{aligned}$$

where:

- b = core conversion ratio, atoms of plutonium-239 produced in the core per atom uranium-235 destroyed in the core
- B = burnup, number of atoms fissioned per number of assembly atoms
- C_b = total cost per blanket element to process and reprocess, dollars per element.
- C_f = costs for reprocessing fuel elements that are reducible to a per g of uranium-235 contained basis, dollars per g of uranium-235 contained
- C_p = costs for recovering plutonium that are reducible to a per g of plutonium-239 contained basis, dollars per g of plutonium-239
- C_n = cost to fabricate a fuel element, dollars per element

- C_s = steam revenue, dollars per MWD
 C_u = reprocessing costs reducible to a per g contained uranium basis, dollars per g uranium
 D = over-all fixed charges, dollars per day
 f = fuel cost, dollars per g of uranium-235 contained
 F = burnup achieved/burnup allowed
 g = blanket conversion ratio, atoms plutonium-239 produced in the blanket per atom uranium-235 destroyed in the core
 I = revenue from plutonium, dollars per g of plutonium-239
 J = diluent ratio, number of assembly atoms per number of uranium-235 atoms
 L = average lifetime of radial blanket, days
 L_f = fractional losses in fuel cycle, atoms uranium-235 lost per atoms uranium-235 cycled
 m = number of atoms uranium-235 destroyed per atoms fissioned
 M = initial core load, number g uranium-235

Note: $M = \frac{\text{critical mass}}{1 - 1/2 \text{ (fraction of fissionable material destroyed during the core life)}}$

 N = number of core elements
 N_b = number of blanket elements
 P = effective core power level, megawatts (includes 0.8 plant factor)
 P_t = effective total power level, megawatts (includes 0.8 plant factor)
 R = revenue deficiency for a 6 per cent return
 U = ratio of total uranium to uranium-235 in the system
 X = fuel economy factor, number g uranium-235 destroyed per MWD.

The expression for revenue deficiency is made up of eight terms. These are respectively (1) the fixed costs, (2) the fuel-replacement costs, (3) the core-reprocessing and -fabrication cycle, (4) plutonium-recovery costs from the core, (5) the blanket-cycle costs, (6) fuel-inventory costs, (7) income from plutonium, and (8) income from steam.

This equation does not accurately include some of the effects which may be important to the final economic considerations. For instance, a variation between plutonium values in the core and blanket is not adequately taken into account. The area of reprocessing is considerably simplified as are the total blanket charges. Nevertheless, the analysis gives a fair basis for comparison between Core II systems and allows for rapid calculations to outline trend information regarding the effects of the various parameters used.

As an illustration of the derived equation, the revenue deficiency term is calculated below for Core I. Figures for critical mass and conversion rates given in APDA-115 are used.

<u>Term</u>	<u>Dollars per Day^(a)</u>
Fixed costs	\$17,730
Fuel replacements	5,180
Core-element reprocessing	1,860
Core fabrication	9,280
Core plutonium recovery	210
Blanket-cycle cost	3,430
Fuel inventory	<u>5,320</u>
Total costs	\$43,010
Plutonium income	\$15,550
Steam income	<u>8,300</u>
Total income	\$23,850
Revenue deficiency for 6 per cent return	\$19,160

Important Terms in the Economic Equation

Within the framework of the ground rules for Core II studies only a limited number of parameters which affect the system economics can be varied. Of these a few, bounded by physical limitations, are restricted to narrow ranges. Examples of the factors which are fixed by the ground rules are C_b , C_f , C_p , D , F , and L_f ; examples of parameters which vary over only limited ranges are f , I , m , N , N_b , X . In this latter example the use of a 139 subassembly core is assumed. The remaining dozen or so quantities are really the ones remaining for system optimization. The following paragraphs will show the influence of the more important parameters for core subassemblies composed of parallel plates of zirconium-clad uranium-10 w/o molybdenum fuel. This design is selected since a number of the important economic characteristics of this reactor system become apparent.

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(a) A 2.5 average a/o burnup and a subassembly-fabrication cost of \$6000 is assumed. Full value plutonium income is also assumed.

Burnup

Probably the single most important variable for economic operation is the extent of burnup of the uranium-235 atoms in the core subassemblies. To allow for a better understanding of the underlying effects, the parameters J , B , m , and F are combined as $JBmF$ to give the per cent of uranium-235 destroyed during the life of a core subassembly. Accordingly, the per cent uranium-235 destroyed during the core life can be improved by designs which allow greater J and m values for a given atomic per cent burnup, B . This fact will be utilized in sections outlined below.

A typical curve of revenue deficiency versus burnup B appears in Figure 1. From this curve it is immediately evident that a region of interest for this parameter is defined. At values of burnup near 1 a/o the improvement in over-all plant economics for a small change in burnup is great. Whereas at burnups near 4 a/o the improvement is small. The question is an academic one at such large burnups but it is clear that other factors would take on leading significance if the "knee" in the curve could be exceeded.

The leveling off of economic improvement with increased burnup is due almost entirely to the increased core life. Fewer subassemblies are reprocessed and fabricated per unit time for high burnups. This reduces all the cost terms except the fixed charges, D .

Power Generation

The importance of core power on economics is not as easily foreseen as was the corresponding importance of burnup. Entirely new system designs are required for each new power considered. The design changes for higher powers are always in the direction of reducing the total fuel material in the core. It is expected therefore, that at some high power one of two things will happen. First, the material available may have insufficient uranium-238 to accommodate a Doppler temperature compensation of the reactivity on thermal excursions so the systems will be rejected from consideration on this score. Secondly, the equivalent core life may become so short because of a reduced ratio of total fuel atoms to uranium-235 atoms that the reprocessing and fabrication costs outweigh the improved income figures, i.e., the factor J is reduced so that only a small per cent of the uranium-235 is destroyed at the time the maximum atomic per cent burnup, B , is achieved. This latter effect is illustrated in Figure 2 and is typical for the alloy systems.

It is important to note on Figure 2 the region of optimum economic performance. This optimum power is of course a function of the subassembly fabrication cost since this controls the cost end of the total revenue. It is also a function of the sodium temperature since this will affect the thickness of the fuel plates and, hence, J , for a given maximum fuel temperature.

The ceramic systems considered showed continuing improvement in the economics with power as opposed to the minimum indicated in the alloy. The maximum powers were always limited by feasibility of fabrication or by a uranium-238/uranium-235 ratio of less than 1.

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FIGURE 1. TYPICAL REVENUE DEFICIENCY VERSUS BURNUP

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FIGURE 2. REVENUE DEFICIENCY VERSUS POWER
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Core-Element Fabrication Costs

The importance of core-fabrication costs is of course coupled to the core life and, hence, to burnup. If relatively few subassemblies are required per unit time then quite expensive ones are tolerable. Conversely, for low-burnup systems (short lived) the emphasis is placed on inexpensive subassemblies.

The curves of Figure 3 show the relative importance of subassembly-fabrication costs as a function of burnup for the alloy core. This is typical of all other systems. In the range of 2 and 3 atomic per cent burnup efforts directed toward reduced fabrication costs are justified.

Plutonium Buy-Back Price

Although the ground-rule value of plutonium buy-back price, I , has been fixed, it is of interest to consider the importance of this quantity insofar as future planning may be affected. To this end, the curves of Figure 4 have been prepared. These curves show the relative importance of I for various powers. The conclusion to be drawn from these curves is that, under the ground rules specified, a serious percentage change in I can better be absorbed in a low-power system than in a high-power design. This fact is a result of the generally delicate balance of income and expenses over the entire power range. A given percentage change in one figure, such as I , is quite drastic in the absolute sense when this figure itself is large as in the case of high-power systems. The same percentage change is less important to the revenue term if the lower power system is considered.

Blanket-Cycle Time

Using the calculated blanket-cycle time for one alloy design as a basis, the curve of Figure 5 has been prepared to show the variation in economics with changes in this figure. It seems clear that this term is not really significant in the range studied. For short core lives the importance is increased but it never appears to approach some of the other parameters. This would imply that this area need not be investigated with the same emphasis as some of the others pointed out above.

It is worth mentioning that no value is given to plutonium in this part of the blanket cycle. Certainly if a revenue-deficiency term were applied to account for an inventory of unrecovered plutonium the picture described above would alter considerably.

Losses from the System

At several points in the core fuel cycle an unrecoverable loss of fuel is expected. The value of this loss has been in the neighborhood of 1 per cent of the throughput. It is important economically in raising the new-fuel requirement and in depleting the possible plutonium revenue. A typical value for the revenue deficiency associated with these losses for an alloy system operating at 2 a/o burnup and a 616-megawatt core power is about \$1000 per day. Apparently this loss figure is unimportant for losses in the range of 1 per cent when it is compared to other economic effects.

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FIGURE 3. REVENUE DEFICIENCY VERSUS SUBASSEMBLY FABRICATION COST
Alloy system; 616-megawatt core power.

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FIGURE 4. VARIATION IN REVENUE DEFICIENCY WITH PLUTONIUM PRICE

Alloy system; \$2000 subassembly; 2 a/o burnup.

FIGURE 5. REVENUE DEFICIENCY CHANGE VERSUS OUT-OF-PILE TIME OF BLANKET SUBASSEMBLIES

Alloy core; \$4000 subassembly; 616-megawatt core power.

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Steam Price

The variation of revenue with steam temperature (hence price) is a complicated function for the alloy systems. Like the variation with power, a completely new sub-assembly design is required for each change in steam temperature. The illustrative curves of Figure 6 have been prepared to show what happens to the revenue for these design changes in the case of the alloy system. It is to be noted that the curves appearing here do not represent design systems and are to be interpreted as trends only.

Since higher steam temperatures go hand in hand with systems having less fuel in the subassemblies it is to be expected that the equivalent burnup of uranium-235 atoms for a given atomic per cent burnup will become a factor. This is apparently the case; at high powers the system changes cause corresponding changes in the burnup which are really important to the economics. That is, the smaller volume fractions of fuel that go with the higher steam temperatures give rise to a sufficient change in J to yield an over-all poorer economic picture. At lower powers, where the core life is fairly long and increases or decreases in J are relatively insignificant, the picture rights itself; that is, higher steam temperatures go with better economics. Unfortunately, the design region for alloy systems is in the range where these two effects cross over.

The effect of changing steam temperature for the ceramic systems is not radical. It resembles the lower power region of the alloy systems. That is, the higher temperature steam is always more economical.

Interrelationships Between the Economic Analysis and the Materials and Design Studies

Core-Physics Studies

The core-physics studies provided the economics analysis with the critical masses M , conversion ratios b and g , and fuel-economy factors X , for all the systems considered. The physics analysis also provided valuable checks on the validity of the assumptions made in the economic analysis. This interrelation effect is described in the section on physics analysis below.

Fuel-Element Engineering Design

The important engineering design restrictions are laid down by the ground rules for Core II. However, within the framework of the ground rules there is sufficient latitude to accommodate some of the important conclusions from the economic analysis. The economic significance of burnup makes it one of the most important design considerations. Design temperatures were limited in the alloy base systems to provide for a more favorable environment from a radiation-damage standpoint. Correspondingly, wherever it was possible, systems having the greatest "fuel" material were designed. This gives rise to a larger J factor.

In the case of the cermet design it was found that an optimum choice in volume per cent dispersed fuel could be expected. This optimum was found to be the minimum

FIGURE 6. ILLUSTRATIVE CURVES OF THE EFFECT OF STEAM TEMPERATURE ON REVENUE DEFICIENCY FOR ALLOY SYSTEMS

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volume per cent dispersed fuel which was fully enriched. Designs on both sides of this optimum were run in order to bound it. Similarly, the location of optimum powers for the alloy design which were dictated by economic consideration were closely approached.

Because of the relative importance of fuel-element fabrication costs, a number of variations on some of the basic designs were considered. Many of these more inexpensive designs proved to give poorer over-all economics because of factors other than fabrication cost.

A final tie-up between economics and design considerations is worth mentioning. Whereas, basically, higher steam temperatures should give better economics, it was found that this was true for only a limited range of burnup. The reasons for this are pointed out in other sections above. It was necessary to have the subassembly designs to find this out.

Fuel-Element Materials

Basically, all of the fuel-element materials considerations were founded in the trends shown in the economic analysis. Such things as the importance of burnup have displaced factors such as reliability or simplicity as most important. Minimum fabrication costs and reprocessing costs are desired, however, all other things being equal.

The intensive survey given to uranium dioxide fuel was based on the conclusions from the economic analysis that maximum burnup and minimum fabrication costs would outweigh almost all other considerations. However, a significant effect failed to materialize, as will be shown later.

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GENERAL CONSIDERATIONS OF POTENTIAL FUEL MATERIALS

As indicated in the previous section, many factors affect the economics of a reactor fuel system; among these, radiation stability, fabrication costs, and density exert very significant influences. Since, ultimately, radiation stability is the most important of these, this factor dominated the evaluation of potential materials. Fabrication was considered only from the viewpoint of possible influence on radiation stability or the feasibility of preparation.

The evaluation of possible fuel materials is reported in three sections; alloy, ceramic, and cermet-type fuels have been considered. As a result of this evaluation the following general conclusions may be stated.

- (1) Based on available data, the uranium-10 w/o molybdenum alloy is the most promising fuel alloy. However, variations in composition, design, or method of fabrication could conceivably improve upon the known irradiation performance of this alloy.
- (2) Uranium dioxide is the only ceramic material that can be considered at this time, since information on other ceramic materials is too limited for possible consideration.
- (3) A cermet, consisting of UO_2 dispersed in a uranium-10 w/o molybdenum matrix, offers promise as a means of localizing radiation damage in an element and is a possible method for improving the economics of the alloy-fuel system.

Alloys

The alloy systems of primary interest in this program are those of uranium with molybdenum, niobium, and zirconium. A great many other alloys have been studied and more data are available, but the metastable gamma-type alloys appear to be most suitable for high-temperature use. Alpha-phase-type alloys may also be attractive, but are subject to transformation to the beta phase at elevated temperatures. This transformation to the beta phase will tend to destroy the cast or heat-treated structure which is generally necessary for stability in this type of alloy. Transformation will also tend to facilitate the release of fission gases, causing excessive swelling. In addition, the interaction of the highly anisotropic alpha-uranium grains of different orientations may be the cause of nucleation of the noble fission gases at high burnups even though the temperature has been kept below the beta region.

These statements seem damning to alpha-type alloys, but should not be interpreted so. Instead, they represent the reasons that metastable, or stabilized gamma-type alloys, properly developed, are expected to have better performance, since the body-centered-cubic gamma structures are isotropic and may not go through a transformation, but tend to be reverted to or kept in the gamma by exposure to irradiation.

The gamma is not retained in alloys containing less than about 7 w/o molybdenum, the gamma phase transforming by a martensitic-type reaction on quenching to yield a

distorted alpha-uranium structure. The relatively poor radiation stability of the gamma-quenched 5 w/o molybdenum alloy is consequently not a property of the gamma phase. The alloys which can be considered in the molybdenum system are therefore those of 8 w/o molybdenum or more. Because of rather similar characteristics in the niobium system, alloys of less than about 8 w/o niobium cannot be considered.

Uranium-Niobium Alloys

Uranium-niobium alloys are found to exhibit the radiation-induced gamma-phase reversion phenomenon characteristic of the uranium-10 w/o molybdenum alloy⁽¹⁾. A uranium-10 w/o niobium alloy has a density of 16.5 g per cm³, only slightly less than the 17.2 g per cm³ of a uranium-10 w/o molybdenum alloy. However, the strength of uranium-niobium alloys is less than that of uranium-molybdenum alloys, and, probably of much greater importance, the thermal stability of uranium-niobium alloys is much less than that of uranium-molybdenum alloys. Transformation in a uranium-10 w/o niobium alloy occurs at the C-curve nose in a matter of minutes⁽²⁾, while transformation in uranium-molybdenum alloys is delayed to times represented by hours⁽³⁾. Since thermal stability is an important factor in the proposed fuel application, the uranium niobium alloys are not regarded as being as promising as the uranium-molybdenum alloys. However, by increasing the niobium content or by ternary alloying to increase the thermal stability of the uranium-10 w/o niobium alloy, the potential of these alloys should be increased.

Uranium-Zirconium Alloys

In the zirconium alloys the gamma phase can be retained on quenching; however, it transforms very rapidly, and in this system it is more practicable to consider the alloys for use in the transformed or epsilon-phase condition. Essentially, single-phase epsilon-alloy behavior can be obtained with additions of between 40 to 50 w/o zirconium. However, densities for alloys in this range are 11 to 10 g per cm³. Since high density is a requirement for Core II application, an immediate limitation to the consideration of these alloys is introduced.

These alloys with zirconium have shown reasonable performance at high burnups, but are limited by the transformation from epsilon to gamma at temperatures just above 600 C. Thus, high burnups can only be expected if the alloys are kept below 600 C.

Uranium-Molybdenum Alloys

Alloys of the uranium-molybdenum system exhibit the best irradiation stability of high-uranium alloys irradiated to date; furthermore, gamma-treated and quenched uranium-10 w/o molybdenum specimens appear to show the least growth when exposed to irradiation. Density changes of about 2.5 per cent per a/o burnup have been reported for bare specimens receiving up to 1.2 a/o burnup⁽⁴⁾, at calculated central metal temperatures up to 600 C.

(1) References at end.

More recently clad coextruded uranium-10 w/o molybdenum specimens have been irradiated to burnups of 2 a/o and experienced density changes averaging about 2.5 per cent per a/o burnup^(5, 6). However, clad coextruded specimens have been found to be split at the ends when irradiated to burnups between 2.0 and 2.7 a/o⁽⁶⁾. The reason for this behavior of the reference Core I alloy pins is not clear, but it should be noted that these particular clad fuel-alloy specimens are not believed to be metallurgically ideal material. First, the coextruded fuel alloy is known to be inhomogeneous, and, as a consequence of the homogenizing heat-treatment limitations introduced by the necessity to limit diffusion into the coextruded cladding, homogenization is not feasible after coextrusion⁽⁷⁾. Also, the enriched material used in these specimens was not of high quality.

In the specimens which split at the ends, volume changes of 2.1 to 7.4 per cent per total a/o burnup were observed. Calculated average central-core temperatures were 340 to 385 C. The meaning of this generally increased rate of swelling is not fully understood, but several factors may be involved. In addition to the inhomogeneity mentioned above and the rather low quality of the enriched uranium, variations in fabrication used heat-treatment variables other than those affecting homogeneity may be important. Another factor may be the conditions of irradiation, since it is probable that some critical flux (for a given neutron spectrum) may be required to maintain the alloy in the gamma condition. This is important if, as assumed, transformation is undesirable.

It is encouraging that near theoretical density changes of about 2.5 per cent per total a/o burnup have been accomplished up to 2 a/o. It is also encouraging that the values of density change observed and reported in Table 1 are within the scatter of values obtained in other work⁽⁸⁾. This observation lends further support to the belief that process and radiation variables are responsible. Thus, it cannot be concluded that the swelling itself represents failure of these samples, since the results are within the scatter of other experiments. It is important, however, to minimize the rate of swelling.

Failure did occur in the above-mentioned samples by splitting of the cladding. It is likely that, if swelling of the core can be minimized and maintained at about 2.5 per cent per a/o burnup to higher burnups, the limiting feature of the zirconium-clad pin design may be the cladding itself. It should be observed that large volume changes are possible without splitting of the cladding, even at the temperatures of these irradiations. For example, Specimens 10CT-8-3 (8-mil cladding thickness) and 10CT-4-25 (4-mil cladding thickness) decreased in density a total of 15.1 and 15.5 per cent without ruptures forming. Also, the ends received higher than average burnups and were unclad, so conditions were favorable for end splitting to occur.

In addition to the data reported above, clad coextruded 9 and 12 w/o molybdenum alloy specimens have been irradiated to burnups of 3.12 and 3.04 a/o with density decreases on the order of 3.5 per cent per total a/o burnup being recorded⁽⁸⁾.

Brief but severe experiments at Battelle indicate that appreciable swelling is induced by heat treating irradiated samples 100 to 200 hr at 575 C⁽⁹⁾. This swelling probably results from the accumulation of fission gases during transformation. Since the fuel alloy will be held at rather low temperatures during reactor shutdown, the initiation of transformation in the reactor is unlikely.

TABLE 1. RESULTS OF THE IRRADIATION OF CAPSULE BMI-9-9, CLADDING-THICKNESS SERIES

Specimen ^(a)	Capsule	Burnup ^(b) , total a/o	Calculated Average Central-Core Temperature ^(c) , C	Dimensional Change, per cent				Density Decrease Per Total a/o Burnup, per cent	Comments on Macroscopic Examination
				Diameter Increase	Length Increase	Volume Increase	Density Decrease		
10CT-4-26	9-9B	2.7	385	9.5	5.1	26.0	17.3	6.4	Crack at one end
10CT-8-3	9-9B	2.1	385	7.9	6.8	24.3	15.1	7.2	Good condition
10CT-11-3	9-9B	2.4	385	5.7	6.9	13.2	9.7	4.2	One end split
10CT-4-25	9-9A	2.1	340	8.3	4.8	22.9	15.5	7.4	Good condition
10CT-8-2	9-9A	2.1	340	3.9	2.1	10.2	8.2	3.9	One end split
10CT-11-2	9-9A	2.0	340	2.3	0.4 ^(d)	5.1	4.2	2.1	Both ends split

(a) The specimens were arranged top to bottom according to their position in the capsules. The cladding thickness is identified by the digit following the CT portion of the specimen number. These digits will be either 4, 8, or 11, to identify cladding thickness of 4.7, 8.1, and 11 mils, respectively. Specimens were coextruded and heat treated 1 hr at 800 C, quenched, and held 15 min at 350 C.

(b) The specimen burnups for Capsule BMI-9-9 were calculated from dosimeter data.

(c) Average specimen central-core temperatures were calculated from dosimeter data.

(d) Value appears to be low, and there is no obvious explanation for this.

The most needed information on the 10 w/o molybdenum alloy at present is a knowledge of its behavior at high temperatures. Data at burnups of less than 1 total a/o burnup and calculated central metal temperatures up to 600 C are favorable; however, there is an uncertainty of the actual temperature. Data obtained at A. T. C. in a high-temperature capsule which is reported to have operated at volume average temperatures from 1240 to 1370 F showed density decreases of 22 to 24 per cent at a burnup of about 1.65 total a/o⁽¹⁰⁾. The maximum central fuel temperature was about 1395 to 1400 F. Certainly swelling is going to be a problem at some temperature, but the limits are too uncertain at present. Increased strength at these high temperatures may be expected to extend the favorable range of conditions of temperature and burnup upward.

Based on present technology, a number of means of improving the radiation stability of the 10 w/o molybdenum alloy can be considered. These are discussed in detail in the paragraphs that follow.

Fabrication, Homogenization Alloy, and Heat Treatment. Homogenization can be achieved by heat treating at the appropriate point in the fabrication process. Generally, a heat treatment of 24 hr at 900 C is adequate for wrought material. However, shorter times at higher temperatures can be substituted. The irradiation performance should be extended by assuring uniformity and by achieving the high-temperature properties of the base alloy throughout.

It is known too that the fabrication and homogenization cycle have considerable effect on the rate of transformation. That is, the material coextruded and treated 1 hr at 800 C transforms much more rapidly than does hot-rolled material treated 24 hr at 900 C.

Quality Control of Starting Materials. It is a characteristic of the enriched metal that the quality varies from batch to batch. Some specifications are needed and should be based on an understanding of the effects of the impurities on performance.

Design of Subassembly. It is probable that design can aid to some extent in extending the life of this fuel. For example, it is expected that flat plates will grow largely in a direction perpendicular to the plate, minimizing the stress which can cause rupture of the zirconium cladding. As indicated earlier, the rupture of the cladding may be a limiting feature of the pin design.

Alloy Additions or Changes. Ternary or quaternary alloy additions may become important if difficulty is encountered in getting the desired burnups, especially at high temperatures. The alloy additions would be added to give additional strength at high temperatures to resist swelling. Increased thermal stability, or resistance to transformation at lower temperatures may also be helpful.

Increases in molybdenum content may also be helpful, since transformation of a 15 w/o molybdenum alloy occurs by ordering, where the atom movements would be less extensive than in the precipitation of alpha phase that occurs in the 10 w/o alloy. The casting and fabrication problems increase in difficulty in this range of composition, but may not prohibit consideration of higher molybdenum content.

Porous Alloy Fuel. Fission-gas atoms which are created within a metallic structure will migrate toward available imperfections and surfaces. If the surface happens to be a microcrack, high gas pressures can be developed within the crack and this in turn results in high stress concentrations. A ductile material can absorb these internal stresses through plastic deformation; however, a brittle material, i.e., one subjected to high burnups, is likely to fail through the propagation of these internal cracks. One possible method of preventing this frequent mode of failure is to build into the element sufficient porosity and surface area to serve as a sink for fission gases, thus preventing high pressures from being created in very localized areas.

If such a program were pursued every effort should be made to control not only the amount and distribution of the porosity, but also to render it as nearly spherical as possible. In many cases, this is obtained by a heat treatment at the sintering temperature of the material in question.

Spiked Alloy Fuel. The uranium-10 w/o molybdenum reference alloy will of necessity be partially enriched. This provides an opportunity for considering a unique procedure for preparing a single-phase alloy in which the advantages of a cermet and those of a wrought fuel are combined into a single system. If fully enriched uranium-10 w/o molybdenum powder is mixed with depleted or natural uranium-10 w/o molybdenum and fabricated into a dense fuel element, a form of dispersion or "spiked" alloy would result in which the enriched phase is uniformly dispersed in a depleted or natural matrix. During irradiation, the severest damage would be isolated and hopefully contained within the highly enriched areas. Arguments advanced in the section devoted to cermets concerning particle size and volume loading would be applicable here. Advantages of this type system over a conventional cermet might be improved fabricability and improved strength.

Ceramics

A number of ceramic materials can be considered as fuel for nuclear reactors. These include the carbides, oxides, nitrides and various intermetallic compounds of uranium. Typical of the latter are U_6Fe and UFe_2 . Except for uranium dioxide (UO_2), little is known of the irradiation behavior of these uranium compounds, and physical-property data are sparse. Isolated tests on uranium carbide and uranium silicides do not indicate particularly superior irradiation properties for these materials.

Owing to its low cost, availability, and corrosion resistance in high-temperature water, UO_2 has received far more attention than the others.

A summary of some of the properties of UO_2 are listed in Table 2. Many of the properties of UO_2 are dependent upon its actual density and method of fabrication. It is characterized as a brittle ceramic of low thermal conductivity and low density relative to other uranium compounds.

From the physical properties it is apparent that fuel elements utilizing UO_2 as a solid body in high-heat-flux applications must be anticipated to be subject to cracking, fragmentation, resintering, and melting. To overcome these difficulties, slurries or pastes of UO_2 in sodium or dispersions of UO_2 in metal (cermet form) offer attractive alternatives.

TABLE 2. PROPERTIES OF UO_2 (a)

Crystal Structure	Face-centered cubic
Theoretical Density	10.9 g per cm^3
Melting Point	2800 C
Conductivity at 100 Per Cent Density and 20 C	0.02 cal/(C)(cm)(sec)
Conductivity at 100 Per Cent Density and 1000 C	0.008 cal/(C)(cm)(sec)
Thermal Expansion (0 to 1000 C)	10×10^{-6} per C
Specific Heat (0 to 2000 C)	0.056 cal/(g)(C)

(a) See Reference (11).

Data representative of the irradiation behavior of UO_2 are given in Table 3. Figure 7 is a plot of the fission gas released in these tests as a function of temperature. The following trends will be noted:

- (1) There appears to be no correlation between amount of fission gas present (exposure) and the percentage of gas released. This implies that the fission process, neutron irradiation, and general radiation effects do not accelerate the rate of diffusion or release of fission gases from the UO_2 . This may or may not be true at higher burnup.
- (2) The fraction of fission gas released increases markedly with temperature, particularly above the homologous temperature (0.6 of the absolute melting temperature). This temperature is significant for many diffusion-controlled processes; diffusion rates are evidently very rapid above this temperature. This is in agreement with an analysis by Lustman⁽¹²⁾, who states that the release of fission gases by UO_2 is temperature sensitive and controlled by gas mobility in the solid state.
- (3) There appears to be little, if any, correlation between the fraction of fission gas release and density. This implies that the distance a gas atom must travel to a free surface is relatively uniform in all of the samples reported. If any effect of density on the fraction of gas release does exist, it is overshadowed by the range of experimental error and the temperature effect.

These trends suggest that the heat flux and maximum temperature must be carefully controlled in the design of a UO_2 fuel system, and that jacketing for the fuel element must be essentially a pressure vessel for the fission gases. Based upon the burnup levels reported thus far, it would appear that burnup will be limited only by the safe pressures of the fuel jacket. Whether or not this is the only limitation under very high burnup is yet to be determined.

TABLE 3. FISSION GAS RELEASE BY URANIUM DIOXIDE
AS A RESULT OF IRRADIATION

Estimated Exposure, MWD/T	Estimated Central Temperature, C	Gas Release, per cent of theoretical	Specimen Density, per cent of theoretical	Reference No.
2,410	(a)	0.05	--	(13)
5,870	1500	0.11	95	(9)
275	1300	0.40	93	(14)
350	1300	0.58	95	(9)
950	2000	1.1	93.5	(9)
1,500	1240	3.1	91	(14)
10,900	1580	5.0	94	(14)
1,680	Melted	8.4	95	(9)
13,500	1850	8.9	90	(14)
1,650	1700	9.5	80	(9)
2,060	1850	16	95	(9)
2,780	2300	23	95	(9)
4,440	(b)	23	--	(13)
1,950	Melted	27	95	(9)
2,400	Melted	44	--	(15)

(a) Low heat rate: 450,000 Btu/(hr)(ft²).

(b) High heat rate: 830,000 Btu/(hr)(ft²), probably melted.

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FIGURE 7. EFFECT OF TEMPERATURE ON THE RATE OF GAS RELEASE BY UO_2 UNDER IRRADIATION

Dispersions

A considerable amount of irradiation data has been obtained on dispersion-type fuel elements containing 25 to 30 volume per cent UO_2 dispersed in a stainless steel matrix and clad with stainless steel. In most of the experiments, surface temperatures have not exceeded 900 F; however, there are sufficient data in the temperature range of 1000 to 1100 F to indicate a "probable life expectancy" for this type of fuel element. Fuel pins, 160 mils in diameter, containing 25 w/o UO_2 have been irradiated to a burnup of 28 per cent of the uranium-235 atoms at surface temperatures of 1130 F with no reported failures. Flat-plate elements, also containing 25 w/o UO_2 , were irradiated to 30 per cent burnup of the uranium-235 atoms at 1100 F surface temperatures with failures occurring only in those specimens containing a very fine dispersion ($3\ \mu$) of UO_2 particles. Specimens from the same experiment prepared with coarser particles ($30\text{-}\mu$ average) showed much improved irradiation stability. This greater stability has been attributed to the smaller percentage of fission-product atoms which have emanated from the larger particles as well as the increased distance between coarser UO_2 particles providing a relatively undamaged metal skeleton. From these experiments with fully enriched UO_2 dispersed in stainless steel, it appears that approximately 30 per cent burnup of the uranium-235 atoms represents a permissible service life for this type of element operating at surface temperatures of 1100 F. This corresponds to a burnup of approximately 1.65 per cent of all the atoms in the fuel core. It is particularly interesting to note when translating to atomic per cent burnup, that approximately 9 per cent of all the atoms in the UO_2 phase have been destroyed. This is considerably higher than could be expected with UO_2 in bulk form (a PWR pin) without extreme damage occurring in the UO_2 .

The UO_2 -stainless steel system is not suitable for the APDA concept because of the limited amount of uranium which can be loaded. This limits the conversion ratio to an unacceptable value. However, a fuel system in which enriched UO_2 is dispersed in a uranium-10 w/o molybdenum metallic matrix overcomes this difficulty. The method of fabrication would be similar and the expected performance under irradiation can be compared to the UO_2 -stainless steel system owing to the excellent high-temperature-strength properties of this alloy. In Table 4, several systems are compared on the basis of energy produced in an equivalent volume of fuel after a designated burnup. A value of unity was assigned to the uranium-10 w/o molybdenum alloy after a burnup of 3 a/o to provide a comparison. A cermet fuel in which 20 volume per cent UO_2 is dispersed in a depleted uranium-alloy matrix will probably withstand irradiation damage to an extent comparable to the UO_2 -stainless steel element of 9 a/o burnup of the UO_2 phase. This gives an energy factor of 0.96. By adjusting the enrichment in the matrix so that a burnup of only half that assigned to the reference alloy is obtained in the matrix, the energy factor is increased to 1.3. If the UO_2 loading is increased to 40 volume per cent and the matrix is depleted, an energy factor of 1.9 is realized if the UO_2 phase can still withstand 9 a/o burnup. However, as the oxide loading is increased, the transverse tensile strength is decreased and fabrication becomes increasingly more difficult. Since the mode of failure of these cermet materials during irradiation appears to be a transverse tensile failure, the loss in physical strength is serious. High loadings have further disadvantages for reasons illustrated in Figures 8 and 9. In Figure 8, the relationship between oxide loading, particle size, and the minimum distance between particles is plotted assuming the oxide particles are spherical and are packed in a cubic formation. It can be seen that with a 40 volume per cent loading oxide particles of $150\ \mu$ in diameter must be used before a continuous matrix - free from fission products emanating from the UO_2 - is obtained. With a

FIGURE 8. RELATIONSHIPS AMONG OXIDE LOADING, PARTICLE SIZE, AND MINIMUM DISTANCE BETWEEN PARTICLES

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FIGURE 9. EFFECT OF PARTICLE SIZE AND LOADING ON VOLUME PER CENT OF OXIDE PLUS DAMAGED METAL MATRIX

20 volume per cent loading, 45 mils of unaffected matrix is available. It is the unaffected region that will make the greatest contribution to the strength of these cermet. The advantage of coarse UO_2 particles is clearly indicated.

TABLE 4. COMPARISON OF SYSTEMS ON THE BASIS OF UNIT ENERGY/EQUIVALENT VOLUME OF FUEL AFTER A DESIGNATED BURNUP

Systems	Energy/Equivalent Volume
Uranium-10 w/o molybdenum 3 a/o burnup	1
UO_2 -80 v/o (uranium-10 w/o molybdenum) 9 a/o burnup UO_2 , 1.5 a/o burnup uranium-10 w/o molybdenum	1.3
UO_2 3 a/o burnup	1.5
UO_2 -80 v/o (uranium-10 w/o molybdenum) 9 a/o burnup UO_2 , 0 burnup uranium-10 w/o molybdenum	0.96
UO_2 -60 v/o (uranium-10 w/o molybdenum) 9 a/o burnup UO_2 , 0 a/o burnup uranium-10 w/o molybdenum	1.9

Figure 9 illustrates in another manner the advantages of the lowest permissible loading and the largest permissible particle size of the oxide phase. It is shown that with small particles the volume of highly damaged matrix material becomes a significant fraction of the total fuel volume. It is the total volume (oxide plus damaged matrix) that should be used in computing thermal conductivities etc., since the conductivity and strength of the damaged region may be impaired.

It would appear, therefore, that serious consideration should be given to pursuing the study of the cermet or dispersion-type element as a possible method of improving the economics of the reference fuel system. In addition, it is felt that the advantages for keeping the volume loading of the oxide phase within definite limits and the particle size at a maximum are clearly demonstrated.

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FUEL-SUBASSEMBLY DESIGN CONSIDERATIONS

General Method of Core-Pressure Drop and Heat-Transfer Analyses

Any system designed must meet the ground rules for both pressure-drop and temperature limitations; however, because of the complexity of the pressure-drop and temperature equations it is difficult to combine the two conditions in such a way that a single set of equations will meet all the necessary conditions. Therefore, the general method used was to determine for various possible coolant flow rates the plate or pin sizes and spacings as functions of the number of plates or pins in order to be consistent with the pressure-drop requirements. The dimensions obtained in this manner are then substituted into the temperature equations for calculation of the maximum fuel and cladding temperatures. The latter process is repeated until a system that meets the temperature requirements is found; this system then meets all pertinent ground rules.

The general method used in these analyses is more specifically described in the sections following.

Method of Core-Pressure-Drop Analysis

The ground rule primarily applicable to the core-pressure drop is C.4 (Revision No. 1, April 25, 1957, Appendix A), which states that the core-pressure drop may be 65 psi for any coolant flow rate chosen. For a given flow rate, this condition can be met for any number of fuel plates per subassembly by simultaneously varying the dimensions and spacing of the fueled members, the maximum number of plates or pins is then determined only by the minimum fabricable sizes of the fuel plates if thermal and physics limitations are not considered.

The following assumptions were used in the calculation of the pressure drop in the core section of a subassembly:

- (1) Clad surfaces are relatively smooth; therefore, to facilitate analytical determination of the flow-channel dimensions as functions of flow rates and number of plates, the Fanning friction factor is used. This friction factor is based on the Reynolds number of the particular single flow channel considered.
- (2) Entrance and exit losses total one-half the velocity head.
- (3) Pressure drop due to spacers is included directly as an effect on the equivalent diameter of the flow channel.
- (4) Hydrodynamic properties of sodium at 800 F are applied.
- (5) No fouling factor or extra allowance is required.
- (6) Sufficient sodium can be leaked between subassemblies to create a longitudinal pressure drop that will maintain the maximum

pressure drop across the half-fueled end plates at a safe value from the stress viewpoint. In addition to this effect, the sodium leakage between subassemblies should help to prevent any bowing of the elements in the direction to give positive reactivity effects.

The derivation of the pressure-drop equation for half-fueled end plate systems is given in Appendix C. A sample of results based on these equations is given in Table 5 for a parallel-plate design subassembly.

TABLE 5. SAMPLE PLATE THICKNESSES AND SPACINGS FOR PLATE SYSTEMS HAVING LIVE END PLATES AND TWO LONGITUDINAL 1/16-IN. RIBS AT THIRD POINTS BETWEEN PLATES

Pressure drop is 65 psi and mass flow rate through core is 20×10^6 lb per hr.

Number of Plates (Including End Plates)	Total Regular Plate Thickness, in.	Total End Plate Thickness, in.	Space Between Adjacent Plates, in.
19	0.085	0.047	0.061
20	0.080	0.045	0.059
21	0.075	0.042	0.057
22	0.071	0.040	0.055
23	0.066	0.038	0.054
24	0.063	0.036	0.052

Method of Temperature Analysis

The ground rules primarily applicable to the heat-transfer design specify the general over-all design and performance requirements of the fuel subassemblies and give the range of usable coolant temperatures. In addition, the physics limitations on the minimum ratio of uranium-238 to uranium-235 in the fuel impose an upper limit on the number of fuel plates per subassembly because total fuel volume decreases as the number of plates increases if the cladding thickness and pressure drop are constant.

The following assumptions were made in the calculation of the maximum temperatures in the core:

- (1) Heat conduction in the fuel and cladding occur only in the direction normal to the cladding surface; any conduction in other directions would tend to reduce the maximum temperature of the cladding and fuel.
- (2) The film coefficient for forced-convection heat transfer to sodium is represented by the empirical equation⁽¹⁶⁾

$$N_{Nu} = 0.625 N_{Pe}^{0.4} .$$

The derivation of the temperature equation for half-fueled end-plate systems is given in Appendix C.

Figure 10 shows the approximate effect on subassembly design of varying the reactor power and/or maximum allowable fuel temperature for both pin and plate systems. Using this illustration, the following trends can be noted:

- (1) For the same increase in reactor power, the required number of pins increases much faster than the required number of plates. Therefore, it can be expected that increases in fabrication costs of a pin system will be greater than those of a corresponding plate system.
- (2) The maximum allowable fuel temperature has a very large effect on the design. As this maximum temperature is decreased, the number of pins or plates increases and the fuel size decreases, which causes an over-all decrease in fuel volume in the core. This volume decrease means fewer fuel atoms per subassembly; the atomic per cent burnup then has to be balanced against the maximum allowable fuel temperature to obtain the optimum system.
- (3) For any fuel material and reasonable reactor power, a system can be designed to meet the specified pressure-drop and temperature limitations imposed by the ground rules.

Some consideration has been given to orificing the core to obtain the same mixed-mean coolant outlet temperature from each subassembly even though the radial maximum-to-average heat-generation rate is about 1.26. To accomplish this, the coolant flow rate through any given subassembly must be proportional to the quantity of heat generated in that subassembly. This means that, if the subassembly design is not varied within a single core and the ground-rule pressure-drop and temperature limitations are to be observed, the subassembly generating the most heat is the only one not orificed. It then appears that orificing might allow a larger fuel volume in the core because of the resulting decrease in coolant temperature in the subassembly generating the most heat; however, the gross effects of orificing cannot be evaluated until an accurate calculation of the heat generated in each subassembly is made available. The spherical physics calculations made for Core II are unsuitable for use in this type of calculation.

In addition, the possible 26 per cent total advantage on the coolant-temperature rise in the hottest subassembly can be gained only if the orificing is done with 100 per cent accuracy. Such accuracy is impossible to attain for the complete core; therefore, additional hot-channel factors must be introduced to account for all the inaccuracies of orifice sizing that might occur. If it is assumed that all other hot-channel factors are about the same as those described for System A, the comparisons in Table 6 can be made for the coolant-temperature-rise hot-channel factors. Similar factors must also be introduced for the film-temperature rise.

There are several other problems associated with orificing, such as providing for variation of orifice size to account for uneven burnup in the core. These additional problems tend to further decrease the net possible advantage obtainable.

As the scope of the present program did not permit a more complete evaluation of the effects of orificing, consideration of orificing was terminated at this point.

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FIGURE 10. EFFECTS ON DESIGN OF VARYING REACTOR POWER AND/OR MAXIMUM ALLOWABLE FUEL TEMPERATURE IF Δp_{core} , t_i , AND Δt ARE FIXED

Shown for uranium-10 w/o molybdenum fuel material.

TABLE 6. COMPARISON OF HOT-CHANNEL FACTORS WITH
AND WITHOUT ORIFICING

	Unorificed System A	Orificed System With Heat-Generation-Rate Prediction On Indicated Basis	
		Computer Calculations	Critical Assembly
Factor for Indicated Abnormal Heat-Transfer Condition			
Inaccuracy of Heat-Generation Prediction for Any Given Subassembly	--	1.20	1.05
Inaccuracy of Orifice Pressure-Drop Calculation	--	1.05	1.05
Manufacturing of Coolant, External Causes	1.03	1.03	1.03
Manufacturing Tolerances on Orifice Size	--	1.03	1.03
Product	1.03	1.34	1.17
Resulting Applied Hot-Channel Factor	1.45	1.70	1.56
Increase Over Applied HCF for System A, per cent	0.0	17.2	7.6
Net Possible Coolant ΔT Advantage Over Unorificed System A, per cent	--	8.8	18.4

CORE-PHYSICS CONSIDERATIONS

Certain quantities from the results of reactor-physics calculations were required as input information to the economic analysis. This information included; fuel loading, fuel burnout, plutonium-239 production, plutonium-240 content, core fissions per uranium-235 atom destroyed, and blanket life. In addition, it was necessary to know the fuel enrichment, since safety requirements specified a minimum atomic ratio (uranium-238 to uranium-235) of one.

The basic physics calculations were performed by APDA and the results of the calculations were used to determine the quantities required in the economic analyses.

The systems sent to APDA for calculating were arrived at from engineering design considerations. Not all systems included in the economics-parameter studies were sent in for calculations, as a method was developed to estimate the pertinent quantities from information at hand.

Preliminary Investigations

The reactor-physics calculations done by APDA were performed on an IBM-650 and employed the PROD II Code developed at KAPL. Ten energy groups were used, and the calculations were done for a spherical reactor system. The ten-group cross sections used have been tested by calculations based on existing fast reactor systems and the results were found to be satisfactory*. The output of these calculations included the critical mass for the sphere (this was corrected to apply to the cylindrical geometry), the initial core and blanket conversion ratios, neutron-flux spectrum in core and blanket, a neutron balance for the system, and spatially integrated fluxes.

In order to determine the degree of complexity required for reliable input to the economic analysis, a rather detailed analysis was done in processing the results of the first few systems received from APDA. The core spectrum was found to be nearly constant, so average cross sections were obtained for the core. The blanket was divided into annular regions, and average cross sections were found for each region. The following equations were solved for the core at various burnups, and for each blanket region at an average burnup of 0.2 per cent or for 5 years of irradiation time, whichever occurs first.

$$\frac{dN_{28}}{dt} = 0$$

$$\frac{dN_{25}}{dt} = -N_{25} \sigma_{a25} \phi$$

$$\frac{dN_{49}}{dt} = N_{28} \sigma_{c28} \phi - N_{49} \sigma_{a49} \phi$$

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*Alexanderson, E. L., private communication.

$$\frac{dN_{40}}{dt} = N_{49} \sigma_{c49} \phi$$

$$\frac{dF}{dt} = N_{49} \sigma_{f49} \phi + N_{25} \sigma_{f25} \phi + N_{28} \sigma_{f28} \phi .$$

The solutions to these equations yield:

$$B.U. = \frac{N_{O25}}{N_f} \left\{ \frac{\sigma_{f25}}{\sigma_{a25}} \left(1 - \frac{N_{25}}{N_{O25}} \right) - \left(\frac{A}{B} \frac{\sigma_{f49}}{\sigma_{a25}} + \frac{\sigma_{f28}}{E_o \sigma_{a25}} \right) \ln \frac{N_{25}}{N_{O25}} - \frac{A}{B^2} \frac{\sigma_{f49}}{\sigma_{a25}} \left[1 - \left(\frac{N_{25}}{N_{O25}} \right)^B \right] \right\}$$

$$\frac{N_{49}}{N_{O25}} = \frac{A}{B} \left[1 - \left(\frac{N_{25}}{N_{O25}} \right)^B \right] ,$$

$$\frac{N_{40}}{N_{49}} = - \frac{\sigma_{c49}}{\sigma_{a25}} \frac{\left\{ \ln \frac{N_{25}}{N_{O25}} + \frac{1}{B} \left[1 - \left(\frac{N_{25}}{N_{O25}} \right)^B \right] \right\}}{1 - \left(\frac{N_{25}}{N_{O25}} \right)^B} ,$$

where

$$A = \frac{\sigma_{c28}}{\sigma_{a25} E_o} , B = \frac{\sigma_{a49}}{\sigma_{a25}} , \text{ and } E_o = \frac{N_{O25}}{N_{28}}$$

B.U. = atoms fissioned per fuel atom

N_{O25} = atoms uranium-235 per cm^3 initially

N_{25} = atoms uranium-235 per cm^3

N_{49} = atoms plutonium-239 per cm^3

N_{28} = atoms uranium-238 per cm^3

N_{40} = atoms plutonium-240 per cm^3

F = fissions per cm^3

N_f = fuel plus diluent atoms per cm^3 .

The irradiation time for any blanket region was found by assuming the power in the region varied linearly with time. The fission rate in a region was found initially and after 0.2 a/o burnup had been achieved and the average power was determined from this.

To find the "dwell-time", T_j , for blanket elements, it is convenient to define

P_{1j} = initial power in the j th blanket region

F_{1j} = initial fission rate in j th region

F_{2j} = final fission rate in the j th region

F_{aj} = average fission rate in the j th region = $\frac{F_{1j} + F_{2j}}{2}$

$N_{25}V_{25j}$ = number of uranium-235 atoms in the j th region.

Then

$$T_j = \frac{184 \left(\frac{\text{mev}}{\text{fiss}} \right) \times 1.6 \times 10^{-19} \frac{\text{megawatt-sec}}{\text{mev}}}{3.15 \times 10^7 \text{ sec/yr}} \times \frac{N_{25} V_{25j}}{P_{1j} \frac{F_{aj}}{F_{1j}}}$$

For those regions where T_j is greater than 5 years, as calculated from the above equation, T_j was set equal to 5 years, and the burnup was found. The final fission rate in these regions was estimated by assuming the power to be constant at its initial value.

On the basis of these calculations, it was possible to reach several conclusions pertinent to the calculation of the necessary economic parameters. These conclusions are listed below.

- (1) N_{49} , N_{40} , and B.U. vary nearly linearly with the fraction of N_{25} destroyed throughout the range of interest. This means that quantities such as core fissions per uranium-235 atom destroyed, and grams of uranium-235 destroyed per megawatt-day do not vary with burnup. So initial values may be used. Further, initial plutonium-production rates are applicable in both core and blanket.
- (2) As was previously mentioned, the core spectrum is nearly constant in space so that the plutonium content is a function only of the uranium-235 destroyed.
- (3) In the range of interest, the plutonium-240 content in the core and blanket was low enough to have only a small effect on the economics.
- (4) The blanket power of any region (assuming constant flux in the blanket) varies considerably over the lifetime of that region. This makes the time average of the blanket power difficult to obtain from spherical calculations.
- (5) If it is assumed that the ratio of initial blanket power to average blanket power is a constant for all systems, and if the blanket lifetime is inversely proportional to average blanket power, then to a good degree of approximation, the blanket lifetime is inversely proportional to the product:

Blanket conversion ratio x core power x grams uranium-235 destroyed per megawatt-day.

These conclusions lead to the following definitions and decisions:

- (a) It would be assumed that each core element would reach its allowable burnup, and since the core spectrum was nearly constant in space, the "average core" uranium-235 content and plutonium-239 content at reprocessing is that commensurate with the allowable burnup.
- (b) Since burnup, and plutonium concentration varied nearly linearly with the uranium-235 fraction destroyed, initial conversion-ratio values in both core and blanket could be used to predict "average" plutonium concentrations at reprocessing. Also, the number of fissions per uranium-235 atom destroyed does not vary with burnup.
- (c) The plutonium-240 content could be neglected in all but the final economics.
- (d) Consistent with Item (a) above, it is assumed that the core is at some equilibrium condition such that each element is loaded with excess fuel initially, and the average amount of fuel in the core at any time is just the calculated critical mass. Under this assumption, it can be shown that the mass of fuel in a core made up entirely of fresh fuel elements is given approximately by:

$$M = \frac{CM}{1 - \frac{JBm}{2} (1-b)} ,$$

where:

CM = average fuel loading,

J = fuel atoms per uranium-235 atom

b = core conversion ratio

B = burnup (atomic per cent)

m = uranium-235 destroyed per core fissions.

Method for Estimating Economic Parameters

Since it did not appear feasible to perform the multigroup machine calculations for every system to be considered in the parameter study, it was necessary to develop a reasonably accurate method of estimating the required quantities. This task was greatly facilitated by a criticality parameter study on fast reactors done by Nims and Dalton of APDA. This work gave critical masses and conversion ratios for reactors

of different size and different sodium-volume fractions. All systems studied had uranium-10 w/o molybdenum alloy for fuel and contained 15 volume per cent stainless steel. By using this report and APDA calculations as check points, the following method was arrived at for estimating critical masses and conversion ratios:

- (1) Find an equivalent sodium volume for use in the Nims-Dalton study.

- (a) For uranium-10 w/o molybdenum,

Equivalent volume fraction sodium = 1 - volume
fraction fuel alloy - 0.15.

- (b) For UO_2 systems,

Equivalent volume fraction sodium = 1 - 1.2 volume
fraction uranium in UO_2 - 0.15.

- (c) For UO_2 -(uranium-10 w/o molybdenum),

Equivalent volume fraction sodium = 1 - volume
fraction uranium-10 w/o molybdenum - 1.2 volume
fraction uranium in UO_2 - 0.15.

These recipes are based on the assumption that all materials other than sodium and uranium can be treated by taking the first 15 volume per cent of these to be the 15 volume per cent stainless steels assumed in the Nims-Dalton study. The uranium not in uranium-10 w/o molybdenum form was multiplied by 0.2, and, since uranium-10 w/o molybdenum is about 20 volume per cent molybdenum, this amount of material was associated as molybdenum equivalent for the uranium as UO_2 . The remaining material was assumed to be equivalent to sodium, and added to it to give the equivalent sodium volume.

- (2) Assuming a core volume of 504 liters for the 139 element cores, the critical mass and conversion ratios were obtained from the Nims-Dalton study. The atomic ratio of uranium-238 to uranium-235 was calculated on the basis of this critical mass.
- (3) To obtain better agreement with APDA results, it was necessary to apply somewhat arbitrary corrections to the values found as in (2) above.

- (a) Uranium-10 w/o molybdenum systems,

Increase the critical mass by 20 per cent. This includes the correction for uranium-235 burnup (~5 per cent) previously mentioned. Decrease the total conversion ratio by subtracting 0.05.

(b) UO₂ systems,

Increase the critical mass by 10 per cent .
Do not change the total conversion ratio.

(c) Cermet systems,

Increase the critical mass by 15 per cent.
Subtract 0.04 from the conversion ratio.

- (4) Correct the core conversion ratio (in all cases) by the new N_{28}/N_{25} caused by the critical-mass change.

In addition to these quantities, the following core quantities were assumed constant in all but the final economics. The constants used were representative values obtained from calculated cases and did not vary greatly from case to case.

- (1) Grams uranium-235 destroyed per megawatt-day = 1.32.
(2) Atoms uranium-235 destroyed per total atom fissioned = 1.10.

Although this method is somewhat arbitrary, it was felt that the results were adequate for the purpose, and the method seems justified when compared with the alternative of having 10-group machine calculations done for all the systems considered.

EVALUATION OF FUEL SYSTEMS

Fuel Systems Considered

As the consideration of potential fuel materials progressed, attempts were made to design fuel elements with these materials, to establish fabrication methods, and to evaluate the economics of these systems. The materials receiving the most attention and carried to various stages of consideration are summarized in the paragraphs below.

Alloys

As discussed previously, the most promising alloy for this application appears to be uranium-10 w/o molybdenum or a ternary gamma-phase alloy. This alloy in a zirconium-clad parallel-plate design was carried through the final detailed analysis. As one of the fuel systems recommended for study in Phase II, it is discussed in detail in a later section.

Ceramics

Active consideration was given to UN, UC, and UO_2 as potential ceramic fuels. Both UN and UC appeared attractive because of their relatively high density and thermal-conductivity characteristics. However, they were abandoned from further consideration early in the study for the following reasons:

- (1) Available physical-property information was insufficient to permit a satisfactory design study and evaluation.
- (2) There were almost no radiation data available. Hence, the economic analysis which required a reference burnup fraction could not be made.
- (3) It was recognized that the reactive nature of these materials would lead to considerably higher fabrication costs than UO_2 or wrought alloy materials. Thus, further consideration of these materials would not be justified at the present time until radiation results indicated that high burnup was achievable to offset the increased fabrication costs.

It should be noted that these materials were passed over primarily from lack of information rather than from inherent deficiencies.

Uranium dioxide, on the other hand, received considerable attention through the entire study. Several forms of UO_2 fuel elements received design and economic evaluation:

- (1) Stainless steel pins filled with UO_2 particles and sodium. This fuel form was considered attractive from the standpoint of fabrication cost and maintaining relatively low UO_2 temperatures. Experimental evidence from other programs indicated a maximum

of about 65 volume per cent of UO_2 could be achievable with proper sizing of the UO_2 particles. However, the tendency for the UO_2 particles to sinter and agglomerate with a consequent change in the uranium distribution within the core was an unknown factor which did not lend itself to analysis. A preliminary economic analysis indicated no improvement of a 65 per cent dense paste over 90 per cent dense solid UO_2 fuel designs. Densities below 65 per cent were less attractive economically. For these reasons, the final effort on UO_2 fuel was devoted to 90 per cent dense bodies.

- (2) Stainless steel pins filled with UO_2 particles and helium gas. This fuel design received initial consideration as a more economical form of UO_2 fuel than (1) above. However, the relatively low effective thermal conductivity of this system required prohibitively small pins with an extremely large number of pins per subassembly.
- (3) Stainless steel pins containing 90 per cent dense pellets of UO_2 -sodium filled. This fuel design survived to the final evaluation, and is discussed in detail in the next section.
- (4) Plates of 90 per cent dense UO_2 in a stainless steel radiator-type subassembly-sodium filled. This fuel design also survived to the final evaluation and is discussed in detail in the next section.

Cermet or Dispersion Fuels

As discussed previously, two cermet-type fuels were considered to be potential fuels for this reactor system:

- (1) UO_2 dispersed in stainless steel. A parallel-plate design using stainless steel-clad UO_2 dispersion in stainless steel fuel was made which met all of the engineering and physics ground rules save one. Owing to the limited amount of uranium which could be introduced into the core in this form, the calculated conversion ratio of this design was only 0.94. The increase in UO_2 which might be required to obtain a net conversion ratio of one or greater appeared to result in a UO_2 -stainless steel ratio too high to take advantage of the burnup fractions achievable for lower UO_2 -content cermets. For these reasons, further consideration of this fuel system was abandoned.
- (2) UO_2 dispersed in uranium-10 w/o molybdenum or ternary gamma-phase alloy. This fuel in a zirconium-clad parallel-plate design survived through the final detailed analysis. As one of the fuel systems recommended for study in Phase II, it is discussed in detail in a later section.

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Fuel Systems Carried to the Final Evaluation

Four fuel-subassembly designs were carried through to the final evaluation. These were:

- (1) Zirconium-clad zirconium-10 w/o molybdenum alloy parallel plates
- (2) Zirconium-clad, 30 volume per cent UO_2 dispersion in uranium-10 w/o molybdenum parallel plates
- (3) 90 per cent dense UO_2 pellets in stainless steel tubes, sodium bonded
- (4) 90 per cent dense UO_2 plates in a stainless steel radiator-type sub-assembly, sodium filled.

Subassembly configurations and dimensions which met the engineering and physics ground rules are summarized in Figures 11, 12, and 13 and Tables 7 and 8.

The plant economics for these systems was compared at a core power level of 616 megawatts as being close to the minimum power level for economic operation under the ground rules.

A summary of the performance and the consequent economics of these systems is given in Table 9.

It is evident from this table that no one system is significantly more favorable than the others from the standpoint of economics. Furthermore, since the revenue deficiency or surplus amounts to the difference between two relatively large amounts (total revenue less total costs) the uncertainties in the estimates making up these totals, e. g. , burnup fraction which is actually achievable, reprocessing unit costs, etc., could easily have a greater effect than the revenue deficiencies or surplus indicated. Accordingly, the selection of the fuel system to be recommended for further development for Core II was based primarily on the technical feasibility of achieving the required performance. Five criteria were considered in arriving at the selection:

- (1) Ability to provide over-all satisfactory performance
- (2) Ability to meet or exceed the burnup fractions required
- (3) Degree of inherent reactor safety
- (4) Likelihood of low-cost fabrication production
- (5) Long-range potential for fast reactor systems.

None of these criteria can be completely evaluated quantitatively; a combined opinion based on past experience with other fuel systems was applied to arrive at the final selection. In doing this the uncertainties in the designs and assumed performance and the magnitude and likelihood of solving the development problems were stressed. A summary of these factors is as follows:

TABLE 7. DEFINITION OF SYMBOLS NOT SHOWN IN FIGURE 11, 12, OR 13

Symbol	Definition and Units
L	Length of fuel in plate or pin (30.5 in.)
n	Number of fuel plates or pins per subassembly (not including end plates)
Q_t	Core power (excluding blanket power) (megawatts)
t_i	Coolant inlet temperature (550 F for alloy and cermet; 650 F for UO_2)
t_{max}	Maximum temperature (F)
$\overline{\Delta t}$	Mixed-mean coolant temperature rise in core (350 F)

TABLE 8. RESULTS OF FINAL DESIGN ENGINEERING

Description	System	Q _T , megawatts	Fuel	Cladding	n	b, in.	j, in.	d, in.	m, in.
			T _{max} , F	T _{max} , F					
Alloy plates (see Figure 1), uranium-10 w/o molybdenum	B2-2	616	1298	-	19	.059	-	-	.006
						.055			.004
	B2-2A	770	1298	-	20	.066	-	-	.006
						.062			.004
	B2-5	924	1292	-	21	.071	-	-	.006
						.067			.004
Cermet plates (see Figure 1), 30 volume per cent UO ₂ in uranium-10 w/o molybdenum	L1-2	616	1291	-	23	.053	-	-	.006
						.049			.004
Ceramic plates ^(b) (see Figure 2), 90 per cent dense UO ₂	D2-3	616	3516	1259	17	.074	.046	-	.008
						.070	.042		.006
Ceramic pins (see Figure 3), 90 per cent dense UO ₂ pellets	G2-6	616	3511	1256	529	.109	.055	.082	.006
						.105	.051	.080	.004

(a) Estimated.

(b) Estimated; accounts for burnup.

(c) Leaks fission gas to coolant.

CALCULATIONS FOR APDA CORE II

D, in.	w ₁ , in.	w ₂ , in.	Plate(a) Uranium-238/ Uranium-235 Ratio	Uranium-238/ Uranium-235 Ratio(a) in UO ₂	Uranium-238/ Uranium-235 Ratio(a) in Uranium-10 w/o Molybdenum	Initial(b) Uranium-235 Loading, kg
.066 .064	-	-	4.12	-	4.12	660
.053 .051	-	-	3.54	-	3.54	625
.042 .040	-	-	2.89	-	2.89	600
.050 .048	-	-	-	.086	22.80	621
.053 .051	.088 .084	.060 .056	1.50	1.50	-	516
.068 .066	-	-	1.22	1.22	-	505

FIGURE 11. SKETCH OF PRESSURE-BONDED ALLOY OR CERMET SUBASSEMBLY WITH LIVE END PLATES

FIGURE 12. SKETCH OF RADIATOR-TYPE SUBASSEMBLY WITH SODIUM-BONDED CERAMIC PLATES

FIGURE 13. SKETCH OF PIN-TYPE-SUBASSEMBLY WITH SODIUM-BONDED CERAMIC PLATES

TABLE 9. PLANT ECONOMICS FOR THE VARIOUS FUEL SYSTEMS

616-Megawatt Maximum Core Power

	Alloy Systems		Oxide Systems	
	Wrought	Cermet	Pin	Plate
Fuel Required, kg	660	621	505	516
Critical Mass, kg	626	576	462	470
Conversion Ratio				
Core	0.42	0.414	0.16	0.18
Blanket	0.72	0.712	0.905	0.90
Fuel Atoms per Uranium-235 Atom	6.54	7.16	6.65	7.50
Initial Enrichment, a/o	0.195	{ 0.0445-Mo 0.913-O ₂	0.45	0.40
Grams Uranium-235 Destroyed per MWD	1.32	1.325	1.448	1.448
Core Life, days	180.5	253	141.6	163.5
Temperature, F				
Inlet, Sodium	550	550	650	650
Outlet, Sodium	900	900	1000	1000
Steam	800	800	900	900
Maximum Fuel	1300	1300	3500	3500
Burnup, a/o				
Maximum	0.03	{ 0.08-O ₂ 0.015-Mo	0.03	0.03
Average	0.0247	{ 0.066-O ₂ 0.01236-Mo	0.0251	0.0251
Fuel Thickness, in.	0.065	0.049	0.067	0.052
Number of Plates or Pins	19 + 2 half	23 + 2 half	529	17
Blanket/Total Power Ratio	0.10	0.10	0.135	0.135
Subassembly Cost, dollars	2660	3510	3680	3120
Costs, dollars				
Fixed	19,000	19,000	19,000	19,000
New Fuel	11,650	11,370	13,030	12,800
Core Cycle	4,960	5,640	7,120	7,040
Fabrication	2,050	1,930	3,610	2,570
Blanket Cycle	6,050	6,000	7,620	7,590
Inventory	8,780	6,490	7,870	7,050
Total	52,490	50,430	58,250	56,050
Revenue, dollars				
Plutonium	32,040	31,700	33,470	33,800
Steam	20,180	20,180	23,200	23,200
Total	52,220	51,880	56,670	57,000
Revenue Deficiency for 6 Per Cent Return	+270	-1,450	+1,580	-950

Uranium-10 w/o Molybdenum or Ternary
Gamma-Phase Systems (Including Both
Wrought and UO₂ Dispersed Fuel Systems)

- (1) Over-all performance. The major uncertainty is considered to be the effect of the stresses and strains in the zirconium cladding. This problem appears to be solvable by straightforward methods.
- (2) Achievable burnup fractions. The burnup fractions assumed for both the wrought and cermet designs have yet to be proven in irradiation experiments. It is believed that this required burnup is achievable through reasonable development.
- (3) Reactor safety. The safety of the alloy system is believed to be essentially equivalent to Core I.
- (4) Low-cost fabrication. The ability to achieve low-cost fabrication of these subassemblies is based on the use of the pressure-bonding technique, and on the use of reasonably streamlined melting procedures for alloy preparation. In addition, the economic feasibility of the cermet fuel depends upon developing low-cost alloy powder. There is every reason to believe these developments can be achieved.
- (5) Long-range potential. It is believed that the primary potential of the metal-type system, either wrought or dispersion fuels, lies in the increased conversion ratio possible. Of possibly greater importance in the case of dispersion fuel is the potential of extremely high burnup fraction of the uranium-235.

UO₂ Systems (Including Both Pin
and Plate Configurations)

- (1) Over-all performance. The uncertainties in over-all performance lie in the behavior of the UO₂ bodies under irradiation and high heat flux. It is certain that the bodies will fragment from thermal stress. Whether this gives rise to local increased temperatures, general shifting of the fuel within the elements, and an increased release of fission gas from the UO₂ crystals is not known. These effects do not lend themselves to analysis.
- (2) Achievable burnup fractions. Based upon radiation tests at lower burnup fractions, it was assumed that the only important radiation effects of UO₂ in this application were in the release of fission gas from the UO₂ itself. Thus, the pin configuration was designed as a pressure container to hold in the estimated release of fission gas. This was estimated to be 1000 psi with 20 per cent release of fission gas. The plate configuration required that the fission gas be released to the main coolant stream. The major uncertainty in the achievable burnup lies in the ability to accommodate the

fission gas released from the UO_2 in a practical and economical matter.

- (3) Reactor safety. There appear to be two prime uncertainties relative to reactor safety with UO_2 fuel. The first is the effect, if any, of any change in distribution of the fuel within the core owing to fragmentation of the UO_2 and its consequent effects. The second is that the maximum temperature in the fuel reaches the melting point of UO_2 at 50 per cent over the design power level rather than 70 per cent in the case of Core I and the metal system. The importance of this could not be readily determined. It should be noted that designs which permitted a greater factor of safety in this respect would not be as favorable economically.
- (4) Low-cost fabrication. The reference fabrication methods for the UO_2 systems are given in Appendix D. It was assumed that the required tolerances of the UO_2 bodies could be achieved without a machining operation. This assumption is considered reasonable but has not been achieved hitherto. If machining were required the fabrication cost could increase by almost an order of magnitude.
- (5) Long-range potential. The primary potential of UO_2 ceramic fuel is believed to rest with the expectation that significantly lower reprocessing costs are inherently possible with ceramic fuels. Whether or not this potential is ultimately realized depends upon the relative development of reprocessing techniques for metal and ceramic fuels. It should be noted that this potential was not taken advantage of in the present study.

The potential for a high burnup fraction of uranium-235 is also present in the UO_2 systems. However, it is contingent upon containment or dissipation of the fission gases outside the UO_2 material.

After consideration of these factors in detail together with the over-all goals of the Core II program it was concluded that the uranium-10 w/o molybdenum fuel systems held the most promise for meeting the objectives of the program. If the dispersion fuel can be satisfactorily developed, it holds by far the most long-range promise. If the dispersion fuel is not capable of its anticipated performance, the wrought fuel offers the most promise of satisfactory performance, both economic and technical, with the least extrapolation over present technology.

The reference designs and fabrication methods are discussed and evaluated in detail in the next section.

DETAILED EVALUATION OF THE RECOMMENDED FUEL SYSTEM

General Description

The reference subassembly design is an integral assembly composed of parallel zirconium-clad fuel plates equally spaced. Sodium coolant flows between the plates. The plates extend the full width and length of the core portion of the subassembly. They are held at the sides by integral attachment to zirconium side plates. The end fuel plates are half fueled. Zirconium reinforcing spacers run the length of the coolant channels. A cross section of the subassembly is shown in Figure 14. Dimensions for the subassembly for either wrought or dispersion fuel are given in Table 10.

The end fittings and axial-blanket portions of the subassembly are identical to the PRDC Core I design.

The Reference Method of Fabrication

Reference Process for Uranium-10 w/o Molybdenum Wrought-Alloy Fuel

The gas pressure-bonding process has been selected as the reference fabrication process for the manufacture of flat-plate uranium-10 w/o molybdenum subassemblies. In this technique, all of the components are fabricated to final size prior to bonding. They are then cleaned, assembled, sealed in a vacuum can, and subjected to gas pressure at an elevated temperature. Excellent channel and dimensional control is obtained. The process is potentially much more economic than the currently used roll-bonding and assembly-welding process as the core material is clad and the components are bonded to form an assembly in one operation.

Melting Operation. The core alloy will be prepared by a single vacuum-induction melting technique. Zirconia crucibles will be used and slab-type ingots will be cast. It is anticipated that after approximately 1 year of development ingots weighing 150 lb will be prepared.

A longer range objective is to prepare ingots weighing about 300 lb. The ultimate maximum ingot weight would of course be governed by critical-mass considerations. Development of multiple-casting techniques for the pouring of several ingots from a large melt is also a long-range objective. The larger melt and multiple-casting process would be reflected in a higher over-all melting yield and lower cost per pound of usable alloy.

Core-Alloy Fabrication. The rectangular slab-type ingots will be hot rolled directly without a forging operation. A salt bath will be employed to minimize surface oxidation and decrease the reheat time between roll passes. The ingot will be hot rolled to an approximate 0.100-in. sheet. It will then be surface conditioned, cold rolled, and again surface conditioned to final thickness. The finished strip will be

FIGURE 14. SKETCH OF REFERENCE SUBASSEMBLY

TABLE 10. DESCRIPTION OF THE REFERENCE DESIGNS
FOR ALLOY AND CERMET FUELS

	System Designation	
	A	B
Fuel Material	U-10 w/o Mo	30 volume per cent UO ₂ in U-10 w/o Mo
Number of Regular Plates (n)	19	23
Number of End Plates	2	2
Fuel Thickness in Regular Plate (a), in.	0.065	0.049
Fuel Thickness in End Plate $\left(\frac{a}{2}\right)$, in.	0.032	0.024
Cladding Thickness (m), in.	0.005	0.005
Total Thickness of Regular Plate (a + 2m), in.	0.075	0.059
Total Thickness of End Plate $\left(\frac{a}{2} + 2m\right)$, in.	0.042	0.034
Fuel Length in Plate (L), in.	30.5	30.5
Over-All Length of Plate (L _p), in.	31-3/16	31-3/16
Space Between Plates (d), in.	0.057	0.051
Subassembly Pitch, in.	2.693	2.693
Over-All Subassembly Length Including Blanket, in.	98.625	98.625
Core Composition in Fuel Section of Subassembly, volume per cent:		
Uranium as UO ₂	--	6.09
Uranium as Uranium-10 w/o Molybdenum	36.56	23.15
Oxygen as UO ₂	--	5.85
Molybdenum as Uranium-10 w/o Molybdenum	7.43	4.71
Zirconium	18.04	19.68
Sodium	37.97	40.52

straightened by roller leveling, sheared, and edged machined to final dimensions preparatory to pressure bonding.

Pressure Bonding. As-rolled zirconium and Ti-Namel strip will be utilized in the pressure-bonding process. The strip material purchased in the as-cold rolled condition to final-thickness dimension will be sheared and blanked to the desired size.

All of the components will be vapor degreased, washed, and assembled in a jig. The assembly will be enclosed in a light stainless steel container which will be evacuated and sealed. The sealed assembly will then be subjected to a helium pressure of 10,000 psi at an elevated temperature.

The components will be assembled as shown in Figure 15. After pressure bonding, the stainless envelope will be stripped and the Ti-Namel strips will be pickled preferentially to form channels in the assembly. Ribs will be incorporated into the assembly by use of spaced zirconium strips in the Ti-Namel plate. During pressure bonding, the ribs will form a metallurgical bond to the two adjacent fuel-element-cladding surfaces.

Nondestructive Testing. An electrical method, based on the principle of the potentiometer, will be used for nondestructively testing for bond integrity. This method is based on the measurement of potential difference between a pair of probes when a constant current is flowing through another pair of probes which are very close to the potential probes. The feasibility of this technique has been demonstrated for use in other Zircaloy cladding systems and an automatic bond-testing machine was built that could inspect an entire assembly.

Cost Estimates. A fabrication flowsheet for the flat-plate uranium-10 w/o molybdenum fuel system is presented in Figure 16. Fabrication and process costs for the entire operation have been compiled in Table 11. The estimates for labor, overhead, general administration, and profit are based on the experienced operation of a typical manufacturing plant producing a product of comparable quality and cost. Special handling because of criticality, health safety, and accountability have been considered in the cost estimates. The fabrication cost of the axial-blanket portion of the sub-assemblies has not been included. However, this cost, estimated by APDA to be about \$755, was included in the total economic analysis.

256 357

FIGURE 15. ASSEMBLY OF COMPONENTS IN THE PRESSURE BONDING PROCESS

FIGURE 16. FABRICATION FLOWSHEET FOR FLAT-PLATE URANIUM-10 w/o MOLYBDENUM SYSTEM
BATTELLE MEMORIAL INSTITUTE

TABLE 11. COST ESTIMATES FOR URANIUM-10 w/o
MOLYBDENUM FLAT-PLATE ASSEMBLY

<u>Fabrication - Fuel Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Thickness	0.065
Core Width	2.449
Core Length	30.5
Number of Plates per Assembly	19 and 2 end plates
Number of Assemblies per Core	139
Uranium-235 Enrichment, a/o	19.53
Total Weight of Alloy, lb	8,373
Total Weight of Uranium, lb	7,536
Total Weight of Molybdenum, lb	837
Fabrication Yield Without Recycle of Remeltable Material, per cent	67.0
Over-All Yield With Recycle of Remeltable Material, per cent	82.8
Total Alloy Required Without Recycle, lb	12,500
Total Alloy Required-Recycled, lb	10,112
Total Uranium Required-Recycled, lb	9,101
Total Molybdenum Required-Recycled, lb	1,011

Fabrication and Material Costs

Material Cost, dollars per assembly	
Molybdenum Metal	51.00
Materials Used in Melting; Crucibles, Stoppers, etc. (\$0.67 per lb Alloy)	60.00
Ti-Namel Steel Spacers	5.00
Zirconium Cover Plates	110.00
Zirconium Channel Strips	25.00
Zirconium Picture Frames	450.00
Materials Used in Pressure Bonding; Stainless Sheet, End Spacers, etc.	20.00
Materials Used in Cleaning of Components; Ether, Pickle Solutions, etc.	10.00
Total	731.00

Fabrication Cost, man-hours per assembly	
Melting of Alloy and Analytical Analysis (0.17 Man-Hours per Lb for Virgin Metal)	10.2
(0.21 Man-Hours per Lb for Recycled Metal)	6.3
Shearing, Edge Machining, Cleaning, and Assembly of Ti-Namel Spacers	8.5
Shearing, Pickling, Cleaning, and Assembly of Zirconium Cover Plates	4.0
Shearing, Pickling, Cleaning, and Assembly of Zirconium Channel Strips	4.0

TABLE 11. (Continued)

<u>Fabrication and Material Costs</u> (Continued)	
Fabrication Cost, man-hours per assembly (continued)	
Shearing, Machining, Cleaning, and Assembly of Zirconium Picture Frames	7.5
Fabrication of Uranium Alloy to Sheet: Hot Rolling, Cold Rolling, Roller Leveling, Shearing, Edge Machining, Cleaning, and Assembly	27.0
Pressure Bonding: Preparation of Stainless Steel Can, Assembly, Evacuation, and Decanning	24.5
Final Machining, Cleanup, and Inspection	12.0
Nondestructive Testing	8.0
Joining of Assembly to Blanket Structure	<u>10.0</u>
Total	124.0
<u>Estimated Total Costs</u>	
Total Cost, dollars per assembly	
Direct Materials	731.00
Direct Labor (\$2.20 per Hr)	272.80
Overhead (250 Per Cent)	<u>683.00</u>
Total	1,686.80
General Administration (3 Per Cent)	50.60
Profit (10 Per Cent)	<u>168.68</u>
Grand Total	1,906.08

Reference Process for 30 Volume Per Cent
UO₂-(Uranium-10 w/o Molybdenum) Fuel

The reference fabrication process selected for the flat-plate subassemblies using UO₂ dispersion fuel can be broken down into three major operations: the preparation of the uranium-10 w/o molybdenum alloy powder, the fabrication of the alloy powder-UO₂ mixtures into a high-density core strip, and the pressure bonding of this strip and other components into a flat-plate subassembly. Except for alloy-powder preparation and core-strip fabrication, all the operations are identical to the wrought-alloy fuel system described above.

Alloy-Powder Preparation. The alloy powder will be prepared by a shot-casting operation. After 1 year of development, 50-lb uranium-10 w/o molybdenum ingots, prealloyed by a consumable-electrode arc-melting operation, will be induction melted in a ceramic-washed graphite crucible and bottom poured on a rapidly rotating ceramic disk. The shotting operation will be accomplished in an inert atmosphere and the powder product collected and screened.

The objectives of further development would be the omission of the prealloying arc-melting step and the use of zirconium crucibles that could be fitted with appropriate and reliable pouring nozzles. It would also be desirable to increase the size of the shotting operation to the 200 to 300-lb range with the accompanying decrease in cost per pound of powder.

Core-Strip Fabrication. The uranium-10 w/o molybdenum alloy powder and UO₂ powder will be weighed and blended in the proper amounts and charged into a hopper over a set of rolls for continuous compacting into core strip. Densities of 80 to 85 per cent of theoretical are expected from this operation. The rolled strip is annealed or sintered in vacuum and cold rolled to the final thickness and anticipated density of 95 per cent of theoretical.

The finished strip will be straightened by roller leveling, sheared, and edged machined to final dimensions preparatory to pressure bonding.

Cost Estimates. A fabrication flowsheet for the reference flat-plate 30 volume per cent UO₂-(uranium-10 w/o molybdenum) fuel system is presented in Figure 17. Fabrication and materials costs for the entire operation have been compiled in Table 12. The estimates for labor, overhead, general administration, and profit are based on the experienced operation of a typical manufacturing plant producing a product of comparable quality and cost. Special handling because of criticality, health, safety, and accountability have been considered in the cost estimates. The fabrication costs of the axial-blanket portion of the subassemblies have not been included.

FIGURE 17. FABRICATION FLOWSHEET FOR FLAT-PLATE 30 VOLUME PER CENT UO_2 URANIUM-10 w/o MOLYBDENUM SYSTEM

TABLE 12. COST ESTIMATES FOR 30 VOLUME PER CENT UO_2 -(URANIUM-10 w/o MOLYBDENUM) FLAT-PLATE ASSEMBLY

<u>Fabrication - Fuel Material Breakdown</u>	
	<u>System B</u>
Dimensions of Fuel Core, in.	
Core Thickness	0.049
Core Width	2.449
Core Length	30.5
Number of Plates per Assembly	23 and 2 end plates
Number of Assemblies per Core	139
Uranium-235 Enrichment, per cent	
UO_2	91.3
Uranium-10 w/o Molybdenum Alloy	4.45
Total Weight of Alloy in Core, lb	5,310
Total Weight of Uranium, lb	4,780
Total Weight of Molybdenum, lb	530
Total Weight of UO_2 in Core, lb	1,431
Total Weight of Uranium in UO_2 , lb	1,262
Over-All Yield With Recycle of Material, per cent	
Alloy	57.0
UO_2	88.3
Total Material Required With Recycle, lb	
Total Alloy	9,320
Total Uranium	8,390
Total Molybdenum	930
Total UO_2	1,620
Total Uranium	1,430
Fabrication Yield of Alloy Without Recycle of Remeltable	
Material, per cent	50.5
Total Alloy Required Without Recycle of Remeltable Material, lb	10,500
<u>Material Costs</u>	
Material Costs per Assembly, dollars	
Molybdenum	50.00
Materials Used in Melting; Crucibles, Stoppers, etc.	210.00
Ti-Namel Steel Spacers	5.00
Zirconium Cover Plates	115.00
Zirconium Channel Strips	25.00
Zirconium Picture Frames	410.00
Materials Used in Cleaning of Components	10.00
Materials Used in Pressure Bonding Stainless Sheet,	20.00
End Spacers, etc.	
Total	845.00

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TABLE 12. (Continued)

<u>Fabrication Costs</u>	
	<u>System B</u>
Fabrication Time, man-hours per assembly	
Arc Melting of Alloy, Induction Melting, Shotting, and Analysis (1.26 Man-Hours per Lb)	101.5
Alloy Powder, Sizing, Weighing, and Blending With UO_2	24.0
Fabrication of Powder to Strip, Vacuum Anneal, Cold Rolling, Roller Leveling, Edge Machining, and Cleaning	6.0
Shearing, Pickling, Cleaning, and Assembly of Ti-Namel Spacers, Zirconium Cover Plates, Channel Strips, and Picture Frames	24.5
Pressure Bonding; Preparation of Stainless Steel Can, Assembly, Evacuation, and Decanning	24.5
Final Machining, Cleanup, and Inspection	12.0
Nondestructive Testing	8.0
Joining of Assembly to Blanket Structure	<u>10.0</u>
Total	206.5
<u>Total Costs</u>	
Total Costs per Assembly, dollars	
Direct Materials	845.00
Direct Labor (\$2.20 per Hr)	454.30
Overhead (250 Per Cent)	<u>1,135.70</u>
Total	2,435.00
General Administration (3 Per Cent)	73.05
Profit (10 Per Cent)	<u>243.50</u>
Grand Total	2,751.55

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Performance Data for the Reference Fuel System

The assumptions described earlier and the equations derived in Appendix C were used in the design of the reference fuel system with three different fuel materials; these systems are described in Table 10. Performance data are given in sections that follow.

Heat Transfer and Fluid Flow

Heat-transfer and fluid-flow performance data for the reference systems described in Table 10 are given in Table 13. The assumed volumetric heat-generation-rate distribution function used in temperature calculations for all the alloy systems is shown in Figure 18, where it is compared with the results of a physics calculation for a uranium-10 w/o molybdenum plate system.

An initial calculation was made for each system using a set of estimated hot-channel factors; the dimensions and physics results of the first calculation were then used to derive improved hot-channel factors for use in the final calculation. The hot-channel factors derived for final calculation of each system are given in Table 14. The method of obtaining these factors is described in Appendix C.

Temperature distributions in the systems are shown in Figures 19 and 20. The temperature limitations set for each system allow a slow transient to at least 1.7 times design power without melting of the cladding or core if the coolant inlet temperature is maintained at its design value.

Table 15 gives the physical properties of the various materials at their approximate average operating temperatures.

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TABLE 13. HEAT-TRANSFER AND FLUID-FLOW PERFORMANCE DATA FOR THE REFERENCE DESIGNS WITH ALLOY AND CERMET FUELS

	System ^(a)	
	A	B
Fuel Material	U-10 w/o Mo	30 volume per cent UO ₂ in U-10 w/o Mo
Power, megawatts		
Total Reactor	684	684
Core	616	616
Blanket	68	68
Number of Fuel Subassemblies	139	139
Core Surface Area, ft ²	2890	3468
Heat Flux, 10 ⁶ Btu/(hr)(ft ²)		
Average at Core Surface	0.727	0.606
Maximum at Core Surface	1.115	0.930
Maximum-to-Average at Core Surface	1.534	1.534
Total Coolant Flow Through Fuel Subassemblies Only, 10 ⁶ lb per hr	20	20
Average Coolant Velocity Between Fuel Plates, ft per sec	40.6	37.8
Coolant Pressure Drop Across Fuel Section of Subassembly, psi	65	65
Temperature, F		
Coolant Inlet	550	550
Mixed-Mean for Coolant Outlet From Core	900	900
Mixed-Mean for Coolant Outlet From Hottest Subassembly ^(b)	1191	1191
Maximum for Coolant in Hottest Subassembly ^(b)	1209	1207
Maximum for Cladding in Hottest Subassembly ^(b)	1231	1227
Maximum for Core in Hottest Subassembly ^(b)	1298	1291

(a) System designation letters are same as those used in descriptions given in Table 10.

(b) Includes hot-channel factors.

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FIGURE 18. VOLUMETRIC HEAT-GENERATION-RATE DISTRIBUTION IN HOTTEST SUBASSEMBLY, BASED ON MACHINE PHYSICS CALCULATIONS FOR URANIUM-10 w/o MOLYBDENUM PLATE SYSTEM

TABLE 14. HOT-CHANNEL FACTORS USED IN TEMPERATURE CALCULATIONS
FOR SYSTEMS A AND B

Abnormal Heat-Transfer Condition	Value for Indicated Hot-Channel Factor							
	System A				System B			
	Δt	Δt_f	Δt_c	Δt_m	Δt	Δt_f	Δt_c	Δt_m
Plate spacing maximized or minimized (± 0.002 in.)	1.06	1.01	1.00	1.00	1.06	1.01	1.00	1.00
Core thickness maximized (± 0.002 in.)	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04
Cladding thickness maximized one side, minimized other side (± 0.001 in.)	1.02	1.02	1.02	1.05	1.03	1.03	1.03	1.07
Maldistribution of coolant, external causes	1.03	1.00	1.00	1.00	1.03	1.00	1.00	1.00
Inaccuracy of power measurement	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
Inaccuracy of thermal-conductivity values	1.02	1.02	1.05	1.10	1.02	1.02	1.05	1.10
Inaccuracy of film-coefficient value	1.00	1.30	1.00	1.00	1.00	1.30	1.00	1.00
Product I	1.33	1.60	1.26	1.36	1.36	1.64	1.28	1.40
Closing off of coolant channels due to radiation damage	1.16	1.00	1.00	1.00	1.06	1.00	1.00	1.00
Excess heat generation in new subassembly	1.07	1.07	1.07	1.07	1.16	1.16	1.16	1.16
Product II	1.24	1.07	1.07	1.07	1.23	1.16	1.16	1.16
Applied hot-channel factor (Product II $\times \frac{1+\text{Product I}}{2}$)	1.45	1.39	1.21	1.26	1.45	1.53	1.32	1.39

239
070

FIGURE 19. TEMPERATURE DISTRIBUTIONS IN HOTTEST SUBASSEMBLY, INCLUDING HOT-CHANNEL FACTORS

System A (uranium-10 w/o molybdenum plate system, zirconium clad, live end plates).
 $Q_t=684$ megawatts; $Q_c=616$ megawatts; $t_i=550$ F; $\Delta t=350$ F; $\Delta p_{core}=65$ psi.

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140 071

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FIGURE 20. TEMPERATURE DISTRIBUTIONS IN HOTTEST SUBASSEMBLY, INCLUDING HOT-CHANNEL FACTORS

System B (cermet plate system, 30 volume per cent UO_2 in uranium-10 w/o molybdenum, zirconium clad, live end plates).

$Q_t=624$ megawatts; $Q_c=616$ megawatts; $t_i=550$ F; $\bar{\Delta t}=350$ F; $\Delta p_{\text{core}}=65$ psi.

TABLE 15. PHYSICAL PROPERTIES OF MATERIALS AT APPROXIMATE
AVERAGE OPERATING TEMPERATURES

Material	Temperature, F	k, Btu/(hr)(ft)(F)	ρ , lb per ft ³	μ , lb/(hr)(ft)	C_P , Btu/(lb)(F)	Source of Data
Sodium	800	39.2	53.2	0.66	0.305	TID-5277
Uranium-10 w/o molybdenum		15				Battelle data
30 volume per cent UO ₂ in uranium-10 w/o molybdenum		10.5(a)				--
Zirconium	1000	9.7				BMI-65

(a) Calculated; assumes no contribution from UO₂.

Physics Analysis

On the basis of both extrapolations, utilizing the method outlined in a previous section, and various "touch-down" points supplied by machine calculations performed by APDA, the following values are estimated for the reference-alloy plate-type systems.

	System A, U-10 w/o Mo	System B, 30 volume per cent UO ₂ in U-10 w/o Mo
Critical Mass, kg	626	576
Initial Fuel Loading, kg	660	621
Enrichments		
UO ₂	--	0.913
Uranium-10 w/o molybdenum alloy	0.195	0.0445
Conversion Ratios		
Core	0.42	0.414
Blanket	0.72	0.712
Total	1.14	1.126
Fuel-Utilization Factors		
Uranium-235 destroyed per atom fissioned in alloy	1.10	0.7
Uranium-235 destroyed per atom fissioned in UO ₂		1.23
Uranium-235 destroyed per atom fissioned in system	1.10	1.10

The analysis of the wrought-alloy plate system was completely straightforward. The cermet system requires some explanation. The condition imposed in this system is that the oxide fuel and the uranium-molybdenum matrix-fuel reach prescribed burn-up fractions simultaneously. This is accomplished provided the following relations are satisfied:

$$B_m (\text{U - 10 w/o Mo matrix}) = \left(\frac{N_{25}\bar{\sigma}_{25}\phi + N_{28}\bar{\sigma}_{28}\phi}{N_{25} + N_{28} + N_{\text{Mo}}} \right)_{\text{U-10 Mo}} = \frac{\bar{\sigma}_{25}\phi + (U_m - 1)\bar{\sigma}_{28}\phi}{J_m},$$

$$B_o (\text{UO}_2 \text{ fuel}) = \left(\frac{N_{25}\sigma_{25}\phi + N_{28}\sigma_{28}\phi}{N_{25} + N_{28} + N_o} \right)_{\text{UO}_2} = \frac{\sigma_{25}\phi + (U_o - 1)\sigma_{28}\phi}{J_o}$$

where

N_{25} , N_{28} = atomic densities of uranium-235 and uranium-238, respectively

B_m , B_o = atomic per cent burnups in uranium-10 w/o molybdenum and oxide, respectively

σ_{25} , σ_{28} = average microscopic fission cross sections for uranium-235 and uranium-238, respectively

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U_m, U_o = reciprocal of enrichment in uranium-10 w/o molybdenum and UO_2 , respectively

J_m, J_o = ratios of total fuel atoms to uranium-235 atoms in uranium-10 w/o molybdenum and oxide phases, respectively.

These relations may be solved for the ratio

$$\frac{J_m}{J_o} = \left\{ \frac{\sigma_{25} + (U_o - 1)\sigma_{28}}{\sigma_{25} + (U_m - 1)\sigma_{28}} \right\} \frac{B_m}{B_o}.$$

The process at this point is iterative. For a given burnup ratio, an assumption is made regarding the enrichments in the uranium-10 w/o molybdenum and UO_2 ; this allows for an evaluation of the ratio of J factors. In turn, this ratio is solved simultaneously with the condition that the sum of the fuel contents in both materials be equal to the expected initial fuel loading in the elements. The solution here gives a value for the enrichments in uranium-10 w/o molybdenum and UO_2 that may be compared with the assumed values. The process may then be repeated until the two sets of values agree. For these computations values of σ_{25} and σ_{28} obtained from APDA calculations were used.

Values for the fuel utilization in the slightly enriched uranium-10 w/o molybdenum matrix and the oxide were obtained using the following relation for both the uranium-10 w/o molybdenum and oxide.

$$\frac{\text{atoms uranium-235 destroyed}}{\text{atoms fissioned}} = \frac{(1 + \alpha) N_{25}\sigma_{25}}{N_{25}\sigma_{25} + N_{28}\sigma_{28}},$$

where

α = ratio of nonfission to fission cross section in the uranium-235.

The fuel utilization for the cermet fuel is then found as

$$\frac{\text{atoms } U^{235} \text{ destroyed}}{\text{atoms fissioned}} = \frac{(U^{235} \text{ destroyed})_{\text{oxide}} + (\text{atoms destroyed})_{U-10 Mo}}{(\text{atoms fissioned})_{\text{oxide}} + (\text{atoms fissioned})_{U-10 Mo}},$$

which is

$$\frac{U^{235} \text{ destroyed}}{\text{atoms fissioned}} = \frac{\left(\frac{U^{235} \text{ destroyed}}{\text{atoms fissioned}} \right)_{\text{oxide}} + \left(\frac{U^{235} \text{ destroyed}}{\text{atoms fissioned}} \right)_{U-10 Mo}}{1 + \left(\frac{\text{atoms fissioned}}{\text{atoms fissioned}} \right)_{U-10 Mo} / \left(\frac{\text{atoms fissioned}}{\text{atoms fissioned}} \right)_{\text{oxide}}}.$$

But these terms evaluate to give

$$\frac{U^{235} \text{ destroyed}}{\text{atoms fissioned}} = \frac{1.22 + 0.23}{1 + 0.32} = \frac{1.45}{1.32} = \frac{1.10}{1}.$$

A detailed analysis of the safety aspects of operating with the alloy Systems A or B was considered as being outside of the scope of the present work. However, since these systems are fundamentally similar to Core I it may be expected that a similar type of transient behavior should result. There is a problem peculiar to the cermet design. This is that a fully enriched fuel form is being used in the oxide. It is known

that slightly positive temperature coefficients of reactivity can be expected for fuel systems having an intimately mixed uranium-238/uranium-235 ratio somewhat less than unity (Doppler effect). All of the systems considered throughout the Core II program were examined from this standpoint. The question of intimate mixing was considered in the cermet systems. It was found that although the dispersed fuel particles are fully enriched, the heat-transfer rate for assumed transients of 0.1 and 1.0 sec is such that the matrix temperatures are comparable to the oxide fuel temperatures for all times. This would lead to the conclusion that the intimate-mixing requirement is approached and that an over-all uranium-238/uranium-235 ratio for the system rather than the oxide portion alone may control the temperature coefficient of reactivity. Furthermore, the prompt reactivity effect of thermal expansion of the fuel is believed to be many times larger than the Doppler effect and it is negative. This leads to at least the qualitative insurance of acceptable transient behavior for this system.

Strength Considerations

The thermal-stress distributions in the A and B reference systems were obtained according to the general method described in Appendix E and are presented in Figures 21 through 24. The values of the physical constants used in the analyses are listed in Table 16.

Discussion of Results. Comparison of the stresses plotted in Figures 21 through 24 with the yield strengths of the materials listed in Table 16 shows that the stresses in the cladding exceed the yield strengths over a large portion of the plates in each system. It is expected that the material will tend to deform plastically in these areas. The analysis is not able to predict the behavior beyond this point. For purposes of comparison the values of thermal stress were calculated at various points in the pin-type reference fuel element for Core I according to the general method already described. The results are tabulated below:

Element	Axial Position	Stress at Indicated Radial Position, 10 ³ psi			
		Center Line of Core	Core-Cladding Interface		Outer Surface Cladding
			Core	Cladding	
Pin-type Reference fuel Element Core 1	$\frac{Z}{L} = 0.8$	-17.6	14.4	23.9	27.6

Comparing these cladding stresses with those presented in Figures 21 through 24 indicates that they are of the same magnitude; from this it is inferred that the behavior of the cladding in the two instances will also be similar.

Bowing (in the transverse plane) of end and regular fuel plates as a result of differential expansion and asymmetric temperature and pressure distributions was examined in all the systems. The use of spacers as indicated in Figure 14 seems sufficient for controlling any bowing which may develop.

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FIGURE 21. THERMAL STRESS DISTRIBUTIONS IN REGULAR FUEL PLATES, SYSTEM A

Cross-sectional stress distribution at indicated Z/L .

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FIGURE 22. THERMAL STRESS DISTRIBUTIONS IN HALF-FUELED END PLATES, SYSTEM A

Cross-sectional stress distribution at indicated Z/L .

820 078 000

FIGURE 23. THERMAL STRESS DISTRIBUTIONS IN REGULAR FUEL PLATES, SYSTEM B

Cross-sectional stress distribution at indicated Z/L .

FIGURE 24. THERMAL STRESS DISTRIBUTIONS IN HALF-FUELED END PLATES, SYSTEM B

Cross-sectional stress distribution at indicated Z/L .

TABLE 16. PHYSICAL PROPERTIES OF MATERIALS AS USED IN THERMAL STRESS ANALYSIS

Material	Modulus of Elasticity, E, 10^6 psi	Linear Coefficient of Expansion, 10^{-6} per F	Poisson's Ratio, ν	Assumed Stress-Free Temperature ^(a) , F	Approximate Yield Strengths, psi	
Zirconium (sponge)	10.5	3.05	0.34	1022	30,000 at ~70 F	10,500 at 572 F
U-10 w/o Mo	12.5	8.44	0.20	1022	90,000 at 500 F	54,000 at 1100 F
30 volume per cent UO ₂ -(U-10 w/o Mo)	~12.5	~8.44	0.20	1022	~63,000 at 500 F	~38,000 at 1100 F

(a) Stress-free temperature of zirconium cladding was assumed equal to that of the core material so that the entire subassembly might be considered stress-free at this temperature.

The calculated values of the stress and the related maximum bowing deflections are conservative to the extent that they follow from the temperature distributions utilized. The latter were derived for the hottest subassembly and include the applied hot-channel factors. The values of stress and deflection are also highly dependent upon the stress-free temperature assumed. This value depends to a great extent upon the fabrication history of the assemblies in question. A study of Figures 21 through 24 indicates the effect on the stresses of the relative values between the stress-free temperature and the plate's temperature distribution.

The stress problems associated with growth accompanying burnup have yet to be solved. The difficulty lies in determining the amount of growth as a function of position in the plates. The magnitude of this effect cannot be accurately assessed until radiation-test results are available.

The Economic Analysis of the Reference Systems

For the reasons outlined in Appendix B, the economic equation cannot be applied to give an accurate quantitative picture of the power-plant economics. The deviation of the final numbers from the equation may be expected to be small for each of the six cost items and two revenue items. However, the final selection of an operating power for intercomparisons between systems was done on the basis that it be the minimum power for economic performance. This implies that the sum of cost and revenue terms will come near zero. Consequently, small errors in the relatively large terms can sum up to a large percentage error in the final revenue-deficiency term. The following methods are considered to be more accurate in giving the economic potential of the reference system.

The basic six cost terms and two revenue terms for the two systems will be outlined below:

Wrought-Alloy Plates, Fuel System A

- (1) Fixed costs per day = \$19,000.

This item has been increased from the corresponding Core I value in that a higher plant investment is required for a 684-megawatt power plant. The increase occurs in the 6 per cent return figure to stock and bond holders, in the Federal income tax, and in the amortization, insurance, etc. These increases were all taken to be proportioned to the ratio of new depreciable investment to the Core I value. The value of the depreciable investment at a power of 684 megawatts is obtained by a linear interpolation using 7×10^6 over the range from 430 to 900 megawatts.

- (2) Fuel-Replacement costs per day = \$11,650.

For both this item and the core-cycle costs which follow the flow diagram of Figure 25 is used. Only the fabrication portion of this diagram was developed by Battelle:

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FIGURE 25. FUEL-CYCLE FLOW DIAGRAM FOR WROUGHT ALLOY FUEL SYSTEM A

Note: Diagram shows throughputs and losses used in economic studies, based on M = 660 kg, enrichment = 0.195.

Kg of Uranium-235

(a) Fuel burned during the core life	117.4
(b) Fuel lost in the fabrication cycle	7.9
(c) Losses in reprocessing	5.4

The enrichments for Items (a) and (b) are 19.5 per cent. The enrichment for Item (c) is 16.9 per cent. Therefore, using unit costs at these enrichments, the total cost for new fuel per core cycled is $\$2.1 \times 10^6$. Based on a core life of 180.5 days, this gives a daily cost of \$11,650.

(3) Fuel-cycle costs per day = \$6,320.

The flow diagram (Figure 25) for the wrought-alloy system gives the items considered in the entire fuel-cycle costs. These are tabulated below on a per core throughput basis:

Flow-Diagram Item (Figure 25)	Basis	Cost per Core Throughput ^(a) , dollars
1	\$50 per kg uranium-235	39,600
2	\$200 per kg uranium-238	158,300
T-1	\$10 per kg uranium	40,600
3	\$2660 per subassembly	370,000
T-2	\$50 per subassembly	6,950
T-3	\$10 per kg uranium	6,750
4	\$250 per kg uranium-235	<u>33,000</u>
	Subtotal fabrication cycle	655,200 per core
T-4	\$250 per subassembly	34,800
10	\$102 per kg uranium	327,000
T-5	\$20 per kg uranium-235	10,840
12	\$35 per kg uranium	<u>112,000</u>
	Subtotal reprocessing cycle	484,640
	Grand total per core	1,139,840

(a) Losses are included in new fuel.

Then for a 180.5-day core life this amounts to \$6,320 per day.

(4) Core-plutonium recovery costs per day = \$690.

The plutonium lost from core subassemblies equals the core and estimated axial-blanket conversion ratio times the loss of fuel. This is about 634 g of plutonium. The cost is \$28,030 per core cycle.

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The cost for plutonium recovery is made up of the reduction step at \$1.50 per g of plutonium and a transportation charge of \$0.02 per g of plutonium applied to the total core and axial-blanket plutonium processed. There is about 6.35 kg of plutonium so the cost per core cycle is \$96,300.

Then the total cost per core cycle is \$124,330. Based on a 180.5-day core life this is about \$690 per day.

- (5) Radial-blanket-cycle costs per day = \$6,050.

The radial-blanket costs are exactly as outlined in Appendix B.

- (6) Fuel-inventory charges per day = \$8,780.

Again utilizing the core-cycle diagram, the dwell times of the fuel and fuel unit value can be inferred. These combine as follows:

	<u>10⁶ Dollars</u>
(a) Value of fuel in fabrication for 60 days at an enrichment of 19.5 per cent	4.25
(b) Value of fuel in the core for 180 days at an average enrichment of 18 per cent	9.75
(c) Value of fuel in the reprocessing cycle for 240 days at an enrichment of 16.9 per cent	<u>11.5</u>
Total fuel value in the system	25.5

Based on a 12.56 per cent yearly inventory charge, this is \$8,780 per day.

- (7) Plutonium revenue per day = -\$32,040.

This term is derived in exactly the way specified in Appendix B. A value of \$44.2 per g of plutonium is given to blanket plutonium and a value of \$41.5 per g of plutonium to the core material. This downgrades the core plutonium based on estimated plutonium-240 content (no exact calculations for core-plutonium quality were made).

- (8) Steam revenue per day = \$20,180.

This term is determined exactly as specified in Appendix B. A ratio of blanket power to total power of 10 per cent is used.

The final revenue-deficiency term is then calculated as the algebraic sum of the eight factors developed above. This amounts to about \$270 per day.

Cermet Plates, Fuel System B

- (1) Fixed costs per day = \$19,000.

The same reasons apply here as outlined above for the wrought-alloy system.

- (2) Fuel-replacement costs per day = \$11,370.

The core-cycle diagram for the cermet system is outlined in Figure 26.

Again only the fabrication portion of the diagram was developed by Battelle.

	<u>Amount, kg</u>
(a) Fuel consumed during the core life	
In oxide form	139.3
In uranium-10 w/o molybdenum form	25.5
(b) Fuel lost in fabrication during the core life	
In oxide(1/2 per cent processed basis)	2.96
In uranium-10 w/o molybdenum (1 per cent processed basis)	1.69
(c) Fuel lost in reprocessing during the core life	
In a mixed form of oxide and metal	4.5

The enrichment for the oxide material is taken to be 91.3% in Steps (a) and (b). The enrichment of the uranium-10 w/o molybdenum for Steps (a) and (b) is taken as 4.45%. In the reprocessing step a homogeneous mixture at an average enrichment of about 18% is assumed.

These assumptions give a new fuel cost for the complete core cycle of $\$2.88 \times 10^6$. Based on a core life of 253 days, this is \$11,370 per day.

- (3) Fuel-cycle costs per day = \$6,890.

The flow diagram (Figure 26) for the cermet system gives the items considered and the percentage throughputs. These are tabulated below.

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FIGURE 26. FUEL-CYCLE FLOW DIAGRAM FOR CERMET FUEL SYSTEM B

Note: Diagram shows throughputs and losses used in economic studies, based on $M_{U-10-Mo} = 96.8$ (enr. = 0.0445) and $M_{UO_2} = 524.2$ (enr. = 0.913).

Flow-Diagram Item (Figure 26)	Basis	Cost per Core Throughput, dollars
1	\$50 per kg uranium-235 (metal form)	8,480
2	\$836 per kg UO ₂ (fluoride to oxide)	615,000
	\$200 per kg uranium-235 (fluoride to metal)	33,920
T-1	\$10 per kg uranium	44,490
3	\$3506 per subassembly	487,500
T-2	\$50 per subassembly	6,950
T-3	\$10 per kg uranium	17,050
4	\$250 per kg uranium-235 (U-10 w/o Mo form)	18,150
	\$1500 per kg uranium-235 (oxide form)	102,000
	Subtotal, fabrication cycle	1,333,540
T-4	\$250 per subassembly	34,800
10	\$153 per kg uranium (in oxide form)	66,500
	\$102 per kg uranium (in U-10 w/o form)	213,000
T-5	\$20 per kg uranium-235	9,120
12	\$35 per kg uranium	88,000
	Subtotal, reprocessing cycle	411,420
	Grand total per core cycle	1,744,960

Based on a 253-day core life this amounts to \$6,890 per day.

- (4) Core plutonium-recovery costs per day = \$680.

The plutonium lost per core cycle is made up from the core elements and the axial-blanket elements. This is about 875 g. Based on a unit value of \$44.2 per g this amounts to a loss of \$38,700 per cycle.

Similarly, the recovery costs (Items 11 and T-6) amount to \$133,000 per cycle. Based on a 253-day core life this is a cost of \$680 per day.

- (5) Radial-blanket-cycle costs per day = \$6,000.

This value is derived by the method specified in Appendix B.

- (6) Fuel-inventory charges per day = \$6,490.

Using the flow diagram of Figure 26 and the throughput information contained therein, the fuel material and dwell times are set up to give daily costs as follows:

10⁶ Dollars

(a) In fabrication for 60 days

Uranium-10 w/o molybdenum at 4.45 per cent enrichment	0.54
---	------

UO ₂ at 9.13 per cent enrichment	2.38
---	------

(b) In core for core life

Uranium-10 w/o molybdenum at about 4.45 per cent enrichment	1.13
---	------

UO ₂ at about 92 per cent enrichment	7.75
---	------

(c) In the reprocessing cycle for 240 days it is considered that a homogeneous mixture of the materials is present. This is at an average enrichment of 18.10 per cent.

The worth of fuel is then	<u>7.06</u>
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Average total fuel value in the system	18.86
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Based on a 12.56 per cent yearly inventory charge this is \$6,490 per day.

(7) Plutonium income per day = \$31,700.

This term is derived in the same manner as for the wrought material.

(8) Steam income per day = \$20,180.

The term is exactly the same as the steam revenue for the wrought-alloy system.

The final revenue deficiency for this system is again the algebraic sum of the cost and revenue factors listed above. This amounts to \$1,450 per day.

Several possibilities present themselves which may lead to improvements in the revenue-deficiency term. Considering the wrought-alloy system first, improvement can be obtained by operating a slightly higher power. It is to be remembered that large increases in power do not lead to better economics under the present ground rules. However, some improvement can be expected for relatively small power increases. The influence of burnup is very important. Any variation in materials to improve this quantity will affect the core life and, hence, the economic picture quite drastically.

It is expected that the cermet system can be improved in the same ways as the wrought system. In addition, the adjusting of volume percentages of oxide dispersed

apparently gives another degree of freedom to this design. It is believed that designs which will optimize the percentage of fuel destroyed during the core life can be found with this added degree of freedom.

Discussion of Problems and Regions of Uncertainty

Materials Behavior

Fuel. The most important question to be answered in the recommended fuel system is whether the estimated burnups can be achieved at the required temperatures. Perhaps the most information is needed at the maximum temperature, but assurance must be obtained as well that the fuel operating at some lower temperature will also behave well. For example, it is conceivable that performance would be adequate at the highest temperature, but that rupture of the cladding might become a problem at some lower temperature because of an inadequate ductility of the cladding material. This does not seem likely on the basis of present data, but must be proven. It is expected that the flat-plate design will minimize the tendency of the cladding to rupture, since most of the growth associated with the accumulation of fission products can be expected to occur in the thickness direction of the fuel plates. Generally speaking, in gamma-phase uranium alloys, the strength begins to drop off rapidly at temperatures above 1300 F, but the uranium-9 to 12 w/o molybdenum alloys have ultimate tensile strengths of about 20,000 psi at 1275 F. No good data are available on their creep properties. This strength indicates that the alloys should perform well up to high temperatures, but it is expected that the most improvement in performance can be obtained by further alloying to increase the high-temperature strength and resistance to creep. An additional advantage of the alloying may be in further improvement of the thermal stability or resistance to transformation of the fuel. Thus, it is reasonable to expect that high burnups at temperatures up to 1300 F will be feasible.

There are too few data available to make a direct comparison of the irradiation performance that can be expected from such an alloy and from a dispersion using this alloy as a matrix. However, by enriching the matrix to achieve a burnup of one-half that expected in the wrought-alloy fuel, and, assuming an irradiation of the dispersed UO_2 particles equivalent to 8 total a/o of the UO_2 , an appreciably higher total burnup results than has been considered for the wrought alloy. The validity of this comparison can only be determined by irradiation testing. However, recent data obtained on the dispersion-type material gave excellent results at 0.6 to 0.7 total a/o burnup (based on the matrix and UO_2) and at a calculated maximum central metal temperature of about 400 F, giving promise for further development. The assumed burnup of 8 total a/o for the UO_2 is based on data for the stainless steel- UO_2 type dispersion.

In the dispersion-type fuel, one might be concerned about reaction between the UO_2 particles and the zirconium cladding, or perhaps by the possibility of the accumulation of fission gas in the region between the UO_2 particles and the cladding, causing a bumping or blistering problem. First, the reaction between zirconium and UO_2 does not become a problem until about 1300 F. Since the zirconium is likely to be at considerably lower temperatures, this does not appear a serious difficulty. If it is a problem, it may be solved by using a fuel which would not react with the zirconium,

such as uranium monocarbide, but this does not appear to be necessary. In either case, a solution might be achieved by leaching the surface particles of UO_2 at some stage of the fabrication before the cladding operation.

The feasibility of making alloy powder has been demonstrated through preliminary laboratory experiments. On the basis of this study and on known technology of uranium-powder production, it is felt that the economical production of alloy powder is possible.

Containers. Zirconium has been shown to have adequate corrosion resistance in sodium, if the oxygen content of the sodium is controlled and kept to a low value. It will be important, however, to control the fabrication processes to give an excellent surface with a minimum of defects or contamination.

Study of the stress in the fuel system indicates that plastic deformation is expected in the zirconium cladding. This may be desirable in order to prevent rupture of the cladding as the fuel swells. On the other hand, rupture may not be a problem in the flat-plate design and a high-strength zirconium alloy corrosion resistant in sodium may be desirable. It is generally thought that Zircaloy exhibits unsatisfactory corrosion resistance in sodium because of its tin content. On the other hand, a number of zirconium alloys having good high-temperature strength are available. If needed, one of these alloys can be selected, after preliminary corrosion tests in sodium, for use in this system.

Fabrication Problems

Wrought-Alloy Fuel. The problems that must be solved in the development of a fabrication process for preparing the zirconium-clad uranium-10 w/o molybdenum assemblies are discussed below. None appears to be a major obstacle. They primarily involve planned modifications in techniques that will significantly reduce the cost of fabrication. It is believed that all of the problems discussed will be solved during the development phase of the program.

Melting Operation. Uranium-molybdenum alloys are currently prepared by a duplex induction-arc-melting process. The development of a single induction-melting process will result in a significant cost reduction. The problem will be one of developing a technique that will be capable of consistently producing homogeneous alloy ingots. A mold design must also be developed to produce ingots that will require a minimum amount of conditioning preparatory to hot rolling. A relatively large amount of reject material must be recycled through the melting operation. This will necessitate the development of techniques for treating the scrap to obtain the maximum yield.

Core-Alloy Fabrication. The fabrication of the core-alloy ingot into sheet is one of the major cost items. This operation generates the largest amount of material that must be recycled or reprocessed. The prime problem is to prevent surface oxidation during fabrication. This will be dependent on the selection or development of a protective noncontaminating salt bath.

Pressure Bonding. The feasibility of the pressure-bonding process has been fully demonstrated on Zircaloy-clad subassemblies measuring 1 by 3 by 10 in. It has also been shown that the best bonds are obtained by using as-machined components. Work is now being conducted to develop cleaning techniques that will permit use of as-rolled surfaces. It has been demonstrated that good bonding can be achieved by a light abrasion of as-rolled material. It is anticipated that a low cost operation for the surface conditioning of as-rolled material will be developed.

The problem of scaling-up to full-size assemblies must be solved. This appears to offer no more problems than a usual scale-up operation. Since the operation involves no significant reduction of components, the dimensional control is governed by the dimensional tolerances specified for the individual components.

Nondestructive Testing. Nondestructive testing of a finished subassembly will offer more problems than encountered in the testing of single fuel elements. It will be necessary to test defected assemblies in order to demonstrate the effectiveness of the proposed testing technique.

Melting Operation. Uranium-molybdenum alloy powder is presently prepared in relatively small quantities by a procedure that involves alloying, fabrication, hydriding, attrition, and vacuum degassing. The use of the pregassed shotting process represents a large step forward in applying technology presently used in industrial super-alloy powder production. However, this step is one of applying known techniques rather than the development of new processes. The principal problems will be connected with the adaptations of the process to the protective atmosphere required and the selection of materials that will withstand the erosive and corrosive conditions present. For these reasons prealloying has been proposed to reduce the higher temperatures and holding times that would be involved in the alloying of molybdenum and uranium in the induction-melting operations. The subjection of the pouring nozzle to these lower temperatures should sustain its life and permit greater control of the shotting operation.

This control of the shotting operation will receive the greatest attention in the development program. Conditions for the manufacture of the highest yields of the desired sizes will be determined.

Core-Strip Preparations. The fabrication of the mixed powders of UO_2 and uranium-10 w/o molybdenum into a continuous strip of core material is an area of some uncertainty. This technique has proven successful in the fabrication of metallic strip from ductile powders and the process is being used successfully for preparing UO_2 -stainless steel core strip. Some of the major problems appear to be (1) obtaining 0.055-in.-thick core strip of adequate density, (2) reducing edge-trim losses, and (3) preparing surface sufficiently clean and nonreactive for successful pressure bonding.

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Variations and Modifications in the Reference Fuel Systems

The evaluation of a new reactor fuel system must include an estimate of the possible variations or modifications which might be needed to achieve the required end performance. Since it is seldom that one is clairvoyant enough to anticipate and solve all of the technical problems attendant upon the development of a core fuel system, one important measure of the potential of the system is its inherent adaptability to avoid or overcome deficiencies as they develop.

Discussed below are some of the alternatives which might be employed in the development of the reference fuel systems.

Materials

It is felt that both the dispersion-type fuel proposed and the wrought alloy fuel, which is the alternate, can be improved through ternary or quaternary alloying. It is expected that little development would be required to select the alloy addition or additions, since some information is already available from research being supported by the AEC.

Numerous variations in technology have been considered in arriving at the recommended alloy system. Various alloys other than molybdenum have been considered; the effect of fabrication and heat-treatment processes, the quality of the starting material, the design of the subassembly, and other variations such as intentional porosity and "spiking" of the fuel by dispersion of enriched particles in a low-enrichment or depleted matrix have also been considered. The recommendations represent what is considered to be the most attractive and best established route.

Cladding materials other than zirconium or zirconium alloys have also been considered, and materials such as molybdenum, tantalum, or niobium may be used, if necessary. It is felt that the use of these cladding materials in the pressure-bonding process is quite feasible.

Another factor which has been considered is the adaptability of the recommended system to the use of plutonium as a fuel. It appears that either the dispersion or wrought alloy is suitable. Use of plutonium in the dispersion-type fuel would be accomplished through the use of plutonium oxide or other refractory compounds in place of the UO_2 . Use of plutonium might have some advantage in the dispersion, since the smaller amount of plutonium required for criticality would permit a smaller volume of the dispersed fuel to be used making both the fabrication and the performance of the dispersion attractive compared to uranium-containing dispersions. Recent studies at Argonne on uranium-plutonium-molybdenum and uranium-plutonium-"fissium" type alloys indicate that the alloy approach is also attractive. Addition of either molybdenum or fissium tends to stabilize the gamma phase in alloys containing 20 w/o plutonium. While this research has been done on cast samples, it is apparent that a good wrought fuel should be feasible with some adjustments. Of course, the use of plutonium would be complicated by the necessity for special handling facilities.

Fabrication

The most feasible alternative to the pressure-bonding process is roll bonding. In this process, each fuel element is individually roll clad and then assembled and welded to form a subassembly. In the current practice, the fuel-element components are assembled into a steel container which is evacuated and sealed prior to rolling. A protective steel container is used to prevent contamination of the zirconium. This is of significant importance in pressurized water reactors where oxygen and nitrogen contamination has a very deleterious effect on the corrosion resistance of the zirconium.

For this application, it may be feasible to roll bond the fuel elements without the use of steel jackets. Corrosion tests of as-rolled zirconium components would demonstrate the effects of slight contamination on the corrosion resistance of zirconium in sodium. The elimination of steel jackets would significantly reduce the costs of the roll-bonding operation.

At one stage of the development program for the dispersion fuel, several alternate methods for preparing the core strip will be under investigation. These will include various combinations of cold compacting, sintering, resintering, cold rolling, and hot rolling both bare and in jackets. For instance, it is expected that the uranium-10 w/o molybdenum alloy powder will be ductile. Thus, it is possible that the mixture of UO_2 and uranium-10 w/o molybdenum alloy powders can be cold pressed, sintered, and recoined to a high density, then fitted in a restraining frame and cold rolled with intermittent anneals to strip of theoretical density.

Design

Several variations in subassembly design were considered in arriving at the reference. The utilization of zirconium-clad fuel pins similar to the Core I design was found to be less attractive than plates from the standpoint of the amount of total uranium which could be introduced into the core commensurate with the heat-transfer and fluid-flow requirements. This resulted in considerably less favorable economics for the same total atom per cent burnup.

Consideration was given to the utilization of the stainless steel radiator-type design, proposed for the UO_2 flat-plate design discussed in a preceding section. This design was considered feasible, if a satisfactory coating could be applied either to the inside of the stainless steel channels or the surface of the fuel plates in order to avoid the uranium-iron eutectic which occurs at about 1375 F. While such a development appeared possible, this design was not favored for the reference for two reasons:

- (1) The estimates of the fabrication costs of the radiator design compared with the pressure-bonded subassembly favored the latter.
- (2) The radiator design involved three layers of material (stainless steel, coating, and sodium) between the active fuel and the coolant stream as compared to the single zirconium cladding in the reference design. This factor favored the reference design from the standpoint of total economics.

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However, in the event that the thermal stresses and strains in the reference design become prohibitive, the radiator design offers an alternative solution.

The inherent advantage in the pressure-bonding process applied to a metal system is the large degree of variation or refinement in design which is feasible with only slight fabrication-cost differentials. It may well be that future technology in reactor-core design will consider flat parallel plates as a primitive design. If, as may be expected, the future trend in core designs is in the direction of reducing the hot-channel factors by various measures, the design refinements available from pressure-bonding fabrication will be ideally suited to take advantage of them.

Economic Comparison of the Reference Design With a Coextruded Pin Design at 430 Megawatt Reactor Power Level

As one indication of the potential advantages to be gained from the reference design and fuel materials, it is informative to compare the reference system with a coextruded pin design at a total reactor power level of 430 megawatts. Essentially this amounts to an evaluation of the economic advantages to be gained with the reference system in the present PRDC plant over the Core I system.

Designs were made for the pin-type subassembly with U-10 w/o Mo alloy fuel and for the plate-type subassembly with both U-10 w/o Mo alloy fuel and UO_2 - U-10 w/o Mo cermet fuel. The description of these designs is given in Table 17. The estimated fabrication costs for the three designs are given in Tables 18, 19, and 20. The same fuel burnup fractions were used in both pin and plate designs as were assumed in the reference fuel system. The results of the economic analysis are shown in Table 17.

The economic results indicate that, while none of the systems has a zero revenue deficiency, the use of the pressure-bonded flat plate design with cermet fuel in the PRDC reactor could result in savings of about \$2.9 million per year. This economic potential accrues from two primary factors:

- (a) The greatly reduced fabrication cost from the use of the flat plate design and the pressure bonding fabrication technique.
- (b) The increased amount of diluent material which can be loaded in plate type subassemblies.

It should be noted that in the case of the Cermet fuel there is available a range of Uranium 235 enrichment and the corresponding volume per cent of the UO_2 required. The choice of 30 v/o UO_2 was selected arbitrarily and was not optimized on the basis of economics.

TABLE 17. PLANT ECONOMICS FOR THE VARIOUS ALLOY FUEL SYSTEMS
430 MEGAWATT MAXIMUM REACTOR POWER

	Wrought Pin	Wrought Plate	Cermet
Fuel Required, kg	655	707	645
Critical Mass, kg	621	674	591
Conversion Rates			
Core	0.41	0.55	0.50
Blanket	0.73	0.65	0.67
Fuel Atoms per Uranium-235 Atom	6.44	7.70	9.41
Initial Enrichment, %	0.198	0.1652	{ 0.0332-Mo 0.690-O ₂
Uranium-235 Destroyed per MWD	1.32	1.32	1.275
Core Life, days	281	363	555
Temperature, F			
Inlet, sodium	550	550	550
Outlet, sodium	900	900	900
Steam	800	800	800
Maximum Fuel	1300	1300	1300
Burnup, a/o			
Maximum	0.03	0.03	{ 0.08-O ₂ 0.015-Mo
Average	0.0248	0.0248	{ 0.066-O ₂ 0.01236-Mo
Fuel Thickness, in.	0.111	0.092	0.081
Number of Plates or Pins	324	17 + 2 half	19 + 2 half
Blanket/Total Power Ratio	0.10	0.10	0.10
Subassembly Cost, dollars	12,170	2,680	4,285
Cost, dollars			
Fixed	17,730	17,730	17,730
New Fuel	7,300	7,100	6,760
Core Cycle	3,450	3,030	3,470
Fabrication	6,010	1,030	1,070
Blanket Cycle	3,860	3,440	3,540
Inventory	6,760	6,210	4,430
Total	45,110	38,540	37,000
Revenue, dollars			
Plutonium	20,100	21,050	19,930
Steam	12,680	12,680	12,680
Total	32,780	33,730	32,610
Revenue Deficiency for 6 Per Cent Return	+12,330	+4,810	+4,390

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TABLE 18. FABRICATION COST ESTIMATES FOR URANIUM-10 w/o MOLYBDENUM
PIN-TYPE FUEL ASSEMBLY DESIGN FOR 430 MW REACTOR POWER

<u>Fabrication - Fuel Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Diameter	0.111
Clad Thickness	0.005 ± 0.001
Core Length	30.5
Number of Elements per Assembly	324
Number of Assemblies per Core	139
U-235 Enrichment, per cent	19.80
Total Weight of Uranium, lb	7,290
Total Weight of Molybdenum, lb	810
Fabrication Yield Without Recycle of Remeltable Material, per cent	55
Over-All Yield With Recycle of Remeltable Material, per cent	70
Total Alloy Required Without Recycle, lb	14,750
Total Alloy Required-Recycled, lb	11,550
Total Uranium Required-Recycled, lb	10,400
Total Molybdenum Required-Recycled, lb	1,150
<u>Fabrication and Material Costs</u>	
Material Cost, dollars per assembly	
Molybdenum Metal	64.00
Materials Used in Melting; Crucibles, Stoppers, etc. (\$.67 per lb Alloy)	72.00
Extrusion Containers, Lead Cones, etc.	270.00
Zirconium Metal for Sleeves, End Plugs, etc.	340.00
Materials Used in Cleaning of Components; Ether, Pickle Solutions, etc.	10.00
Stainless Steel Assembly	350.00
Total	1,106.00
Fabrication Cost, man-hours per assembly	
Melting of Alloy and Analytical Analysis (0.17 Man-Hours per Lb for Virgin Metal)	10.0
(0.21 Man-Hours per Lb for Recycled Metal)	10.5
Conditioning of Alloy for Primary Extrusion	5.0
Machining, Cleaning, Assembly, and Welding of Extrusion Components for Primary Extrusion	8.0
Primary Extrusion	4.0
Dejacket and Straighten Core Alloy	3.0
Machine Core Alloy for Final Extrusion	220.0
Machine, Clean, Assemble, and Weld Components for Final Extrusion	340.0
Final Extrusion and Dejacketing Operation	125.0
Alternate Swage and Anneal Treatments to Reduce Coextruded Material to Final Size	250

TABLE 18. (Continued)

<u>Fabrication and Material Costs</u> (Continued)	
Fabrication Cost, man-hours per assembly (continued)	
Weld End Plugs	125
Nondestructive Testing	50
Assemble to Form Assembly	8
Join to Blanket Assembly	<u>10</u>
Total	1,168.5
<u>Estimated Total Costs</u>	
Total Cost, dollars per assembly	
Direct Materials	1,106.00
Direct Labor	2,570.70
Overhead (250 Per Cent)	<u>6,426.75</u>
Total	10,103.45
General Administration (3 Per Cent)	303.00
Profit (10 Per Cent)	<u>1,010.00</u>
Grand Total	11,416.45

TABLE 19. FABRICATION COST ESTIMATES FOR URANIUM-10 w/o MOLYBDENUM
FLAT-PLATE ASSEMBLY DESIGN FOR 430 MW REACTOR POWER

<u>Fabrication - Fuel Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Thickness	0.092
Core Width	2.449
Core Length	30.5
Number of Plates per Assembly	17 and 2 end plates
Number of Assemblies per Core	132
U-235 Enrichment, per cent	16.52
Total Weight of Alloy, lb	10,500
Total Weight of Uranium, lb	9,450
Total Weight of Molybdenum, lb	1,050
Fabrication Yield Without Recycle of Remeltable Material, per cent	71.0
Over-All Yield With Recycle of Remeltable Material, per cent	84.0
Total Alloy Required Without Recycle, lb	14,800
Total Alloy Required-Recycled, lb	12,500
Total Uranium Required-Recycled, lb	11,250
Total Molybdenum Required-Recycled, lb	1,250
<u>Fabrication and Material Costs</u>	
Material Cost, dollars per assembly	
Molybdenum Metal	63.00
Materials Used in Melting; Crucibles, Stoppers, etc. (\$.67 per Lb Alloy)	71.00
Ti-Namel Steel Spacers	5.00
Zirconium Cover Plates	110.00
Zirconium Channel Strips	25.00
Zirconium Picture Frames	450.00
Materials Used in Pressure Bonding; Stainless Sheet, End Spacers, etc.	20.00
Materials Used in Cleaning of Components, Ether, Pickle Solutions, etc.	<u>10.00</u>
Total	754.00
Fabrication Cost, man-hours per assembly	
Melting of Alloy and Analytical Analysis (0.17 Man-Hours per Lb Virgin Metal)	12.9
(0.21 Man-Hours per Lb Recycled Metal)	6.5
Shearing, Edge Machining, Cleaning, and Assembly of Ti-Namel Spacers	8.5
Shearing, Pickling Cleaning, and Assembly of Zirconium Cover Plates	4.0
Shearing, Pickling, Cleaning, and Assembly of Zirconium Channel Strips	4.0

TABLE 19. (Continued)

<u>Fabrication and Material Costs</u> (Continued)	
Fabrication Cost, man-hours per assembly (continued)	
Shearing, Machining, Cleaning, and Assembly of Zirconium Picture Frames	7.2
Fabrication of Uranium Alloy to Sheet: Hot Rolling, Cold Rolling, Roller Leveling, Shearing, Edge Machining, Cleaning, and Assembly	26.0
Pressure Bonding: Preparation of Stainless Steel Can, Assembly, Evacuation, and Decanning	24.5
Final Machining, Cleanup, and Inspection	12.0
Non-Destructive Testing	8.0
Joining of Assembly to Blanket Structure	<u>10.0</u>
Total	123.6
<u>Estimated Total Costs</u>	
Total Cost, dollars per assembly	
Direct Materials	754.00
Direct Labor (\$2.20 per Hr)	271.92
Overhead (250 Per Cent)	<u>677.80</u>
Total	1,703.72
General Administration (3 Per Cent)	51.10
Profit (10 Per Cent)	<u>170.37</u>
Grand Total	1,924.19

TABLE 20. FABRICATION COST ESTIMATES FOR 30 VOLUME PER CENT
 UO_2 -(URANIUM-10 w/o MOLYBDENUM) FLAT-PLATE
 ASSEMBLY DESIGN FOR 430 MW REACTOR POWER

<u>Fabrication - Fuel Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Thickness	.081 $\pm$.001
Core Thickness (end plates)	.040 $\pm$.001
Core Width	2.449
Core Length	30.5
Clad Thickness	.005 $\pm$.001
Number of Plates per Assembly	19
Number of Fueled End Plates per Assembly	2
Number of Assemblies per Core	139
U-235 Enrichment, per cent	
UO_2	69.0
U-10 w/o Mo Alloy	3.32
Total Weight of Alloy in Core, lb	7,350
Total Weight of Uranium, lb	6,615
Total Weight of Molybdenum, lb	735
Total Weight of UO_2 in Core, lb	1,966
Total Weight of Uranium in UO_2 , lb	1,732
Over-All Yield With Recycle of Material, per cent	
(a) Alloy	57.0
(b) UO_2	88.3
Total Material Required With Recycle	
(a) Total Alloy, lb	12,890
(1) Total Uranium, lb	11,600
(2) Total Molybdenum, lb	1,290
(b) Total UO_2 , lb	2,234
(1) Total Uranium, lb	1,969
Fabrication Yield Without Recycle of Remeltable Material	
(a) Alloy	50.5
Total Alloy Required Without Recycle of Remeltable Material, lb	14,520

Fabrication and Material Costs

Material Cost, dollars per assembly	
Molybdenum	63.50
Materials Used in Melting; Crucibles, Stoppers, etc.	266.70
Ti-Namel Steel Spacers	6.35
Zirconium Cover Plates	146.00
Zirconium Channel Strips	31.75
Zirconium Picture Frames	521.00
Materials Used in Cleaning of Components	10.00
Materials Used in Pressure Bonding Stainless Sheet, End Spacers, etc.	25.00
Total	1,070.30

TABLE 20. (Continued)

<u>Fabrication and Material Costs</u> (Continued)	
Fabrication Time, man-hours per assembly	
Arc Melting of Alloy, Induction Melting, Shotting, and Analysis (1.26 Man-Hours per Lb)	128.9
Alloy Powder, Sizing, Weighing, Blending With UO_2	30.5
Fabrication of Powder to Strip, Vacuum Anneal, Cold Rolling, Roller Leveling, Edge Machining, Cleaning	7.6
Shearing, Pickling, Cleaning, and Assembly of Ti-Namel Spacers, Zirconium Cover Plates, Channel Strips, and Picture Frames	31.0
Pressure Bonding; Preparation of Stainless Steel Can, Assembly, Evacuation, and Decanning	31.0
Final Machining, Cleanup, and Inspection	15.3
Non-Destructive Testing	10.0
Joining of Assembly to Blanket Structure	12.5
Total	266.8
<u>Estimated Total Costs</u>	
Total Cost, dollars per assembly	
Direct Materials	1,070.30
Direct Labor (\$2.20 per Hr)	586.96
Overhead (250 Per Cent)	1,467.40
Total	3,124.66
General Administration (3 Per Cent)	93.74
Profit (10 Per Cent)	312.47
Grand Total	3,530.77

DISCUSSION

Several points are worthy of discussion concerning this study.

It is apparent that the reference design is "optimized" to the ground rules. This was required simply because the task of obtaining economic power from a realistic reactor system is not so easy as to permit wide latitude in the choice of design and fabrication parameters. Thus it should be remembered that the reference design dimensions, fabrication cost figures, and plant economics, are tied to a \$44 per g of plutonium value and a slightly modified PRDC plant.

The results of the study showed that fuel burnup, in particular, the burnup fraction of uranium-235, was an extremely strong factor in plant economics. Furthermore, in order to evaluate and compare fuel systems it was necessary to specify a burnup for each system. It is a fact that if only fully proven burnup fractions were specified, no fuel system (nor probably any reactor plant in the country) would have much hope for economic power production. Thus it was necessary and completely proper to specify burnup on the basis of what might reasonably be expected to be achieved through development of the fuel materials. Within present technology the guides for doing this are few. No one can say with certainty, for example, that 2.5 a/o burnup is satisfactory but 3 a/o is not; yet the total plant economics might be changed by several thousand dollars a day if the higher burnup is possible. The normal temptation to "help the economics" by assuming an unrealistically high burnup is a serious threat to the value of a study such as this one. It should be noted, however, that over-conservation is just as dangerous in this respect, for it could lead to a design optimization which could not take full advantage of increased burnup if it were achievable. The Battelle staff was acutely conscious of these factors during the course of the study.

In spite of these apparent uncertainties in the quantitative economic results of the study, it is believed that the study is valuable and worth while. It has shown the relative importance of the complex factors making up the total plant performance, and indicated the direction for the development work in the next phase of the program.

Battelle is convinced that the recommended fuel system will contribute a significant improvement in future fast power breeder reactor economics.

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APPENDIX A

COPY OF GROUND RULES FOR CORE-II
DESIGN AND DEVELOPMENT

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ATOMIC POWER DEVELOPMENT ASSOCIATES, INC.

SUMMARYGround Rules for Core-II Design
and Development - Phase IC
O
P
YObjective

This program has the goal of demonstrating economic profitable power production from a fixed fuel fast breeder power reactor using aqueous reprocessing. This shall be accomplished by a five-year design and development effort including fuel subassembly tests in the PRDC reactor. Following sufficient test evaluation, the program will terminate with the delivery of one fabricated core load or partial core load as may be agreed by the parties concerned.

Phase I of this program consists of design studies, minor test work, and evaluation of materials to enable the contractor to recommend a fuel subassembly design, fuel and structural materials, and a basic fabrication technique for further development in Phase II. An alternate design, fuel material, and fabrication technique may be recommended if it is a strong second choice to the one selected above.

Phase I studies shall be conducted on the basis of the original Ground Rules herein summarized, plus revisions as made, based on the limitations of the PRDC reactor plant design. If it becomes apparent that "economic" power cannot be achieved with these limitations, then the studies shall turn to a second approach which would be the design of a core for another fast power reactor not limited by the design conditions of the PRDC plant.

APDA will supply the following information for use in the Core-II program, as required by the contractor:

1. Basic physics calculations.
2. Fuel cycle cost rates except fabrication cost.
3. Other economic data.

Ground RulesA. General

1. U-235 is the fissionable material. Preference shall be given to designs that may utilize plutonium as the fissionable material eventually.
2. The net breeding ratio of the entire fuel and blanket cycle shall not be less than 1.0.

3. Core and blanket material shall be aqueously processed by the AEC or a commercial contractor. Consideration shall be given to the problem of disassembly of core element designs and to the problem of separating fissionable and fertile materials from other elements for simple processing.
4. Phase I shall be completed by July 31, 1957.

B. Physics and Safety

1. The designs considered shall have sufficient negative coefficients of reactivity to prevent melting of core materials under all credible accident conditions studied.
2. The ratio of U-238 to U-235 intimately mixed in the core fuel material shall not be less than 1.8.

C. Reactor Plant Performance

1. Minimum heat power output is 430 mw. Maximum power output is not restricted.
2. Maximum design average sodium outlet temperature from the reactor is 1000 F.
3. The mixed mean temperature rise of the primary coolant from reactor inlet to reactor outlet shall not exceed 350 F at design power.
4. The coolant flow rate through the reactor shall not exceed 150% of PRDC design flow rate, i.e., shall not exceed 19.8×10^6 lbs/hr. Approximately 90% of the total flow rate is available for core flow.
5. Pressure drop through core subassemblies is limited to 65 psi.

D. Subassembly and Core Design

1. Core subassemblies must retain same handling and support sections as shown on APDA Drawing No. 6XN-1074.
2. Subassembly housings are on a square pitch of 2.693".
3. Number of core fuel subassemblies may be between 91 and 139.

E. Reactor Plant Cost Data

1. U-235 cost and plutonium revenue shall be calculated from AEC Price Schedule dated January 5, 1955.
2. Annual plant factor - 80%.
3. Steam revenue value - $\$0.45/10^6$ Ctu.

4. Blanket cycle time - based on 0.2% burn-up or 5 years residence time whichever occurs first.
5. Core and blanket processing cycle costs (except fabrication) shall be calculated from unit charges on the fuel and blanket flow diagram, APDA Drawing No. 6XN-880-B, dated November 25, 1955. Fuel material throughput should be calculated by assuming monthly unloading of core subassemblies.
6. Fuel inventory charge - 12.5% annually. Total inventory to be determined from reprocessing flow chart cycle time and core life calculated.
7. Annual production expenses (exclusive of fuel and blanket cycle costs) - \$1,651,500.
8. Plant investment - \$32,226,000.
9. Plant fixed charges (except return on investment).

Depreciation	- \$ 826,000 (20 year plant life)
Insurance	- 133,900
State, local, and miscellaneous taxes	- <u>633,900</u>

TOTAL - \$1,593,800

10. Return on plant investment - 6%

50% bonds at 4-1/2%
50% stocks at 7-1/2%

11. Federal income tax - to be calculated as $52/48 \times (\text{stock return less depreciation in excess of amortization})$.

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GROUND RULES - CORE-IIRevision No. 1 - April 25, 1957

Paragraph Reference:

- A. 4 Phase I shall be completed by December 31, 1957.
- B. 2 The ratio of U-238 to U-235 intimately mixed in the core fuel material shall be not less than 1.0.
- B. 3 Maximum reactivity change due to burn-up that may be permitted by the PRDC reactor is \$1.20.
- C. 2 For a maximum sodium outlet temperature from the reactor of 1000 F, the maximum steam temperature shall be 100 F less, or 900 F.
- C. 4 Advantage may be taken of orificing to obtain uniform heat removal and increase coolant flow rates above 150% of PRDC design rate. In this case, maximum coolant flow rate allowable is that which may be attained with a 65 psi drop through the core section of a subassembly.
- D. 4 The number of control subassemblies will remain the same as present PRDC design, i.e., ten containing no fissionable material.
- E. 3 Steam revenue value based on temperature of steam:
800 F	-	\$0.45/10 ⁶ Btu
900 F	-	\$0.50/10 ⁶ Btu
1000 F	-	\$0.55/10 ⁶ Btu
- E. 5 Fuel material throughput should be calculated based on burn-up and power produced.

PROCESSING COSTS

Referred to Drawing 6XN-880-B

Core

Items 9 and 10	- \$102/kg total uranium
Fixed cost - plant turnaround	- \$122,500/year
L-5 U and Pu	- 1 per cent loss
Item 11	- May be classified yet
L-8	- Included in L-5
Item 12	- \$35/kg total uranium

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A-5 and A-6

Blanket

Items 109-1, 109-2, 109-3, 110-	- \$15.30/kg total uranium
Storage containers for U-Nitrate	- ?
L-7 Pu	- 1 per cent
Item 11	- May still be classified
L-8 Pu	- Included in L-7 Pu

E.6 Fuel inventory charge - 12.56% annually.

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APPENDIX B

REACTOR SYSTEMS ECONOMIC EQUATION

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APPENDIX B

REACTOR SYSTEMS ECONOMIC EQUATION

The relationship developed below was intended to serve as a guide in the thinking regarding design and materials for the APDA Core II. As such, it is not expected that entirely accurate over-all economics for any one system will result from its use. It is believed to be a relatively powerful tool with which to unravel some of the interdependencies of the materials and design parameters. The trend information reported in the text can be obtained quite rapidly by using the relation developed below. A sample calculation for Core I is also demonstrated in the text.

The economic equation is really a compromise between the desire for a precise economic picture for all fuel systems considered and the necessity to obtain the economic outlook for a great number and variety of systems. The analysis is carried out in as gross parameters as possible which will yet allow for an understanding of the underlying phenomena. Any unnecessary complications such as trying to include the detailed parameters of flow rates, temperatures, elements per subassembly, etc., have been avoided.

Common Groupings

The notation has been described in the body of the report. Using this notation, some of the important common groupings are:

Fuel consumed per day = PX

Fraction of fuel burned up during core life = JBmF

Average core life = $\frac{JBmF}{PX} M$

Quantity of 25 per day requiring reprocessing = $\frac{(1 - JBmF)M}{\frac{JBmFM}{PX}} = \frac{PX}{JBmF} (1 - JBmF)$.

General Economic Equation

Revenue deficiency per day =

- (1) Fixed costs per day
- plus (2) Fuel-replacement costs per day
- plus (3) Entire core-fabrication and reprocessing-cycle costs per day
- plus (4) Core-plutonium recovery costs per day

- plus (5) Entire radial-blanket cycle costs per day
 plus (6) Fuel-inventory costs per day
 minus (7) Income from plutonium per day
 minus (8) Income from steam per day.

Derivation of Cost and Revenue Factors

1. Fixed Costs Per Day

This factor includes:

- (a) Amortization
- (b) Insurance
- (c) State and local taxes, etc.
- (d) Fixed production expenses
- (e) Fixed costs in the fuel and blanket cycles as applicable
- (f) Fixed per cent on depreciable investment.

Item (f) is a 6 per cent return for stocks and bonds on depreciable investment plus Federal income tax. The Federal income tax is taken as 52/48 (stock return on plant investment minus depreciation in excess of amortization).

2. Fuel-Replacement Costs Per Day

Fuel replacement costs per day = fuel unit cost (fuel consumed per day
 + fuel lost in reprocessing per day)

Fuel consumed per day = PX

Fuel lost per day = fuel reprocessed x $\frac{\text{g uranium-235 lost}}{\text{g uranium-235 reprocessed}}$

$$= \frac{PX}{JBmF} (1 - JBmF) L_f .$$

Thus,

Fuel-replacement cost per day = $f \left\{ PX + \frac{PXL_f}{JBmF} (1 - JBmF) \right\}$

simplifying this is $PX f \left\{ 1 + L_f \frac{(1 - JBmF)}{JBmF} \right\} .$

Comments:

- (a) The uranium-238 atoms to be replaced are included by the choice of (f).
- (b) By its definition the losses L_f will include both fabrication and reprocess losses. The reprocessed material should have its own loss and likewise the fabrication cycle. This complication is not considered to be significant enough to be included in the economic analysis.

3. Entire Core-Fabrication and Reprocessing Cycle Costs Per Day

It is assumed that all costs except those accounted for in the fabrication step itself can be reduced to a per g contained uranium-235, C_f or a per g contained uranium basis, C_u . Then:

Entire fuel-cycle costs per day = contained uranium-235 processed per day x C_f + contained uranium processed per day x C_u + fabrication cost per core subassembly x number of subassemblies required per day

$$= \frac{PX}{JBmF} (1 - JBmF) C_f + \frac{PX}{JBmF} (U - JBmF) C_u + \frac{PXN}{JBmF} C_n$$

simplifying this, it becomes

$$\frac{PX}{JBmF} \left\{ (1 - JBmF) C_f + (U - JBmF) C_u + NC_n \right\} .$$

Comments:

This term includes only the quantities of materials (uranium-235 and total uranium) that leave the core on the average. (Note: $M(1 - JBmF)$ is approximately the contained uranium-235 in spent core and $M(U - JBmF)$ is approximately total uranium in spent core.) The fabrication cycle includes costs per g uranium-235 and per g total uranium which should be based on somewhat more material to include new fuel. However, since a large share of the entire cycle costs is incurred with spent fuel the exclusion of the new fuel seems justified to the accuracy required for this type of analysis.

4. Core-Plutonium Recovery Costs Per Day

This factor includes the cost of processing to recover the core plutonium and also the cost for losses of core plutonium.

Core-plutonium recovery costs per day = plutonium loss per day x unit revenue for plutonium + quantity of plutonium processed per day x unit process costs

$$= PXbL_fI + PXb(1 - L_f) C_p$$

$$= PXb \left\{ L_f(I_c - C_p) + C_p \right\} .$$

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Comments:

- (a) The loss figure used here is taken to be the same as the fuel losses. It is, therefore, in a conservative direction.
- (b) By using the core plutonium alone the costs here do not include the end blanket throughput. It is not generally possible to find the end blanket portion of the total blanket plutonium so that this assumption is necessary. Inasmuch as this entire term is quite small, the error is considered to be acceptable.

5. Entire Radial-Blanket Cycle Costs Per Day

It is assumed that since the identical radial blanket is being used for all Core II designs as is considered in Core I, the only change will be in the average blanket life-time. This gives a cost per day based on Core I costs per day times a ratio of blanket lives. The average blanket life is determined from the consideration that

$$L \propto \frac{1}{\text{blanket power density}}$$

Since the blanket power density is in general proportional to the average blanket plutonium production rate then

$$L \propto \frac{N_b}{P X_g}$$

A constant of proportionality can be determined by reference to Core I

$$L_0 = \frac{k N_0}{P_0 X_{0g0}}$$

where the zero subscript denotes Core I values. Solving for the constant gives

$$k = L_0 P_0 X_{0g0} / N_0$$

Then for any system

$$L = \frac{L_0 P_0 X_{0g0} N_b}{P X_g N_0}$$

hence, the cost per day is simply

$$\frac{C_b N_b}{L}$$

Comments:

- (a) The demonstration that plutonium production rates are proportional to blanket powers is possible only for the entire blanket. Individual radial regions do not satisfy this condition. The assumption is, therefore, only correct insofar as the radial neutron-energy spectra among the systems remain constant. Theoretical verification of this has been possible for about 10 radically different cases studied.
- (b) Plutonium losses are included in C_b .

6. Fuel-Inventory Costs Per Day

The total fuel inventory includes a core of average critical mass and fuel in re-processing. This latter is $\frac{\text{out-of-pile time}}{\text{in-pile time}}$ x average fuel in core at reprocessing or

$$\frac{270 \text{ PX}}{\text{JBmFM}} (M - \text{JBmFM})$$

where the average out-of-pile time is taken as 270 days. This allows about 1 month average for fabrication.

The average critical mass is the fuel loading minus half of the allowed burnup. That is

$$M \left(1 - \frac{\text{JBmF}}{2} \right) .$$

The total fuel inventory is then

$$M \left\{ 1 + \frac{270 \text{ PX}}{\text{JBmFM}} (1 - \text{JBmF}) - \frac{\text{JBmF}}{2} \right\} .$$

The fixed yearly interest rate for the value of this fuel is made up of 6 per cent by stocks and bonds, Federal income tax at 52/48 of the stock return, and additional interest rate to give a total of 12.56 per cent.

Thus, the fuel-inventory costs per day are

$$\frac{0.1256}{365} fM \left\{ 1 + \frac{270 \text{ PX}}{\text{JBmFM}} (1 - \text{JBmF}) - \frac{\text{JBmF}}{2} \right\} .$$

Comments:

- (a) Refinements on the inventory costs must include excess fuel in fabrication plant and a dwell time in excess of 1 month for this fuel. Inasmuch as these numbers cannot be generally specified, they represent an error to this term.
- (b) No inventory charge is made for unrecovered plutonium.

7. Income From Plutonium Per Day

The plutonium losses and recovery costs are included in Items 4 and 5 above. Therefore, the plutonium income is based on the unit revenue I_c and I_b for core and blanket plutonium, respectively, and the over-all conversion ratio $(b + g)$. Plutonium income per day = $\text{PX} (I_c b + I_b g)$. If it is further assumed that

$$I_c = I_b = I$$

then this becomes

$$\text{PXI} (b + g) .$$

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Comments:

The assumption of $I_c = I_b$ is quite good except at very high burnups where the quality of the plutonium (per cent 240 content) is large. Estimates on most systems in the design burnup range showed the quality in the blanket to be less than 0.5 per cent and in the core to be less than about 2 per cent.

8. Income From Steam Per Day

By the definition of the terms this is simply

$$C_s P_t .$$

APPENDIX C

DERIVATION OF EQUATIONS FOR ENGINEERING ANALYSES OF PARALLEL-
PLATE SUBASSEMBLY DESIGN SYSTEMS HAVING LIVE END PLATES

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APPENDIX C

DERIVATION OF EQUATIONS FOR ENGINEERING ANALYSES OF PARALLEL-
PLATE SUBASSEMBLY DESIGN SYSTEMS HAVING LIVE END PLATES

The following sections give the derivations of the pressure-drop, heat-transfer, and thermal-stress equations used in the engineering analyses of plate systems having live end plates.

The coordinate system used in these derivations is shown in the sketch of Figure C-1; nomenclature and units are given in Table C-1.

Derivation of General Pressure-Drop Equations

If it is assumed that the flow boundaries are smooth and that the total combined entrance and exit losses are one-half velocity head, the pressure-drop equation to be used is

$$\Delta p = 4f \frac{L_p}{D_e} \frac{W^2}{2g\rho A^2} + \frac{W^2}{4g\rho A^2}, \quad (C-1)$$

where f is given by

$$f = 0.046 N_{Re}^{-0.2}. \quad (C-2)$$

If two ribs each r thick are spaced at third points in a flow channel F wide, and if d is the space between adjacent plates, the width of a single flow channel is then

$$c = \frac{F-2r}{3}. \quad (C-3)$$

The equivalent diameter of a single flow channel is then

$$D_e = \frac{2cd}{c+d}. \quad (C-4)$$

There are N subassemblies each with $3(n+1)$ flow channels of this equivalent diameter. Therefore, the mass flow rate through the single channel being considered is

$$W = \frac{W_t}{3N(n+1)}. \quad (C-5)$$

Substitution of Equations (C-2), (C-3), (C-4), and (C-5) into Equation (C-1) gives the relationship

$$1.333 g\rho d^3 \Delta p (F-2r)^3 = 0.0532 \mu^{0.2} L_p (F-2r + 3d)^{1.2} \left[\frac{W_t}{N(n+1)} \right]^{1.8} + 0.333 (F-2r)d \left[\frac{W_t}{N(n+1)} \right]^2. \quad (C-6)$$

FIGURE C-1. SKETCH SHOWING COORDINATE SYSTEM USED FOR HEAT TRANSFER AND THERMAL STRESS ANALYSES

Ribs between plates not shown.

TABLE C-1. NOMENCLATURE AND UNITS USED IN ENGINEERING ANALYSES

Symbol	Definition	Units
A	Cross-sectional flow area of a single flow channel	In. ²
a	Core thickness in regular fuel plate	In.
B	Defined = $\left(\frac{\alpha E}{1-\nu}\right)_m \frac{g}{k_m}$	Lb per in. ⁴
C	Constant	As required
c	Distance between ribs or rib and sideplate	In.
c _p	Specific heat of coolant	Btu/(lb)(F)
D _e	Equivalent diameter of a single flow channel = $\frac{4A}{p}$	In.
d	Space between two fuel plates	In.
E	Modulus of elasticity	Lb per in. ²
F	Total width of core in fuel plate	In.
f	Friction factor = $0.046 N_{Re}^{-0.2}$	None
g	Constant in Newton's laws of motion	In. per hr ²
H	Constant in volumetric heat-generation rate-distribution function for hottest subassembly	None
h	Film coefficient for convective heat transfer from cladding to coolant	Btu/(hr)(in. ²)(F)
J	Constant in volumetric heat-generation rate-distribution function for hottest subassembly	None
k	Thermal conductivity	Btu/(hr)(in.)(F)
L	Length of core in plate	In.
L _p	Total length of plate	In.
m	Cladding thickness	In.
N	Number of fuel subassemblies	None
N _{Nu}	Nusselt number = $\frac{hD_e}{k}$	None
N _{Pe}	Peclet number = $N_{Re} N_{Pr}$	None
N _{Re}	Reynolds number $\frac{D_e W}{\mu A}$	None

TABLE C-1. (Continued)

Symbol	Definition	Units
n	Number of regular plates per subassembly (excludes end plates)	None
P	Wetted perimeter of area A	In.
Δ_p	Pressure drop in subassembly, excluding blanket	Lb per in. ²
Q_c	Core power, excluding power generated in blanket	Btu per hr
Q_z	Heat transferred to coolant in a single flow channel up to point z	Btu per hr
q	Volumetric heat-generation rate in hottest subassembly	Btu/(hr)(in. ³)
$\bar{q}$	Average volumetric heat-generation rate in fuel = $\frac{Q_c}{V_m}$	Btu/(hr)(in. ³)
q''	Heat flux at outer cladding surface	Btu/(hr)(in. ²)
S_o	Outside dimension of a subassembly	In.
T	Relative temperature = $t - t_s$	F
$T_{a/2}$	Relative temperature at cladding-core interface	F
t	Temperature	F
t'_m	Temperature along z axis in fuel	F
t_o	Temperature at outer surface	F
t_s	Stress-free temperature of subassembly	F
Δt_z	Temperature rises at any fixed point z	F
$\bar{\Delta t}$	Mixed-mean coolant temperature rise in core	F
V_m	Total fuel volume in core	In. ³
W	Mass flow rate through a single coolant channel	Lb per hr
W_t	Total mass flow rate through N fuel subassemblies	Lb per hr
$\frac{z}{L}$	Fraction of core length, measured from coolant inlet end	None
$\frac{z_m}{L}$	Fraction of core length to location of maximum core temperature	None
α	Linear coefficient of expansion in thermal-stress derivation	F ⁻¹

TABLE C-1. (Continued)

Symbol	Definition	Units
α	Applied hot-channel factor for coolant-temperature rise	None
β	Applied hot-channel factor for film-temperature rise	None
γ	Applied hot-channel factor for cladding-temperature rise	None
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	Shear strains ^(a)	In. per in.
δ	Applied hot-channel factor for temperature rise in core half thickness	None
∇^2	Operator = $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	In. ⁻²
$\epsilon_x, \epsilon_y, \epsilon_z$	Strains in x, y, and z directions, respectively	In. per in.
μ	Coolant viscosity	Lb/(hr)(in.)
ν	Poisson's ratio	None
ρ	Coolant density	Lb per in. ³
$\sigma_x, \sigma_y, \sigma_z$	Thermal stresses in x, y, and z directions, respectively	Lb per in. ²
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stresses ^(a)	Lb per in. ²

(a) First subscript on symbol indicates direction of normal to plane on which stress component acts; last subscript indicates direction of stress component.

Note: The use of an additional subscript c, f, or m indicates evaluation for the cladding, film, or core, respectively, when the subscript is used with the symbol for a temperature or physical property; the use of no subscript indicates evaluation for the coolant.

For an assumed average coolant temperature all physical properties of the coolant are known. Therefore, for fixed pressure drop, total coolant channel width, rib width, and number of subassemblies, the plate spacing is determined as a function of the number of fuel plates and the coolant flow rate through the subassemblies. For this case, the total coolant channel width has been chosen equal to the core width in the plate.

The total of the spaces and plate thickness must equal the outside dimension of the subassembly; therefore,

$$b = \frac{S_o - (n+1)d - 2m}{n+1} \quad (C-7)$$

and

$$a = b - 2m \quad (C-8)$$

The cross-section dimensions of the desired system are then completely described by Equations (C-3), (C-6), (C-7), and (C-8).

Derivation of General Heat-Transfer Equations for Hottest Subassembly

Volumetric Heat-Generation Rate Distribution

On the basis of curves obtained from the physics analyses of the various systems considered, it was found that the ratio of the volumetric heat-generation rate in the hottest subassembly to the average rate in all the fuel in the core could be adequately described by a set of equations of the form

$$\frac{q}{\bar{q}} = \begin{cases} H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) & 0.0 \leq \frac{z}{L} \leq 0.4 \\ H + J & 0.4 \leq \frac{z}{L} \leq 0.6 \\ H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-9)$$

for all cases of interest. However,

$$\bar{q} = \frac{Q_c}{V_m} \quad (C-10)$$

Therefore, substituting Equation (C-10) into Equation (C-9)

$$q = \begin{cases} \frac{Q_c}{V_m} \left[H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) \right] & 0.0 \leq \frac{z}{L} \leq 0.4 \\ \frac{Q_c}{V_m} [H + J] & 0.4 \leq \frac{z}{L} \leq 0.6 \\ \frac{Q_c}{V_m} \left[H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) \right] & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-11)$$

Coolant-Temperature Rise

In the plate design with live end plates and ribs at third points between plates, there are N subassemblies, n plates, and $3(n+1)$ equal coolant channels. The flow through a single coolant channel is then

$$W = \frac{W_t}{3N(n+1)} .$$

But

$$W_t \equiv \frac{Q_c}{c_p \Delta t} ,$$

therefore,

$$W = \frac{Q_c}{3Nc_p \Delta t(n+1)} . \quad (C-12)$$

However,

$$W \equiv \frac{Q_z}{c_p \Delta t_z} . \quad (C-13)$$

Equating Equations (C-12) and (C-13),

$$\Delta t_z = \frac{3Q_z}{Q_c} N \Delta t(n+1) . \quad (C-14)$$

The quantity Q_z is defined as the total heat transferred into the single coolant stream considered between the inlet and point z ; therefore,

$$Q_t = \int_{v_z} q dV_z ,$$

where

$$dV_z = \frac{V_m}{3LN(n+1)} dz ,$$

from which

$$Q_z = \frac{V_m}{3LN(n+1)} \int_0^z q dz . \quad (C-15)$$

The $(n+1)$ term appears because the two half-thick fueled end plates contribute a fuel volume equal to that of one regular plate.

Substituting Q_z from Equation (C-15) and q from Equation (C-11) into Equation (C-14) gives the resulting coolant-temperature distribution

$$\Delta t_z = \begin{cases} J\bar{\Delta t} \left[0.3027 + \frac{H}{J} \frac{z}{L} - 0.3183 \cos \left(0.1\pi + \frac{\pi z}{L} \right) \right] & 0.0 \leq \frac{z}{L} \leq 0.4 \\ J\bar{\Delta t} \left[\frac{z}{L} \left(\frac{H}{J} + 1 \right) - 0.0973 \right] & 0.4 \leq \frac{z}{L} \leq 0.6 \\ J\bar{\Delta t} \left[0.5027 + \frac{H}{J} \frac{z}{L} + 0.3183 \cos \left(1.1\pi - \frac{\pi z}{L} \right) \right] & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-16)$$

Film-Temperature Rise

The convective-heat-transfer coefficient

$$h = 0.625 \frac{k}{D_e} N_{Pe}^{0.4} \quad (C-17)$$

is defined such that the heat flux

$$q'' \equiv h \Delta t_{fz} \quad (C-18)$$

But, in addition,

$$q'' \equiv \frac{q V_m}{A_h} \quad (C-19)$$

Equating Equations (C-18) and (C-19) and substituting q from Equation (C-11) and h from Equation (C-17) in the result:

$$\Delta t_{fz} = \begin{cases} \frac{1.880cd \bar{\Delta t}^{0.4}}{FL} \left[\frac{Q_c}{kN(c+d)(n+1)} \right]^{0.6} \left[H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) \right] & 0.0 \leq \frac{z}{L} \leq 0.4 \\ \frac{1.880cd \bar{\Delta t}^{0.4}}{FL} \left[\frac{Q_c}{kN(c+d)(n+1)} \right]^{0.6} [H + J] & 0.4 \leq \frac{z}{L} \leq 0.6 \\ \frac{1.880cd \bar{\Delta t}^{0.4}}{FL} \left[\frac{Q_c}{kN(c+d)(n+1)} \right]^{0.6} \left[H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) \right] & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-20)$$

Fuel-Temperature Rise

The general differential equation describing heat conduction with heat generation in a solid is

$$\nabla^2 t_m = - \frac{q}{k_m},$$

which, if conduction in the y and z directions is assumed negligible, can be written

$$\frac{d^2 t_m}{dx^2} = - \frac{q}{k_m}.$$

This equation has the solution

$$t_m = -\frac{qx^2}{2k_m} + C_1x + C_2. \quad (C-21)$$

The boundary conditions to be imposed on this equation are

$$\frac{dt}{dx} = 0 \text{ at } x = 0$$

and

$$t_m = t_{mo} \text{ at } x = \pm \frac{a}{2}.$$

The required values of the constants of Equation (C-21) are then

$$C_1 = 0,$$

and

$$C_2 = t_{mo} + \frac{qa^2}{8k_m}.$$

Substitution of these constants into Equation (C-21) gives the temperature distribution in the core

$$t_m = \frac{q}{2k_m} \left(\frac{a^2}{4} - x^2 \right) + t_{mo}.$$

The temperature drop from the center to the outside of the core is then, with the substitution of q from Equation (C-11),

$$\Delta t_{mz} = \begin{cases} \frac{0.125 Q_{ca}^2}{k_m V_m} \left[H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) \right] & 0.0 \leq \frac{z}{L} \leq 0.4 \\ \frac{0.125 Q_{ca}^2}{k_m V_m} [H + J] & 0.4 \leq \frac{z}{L} \leq 0.6 \\ \frac{0.125 Q_{ca}^2}{k_m V_m} \left[H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) \right] & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-22)$$

Clad-Temperature Rise

The general differential equation describing heat conduction without heat generation in a solid is

$$\nabla^2 t_c = 0,$$

which, if conduction in the y and z directions is assumed negligible, can be written

$$\frac{d^2 t_c}{dx^2} = 0.$$

This equation has the solution

$$t_c = C_1 x + C_2. \quad (C-23)$$

The boundary conditions to be imposed on Equation (C-23) are

$$t_c = t_{co} \text{ at } x = \frac{a}{2} + m$$

and

$$k_c \left(\frac{dt_c}{dx} \right) = k_m \left(\frac{dt_m}{dx} \right) \text{ at } x = \frac{a}{2}.$$

The required values of the constants in Equation (C-23) are then

$$C_1 = \frac{qa}{2k_c},$$

and

$$C_2 = t_{co} + \frac{qa}{2k_c} \left(\frac{a + 2m}{2} \right).$$

Substitution of these constants into Equation (C-23) gives the temperature distribution in the clad

$$t_c = t_{co} + \frac{qa}{2k_c} \left(\frac{a + 2m}{2} - x \right).$$

The temperature drop across the clad is then, with the substitution of q from Equation (C-12),

$$\Delta t_{cz} = \begin{cases} \frac{0.500 \text{ am} Q_c}{k_c V_m} \left[H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) \right] & 0.0 \leq \frac{z}{L} \leq 0.4 \\ \frac{0.500 \text{ am} Q_c}{k_c V_m} [H + J] & 0.4 \leq \frac{z}{L} \leq 0.6 \\ \frac{0.500 \text{ am} Q_c}{k_c V_m} \left[H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) \right] & 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (C-24)$$

Hot-Channel Factors

The abnormal heat-transfer conditions considered in the determination of the applied hot-channel factors are:

- (1) Plate spacing minimized or maximized
- (2) Material thickness maximized
- (3) Cladding thickness maximum on one side and minimum on other side
- (4) Maldistribution of coolant to subassemblies due to causes external to the subassemblies

- (5) Inaccuracy of power measurement
- (6) Inaccuracy of thermal conductivity values
- (7) Inaccuracy of film coefficient value.
- (8) Closing-off of coolant channels due to radiation damage
- (9) Excess heat generation in a new subassembly introduced into a core in which some burnup has occurred.

The factor for each of these is determined by assuming that only the stated abnormal condition exists with all other conditions being normal; the hot-channel factor for each temperature rise (coolant, film, cladding, and core) is then analytically determined as the ratio of the given temperature rise in the "hot" case to that in the normal case.

The first seven items can produce "cool" cases as well as hot ones; the probability that the product of the factors for these items is a given amount less than unity is the same as the probability that it will be the same amount greater than unity if a normal probability distribution is assumed. Therefore, if the factors calculated have their maximum possible values, the applied hot-channel factor due to the first seven items listed is approximately the average of the most probable hot-channel factor (unity) and maximum possible hot-channel factor. The factors due to the remaining two items are certain to occur and to be detrimental. Denoting the product of maximum factors caused by the first seven items is Product I and the product of the maximum factors caused by the last two items is Product II, the over-all applied hot-channel factors to be used are calculated as

$$\text{Applied HCF} = \text{Product II} \times \frac{1 + \text{Product I}}{2} \quad (\text{C-25})$$

Temperature Distribution Along Center of Core

The temperature distribution along the center of the core is

$$t_m' = t_i + \alpha \Delta t_z + \beta \Delta t_{fz} + \gamma \Delta t_{cz} + \delta \Delta T_{mz}$$

where α , β , γ , and δ are the applied hot-channel factors for the coolant-, film-, clad-, and material-temperature rises, respectively. Substituting the appropriate expressions for the temperature rises from Equations (C-16), (C-20), (C-22), and (C-24):

$$t_m' = \begin{cases} t_i + \alpha J \Delta t \left[0.3027 + \frac{H}{J} \frac{z}{L} - 0.3183 \cos \left(0.1\pi + \frac{\pi z}{L} \right) \right] \\ + \left[\frac{1.880 \beta c d \Delta t^{0.4}}{F L} \left\{ \frac{Q_c}{k N(n+1)(c+d)} \right\}^{0.6} + \frac{Q_c}{V_m} \left\{ \frac{0.500 \gamma a m}{k_c} + \frac{0.125 \delta a^2}{k_m} \right\} \right] \\ \times \left[H + J \sin \left(0.1\pi + \frac{\pi z}{L} \right) \right] \quad 0.0 \leq \frac{z}{L} \leq 0.4 \\ \\ t_i + \alpha J \Delta t \left[\frac{z}{L} \left(\frac{H}{J} + 1 \right) - 0.0973 \right] \\ + \left[\frac{1.880 \beta c d \Delta t^{0.4}}{F L} \left\{ \frac{Q_c}{k N(n+1)(c+d)} \right\}^{0.6} + \frac{Q_c}{V_m} \left\{ \frac{0.500 \gamma a m}{k_c} + \frac{0.125 \delta a^2}{k_m} \right\} \right] \\ \times \left[H + J \right] \quad 0.4 \leq \frac{z}{L} \leq 0.6 \\ \\ t_i + \alpha J \Delta t \left[0.5027 + \frac{H}{J} \frac{z}{L} + 0.3183 \cos \left(1.1\pi - \frac{\pi z}{L} \right) \right] \\ + \left[\frac{1.880 \beta c d \Delta t^{0.4}}{F L} \left\{ \frac{Q_c}{k N(n+1)(c+d)} \right\}^{0.6} + \frac{Q_c}{V_m} \left\{ \frac{0.500 \gamma a m}{k_c} + \frac{0.125 \delta a^2}{k_m} \right\} \right] \\ \times \left[H + J \sin \left(1.1\pi - \frac{\pi z}{L} \right) \right] \quad 0.6 \leq \frac{z}{L} \leq 1.0 \end{cases} \quad (\text{C-26})$$

Maximum Fuel Temperature

The maximum fuel temperature will occur at the center of the core at the point where

$$\left(\frac{dt'_m}{dz} \right)_{z_m} = 0 .$$

Because the heat-generation rate is constant in $0.4 \leq \frac{z}{L} \leq 0.6$, the maximum fuel temperature must occur in $0.6 \leq \frac{z}{L} \leq 1.0$. Therefore, denoting the fraction of core length to the point where this maximum occurs as $\frac{z_m}{L}$ and taking the required derivative in the interval of interest:

$$\left[\frac{5.906\beta cd}{\alpha JFL} \left\{ \frac{Q_c}{kN\Delta t(n+1)(c+d)} \right\}^{0.6} + \frac{1.571Q_c}{\alpha JV_m\Delta t} \left\{ \frac{\gamma a m}{k_c} + \frac{0.250\delta a^2}{k_m} \right\} \right] \cos \left(1.1\pi - \frac{\pi z_m}{L} \right) - \sin \left(1.1\pi - \frac{\pi z_m}{L} \right) = \frac{H}{J} \quad (C-27)$$

Equation (C-22) must be solved for $\frac{z_m}{L}$ by trial and error; substitution of the value of $\frac{z_m}{L}$ in the portion of Equation (C-21) valid for $0.6 \leq \frac{z}{L} \leq 1.0$ gives the maximum fuel temperature in the hottest subassembly.

APPENDIX D

REFERENCE FABRICATION PROCESSES FOR UO₂ FUEL SYSTEMS

APPENDIX D

REFERENCE FABRICATION PROCESSES FOR UO₂ FUEL SYSTEMSPin-Type Uranium Dioxide Fuel System

The process for the manufacture of pin-type uranium dioxide fuel elements can be considered more of an assembly operation rather than a fabrication process. All of the components are prepared individually and then assembled to form a sealed and unbonded fuel element. Uranium dioxide, prepared in pellet form, is loaded into a stainless steel tube. The tube is filled with sodium and then sealed by welding.

Fabrication of Uranium Dioxide Pellets

Uranium dioxide pellets will be prepared by a pressing technique. The press technique is considered more economic than the extrusion process when small specimens and batches are pressed. A readily sinterable grade of oxide that can be sintered at a temperature in the vicinity of 3000 F will be used.

In the initial step the as-received uranium dioxide will be mixed with a binder. This mixture will then be dried and disintegrated to form a free-flowing powder. The powder will be fed into a press and pellets will be pressed at the rate of 3600 per hr for each press under a pressure of approximately 20 tsi. These pellets will be fed on a conveyor through a hydrogen furnace for firing to a density of 90 per cent.

Sodium-Filling and Assembly Operations

The uranium dioxide pellets contained in a holding and alignment fixture will be loaded into a double-chambered dry box. One side of the dry box can be evacuated and the other side is outgassed and filled with helium. The oxide pellets are outgassed, transferred to the helium-filled chamber, and fed into stainless steel tubes which are sealed on one end. The tubes are then capped and transferred to another dry box for sodium filling.

Cast sticks of sodium approximately 8 in. long will be inserted into the stainless tubes above the uranium dioxide pellets. The tubes containing the oxide pellets and sodium will be heated above the melting point of sodium. After the sodium flows to fill the void space, end plugs will be inserted and the tubes will be sealed by Heliarc welding.

Leak Testing

It is proposed to nondestructively test the sodium-filled pins by detecting helium leakage with a mass spectrometer. The sealed pins will be loaded into a chamber which is pressurized with 500 psi of helium. The purpose of the pressure treatment is to force the helium gas through small leaks in the cladding. It is estimated that in a period of 1 hr sufficient helium would be forced through a 1- μ in. hole to increase the

internal pressure by 1 atm. After the pressure treatment the chamber will be evacuated and sniffed for helium.

Assembly

The pins will be assembled into plastic spacers to facilitate loading into the stainless steel subassembly. After loading, stainless steel wires will be inserted at intervals to space the fuel elements and the plastic spacers will be dissolved.

Cost Estimates

A flowsheet for the fabrication operation is presented in Figure D-1. Complete estimates of fabrication and material costs are contained in Table D-1.

Problems in Fabricating Pin-Type Uranium Dioxide Fuel Elements

The problems anticipated in the fabrication of pin-type uranium dioxide fuel elements are discussed.

Fabrication of Uranium Dioxide Pellets. The most important factor in the production of uranium dioxide pellets is the availability of a readily sinterable grade of uranium dioxide. This material must be uniform as it affects the sintering operation and ultimate density of the finished product.

An uncertainty in the operation is the utilization of as-sintered uranium dioxide pellets. It is believed that if uniform readily sinterable oxide is available it will be very feasible to produce pellets to a ± 0.002 -in. tolerance. Machining would make the operation prohibitive from a cost standpoint because of the large number of pellets that would have to be processed.

Sodium-Filling and Assembly Operations. There are several problems that must be solved in the sodium-filling and encapsulation operation. One problem involves the evacuation of the oxides of reactive gases. Sufficient sodium must also be encapsulated to completely envelope the pellets. Another problem may involve the final welding operation. Wetting of the stainless steel surfaces by the sodium may affect the quality of the weld.

Leak Testing. The final nondestructive test is dependent on the penetration of helium into minute holes in the stainless steel. It is possible that the sodium will plug these holes and prevent the helium from being forced into the pin.

FIGURE D-1. FABRICATION FLOWSHEET FOR PIN-TYPE URANIUM DIOXIDE FUEL SYSTEM

TABLE D-1. COST ESTIMATES FOR PIN-TYPE URANIUM DIOXIDE FUEL SYSTEM

<u>Fabrication - Fuel-Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Diameter	0.067
Core Length	30.5
Number of Elements per Assembly	529
Number of Assemblies per Core	139
Uranium-235 Enrichment, a/o	45.05
Total Weight of UO ₂ in Core, lb	2,809
Total Weight of Uranium in Core, lb	2,477
Fabrication Yield Without Recycle, per cent	80
Fabrication Yield With Recycle, per cent	95
Weight of UO ₂ Required With Recycle, lb	2,957
Weight of Uranium Required With Recycle, lb	2,607
<u>Material Costs</u>	
Cost per Assembly, dollars	
Stainless Steel Tubing \$24 per 100 ft (40 in. per pin)	423.30
Materials Used in Pressing and Sintering Operations; Hydrogen, Binder, etc.	52.90
Materials Used in Evacuation, Sodium Filling, and Encapsulation, Leak Detection; Helium, End Plugs, Sodium, etc.	211.60
Stainless Steel Subassembly	350.00
Total	1,037.80
<u>Fabrication Costs</u>	
Man-Hours per Assembly	
Pressing, Sintering, and Inspection of UO ₂ Pellets (0.104 per Element)	70
Evacuation, Sodium Filling, Encapsulation, and Leak Detection (0.4 per Element)	212
Isotopic and Analytical Analysis, and Accountability Control	10
Assembly of Elements to Form Subassembly	8
Joining of Assembly to Blanket Structure	10
Total	310

TABLE D-1. (Continued)

<u>Total Costs</u>	
Cost per Assembly, dollars	
Direct Materials	1,037.80
Direct Labor (2.20 per hr)	682.00
Overhead (250 per cent)	<u>1,705.00</u>
Total	2,587.00
General Administration (3 per cent)	77.61
Profit (10 per cent)	<u>258.70</u>
Grand Total	2,923.31

285 135

Alternate Methods for Fabricating Pin-Type Uranium Dioxide Fuel Elements

There is no alternative method for the fabrication of the pin-type uranium dioxide fuel elements, however, it is possible to make many modifications of the reference process. In the event the oxide pellets distort beyond the specified tolerance of ± 0.002 in. during the sintering operation it may be possible to control dimensions more accurately by use of a hot-pressing technique. This would of course increase the costs significantly. It is estimated that the costs for preparing the pellets would be increased by approximately 500 per cent. Machining of the pellets would increase the costs by even a greater amount. Extrusion of the oxide is a longer range development, however, if dimensional tolerances cannot be achieved in the manufacture of small pellets it would be difficult to obtain them through an extrusion operation.

Flat-Plate Uranium Dioxide Fuel System

The reference fabrication process for preparing flat-plate uranium dioxide fuel subassemblies is very similar to that proposed for the pin-type oxide system. The oxide core is pressed and sintered and then loaded into a brazed radiator-type stainless steel subassembly. The assembly is then filled with sodium and sealed. It is proposed to intentionally incorporate voids into the fuel portions of the assembly to allow for the escape of fission gases.

Fabrication of Uranium Dioxide Platelets

The same technique will be used for the preparation of uranium dioxide platelets as will be used for the pellets. The oxide will be mixed with a binder, dried, and disintegrated to form a free-flowing powder. This powder will be pressed into plates 2.45 by 1 by 0.052 in. using a pressure of approximately 20 tsi. After pressing, the plates will be fired to high density in a hydrogen atmosphere.

The process is based on the assumption that the thickness of the plates can be controlled through the sintering operation to a tolerance of $+0.003$ and -0.000 in. If tolerances cannot be maintained and the specimens must be machined, the cost for preparing the plates will probably be prohibitive.

Brazing of Stainless Steel Subassemblies

The radiator-type stainless steel subassembly will be fabricated by brazing. The stainless components will be assembled, jigged, and brazed in a hydrogen atmosphere, using a high-temperature stainless steel braze material.

Sodium-Filling and Assembly Operation

The sintered oxide plates will be loaded into the brazed stainless steel subassembly which will be opened at one end. After loading, the assembly will be placed in a vacuum chamber to remove reactive gases. The evacuated assembly will then be transferred to a helium-filled dry box for sodium filling. Liquid sodium will be poured into

the channels above the oxide core. The assembly will be heated to facilitate the flow of sodium into all void space. After sodium filling, end plugs will be inserted into the fuel channels and the channels will be sealed by welding. Since the assemblies will be intentionally defected to allow release of fission gases, there will be no need for leak testing.

Cost Estimates

The flowsheet for the fabrication of the flat-plate oxide system is similar to that of the pin-type oxide system presented in Figure D-1. Estimates of material and fabrication costs are contained in Table D-2. The estimate for the stainless steel subassembly was obtained from the Ferrotherm Company. It is a preliminary estimate and represents the maximum anticipated cost.

Problems in Fabricating Flat-Plate Uranium Dioxide Fuel Assemblies

The problems that will be encountered in the fabrication of the flat-plate uranium dioxide fuel assemblies are discussed.

Fabrication of Uranium Dioxide Plates. There is considerable uncertainty that the uranium dioxide can be pressed and sintered to the specified tolerances. This doubt exists, although it is assumed that a uniform reactive grade of oxide will be available.

If the tolerance cannot be achieved, it may be necessary to use a hot-pressing technique or machining operation. This would increase the costs by a factor of from 3 to 10 times the specified costs.

Brazing of Stainless Steel Subassemblies. There appears to be only one problem associated with the actual brazing operation, and it is one of maintaining dimensional tolerances. Improvement can be obtained by the use of spacers and special jigging; however, this is also reflected in an increased cost of brazing.

A possible problem is the corrosion resistance of the braze material in liquid sodium. This would most likely never be a serious problem unless the sodium becomes contaminated with oxygen.

Sodium-Filling and Assembly Operation. The only problem to be encountered in this operation is one common to all operations of this type and that is one of filling all of the void space with sodium. Welding is not considered to be a problem as the assemblies will be intentionally defected.

TABLE D-2. COST ESTIMATES FOR FLAT-PLATE
URANIUM DIOXIDE FUEL SYSTEM

<u>Fabrication - Fuel-Material Breakdown</u>	
Dimensions of Fuel Core, in.	
Core Thickness	0.052
Core Width	2.484
Core Length	30.5
Number of Elements per Assembly	17
Number of Assemblies per Core	139
Uranium-235 Enrichment, a/o	40.00
Total Weight of UO_2 in Core, lb	3,315
Total Weight of Uranium in Core, lb	2,922
Fabrication Yield Without Recycle, per cent	80
Fabrication Yield With Recycle, per cent	95
Weight of UO_2 Required With Recycle, lb	3,489
Weight of Uranium Required With Recycle, lb	3,076
<u>Fabrication and Material Costs</u>	
Cost per Assembly, dollars	
Stainless Steel Subassembly	750.00
Materials Used in Pressing and Sintering Operations; Hydrogen, Binder, etc.	50.00
Materials Used in Evacuation, Sodium Filling, and Encapsulation; Helium, Channel Plugs, Sodium, etc.	160.00
Total	960.00
<u>Fabrication Costs</u>	
Man-Hours per Assembly	
Pressing, Sintering, and Inspection of UO_2 Plates	66
Evacuation, Sodium Filling, and Encapsulation	50
Isotopic and Analytical Analysis, and Accountability Control	10
Joining of Assembly to Blanket Structure	10
Total	136

TABLE D-2. (Continued)

<u>Total Costs</u>	
Cost per Assembly, dollars	
Direct Materials	960.00
Direct Labor (\$2.20 per hr)	299.20
Overhead (250 per cent)	<u>748.00</u>
Total	2,007.20
General Administration (3 per cent)	60.00
Profit (10 per cent)	<u>200.00</u>
Grand Total	2,267.20

Alternate Methods for Fabricating Flat-Plate Uranium Dioxide Assemblies

The only feasible alternative method for fabricating this system involves modification of the method for preparing the flat-plate uranium dioxide plates. It may be possible to extrude the oxide followed by sintering, but it would be extremely difficult to maintain tolerances. Machining of relatively large, thin plates of oxide is not feasible because the oxide is extremely brittle. As mentioned previously, the reference pressing technique can be modified by use of hot pressing or machining if the specified tolerances cannot be achieved in a cold-pressing and sintering operation. These operations would increase the costs considerably. This is especially true of the machining operation where the reject rate would be relatively high because of the brittle nature of the oxide.

A long-range development would involve adding small amounts of a metallic phase to increase the thermal conductivity of the oxide and to add some ductility. A program is now being conducted at Battelle to study such materials. Preliminary results indicate that extensive improvement can be made by the addition of small amounts of metal.

APPENDIX E

STRESS ANALYSIS OF THE REFERENCE FUEL SUBASSEMBLY DESIGNS

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APPENDIX E

STRESS ANALYSIS OF THE REFERENCE FUEL SUBASSEMBLY DESIGNSGeneral Discussion

A qualitative examination of the proposed core fuel systems reveals certain situations which if realized would result in the failure of part or all of a particular subassembly. It is the purpose of this general analysis to determine to what extent these situations would develop during operation of the core.

The greatest difficulty arises from a difference in the linear coefficients of expansion of the materials involved. In a given plate this difference may result in stresses of sufficient magnitude to rupture the cladding or the core. A differential expansion among several plates could cause buckling and, hence, coolant-channel restriction. It may also lead to excessive shear concentrations in the sides and ends of the plates in the vicinity of the end of the fuel region.

Asymmetric temperature and pressure distributions must also be considered as contributing to plate bowing. In addition, thermal gradients across an entire subassembly may result in the bowing of the subassembly as a unit. Finally, growth accompanying burnup and permanent elongation resulting from creep phenomena, if excessive, may cause rupture of both cladding and core.

The problem of the differential expansion among plates is further complicated by the difficulty and uncertainty in determining the restraint imposed upon the plates. For the most part the fuel systems are such that all plates are identical except for the end plates which may be active or inactive. In either case the end plates are most likely to react to temperature distributions to a lesser degree than the regular fuel plates (coefficient of expansion of core is usually greater than that of cladding). It is obvious that the degree of restraint imposed upon the fuel plates varies as one moves away from the end plates toward the center of the subassembly.

The behavior of the subassembly as a unit under the influence of a flux gradient may depend upon its orientation to the gradient. This situation is difficult to analyze quantitatively. It will be discussed later.

General Method of Analysis

The basic assumptions regarding the properties of the materials and the plates as a whole are that the materials are homogeneous, isotropic, and elastic; the values of the modulus of elasticity, E , Poisson's ratio, ν , and the linear coefficient of expansion, α , are independent of the temperature over the range encountered; there are no body or surface forces, and the bond between the core and the cladding is completely integral. The temperature distributions, T , are those relative to the temperature at which the plates and subassemblies may be considered stress-free. Further conditions placed upon the plates are that they are completely unrestrained, they are under the influence

of temperature distributions which are functions of position in the thickness of the plate and independent of position along the length and width, and they are infinite in size in the directions along the length and width.

The stress-temperature-position relations are then derived by a straightforward application of these assumptions and conditions to the stress-strain relations and the general equations of equilibrium and compatibility. (17)*

It is necessary to further qualify these conditions regarding size, restraint, and temperature distributions and to examine any limitations they may place on the resultant stress relations. As was previously mentioned the degree of restraint on the regular fuel plates decreases as one moves toward the center of the subassembly. The difficulty in determining the degree of restraint for each plate forces one to choose from the available extremes of restraint and then to make inferences from the results as to the actual stress picture, having borne in mind the proximity of the end plate to the regular fuel plate in question. The extremes are fully restrained and completely unrestrained expansion. In the case of active end plates, particularly those whose core thickness is at least half that of the regular fuel plates, it seems reasonable to expect that their behavior would not be greatly different from that of the regular fuel plates. Moreover, the ratio of regular fuel plates to end plates will probably be in the vicinity of 10. It seems then, that the unrestrained condition is the better choice since it appears to apply to the majority of the plates.

The use of the general two-dimensional temperature distribution (T is a function of plate length and thickness) in the derivation leads to a fourth-order differential equation which can be solved only by analog or numerical methods and then not without considerable time and effort. This effort hardly seems justified since the temperature gradients in the longitudinal direction are small and the ratio of plate length to thickness is quite large. For this reason, it was assumed that the temperature is a function of plate thickness only. Under these circumstances, it is to be expected that for a given change in position the change in the stress in the direction along the length is much less than that in the direction of the thickness. Therefore, for all practical purposes, if the resultant stress relation is evaluated for points along the length which are separated by distances that are large compared with the plate's thickness, it may be considered as indicating the stresses as functions of both length and width.

The usual condition that all normal stresses and associated shears go to zero at the physical boundaries of the plate again severely complicates the derivation. Choosing the length and width extremely large and examining only a small area of this semi-infinite plate eliminates this difficulty. The associated condition for this case is that there are no net forces on the plate and that the strain is constant over the plate. It is believed that the limitation induced by this semi-infinite condition results in a stress relation which is valid everywhere except within a distance equal to several plate thicknesses from the ends and edges.

*References appear at end of text.

The complete derivation of the stress-temperature-position relation according to the foregoing assumptions and conditions is presented below.

The criterion for buckling specified is that the force on a plate resulting from a differential strain between it and the adjacent plate(s) be equal to or greater than the critical force for buckling. The latter is obtained from the theory of the buckling of thin plates and includes the effects of plate thickness, area, materials, and manner of support.⁽¹⁸⁾ The force resulting from a differential strain, $\Delta\epsilon$, is simply

$$F_{\Delta\epsilon} = \Delta\epsilon E S$$

where S is the area of the edge or side of the plate. The degree of restraint on each of the regular fuel plates enters here in determining the $\Delta\epsilon$ and eventually the deflection of the plate should it buckle. Because of the uncertainty of this degree of restraint, the analysis of buckling presented here is restricted to the worst situation. This situation is most likely to occur at the regular fuel plate adjacent to the end plates. It is for this plate that the $\Delta\epsilon$'s and resulting deflections are calculated. If according to the buckling criterion either the end plate or a regular fuel plate should buckle (depending upon the temperature with respect to the stress-free temperature), the deflection is calculated in a first approximation as follows. The unrestrained expansion for each plate is calculated. The larger is then considered to be "compressed" until it occupies the available space, the latter being determined by the smaller plate. The net expansion of the larger plate then allows one to calculate the bowing for the assumed resultant shape. In these cases where the ends of the plate are rigidly attached to the side plates the buckled plate will be bow-shaped with the tangent to the plate surface at its ends and sides normal to the surface of the side plates. In this curve, it is usually assumed that the points of inflection occur at the quarter-length positions. Assuming now that the middle half of this curve accommodates the net expansion and further assuming that this part of the curve approximates the arc of a circle of large radius, an order of magnitude of the deflection of the bowed plate may then be obtained. Since the end plates will yield slightly these values are considered to be conservative.

In addition to the plate deflections outlined above, bowing of the plates may result from asymmetric temperature and pressure distributions. The bending moment due to the asymmetric temperature distribution is readily obtainable from the calculation of the stress and can be easily converted to an equivalent uniform load on the plate. This together with the pressure-induced load gives the total load on the plate. Beam theory provides the necessary expression for using this known load to calculate the deflection and the associated stresses.^(19,20)

The problem of excessive shear concentrations in the cladding in the vicinity of the edges of the core where the plates join the side plates can be essentially eliminated by rounding or chamfering the edges of the core and by rounding the ends of the coolant channels.

Derivation of Stress-Temperature-Position Relation

The nomenclature and coordinate system used are given in Figure C-1 and Table C-1. Assume:

- (1) Core and cladding materials are homogeneous, isotropic, and elastic.
- (2) $E, \nu, \alpha \neq f(T)$ in the range encountered.
- (3) No surface or body forces.
- (4) Bond between core and cladding is completely integral.
- (5) Temperature distribution, T , is that relative the stress-free temperature, and is a function of x only.
- (6) Plates are completely unrestrained.
- (7) Dimensions in y and z directions approach ∞ .

From (5)

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0 ,$$

Therefore

(E-1)

$$\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial z} = 0 \text{ where } \psi = \sigma_x, \sigma_y, \sigma_z, \epsilon_x, \epsilon_y, \epsilon_z, \tau_{xy}, \tau_{yz}, \tau_{xz} .$$

General Equilibrium Equations (Relation to insure no rotation or translation of the body)

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (E-2)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} = 0 \quad (E-3)$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0 . \quad (E-4)$$

Stress-Strain Relations (A form of Hookes' law relating stress and strain)

$$\epsilon_x = \frac{1}{E} \left\{ \sigma_x - \nu (\sigma_y + \sigma_z) \right\} + \alpha T \quad (E-5)$$

$$\epsilon_y = \frac{1}{E} \left\{ \sigma_y - \nu (\sigma_x + \sigma_z) \right\} + \alpha T \quad (E-6)$$

$$\epsilon_z = \frac{1}{E} \left\{ \sigma_z - \nu (\sigma_y + \sigma_x) \right\} + \alpha T . \quad (E-7)$$

Shear-Strain Relations

$$\gamma_{xy} = 2 \frac{(1 + \nu)}{E} \tau_{xy} \quad (E-8)$$

$$\gamma_{xz} = 2 \frac{(1 + \nu)}{E} \tau_{xz} \quad (E-9)$$

$$\gamma_{yz} = 2 \frac{(1 + \nu)}{E} \tau_{yz} . \quad (E-10)$$

General Compatability Equations (Insures that displacements are continuous)

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2}; \quad \frac{2 \partial^2 \epsilon_x}{\partial y \partial z} = \frac{\partial}{\partial x} \left\{ - \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right\} \quad (E-11)$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial z} = \frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2}; \quad \frac{2 \partial^2 \epsilon_z}{\partial x \partial y} = \frac{\partial}{\partial z} \left\{ \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right\} \quad (E-12)$$

$$\frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^2 \epsilon_z}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial z^2}; \quad \frac{2 \partial^2 \epsilon_y}{\partial x \partial z} = \frac{\partial}{\partial y} \left\{ \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right\} . \quad (E-13)$$

Applying Equation (E-1) to Equations (E-2) through (E-13) gives the following pertinent expressions:

Equilibrium Conditions

$$\frac{\partial \sigma_x}{\partial x} = 0 \quad (E-14)$$

$$\frac{\partial \tau_{xy}}{\partial x} = 0 \quad (E-15)$$

$$\frac{\partial \tau_{xz}}{\partial x} = 0 . \quad (E-16)$$

Compatibility Equations

$$\frac{\partial^2 \epsilon_y}{\partial x^2} = 0 \quad (\text{E-17})$$

$$\frac{\partial^2 \epsilon_z}{\partial x^2} = 0 \quad (\text{E-18})$$

Substitution of the stress-strain relations and Equation (E-14) into Equations (E-17) and (E-18) yields:

$$\frac{\partial^2 \epsilon_y}{\partial x^2} = \left\{ \frac{\partial^2 \sigma_y}{\partial x^2} - \nu \frac{\partial^2 \sigma_z}{\partial x^2} \right\} + \alpha E \frac{\partial^2 T}{\partial x^2} = 0 \quad (\text{E-19})$$

$$\frac{\partial^2 \epsilon_z}{\partial x^2} = \left\{ \frac{\partial^2 \sigma_z}{\partial x^2} - \nu \frac{\partial^2 \sigma_y}{\partial x^2} \right\} + \alpha E \frac{\partial^2 T}{\partial x^2} = 0 \quad (\text{E-20})$$

Solving Equations (E-19) and (E-20) simultaneously

$$\frac{\partial^2 \sigma_z}{\partial x^2} = \frac{\partial^2 \sigma_y}{\partial x^2} = \frac{\alpha E}{1-\nu} \frac{\partial^2 T}{\partial x^2} \quad (\text{E-21})$$

Assume for simplicity that $\sigma_z = \sigma_y = \sigma$.

The following conditions must now be applied to the solution of Equation (E-21).

$$(1) \frac{\partial \sigma_m}{\partial x} = 0 \quad \text{at } x = 0,$$

$$(2) \epsilon_m = \epsilon_c = C \quad \text{for all } x,$$

$$(3) \int_{-b/2}^{b/2} \sigma \, dx = 0,$$

(4) Uniform heat generation in core; no heat generation in cladding.

In fuel region Equation (E-21) can be rewritten as

$$\frac{\partial^2 \sigma_m}{\partial x^2} = - \left(\frac{\alpha E}{1-\nu} \right)_m \frac{\partial^2 T}{\partial x^2} = \left(\frac{\alpha E}{1-\nu} \right)_m \frac{q}{k} = B \quad \text{for } -\frac{a}{2} \leq x \leq +\frac{a}{2}. \quad (\text{E-22})$$

Integrating Equation (E-22):

$$\frac{\partial \sigma_m}{\partial x} = B_y + C_1.$$

From Condition (1), it is easily seen that $C_1 = 0$. Substitution of $C_1 = 0$ and integrating again yields:

$$\sigma_m = \frac{Bx^2}{2} + C_2 \quad \text{for } -\frac{a}{2} \leq x \leq \frac{a}{2} . \quad (\text{E-23})$$

In the cladding region Equation (E-21) becomes

$$\frac{\partial^2 \sigma_c}{\partial x^2} = 0 \quad \text{for } \pm \frac{a}{2} \leq x \leq \pm \frac{b}{2} , \quad (\text{E-24})$$

since no heat is generated in this region. Integrating Equation (E-24) twice yields

$$\sigma_c = C_3 x + C_4 \quad \text{for } \pm \frac{a}{2} \leq x \leq \pm \frac{b}{2} . \quad (\text{E-25})$$

Since we have assumed $\sigma_y = \sigma_z$, the expression for strain in the y and z direction becomes

$$\epsilon = \frac{1-\nu}{E} \sigma + \alpha T .$$

Using this expression and Condition (2), one obtains the following relations:

$$\left(\frac{1-\nu}{E} \right)_c (C_3 x + C_4) + \alpha_c T_c = \left(\frac{1-\nu}{E} \right)_c \left(C_3 \frac{a}{2} + C_4 \right) + \alpha_c T_{a/2} , \quad (\text{E-26})$$

$$\left(\frac{1-\nu}{E} \right)_m \left(B \frac{x^2}{2} + C_2 \right) + \alpha_m T_m = \left(\frac{1-\nu}{E} \right)_m + \left(\frac{B(a)^2}{8} + C_2 \right) + \alpha_m T_{a/2} , \quad (\text{E-27})$$

$$\left(\frac{1-\nu}{E} \right)_c \left(C_3 \frac{a}{2} + C_4 \right) + \alpha_c T_{a/2} = \left(\frac{1-\nu}{E} \right)_m + \left(\frac{B(a)^2}{8} + C_2 \right) + \alpha_m T_{a/2} . \quad (\text{E-28})$$

It is easily shown that Equation (E-27) is trivial.

Solving Equation (E-26) for C_3 ,

$$C_3 = \left(\frac{\alpha E}{1-\nu} \right)_c \left(\frac{T_{a/2} - T_c}{y - a/2} \right) .$$

If we choose

$$T_c = T_{a/2} \frac{y - b/2}{a/2 - b/2}$$

C_3 becomes

$$C_3 = \left(\frac{\alpha E}{1-\nu} \right)_c \frac{T_{a/2}}{b/2 - a/2} .$$

Substituting this value for C_3 into Equation (E-28), one obtains the expression

$$\alpha_c a \frac{T_{a/2}}{b-a} + \left(\frac{1-\nu}{E} \right)_c C_4 = \left(\frac{1-\nu}{E} \right)_m \left(\frac{Ba^2}{8} \right) + \left(\frac{1-\nu}{E} \right)_m C_2 + T_{a/2} (\alpha_m - \alpha_c) .$$

Solving for C_4 and C_2 :

$$C_4 = \left(\frac{E}{1-\nu} \right)_c \left(\frac{1-\nu}{E} \right)_m C_2 + \left(\frac{E}{1-\nu} \right)_c \left(\frac{1-\nu}{E} \right)_m \frac{Ba^2}{8} + \left(\frac{E}{1-\nu} \right)_c T_{a/2} (\alpha_m - \alpha_c) - \left(\frac{E}{1-\nu} \right)_c \alpha_c a \frac{T_{a/2}}{b-a} \quad (E-29)$$

$$C_2 = \left(\frac{E}{1-\nu} \right)_m \left(\frac{1-\nu}{E} \right)_c C_4 - \frac{Ba^2}{8} - \left(\frac{E}{1-\nu} \right)_m T_{a/2} (\alpha_m - \alpha_c) + \left(\frac{E}{1-\nu} \right)_m \alpha_c a \frac{T_{a/2}}{b-a} . \quad (E-30)$$

Applying Condition (3) now one obtains the equation

$$\begin{aligned} \int_{-b/2}^{-a/2} \left[- \left(\frac{\alpha E}{1-\nu} \right)_c \frac{2T_{a/2}}{b-a} x + C_4 \right] dx + 2 \int_0^{a/2} \left[\frac{Bx^2}{2} + C_2 \right] dx \\ + \int_{a/2}^{b/2} \left[\left(\frac{\alpha E}{1-\nu} \right)_c \frac{2T_{a/2}}{b-a} x + C_4 \right] dx = 0 . \end{aligned}$$

Integration and simplification yields

$$\left(\frac{\alpha E}{1-\nu} \right)_c T_{a/2} \frac{b+a}{2} + (b-a) C_4 + a C_2 + \frac{Ba^3}{24} = 0 . \quad (E-31)$$

Substitution of Equation (E-30) into Equation (E-31) yields

$$\begin{aligned} C_4 \left[\frac{b}{2} + \frac{a}{2} \left\{ \left(\frac{E}{1-\nu} \right)_m \left(\frac{1-\nu}{E} \right)_c - 1 \right\} \right] = - \left(\frac{\alpha E}{1-\nu} \right)_c \frac{T_{a/2}}{4} (b+a) + \frac{Ba^3}{24} \\ + \frac{a}{2} \left(\frac{E}{1-\nu} \right)_m T_{a/2} \left[\alpha_m - \frac{b}{b-a} \alpha_c \right] . \quad (E-32) \end{aligned}$$

Substitution of Equation (E-29) into Equation (E-31) likewise yields

$$C_2 \left[\frac{b}{2} + \frac{a}{2} \left\{ \left(\frac{E}{1-\nu} \right)_m \left(\frac{1-\nu}{E} \right)_c - 1 \right\} \right] = \left(\frac{E}{1-\nu} \right)_m \frac{b-a}{2} \left[- \frac{Ba^2}{4} \left\{ \left(\frac{1-\nu}{E} \right)_c \frac{a}{6(b-a)} + \left(\frac{1-\nu}{E} \right)_m \frac{1}{2} \right\} \right. \\ \left. \frac{T_{a/2}}{2} (\alpha_c - 2\alpha_m) \right] . \quad (E-33)$$

Equation (E-33) together with Equation (E-23) gives the relation for the stresses in the fuel region. It can easily be shown that the resulting expression is equivalent to the following:

$$\sigma_m = - \left(\frac{\alpha E}{1-\nu} \right)_m T_m + \frac{\left(\frac{E_m}{E_c} \frac{1-\nu_c}{1-\nu_m} \right) \left[\int_0^{a/2} \left(\frac{\alpha E}{1-\nu} \right)_m T_m dx + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-\nu} \right)_c T_c dx \right]}{\frac{b}{2} + \frac{a}{2} \left[\frac{E_m}{E_c} \frac{1-\nu_c}{1-\nu_m} - 1 \right]}$$

This expression is identical to the expression derived by C. O. Smith for the same conditions and reported in Nuclear Science and Engineering, Vol. 2, No. 2, 1957. Since this expression is in a form which is somewhat easier to handle, it was used in preference to that just derived. It has been inferred by this identity that the other expressions derived by C. O. Smith also apply. They are listed here for convenience:

$$\sigma_c = - \left(\frac{\alpha E}{1-\nu} \right)_c T_c + \frac{\left[\int_0^{a/2} \left(\frac{\alpha E}{1-\nu} \right)_m T_m dx + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-\nu} \right)_c T_c dx \right]}{\frac{b}{2} + \frac{a}{2} \left[\frac{E_m}{E_c} \frac{1-\nu_c}{1-\nu_m} - 1 \right]}$$

For those cases where T_m and T_c are not symmetrical, the following expressions apply

$$\sigma_m = - \left(\frac{\alpha E}{1-\nu} \right)_m T_m + \frac{3x}{2} \left(\frac{E_m}{E_c} \frac{1-\nu_c}{1-\nu_m} \right) \\ \frac{\left[\int_{-b/2}^{-a/2} \left(\frac{\alpha E}{1-\nu} \right)_c x T_c dx + \int_{-a/2}^{a/2} \left(\frac{\alpha E}{1-\nu} \right)_m x T_m dy + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-\nu} \right)_c x T_c dx \right]}{\left(\frac{b}{2} \right)^3 + \left(\frac{a}{2} \right)^3 \left[\frac{E_m}{E_c} \frac{1-\nu_c}{1-\nu_m} - 1 \right]}$$

$$+ \frac{1}{2} \left(\frac{E_m}{E_c} \frac{1-v_c}{1-v_m} \right) \frac{\left[\int_{-b/2}^{-a/2} \left(\frac{\alpha E}{1-v} \right)_c T_c dx + \int_{-a/2}^{a/2} \left(\frac{\alpha E}{1-v} \right)_m T_m dx + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-v} \right)_c T_c dx \right]}{\frac{b}{2} + \frac{a}{2} \left[\frac{E_m}{E_c} \frac{1-v_c}{1-v_m} - 1 \right]}$$

$$\sigma_c = - \left(\frac{\alpha E}{1-v} \right)_c T_c$$

$$+ \frac{3x}{2} \frac{\left[\int_{-b/2}^{-a/2} \left(\frac{\alpha E}{1-v} \right)_c x T_c dx + \int_{-a/2}^{a/2} \left(\frac{\alpha E}{1-v} \right)_m x T_m dx + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-v} \right)_c x T_c dx \right]}{\left(\frac{b}{2} \right)^3 + \left(\frac{a}{2} \right)^3 \left[\frac{E_m}{E_c} \frac{1-v_c}{1-v_m} - 1 \right]}$$

$$+ \frac{1}{2} \frac{\left[\int_{-b/2}^{-a/2} \left(\frac{\alpha E}{1-v} \right)_c T_c dx + \int_{-a/2}^{a/2} \left(\frac{\alpha E}{1-v} \right)_m T_m dx + \int_{a/2}^{b/2} \left(\frac{\alpha E}{1-v} \right)_c T_c dx \right]}{\frac{b}{2} + \frac{a}{2} \left[\frac{E_m}{E_c} \left(\frac{1-v_c}{1-v_m} \right) - 1 \right]}$$